LONG ANSWER QUESTIONS:

UNIT-II

1. a) Describe the Watt's parallel mechanism for straight line motion and derive the condition under which the straight line is traced.
   b) Discuss about Scott Russul straight line mechanism.

2. a) Give a neat sketch of the straight line motion 'Hart mechanism.' Prove that it produces an exact straight line motion.
   b) Sketch an intermittent motion mechanism and explain its practical applications

3. a) Two inclined shafts are connected by means of a universal joint. The speed of the driving shaft is 1000 r.p.m. If the total fluctuation of speed of the driven shaft is not to exceed 12.5% of this, what is the maximum possible inclination between the two shafts? With this angle, what will be the maximum acceleration to which the driven shaft is subjected and when this will occur?
   b) What is the condition for correct steering? Sketch and show the two main types of steering gears and discuss their relative advantages

4. a) Explain why two Hooke's joints are used to transmit motion from the engine to the differential of an automobile.
   b) The angle between the axes of two shafts connected by Hooke's joint is 18°. Determine the angle turned through by the driving shaft when the velocity ratio is maximum and unity.

5. A circle with EQ' as diameter has a point Q on it circumference. P is a point on EQ produced such that if Q turns about E, EQ. EP is constant. Prove that P moves in a straight line perpendicular to EQ'.

6. Sketch a pantograph, explain its working and show that it can be used to reproduce to an enlarged scale a given figure.

7. What is the difference between copied and generated straight line motions? Give example for each of them.

8. a) Prove that the peaucellier mechanism generates a straight-line motion.
   b) The track arm of a Davis steering gear is at a distance of 185 mm from the front main axle whereas the difference between their lengths is 90 mm. If the distance between steering pivots of the main axle is 1.2 m, determine the length of the chassis between the front and the rear wheels. Also find the inclination of the track arms to the longitudinal axis of the vehicle.

9. Design a pantograph for an indicator to be used to obtain the indicator diagram of an engine. The distance between the fixed point and the tracing point is 160 mm. The indicator diagram should be four times. The gas pressure inside the cylinder of the engine.

10. a) Show that the pantograph can produce paths exactly similar to the ones traced out by a point on a link on an enlarged or a reduced scale.
b) Two shafts are connected by a Hooke's joint. The driving shaft revolves uniformly at 500rpm. If the total permissible variation in speed of a driven shaft is not to exceed = 6% of the mean speed, find the greatest permissible angle between the centerlines of the shafts. Also determine the maximum and minimum speeds of the driven shaft.

11. a) Sketch and Describe the Scott-Russel and Robert's straight-line motion mechanisms.
b) For an Ackermann steering gear, derive the expression for the angle of inclination of the track arms to longitudinal axis of the vehicle.

12. a) Sketch and describe the peaucellier and Hart straight-line motion mechanisms
b) The driving shaft of Hooke's joint runs at a uniform speed of 280 r.p.m and the angle _ between the shaft axes is 20o. The driven shaft with attached masses has a mass of 60 kg at a radius of gyration of 15 cm. If a steady torque of 200 N-m resists rotation of the driven shaft, find the torque required at the driving shaft, when _ = 450; g=981 cm/sec2. At what value of _ will the total fluctuation of speed of the driven shaft be limited to 28 rpm.

13. a) Sketch a pantograph, explain its working and show that it can be used to reproduce to an enlarged scale a given figure?
b) Explain with neat sketch about Hart mechanism.

14. Explain Ackermans steering gear mechanism with neat sketch.
15. How can we ensure that a Tchebicheff mechanism traces an approximate straight line. Prove?
16. What is a Scott-Russel mechanism? What is its limitations how it is modified.
17. With reference to a Paucullier mechanism, Show that it can be used to trace a straight line. Prove mathematically?

SHORT ANSWER QUESTIONS:

1. What are the limitations of Scott Russell mechanism?
2. What is a pantograph?
3. Explain about Davis Steering gear?
4. Differentiate between Davis and Ackermann steering gears.
5. What is a Hooke's joint? What are its applications?
6. Derive the condition for correct steering.
7. Explain Scott Russul mechanism with necessary equation
8. Explain Hooke's joint with necessary derivation.
9. What are the disadvantages of a Davis steering gear mechanism?
10. Draw the polar velocity diagram for Hooke's joint.
11. What are the applications of a pantograph?
12. Explain about Grasshopper mechanism.
13. Explain about pantograph.

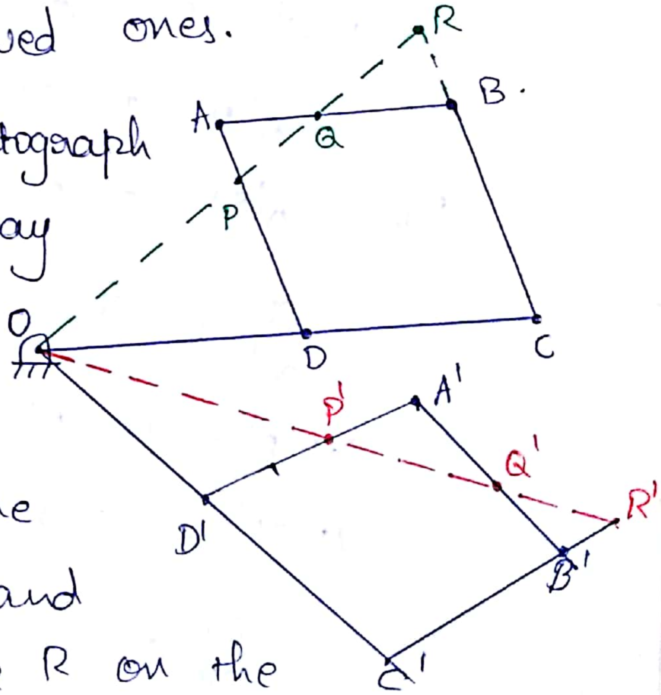
2. LOWER PAIR MECHANISMS.

When the two elements of a pair have a surface contact and a relative motion takes place, the surface of one element slides over the surface of the other, the pair is formed is known as "lower pair."

Lower pairs usually comprise turning and sliding pairs.

Pantograph:-

A pantograph is a four-bar linkage used to produce paths exactly similar to the ones traced out by a point on the linkage. The paths produced are an enlarged or reduced scale and may be straight or curved ones.



Four links of a pantograph are arranged in such a way that a parallelogram ABCD is formed. Thus, AB = DC and BC = AD.

If some point O in one of the links is fixed and three other points P, Q & R on the other three links are located in such a way that OPQR is a straight line, it can be shown that the points P, Q and R always move parallel and similar to each other

over any path, straight or curved.
Their motions will be proportional to their distances from the fixed point.

let the linkage be moved to another position so that A moves to A' and so on.

