Unit-II: Fluid Kinematics and Dynamics

FLUID KINEMATICS

Fluid Kinematics gives the geometry of fluid motion. It is a branch of fluid mechanics, which describes the fluid motion, and it's consequences without consideration of the nature of forces causing the motion. Fluid kinematics is the study of velocity as a function of space and time in the flow field. From velocity, pressure variations and hence, forces acting on the fluid can be determined.

VELOCITY FIELD

Velocity at a given point is defined as the instantaneous velocity of the fluid particle, which at a given instant is passing through the point. It is represented by V=V(x,y,z,t). Vectorially, V=ui+vj+wk where u,v,w are three scalar components of velocity in x,y and z directions and (t) is the time. Velocity is a vector quantity and velocity field is a vector field.

Fluid Mechanics is a visual subject. Patterns of flow can be visualized in several ways. Basic types of line patterns used to visualize flow are streamline, path line, streak line and time line.

(a) Stream line is a line, which is everywhere tangent to the velocity vector at a given instant.
(b) Path line is the actual path traversed by a given particle.
(c) Streak line is the locus of particles that have earlier passed through a prescribed point.
(d) Time line is a set of fluid particles that form a line at a given instant.

Streamline is convenient to calculate mathematically. Other three lines are easier to obtain experimentally. Streamlines are difficult to generate experimentally. Streamlines and Time lines are instantaneous lines. Path lines and streak lines are generated by passage of time. In a steady flow situation, streamlines, path lines and streak lines are identical. In Fluid Mechanics, the most common mathematical result for flow visualization is the streamline pattern – It is a common method of flow pattern presentation.

Streamlines are everywhere tangent to the local velocity vector. For a stream line, (dx/u) = (dy/v) = (dz/w). Stream tube is formed by a closed collection of streamlines. Fluid within the stream tube is confined there because flow cannot cross streamlines. Stream tube walls need not be solid, but may be fluid surfaces

METHOD OF DESCRIBING FLUID MOTION

Two methods of describing the fluid motion are: (a) Lagrangian method and (b) Eularian method. A single fluid particle is followed during its motion and its velocity, acceleration etc. are described with respect to time. Fluid motion is described by tracing the kinematics behavior of each and every individual particle constituting the flow. We follow individual fluid particle as it moves through the flow. The particle is identified by its position at some instant and the time elapsed since that instant. We identify and follow small, fixed masses of fluid. To describe the fluid flow where there is a relative motion, we need to follow many particles and to resolve details of the flow; we need a large number of particles. Therefore, Langrangian method is very difficult and not widely used in Fluid Mechanics.

EULARIAN METHOD

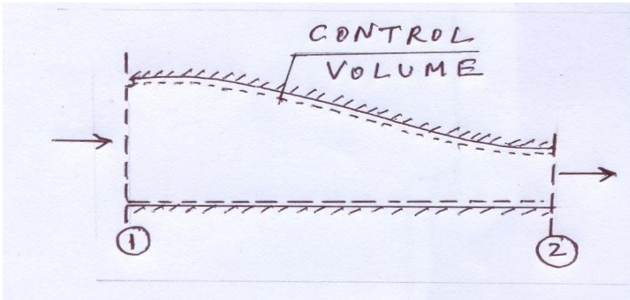


Fig. Eulerian Method

The velocity, acceleration, pressure etc. are described at a point or at a section as a function of time. This method commonly used in Fluid Mechanics. We look for field description, for Ex.; seek the velocity and its variation with time at each and every location in a flow field. Ex., V=V(x,y,z,t). This is also called control volume approach. We draw an imaginary box around a fluid system. The box can be large or small, and it can be stationary or in motion.

TYPES OF FLUID FLOW

1. Steady and Un-steady flows
2. Uniform and Non-uniform flows
3. Laminar and Turbulent flows
4. Compressible and Incompressible flows
5. Rotational and Irrotational flows
6. One, Two and Three dimensional flows

STEADY AND UNSTEADY FLOW

Steady flow is the type of flow in which the various flow parameters and fluid properties at any point do not change with time. In a steady flow, any property may vary from point to point in the field, but all properties remain constant with time at every point.[$\partial V/\partial t$] x,y,z= 0; [$\partial p/\partial t$]x,y,z =0. Ex.: V=V(x,y,z); p=p(x,y,z) . Time is a criterion.

Unsteady flow is the type of flow in which the various flow parameters and fluid properties at any point change with time. [$\partial V/\partial t$] x,y,z$\neq$0 ; [$\partial p/\partial t$] x,y,z$\neq$0, Eg.:V=V(x,y,z,t), p=p(x,y,z,t) or V=V(t), p=p(t) . Time is a criterion

UNIFORM AND NON-UNIFORM FLOWS

Uniform Flow is the type of flow in which velocity and other flow parameters at any instant of time do not change with respect to space. Eg., V=V(x) indicates that the flow is uniform in ‘y’ and ‘z’ axis. V=V (t) indicates that the flow is uniform in ‘x’, ‘y’ and ‘z’ directions. Space is a criterion.

Uniform flow field is used to describe a flow in which the magnitude and direction of the velocity vector are constant, i.e., independent of all space coordinates throughout the entire flow field (as opposed to uniform flow at a cross section). That is, $[\partial V/\partial s]_{t=constant} = 0$, that is ‘V’ has unique value in entire flow field.

Non-uniform flow is the type of flow in which velocity and other flow parameters at any instant change with respect to space.

