

\* FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics, also known as the conservation of energy principle. It states that during any process, if the energy disappears in one form, it appears in other form, but its total quantity remains always constant.

That is, the energy can be neither created nor destroyed, it can only change its form.

For example, for the energy interaction between a system and its surroundings, the energy lost by a system must be exactly equal to the amount of energy gained by the surroundings.

The first law can be proved mathematically, but no process in nature is known to have violated the first law of thermodynamics.

It is the relation of energy balance and is applicable to any kind of system [open or closed] undergoing any kind of process.

Example 1:-

Let us consider a process that involves only heat transfer but no work interaction. A hot potato taken from oven is exposed to room air. As a result of heat transfer from the hot potato, its energy will decrease. In absence of other effects, the decrease of total energy of the potato becomes equal to the amount of heat transferred to its surroundings. Therefore,

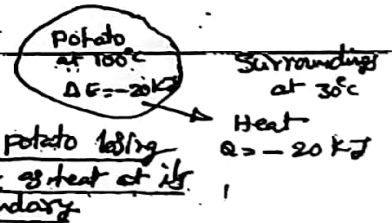
the principle of conservation of energy becomes can be

expressed as  $-\Delta E = -Q$

where  $\Delta E = E_2 - E_1$ .

In the absence of any work interaction between a system and its surroundings, the amount of net heat transfer is equal to change in the energy of a system.

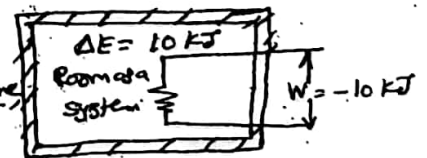
$Q = \Delta E$  when  $W = 0$ .



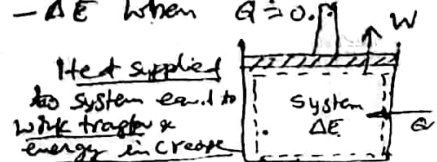
Example 2:- Consider a well-insulated room heated by an electric heater. As a result of electrical work done,

the energy of the room will increase. Since the room is adiabatic and cannot have any heat interaction with its surroundings, the conservation of energy principle dictates that electrical work done on the room must be equal to increased energy of the room.  $-W = \Delta E$ .

For an adiabatic process, the amount of work done is equal to the change in energy of the system.  $W = -\Delta E$  when  $Q = 0$ .



Example 3:- Consider work and heat transfer simultaneously. The gas in cylinder is heated. As the energy of the gas increases, its pressure and temperature also increases.



As then the gas will expand and work will be done at the boundary of the system. The conservation of energy principle reveals that

$$Q = W + \Delta E \quad \text{where } Q = \text{Net heat transfer across system boundaries}$$

$$Q - W = \Delta E$$

$$W = \text{Net work transfer}$$

$$\Delta E = \text{Net change in total energy of the system } (E_2 - E_1)$$

The total energy of the system is the sum of internal energy  $U$ , potential energy  $PE$  and kinetic energy  $KE$ .

$$\Delta E = \Delta U + \Delta PE + \Delta KE \quad (kJ)$$

Mostly closed systems in practice are stationary, they don't have kinetic energy and potential energy during a process.

The first law of thermodynamics is reduced to

$$\boxed{Q - W = \Delta U} \quad kJ$$

### Mechanisms of Energy Transfer:-

The energy can be transferred to or from a system in three forms. heat, work and mass flow.

① Heat Transfer:- (a) Heat transferred to a system increases the energy of the molecules and thus an increase in the internal energy of the system, and heat transfer from a system decreases the energy of molecules and this results in decrease of internal energy.

② Work Transfer (W):- Work refers to the transfer of energy due to potential difference other than temperature difference between a system and its boundary.

A rising piston, rotating shaft and an electric wire carrying current crossing the system boundary - all these energy transfers are associated with work interactions. The work transfer to a system increases the energy of the system and work transfer from a system decreases energy.

③ Mass flow  $m$ :- When mass enters the system, the energy of the system increases because mass carries energy with it. Similarly, when some mass leaves the system, the energy of the system decreases, because the leaving mass takes some energy with it.

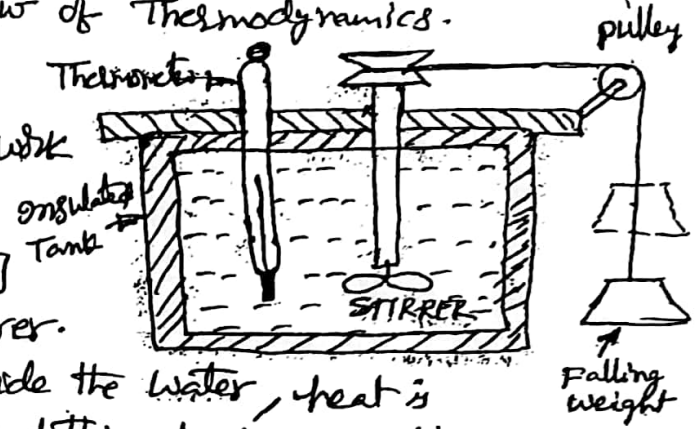


First Law of Thermodynamics for a cyclic process

- Joule's Experiment -

Joule conducted several experiments which led to the formulation of the First law of Thermodynamics.

When weight is allowed to fall to a certain distance, work is done on the water [product of weight and distance moved] through the rotation of stirrer.



When the stirrer rotates inside the water, heat is produced due to friction and this heat generation is measured by the measurement of rise in temperature using thermometer fixed in container.

Further, the insulation from the tank was removed and the whole system was transferred placed in a water bath. The heat was transferred from the system in order to bring same initial state conditions.

During a complete cycle, there was net work input and net heat output from the system.

"Joule" found in his experimental observations that, Whenever a closed system undergoes a cycle, the work input to the system is proportional to the net heat output.

It is expressed as  $\oint \delta W \propto \oint \delta Q$   
 $\oint \delta W = J \oint \delta Q$  ;  $\frac{1 \text{ cal}}{4.186 \text{ J}}$   $\oint \delta W = \oint \delta Q$

Where, J = constant of proportionality called mechanical equivalent of heat. = 25 Value 8 1 in S.I units.

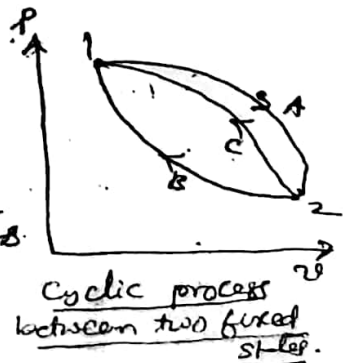
Internal Energy (E) - A property of system

Consider a system undergoing a change of state from 1 to 2 along path A and returning from 2 to 1, along path B. They complete cycle (A-B-A).

Applying first law of Thermodynamics for cyclic process.

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_B = \int_1^2 \delta W_A + \int_2^1 \delta W_B$$

$$\int_1^2 \delta Q_A - \int_1^2 \delta Q_B = \int_1^2 \delta W_A - \int_1^2 \delta W_B$$



Re-arranging, we get

$$\int_1^2 \delta Q_A - \int_1^2 \delta W_A = \int_1^2 \delta Q_B - \int_1^2 \delta W_B ; \text{The quantity } \delta Q - \delta W \text{ is } dE.$$

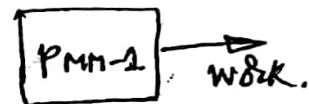
(Q)  $\Delta E_A = \Delta E_B$ ; similarly for cyclic process 1-A-2-C-1,  
 $\Delta E_A = \Delta E_C$ .

The change in energy between two states <sup>does not</sup> depends upon path followed by process, depends between two states 1 & 2

### Perpetual Motion Machine of First Kind - PMM 1

A device that violates the first law of Thermodynamics by producing work from nothing is called a perpetual motion machine of the first kind. PMM1.

It is defined as a machine which produces work energy without consuming an equivalent of energy from other source - it is impossible in actual practice, because no machine can produce energy of its own without consuming any other form of energy.



**PROB.** A system is composed of a gas contained in a cylinder fitted with a piston. The gas expands from state 1 for which  $E_1 = 75 \text{ kJ}$  to a state 2 for which  $E_2 = -25 \text{ kJ}$ . During the expansion, the gas does  $60 \text{ kJ}$  of work on the surroundings. Determine the heat transferred to (Q) from the system during the process.

**SOL.** To find, the amount of heat transferred,  
Analysis, according to the First law of Thermodynamics for a process,

$$Q - W = \Delta E$$

(Q)  $Q = W + \Delta E$  — (1)  
 where  $\Delta E = E_2 - E_1 = -25 - 75 = -100 \text{ kJ}$

Using the equation (1),  $Q = W + \Delta E$

here  $W = \text{work done by gas} = +60 \text{ kJ}$ .

$$\therefore Q = 60 - 100 = -40 \text{ J (Heat is rejected)}$$



**PROB** A system undergoes a cyclic process composed of four processes 1-2, 2-3, 3-4 and 4-1. The energy transfers tabulated as

process	Q kJ/min	W kJ/min	$\Delta U$ kJ/min
1-2	400.0	150.0	—
2-3	200.0	—	300.0
3-4	-200	—	—
4-1	0	75	—

Complete the table and determine the power output.

For process 1-2;  $Q - W = \Delta U$   
 $400 - 150 = \Delta U$  ;  $\Delta U_{1-2} = 250 \text{ kJ/min.}$

For process 2-3;  $Q - W = \Delta U$  ;  $W_{2-3} = Q - \Delta U = 200 - 300 = -100 \text{ kJ/min.}$

For process 4-1;  $Q - W = \Delta U$  ;  $\Delta U_{4-1} = 0 - 75 = -75 \text{ kJ/min.}$

For a cyclic process,  $\oint \delta Q = \oint \delta W$

$$Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} = W_{1-2} + W_{2-3} + W_{3-4} + W_{4-1}$$

$$400 + 200 - 200 + 0 = 150 - 100 + W_{3-4} + 75$$

$$\therefore W_{3-4} = 275 \text{ kJ/min.}$$

For process 3-4;  $Q - W = \Delta U$  ;  $-200 - 275 = \Delta U_{3-4} = -475 \text{ kJ/min.}$

process	Q kJ/min	W kJ/min	$\Delta U$ kJ/min
1-2	400	150	250
2-3	200	-100	300
3-4	-200	275	-475
4-1	0	75	-75

**Limitations or restrictions of First Law of Thermodynamics:-**

① When a closed system undergoes a thermodynamic cycle, the net heat transfer is equal to the net work transfer. This statement does not satisfy the direction of flow of heat and work [i.e., whether the heat flows from a hot body to a cold body (or) from a cold body to a hot body]. It does not give any condition under which these transfers takes place.

② The heat energy and mechanical work are mutually convertible. Though the mechanical work can be fully

Converted into heat energy, but only a part of heat energy can be converted into mechanical work. This means that the heat energy and mechanical work are not fully mutually convertible. In other words, there is a limitation on conversion of one form of energy into other.

A machine which violates the first law of Thermodynamics is known as perpetual motion machine of the first kind PMM-1. It is defined as a machine which produces work energy without consuming an equivalent energy from other source. Such a machine is impossible.

**PROB.** A stationary mass of gas is compressed without friction from an initial state of  $0.3 \text{ m}^3$  and  $0.105 \text{ MPa}$  to a final state of  $0.15 \text{ m}^3$  and  $0.105 \text{ MPa}$ . The pressure is remaining constant during process. There is a transfer of  $37.6 \text{ kJ}$  of heat from the gas during the process. How much does the internal energy of the gas change.

**SOL.** First law for a system in a process

$$Q = \Delta U + W$$

$$Q_{1-2} = \Delta U_{1-2} + W_{1-2} = \Delta U + P(V_2 - V_1)$$

$$\text{Since } W_{1-2} = P(V_2 - V_1) = 0.105 \times 10^6 [0.15 - 0.3] = -15,750 \text{ J} \quad (\text{negative})$$

$$Q_{1-2} = -37.6 \times 10^3 \text{ J} \quad (\text{negative}) \text{ since transfer of heat from gas.}$$

$$\therefore -37.6 \times 10^3 = \Delta U + (-15,750)$$

$$\therefore \Delta U = -37.6 \times 10^3 + 15,750 = -21,850 \text{ J only} \quad (\text{decrease})$$

**PROB.** In a general compression process  $2 \text{ kJ}$  of mechanical work is supplied to  $5 \text{ kg}$  of working substance and  $800 \text{ J}$  of heat is rejected to the cooling jacket. Calculate the change in specific internal energy.

**SOL.** For first law of thermodynamic process.

$$Q = \Delta U + W$$

$$Q = 800 \text{ J} \quad (\text{negative}) \text{ rejected, } W = 2000 \text{ J} \quad (\text{negative}) \text{ supplied, } -800 = \Delta U - 2000$$

$$\text{change in internal energy } \Delta U = 1200 \text{ J.}$$

$$\text{specific internal energy change} = \frac{\Delta U}{m} = \frac{1200}{5} = 240 \text{ J/kg.}$$



**PROB.** A fluid confined in a cylinder by a spring-loaded frictionless piston so that the pressure in the fluid is a linear function of the volume  $p = a + bV$ . The internal energy of the fluid is given by the following equation  $U = 34 + 3.15 pV$  where  $U$  is in kJ,  $p$  is in kPa and  $V$  is in  $m^3$ . If the fluid changes from an initial state of 170 kPa,  $0.03 m^3$  to a final state of 400 kPa &  $0.06 m^3$ , without any work other than that done on the piston. Find the direction & magnitude of the work and heat transfer.

**SOL.**  $p_1 = 170 \text{ kPa}$ ,  $V_1 = 0.03 m^3$   $U = 34 + 3.15 pV$   
 $p_2 = 400 \text{ kPa}$ ,  $V_2 = 0.06 m^3$

Change in internal energy,  $U_2 - U_1 = 3.15 [p_2 V_2 - p_1 V_1]$   
 $= 3.15 [400 \times 10^3 \times 0.06 - 170 \times 10^3 \times 0.03] = 59.5 \text{ kJ}$

The relation between  $p$  and  $V$  is given by,

$p = a + bV$   
 at  $p_1, V_1$ ;  $170 = a + b \times 0.03$  — (1)  
 at  $p_2, V_2$ ;  $400 = a + b \times 0.06$  — (2)      Solving (1) & (2),

$a = -60$ ;  $b = 7667$   
 $\therefore$  work transfer,  $W_{1-2} = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} (a + bV) dV$   
 $= a(V_2 - V_1) + b \left[ \frac{V_2^2 - V_1^2}{2} \right]$

$= (V_2 - V_1) \left[ a + b \left( \frac{V_2 + V_1}{2} \right) \right]$   
 $= (0.06 - 0.03) \left[ -60 + \frac{7667}{2} (0.092) \right]$   
 $= 8.55 \text{ kJ}$

Work done by the system, the magnitude being 8.55 kJ.

Heat transfer involved is given by

$Q_{1-2} = U_2 - U_1 + W_{1-2} = 59.5 + 8.55 = 68.05 \text{ kJ}$

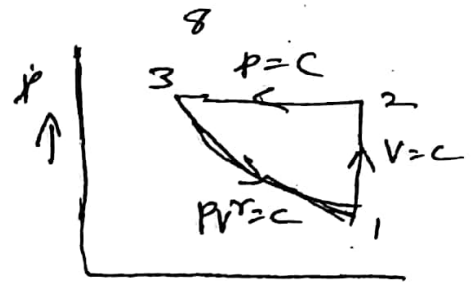
**PROB.** A system receives 200 kJ of heat at constant volume process and rejects 220 kJ of heat at constant pressure and 40 kJ of work is done on the system. The system is brought to its original state by an adiabatic process. Calculate the adiabatic work if the initial & internal energy is 240 kJ, then calculate the value of internal energy at all points.

$$Q_{1-2} = 200 \text{ kJ, heat is received}$$

$$Q_{2-3} = -220 \text{ kJ (heat rejected)}$$

initial internal energy,  
 $U_1 = 240 \text{ kJ}$

Work done during the process,  $W_{2-3} = -40 \text{ kJ}$ .  
 (20 kJ done on the system)



process 1-2      constant volume process  
 $W_{1-2} = 0$       since  $\int p dV \Rightarrow \because V_1 = V_2$

$$\therefore Q_{1-2} = \Delta U + W = \Delta U$$

$$200 = U_2 - U_1$$

$$\therefore U_2 = 200 + U_1 = 200 + 240 = 440 \text{ kJ}$$

process 2-3       $Q_{2-3} = -220 \text{ kJ}$

According to first law for a process 2-3,

$$Q_{2-3} = \Delta U + W_{2-3}$$

$$-220 = \Delta U - 40$$

$$\therefore \Delta U = -220 + 40 = -180 \text{ kJ}$$

$$U_3 - U_2 = -180 \text{ kJ}$$

initial energy at state 3,  
 $\therefore U_3 = -180 + U_2 = -180 + 440 = 260 \text{ kJ}$

for process (3-1) adiabatic process:-

$$Q_{3-1} = 0$$

For cyclic process 1-2-3, we have

$$\oint da = \oint dw$$

$$Q_{1-2} + Q_{2-3} + Q_{3-1} = W_{1-2} + W_{2-3} + W_{3-1}$$

$$200 - 220 + 0 = 0 - 40 + W_{3-1}$$

Adiabatic work:  $W_{3-1} = -20 + 40 = 20 \text{ kJ}$



## TEMPERATURE:-

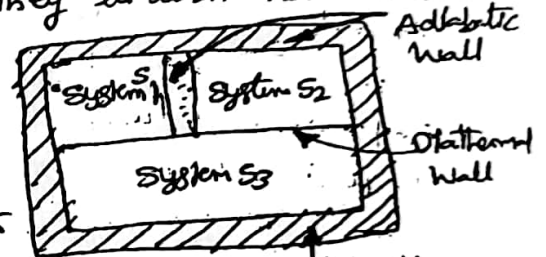
Temperature can be defined as a measure of hotness or coldness. The property which distinguishes Thermodynamics from other sciences is temperature. One might say that temperature bears as important a relation to thermodynamics as force to statics or velocity to dynamics. Temperature is associated with the ability to distinguish hot from cold.

When two bodies are at different temperatures are brought into contact, after some time, they attain a common temperature and are then said to exist in thermal equilibrium.

### \* Zeroth Law of Thermodynamics

It states that when two systems are in thermal equilibrium with a third system, they in turn have thermal equilibrium with each other.

Consider two systems  $S_1$  and  $S_2$  which are separated by an adiabatic wall, and a third system  $S_3$  is in contact with both the systems  $S_1$  and  $S_2$ .



These systems are in thermal equilibrium.

If systems  $S_1$  and  $S_2$  are individually in thermal equilibrium

with a third system  $S_3$ , then the systems  $S_1$  and  $S_2$  will also be in thermal equilibrium with each other, even though they are not in contact.

### Measurement of Temperature:-

The measurement of temperature depends upon establishment of thermal equilibrium between a system and the device used to measure the temperature. The sensing device should have at least one measurable property that changes with change in temperature. Such a property is called a thermometric property. The substance which shows the changes in the thermometric property is called "Thermometric Substance".

List of Thermodynamic Thermometric properties and devices are given below

- ① Change in dimension:- Expansion or Contraction of metal such as mercury-in-glass thermometer.
- ② Change in electrical resistance of metals & semi-conductors such as Resistance thermometer & thermistors

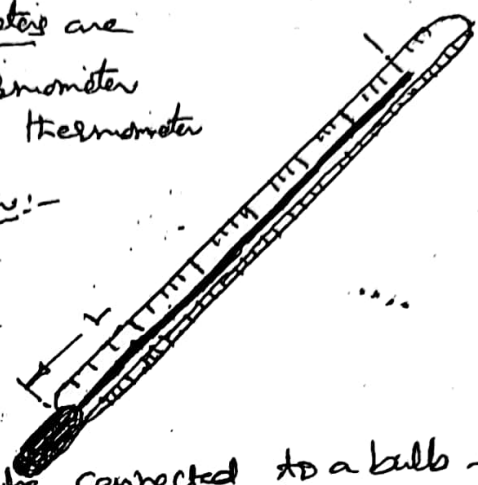
- ③ Thermoelectric emf between cold and hot junctions, such as thermocouples.
- ④ Change in intensity and colour of emitted radiation such as pyrometers.