In Δ^{les} ODP & OCR
O, P & R lie on same straight line.

$\angle DOP = \angle COR$
$\angle ODP = \angle OCR$.

$\therefore$ Δ^{les} are similar.

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR}.$$

Now $A'B' = AB = DC = D'C'$
$B'C' = BC = AD = A'D'$

$\therefore$ $A'B'C'D'$ is again a parallelogram.

In Δ^{les} $OD'P'$ and $OC'R'$

$$\frac{OD'}{OC'} = \frac{OD}{OC} = \frac{DP}{CR} = \frac{D'P'}{C'R'}$$

$\angle OD'P' = \angle OC'R'$. O, P' & R' lie on a straight line.
$\angle D'OP' = \angle C'OR'$

$$\frac{OP}{OR} = \frac{OD}{OC} = \frac{OD'}{OC'} = \frac{OP'}{OR'}.$$

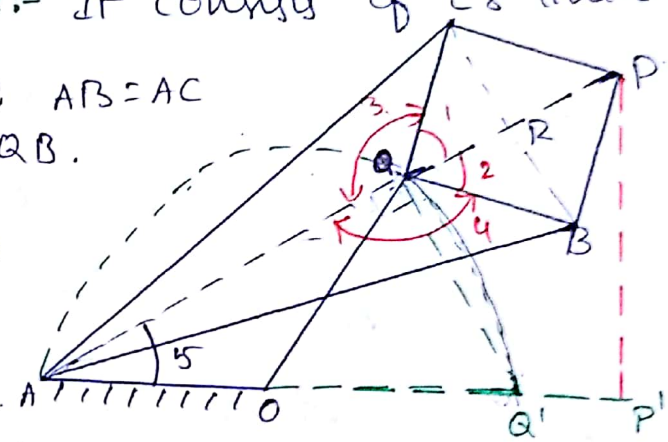
This shows that as the linkage is moved, the ratio of the distances of P & R from the fixed point remains same or two points are displaced proportional to their distances from the fixed point. This will be true for all the positions of the links. Thus, P & R will trace exactly similar paths.

Similarly P & Q trace similar paths. Thus, P, Q & R trace similar paths when the linkage is given motion.

Straight-line Mechanisms:-

1. Paucellier Mechanism:- It consists of (8 links.) such that, OA = OQ ; AB = AC and BP = PC = CQ = QB.

OA is the fixed link and OQ is a rotating link.



As link OQ rotates around O, P moves in a straight line $\perp^{r}$ to OA.

Since BPCQ is a rhombus,

QP always bisects the angle BQC.

i.e $\angle 1 = \angle 2 \rightarrow$ (i) in all positions.

in Δ^{les} AQC & AQB,

AQ is common, AC = AB, QC = QB.

$\therefore$ Δ^{les} are ~~congurrent~~ congruent in all positions.

$\angle 3 = \angle 4 \rightarrow$ (ii).

(i) + (ii) $\Rightarrow$ $\angle 1 + \angle 3 = \angle 2 + \angle 4$.

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^\circ$

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4 = 180^\circ$

③

∴ A, Q & P lie on a straight line.

Let PP' be the perpendicular on AO Produced.

Δ^{les} AQQ' & APP' are similar b/c

$\angle 5$ is common and $\angle AQQ' = \angle AP'P = 90°$.

$$\therefore \frac{AQ}{AP'} = \frac{AQ'}{AP}.$$

$$\Rightarrow AQ' \cdot AP' = (AQ)(AP)$$

$$= (AR - RQ)(AR + RP).$$

$[\because RQ = RP]$

$$= (AR - RQ)(AR + RQ)$$

$$= (AR)^2 - (RQ)^2$$

$$AQ' \cdot AP' = [(AC)^2 - (CR)^2] - [(CQ)^2 - (CR)^2].$$

$$AQ' \cdot AP' = (AC)^2 - (CR)^2 - (CQ)^2 + (CR)^2$$

$$AQ' \cdot AP' = (AC)^2 - (CQ)^2$$

$$AP' = \frac{(AC)^2 - (CQ)^2}{AQ'}$$

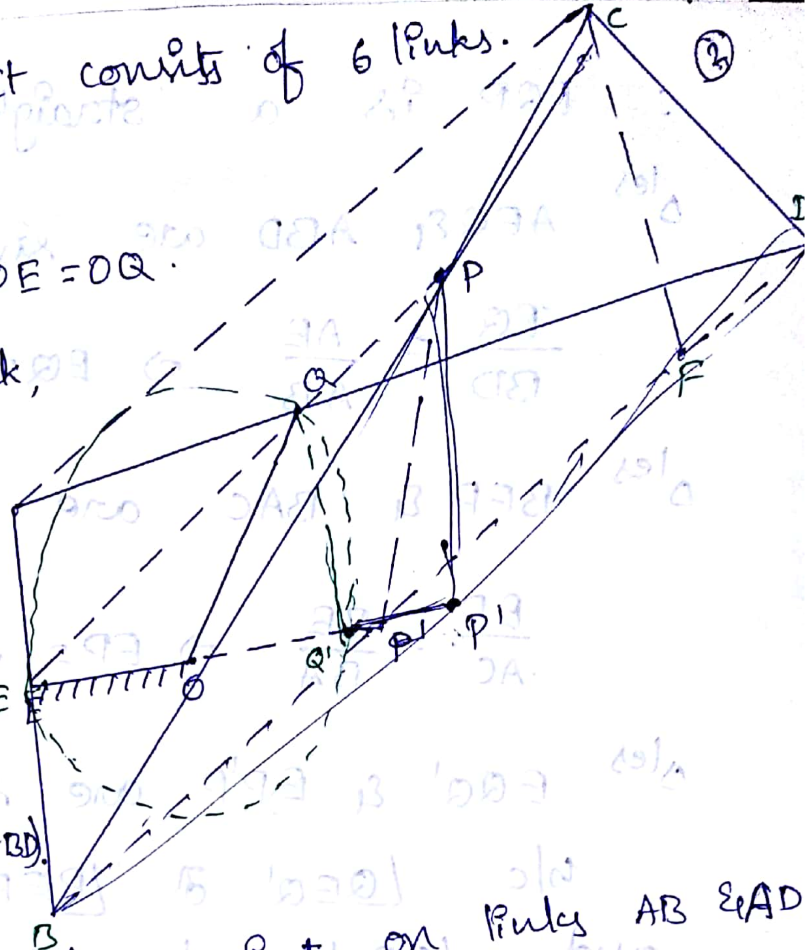
$AP' =$ Constant as AQ', AC, CQ are fixed.

This means that the projection of P & AQ produced is constant for all the configurations.