$[\partial V/\partial s]_{t=constant}$ is not equal to zero. Distance or space is a criterion

LAMINAR AND TURBULANT FLOWS

Laminar Flow is a type of flow in which the fluid particles move along well-defined paths or stream-lines. The fluid particles move in laminas or layers gliding smoothly over one another. The behavior of fluid particles in motion is a criterion. Turbulent Flow is a type of flow in which the fluid particles move in zigzag way in the flow field. Fluid particles move randomly from one layer to another. Reynolds number is a criterion. We can assume that for a flow in pipe, for Reynolds No. less than 2000, the flow is laminar; between 2000-4000, the flow is transitional; and greater than 4000, the flow is turbulent.

COMPRESSIBLE AND INCOMPRESSIBLE FLOWS

Incompressible Flow is a type of flow in which the density (ρ) is constant in the flow field. This assumption is valid for flow Mach numbers with in 0.25. Mach number is used as a criterion. Mach Number is the ratio of flow velocity to velocity of sound waves in the fluid medium

Compressible Flow is the type of flow in which the density of the fluid changes in the flow field. Density is not constant in the flow field. Classification of flow based on Mach number is given below:

$M < 0.25$ – Low speed
$M <$ unity – Subsonic
M around unity – Transonic
$M >$ unity – Supersonic
$M >>$ unity, (say 7) – Hypersonic

ROTATIONAL AND IRROTATIONAL FLOWS

Rotational flow is the type of flow in which the fluid particles while flowing along stream-lines also rotate about their own axis.

Ir-rotational flow is the type of flow in which the fluid particles while flowing along stream-lines do not rotate about their own axis.

ONE, TWO AND THREE DIMENSIONAL FLOWS

The number of space dimensions needed to define the flow field completely governs dimensionality of flow field. Flow is classified as one, two and three- dimensional depending upon the number of space co-ordinates required to specify the velocity fields.

One-dimensional flow is the type of flow in which flow parameters such as velocity is a function of time and one space coordinate only.
For Ex., $V=V(x,t)$ – 1-D, unsteady ; $V=V(x)$ – 1-D, steady

Two-dimensional flow is the type of flow in which flow parameters describing the flow vary in two space coordinates and time.
For Ex., $V=V(x,y,t)$ – 2-D, unsteady; $V=V(x,y)$ – 2-D, steady

Three-dimensional flow is the type of flow in which the flow parameters describing the flow vary in three space coordinates and time.

For Ex., V=V(x,y,z,t) – 3-D, unsteady ; V=V(x,y,z) – 3D, steady

CONTINUITY EQUATION

Rate of flow or discharge (Q) is the volume of fluid flowing per second. For incompressible fluids flowing across a section,

Volume flow rate, Q= A ×V m3/s where A=cross sectional area and V= average velocity.

For compressible fluids, rate of flow is expressed as mass of fluid flowing across a section per second.

Mass flow rate (m) =(ρAV) kg/s where ρ = density.

Fig. Continuity Equation

Continuity equation is based on Law of Conservation of Mass. For a fluid flowing through a pipe, in a steady flow, the quantity of fluid flowing per second at all cross- sections is a constant.

Let v 1=average velocity at section [1], ρ1=density of fluid at [1], A1=area of flow at [1]; Let v2, ρ2, A2 be corresponding values at section [2].

Rate of flow at section [1]= ρ 1 A1 v1

Rate of flow at section [2]= ρ 2 A2 v2

? 1 A1 v1= ρ2 A2 v2

This equation is applicable to steady compressible or incompressible fluid flows and is called Continuity Equation. If the fluid is incompressible, 1 = ?2 and the continuity equation reduces to A 1 v1= A2 v2

For steady, one dimensional flow with one inlet and one outlet,

ρ 1 A1 v1− ρ2 A2 v2=0

For control volume with N inlets and outlets

_ i=1N (ρi Ai vi) =0 where inflows are positive and outflows are negative .

Velocities are normal to the areas. This is the continuity equation for steady one dimensional flow through a fixed control volume

When density isx2+ay2+az2)1/2

constant, _

Problem 1.0

Given the velocity field V= (4+xy+2t)i + 6x3j + (3xt2+z)k. Find acceleration of a fluid particle at (2,4,-4) at t=3.

[dV/dt]=[∂V/∂t]+u[∂V/∂x]+v[∂V/ ∂y]+w[∂V/∂z]

u. (4+xy+2t); v=6x3; w= (3xt2+z)

[∂V/∂x]= (yi+18x2j+3t2k); [∂V/∂y]= xi; [∂V/∂z]= k; [∂V/ ∂t] = 2i+6xtk . Substituting,

[dV/dt]= (2+4y+xy2+2ty+6x4)i + (72x2+18x3y+36tx2)j + (6xt+12t2+3xyt2+6t3+z+3xt2)k

The acceleration vector at the point (2,4, -4) and time t=3 is obtained by substitution,

a. 170i+1296j+572k; Therefore, a x=170, ay=1296, az=572

b. Resultant |a|= [1702+12962+5722]1/2 units = 1426.8 units.

VELOCITY POTENTIAL AND STREAM FUNCTION

Velocity Potential Function is a Scalar Function of space and time co-ordinates such that its negative derivatives with respect to any direction give the fluid velocity in that direction.

_ = _ (x,y,z) for steady flow.

u. -(∂_/∂x); v= -(∂_/∂y); w= -(∂_/ ∂z) where u,v,w are the components of velocity in

x,y and z directions.