Commonly used thermometers are

- a) liquid-in-glass thermometer
- b) constant volume gas thermometer
- c) liquid-in-glass thermometer:-

It works on expansion (or) contraction of a thermometric substance with temperature.

It consists of a uniform diameter glass capillary tube connected to a bulb filled with a liquid at one end [mercury]. As the temperature increases, the liquid expands in volume and rises in the capillary. The height of liquid column is calibrated into a temperature scale, which may be read. mercury, alcohol, ether are commonly used.



Constant volume gas thermometer

The thermometer consists of a

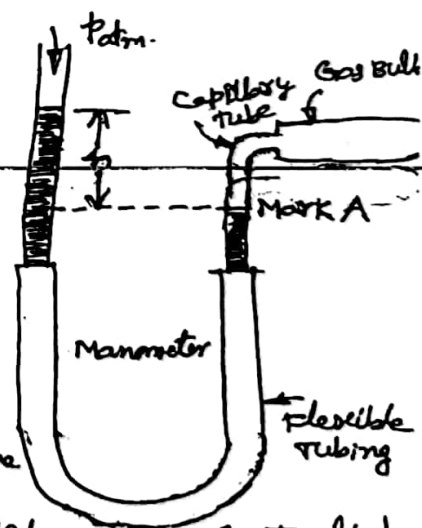
- i) bulb which encloses a fixed mass of an ideal gas
- ii) capillary tube
- iii) U-tube manometer with flexible bend.

The capillary tube connects the gas bulb to one limb of the manometer; the other limb being open to atmosphere.

The flexible bend of the manometer helps to raise (or) lower the limb open to atmosphere

so that mercury can be made to stand at fixed mark A in the limb communicating with the bulb. This is to ensure that volume of the gas in the bulb is maintained at constant value.

During operation, the bulb communicates with a constant temperature bath. There is exchange of heat between the bath and bulb until both attain the thermal equilibrium [Equality of temp]. Due to heat transfer to the bulb, the gas in the bulb expands and pushes the mercury downwards in right limb. The flexible tubing is then adjusted so that mercury level again attains the position of fixed mark A.





The difference in the level of mercury in the two limbs is recorded, and absolute pressure  $p$ , is determined by hydrostatic equation.

$$p = p_{\text{atm}} + \rho g h, \text{ where } \rho = \text{density, } h = \text{difference of mercury level}$$

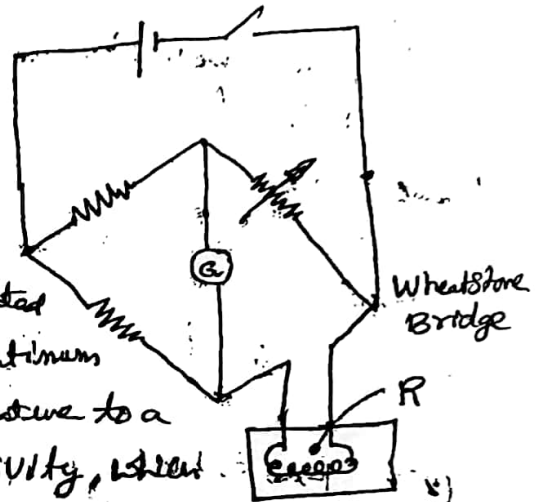
Firstly, the gas bulb is placed in a constant temp. bath at the triple point (273.15 K) temp and corresponding pressure  $p_{TP}$  is calculated.

Now, the bulb is brought in contact with a system whose temp.  $T$  is to be measured and measure the pressure,  $p$ .

∴ The new temperature,  $T = 273.15 \times \frac{p}{p_{TP}}$

### Electrical resistance thermometer

In the resistance thermometer, the change in resistance of a metal wire due to its change in temperature is the thermometric property. The wire, frequently platinum, may be incorporated in a Wheatstone bridge circuit. The platinum resistance thermometer measures temperature to a high degree of accuracy and sensitivity, which makes it suitable as a standard for the calibration of other thermometers.



On a restricted range, the following quadratic equation is often used.

$$R = R_0 (1 + At + Bt^2)$$

where  $R_0$  = resistance of platinum wire when it is surrounded by melting ice

$A, B$  are constants.

TEMPERATURE SCALE:— To measure the temperature of a system, some numerical values are assigned on the thermometer. These numerical values on thermometer together are called temperature scale.

To give numerical value to the temperature of a body we have to define a scale of temperature. This is to choose two fixed point temperatures are the melting point of ice and the other boiling point of water (steam) at 1 atm as fixed temperature point.

We assign arbitrary  $T_1$  to ice point and  $T_2$  to steam point, correspondingly let the length of the mercury column are  $l_1$  and  $l_2$  respectively. The temperature correspondingly to any length  $l$  may be defined assuming linear relation—

$$\therefore t = a l + b.$$

Thus a change of one degree in temperature <sup>12</sup> means a change of  $\frac{l_2 - l_1}{l_2 - l_1}$  in the length of mercury column.

There are four different scales:-

① Centigrade scale:- It is also known as Celsius scale.

on this, melting point of ice is marked as  $0^\circ\text{C}$  and boiling point of water as  $100^\circ\text{C}$ , under atmospheric pressure.

Space between two points is divided into 100 equal parts. It is denoted by  $^\circ\text{C}$ .

Linear Equation

$$t = at + b$$

$$0 = al_1 + b$$

$$100 = al_2 + b$$

$$a = \frac{100}{l_2 - l_1}$$

$$b = -al_1$$

$$= -\frac{100}{l_2 - l_1} \times l_1$$

$$t^\circ\text{C} = \frac{100 \times l - 100 l_1}{l_2 - l_1} = \frac{100(l - l_1)}{l_2 - l_1} \checkmark$$

② Fahrenheit scale

32 and  $212^\circ\text{F}$  are ice point and steam point respectively.

$$^\circ\text{F} = \frac{180}{l_2 - l_1} (l - l_1) + 32$$

③ Absolute temperature (Kelvin)

$$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$$

④ Rankine scale  $T(\text{R}) = T(^{\circ}\text{F}) + 459.67$

**PROB** The pressure in a constant gas thermometer is measured as 32 mm of Hg above atmospheric pressure at triple point. Determine the temp in  $^\circ\text{C}$ , when the pressure is 76 mm of Hg above atmospheric pressure. The barometer reads 752 mm of Hg.

**SOL** Absolute pressure at triple point,  $P_{TP} = 752 + 32 = 784$  mm of Hg.

Absolute pressure at given pressure,  $p = 752 + 76 = 828$  mm of Hg.

Triple point temperature,  $T_{TP} = 273.15\text{K}$ .

To find Temperature  $T$  at pressure  $p$ .

Analysis:- For constant volume gas thermometer,  $\frac{P_{TP}}{T_{TP}} = \frac{p}{T}$

$$\text{①), } T = T_{TP} \times \frac{p}{P_{TP}} = 273.15 \times \frac{828}{784} = 288.48\text{K}$$

$$\text{Celsius temperature, } T = 288.48 - 273.15 = 15.33^\circ\text{C}$$



**PROB.** The temperature scale of a certain thermometer is given by the relation  $t = A \ln p + B$ , where A and B are constants and p is the thermometric property of the fluid in thermometer. At ice point and steam point, if the thermometric property is found to be 1.5 and 7.5 respectively, what will be the temperature corresponding to the thermometric property of 3.5 on Celsius scale?

**Sol.**  
 Initial property  $p_1 = 1.5$   
 Final property  $p_2 = 7.5$   
 Ice-point temperature  $T_1 = 0^\circ\text{C}$   
 Steam-point temperature  $T_2 = 100^\circ\text{C}$   
 Thermometric property  $p_3 = 3.5$

To find The temperature corresponding to thermometric property of 3.5

**Analysis** - The relation for temperature is given as  
 $t = A \ln p + B$

At ice point,  $0 = A \ln(1.5) + B$  — (1)

(1)  $0 = 0.405 A + B$

At steam point,  $100 = A \ln(7.5) + B$

(2)  $100 = 2.015 A + B$  — (2)

Solving (1) & (2); subtracting equation (1) from (2), we get

$100 = 1.61 A$ ;  $A = 62.13$ ;  $B = -25.2$

Using A and B for thermometric property at 3.5

$t = 62.13 + \ln(3.5) - 25.2 = 54.64^\circ\text{C}$

**PROB.** A new temperature scale in degree N is to be defined with boiling and freezing points on the scale are  $400^\circ\text{N}$  and  $100^\circ\text{N}$  respectively. Correlate this with (i) Centigrade scale (ii) Fahrenheit scale

(b) What will be the reading on new scale corresponding to  $60^\circ\text{C}$ .

**Sol.** Centigrade scale melting point is  $0^\circ\text{C}$ ; Boiling point  $100^\circ\text{C}$ .  
 under Atmospheric pressure.

The relationship between temperature & length is given by

$t = a l + b$  — (1)

$0 = a l_1 + b$  — (2)

$100 = a l_2 + b$  — (3)

Subtracting (3) from (2), we get;  $a = \frac{100}{l_2 - l_1}$ ,  $b = -\frac{100 l_1}{l_2 - l_1}$

Substituting a & b in equation (1),  $t^\circ\text{C} = \frac{100 (l - l_1)}{l_2 - l_1}$

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For given temperature scale, the boiling and freezing points are  $400^\circ\text{N}$  and  $100^\circ\text{N}$  respectively.

We have  $t = a l + b$ .

$$100 = a l_1 + b \quad \text{--- (4)}$$

$$400 = a l_2 + b \quad \text{--- (5)}$$

By solving the above equation,

$$a = \frac{300}{l_2 - l_1}; \quad b = 100 - \frac{300 l_1}{l_2 - l_1}$$

Now substituting the values of  $a$  and  $b$  in equation (1), we get

$$\begin{aligned} t^\circ\text{N} &= \frac{300}{l_2 - l_1} l + 100 - \frac{300 l_1}{l_2 - l_1} = \frac{300}{l_2 - l_1} (l - l_1) + 100 \\ &= 3 \left( \frac{100}{l_2 - l_1} \right) (l - l_1) + 100 \end{aligned}$$

By substituting  $\frac{100}{(l_2 - l_1)} (l - l_1) = t^\circ\text{C}$

$$\therefore \boxed{t^\circ\text{N} = 3t^\circ\text{C} + 100}$$

For a Fahrenheit scale, the boiling and freezing points are  $212^\circ\text{F}$  and  $32^\circ\text{F}$  respectively.

We have  $t = a l + b$ .

$$32 = a l_1 + b \quad \text{--- (7)}$$

$$212 = a l_2 + b \quad \text{--- (8)}$$

$$a = \frac{180}{l_2 - l_1} \quad b = 32 - \frac{180 l_1}{l_2 - l_1}$$

Solve the above equation,

$$t^\circ\text{F} = \frac{180 l}{l_2 - l_1} + 32 - \frac{180 l_1}{l_2 - l_1} = \frac{180 (l - l_1)}{l_2 - l_1} + 32 \quad \text{--- (9)}$$

We have  $t^\circ\text{N} = 3 \left( \frac{100}{l_2 - l_1} \right) (l - l_1) + 100$

$$= 300 \left( \frac{t^\circ\text{F} - 32}{180} \right) + 100$$

$$\therefore \boxed{t^\circ\text{N} = \frac{5}{3} (t^\circ\text{F} - 32) + 100}$$

(b) The reading on new scale corresponding to  $60^\circ\text{C}$  is

$$t^\circ\text{C} = 60^\circ\text{C}$$

relationship between  $t^\circ\text{N}$  and  $t^\circ\text{C}$  is

$$t^\circ\text{N} = 3t^\circ\text{C} + 100 = 3 \times 60 + 100 = \underline{280^\circ\text{N}}$$

## properties of perfect gases

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A perfect gas (or an ideal gas) may be defined as a state of a substance, whose evaporation from its liquid state is complete, and strictly obey all the gas laws under all conditions of temperature & pressure.

### Laws of perfect gases:-

The physical properties of a gas are controlled by the following three variables.

- 1) pressure exerted by the gas
- 2) temperature
- 3) Volume occupied by gas.

Laws: 1) Boyle's Law 2) Charles's Law 3) Gay-Lussac's Law.

Boyle's Law:- The volume of absolute pressure of a given mass of a perfect gas varies inversely as its absolute volume, when the temperature remains constant.

$$p \propto \frac{1}{V} \quad \text{or} \quad pV = \text{constant} \quad \text{or} \quad p_1 V_1 = p_2 V_2 = p_3 V_3 = \text{constant}$$

Charles's Law:- The volume of a given mass of a perfect gas varies directly as its absolute temperature when absolute pressure remains constant.

$$V \propto T \quad \text{or} \quad \frac{V}{T} = \text{constant} \quad \text{or} \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{V_3}{T_3} = \text{constant}$$

Gay-Lussac's Law:- The absolute pressure of a given mass of a perfect gas varies directly as its absolute temperature when volume remains constant.

$$p \propto T \quad \text{or} \quad \frac{p}{T} = \text{constant} \quad \text{or} \quad \frac{p_1}{T_1} = \frac{p_2}{T_2} = \frac{p_3}{T_3} = \text{constant}$$

General gas equation:- Which obeys both Boyle's & Charles's Law.

$$p \propto \frac{1}{V} \quad \text{or} \quad V \propto \frac{1}{p} \rightarrow \text{Boyle's Law}$$

$$V \propto T \rightarrow \text{Charles's Law}$$

$$\text{or} \quad V \propto \frac{T}{p} \quad \text{or} \quad \frac{pV}{T} = \text{constant}$$

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3} = \text{constant}$$

Characteristic gas equation:-  $pV = nRT$ .

where  $R =$  characteristic gas constant.  $0.287 \text{ kJ/kg K}$ .

Joule's Law:- The change of internal energy is proportional to change of temperature  $\Delta U \propto \Delta T$

$$\therefore \Delta U = m c_p (T_2 - T_1) \checkmark$$



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Specific heats:- The amount of heat required to raise the temperature of its unit mass through one degree. Solids & liquids have one specific heat while gases have two specific heats ( $C_p$  &  $C_v$ ).

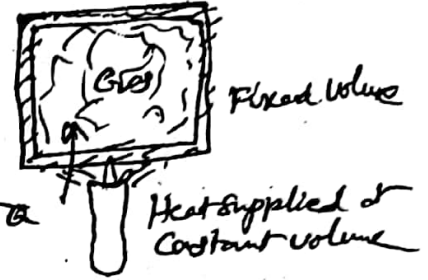
Specific heat at constant volume ( $C_v$ ):-

It is the amount of heat required to raise the temperature of a unit mass of gas through one degree when it is heated at constant volume.  $C_v$ ,  $\text{kJ/kgK}$ .

Heat supplied at constant volume

$$Q_{1-2} = m \times \text{sp. heat at constant vol} \times \text{rise in temperature}$$

$$= m C_v (T_2 - T_1)$$



Specific heat at constant pressure ( $C_p$ ):-

It is the amount of heat required to raise the temperature of a unit mass of a gas through one degree when it is heated at constant pressure.

Heat supplied,  $Q = m C_p (T_2 - T_1)$ .



Enthalpy of a Gas,  $h$ : It is the sum of the internal energy ( $U$ ) and the product of pressure and volume ( $PV$ ).

Since  $(U + PV)$  is made entirely of properties, therefore enthalpy ( $h$ ) is also a property.

Relation Between two specific heats:-

Consider a gas enclosed in a container and being heated at constant pressure from initial state 1 to final state 2.

Heat supplied at constant pressure:-

$$Q_{1-2} = m C_p (T_2 - T_1)$$

Heat utilized for external work,  $W_{1-2} = P(V_2 - V_1)$

increase in internal energy,  $dU = m C_v (T_2 - T_1)$

$$Q_{1-2} = W_{1-2} + \Delta U_{1-2}$$

$$m C_p (T_2 - T_1) = m R (T_2 - T_1) + m C_v (T_2 - T_1)$$

$$C_p - C_v = R$$

$$\frac{C_p}{C_v} = \gamma \text{ (adiabatic index)}$$

Non-Flow processes:- The thermodynamic processes which do not involve the mass flow across their boundary (closed systems).

The commonly used non-flow processes are

- 1) constant-volume process
- 2) constant-pressure process
- 3) constant-temperature process
- 4) polytropic process
- 5) Adiabatic process. (Isentropic process)

1) Constant volume process (Isochoric process) :-

If a gas is confined in a closed cylinder, its volume cannot change when heated or cooled.

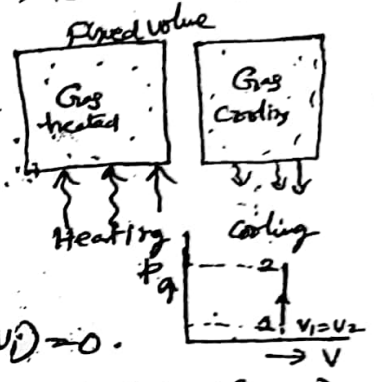
The temperature & pressure increased with heat addition. The internal energy of gas increases.

Charles Law for constant volume process.

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Work done by gas  $W_{1-2} = \int P dV = P(V_2 - V_1) = 0$ .

Heat supplied  $Q_{1-2} = (U_2 - U_1) + W_{1-2} = U_2 - U_1 = m C_V (T_2 - T_1)$ .



2) Constant pressure process (Isobaric process) :-

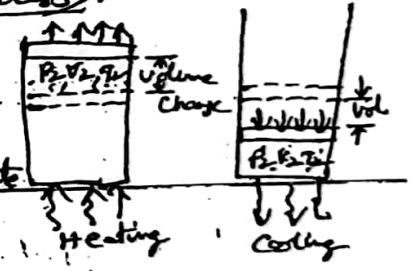
If heat is supplied to a gas under constant pressure, the volume of the gas will increase in direction proportional to the change in the absolute temperature of the gas.

The relationship between pressure, volume and temperature is

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} ; \text{ since } P_1 = P_2 ; \frac{V_1}{T_1} = \frac{V_2}{T_2} = \frac{V}{T} = \text{const.}$$

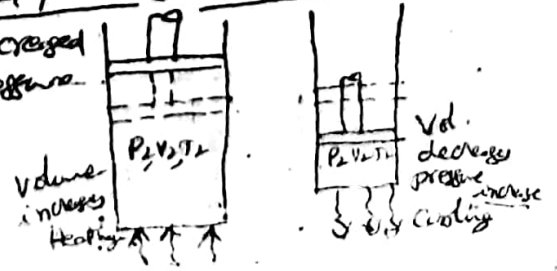
Work done by the gas  $W_{1-2} = P(V_2 - V_1) = m R (T_2 - T_1)$

Heat supplied  $Q_{1-2} = (U_2 - U_1) + W_{1-2}$   
 $= m C_V (T_2 - T_1) + m R (T_2 - T_1)$   
 $= m C (T_2 - T_1) (C_V + R)$   
 $= m C_P (T_2 - T_1)$



3) Constant Temperature process (Isothermal process) (Boyle's process)

If the volume of a gas is increased or decreased with constant temperature, the absolute pressure will vary inversely with volume. This is Boyle's Law for constant temperature process.





The process is also called as isothermal (constant temperature) process.

Boyle's law for a constant temperature process is

$$P_1 V_1 = P_2 V_2$$

The relationship between pressure, volume and temperature is

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{Since } T_1 = T_2 \quad P_1 V_1 = P_2 V_2 = PV = \text{constant}$$

Work done by the gas  $W_{1-2} = \int_{V_1}^{V_2} \frac{c}{V} dV = c \int_{V_1}^{V_2} \frac{1}{V} dV$

$$= P_1 V_1 \left[ \log_e V \right]_{V_1}^{V_2} = P_1 V_1 \log_e \frac{V_2}{V_1}$$

Heat supplied,  $Q_{1-2} = W_{1-2} + \Delta U$   $\because \Delta U = m C_V (T_2 - T_1) = 0; T_1 = T_2$

$$= W_{1-2} = P_1 V_1 \log_e \frac{V_2}{V_1}$$

**ADIABATIC PROCESS:** In an adiabatic process, the gas changes its condition without the transfer of heat to (or) from the surroundings. Pressure, volume and temperature of the gas vary during adiabatic process.