Thus, PP' is always a normal to AO Produced

∴ P moves in a straight line $\perp^{r}$ to AO.

(2) Hart Mechanism:- It consists of 6 links such that



AB = CD, AD = BC & OE = OQ.

OE is the fixed link, and OQ is the rotating link.

The links are arranged in such a way that ABDC is a trapezium (AC ∥ BD).

Pins, E & Q are the points on links AB & AD respectively, and the point P on the link CB are located in such a way that

$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB}.$$

OQ rotates about O, P moves in a line $\perp$r to EO produced.

in Δ^{le} ABD, $\frac{AE}{AB} = \frac{AQ}{AD}$.

$\therefore$ EQ is parallel to BD and thus $\parallel^{le}$ to AC.

in Δ^{le} ABC, $\frac{AE}{AB} = \frac{CP}{CB}$.

$\therefore$ EP is $\parallel^{le}$ to AC and thus $\parallel^{le}$ to BD.

EQ, EP are both $\parallel^{le}$ to AC & BD and have a point E in common.

$\therefore$ EQP is a straight line.

Δ^{les} AEQ & ABD are similar ($\because EQ \parallel BD$).

$$\frac{EQ}{BD} = \frac{AE}{AB} \Rightarrow EQ = BD \times \frac{AE}{AB} \rightarrow (ii)$$

Δ^{les} BEP & BAC are similar ($\because EP \parallel AC$).

$$\frac{EP}{AC} = \frac{BE}{BA} \Rightarrow EP = AC \times \frac{BE}{AB} \rightarrow (iii)$$

Δ^{les} EQQ' & EP'P are similar,

b/c $\angle QEQ'$ & $\angle PEP'$ is common

and $\angle EQQ' = \angle EP'P = 90^\circ$

$$\therefore \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

$$EQ' \times EP' = EQ \times EP$$

$$= \left(BD \times \frac{AE}{AB}\right) \times \left(AC \times \frac{BE}{AB}\right)$$

$$EP' = \frac{AE \times BE}{(EQ')(AB)^2} \left[(BD) \times (AC)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2} \left[(BF + FD)(BF - FD)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2} \left[(BF)^2 - (FD)^2\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2} \left[\{(BC)^2 - (CF)^2\} - \{CD^2 - CF^2\}\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2} \left[(BC)^2 - (CD)^2\right]$$

$=$ constant.

Thus, EP' is always.

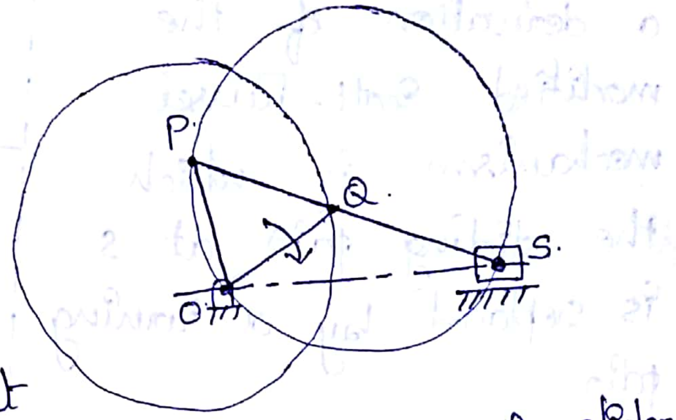
$\therefore$ the projection of P on EO produced is always the same point & P moves in a straight line $\perp^r$ to EO.

Scott-Russel Mechanism :- It consists of 3 movable links; OQ, PS & slider S.

OQ is the crank.

The links are connected in such a way that

$QO = QP = QS$.



It can be proved that P moves in a straight line $\perp^r$ to OS as the slider S moves along OS.

As $QO = QP = QS$, a circle is passing through O, P & S with PS as the diameter and Q as the centre.

Now, O lies on the circumference of the circle and PS is the diameter.

$\therefore \angle POS$ is a right angle.

The path of P is through the joint O which is not desirable. This can be avoided if the links are proportioned in a way that QS is the mean proportional b/w OQ & OP.

i.e $\frac{OQ}{QS} = \frac{QS}{QP}$.

In this case P will approximately traverse a straight line $\perp^r$ to OS and that also for small movements of S a for small values of angle θ.

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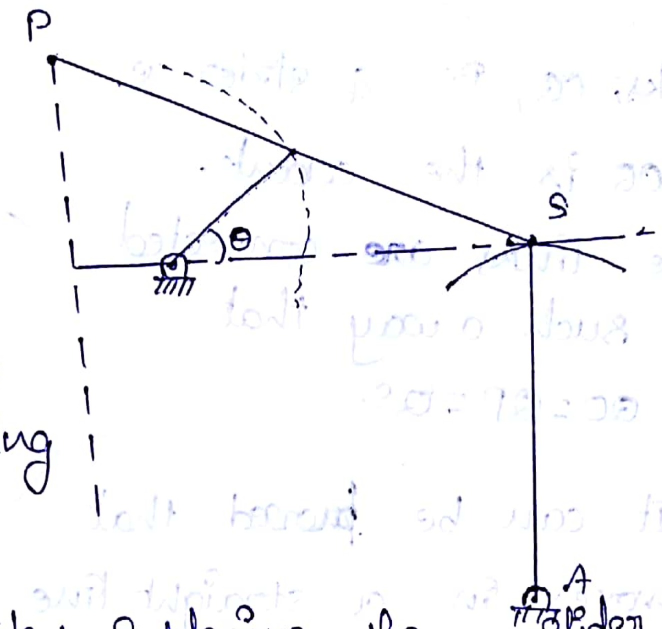
Grass-Hopper Mechanism:-

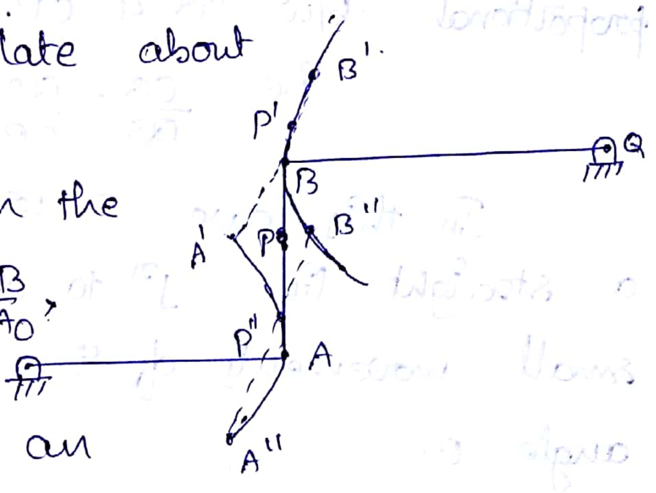


This mechanism is a derivation of the modified Scott-Russel mechanism in which the sliding pair at S is replaced by a turning pair.