In cylindrical co-ordinates, the velocity potential function is given by u u r. (?_/?r),

_ = (1/r)(∂_/∂_)

The continuity equation for an incompressible flow in steady state is

(∂u/∂x + ∂v/∂y + ∂w/∂z) = 0

Substituting for u, v and w and simplifying,

(∂2_ /∂x2 + ∂2_ /∂y2 + ∂2_/∂z2) = 0

Which is a Laplace Equation. For 2-D Flow, (∂2_ /∂x2 + ∂2_ /∂y2) =0

If any function satisfies Laplace equation, it corresponds to some case of steady incompressible fluid flow.

IRROTATIONAL FLOW AND VELOCITY POTENTIAL

Assumption of Ir-rotational flow leads to the existence of velocity potential. Consider the rotation of the fluid particle about an axis parallel to z-axis. The rotation
component is defined as the average angular velocity of two infinitesimal linear
segments that are mutually perpendicular to each other and to the axis of rotation.

Consider two-line segments _x, _y. The particle at P(x,y) has velocity components u,v in the x-y plane.

The angular velocities of _x and _y are sought.
The angular velocity of (_x) is {[v+ (∂v/∂x) _x –v] / _x} = (∂v/∂x) rad/sec

The angular velocity of (_y) is -{[u+ (∂u/∂y) _y –u] / _y} = -(u/y) rad/sec Counter clockwise direction is taken positive. Hence, by definition, rotation
component (_ _ z. is _z= 1/2 {(?v/?x)- (?u/?y)}. The other two components are
x. 1/2 {(?w/?y)- (?v/?z)}
y. 1/2 {(?u/?z)- (?w/?x)}
The rotation vector = _ = i_ x +j_y +k_z.
The vorticity vector(Ω) is defined as twice the rotation vector = 2_

PROPERTIES OF POTENTIAL FUNCTION

_ z. 1/2 {(?v/?x)- (?u/?y)}
_ x. 1/2 {(?w/?y)- (?v/?z)}
y. 1/2 {(?u/?z)- (?w/?x)};
Substituting u=- (∂_/∂x); v=- (∂_/∂y); w= - (∂ _/∂z) ; we get
_ z. 1/2 {(?/?x)(- ?_/?y) - (?/?y)(- ?_/?x)}
= ½{-(∂2_/∂x∂y)+ (∂2_/ ∂y∂x)} = 0 since _ is a continuous function.
Similarly, _ x=0 and _y =0
All rotational components are zero and the flow is irrotational.– Therefore, irrotational flow is also called as Potential Flow.

If the velocity potential (_) exists, the flow should be irrotational. If velocity potential function satisfies Laplace Equation, It represents the possible case of steady, incompressible, irrotational flow. Assumption of a velocity potential is equivalent to the assumption of irrotational flow.

Laplace equation has several solutions depending upon boundary conditions.
If _ 1 and _2 are both solutions, _1+ _2 is also a solution
2(_1)=0, ∇ 2(_2)=0, ∇ 2(_1+ _2)=0
Also if _
is a solution, C_1 is also a solution (where C=Constant)

STREAM FUNCTION (_)

Stream Function is defined as the scalar function of space and time such that its partial derivative with respect to any direction gives the velocity component at right angles to that direction. Stream function is defined only for two dimensional flows and 3-D flows with axial symmetry.

(∂_/∂x) = v ; (∂_/∂y) = -u

In Cylindrical coordinates, u r = (1/r) (∂_/∂_) and u_ = (∂_/∂r)

z = 0. Hence for 2-D flow, (∂2 _ /∂x2)+ (∂2_ /∂y2)

PROPERTIES OF STREAM FUNCTION

1.If the Stream Function (_) exists, it is a possible case of fluid flow, which may be rotational or irrotational.
2.If Stream Function satisfies Laplace Equation, it is a possible case of an irrotational flow.

EQUI-POTENTIAL & CONSTANT STREAM FUNCTION LINES

On an equi-potential line, the velocity potential is constant, _=constant or d(_)=0. _ = _(x,y) for steady flow.
d(_) = (∂_/∂x) dx + (∂_/∂y) dy.
d(_) = -u dx – v dy = -(u dx + v dy) = 0.
For equi-potential line, u dx + v dy = 0
Therefore, (dy/dx) = -(u/v) which is a slope of equi-potential lines
For lines of constant stream Function,
_ = Constant or d(_)=0, _ = _(x,y)
d(_) = (∂_/∂x) dx + (∂_/∂y) dy = vdx - udy
Since (∂_/∂x) = v; (∂_/∂y) = -u
Therefore, (dy/dx) = (v/u) = slope of the constant stream function line. This is the slope of the stream line.

The product of the slope of the equi-potential line and the slope of the constant stream function line (or stream Line) at the point of intersection = -1.
Thus, equi-potential lines and streamlines are orthogonal at all points of intersection.