The relationship between pressure, volume & temperature is  $P_1 V_1^\gamma = P_2 V_2^\gamma = c$ , constant

Work done during adiabatic process  $W_{1-2} = \int_1^2 P dV$

$$= \int_1^2 \frac{c}{V^\gamma} dV = c \int_1^2 \frac{dV}{V^\gamma} = c \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_1^2$$

$$= \frac{P_2 V_2^\gamma \cdot V_2^{-\gamma+1} - P_1 V_1^\gamma \cdot V_1^{-\gamma+1}}{-\gamma+1} = \frac{P_2 V_2 - P_1 V_1}{\gamma-1}$$

[Take  $C_V = \frac{R}{\gamma-1}$ ]

$= - \left( \frac{P_2 V_2 - P_1 V_1}{\gamma-1} \right)$ , -ve sign denotes work done on the gas.

Heat supplied (or) Heat transferred,  $Q_{1-2} = 0$  (Adiabatic process  $Q = 0$ )

**POLYTROPIC PROCESS:**

An expansion process in which the energy to do the work is supplied partly from an external source and partly from the gas itself is known as polytropic process that will follow a path which will fall in between those of the isothermal and adiabatic processes.

$$PV^n = c \quad ; \quad P_1 V_1^n = P_2 V_2^n = PV^n = c$$

Work done during polytropic process,  $W_{1-2} = \frac{P_2 V_2 - P_1 V_1}{n-1} = \frac{m R (T_2 - T_1)}{n-1}$

$$\frac{T_1}{T_2} = \left( \frac{P_1}{P_2} \right)^{\frac{n-1}{n}} ; \quad \frac{T_1}{T_2} = \left( \frac{V_2}{V_1} \right)^{n-1}$$

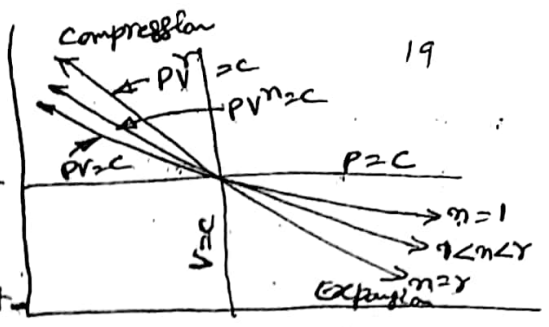
Heat transferred,  $Q_{1-2} = W_{1-2} + \Delta U_{1-2}$

$$= \frac{m R (T_2 - T_1)}{n-1} + m C_V (T_2 - T_1) = \frac{m R (T_2 - T_1)}{n-1} + m \times \frac{R}{\gamma-1} (T_2 - T_1)$$

$$= \frac{m R (T_2 - T_1)}{n-1} \left[ \frac{1}{n-1} + \frac{1}{\gamma-1} \right] = \frac{\gamma-n}{\gamma-1} \times \frac{m R (T_2 - T_1)}{n-1}$$



an isothermal compression process, ↑ energy is transferred as heat from the gas to an external sink at the same rate that work is being done on the gas. The internal energy is constant.

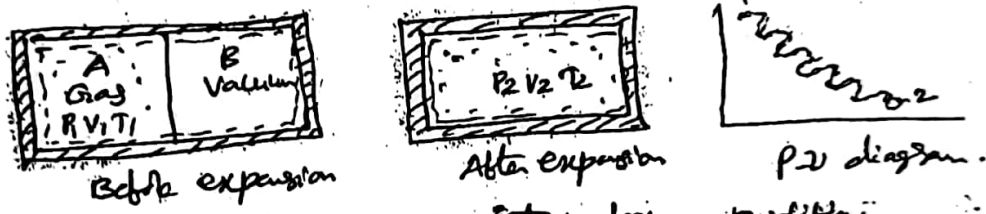


During an adiabatic process, there is no transfer of heat energy. Hence an amount of energy is equal to the amount of work done on the gas is set up in the gas as an increase in the internal energy and temperature of gas increases.

Comparison of isentropic, isothermal & polytropic process.

Free Expansion [or Unrestricted Expansion] process:-

The free expansion process is an irreversible non-flow process. A free expansion occurs when a fluid is allowed to expand suddenly into a vacuum chamber through an orifice of large dimensions.



Consider two chambers A and B separated by a partition. If the partition is removed, the gas will expand freely and occupy the whole space. By this volume increases to  $V_2$ , pressure & temp. decreases to  $P_2, T_2$ .

Since there is no expansion of boundary of the system, no work is done.  $\therefore Q_{1-2} = 0, W_{1-2} = 0$  and  $dU = 0; dH = 0$ .

S.No.	Type of reversible non-flow process	P-2-T relation	Work done $W_{1-2}$	Change in internal energy $\Delta U = U_2 - U_1$	Heat supplied $Q = W + \Delta U$	Change in enthalpy $dH = h_2 - h_1$
1.	Constant volume process	$\frac{P_1}{T_1} = \frac{P_2}{T_2}$	0	$mC_V(T_2 - T_1)$	$mC_V(T_2 - T_1)$	$mC_P(T_2 - T_1)$
2.	Constant pressure process	$\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$P(V_2 - V_1)$	$mC_V(T_2 - T_1)$	$mC_P(T_2 - T_1)$	$mC_P(T_2 - T_1)$
3.	Constant temp (or) isothermal	$P_1V_1 = P_2V_2$	$P_1V_1 \log_e \frac{V_2}{V_1}$	0	$P_1V_1 \log_e \frac{V_2}{V_1}$	0
4.	Adiabatic (or) isentropic $PV^\gamma = c$	$P_1V_1^\gamma = P_2V_2^\gamma$ $\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$ $= \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$	$\frac{P_2V_2 - P_1V_1}{\gamma-1}$	$mC_V(T_2 - T_1)$	0	$mC_P(T_2 - T_1)$
5.	polytropic process $PV^n = c$	$P_1V_1^n = P_2V_2^n$	$\frac{P_2V_2 - P_1V_1}{n-1}$	$mC_V(T_2 - T_1)$	0	$\frac{\gamma-n}{\gamma-1} mC_P(T_2 - T_1)$

**PROB.** A certain gas occupies a space of  $0.3 \text{ m}^3$  at a pressure of 2 bar and a temperature of  $77^\circ\text{C}$ . It is heated at constant volume, until the pressure is 7 bar. Determine 1) Temperature at end of the process 2) mass of gas 3) change in internal energy 4) change in enthalpy during the process.  
 Assume  $C_p = 1.005 \text{ kJ/kgK}$ ,  $C_v = 0.712 \text{ kJ/kgK}$ ,  $R = 287 \text{ J/kgK}$ .

**SOL.**  
 $V_1 = 0.3 \text{ m}^3$ ,  $P_1 = 2 \text{ bar} = 2 \times 10^5 \text{ N/m}^2$ ,  $T_1 = 77^\circ\text{C} = 350 \text{ K}$   
 $P_2 = 7 \text{ bar} = 7 \times 10^5 \text{ N/m}^2$ ,  $C_p = 1.005 \text{ kJ/kgK}$ ,  $R = 287 \text{ J/kgK}$

1) Temperature at the end of the process,  $T_2$

We know that,  $\frac{P_1}{T_1} = \frac{P_2}{T_2}$   
 $\therefore T_2 = \frac{P_2}{P_1} \times T_1 = \frac{7 \times 10^5}{2 \times 10^5} \times 350 = 1225 \text{ K} = 1225 - 273 = 952^\circ\text{C}$

2) Mass of the gas,

Let  $m =$  mass of the gas  
 we know that,  $P_1 V_1 = mRT_1$ ;  $m = \frac{P_1 V_1}{RT_1} = \frac{2 \times 10^5 \times 0.3}{287 \times 350} = 0.597 \text{ kg}$ .

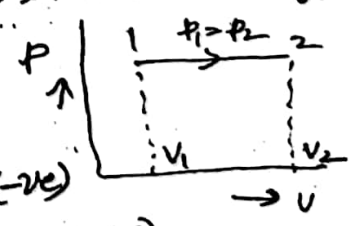
3) change in internal energy:-

$dU = U_2 - U_1 = m C_v (T_2 - T_1) = 0.597 \times 0.712 (1225 - 350) = 372 \text{ kJ}$

4) change in enthalpy,  $dH = H_2 - H_1 = m C_p (T_2 - T_1)$

$= 0.597 \times 1.005 (1225 - 350) = 525 \text{ kJ}$ .

**PROB.** A closed system undergoes a reversible process at a constant pressure process of 3.5 bar and its volume changes from  $0.15 \text{ m}^3$  to  $0.06 \text{ m}^3$ . 25 kJ of heat is rejected by the system during the process. Determine the change in internal energy of the system.



**SOL.** pressure,  $p = 3.5 \text{ bar} = 3.5 \times 10^5 \text{ N/m}^2$   
 $V_1 = 0.15 \text{ m}^3$ ,  $V_2 = 0.06 \text{ m}^3$ .

Heat rejected by system,  $Q = -25 \text{ kJ} (-ve)$

Work done  $= p(V_2 - V_1) = 3.5 \times 10^5 (0.06 - 0.15) = -31,500 \text{ J} = -31.5 \text{ kJ} (-ve)$  (work done on the system)

According to First law of thermodynamics.

$Q = \Delta U + W$   
 $-25 = \Delta U + (-31.5) \Rightarrow \Delta U = 31.5 - 25 = 6.5 \text{ kJ}$



**PROB** A quantity of air has a volume of  $0.4 \text{ m}^3$  at a pressure of  $5 \text{ bar}$  and a temperature of  $80^\circ\text{C}$ . It is expanded in a cylinder at a constant temperature to a pressure of  $1 \text{ bar}$ . Determine the amount of work done by the air during expansion.

**SOL** Given,  $v_1 = 0.4 \text{ m}^3$ ,  $p_1 = 5 \text{ bar} = 5 \times 10^5 \text{ N/m}^2$ ,  $T_1 = 80^\circ\text{C} = 80 + 273 = 353 \text{ K}$   
 $p_2 = 1 \text{ bar} = 1 \times 10^5 \text{ N/m}^2$ .

Work done by the air during (expansion)

$$W_{1-2} = p_1 v_1 \ln \frac{v_2}{v_1} = p_1 v_1 \ln \frac{p_1}{p_2} = 5 \times 10^5 \times 0.4 \ln \frac{5}{1}$$

$$= 3.218 \times 10^5 \text{ J} = 321.8 \text{ kJ} \quad \checkmark \text{ (work done by system is +ve)}$$

**PROB.**  $0.1 \text{ m}^3$  of air at a pressure of  $1.5 \text{ bar}$  is expanded isothermally to  $0.5 \text{ m}^3$ . Calculate the final pressure of the gas and heat supplied during the process.

**SOL** Given,  $v_1 = 0.1 \text{ m}^3$ ,  $p_1 = 1.5 \text{ bar} = 1.5 \times 10^5 \text{ N/m}^2$ ,  $v_2 = 0.5 \text{ m}^3$ .

Final pressure of the gas,

We know that,  $p_1 v_1 = p_2 v_2$

$$p_2 = \frac{p_1 v_1}{v_2} = \frac{1.5 \times 10^5 \times 0.1}{0.5} = 0.3 \times 10^5 \text{ N/m}^2 = 0.3 \text{ bar}$$

Work done by the system,  $W_{1-2} = p_1 v_1 \ln \frac{v_2}{v_1} = p_1 v_1 \ln \frac{p_1}{p_2}$

$$= 1.5 \times 10^5 \times 0.1 \ln \left( \frac{1.5}{0.3} \right) = 24.115 \text{ J}$$

$$= 24.115 \text{ kJ} \quad \checkmark$$

Heat supplied,  $Q = \Delta U + W = 0 + 24.115$   $\Delta U = 0$ , since  $T_1 = T_2$

$$= 24.115 \text{ kJ} \quad \checkmark$$

**PROB.** A system contains  $0.15 \text{ m}^3$  of a gas at a pressure of  $3.8 \text{ bar}$  and  $150^\circ\text{C}$ . It is expanded adiabatically till the pressure falls to  $1 \text{ bar}$ . The gas is then heated at a constant pressure till its enthalpy increases by  $70 \text{ kJ}$ . Determine the total work done. Take,  $C_p = 1 \text{ kJ/kgK}$ ,  $C_v = 0.714 \text{ kJ/kgK}$ .

**SOL** Adiabatic index,  $\gamma = \frac{C_p}{C_v} = \frac{1}{0.714} = 1.4$

For an Adiabatic process 1-2,  $p, v, T$  relations are

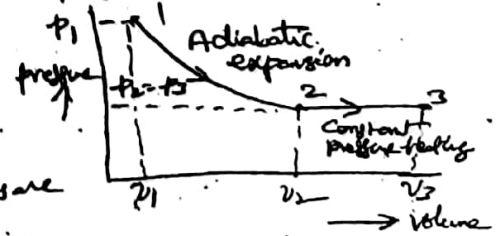
$$p_1 v_1^\gamma = p_2 v_2^\gamma \quad \text{or} \quad \frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}}$$

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

Final volume,  $v_2 = v_1 / \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}}$

$$= 0.15 / \left( \frac{1}{3.8} \right)^{\frac{1}{1.4}}$$

$$= 0.39 \text{ m}^3$$



Given,  $v_1 = 0.15 \text{ m}^3$   
 $p_1 = 3.8 \text{ bar} = 3.8 \times 10^5 \text{ N/m}^2$   
 $T_1 = 150^\circ\text{C} + 273 = 423 \text{ K}$   
 $p_2 = 1 \text{ bar}$  enthalpy  
 $\Delta H = +70 \text{ kJ}$  (increase)



gas constant,  $R = C_p - C_v = 1 - 0.714 = 0.286 \text{ kJ/kg} \cdot \text{K}$

Temperature at state 2,  $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$ ;  $T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$$\Rightarrow T_2 = 423 \left(\frac{1}{3.8}\right)^{\frac{1.4-1}{1.4}} = 288.7 \text{ K}$$

mass of the gas,  $P_1 V_1 = m R T_1$   
 $m = \frac{P_1 V_1}{R T_1} = \frac{0.286 \times 10^6 \times 0.15}{286 \times 423} = 0.47 \text{ kg}$

Increase in enthalpy during the constant pressure process 2-3,

$$dH = m C_p (T_3 - T_2)$$

$$70 = 0.47 \times 1 (T_3 - 288.7); T_3 = \frac{70}{0.47} + 288.7 = 437.6 \text{ K}$$

Since the heating is at constant pressure, therefore,

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3} \Rightarrow \frac{V_2}{T_2} = \frac{V_3}{T_3} \text{ since } P_2 = P_3$$

$$V_3 = \frac{V_2 T_3}{T_2} = \frac{0.39 \times 437.6}{288.7} = 0.59 \text{ m}^3$$

Work done during adiabatic expansion,

$$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{3.8 \times 10^5 \times 0.15 - 1.4 \times 10^5 \times 0.39}{1.4 - 1}$$

$$= 45,000 \text{ J} = 45 \text{ kJ} \checkmark$$

Work done during constant pressure heating,

$$W_{2-3} = P_2 (V_3 - V_2) = 1.4 \times 10^5 (0.59 - 0.39)$$

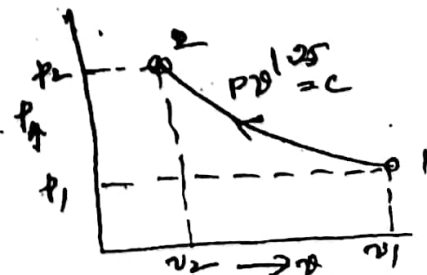
$$= 20,000 \text{ J} = 20 \text{ kJ}$$

$$\text{Total Work done, } W = W_{1-2} + W_{2-3} = 45 + 20 = 65 \text{ kJ} \checkmark$$

**Prob.** A gas mixture obeying perfect gas law has a molecular mass of 26.7. The gas mixture is compressed through a compression ratio of 12 according to the law  $P V^{1.25} = \text{constant}$ , from initial conditions of 0.9 bar and 333K. Assuming a mean molar specific heat at constant volume of 21.1 kJ/kg. find per kg of mass, the work done and heat flow across the cylinder walls. For the above gas, determine the values of characteristic gas constant, molar specific heat at a constant pressure and ratio of specific heats.

**Sol.** Given, Molecular mass,  $M = 26.7$   
 Compression ratio,  $\gamma = \frac{V_1}{V_2} = 12$   
 Initial pressure,  $P_1 = 0.9 \text{ bar} = 0.9 \times 10^5 \text{ N/m}^2$   
 Initial temperature,  $T_1 = 333 \text{ K}$

First find  $V_2, P_2$ ;  
 We know that  $P_1 V_1^\gamma = P_2 V_2^\gamma$   
 $P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma = 0.9 (12)^{1.25} = 20.1 \text{ bar}$



characteristic gas constant,  $R = \frac{\text{universal gas constant}}{\text{molecular weight}}$  23  
 $= \frac{8314}{26.7} = 311.4 \text{ J/kgK}$

and  $p_1 v_1 = mRT_1$

$$v_1 = \frac{mRT_1}{p_1} = \frac{1 \times 311.4 \times 333}{0.9 \times 10^5} = 1.15 \text{ m}^3$$

$$v_2 = \frac{v_1}{12} = \frac{1.15}{12} = 0.096 \text{ m}^3 \quad \left[ \text{Since mass of gas} = 1 \text{ kg, given} \right]$$

Work done work supplied during compression,

$$W_{1-2} = \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{20.1 \times 10^5 \times 0.096 - 0.9 \times 10^5 \times 1.15}{1.25-1}$$

$$= 357840 \text{ J} \quad \text{or} \quad \underline{357.84 \text{ kJ}} \checkmark$$

Heat flow across cylinder walls:

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^{\gamma-1}$$

$$p_2 = p_1 \left( \frac{v_1}{v_2} \right)^{\gamma-1} = 0.9 \left( \frac{1.15}{0.096} \right)^{1.25-1} = 620 \text{ k}$$

specific heat at constant volume,  $c_{v,m} = \frac{C_{v,m}}{M} = \frac{21.1}{26.7} = 0.79 \text{ kJ/kgK}$

change in internal energy,  $dU = U_2 - U_1 = m c_v (T_2 - T_1) = 1 \times 0.79 (620 - 333)$   
 $= 226.7 \text{ kJ}$

Heat flow,  $Q = \Delta U + W = 226.7 + (-357.84)$  [∵ W is -ve, work supplied]  
 across cylinder walls,  $= \underline{-131.1 \text{ kJ}}$  Heat rejected

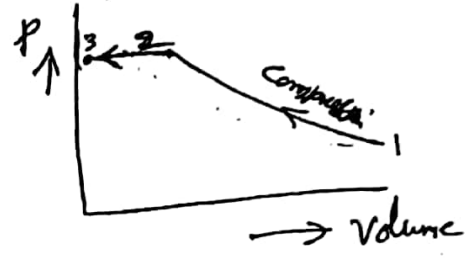
$C_p - C_v = R$ ;  $C_p - 0.79 = 0.3114$ ;  $C_p = 1.1014 \text{ kJ/kgK}$

Mean specific heat at constant pressure,  $C_{p,m} = M C_p = 26.7 \times 1.1014 = 29.4 \text{ kJ/kgK}$

Ratio of specific heats,  $\gamma = \frac{C_p}{C_v} = \frac{1.1014}{0.79} = \underline{1.394} \checkmark$

**PROB** A cylinder contains  $0.115 \text{ m}^3$  of gas at 1 bar and  $90^\circ\text{C}$ . The gas is compressed to a volume  $0.0288 \text{ m}^3$ . The final pressure being 5.67 bar. Calculate

- (a) mass of the gas
  - (b) value of index of compression
  - (c) increase in internal energy
  - (d) Heat transfer during compression.
- of ~~the~~ after compression, the gas is to be cooled at constant pressure to its original temperature of  $90^\circ\text{C}$ , find the work of compression required. Assume  $\gamma = 1.4$  &  $R = 0.3 \text{ kJ/kgK}$ .