This is achieved by replacing the slider with a link AS ⊥r to OS in the mean position. AS is pin-jointed at A.

If the length AS is large enough, S moves in an approximated straight line ⊥r to OS for small angular movements. P again will move in an approximate straight line if QS is the mean proportional b/w OQ & QP; i.e $\frac{OQ}{QS} = \frac{QS}{QP}$.

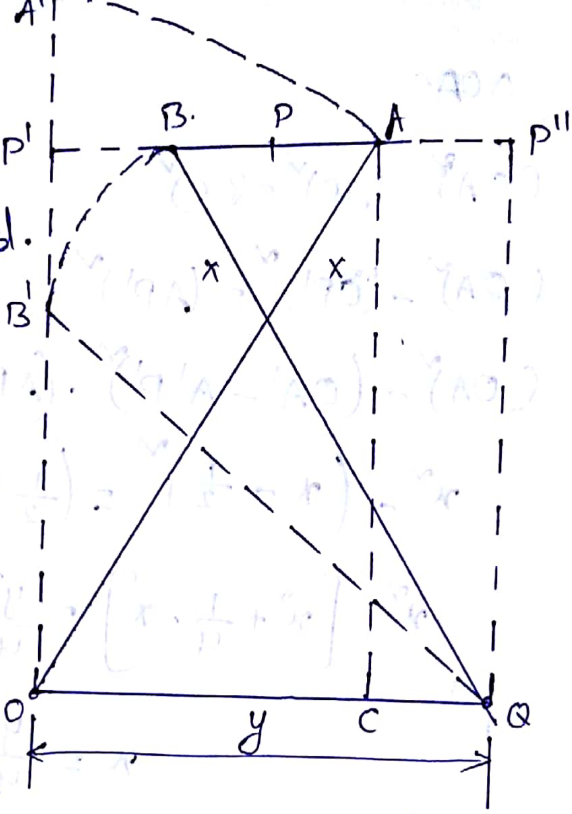
Watt Mechanism:- It has 4 links OQ, OA, QB & AB. OQ is the fixed link.



Links OA & QB can oscillate about centres O & Q respectively.

If P is a point on the link AB such that $\frac{PA}{PB} = \frac{QB}{OA_0}$, then for small oscillations of OA & QB, P will trace an approximately straight line.

Tchebicheff Mechanism:-



It consists of 4 links OA, QB, AB & OQ. OQ is fixed. OA = QB., PA = PB.

The proportions of the links are taken in such a way that P, A & B lie on vertical lines when on extreme positions, i.e when directly above O & Q.

Let AB = 1 unit.

OA = QB = x units

OQ = y units.

When AB is on the extreme left position, A & B assume the positions A' & B' resp.

In $\Delta OQB'$, $(QB')^2 - (OQ)^2 = (OB')^2$.

$$(QB)^2 - (OQ)^2 = (OB')^2 \quad [\because QB' = QB).$$

$$x^2 - y^2 = (OA' - A'B')^2.$$

$$x^2 - y^2 = (x-1)^2$$

$$\cancel{x^2} - y^2 = \cancel{x^2} + 1 - 2x.$$

$$\Rightarrow 2x = 1 + y^2$$

$$x = \frac{y^2+1}{2} \longrightarrow (i)$$

In ΔOAC,

$$(OA)^2 - (AC)^2 = (OC)^2$$

$$(OA)^2 - (OP')^2 = (AP')^2$$

$$(OA)^2 - (OA' - A'P')^2 = (AP + PP')^2$$

$$x^2 - \left(x - \frac{1}{2}\right)^2 = \left(\frac{1}{2} + \frac{y}{2}\right)^2$$

$$x^2 - \left[x^2 + \frac{1}{4} - x\right] = \frac{y^2}{4} + \frac{1}{4} + \frac{y}{2}$$

$$x = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2} \longrightarrow (ii)$$

$(i) = (ii)$

$$\frac{y^2+1}{2} = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2}$$

$$\frac{y^2}{2} + \frac{1}{2} = \frac{y^2}{4} + \frac{y}{2} + \frac{1}{2}$$

$$\frac{y^2}{4} = \frac{y}{2}$$

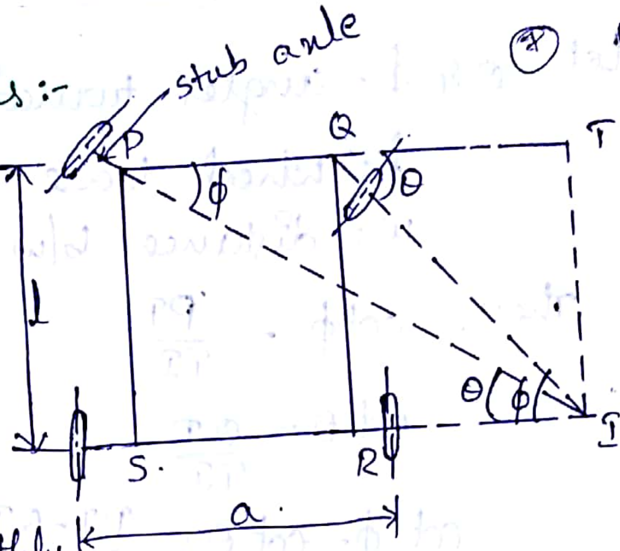
$$y = 2$$

$$\therefore x = \frac{y^2+1}{2} = \frac{5+1}{2} = 2.5$$

Thus, $AB : OQ : OA = 1 : 2 : 2.5$

This ratio of the links ensures that P moves approximately in a horizontal straight line $\parallel^{le}$ to OQ.

Automobile Steering Gears :-



When an automobile takes turn on a road, all the wheels should make concentric circles to ensure that they roll on the road smoothly and there is a line contact b/w the tyres and the surface of the path preventing the excess wear of tyres.

This is achieved by mounting the two front wheels on two short axles, known as stub axles. The stub axles are pin-jointed with the main front axle which is rigidly ~~fixed~~ attached to the rear axle. Thus, the steering is affected by the use of front wheels only.

When the vehicle takes turn towards one side, the front wheel of that side must swing about the pin through a greater angle than the wheel of the other side. If the axes of the stub axles when produced, intersect at a point I on the common axis of the two rear wheels. In that case, all the wheels of the vehicle will move about a vertical axis through I, minimizing the tendency of the wheels to skid. The point I is also the instantaneous centre of the motion of the 4 wheels.

Let θ & ϕ = angles turned by the stud axles

l = wheel base

w = distance b/w the pivots of front axles.