Examples: Uniform flow, Line source and sink, Line vortex
Two-dimensional doublet – a limiting case of a line source approaching a line sink

RELATIONSHIP BETWEEN STREAM FUNCTION AND VELOCITY POTENTIAL

u. -(∂_/∂x), v= -(∂_/∂ y)
u. -(∂_/∂y), v= (∂_/∂x) ; Therefore,
-(∂_/∂x) = - (∂_/∂y) and - (∂_/∂ y) = (∂_/∂x)
Hence, (∂_/∂ x) = (∂_/∂y) and (∂_/∂y) = -(∂_/∂x)

Problem-1
The velocity potential function for a flow is given by _= (x2 –y2). Verify that the flow is incompressible and determine the stream function for the flow.
u=–(∂_/∂x)= –2x , v= – (∂_/∂y)= 2y
For incompressible flow, (∂u/∂x)+ (∂v/∂y)= 0
Continuity equation is satisfied. The flow is 2-D and incompressible and exists.

u. –(∂_/∂y); v= (∂_/∂x) ; (∂_/∂y)= –u = 2x;

_ = 2xy+F(x) + C ; C=Constant

(_/ ∂x)= v = 2y; _ = 2xy+ F(y)+C Comparing we get, _= 2xy+C

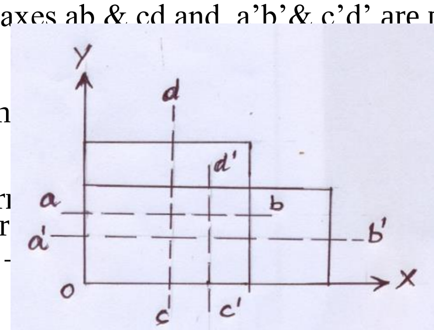
Problem-2.
The stream function for a 2-D flow is given by _ = 2xy. Calculate the velocity at the point P (2,3) and velocity function (_).
Given _ = 2xy; u= –(∂_/∂y) = -2x; v= (∂_/∂x)=2y
Therefore, u= –4 units/sec. and v= 6 units/sec.
Resultant=√(u2+v2) = 7.21 units/sec.
(∂_/∂x)= –u = 2x; _= x2+F(y)+C; C=Constant.
(∂_/∂y) = –v= –2y; _= – y2+F(x)+C,
Therefore, we get, _= (x2 –y2) +C

TYPES OF MOTION

A Fluid particle while moving in a fluid may undergo any one or a combination of the following four types of displacements:

1. Linear or pure translation
2. Linear deformation
3. Angular deformation
4. Rotation.

(1) Linear Translation is defined as the movement of fluid element in which fluid element moves from one position to another bodily – Two axes ab & cd and a'b'& c'd' are parallel.



(2) Linear deformation is defined as deformation tion – axes are parallel, but length changes.

(3) Angular deformation, also called shear defor change in the angle contained by two adjacent sides. The angular
shear strain rate = ½(__ 1 + __2) = ½(?v/?x -

Fig Angular deformation Fig. Rotation

(4) Rotation is defined as the movement of the fluid element in such a way that both its axes (horizontal as well as vertical) rotate in the same direction. Rotational

components are:

_ z. 1/2 {(?v/?x)- (?u/?y)}

_ x. 1/2 {(?w/?y)- (?v/?z)}

y. 1/2 {(?u/?z)- (?w/?x)}. Vorticity (?) is defined as the value twice of the rotation

and is given as 2_

Problem-3.

Find the vorticity components at the point (1,1,1) for the following flow field;

u=2x2+3y, v= – 2xy+3y2+3zy, w= –(3z2¸2) +2xz – 9y2z

? =2_ where **Ω**= Vorticity and _= component of rotation.

Ω **Ω**x. {(?w/?y)- (?v/?z)}= –18yz–3y= –21 units

Ω y. {(?u/?z)- (?w/?x)}= 0–2z= –2 units

z. {(?v/?x)- (?u/?y)}= – 2y –3 = –5 units

KINEMATIC FLUID FLOWS

KINEMATIC:- With Motion of the Fluid with out Considering force is Called Kinematics.

Types of Fluid flows:-

1. Study flow
2. Unstudy flow

1. **Study flow:-** Do not change with respective time of flow like velocity, Pressure & density

$$\left(\frac{du}{dt}\right)_{x_0 y_0 t_0} = 0 ; \left(\frac{dp}{dt}\right)_{x_0 y_0 t_0} = 0 ; \left(\frac{dp}{dt}\right)_{x_0 y_0 t_0} = 0$$

2. **Unstudy flow:-** The Fluid particle with respective change the time of a flow.

$$\left(\frac{du}{dt}\right)_{x_0 y_0 t_0} \neq 0 ; \left(\frac{dp}{dt}\right)_{x_0 y_0 t_0} \neq 0 ; \left(\frac{dp}{dt}\right)_{x_0 y_0 t_0} \neq 0$$

Uniform flow:- The velocity at any given time doesn't change with respective to space.

$$\left(\frac{du}{dg}\right)_{t = Const.} = 0$$

Non-uniform Flow:- The Velocity at any given time changes with respective to Space.

$$\left(\frac{du}{dy}\right)_{\tau = const} \neq 0$$

Compressible flow:~ the density of the Fluid changes from point to point where density is not Constant

$$\rho \neq \text{Constant}$$

In - Compressible flow:~ the density of the Fluid is Constant where density is Const.

$$\rho = \text{Constant}$$

Rotational flow:~ the Fluid particle which flows for an own - Axis.

Irrotational flow:~ the Fluid particle which does n't flow for an own - Axis.

One dimensional flow:~ with flow particles such as velocity Function of time & one space of Cordinate.

$u, v, w \longrightarrow$ velocity Coordinates

$u = f(x)$; $v = 0$; $w = 0$

$u = 0$; $v = f(y)$; $w = 0$

$u = 0$; $v = 0$; $w = f(z)$.

Two dimensional flow:~ is the Flow particle Such as velocity Function of time and two

Squares of Co-ordinates.

$u, v, w \rightarrow$ velocity Co-ordinates

$\rightarrow u = f(x, y)$; $v = f(y, x)$; $w = 0$

$\rightarrow v = 0$; $v = f(y, z)$; $w = f(z, y)$

$u = f(x, z)$; $v = 0$; $w = f(z, x)$

Three dimensional flow:-

is the flow particles such as velocity function of time & three spaces of co-ordinates.