**SOL** Given,  $V_1 = 0.115 \text{ m}^3$   
 $p_1 = 1 \text{ bar}$   
 $T_1 = 90 + 273 = 363 \text{ K}$   
 $V_2 = 0.0288 \text{ m}^3$   
 $p_2 = 5.67 \text{ bar}$   
 $\gamma = 1.4 ; R = 0.3 \text{ kJ/kgK}$

(a) mass of the gas:-  $p_1 V_1 = m R T_1 ; m = \frac{p_1 V_1}{R T_1}$   
 $= \frac{1 \times 10^5 \times 0.115}{(0.3 \times 1000) \times 363}$   
 $= 0.1056 \text{ Kgs} \checkmark$

(b) value of index of compression:-  
 $p_1 V_1^n = p_2 V_2^n$   
 $\therefore n = \frac{\log_e (p_2/p_1)}{\log_e (V_1/V_2)} = \frac{\log_e (5.67/1)}{\log_e (0.115/0.0288)} = 1.253$

(c) increase in internal energy:-  
 $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1} ; T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{n-1} = 363 \left(\frac{0.115}{0.0288}\right)^{1.253-1} = 515.17 \text{ K}$   
 $\Delta U = \text{change in internal energy} = m C_v (T_2 - T_1) = m \left(\frac{R}{\gamma - 1}\right) (T_2 - T_1)$   
 $= 0.1056 \times \frac{0.30}{1.4 - 1} (515.17 - 363)$   
 $= 12.05 \text{ kJ} \checkmark$

(d) Heat Transfer:-  
 $Q = \Delta U + W ; W = \frac{p_2 V_2 - p_1 V_1}{n-1} = \frac{5.67 \times 0.0288 - 1 \times 0.115}{1.253 - 1} 10^5$   
 $= 19.05 \text{ kJ} \text{ (-ve) work done on system}$   
 $\therefore Q = \Delta U + W$   
 $= 12.05 - 19.05 = -7 \text{ kJ} \checkmark$

(e) After compression, it required to cool gas to original temp of  $90^\circ\text{C}$   
 $\frac{p_3 V_3}{T_3} = \frac{p_2 V_2}{T_2} ; p_3 = p_2 ; V_3 = V_2 \times \frac{T_3}{T_2} = \frac{363 \times 0.0288}{515.17} = 0.0202 \text{ m}^3$   
 Additional work required,  $W = p_2 (V_3 - V_2) = 5.67 \times 10^5 (0.0202 - 0.0288)$   
 $= -4.87 \text{ kJ} \checkmark$



\* Application of First law of Thermodynamics to a steady flow process. 25

A flow process constitutes an open system, in which the working substance enters and leaves the control surface of a system. Energy interaction in the form of heat and work may also take place with in the system.

Flow energy (or) flow work refers to work required to push a certain mass of fluid into and out of the control volume.

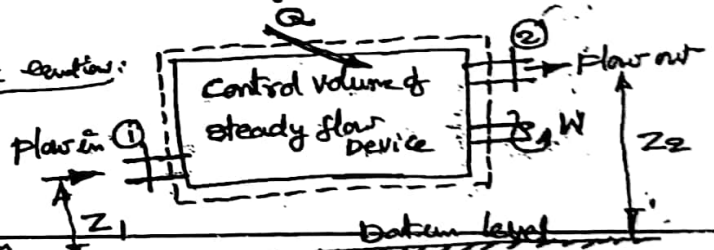
On a steady flow process, the following conditions must be satisfied.

- (a) The rate of mass flow at inlet and outlet is same.
- (b) The rate of heat transfer is constant.
- (c) The rate of work transfer is constant.
- (d) State of the working substance at any point with in the system is same at all points.
- (e) There is no change in the chemical composition of the systems.

If any one of these conditions are not satisfied, then the process is said to be non-steady flow process. In engineering mainly concerned with steady flow process.

Steady flow Energy Balance equation:

Consider an open system through which the working substance flows at a steady rate as shown in figure. The working substance enters the system at section (1) and leaves the system at section (2).



- Let
- $p_1$  = pressure of the working substance entering the system, N/m<sup>2</sup>
  - $v_{s1}$  = specific volume of the substance entering the system, m<sup>3</sup>/kg.
  - $V_1$  = velocity of the working substance entering the system, m/sec.
  - $u_1$  = specific internal energy of the working substance entering the system in J/kg.
  - $z_1$  = Height above datum level for inlet in metres.

Similarly  $p_2, v_{s2}, V_2, u_2$  &  $z_2$  are corresponding values for the working substance leaving the system.

- Let
- $q_{1-2}$  = Heat supplied to the system J/kg
  - $w_{1-2}$  = work delivered by the system J/kg.

The total energy of entering the system per kg of working substance

$$E_1 = \text{internal energy} + \text{flow work} + \text{Kinetic energy} + \text{potential energy} + \text{Heat supplied.}$$

$$= u_1 + P_1 v_{s1} + \frac{V_1^2}{2} + g z_1 + q_{1-2}$$

Similarly, Total energy leaving the system per kg of working substance

$$E_2 = u_2 + P_2 v_{s2} + \frac{V_2^2}{2} + g z_2 + W_{1-2}$$

Assuming no loss of energy during flow, According to First law of Thermodynamics,  $e_1 = e_2$

$$u_1 + P_1 v_{s1} + \frac{V_1^2}{2} + g z_1 + q_{1-2} = u_2 + P_2 v_{s2} + \frac{V_2^2}{2} + g z_2 + W_{1-2}$$

$$\boxed{m \left( h_1 + \frac{V_1^2}{2} + z_1 g + q_{1-2} \right) = m \left( h_2 + \frac{V_2^2}{2} + g z_2 + W_{1-2} \right)}$$

This is known as steady flow energy equation (SFEE).

Note:- on steady flow, mass flow rate is constant entering & leaving the working substance.

- ①  $m = \frac{A_1 V_1}{v_{s1}} = \frac{A_2 V_2}{v_{s2}}$
- ② If K.E & P.E are neglected, SFEE,  $q_{1-2} - W_{1-2} = h_2 - h_1$
- ③ SFEE may be written as,  $q_{1-2} - W_{1-2} = h_2 - h_1 + \left( \frac{V_2^2}{2} - \frac{V_1^2}{2} \right) + g(z_2 - z_1)$

\* Application of steady flow energy equation (SFEE)

① BOILER:- A boiler is a device which supplies heat to water and generates steam. on this system, there is no change in kinetic and potential energies and also there is no work done by the system. KE & P.E & work done is zero.

The SFEE equation,

$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \left( \frac{V_2^2}{2} - \frac{V_1^2}{2} \right) + (z_2 g - z_1 g)$$

∴  $q_{1-2} = h_2 - h_1$

This shows that heat supplied to a boiler increases the enthalpy of the system.

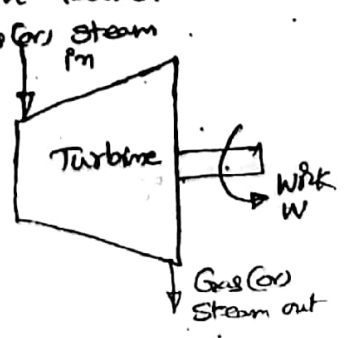
② STEAM TURBINE:-

on steam turbine, steam is passed through the turbine and part of its energy is converted into work in the turbine

The steam leaves the turbine at lower pressure and temperature.

$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

Here,  $q_{1-2} = 0$ , if turbine is insulated,  
 = -ve, if turbine not insulated.



NAME

A steady-flow process can be defined as a process during which all properties of fluid at each location within the system remain constant with respect to time. That is, the fluid properties can change from point to point within the control volume but at any fixed location they remain the same during the entire process. steady flow means no change with time.

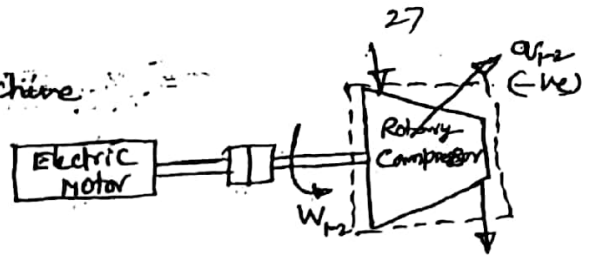
Conditions for steady-flow process:

1. The fluid properties (intensive (or Extensive) within the control volume remain constant at each location with respect to time.
2. The properties of the fluid crossing the boundary (inlet and outlet) remain constant at each point of the boundary.
3. The Mass-flow rate into the system is always equal to the mass flow rate out of the system.
4. Heat and work interactions with the surroundings occur at a steady state.



③ ROTARY COMPRESSOR:-

A rotary compressor is a machine which compresses air (or) gas and supplies the same at moderate pressure in large quantity.



- on compressor,  $Z_1 = Z_2$ , and heat rejected from system

so,  $q_{1-2} = -ve$

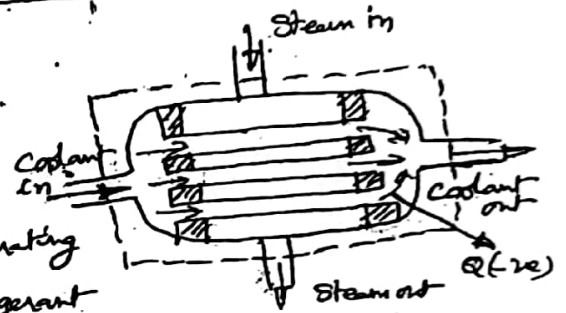
$W_{1-2}$  work supplied to the system,  $-ve$ .

$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

$$\therefore -q_{1-2} - (-W_{1-2}) = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$$

④ CONDENSER:-

A condenser is a device used to condense steam in case of steam power plants using water as the cooling medium, where as in refrigerating systems, it is used to condense refrigerant vapour using air as the cooling medium. There is no change in KE and P.E. Also, there is no work done by the system.



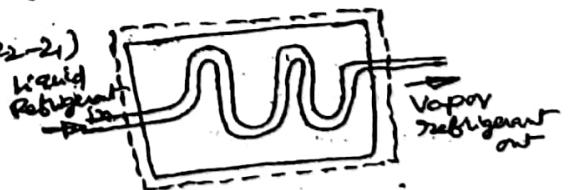
$$\therefore -q_{1-2} = h_2 - h_1$$

$$-q_{1-2} = h_2 - h_1 \quad \text{OR} \quad q_{1-2} = h_1 - h_2$$

⑤ EVAPORATOR:- Evaporator is a device used in refrigeration systems in which liquid refrigerant passes, receives heat and leaves as vapour refrigerant. For such a system, K.E & P.E, work done is zero.

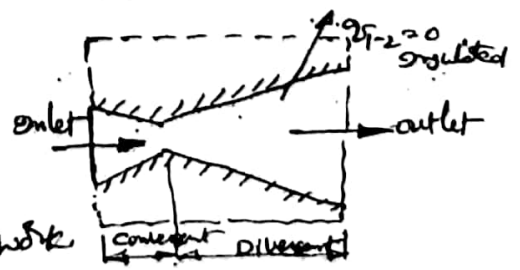
$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$\therefore q_{1-2} = h_2 - h_1$$



⑥ NOZZLE:- A nozzle is a device

which increase the velocity of the working substance at the expense of its pressure drop. The nozzle is insulated, so that no transfer of heat from the system, Further, the system does not deliver any work. There is no change in P.E



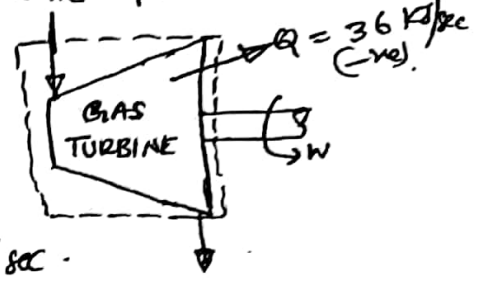
$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$0 = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} \therefore V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

of initial velocity,  $V_1$  is neglected

velocity of steam at exit,  $V_2 = \sqrt{2(h_1 - h_2)}$

**PROB.** A gas turbine receives gas at an enthalpy of 800 kJ/kg and a velocity of 100 m/sec. The gas leaves the turbine at an enthalpy of 380 kJ/kg and a velocity of 150 m/sec. Heat lost to surroundings from the gas is 36 kJ/sec. If the rate of gas flow is 10 kg/sec, Find the power developed by the turbine.



**SOL.** Given, Enthalpy at inlet of turbine,  $h_1 = 800 \text{ kJ/kg}$

velocity at inlet,  $V_1 = 100 \text{ m/sec}$

$h_2 = 380 \text{ kJ/kg}$ ,  $V_2 = 150 \text{ m/sec}$ .  
Heat rejected,  $Q_{1-2} = -36 \text{ kJ/sec}$ .

rate of gas flow,  $\dot{m} = 10 \text{ kg/sec}$ .

**SFEE is**

$$Q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$\frac{Q_{1-2}}{\dot{m}} - \frac{W_{1-2}}{\dot{m}} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + 0$$

$$\frac{-36 \times 10^3}{10} - \frac{W_{1-2}}{10} = (380 \times 10^3 - 800 \times 10^3) + \frac{150^2 - 100^2}{2}$$

$$\frac{-36 \times 10^3 - W_{1-2}}{10} = (-420 \times 10^3 + 6250) = (-413,750)$$

$$W_{1-2} = 4101500 \text{ kJ/sec} \text{ (or) } 4101 \text{ kW}$$

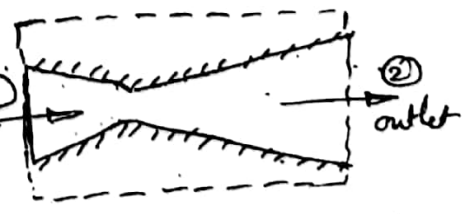
**PROB.** A nozzle receives 1200 kg/hr of steam at 13 MN/m<sup>2</sup>, specific volume  $v_{s1}$  is 142 litres/kg, internal energy 4400 kJ/kg and negligible speed. At exit, the pressure is 100 kN/m<sup>2</sup>, specific volume 1660 litres/kg and internal energy 2200 kJ/kg. Calculate the exit velocity of steam.

**SOL.** Given, mass of steam,  $m = 1200 \text{ kg/hr}$ , inlet pressure at inlet,  $p_1 = 1.3 \text{ MN/m}^2 = 1.3 \times 10^6 \text{ N/m}^2$

specific volume at inlet,  $v_{s1} = 142 \text{ lit/kg} = 0.142 \text{ m}^3/\text{kg}$

internal energy at inlet,  $u_1 = 4400 \text{ kJ/kg}$

similarly,  $p_2 = 100 \times 10^3 \text{ N/m}^2$   
 $v_{s2} = 1660 \text{ lit/kg} = 1.66 \text{ m}^3/\text{kg}$   
 $u_2 = 2200 \text{ kJ/kg}$



ie SFE,  $q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$

$0 = (h_2 - h_1) + \frac{v_2^2}{2}$  ;  $q_{1-2} = 0, w_{1-2} = 0$   
 $v_1 = 0, z_1 = z_2$

$\therefore v_2 = \sqrt{2(h_1 - h_2)}$

$h_1 = u_1 + p_1 v_{s1} = 4400 \times 10^3 + 1.3 \times 10^6 \times 0.142 = 4584600 \text{ J/kg}$

$h_2 = u_2 + p_2 v_{s2} = 2200 \times 10^3 + 100 \times 10^3 \times 1.66 = 2366000 \text{ J/kg}$

$\therefore v_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(4584600 - 2366000)} = 2106 \text{ m/sec}$

**Prob** A blower handles 1 kg/s of air at 20°C and consumes 15 kW. Its inlet and outlet velocities of air 100 m/sec and 150 m/sec respectively. Find the exit air temperature. Assume adiabatic conditions ( $q_{1-2} = 0$ )

**Sol.** Given: mass of air = 1 kg/s, Power consumed,  $W = -15 \text{ kW}$  (-ve)  
 inlet temperature of air,  $T_1 = 20^\circ\text{C}$ ,  $v_1 = 100 \text{ m/sec}$ ,  $v_2 = 150 \text{ m/sec}$

The SFE

$q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$

$\frac{q_{1-2}}{m} - \frac{w_{1-2}}{m} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2}$

$-\frac{(-15) \times 10^3}{1} = (h_2 - h_1) + \frac{150^2 - 100^2}{2}$

$\therefore (h_2 - h_1) = 15000 - 6250 = 8750 \text{ J/kg}$

$8750 = m c_p (T_2 - T_1) = 1 \times 1005 \times (T_2 - 293)$

$\therefore$  Exit air temperature,  $T_2 = 301.706 \text{ K}$  (or  $28.7^\circ\text{C}$ )

**Prob** A steady flow apparatus 140 kJ of work is done by each kg of fluid. The specific volume of fluid, pressure and velocity at the inlet are  $0.37 \text{ m}^3/\text{kg}$ , 600 kPa and 16 m/sec. The inlet is 32 mm above floor and the discharge pipe is at the floor level. The discharge conditions are  $0.62 \text{ m}^3/\text{kg}$ , 100 kPa and 300 m/sec. The total heat loss between inlet and discharge is 9 kJ/kg of fluid. Find whether specific internal energy increase or decrease.

**Sol** Given, work done by fluid,  $W = 140 \text{ kJ}$  (+ve)

$v_{s1} = 0.37 \text{ m}^3/\text{kg}$

$p_1 = 600 \text{ kPa}$

$v_1 = 16 \text{ m/sec}$

$z_1 = 32 \text{ mm}$

$z_2 = 0$

$v_{s2} = 0.62 \text{ m}^3/\text{kg}$

$p_2 = 100 \text{ kPa}$

$v_2 = 300 \text{ m/sec}$

$Q = \text{total heat loss}$

$= 9 \text{ kJ/kg}$  (-ve)



We have SFEE is,

$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$\frac{Q_{12}}{m} - \frac{W_{12}}{m} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$\frac{-9 \times 10^3}{1} - \frac{(140 \times 10^3)}{1} = (h_2 - h_1) + \frac{300^2 - 16^2}{2} + 9.81[0 - 32]$$

$$\therefore (h_2 - h_1) = 104441.92 \text{ J/kg}$$

$$(u_2 + p_2 v_2 - (u_1 + p_1 v_1)) = 104441.92$$

$$\therefore (u_2 - u_1) = 104441.92 - p_2 v_2 + p_1 v_1$$

$$= 104441.92 - 100 \times 10^3 \times 0.62 + 600 \times 10^3 \times 0.37$$

$$\therefore u_2 - u_1 = 264,441.7 \text{ (or } 264.4 \text{ kJ (in case))}$$

**PROB** Steam enters a steam condenser with an enthalpy of 2090 kJ/kg and velocity of 510 m/sec. The condensate leaves the condenser with an enthalpy of 209 kJ/kg and with a velocity of 10 m/sec. determine the heat received by cooling water per kg of steam condensed.

**SOL** Given,  $h_1 = 2090 \text{ kJ/kg}$  The SFEE is  
 $v_1 = 510 \text{ m/sec}$   
 $h_2 = 209 \text{ kJ/kg}$   
 $v_2 = 10 \text{ m/sec}$

$$q_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$\therefore q_{1-2} = (209 - 2090) + \frac{10^2 - 510^2}{2}$$

$$\therefore \text{Heat rejected from steam} = -2011 \text{ kJ/kg} \checkmark$$

**PROB** A gas leaving the turbine jet engine flows steadily into the jet pipe with an enthalpy 960 kJ/kg and velocity 250 m/sec. The exit from the pipe is at enthalpy 860 kJ/kg and exhaust is in line with intake. Neglect heat losses from the system; determine the velocity of the gas leaving the pipe.