Then, $\cot\phi = \frac{PT}{TI}$,

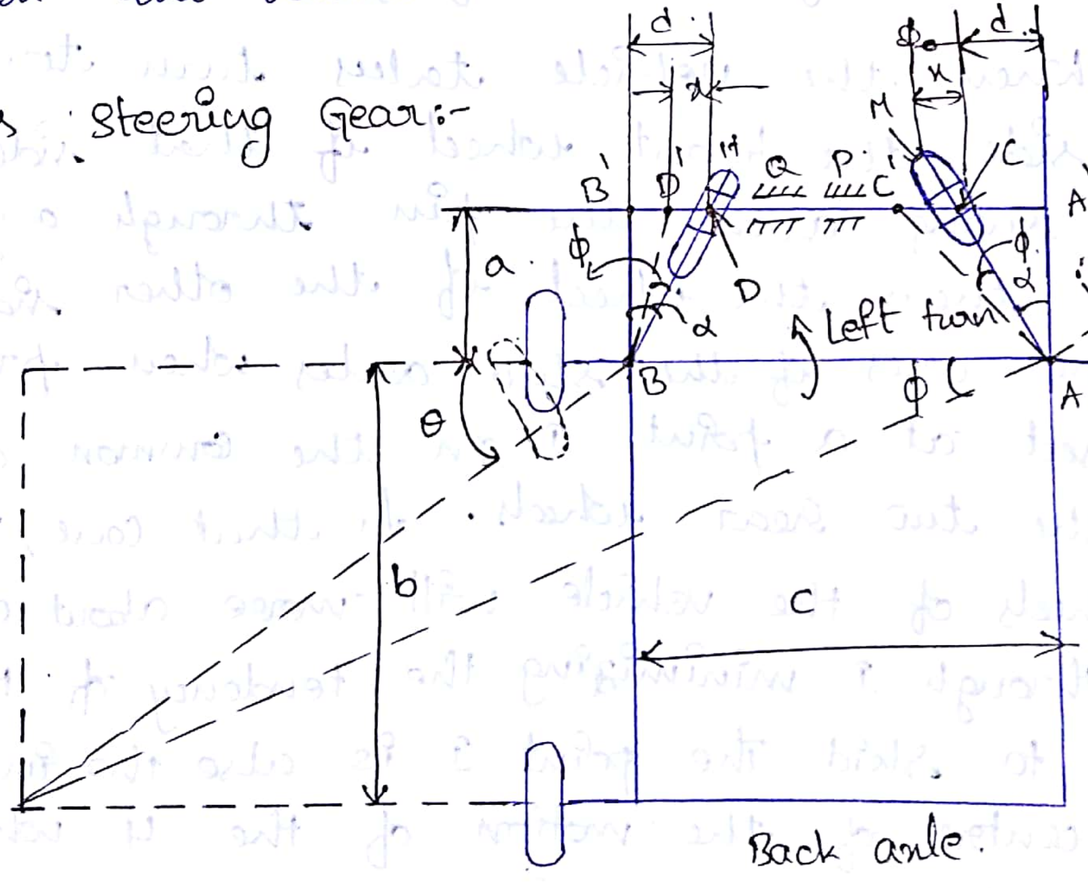
$\cot\theta = \frac{QT}{TI}$

$\cot\phi - \cot\theta = \frac{PT-QT}{TI} = \frac{PQ}{TI} = \frac{w}{l}$.

This is known as the fundamental equation of correct gearing. Mechanisms that fulfill this ~~Mechanism~~ fundamental equation are known as steering gears.

If this condition is satisfied, there won't be any skidding of the wheels, when the vehicle takes a turn.

Davis Steering Gear:-



~~Davis Ste~~

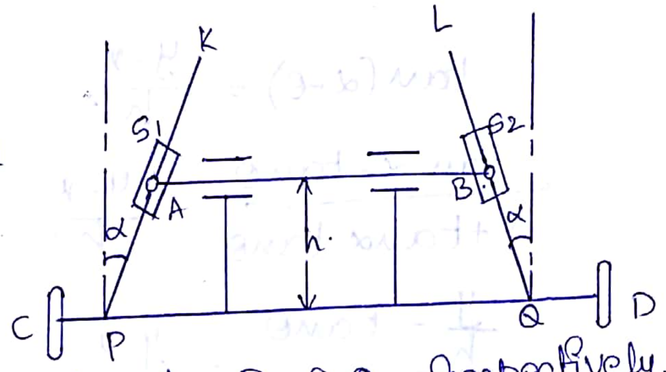
Types of steering gears:-

There are two main types of steering gears:

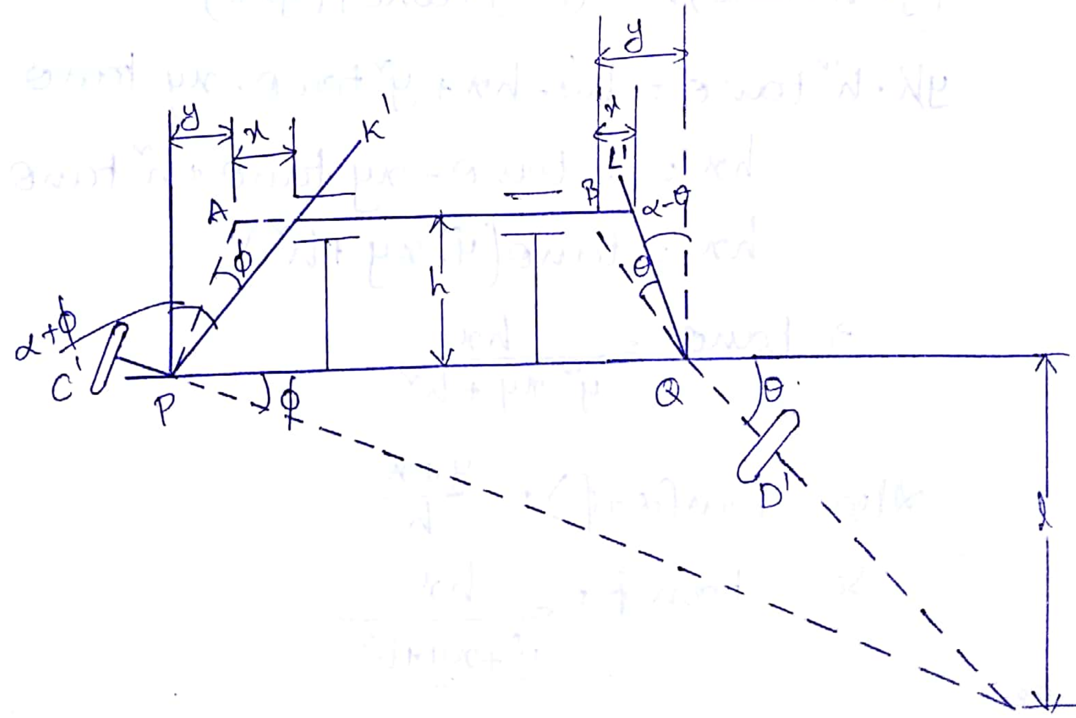
1) Davis steering gear:- It has sliding pairs which means more friction and easy wearing.