$u, v, w \rightarrow$ velocity co-ordinates

$u = f(x, y, z)$, $v = f(y, z, x)$; $w = f(z, x, y)$

Rate of flow:-

Quantity of a fluid flow per second through a section of a pipe or channel.

Q = Discharge = m^3/sec

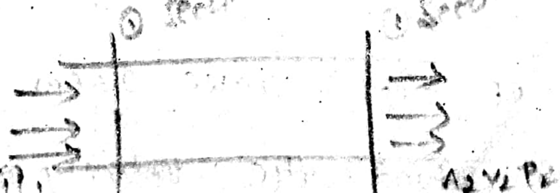
$$Q = A \times V = \frac{\pi}{4} d^2 \times v = m^2 \times m/sec = m^3/sec$$

A = Cross section Area of pipe.

V = velocity.

Continuity Equation:-

Conservation of mass is called continuity equation. The fluid flowing through the pipe at all the cross section.



inlet = outlet

$A_1V_1 = A_2V_2$

$\therefore Q_{in} = Q_{out}$

The diameteores of a pipe 2 sections 20cm, 50cm, respectively. Find the discharge through the pipe if the velocity of water flowing through the pipe at section 1 is 10m/s. determine the velocity at section 2?

Sol:

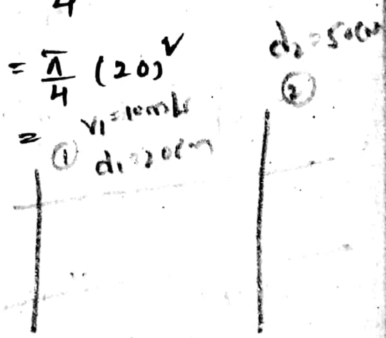
$A_1V_1 = A_2V_2$ $\qquad A = \frac{\pi}{4}(d)^2$

$= \frac{\pi}{4}(20)^2$

$=$

find $Q = 0.003\ m^3/sec$

$V_2 = 1.578$



$Q_{inlet} = A_1V_1$

$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.02)^2 = 0.0003\ m^2$ $\qquad \because d_1 = 20cm = 0.02m$

$Q = A_1V_1 = 0.0003 \times 10 = 0.003\ m^3/sec$

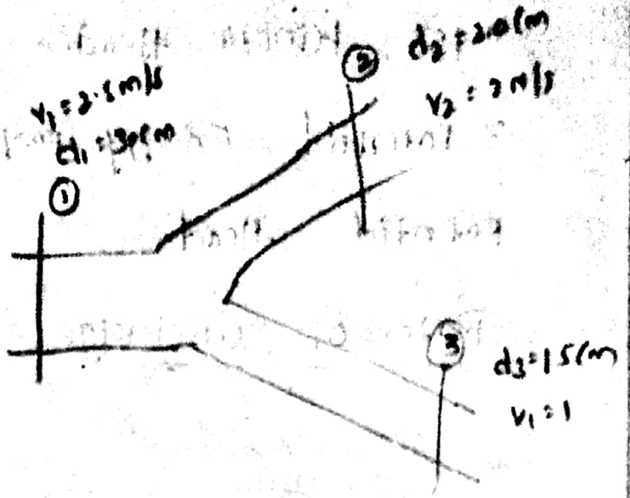
$A_1V_1 = A_2V_2$ $\qquad A_2 = \frac{\pi}{4}(0.05)^2 = 0.0019\ m^2$.

$V_2 = \frac{A_1V_1}{A_2} = \frac{0.003}{0.0019}$

$= 1.578\ m/sec.$

A 30cm dia. pipe conveying water branches into two pipes of dia. 20cm & 15cm average velocity in 30cm pipe is 2.5m/sec. Find the discharge of pipe. Take 20cm pipe velocity is 2m/sec. Find the velocity of 15cm pipe.

Sol Areas



$A_1 = \frac{\pi}{4}(0.3)^2 = 0.07 m^2$

$A_2 = \frac{\pi}{4}(0.2)^2 = 0.031 m^2$

$A_3 = \frac{\pi}{4}(0.15)^2 = 0.017 m^2$

discharges.

$Q_1 = A_1 \times V_1 = 0.07 \times 2.5 = 0.175 \; m^3/sec$

$Q_2 = A_2 \times V_2 = 0.031 \times 2 = 0.062 \; m^3/sec$

$Q_1 = Q_2 + Q_3$

$Q_3 = Q_1 - Q_2 = 0.175 - 0.062 = 0.113 \; m^3/sec$

$Q_3 = A_3 V_3$

$V_3 = \frac{Q_3}{A_3} = \frac{0.113}{0.017} = 6.647 \; m/sec$

Energy Equation :-

They are 3 types of Energy

1. Pressure Energy = "$P/\rho g$"
2. Kinetic Energy = "$V^2/2g$"
3. Potential Energy = "Z"

$P.E + K.E + P.E = Const.$

$\frac{P}{\rho g} + \frac{V^2}{2g} + Z = Const.$ → Bernoulli's Equation.