**SOL** Given,  $h_1 = 960 \text{ kJ/kg}$   
 $v_1 = 250 \text{ m/sec}$   
 $h_2 = 860 \text{ kJ/kg}$   
 $v_2 = ?$

The SFEE is

$$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + 0$$

$$= (860 - 960) \times 10^3 + \frac{V_2^2 - 250^2}{2}$$

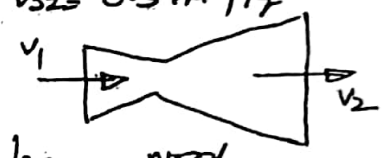
$$\therefore v_2 = 512.34 \text{ m/sec}$$

**PROB** One kg of fluid enters a nozzle with a velocity of 300 m/min and enthalpy of 2990 KJ/kg. The enthalpy of fluid at exit is 2760 KJ/kg. The nozzle is placed horizontally and neglect heat loss from the nozzle.

- Determine
- (a) The Velocity of the fluid at the exit
  - (b) The mass flow rate, if the area of the nozzle is 0.095 m<sup>2</sup> and the specific volume at inlet is 0.19 m<sup>3</sup>/kg
  - (c) The exit area of the nozzle, if specific volume at exit is 0.5 m<sup>3</sup>/kg.

**SOL:** Given,  $m = 1 \text{ kg}$   
 $V_1 = 300 \text{ m/min} = 5 \text{ m/sec}$   
 $h_1 = 2990 \text{ KJ/kg}$   
 $h_2 = 2760 \text{ KJ/kg}$

$A_1 = 0.095 \text{ m}^2$   
 $v_{s1} = 0.19 \text{ m}^3/\text{kg}$   
 $v_{s2} = 0.5 \text{ m}^3/\text{kg}$



i) Exit velocity,  

$$V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

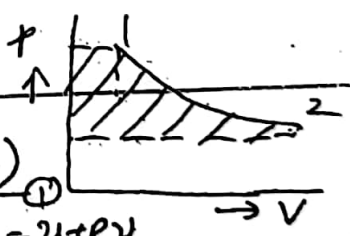
$$= \sqrt{5^2 + 2(2990 - 2760)10^3} = 678 \text{ m/sec}$$

ii) Mass flow rate,  $m = \frac{A_1 V_1}{v_{s1}} = \frac{0.095 \times 5}{0.19} = 2.5 \text{ kg/sec}$

iii) Exit area  $A_2$ ;  $m = \frac{A_2 V_2}{v_{s2}}$  ;  $A_2 = \frac{m v_{s2}}{V_2} = \frac{2.5 \times 0.5}{678} = 0.00184 \text{ m}^2$

Work done in a steady flow process

We know that the steady flow equation for unit mass flow, the differential form is



$\delta q - \delta w = dh + d(ke) + d(pe)$  (1)

We also know that,  $h = u + pv = u + pu$

differentiating the expression,  $dh = du + d(pv) = du + v dp + p dv$

According to first law,  $\delta q - \delta w = du + p dv$

$\delta q = du + p dv$

substituting the dh value in equation (1),

$\delta q - \delta w = \delta q + v dp + d(ke) + d(pe)$  (2)

neglecting ke and pe.

The equation (2) becomes,  $-\delta w = v dp$

$\therefore \delta w = -v dp$   
 on integrating  $W = -\int_1^2 v dp$

## Work done for various steady flow processes

32

① Constant volume process,

$$W_{1-2} = - \int_1^2 v dp = -v(p_2 - p_1) = v(p_1 - p_2)$$

② Constant pressure process

$$W_{1-2} = - \int_1^2 v dp = v(p_1 - p_2) = 0 \text{ since } p_1 = p_2.$$

③ Constant temperature process ( $p_1 v_1 = p_2 v_2 = p v$ ).

$$\begin{aligned} W_{1-2} &= - \int_1^2 v dp = \int_1^2 \frac{p_1 v_1}{p} dp = -p_1 v_1 \int_1^2 \frac{dp}{p} \\ &= -p_1 v_1 [\ln p_2 - \ln p_1] = p_1 v_1 \ln \frac{p_1}{p_2} = p_1 v_1 \ln \frac{v_2}{v_1} \end{aligned}$$

④ Adiabatic process: -  $p v^\gamma = p_1 v_1^\gamma = p_2 v_2^\gamma = c$ ;  $v = v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{\gamma}}$

$$\begin{aligned} W &= - \int_1^2 v dp = - \int_1^2 v_1 \left(\frac{p_1}{p}\right)^{\frac{1}{\gamma}} dp \\ &= v_1 p_1^{\frac{1}{\gamma}} \int_1^2 p^{-\frac{1}{\gamma}} dp = -v_1 p_1^{\frac{1}{\gamma}} \left[ \frac{p^{-\frac{1}{\gamma} + 1}}{-\frac{1}{\gamma} + 1} \right]_1^2 \\ &= \frac{\gamma}{\gamma - 1} v_1 p_1^{\frac{1}{\gamma}} [p_2^{\frac{\gamma-1}{\gamma}} - p_1^{\frac{\gamma-1}{\gamma}}] \\ &= \frac{\gamma}{\gamma - 1} (p_1 v_1 - p_2 v_2) \checkmark \end{aligned}$$

⑤ polytropic process

$$p_1 v_1^n = p_2 v_2^n = p v^n = c$$

$$W_{1-2} = \frac{n}{n-1} (p_1 v_1 - p_2 v_2)$$

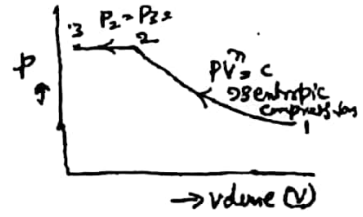


**PROB** A cylinder contains  $0.115 \text{ m}^3$  of gas at 1 bar and  $90^\circ\text{C}$ . The gas is compressed to a volume  $0.0288 \text{ m}^3$ , the final pressure being 5.67 bar. calculate (i) mass of the gas (ii) value of index of compression (iii) increase in internal energy of the gas (iv) heat transfer during compression.

After above compression, gas is to be cooled at constant pressure to its original temperature of  $90^\circ\text{C}$ , find work of compression required. Assume,  $\gamma = 1.4$  and  $R = 0.3 \text{ kJ/kg K}$ .

**SOL** Given,  $V_1 = 0.115 \text{ m}^3$ ,  $P_1 = 1 \text{ bar}$ ,  $T_1 = 90 + 273 = 363 \text{ K}$

$V_2 = 0.0288 \text{ m}^3$ ,  $P_2 = 5.67 \text{ bar}$ ,  $\gamma = 1.4$ ;  $R = 0.3 \text{ kJ/kg K}$ .



(i) mass of the gas (m) we know,  $P_1 V_1 = mRT$

$$m = \frac{P_1 V_1}{RT_1} = \frac{1 \times 10^5 \times 0.115}{0.3 \times 1000 \times 363} = 0.1056 \text{ kg}$$

(ii) Value of index of compression (n):

$$P_1 V_1^n = P_2 V_2^n$$

$$\therefore n = \frac{\log_e P_2/P_1}{\log_e V_1/V_2} = \frac{\log_e \frac{5.67}{1}}{\log_e \frac{0.115}{0.0288}} = 1.253$$

(iii) Increase in internal energy ( $\Delta U$ )

For polytropic compression,  $T_2/T_1 = (V_1/V_2)^{\gamma-1}$ ;  $T_2 = T_1 (V_1/V_2)^{\gamma-1} = 363 \left[ \frac{0.115}{0.0288} \right]^{1.4-1} = 515.17 \text{ K}$

$$\therefore \Delta U = m C_v [T_2 - T_1] = m \frac{R}{\gamma-1} (T_2 - T_1) = 0.1056 \times \frac{0.30}{1.4-1} [515.17 - 363] = 12.05 \text{ kJ}$$

(iv) Heat Transfer (Q)

$$Q = \Delta U + W; W = \frac{P_1 V_1 - P_2 V_2}{\gamma-1} = \frac{1 \times 0.115 - 5.67 \times 0.0288}{1.253-1} \times 10^5 = -19.05 \text{ kJ}$$

Work done on system

$$\therefore Q = 12.05 - 19.05 = -7 \text{ kJ}$$

(v) After compression, it is required to cool gas to original temp. of  $90^\circ\text{C}$ ,  $T_3 = T_1$

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_1}; P_2 = P_3; V_3 = V_2 \times \frac{T_1}{T_2} = 0.0288 \times \frac{363}{515.17} = 0.0202 \text{ m}^3$$

Additional work required,  $W = P_2 [V_3 - V_2] = 5.67 \times 10^5 [0.0202 - 0.0288] = -4.87 \text{ kJ}$

**PROB (2)** Define the first laws of Thermodynamics and discuss why it is known as law of conservation of energy.

**SOL** First law of Thermodynamics states that "heat and work are mutually convertible".

It can also be stated as whenever a system undergoes a cyclic process, the net heat supplied to the system is equal to the net work done by the system. Mathematically,  $\oint \delta Q = \oint \delta W$ .

The above statement valid for system undergoing cycles.

For a system undergoing change of state where both heat and work transfer takes place, the net energy is stored in the system in the form of internal energy.

Mathematically,  $Q - W = \Delta U$  where  $Q$  = Heat supplied to system,  $W$  = Work done by system,  $\Delta U$  = change in internal energy.

Therefore, energy is conserved in this operation, it leads to a conclusion that, First law of thermodynamics is just another statement of law of conservation of energy and perfectly goes by the statement that "the energy neither be created nor destroyed but changes one form to another form."

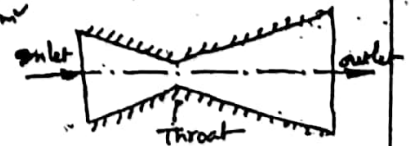
**PROB** A nozzle receives  $1200 \text{ kg/hr}$  of steam at  $1.3 \text{ MN/m}^2$ , specific volume  $142 \text{ lit/sec kg}$ , internal energy  $4400 \text{ kJ/kg}$  and negligible speed. At exit, the pressure is  $100 \text{ kN/m}^2$ , specific volume,  $1660 \text{ lit/kg}$  and internal energy  $2200 \text{ kJ/kg}$ . calculate exit velocity of steam.

**SOL** mass of steam,  $m = 1200 \text{ kg/hr}$ ;  $P_1 = 1.3 \text{ MN/m}^2 = 1.3 \times 10^6 \text{ N/m}^2$

$$v_{s1} = 142 \text{ lit/sec kg} = 142 \times 10^{-3} \text{ m}^3/\text{kg}; u_1 = 4400 \text{ kJ/kg}$$

$$\text{similarly, } P_2 = 100 \text{ kN/m}^2 = 100 \times 10^3 \text{ N/m}^2; v_{s2} = 1660 \text{ lit/kg}$$

$$u_2 = 2200 \text{ kJ/kg}$$



**SPEE** for nozzle,  $g(h_1 - h_2) = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$

$$\therefore V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$$

$$\therefore \text{since } h_1 = u_1 + P_1 v_{s1} = 4400 \times 10^3 + 1.3 \times 10^6 \times 142 \times 10^{-3} = 4584600 \text{ J/kg}$$

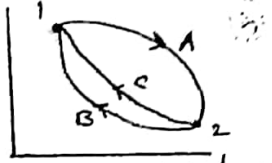
$$h_2 = u_2 + P_2 v_{s2} = 2200 \times 10^3 + 100 \times 10^3 \times 1660 \times 10^{-3} = 2366000 \text{ J/kg}$$

Exit velocity;  $V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(4584600 - 2366000)} = 2106 \text{ m/sec}$

**Prob**

Show that energy is a property of the system.

pressure



**Sol**

Consider a system undergoing a change of state 1 to 2 along path A.  $\uparrow$   
Thus complete cycle 1-A-2-B-1, Applying first law for cyclic process,

$$\int_1^2 \delta Q_A + \int_2^1 \delta Q_B = \int_1^2 \delta W_A + \int_2^1 \delta W_B ; \int_1^2 \delta Q_A - \int_1^2 \delta Q_B = \int_1^2 \delta W_A - \int_1^2 \delta W_B, \text{ cyclic process}$$

By re-arranging, we get

$$\int_1^2 \delta Q_A - \int_1^2 \delta W_A = \int_1^2 \delta Q_B - \int_1^2 \delta W_B ;$$

$$\therefore \Delta E_A = \Delta E_B$$

$$\text{Similarly } \Delta E_A = \Delta E_C$$

The quantity  $Q_A - W_A = \Delta E$   
The change in energy does not depend on path followed but only depends on state 1 and 2.

**Prob**

A closed system undergoes a reversible process at a constant pressure of 3.5 bar and its volume changes from 0.15 m<sup>3</sup> to 0.06 m<sup>3</sup>. 25 kJ of heat is rejected by system during the process. Determine the change in internal energy of the system.

**Sol**

pressure,  $p_1 = p_2 = 3.5 \text{ bar} = 3.5 \times 10^5 \text{ N/m}^2$ ;  $V_1 = 0.15 \text{ m}^3$ ,  $V_2 = 0.06 \text{ m}^3$

Heat rejected by system,  $Q = -25 \text{ kJ} = -25 \times 10^3 \text{ J}$

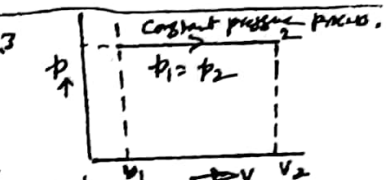
$$\text{Workdone, } W_{1-2} = p[V_2 - V_1] = 3.5 \times 10^5 [0.06 - 0.15]$$

$$= -31,500 \text{ Joules.}$$

According to First law

$$Q = \Delta U + W ; -25 \times 10^3 = \Delta U + (-31,500)$$

$$\therefore \Delta U = 6,500 \text{ Joules}$$



Work is done on system because compression process, volume decreases.

**Prob**

What do you understand by thermometric property and thermometric substance?

**Sol**

The measurement of temperature depends on establishment of thermal equilibrium between a system and the device used to measure the temperature.

Thermometric property:- The sensing device should have at least one measurable physical property, that changes with change in temp. Such a property called Thermometric property.

Thermometric substance:- The substance which shows changes in thermometric property is called Thermometric substance, at & the reference body like thermocouple, thermometer.

S.No.	Physical Property	Thermometric substance (Application)
1.	Pressure, P	Constant volume gas thermometer
2.	Volume, V	Constant pressure gas thermometer
3.	Resistance, R	Electrical resistance thermometer
4.	Thermal emf, E	Thermo-couple
5.	Length, l	Mercury-in-glass thermometer

**Prob**

What are the different scales of temperature? Establish a relationship between centigrade scale & Fahrenheit scale.

**Sol**

Temperature Scale:- To measure the temp. of a system, some numerical values are assigned on the thermometer, these numerical values on thermometer together called temp. scale.

Scale of temperature between two fixed points is one is melting point and other boiling point at 1 atmosphere.

$$t = a \cdot t_b$$

There are 4 different scales: (i) Centigrade scale (ii) Fahrenheit scale (iii) Absolute Kelvin scale (iv) Reaumur scale

(i) Centigrade (or) Celsius scale:- On this, melting point of ice marked as 0°C and boiling point of water at 100°C, under atmospheric pressure. It is divided into 100 equal parts, denoted by °C.

$$t = a \cdot t_b \quad \text{--- (1)}$$

$$0 = a \cdot 1 + b \quad \text{--- (2)}$$

$$100 = a \cdot 100 + b \quad \text{--- (3)}$$

$$\text{From (2) \& (3), } a = \frac{100}{100-1} ; b = -a \cdot 1 = \frac{-100 \cdot 1}{100-1}$$

$$\therefore t^\circ\text{C} = a \cdot t_b = \frac{100 \cdot t_b}{100-1} - \frac{100 \cdot 1}{100-1} = \frac{100(t_b - 1)}{(100-1)} \checkmark$$

(ii) Fahrenheit scale (°F): 32°F & 212°F are ice point & steam point respectively.

$$t = a \cdot t_b \quad \text{--- (1)}$$

$$32 = a \cdot 1 + b \quad \text{--- (2)}$$

$$212 = a \cdot 100 + b \quad \text{--- (3)}$$

$$\text{From (2) \& (3), } a = \frac{180}{100-1} ; b = 32 - \frac{180 \cdot 1}{100-1}$$

$$t^\circ\text{F} = a \cdot t_b = \frac{180 \cdot t_b}{100-1} + 32 - \frac{180 \cdot 1}{100-1} = \frac{180(t_b - 1)}{(100-1)} + 32 \checkmark$$

$$\text{Relation between } ^\circ\text{C} \text{ \& } ^\circ\text{F} \text{ are ; } t^\circ\text{F} = 180 \frac{(t^\circ\text{C} - 1)}{100-1} + 32 = 180 \times \frac{t^\circ\text{C}}{100} + 32 = \frac{9}{5} t^\circ\text{C} + 32 \checkmark$$

**Prob**

What is Mechanical equivalent of heat?

**Sol**

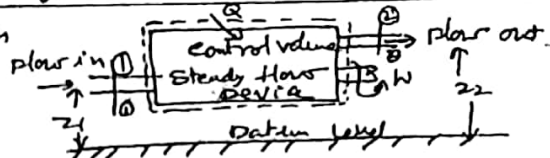
Joule's experiment concluded that there exists a relationship between heat and work, and both are two forms of energy. Joule conducted experiments with different amounts of work with various systems, and concluded that the work input is always proportional to the heat transferred from the system.  $W \propto Q$  (or)  $W = JQ$  where, J = constant of proportionality called Joule's constant.

Sign S.I units ; J = 1



**PROB** Derive the expression for steady flow energy equation

**SOL** Consider an open system through which the working substance flows at a steady rate as shown in figure. The working



substance enters at section ① and leaves system at section ②.

- Let  $p_1$  = pressure of working substance entering the system,  $N/m^2$
- $v_{s1}$  = specific volume " " " " ,  $m^3/kg$
- $V_1$  = velocity of working substance " " " " ,  $m/sec$
- $u_1$  = specific internal energy of working substance at inlet,  $J/kg$
- $z_1$  = Height above datum level for inlet in metres.

Similarly  $p_2, v_{s2}, V_2, u_2$  &  $z_2$  are corresponding values at exit

- Let  $Q_{1-2}$  = Heat supplied into system  $J/kg$
- $W_{1-2}$  = Work delivered by the system,  $J/kg$ .

Total energy of the system entering per kg working substance = Total energy per kg at exit

$$e_1 = e_2$$

$$\frac{u_1 + p_1 v_{s1}}{m_1} + \frac{V_1^2}{2} + g z_1 + Q_{1-2} = \frac{u_2 + p_2 v_{s2}}{m_2} + \frac{V_2^2}{2} + g z_2 + W_{1-2}$$

$$\therefore Q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

**PROB** A blower handles 1 kg/sec of air at 20°C and consumes 1.5 kW. If inlet and outlet velocities of air are 100 m/sec and 150 m/sec respectively. Find the exit air temperature. Assume adiabatic conditions ( $Q=0$ )

**SOL** Power consumed,  $W = -15 \text{ kW} (-15000 \text{ J/sec})$

SPEE,  $Q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$

$$\frac{Q_{1-2}}{\text{mass}} - \frac{W_{1-2}}{\text{mass}} = (h_2 - h_1) + \frac{150^2 - 100^2}{2}$$

$$- (-15) \times 10^3 = (h_2 - h_1) + \frac{150^2 - 100^2}{2}$$

$$\therefore (h_2 - h_1) = m c_p [T_2 - T_1] = 8750 \text{ J/kg}$$

$$= 1 \times 1.005 \times 10^3 [T_2 - (20 + 273)] = 8750$$

Exit Air temp:  $T_2 = 301.706 \text{ K (or) } 28.7^\circ\text{C}$

**PROB** Explain the working of constant volume gas thermometer.

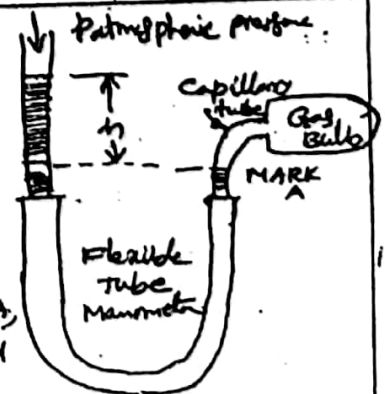
**SOL** The thermometer consists of (i) bulb consists of fixed mass of ideal gas (ii) capillary tube (iii) U-tube manometer.