2) Ackermann steering gear:- It has only turning pairs.

~~Davis~~ Davis steering gear:-



It consists of two arms PK & QL fixed to the stub axles PC & QD to form two similar bell-crank levers CPK & DQL pivoted at P & Q respectively. A cross link or track arm AB, constrained to slide parallel to PQ, is pin-jointed at its ends to two sliders. The sliders S_1 & S_2 are free to slide on the links PK & QL respectively.



During the straight motion of the vehicle, the gear is in the mid position with equal inclination of the arms PK & QL with PQ.

As the vehicle turns right, the cross-arm AB also moves right through a distance x from the mid position. The bell-crank levers assume the positions $C'P'K'$ & $D'QL'$.

Let h = vertical distance b/w AB & PQ.

$$\tan(\alpha-\theta) = \frac{y-x}{h} \qquad \left[\because \tan\alpha = \frac{y}{h}\right]$$

$$\frac{\tan\alpha - \tan\theta}{1+\tan\alpha\tan\theta} = \frac{y-x}{h}$$

$$\frac{\frac{y}{h} - \tan\theta}{1+\frac{y}{h}\cdot\tan\theta} = \frac{y-x}{h}$$

$$\frac{y-h\tan\theta}{h+y\tan\theta} = \frac{y-x}{h}$$

$$(y-h\tan\theta)h = (h+y\tan\theta)(y-x)$$

$$y\cancel{h} - h^2\tan\theta = \cancel{h}y - hx + y^2\tan\theta - xy\tan\theta$$

$$hx = y^2\tan\theta - xy\tan\theta + h^2\tan\theta$$

$$hx = \tan\theta(y^2 - xy + h^2)$$

$$\Rightarrow \tan\theta = \frac{hx}{y^2-xy+h^2}$$

Also, $\tan(\alpha+\phi) = \dfrac{y+x}{h}$

So, $\tan\phi = \dfrac{hx}{y^2+xy+h^2}$

For ~~the~~ correct steering action, $\cot\phi - \cot\theta = \frac{w}{l}$. (A)

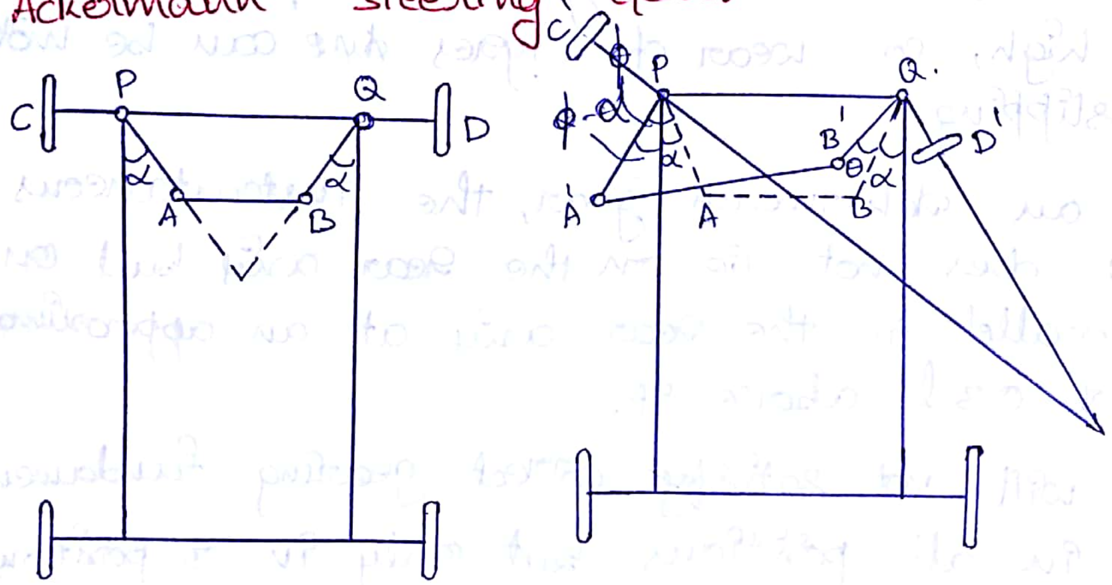
$$\frac{y^2+xy+h^2}{hx} - \frac{y^2-xy+h^2}{hx} = \frac{w}{l}.$$

$$\frac{2xy}{hx} = \frac{w}{l}.$$

$$\frac{y}{h} = \frac{w}{2l}$$

$$\tan\alpha = \frac{w}{2l}$$

Ackermann Steering Gear:-



It is consisting of 4-link mechanism PABQ having 4 turning pairs. PA & QB are two equal arms which are fixed to the stub axles PC & QD to form two similar bell-crank levers CPA & DQB pivoted at P & Q respectively. AB is pin-jointed at the ends to two bell-crank levers.

During the straight motion of the vehicle, the gear is in the mid-position with equal arms PA & QB with PQ. AB is parallel PQ.

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For every value of θ, ϕ value will be varied. The angle ϕ found by drawing the gear ~~the geo~~ may be termed as ϕ_a (ϕ actual).

For theoretical values of $\{\phi_t\}$ for different values of θ, for the given values of w & l can be calculated from, $\cot\phi - \cot\theta = \frac{w}{l}$.

1. For small values of θ, ϕ_a is higher than ϕ_t.
2. For larger values of θ, ϕ_a is lower than ϕ_t.

If the vehicle is taking a sharp turn, θ will be high, so wear of tyres due can be more due to slipping.

In an Ackermann gear, the instantaneous centre I does not lie on the rear axis but on a line parallel to the rear axis at an approximate distance of 0.3 l above it.

It will not satisfy correct gearing fundamental equation in all positions but only in 3 positions. Those are:

1) when the vehicles moves straight
2) when the vehicle moves at a correct angle to right
3) when the vehicle moves at a correct angle to left.

If the fundamental equation is not satisfied slipping of the wheel will takes place.

In all other positions (except these 3), pure rolling is not possible due to slipping of the wheel.

Determination of angle α :-

If the values of PA, PQ & the angle α are known, the mechanism can be drawn to a suitable scale in different positions and the actual angle ϕ can be evaluated for different values of θ.