Varies of Energy :-

1. Pressure Energy per unit weight of Fluid are pressure Head.

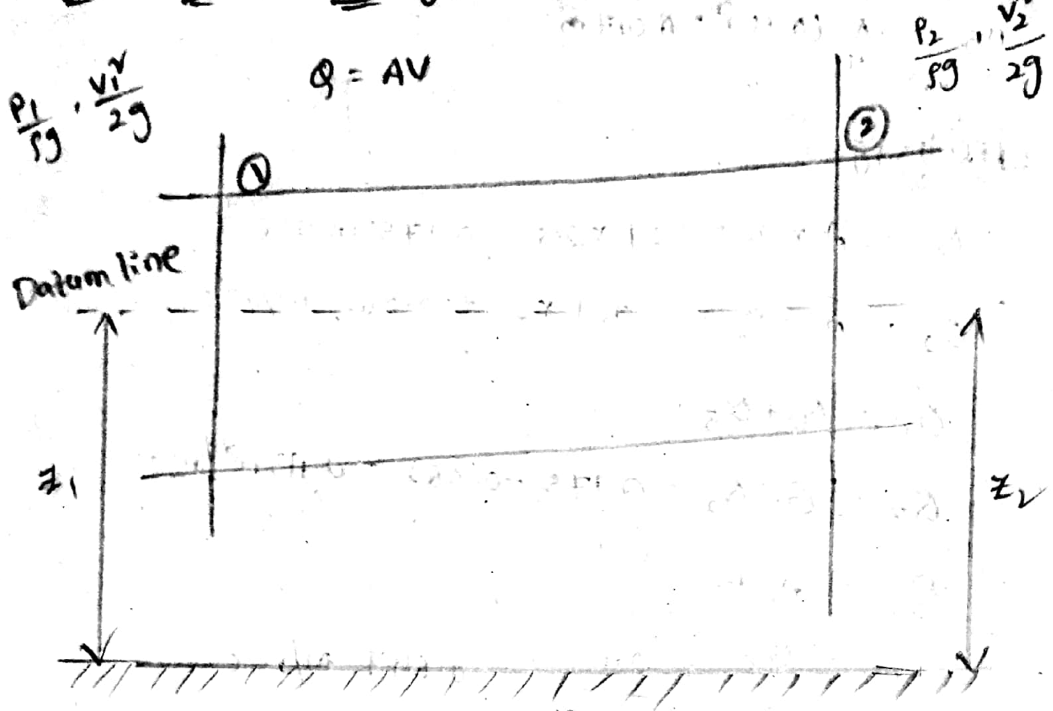
2. Kinetic Energy per unit weight of Fluid

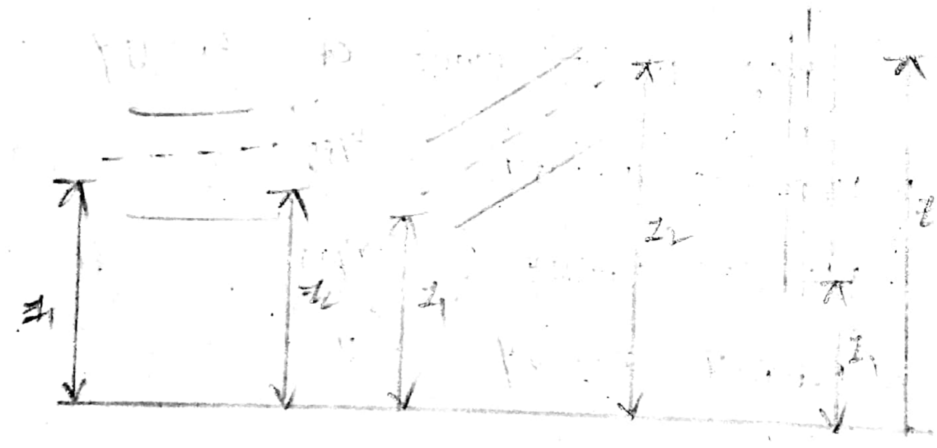
are Kinetic Head.

3. Potential Energy per unit weight of fluid are Potential Head.

Rate of discharge :-

$$Q = AV$$





Continuity Equation : inlet = outlet

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

A 20cm dia. of pipe under Pressure of 20N/cm² and velocity 3m/sec the datum line 5m. above the surface level. Determine the Pressure Head Potential Head & Kinetic Head & total Head.

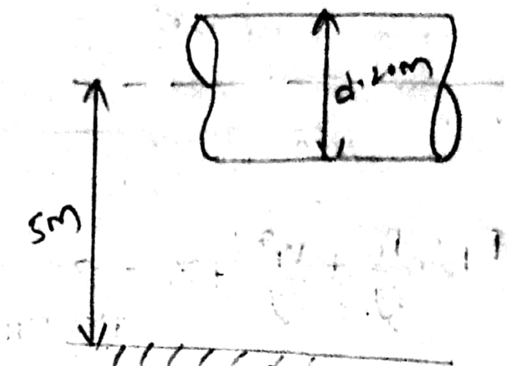
Sol:

$P = 20N/cm^2$

$= 20 \times 10^4 N/m^2$

$V = 3m/sec$

$Z = 5m$



Pressure Energy $\frac{P}{\rho g} = \frac{20 \times 10^4}{1000 \times 9.81} = 20.38$