During operation, the bulb gas bulb communicates with a constant temp. bath. Due to heat exchange between bath and bulb, heat transfer to the gas in bulb takes place and gas will expand and pushes mercury below Mark A. By adjusting flexible tube up to Mark A, the difference in level of mercury in two limbs (h) recorded

$$p = p_{atm} + \rho g h$$

Firstly, the gas bulb is placed at triple point temperature (273.15 K) and  $p_{tp}$  is calculated. [ $p_{tp} = p_{atm} + \rho g h_{tp}$ ]. Now, the bulb is brought in contact with a system whose temp (T) is to be measured and measure p.

$$\therefore \text{The new temperature, } T = 273.15 \times \frac{p}{p_{tp}}$$



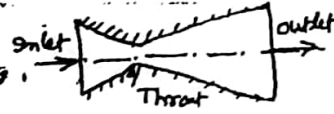


2. (a) Define First law of Thermodynamics and What are the limitations of first law of thermodynamics?  
 (b) A nozzle receives 1200 kg/hr of steam at 1.3 MN/m<sup>2</sup>, specific volume 142lit/kg, internal energy 4400 kJ/kg and negligible speed .At the exit ,the pressure is 100 kN/m<sup>2</sup>,specific volume 1660 lit/kg and internal energy 2200 kJ/kg. Calculate the exit velocity of the steam.

**SOL (a)** First law states that "heat and work are mutually convertible". It can be also stated that as whenever a system undergoes a cyclic process, the net heat supplied to system is equal to net work done by system. Mathematically,  $\oint \delta Q = \oint \delta W \rightarrow$  The statement valid for systems undergoing cyclic processes. For systems undergoing change of state (or process) where heat and work transfer takes place, the net energy ( $\Delta U$ ) is stored in the form of internal energy. Mathematically,  $Q - W = \Delta U$ . Where,  $Q =$  Heat supplied to the system,  $W =$  work done by system,  $\Delta U =$  change in internal energy. Therefore, energy is conserved [Converted from Q to W]. Therefore, it is called Law of Conservation of energy.

**LIMITATIONS OF FIRST LAW:** ① Limitation on conversion of one form of energy to other form ② According to 1st law, no restriction on direction of flow of work and heat, which is not true in reality ③ Work can be completely into heat but reverse is not possible completely. ④ Heat can not flow from cold body to hot body without external work. ⑤ Gas expands from high pressure to low pressure. But reverse is not automatically true.

**SOL (b)** Given,  $m = 1200 \text{ kg/hr}$ ,  $p_1 = 1.3 \text{ MN/m}^2 = 1.3 \times 10^6 \text{ N/m}^2$   
 Specific volume,  $v_{s1} = 142 \text{ lit/kg} = 142 \times 10^{-3} \text{ m}^3/\text{kg}$ ,  $u_1 = 4400 \text{ kJ/kg} = 4400 \times 10^3 \text{ J/kg}$   
 Similarly,  $p_2 = 100 \text{ kN/m}^2 = 100 \times 10^3 \text{ N/m}^2$ ,  $v_{s2} = 1660 \times 10^{-3} \text{ m}^3/\text{kg}$ ,  $u_2 = 2200 \times 10^3 \text{ J/kg}$ .  
 According to SFEE,  $q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$ ;  $V_2 = \sqrt{V_1^2 + 2(h_1 - h_2)}$   
 Since  $h_1 = u_1 + p_1 v_{s1} = 4400 \times 10^3 + 1.3 \times 10^6 \times 142 \times 10^{-3} = 4584600 \text{ J/kg}$ ,  $h_2 = u_2 + p_2 v_{s2} = 2200 \times 10^3 + 100 \times 10^3 \times 1660 \times 10^{-3} = 2366000 \text{ J/kg}$ .  
 $\therefore$  Exit velocity,  $V_2 = \sqrt{2(h_1 - h_2)} = \sqrt{2(4584600 - 2366000)} = 2106 \text{ m/sec}$ .



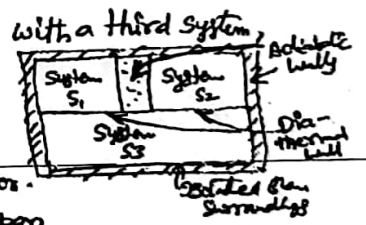
2. (a) What is the Zeroth law of Thermodynamics? Explain how it is applied for the measurement of temperature.

(b) Define the thermometric property? Explain the working of constant volume thermometer.

(c) Establish the correlation between Centigrade and Fahrenheit scale

**SOL (c)**

It states that when two systems are in thermal equilibrium with a third system, then they themselves have thermal equilibrium with each other. If systems  $S_1$  and  $S_2$  are individually in thermal equilibrium with a third system  $S_3$ , then systems  $S_1$  &  $S_2$  also in equilibrium with each other.



Measurement of Temperature

The measurement of temperature depends upon

establishment of thermal equilibrium between a system and a the sensing device to measure temperature.

**SOL (b)**

**THERMOMETRIC PROPERTY:** The sensing device should have atleast one measurable property that changes with change in temperature called thermometric property.

Ex- Change in Dimension, change in electrical resistance, Thermoelectric emf, change in length of

Constant Volume Gas Thermometer consists of (i) Bulb consists of fixed mass of ideal gas.

(ii) Capillary tube (iii) U-tube manometer.

During operation, Gas bulb communicates with constant temp bath.

Due to heat transfer, gas expands pushes mercury below MARK A.

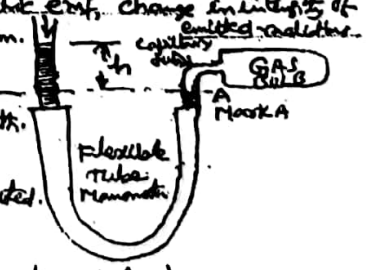
By adjusting flexible tube; mercury again up to MARK A, difference 'h' noted.

$p = p_{atm} + \rho g h$

Firstly, the gas bulb placed placed at Triple point (273.15K);  $p_{tp} = p_{atm} + \rho g h_{tp}$

Now, The gas bulb at unknown temp. bath, pressure measured,  $p = p_{atm} + \rho g h$ .

$\therefore$  New temperature,  $t = 273.15 \times \frac{p}{p_{tp}}$



**SOL (c)**

Centigrade Scale:

Melting point 0°C ; Boiling point 100°C at atmospheric pressure & water. It is divided into 100 equal parts.  
 $t = a_1 t + b$  ①;  $0 = a_1 + b$  ②;  $100 = a_2 + b$  ③; By solving ② & ③,  $a = \frac{100}{12-1}$ ;  $b = \frac{-100 \times 1}{12-1}$

Fahrenheit Scale:

Melting point 32° & Boiling point 212°F  
 $t = a_1 t + b$ ;  $32 = a_1 + b$  ④;  $212 = a_2 + b$  ⑤; By solving,  $a = \frac{180}{212-32}$ ;  $b = 32 - \frac{180 \times 1}{212-32}$   
 $t = 180 \frac{(t-1)}{212-32} + 32$ ; Relation between °C & °F:  $t^{\circ F} = \frac{180}{100} t^{\circ C} + 32$

2. A cylinder contains  $0.115 \text{ m}^3$  of gas at 1 bar and  $90^\circ\text{C}$ . The gas is compressed to a volume  $0.0288 \text{ m}^3$ , the final pressure being 5.67 bar. Calculate

- (i) The mass of the gas
- (ii) The value of index of compression
- (iii) The increase in internal energy of the gas
- (iv) The heat transferred during the process

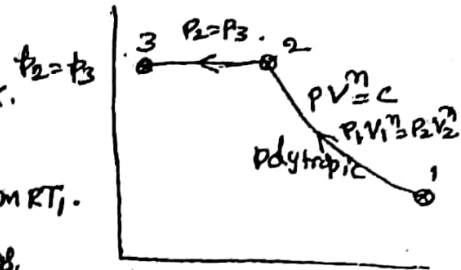
Take,  $\gamma = 1.4$ ;  $R = 0.3 \text{ kJ/kgK}$ .

$$C_V = \frac{R}{\gamma - 1}; C_P = \frac{\gamma R}{\gamma - 1}$$

If, after above compression, the gas is to be cooled at constant pressure to its original temperature of  $90^\circ\text{C}$ , find the work of compression required

**Sol:** Given, initial volume,  $V_1 = 0.115 \text{ m}^3$   
 initial pressure,  $p_1 = 1 \text{ bar}$ ,  $T_1 = 90 + 273 = 363 \text{ K}$ .

Final volume,  $V_2 = 0.0288 \text{ m}^3$ ,  $p_2 = 5.67 \text{ bar}$ .



(a) MASS OF THE GAS (m):- we know Equation of state,  $pV = mRT_1$ .

$$\therefore m = \frac{p_1 V_1}{RT_1} = \frac{(1 \times 10^5) \times 0.115}{(0.3 \times 1000) \times 363} = 0.104 \text{ kg}$$

(b) value of index of compression (n):  $p_1 V_1^n = p_2 V_2^n$ ;  $n = \frac{\log_e (p_2/p_1)}{\log_e (V_1/V_2)} = \frac{\log_e 5.67}{\log_e \frac{0.115}{0.0288}}$

(c) Increase in Internal energy:- ( $\Delta U$ )

we know,  $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$ ;  $T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{n-1} = 363 \left(\frac{0.115}{0.0288}\right)^{1.253-1} = 515.17 \text{ K}$ .

$\therefore \Delta U = \text{change in Internal energy} = m c_V (T_2 - T_1) = m \left(\frac{R}{\gamma - 1}\right) (T_2 - T_1)$   
 $= 0.1056 \times 0.3 \frac{(515.17 - 363)}{1.4 - 1} = 12.0 \text{ kJoule}$

(d) Heat Transfer (Q)

$Q = \Delta U + W$ ;  $W = \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{(1 \times 0.115 - 5.67 \times 0.0288) \times 10^5}{1.253 - 1} = -19,089 \text{ J} = -19.09 \text{ kJ}$   
 $= 12.05 - 19.08 = -7.03 \text{ kJ}$

AFTER COMPRESSION, IT IS REQUIRED TO COOL GAS TO ORIGINAL TEMPERATURE OF  $90^\circ\text{C}$  ( $T_3 = T_1$ )

$\therefore$  PROCESS (2-3)  $\frac{p_2 V_2}{T_2} = \frac{p_3 V_3}{T_3 = T_1} \Rightarrow p_2 = p_3$ ;  $\therefore V_3 = V_2 \frac{T_2}{T_1} = \frac{363 \times 0.0288}{515.17} = 0.0202 \text{ m}^3$ .

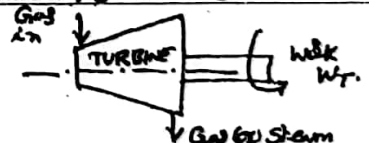
$\therefore$  Additional work required,  $W_{2-3} = p_2 [V_3 - V_2] = 5.67 \times 10^5 [0.0202 - 0.0288] = -4870 \text{ J} = -4.87 \text{ kJ}$  (-ve)

2. (a) Derive the steady flow energy equations for steam turbine and rotary compressor.

(b) The gas leaving the turbine jet engine flows steadily in to the jet pipe with enthalpy 800 kJ/kg and velocity 123 m/s. The exit from the pipe is at enthalpy 450 kJ/kg, and exhaust in line with intake. Neglect heat losses from the system determine the velocity gas leaving the pipe.

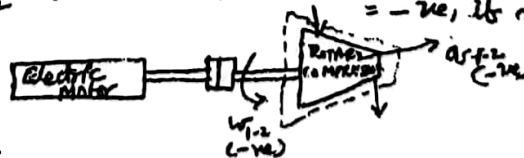
(a) SOLUTION:-

STEAM TURBINE:- In steam turbine, steam (or) gas is passed through turbine, converted into work. The steam leaves with low pressure and low temperature.



$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$ ; Here  $q_{1-2} = 0$ , if turbine is insulated = -ve, if not insulated.

ROTARY COMPRESSOR:-



A rotary compressor is a machine which compresses air (or) gas and supplies at high pressure in large quantity.

In compressor,  $q_{1-2} = -ve$  (heat rejected by system)

$W_{1-2} = -ve$  (work supplied to system).

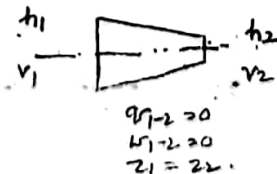
$q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \Rightarrow -q_{1-2} - (-W_{1-2}) = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2}$

(b) SOLUTION:- Given,  $h_1 = 800 \text{ kJ/kg} = 800 \times 10^3 \text{ J/kg}$ ,  $V_1 = 123 \text{ m/sec}$ ;  $h_2 = 450 \text{ kJ/kg} = 450 \times 10^3 \text{ J/kg}$ .

According to SFEE,  $q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$

$0 = 0 = (860 - 760) \times 10^3 + \frac{V_2^2 - 250^2}{2}$

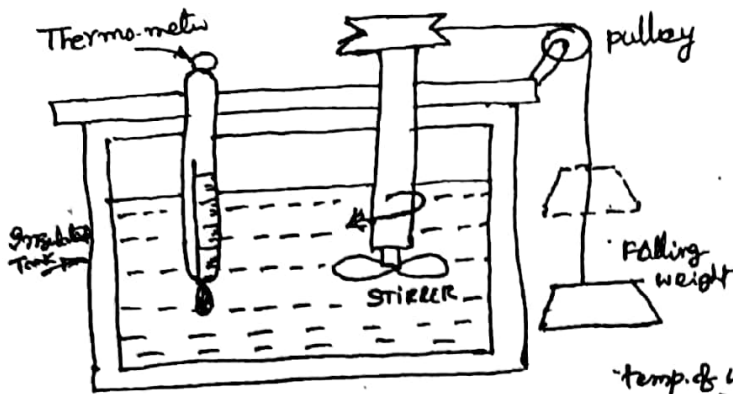
$\Rightarrow V_2 = 512.74 \text{ m/sec}$  ✓





## UNIT-2

Explain Joule's experiments with suitable sketches.



Joule conducted several experiments which led to the formulation of first law.

When weight is allowed to fall to a certain distance, work is done on water through rotation of stirrer. When stirrer rotates inside the water, heat is produced due to friction and is measured by rise in

temp. of water. Further, insulation from tank was removed and the whole system placed in water bath.

The heat transferred from the system in order to bring some initial conditions.

Whenever a closed system undergoes a cycle, the work input to the system is proportional to the net heat output.

$$\oint \delta W = \oint \delta Q \quad \text{or} \quad \oint \delta W = \oint \delta Q \quad T = 1 \text{ in SI units.}$$

**PROB.** Air flows steadily at the rate of 0.5 kg/sec through an air compressor, entering at 7 m/sec velocity, 100 kPa pressure and 0.95 m<sup>3</sup>/kg volume, and leaving at 5 m/s, 700 kPa and 0.19 m<sup>3</sup>/kg. The internal energy of air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in the compressor jacket absorbs heat from the air at the rate of 58 kW. Compute the rate of shaft work input to the air in kW and find

**SOL.** At inlet,  $m = 0.5 \text{ kg/sec}$   
 velocity  $V_1 = 7 \text{ m/sec}$   
 pressure,  $P_1 = 100 \text{ kPa} = 100 \times 10^3 \text{ Pa}$   
 specific volume,  $v_{s1} = 0.95 \text{ m}^3/\text{kg}$

At exit;  $v_2 = 5 \text{ m/s}$ ;  $P_2 = 700 \times 10^3 \text{ Pa}$ ,  $v_{s2} = 0.19 \text{ m}^3/\text{kg}$

$u_2$  is greater than  $u_1$  by 90 kJ/kg; so  $u_2 - u_1 = 90 \text{ kJ/kg}$ .

By applying SPEC;  $q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{1}{2} [v_2^2 - v_1^2] + g(z_2 - z_1)$

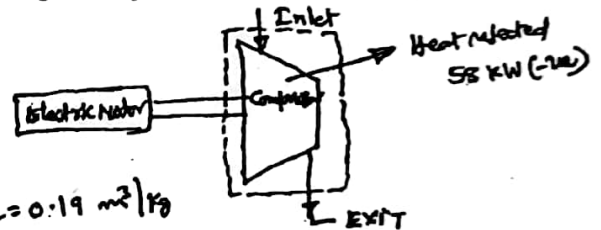
$$\text{here, } q_{1-2} = \frac{Q_{1-2}}{m} = \frac{-58 \times 10^3}{0.5} = -116 \times 10^3 \text{ J/kg (neg)}$$

$$\therefore q_{1-2} - w_{1-2} = (u_2 - u_1) + (P_2 v_{s2} - P_1 v_{s1}) + \frac{1}{2} [v_2^2 - v_1^2] + 0$$

$$-116 \times 10^3 - w_{1-2} = 90 \times 10^3 + (700 \times 10^3 \times 0.19 - 100 \times 10^3 \times 0.95) + \frac{1}{2} [5^2 - 7^2]$$

$$= 90 \times 10^3 + 38 \times 10^3 + (-12)$$

$$\therefore w_{1-2} = 243,988 \text{ W} = 243.98 \text{ kW}$$



**PROB.** What is steady flow process?

**SOL.** In steady flow process the flow rate of mass and energy don't vary with time and properties within the system are invariant with time at every point. The analysis of steady flow process is based on the principle of conservation of energy.

consider an open system through which the working substance flows at a steady rate as shown in figure. The working substance enters at section (1) and leaves system at section (2).



Let  $p_1$  = pressure of working substance entering the system,  $N/m^2$

$v_{s1}$  = sp. volume of working substance entering the system,  $m^3/kg$

$V_1$  = velocity at inlet,  $m/sec$

$e_1$  = specific internal energy at inlet,  $J/kg$

$z_1$  = Height above datum level at inlet in metres.

Similarly,  $p_2, v_{s2}, V_2, z_2, z_2'$  are corresponding values at exit.

Let  $q_{1-2}$  = Heat supplied into the system,  $J/kg$

$w_{1-2}$  = Work delivered by the system,  $J/kg$

Total energy of the system entering per kg of working substance

= total energy per kg at exit

$$e_1 = e_2$$

$$e_1 + p_1 v_{s1} + \frac{V_1^2}{2} + g z_1 + q_{1-2} = e_2 + p_2 v_{s2} + \frac{V_2^2}{2} + g z_2 + w_{1-2}$$

$$\therefore \boxed{q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)}$$

**PROB.** A turbo compressor delivers  $2.33 \text{ m}^3/\text{s}$  at  $0.276 \text{ MPa}$ ,  $43^\circ\text{C}$  which is heated at this pressure to  $430^\circ\text{C}$  and finally expanded in a turbine which delivers  $1860 \text{ kW}$ . During the expansion there is a heat transfer of  $0.09 \text{ MJ/s}$  to the surroundings. Calculate the turbine exhaust temperature if changes in KE and PE negligible.