Projection of BB' on PQ = Projection of AA' on PQ.

$$QB[\sin(\alpha+\theta) - \sin\alpha] = PA[\sin\alpha + \sin(\phi-\alpha)].$$

$$\sin(\alpha+\theta) - \sin\alpha = \sin\alpha + \sin(\phi-\alpha) \qquad [\because QB = PA]$$

$$(\sin\alpha\cos\theta + \cos\alpha\sin\theta) - \sin\alpha = \sin\alpha + (\sin\phi\cos\alpha - \cos\phi\sin\alpha)$$

~~sin α (~~

$$\sin\alpha\cos\theta - 2\sin\alpha + \cos\phi\sin\alpha = \sin\phi\cos\alpha - \cos\alpha\sin\theta$$

$$\sin\alpha\{\cos\theta + \cos\phi - 2\} = \cos\alpha(\sin\phi - \sin\theta)$$

$$\frac{\sin\alpha}{\cos\alpha} = \frac{\sin\phi - \sin\theta}{\cos\theta + \cos\phi - 2}$$

$$\tan\alpha = \frac{\sin\phi - \sin\theta}{\cos\theta + \cos\phi - 2}.$$

Hooke's Joint:- It is commonly known as universal joint, and it is used to connect two non-parallel and intersecting shafts. Application of this joint is in an automobile where it is used to transmit power from the gear box to the rear axle. The driving shaft rotates at a uniform angular speed whereas the driven shaft rotates at a continuously varying angular speed.

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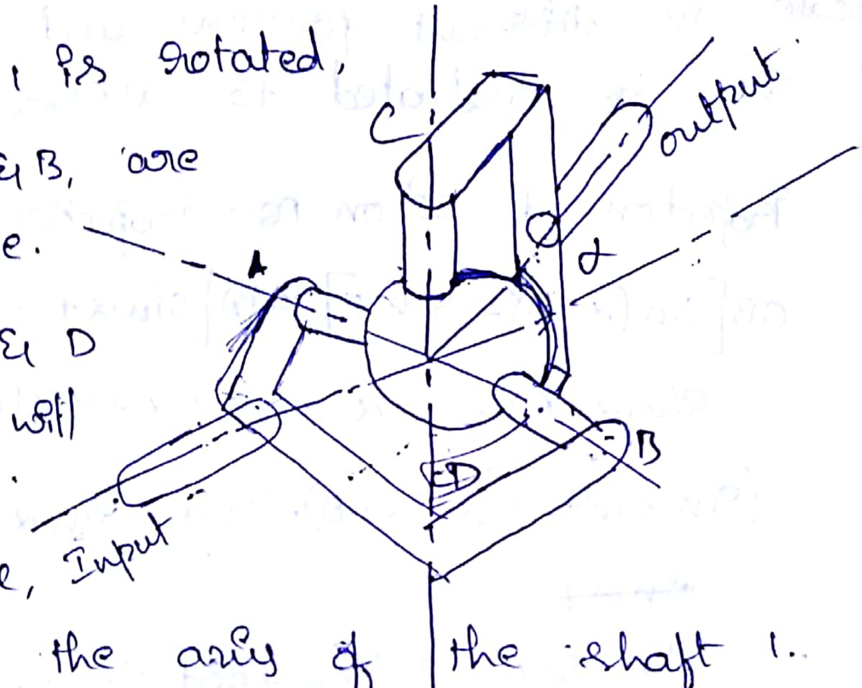
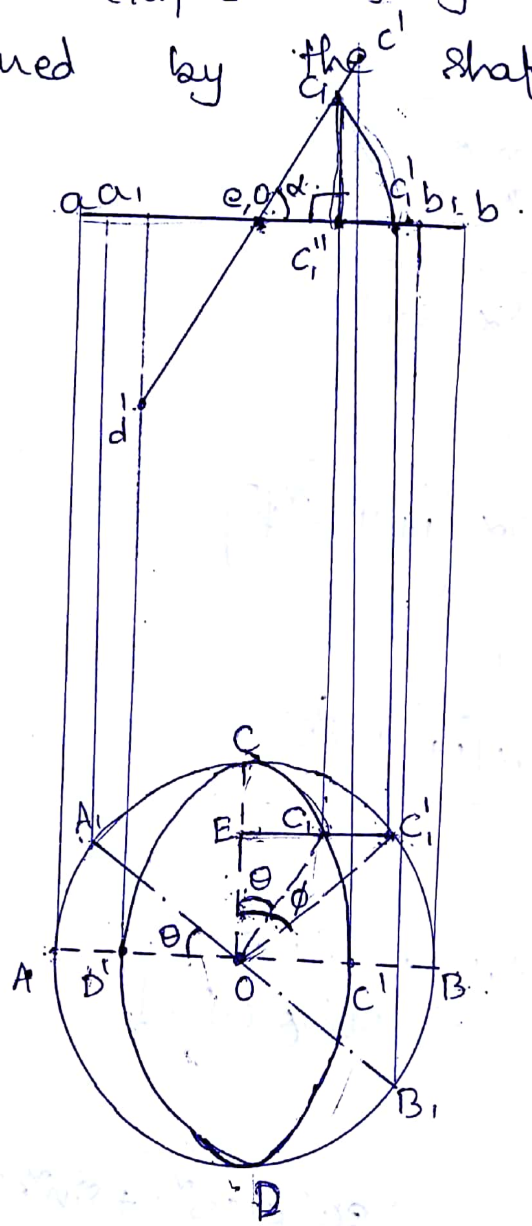
The shafts 1 & 2 rotate in the fixed bearings. A complete revolution of either shaft.



As the shaft 1 is rotated, its fork ends A & B, are rotated in a circle. The fork ends C & D of the shaft 2 will move along the path of an ellipse, if viewed along the axis of the shaft 1.

In the top view, the motion of the fork ends of the shaft 1 is along the line ab whereas that of the shaft 2 on a line c'd' at an angle of α to ab.

Let the shaft 1 rotate through an angle θ so that fork ends assume the positions A_1 & B_1. Now, the angle moved by the shaft 2 would also be θ when viewing along the axis of the shaft 1. Let the fork end C take the position C_1. However, the true angle turned by the shaft 2 would be when it is viewed along its own axis. when viewing along the axis of the shaft 2. Here, C & D move in a circle. The point C_1 lies on (11) a circle at the same height as it is on the ellipse. This gives the true angle ϕ turned by the shaft 2.



$$\tan\phi = \frac{EC_1'}{OE}$$

$$\tan\theta = \frac{EC_1}{OE}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{EC_1'}{EC_1}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{ec_1'}{ec_1''}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{1}{(ec_1''/ec_1')}$$

$$\frac{\tan\phi}{\tan\theta} = \frac{1}{\cos\alpha}.$$

$$\Rightarrow \tan\theta = \tan\phi \cos\alpha.$$

let

ω_1 = angular velocity of driving shaft $= \frac{d\theta}{dt}$

ω_2 = angular velocity of driven shaft $= \frac{d\phi}{dt}$.