Kinetic Energy $\frac{V^2}{2g} = \frac{3 \times 3}{2 \times 9.81} = 0.45$

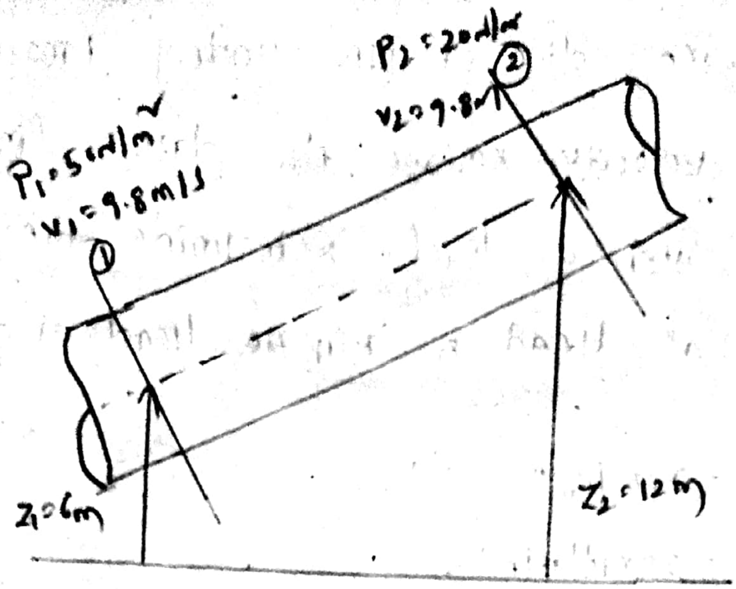
Potential Energy $Z = 5$

Total Head $= \frac{P}{\rho g} + \frac{V^2}{2g} + Z = \text{Constant}$

$= 20.38 + 0.45 + 5 = 25.83m.$

A pipe dia. 300mm of inlet and outlet an inlet pressure of Section 1 50N/m² and datum line 6m outlet pressure of Section 2 20N/m² 9.8m/s is velocity of two section. The datum line at section 2 is 12m Find the total Energy at two sections and Head loss of water.

Sol:-



$$E_1 = \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{50}{1000 \times 9.81} + \frac{9.81^2}{2 \times 9.81} + 6$$

$$= 10.9100$$

$$E_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{20}{1000 \times 9.81} + \frac{9.81^2}{2 \times 9.81} + 12$$

$$= 16.90$$

$$T_{Loss} = E_2 - E_1 = 5.99$$

A pipe long slope down at section 1 is 2m and section 2 is 4m. From 300mm and 600mm dia. at the end carries of discharge 150 lit/sec of water. pressure at section 2 is 80N/m². Find the velocities and pressure at section 1.

Sol:-

Given data:-

$Z_1 = 2m$, $Z_2 = 4m$

Discharge = 150 Lit

$d_1 = 300mm$

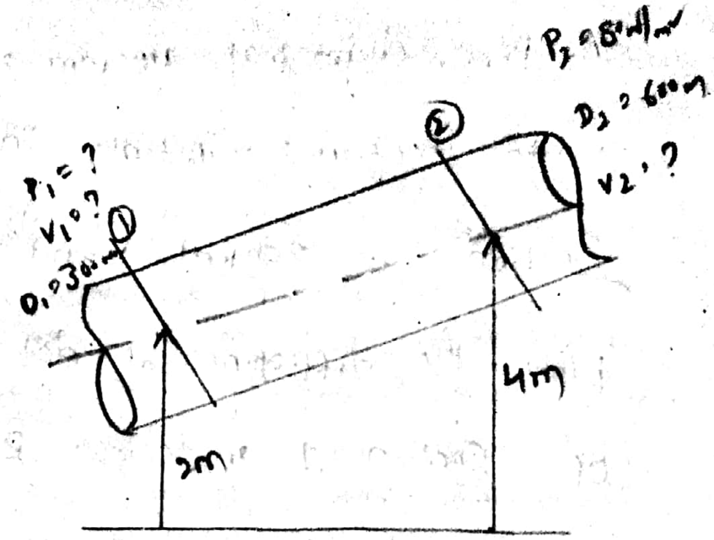
$d_2 = 600mm$

$P_1 = ?$

$V_1 = ?$

$P_2 = 80\ N/m^2$

$V_2 = ?$



Cross Sectional Area.

$$A_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} (0.3)^2 = 0.070 m^2$$

$$A_2 = \frac{\pi}{4} d_2^2 = \frac{\pi}{4} (0.6)^2 = 0.282 m^2$$

$Q = 150$ Lit/sec

$= 0.150\ m^3/sec$

$Q_1 = A_1 V_1$

$$V_1 = \frac{Q}{A_1} = \frac{0.150}{A_1} = \frac{0.150}{0.010} = 2.14\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{0.150}{0.282} = 0.53\ m/s$$

inlet = outlet

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{P_1}{9.81} + \frac{(2.14)^2}{1 \times 9.81} + 2 = \frac{80}{9.81} + \frac{(0.53)^2}{2 \times 9.81} + 4$$

$$P_1 = \frac{(2.14)^2}{1 \times 9.81} + 2 = \frac{80}{9.81} + \frac{(0.53)^2}{2 \times 9.81} + 4$$

$P_1 = 97.4\ N/m^2$

A pipe diameter of 400mm & 600mm at the Section 1 and Section 2. initially intensity of pressure at Section 1 is $350 KN/m^2$ and the pressure 2 is $100 KN/m^2$. Find the difference at datum line and velocities of Section - 1 & Section - 2. The rate of discharge is 400 lit/sec.

Take datum line at Section 1 is 3m.