**Soln**

pressure after compression,  $p_c = 0.276 \times 10^6 \text{ Pa}$

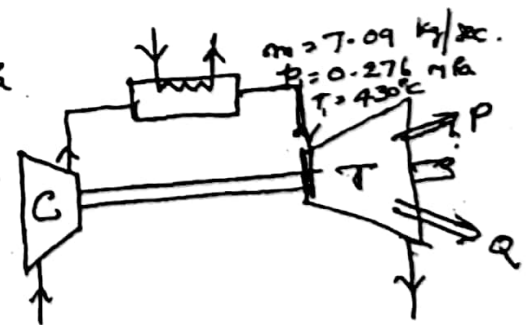
volume flow rate,  $V = 2.33 \text{ m}^3/\text{s}$

temperature,  $T = 43 + 273 = 316 \text{ K}$

$\therefore$  we know,  $pV = mRT$

$\therefore$  mass flow rate,  $m = \frac{pV}{RT}$

$$= \frac{0.276 \times 10^6 \times 2.33}{287 \times 316} = 7.09 \text{ kg/sec} \quad [\text{Take, } R = 287 \text{ J/kgK}]$$



power developed by turbine,  $P = 1860 \text{ kW}$  (given)

$$\text{power developed by turbine per unit mass, } w_{1-2} = \frac{(1860 \times 10^3)}{m} = \frac{1860 \times 10^3}{7.09} = 262.3 \times 10^3 \text{ J/kg}$$

Heat transfer from turbine,  $Q = 0.09 \text{ MJ/s}$

$$\text{Heat transfer from turbine per unit mass, } q_{1-2} = \frac{Q}{m} = \frac{0.09 \times 10^6}{7.09} = 12.64 \times 10^3 \text{ J/kg}$$

Take negative of  $q_{1-2} = -12.64 \times 10^3 \text{ J/kg}$

Apply SFEE to the turbine,

$$q_{T-2} - w_{T-2} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1) \quad [\because KE \& PE \text{ negligible}]$$

$$\therefore q_{T-2} - w_{T-2} = \dots cp [T_2 - T_1]$$

$$-12.69 \times 10^3 - 262.3 \times 10^3 = 1.005 \times 10^3 [T_2 - (43 + 273)]$$

$$-274.99 = 1.005 [T_2 - 316]$$

$$\therefore T_2 = -273.63 + 316 = \underline{42.37 \text{ K}} \checkmark$$

**PROB.** A nozzle is a device for increasing the velocity of a steadily flowing stream. At the inlet to a certain nozzle, the enthalpy of the fluid passing is  $3000 \text{ kJ/kg}$  and the velocity is  $60 \text{ m/s}$ . At the discharge end, the enthalpy is  $2762 \text{ kJ/kg}$ . The nozzle is horizontal and there is negligible heat loss from it. Find the velocity at exit from the nozzle. If the inlet area is  $0.1 \text{ m}^2$  and the specific volume at inlet is  $0.187 \text{ m}^3/\text{kg}$ . Find the mass flow rate.

**SOLUTION**

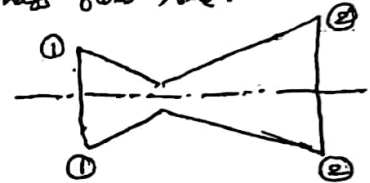
At inlet,  $h_1 = 3000 \text{ kJ/kg}$ ,  $v_1 = 60 \text{ m/s}$

At exit,  $h_2 = 2762 \text{ kJ/kg}$ ,  $v_2 = ?$

$\therefore$  Apply SFEE to the nozzle,  $q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{v_2^2 - v_1^2}{2} + g(z_2 - z_1)$

$$0 = (2762 - 3000) \times 10^3 + \frac{v_2^2 - 60^2}{2}$$

$$\therefore v_2 = \sqrt{238 \times 10^3 + (60)^2} = \sqrt{238000 + 3600} = 491.5 \text{ m/sec.}$$



mass flow rate,  $m = \frac{A_1 v_1}{v_{s1}} = \frac{A_2 v_2}{v_{s2}} = \frac{0.1 \times 60}{0.187} = 32 \text{ kg/sec.}$

**PROB.** During one cycle, the working fluid in an engine engages in two work interactions:  $15 \text{ kJ}$  to the fluid and  $44 \text{ kJ}$  from the fluid, and three heat interactions: two of which are known:  $75 \text{ kJ}$  to the fluid and  $40 \text{ kJ}$  from the fluid. Evaluate the magnitude and direction of third heat transfer?

**SOL.**  $w_{1-2} = -15 \text{ kJ}$ ;  $w_{2-3} = 44 \text{ kJ}$ ;  $w_{3-1} = 0$  (No work transfer).

$q_{1-2} = 75 \text{ kJ}$ ;  $q_{2-3} = -40 \text{ kJ}$ ;  $q_{3-1} = ?$  to find.

We know for cyclic process; Apply first law;  $\oint \delta w = \oint \delta q$ .

$$w_{1-2} + w_{2-3} + w_{3-1} = q_{1-2} + q_{2-3} + q_{3-1}; -15 + 44 = 75 + (-40) + q_{3-1}$$

$$\therefore q_{3-1} = -6 \text{ kJ} \text{ (Heat rejected by the fluid).}$$

**PROB.** Air at  $85 \text{ kPa}$  is a cylinder compressed isothermally from  $85 \text{ kPa}$ ,  $29^\circ\text{C}$  to  $380 \text{ kPa}$ . The initial volume is  $0.423 \text{ m}^3$ . Find heat transfer and change in entropy of air.

$$\therefore \text{Heat transfer, } q = \text{work required for compression} = p_1 v_1 \ln \frac{p_2}{p_1} = 85 \times 0.423 \ln \frac{380}{85} = -53.8 \text{ kJ} \text{ (Heat rejected)}$$

$$\text{change in entropy } (s_2 - s_1) = \frac{dq}{T} = \frac{p_1 v_1}{T_1} \ln \frac{p_2}{p_1} = \frac{85 \times 0.423}{(29 + 273)} \ln \frac{380}{85} = -0.178 \text{ kJ/kgK}$$

# Thermodynamic process table.

S.No:	Thermodynamic process	P-V-T relation	Quasi-static work	Change in internal energy	Heat Transfer	Change in Enthalpy	Change in Entropy
1	Constant volume process	$V_1 = V_2 = C$ (or) $\frac{P_1}{T_1} = \frac{P_2}{T_2}$	$W_{1-2} = 0$	$\Delta U = m C_V [T_2 - T_1]$	$Q = W + \Delta U$ $m C_V [T_2 - T_1]$	$m C_P [T_2 - T_1]$	$m C_V \ln \frac{T_2}{T_1}$
2	Constant pressure process	$P_1 = P_2 = C$ (or) $\frac{V_1}{T_1} = \frac{V_2}{T_2}$	$W_{1-2} = P [V_2 - V_1]$	$m C_P [T_2 - T_1]$	$m C_P [T_2 - T_1]$	$m C_P [T_2 - T_1]$	$m C_P \ln \frac{T_2}{T_1}$
3	Isobaric (or) constant temperature process	$T_1 = T_2 = C$ $P_1 V_1 = P_2 V_2$	$W_{1-2} = P_1 V_1 \ln \frac{V_2}{V_1}$ $= P_1 V_1 \ln \frac{P_2}{P_1}$	0	$P_1 V_1 \ln \frac{V_2}{V_1}$ $P_1 V_1 \ln \frac{P_1}{P_2}$	0	$m R \ln \frac{V_2}{V_1}$ $m R \ln \frac{P_1}{P_2}$
4	polytropic process	$P_1 V_1^n = P_2 V_2^n = C$ $\left(\frac{T_2}{T_1}\right)^{\frac{n}{\gamma-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{n}{\gamma-1}}$ $\left(\frac{V_2}{V_1}\right)^{\frac{n}{\gamma-1}} = \left(\frac{P_1}{P_2}\right)^{\frac{n}{\gamma-1}}$	$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ $= \frac{m R (\gamma - 1) T_1}{\gamma - 1}$	$m C_V [T_2 - T_1]$	$\frac{\gamma - 1}{\gamma - 1} \times W_{1-2}$	$m C_P [T_2 - T_1]$	$\frac{\gamma - 1}{\gamma - 1} m R \ln \frac{V_2}{V_1}$
5	Adiabatic process (or) isentropic process	$P_1 V_1^\gamma = P_2 V_2^\gamma = C$ $\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma}{\gamma-1}}$ $\left(\frac{V_2}{V_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma}{\gamma-1}}$	$W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$ $= \frac{m R (\gamma - 1) T_1}{\gamma - 1}$	$m C_V [T_2 - T_1]$	0	$m C_P [T_2 - T_1]$	0



# UNIT-1: PERFECT GASES

## PERFECT GAS:-

Gases which do not change their phase during thermodynamic process and obey Boyle's law, Charles law and characteristic gas equation, are known as ideal (or) perfect gases.

A gas can be modeled as an ideal gas when it has following features.

- 1) It has no intermolecular forces of attraction (or) repulsion
- 2) It does not change its phase during a thermodynamic process.
- 3) It obeys Boyle's, Charles laws and the characteristic gas equation.

The internal energy of gases decreases rapidly with decreasing pressure and disappears when absolute pressure approaches zero. At zero pressure, all real gases behave in a similar manner and the state of identical behaviour is called ideal state.

Ideal gases:- In actual practice, no gas is completely ideal gas, but many gases treated as an ideal gases like Air, Nitrogen, oxygen, hydrogen, helium, argon, neon, krypton, carbon dioxide.

Boyle's law:- volume (or) specific volume of a gas is inversely proportional to absolute pressure  
 $v \propto \frac{1}{p}$  (or)  $p v = \text{constant}$ ; If a gas changes its state

Charles law:- The specific volume is directly proportional to absolute temperature.  
 $v \propto T$  (or)  $\frac{v}{T} = C$ ; (or)  $\frac{v_1}{T_1} = \frac{v_2}{T_2}$  for a gas changes its state.

Equation of state:- It is observed that thermodynamic properties are interrelated. Any equation that relates the pressure, temperature and specific volume of a substance is known as an equation of state.

$f(p, v, T) = 0$  (or)  $p = f(v, T)$ ;  $v = f(p, T)$  (or)  $T = f(p, v)$ .

Characteristic gas equation:- In any thermodynamic system of an ideal gas, the pressure, temperature and specific volume vary simultaneously. On the basis of the characteristic, an equation is derived with the help of Boyle's and Charles laws.

Boyle's law  $v \propto \frac{1}{p}$  when  $T = \text{constant}$

Charles law,  $v \propto T$  when  $p = \text{constant}$

By combining both relations,  $v \propto \frac{T}{p}$  (or)  $v = \frac{R \cdot T}{p}$

(or)  $p v = R T$  where  $R = \text{characteristic (or) specific gas constant}$ .  
 $v = \text{specific volume}$ .

(or)  $P V = m R T \rightarrow V = \text{Volume of a gas}$ .

(or)  $P V = m R_u T \rightarrow$  where  $m = \text{No. of moles of a gas} = \frac{\text{mass}}{\text{molar wt}}$ .  
 $R_u = \text{Universal gas constant}$

\*  $R_u = 8314 \text{ J/kg mol} \cdot \text{K}$ .

$R_u = M \times R$ ;  $M = \text{molecular mass of any gas}$ .

The relationship between two principle specific heats and characteristic gas constant for

a perfect gas: Let us consider a perfect gas being heated from a temperature  $T_1$  to  $T_2$  at constant pressure.

From a non-flow equation  $Q = \Delta U + W$

$Q = (u_2 - u_1) + p d v$

$Q = (u_2 - u_1) + p [v_2 - v_1] ; m c_p (T_2 - T_1) = m c_v (T_2 - T_1) + \underbrace{m R (T_2 - T_1)}_{\text{Heat supplied}}$



$\therefore \boxed{C_p - C_v = R}$  on dividing both sides by  $C_v$ ,

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v} \quad ; \quad C_v = \frac{R}{\left(\frac{C_p}{C_v} - 1\right)} = \frac{R}{\gamma - 1}$$

similarly  $\frac{C_p}{C_p} = \frac{C_v}{C_p} = \frac{R}{C_p} \quad ; \quad 1 - \frac{1}{\gamma} = \frac{R}{C_p}$

$$\therefore \boxed{C_p = \frac{\gamma R}{(\gamma - 1)}} \quad ; \quad \therefore \boxed{\frac{C_p}{C_v} = \gamma}$$

Relationship between absolute temperature (T) and absolute pressure (P) in Adiabatic process ( $PV^\gamma = C$ )

The thermodynamic properties for a perfect gas are related as

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{and} \quad \frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1} \quad \text{--- (1)}$$

For an isentropic process,  $PV^\gamma = C$  and  $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$\frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}} \quad \text{--- (2)}$$

Substitute (2) in (1)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right) \left(\frac{V_2}{V_1}\right) = \left(\frac{P_2}{P_1}\right) \left(\frac{P_2}{P_1}\right)^{-\frac{1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{1 - \frac{1}{\gamma}} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}$$

$$\therefore \boxed{\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}}}$$

similarly ;  $P_1 V_1^\gamma = P_2 V_2^\gamma \quad ; \quad \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^\gamma \quad \text{--- (3)}$

Substitute (3) in (1)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right) \left(\frac{V_1}{V_2}\right) = \left(\frac{V_1}{V_2}\right)^\gamma \left(\frac{V_2}{V_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} = \left(\frac{V_1}{V_2}\right)^{\frac{\gamma - 1}{\gamma}}$$

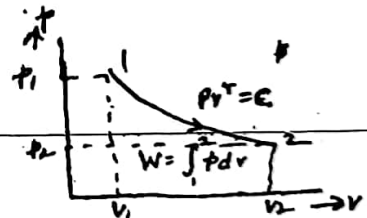
$$\therefore \boxed{\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{\gamma - 1}{\gamma}}}$$

Work done during Adiabatic process The work done during a

non-linear process is expressed as

$$W_{1-2} = \int_1^2 P dV = \int_1^2 \left(\frac{C}{V^\gamma}\right) dV = C \int_1^2 V^{-\gamma} dV$$

$$= C \left[ \frac{V^{-\gamma+1}}{-\gamma+1} \right]_1^2 = \frac{C \cdot V_2^{-\gamma+1} - C \cdot V_1^{-\gamma+1}}{-\gamma+1} = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1} = \frac{mR(T_2 - T_1)}{\gamma - 1}$$



Equations for change of entropy of an ideal gas:-

Let 1 kg of gas at a pressure  $P_1$ , volume  $V_1$ , absolute temperature  $T_1$  & Entropy  $S_1$  be heated to  $P_2, V_2, T_2$  &  $S_2$  respectively. By law of conservation of energy

$dq = du + dw = C_v dT + P dV$   
 Dividing on both sides by  $T$  ;  $\frac{dq}{T} = C_v \frac{dT}{T} + \frac{P dV}{T}$  [since  $\frac{PV}{T} = \frac{R}{C_v}$ ]

Integrating on both sides,  $\int_1^2 \frac{dq}{T} = C_v \int_1^2 \frac{dT}{T} + R \int_1^2 \frac{dV}{V}$

$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}} \rightarrow (T, V) \quad \text{--- (1)}$$

Similarly

$$S_2 - S_1 = C_v \ln \frac{P_2 V_2}{P_1 V_1} + R \ln \frac{V_2}{V_1}$$

$$= C_v \ln \frac{P_2}{P_1} + (C_v + R) \ln \frac{V_2}{V_1}$$

[  $\frac{P_2 V_2}{P_1 V_1} = \frac{P_2 V_2}{P_1 V_1} \cdot \frac{V_1}{V_1}$   
 $\frac{T_2}{T_1} = \frac{P_2 V_2}{P_1 V_1}$  ]

$$\boxed{S_2 - S_1 = C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1}} \rightarrow (P, V) \quad \text{--- (2)}$$

Similarly

$$\boxed{S_2 - S_1 = C_p \ln \frac{T_2}{T_1} + R \ln \frac{P_1}{P_2}} \rightarrow (T, P) \quad \text{--- (3)}$$



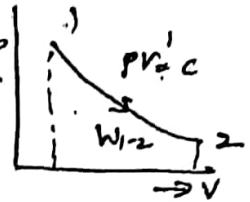
**PROB** 1.5 Kgs of air at pressure of 6.75 bar occupies a volume of 0.23 m<sup>3</sup>. If this air is expanded to a volume of 1.13 m<sup>3</sup>, find the workdone and heat absorbed (or) rejected by the air for each of the following methods of carrying out the process (i) isothermally (ii) Adiabatically.

**SOL** Mass of air,  $m = 1.5$  Kgs.

Initial pressure of air,  $P_1 = 6.75$  bar =  $6.75 \times 10^5$  Pa.

Initial volume of air,  $V_1 = 0.23$  m<sup>3</sup>.

Final volume of air,  $V_2 = 1.13$  m<sup>3</sup>.



(i) Isothermal process

To find final pressure,  $P_2$ ;  $P_1 V_1 = P_2 V_2$  for isothermal process.

$$P_2 = \frac{P_1 V_1}{V_2} = \frac{6.75 \times 10^5 \times 0.23}{1.13} = 1.37 \times 10^5 \text{ Pa.}$$

$$\text{Workdone, } W_{1-2} = P_1 V_1 \ln \left( \frac{V_2}{V_1} \right) = 6.75 \times 10^5 \times 0.23 \ln \left( \frac{1.13}{0.23} \right) = 247.14 \text{ kJ/kg.}$$

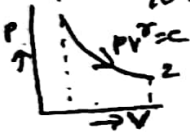
$$\text{Total workdone } W_{1-2} = \text{mass} \times W_{1-2} = 1.5 \times 247.14 = \frac{370.71 \text{ kJ}}{10^3}$$

$$\text{Heat transfer, } Q = W_{1-2} + \Delta U = W_{1-2} = 370.71 \text{ kJ} \quad [\because \Delta U = 0 \text{ for constant temperature process}]$$

(ii) Adiabatic process

To find final pressure,  $P_2$ ;  $P_1 V_1^\gamma = P_2 V_2^\gamma$

$$P_2 = P_1 \left( \frac{V_1}{V_2} \right)^\gamma = 6.75 \times 10^5 \left[ \frac{0.23}{1.13} \right]^{1.4} = 0.726 \times 10^5 \text{ bar.}$$



$$\text{Workdone, } W_{1-2} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{(6.75 \times 0.23 - 0.726 \times 1.13) \times 10^5}{1.4 - 1} = 183 \text{ kJ/kg.}$$

$$\text{Total workdone} = \text{mass of air} \times W_{1-2} = 1.5 \times 183 = 274.5 \text{ kJ}$$

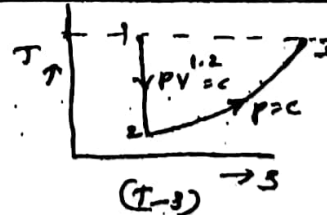
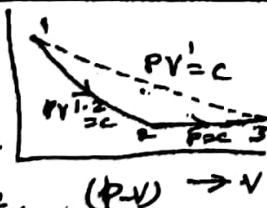
For an Adiabatic process,  $Q = 0$ .

**PROB.** A gas occupies 0.034 m<sup>3</sup> at 600 kPa and 85°C. It is expanded in the non-flow process according to the law  $PV^{1.2} = \text{constant}$  to a pressure of 60 kPa after which it is heated at constant pressure back to its original temperature. Sketch the process on P-V and T-s diagrams and calculate the whole process the workdone, the heat transferred.

Take,  $C_p = 1.047$  and  $C_v = 0.775$  kJ/kgK for the gas.