$$\frac{\omega_2}{\omega_1} = \frac{d\phi/dt}{d\theta/dt}$$

$$\tan\theta = \tan\phi \cos\alpha.$$

Differentiate w.r.t 't'

$$\sec^2\theta \cdot \frac{d\theta}{dt} = \cos\alpha \cdot \sec^2\phi \cdot \frac{d\phi}{dt}$$

$$\sec^2\theta \cdot \omega_1 = \cos\alpha \cdot \sec^2\phi \cdot \omega_2$$

$$\frac{\omega_2}{\omega_1} = \frac{\sec^2\theta}{\cos\alpha \cdot \sec^2\phi}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta \cdot \cos\alpha \cdot \sec^2\phi}$$

$$= \frac{1}{\cos^2\theta \cdot \cos\alpha \cdot \left[1 + \tan^2\phi\right]}$$

$$= \frac{1}{\cos^2\theta \cdot \cos\alpha \left[1 + \frac{\tan^2\theta}{\cos^2\alpha}\right]}$$

$$= \frac{1}{\cos^2\theta \cdot \cos\alpha \left[1 + \frac{\sin^2\theta}{\cos^2\theta \cdot \cos^2\alpha}\right]}$$

$$= \frac{1}{\cos^2\theta \cdot \cos\alpha \left[\frac{\cos^2\theta \cdot \cos^2\alpha + \sin^2\theta}{\cos^2\theta \cos^2\alpha}\right]}$$

$$= \frac{\cos\alpha}{\cos^2\theta \cos^2\alpha + \sin^2\theta}$$

$$= \frac{\cos\alpha}{\cos^2\theta\left(1 - \sin^2\alpha\right) + \sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta}$$

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha}$$

(i) if $\frac{\omega_2}{\omega_1} = 1$

$$\frac{\cos\alpha}{1-\cos^2\theta\sin^2\alpha} = 1$$

$$\cos\alpha = 1-\cos^2\theta\cdot\sin^2\alpha.$$

$$\cos^2\theta\cdot\sin^2\alpha = 1-\cos\alpha$$

$$\cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha}$$

$$\cos^2\theta = \frac{1-\cos\alpha}{1-\cos^2\alpha}$$

$$\cos^2\theta = \frac{\cancel{1-\cos\alpha}}{\cancel{(1-\cos\alpha)}(1+\cos\alpha)}$$

$$\cos^2\theta = \frac{1}{1+\cos\alpha}$$

$$\cos^2\theta = \frac{\sin^2\theta+\cos^2\theta}{1+\cos\alpha}$$

$$\cancel{\cos^2\theta} = \frac{\cancel{\cos^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta}+1\right)}{1+\cos\alpha}$$

$$\cancel{1}+\cos\alpha = \cancel{1}+\tan^2\theta$$

$$\tan^2\theta = \cos\alpha$$

$$\tan\theta = \pm\sqrt{\cos\alpha}.$$

(ii) if $\frac{\omega_2}{\omega_1}$ is minimum.

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$1-\cos^2\theta \cdot \sin^2\alpha$ is to be maximum.

if $\theta = 90^\circ$ or 270°.

$1-\cos^2\theta \cdot \sin^2\alpha = 1$.

$$\Rightarrow \frac{\omega_2}{\omega_1} = \cos\alpha.$$

(iii) if $\frac{\omega_2}{\omega_1}$ is maximum

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$1-\cos^2\theta \cdot \sin^2\alpha$ is to be minimum.

if $\theta = 0^\circ$ or 180°

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos\alpha}.$$

Double Hooke's Joint :-

It is possible to connect two shafts by two Hooke's couplings through an intermediate shaft such that the uneven velocity ratio of the first coupling will be cancelled out by the other one.

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Two parallel or intersecting shafts may be connected by a double universal joint and have uniform output motions, provided that the intermediate shaft makes equal angles with the connected shafts and that the forks on the intermediate shaft are in the same plane.

Let θ be the angle of rotation of driving shaft and let the intermediate shaft rotate through angle φ. At the other end of intermediate shaft, let ψ be the angle of rotation of intermediate shaft for the angle of rotation φ by output shaft.

$$\tan\theta = \tan\phi \cos\alpha \rightarrow (1)$$

$$\tan\psi = \tan\phi \cos\alpha \rightarrow (2)$$

P6:- Two shafts with an included angle of 160° are connected by a Hooke's joint. The driving shaft runs at a uniform speed of 1500 r.p.m. The driven shaft carries a flywheel of mass 12 kg & 100 mm radius of gyration. Find the max angular acceleration of the driven shaft and the maximum torque required.

Sol:- $\alpha = 180° - 160° = 20°$, $N = 1500$ r.p.m ; $m = 12$ kg, $k = 100$ mm

$k = 0.1$ m.

$$\omega_1 = \frac{2\pi N}{60} = \frac{2\pi \times 1500}{60} = 157 \text{ rad/sec.}$$

$$I = mk^2 = 12(0.1)^2 = 0.12 \text{ kg-m}^2.$$

$$\cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha} = \frac{2\sin^2 20}{2-\sin^2 20} = 0.124.$$

$$2\theta = 82.9° \Rightarrow \theta = 41.45°.$$

$$\frac{d\omega_2}{dt} = \frac{\omega^2 \cos\alpha \cdot \sin 2\theta \cdot \sin^2\alpha}{(1-\cos^2\theta \cdot \sin^2\alpha)^2}$$

$$= (157)^2 \cos 20° . x$$

Angular acceleration of the driven shaft :- (14)

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\cos^2\theta \cdot \sin^2\alpha}$$

$$\omega_2 = \omega_1 \cos\alpha \,(1-\cos^2\theta \cdot \sin^2\alpha)^{-1}$$

$$\frac{d\omega_2}{dt} = \omega_1 \cos\alpha \left[-1\,(1-\cos^2\theta \cdot \sin^2\alpha)^{-2} \times 2\cos\theta\sin\theta\sin^2\alpha \cdot \frac{d\theta}{dt}\right]$$

$$\frac{d\omega_2}{dt} = \frac{-\omega_1^2 \cos\alpha \cdot \sin 2\theta \cdot \sin^2\alpha}{(1-\cos^2\theta \cdot \sin^2\alpha)^2}.$$

for max angular acceleration of driven shaft,

$\frac{d\omega_2}{dt}$ w.r.t $\theta = 0$.

$$\cos 2\theta = \frac{\sin^2\alpha\,(2-\cos^2 2\theta)}{2-\sin^2\alpha}.$$

if $\alpha < 30^\circ$.

$$\Rightarrow \cos 2\theta = \frac{2\sin^2\alpha}{2-\sin^2\alpha}.$$

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