**SOL**

Take  $C_p = 1.047$  kJ/kgK  
 $C_v = 0.775$  kJ/kgK



Given,  
 $P_1 = 600 \text{ kPa} = 600 \times 10^3 \text{ Pa}$   
 $V_1 = 0.034 \text{ m}^3$   
 $T_1 = 85 + 273 = 358 \text{ K}$   
 $P_2 = 60 \text{ kPa} = 60 \times 10^3 \text{ Pa}$   
 $P_3 = P_2$   
 $T_3 = T_1 = 85^\circ\text{C} = 358 \text{ K}$

Process (1-2)  $PV^{1.2} = c$

$$P_1 V_1^{1.2} = P_2 V_2^{1.2}$$

$$V_2 = V_1 \left( \frac{P_1}{P_2} \right)^{1/1.2} = 0.034 \left[ \frac{600 \times 10^3}{60 \times 10^3} \right]^{1/1.2} = 0.2316 \text{ m}^3$$

$$W_{1-2} = \int P dV = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} = \frac{600 \times 10^3 \times 0.034 - 60 \times 10^3 \times 0.2316}{1.2 - 1} = 32,520 \text{ J} = 32.52 \text{ kJ}$$

$$Q_{1-2} = \frac{\gamma - 1}{\gamma} \times W_{1-2} = \frac{1.4 - 1.2}{1.4 - 1} \times 32.52 = 16.26 \text{ kJ.}$$

For process (1-2)  $\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}}$ ;  $T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} = 358 \left[ \frac{60 \times 10^3}{600 \times 10^3} \right]^{\frac{1.2 - 1}{1.2}} = 244 \text{ K.}$

For process (2-3)

$$\frac{P_2 V_2}{T_2} = \frac{P_3 V_3}{T_3}; \quad \frac{V_2}{T_2} = \frac{V_3}{T_3}; \quad V_3 = \frac{T_3}{T_2} \times V_2 = \frac{T_1}{T_2} \times V_2$$

$$V_3 = \frac{T_1}{T_2} \times V_2 = \frac{358}{244} \times 0.2316 = 0.3398 \text{ m}^3$$



Work done in the process (2-3) Constant pressure process

(\*)

$$W_{2-3} = P_2 [V_3 - V_2] = 60 \times 10^3 [0.3398 - 0.2315] = 6492 \text{ J} = \underline{6.492 \text{ kJ}}$$

$$Q_{2-3} = m C_p [T_3 - T_2] = 1 \times 1.047 \times [358 - 244] = \underline{119.35 \text{ kJ}}$$

$$\therefore \text{Total Work done, } W_{1-3} = W_{1-2} + W_{2-3} = 32.52 + 6.492 = \underline{39.01 \text{ kJ}}$$

$$\text{Total Heat transferred, } Q_{1-3} = Q_{1-2} + Q_{2-3} = 16.26 + 119.35 = \underline{135.61 \text{ kJ}}$$

**Prob** A gas in a cylinder fitted with a piston undergoes a cycle composed of three processes. First, the gas expands at constant pressure with a heat addition of 42 kJ and a work output of 12.0 kJ. Then, it is cooled at constant volume by a removal of 48 kJ of heat. Finally, an adiabatic process restores the gas to its initial state. Determine,

- The work of the adiabatic process
- The stored energy of the gas at each of the other two states if its stored energy in the initial state is assigned the value of zero.

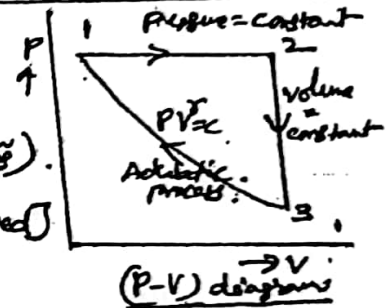
**Sol** Process (1-2) Constant pressure process :-

Heat addition,  $Q_{1-2} = 42.0 \text{ kJ}$  (+ve, heat added to system gas).

Work output,  $W_{1-2} = 12.0 \text{ kJ}$  (+ve, work done by system gas).

Process (2-3) constant volume process

Heat removed,  $Q_{2-3} = -48 \text{ kJ}$  [-ve, due to heat removed from system].



From process (1-2)  $Q_{1-2} = \Delta U_{1-2} + W_{1-2}$

$$\therefore \Delta U_{1-2} = Q_{1-2} - W_{1-2} = 42 - 12 = 30 \text{ kJ}$$

From process (2-3)

$$Q_{2-3} = \Delta U_{2-3} + W_{2-3} = \Delta U_{2-3} + 0 \quad [\because W_{2-3} = 0 \text{ for constant volume}]$$

$$\therefore Q_{2-3} = \Delta U_{2-3} = -48 \text{ kJ}$$

From process (3-1) Adiabatic work process

$$Q_{3-1} = 0 \text{ for adiabatic process}$$

For cyclic process  $\sum F_n$  Three thermodynamic processes 1-2, 2-3, 3-1

$$\oint \delta W = \oint \delta Q$$

$$W_{1-2} + W_{2-3} + W_{3-1} = Q_{1-2} + Q_{2-3} + Q_{3-1}$$

$$12.0 + 0 + W_{3-1} = 42 + (-48.0) + 0$$

$$\therefore W_{3-1} = \text{Adiabatic work} = -18 \text{ kJ} \quad [\text{work is done on the system, compression process}]$$

From process (3-1)

$$Q_{3-1} = W_{3-1} + \Delta U_{3-1}$$

$$\therefore \Delta U_{3-1} = Q_{3-1} - W_{3-1} = 0 - (-18) = 18 \text{ kJ}$$

From process (2-3)

$$Q_{2-3} = W_{2-3} + \Delta U_{2-3}$$

$$\Delta U_{2-3} = -48 \text{ kJ}$$

Since,  $U_1 = 0$  (given) ;  $\therefore U_2 = U_1 + 30 \text{ kJ} ; U_2 = 30 \text{ kJ}$

$$\Delta U_{2-3} = U_3 - U_2 = -48 \text{ kJ} ; U_3 = -48 + U_2 = -48 + 30 = -18 \text{ kJ}$$

Real Gases:- The ideal gas equation of state,  $pV = RT$  can be used with the assumption of no attraction (or) very little attraction force of molecules with in the gas and the volume of molecules is negligibly small compared to volume of gas.

For many gases, at very low pressure & high temperature, the force of attraction and volume of molecules compared to volume of gas are small and real gases obey very close to ideal gas equation.

But at higher pressure, the forces of attraction and volume of molecules are higher (appreciable). Then, the real gas behaviour deviates from ideal-gas behaviour.

Compressibility factor (Z)

The slight modification in the ideal gas equation ( $pV = RT$ ) of state to fit real-gas behaviour with introduction of a correction factor (Z) is called compressibility factor (Z). It is defined as

$$Z = \frac{pV}{RT} = \frac{\text{Actual volume of the gas}}{\text{Volume as predicted by ideal gas equation}} = \frac{V}{\left(\frac{RT}{P}\right)} = \frac{pV}{RT}$$

For an ideal gas,  $Z = 1$ .

For a real gas Z is a function of pressure and temperature.

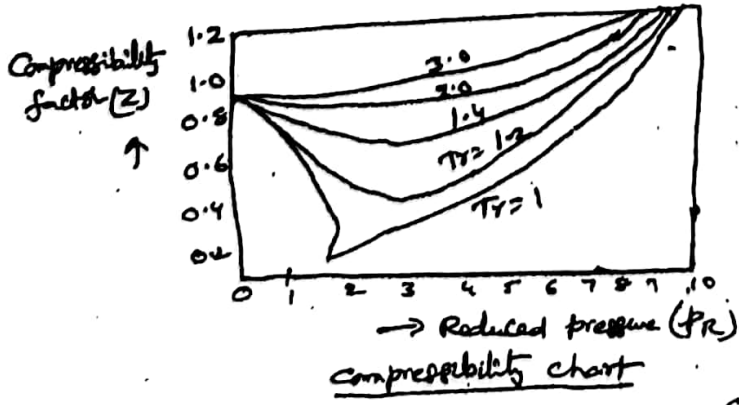
The real gases behave differently at a given temperature and pressure, but they behave very closely at their reduced pressure and reduced temperature.

Reduced pressure,  $P_R = \frac{P_{\text{actual}}}{P_{\text{critical}}} = \frac{P}{P_c}$

Reduced temperature,  $T_R = \frac{T_{\text{actual}}}{T_{\text{critical}}} = \frac{T}{T_c}$

Reduced specific volume,  $V_R = \frac{V_{\text{actual}}}{V_{\text{critical}}} = \frac{V}{V_c}$

Disadvantage of Compressibility chart factor chart:- That a separate chart is needed for each gas. But at the same reduced co-ordinates, the compressibility factor Z is approximately same for all gases. It is called law of corresponding states. Thus  $Z_R = f(P_R, T_R)$ .



→ Reduced pressure (P\_R)  
compressibility chart

Compressibility factor (Z)

$Z = \frac{pV}{RT}$  for ideal gas

$$Z = \frac{P_c V_c}{RT_c} \times \frac{P_2 V_2}{T_2} = Z_c \times \frac{P_2 V_2}{T_2} \rightarrow \text{for real gas.}$$

This chart is useful when the properties of the gas at critical point are known.

The deviation of the real gases from ideal gases can be known by this chart. Accuracy is  $\pm 5\%$  by this chart. If the compressibility factor Z is less than unity, the gas is more compressible than perfect gas.



## Vander Waal's Equation of State:-

Vander Waal's Equation:- In 1873, J.D. Vander Waal's presented an equation of state which was of interest on physical reasoning, introduced two correction constants in the equation of ideal gas equation of state.

$$\left(p + \frac{a}{\bar{v}^2}\right) (\bar{v} - b) = R_u T \quad \text{where } \bar{v} = \text{molar volume in } \text{m}^3/\text{kg mol}$$

$$R_u = \text{universal gas constant} = 8.314 \text{ kJ/kg mol K.}$$

The constant 'a' was introduced to account for the existence of intermolecular attraction, the constant 'b' was introduced to account for volume of molecules and  $R_u$  is universal gas constant. These constants are evaluated from the behaviour of the gas at critical point.

$$\frac{R_u T_c}{p_c \bar{v}_c} = \frac{8}{3}$$

Except at higher pressures, the real gases do not obey Vander Waal's equation in all range pressures and temperatures. Despite of its limitations, the Vander Waal's equation has historical importance because it was the first attempt to model the behaviour of real gases.

**PROB** Determine the pressure of air at  $190^\circ\text{C}$  having a specific volume of  $0.00295 \text{ m}^3/\text{kg}$  by means of  
 (1) Ideal gas equation (2) Vander Waal's equation  
 Take,  $a = 135522 \text{ N}\cdot\text{m}^4/(\text{kg}\cdot\text{mole})^2$ ,  $b = 0.0362 \text{ m}^3/\text{kg}\cdot\text{mole}$ ,  $R = 8314 \text{ J/kg}\cdot\text{mol}\cdot\text{K}$ .

**SOL** Let:  $p =$  pressure of air  
 $T =$  Temperature of air  $= 190^\circ\text{C} = 190 + 273 = 463 \text{ K}$   
 $\bar{v} =$  specific volume of air  $0.00295 \text{ m}^3/\text{kg}$   
 $R_u =$  Universal gas constant of air  $= 8314 \text{ J/kg}\cdot\text{mol}\cdot\text{K}$

Vander Waal's equation constants are  
 $a = 135522 \text{ N}\cdot\text{m}^4/(\text{kg}\cdot\text{mole})^2$ ;  $b = 0.0362 \text{ m}^3/\text{kg}\cdot\text{mole}$ .

Take molecular weight of air,  $M = 29$ .

Ideal gas equation  $pV = mRT$ . Perfect gas equation.

$$pV = mRT$$

$$\Rightarrow p\bar{v} = RT \quad [\text{where, } \bar{v} = \text{specific volume} = \text{volume/mass}]$$

$$p\bar{v} = \left(\frac{R_u}{M}\right) \times T \quad [\text{where } R_u = M \times R; M = \text{molecular mass of gas}]$$

$$\therefore p = \frac{R_u \times T}{\bar{v} \times M} = \frac{8314 \times 463}{29 \times 0.00295} = \frac{44.996 \times 10^6 \text{ N/m}^2}{1.05} = 449.9 \text{ bar}$$

Vander Waal's equation:-

$$\left(p + \frac{a}{\bar{v}^2}\right) (\bar{v} - b) = R_u T \quad [\bar{v} = \bar{v} \times M = 0.00295 \times 29]$$

$$\therefore p = \frac{R_u \cdot T}{\bar{v} - b} - \frac{a}{\bar{v}^2} = \left[ \frac{8314 \times 463}{0.00295 \times 29 - 0.0362} - \frac{135522}{(0.00295 \times 29)^2} \right]$$

$$= 780 \times 10^5 - 180 \times 10^5 = 595 \times 10^5 \text{ Pa}$$

$$\therefore \text{pressure, } p = \frac{595 \times 10^5}{10^5} = 595 \text{ bar} \checkmark$$



**PROB** Determine the pressure of 2 moles of air at 400°C with a total volume of 0.5 m<sup>3</sup>. Use the ideal gas law and Vander Waals law.

**SOL:** Let  $p$  = pressure of air ;  $n$  = No. of moles of air = 2  
 $T$  = Temperature of air = 400°C = 400 + 273 = 673 K ;  $V$  = total volume = 0.5 m<sup>3</sup>  
 $R_u$  = universal gas constant of air = 8314 J / kg-mole K.  
 Vander Waals equation constants for air

Take constants as Assumptions:  
 $a = 1.368 \text{ bar} \left[ \frac{\text{m}^3}{\text{kg-mole}} \right]^2$  ;  $b = 0.0367 \text{ m}^3 / \text{kg-mole}$   
 $= 1.368 \times 10^5 \text{ Pa} \left[ \frac{\text{m}^3}{\text{kg-mole}} \right]^2$

Ideal gas equation  $pV = n \cdot R_u T$  ✓

pressure of air,  $p = \frac{n \cdot R_u \cdot T}{V} = \frac{2 \times 8314 \times 673}{0.5} = 22,381,288 \text{ N/m}^2$

$p = \frac{22,381,288}{10^5} = 223.81 \text{ bar}$

Vander Waals equation:

$\left( p + \frac{a}{v^2} \right) (v - b) = R_u T$

where  $\bar{v} = \frac{V(\text{total volume})}{n(\text{No. of moles})} = \frac{0.5}{2} = 0.25 \text{ m}^3 / \text{kg-mole}$

∴ pressure,  $p = \frac{R_u \cdot T}{(v - b)} - \frac{a}{v^2} = \frac{8314 \times 673}{(0.25 - 0.0367)} - \frac{1.368 \times 10^5}{(0.25)^2}$   
 $= 26,232,170 - 2,188,800$   
 $= 24,043,370 \text{ N/m}^2$   
 $= 240.43 \text{ bar} ✓$

**PROB** Find the molecular weight and gas constant for the gas whose specific heats are as follows.  $C_p = 1.967 \text{ kJ/kgK}$ ,  $C_v = 1.507 \text{ kJ/kgK}$ .

**SOL:** Given that,  $C_p = 1.967 \text{ kJ/kgK}$  and  $C_v = 1.507 \text{ kJ/kgK}$ .

We know, Gas constant,  $R = C_p - C_v = 1.967 - 1.507 = 0.46 \text{ kJ/kgK}$ .

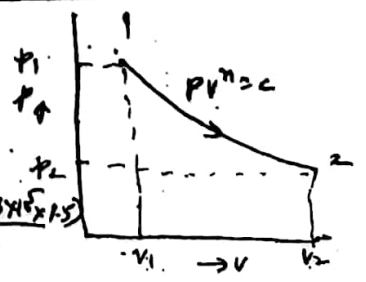
We know universal gas constant,  $R_u = 8.314 \text{ kJ/kgK}$

∴ Molecular weight of the gas,  $M = \frac{R_u}{R} = \frac{8.314}{0.46} = 18.07 \text{ kg/kg-mole} ✓$

**PROB.** 0.3 m<sup>3</sup> of air at pressure 8 bars expands to 1.5 m<sup>3</sup>. The final pressure is 1.3 bar. Assuming the expansion to be polytropic, calculate the heat supplied and change of internal energy. Take,  $\gamma = 1.4$

**SOL:** We know for polytropic expansion process,  $p v^n = c$   
 $p_1 v_1^n = p_2 v_2^n$  ;  $\frac{p_1}{p_2} = \left( \frac{v_2}{v_1} \right)^n$  ;  $n = \frac{\log_e p_1/p_2}{\log_e v_2/v_1}$   
 $\therefore n = \frac{\log_e 8/1.3}{\log_e 1.5/0.3} = 1.129$

(i) Heat supplied,  $Q = \frac{\gamma - n}{\gamma - 1} \times \frac{p_1 v_1 - p_2 v_2}{n - 1} = \frac{(1.4 - 1.129)}{1.4 - 1} \times \frac{(8 \times 10^5 \times 0.3 - 1.3 \times 10^5 \times 1.5)}{1.129 - 1}$   
 $= 236340 \text{ J/kg} = 23.63 \text{ kJ/kg}$



③

Change in internal energy;  $\Delta U = C_v [T_2 - T_1]$   $[C_v = \frac{R}{\gamma - 1}]$

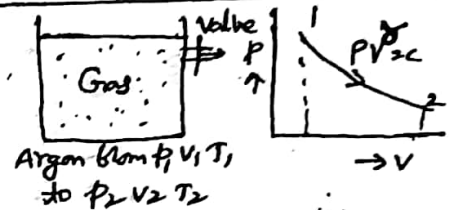
$$\Delta U = \frac{R}{\gamma - 1} \times (T_2 - T_1) = \frac{RT_2 - RT_1}{\gamma - 1} = \frac{P_2 V_2 - P_1 V_1}{\gamma - 1}$$

$$= \frac{1.3 \times 10^5 \times 1.5 - 8 \times 10^5 \times 0.3}{1.4 - 1} = -112,500 \text{ J/kg}$$

$$= -112.5 \text{ kJ/kg}$$

**PROB.** A tank of volume  $1.3 \text{ m}^3$  is filled with argon gas at 6 bar and  $260^\circ\text{C}$ . If the gas within the tank changes its state isentropically when it flows from the tank until the pressure drops to the atmospheric pressure of 1 bar, determine the mass of the gas that has left the tank during the process.

**SOL.** Volume,  $V_1 = 1.3 \text{ m}^3$ , pressure,  $P_1 = 6 \text{ bar}$   
 Temperature,  $T_1 = 260^\circ\text{C} = 533 \text{ K}$   
 pressure,  $P_2 = 1 \text{ bar}$



The gas flows isentropically from 6 bar to 1 bar. From the expansion process of the gas from state 1 to state 2 isentropically, we have,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} ; T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} = 533 \left(\frac{1}{6}\right)^{\frac{0.4}{1.4}} = 319.44 \text{ K}$$

Assume  $R$  for argon =  $0.2081 \text{ kJ/kgK}$

Initial mass,  $m_1 = \frac{P_1 V_1}{RT_1} = \frac{6 \times 10^5 \times 1.3}{0.2081 \times 10^3 \times 533} = 7.0322 \text{ kg}$

Final mass,  $m_2 = \frac{P_2 V_2}{RT_2} = \frac{1 \times 10^5 \times 1.3}{0.2081 \times 319.44} = 1.9556 \text{ kg}$

Mass of the gas which left the tank during the process,  $m = m_1 - m_2 = 7.0322 - 1.9556 = 5.0766 \text{ kg}$

**PROB** Show that  $C_p - C_v = \frac{\beta^2 TV}{K}$

**SOL** From First law of Thermodynamics,  $dQ = du + PdV$ .  
 Since we may write,  $u = \phi(T, V)$ .

$$du = \left(\frac{\partial u}{\partial T}\right)_V \cdot dT + \left(\frac{\partial u}{\partial V}\right)_T \cdot dV$$

$$\therefore dQ = du + PdV = \left(\frac{\partial u}{\partial T}\right)_V \cdot dT + \left\{P + \left(\frac{\partial u}{\partial V}\right)_T\right\} dV = C_v dT + \left\{P + \left(\frac{\partial u}{\partial V}\right)_T\right\} dV$$

This is true for any reversible process, and so, for a reversible constant pressure process,

$$dQ = C_p(dT)_p = C_v(dT)_p + \left\{P + \left(\frac{\partial u}{\partial V}\right)_T\right\} (dV)_p$$

Hence,  $C_p - C_v = \left\{P + \left(\frac{\partial u}{\partial V}\right)_T\right\} \left(\frac{\partial V}{\partial T}\right)_p$

From bondtze function:  $dF = -SdT - PdV$ ;  $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T} \left\{P + \left(\frac{\partial u}{\partial V}\right)_T\right\}$

$$\therefore C_p - C_v = T \cdot \left(\frac{\partial P}{\partial T}\right)_V \cdot \left(\frac{\partial V}{\partial T}\right)_p$$

From  $\beta = \frac{1}{V} \left(\frac{\partial V}{\partial T}\right)_p$ ;  $K = -\frac{1}{V} \left(\frac{\partial V}{\partial P}\right)_T$ ; we have  $C_p - C_v = \frac{\beta^2 TV}{K}$