Sol:-

Given data:-

$d_1 = 400mm = 0.4m$

$d_2 = 600mm = 0.6m$

$P_1 = 350 KN/m^2 = 350 \times 10^3 N/m^2$

$P_2 = 100 KN/m^2 = 100 \times 10^3 N/m^2$

Area of Section 1 $= A_1 = \frac{\pi}{4} d_1^2$

$= \frac{\pi}{4} (0.4)^2 = 0.125$

$A_2 = \frac{\pi}{4} (d_2)^2$

$= \frac{\pi}{4} (0.6)^2 = 0.282$

$Q = 400 Lit/sec = 0.4 m^3/sec.$

$A_1 \times V_1 = 0.4$

$0.125 \times V_1 = 0.4$

$V_1 = 3.2 m/sec.$

$A_2 \times V_2 = 0.4.$

$$V_2 = \frac{0.4}{0.282} = 1.418.$$

By the Energy Equation

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$Z_1 - Z_2 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} - \frac{P_1}{\rho g} - \frac{V_1^2}{2g}.$$

$$Z_1 - Z_2 = \frac{1}{\rho g}(P_2 - P_1) + \frac{1}{2g}(V_2^2 - V_1^2)$$

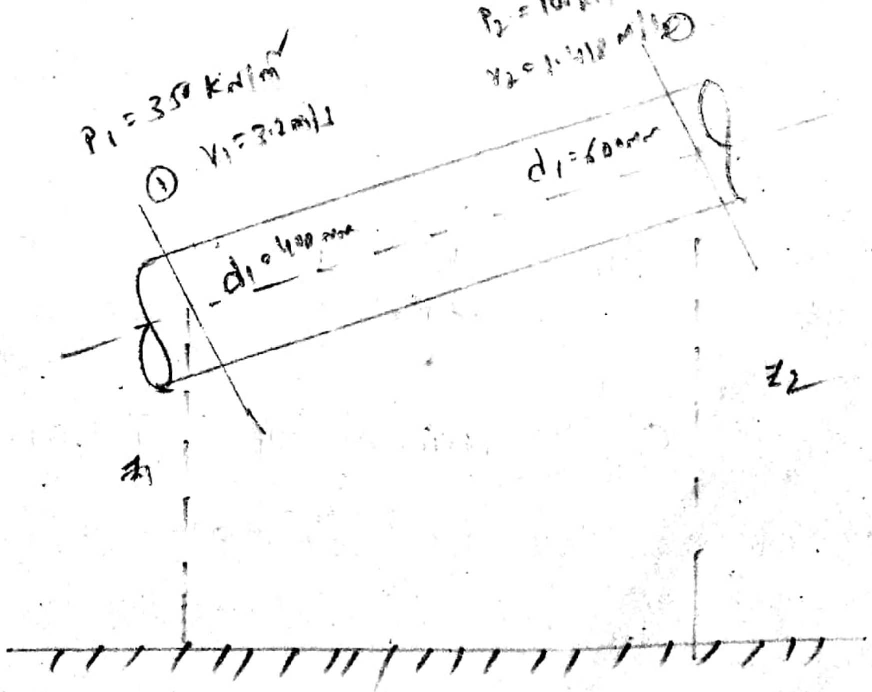
$$Z_1 - Z_2 = \frac{1}{1000 \times 9.81}(100 \times 10^3 - 350 \times 10^3) + \frac{1}{2 \times 9.81}\left[(1.418)^2 - (3.2)^2\right]$$

$$Z_1 - Z_2 = \frac{-256}{9.81} + \frac{1}{19.62}(-8.229)$$

$$= -25.484 - 0.419$$

$$Z_1 - Z_2 = -25.903$$

The diagram for the above data



Take datum line is Section 1. = 3m

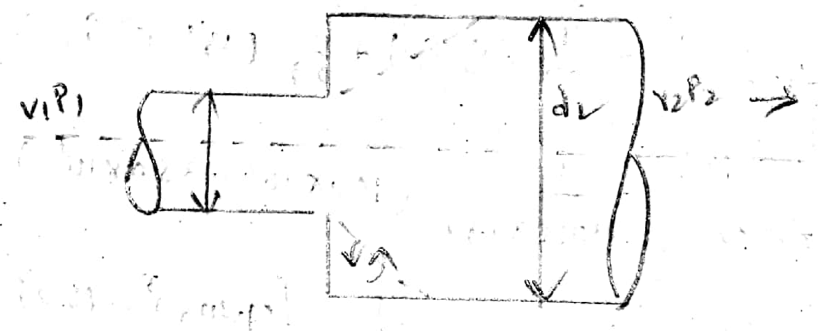
$$z_1 = 3m$$

$$z_1 - z_2 = -25.903$$

$$3 - z_2 = -25.903$$

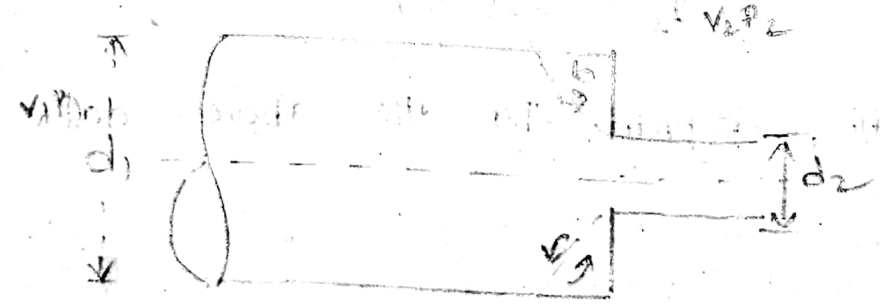
$$z_2 = 28.903$$

Loss of Head due to Sudden Expansion:~



$$H_e = \frac{v^2}{2g} = \frac{(v_1 - v_2)^2}{2g}$$

Loss of Head due to Sudden Contraction:~



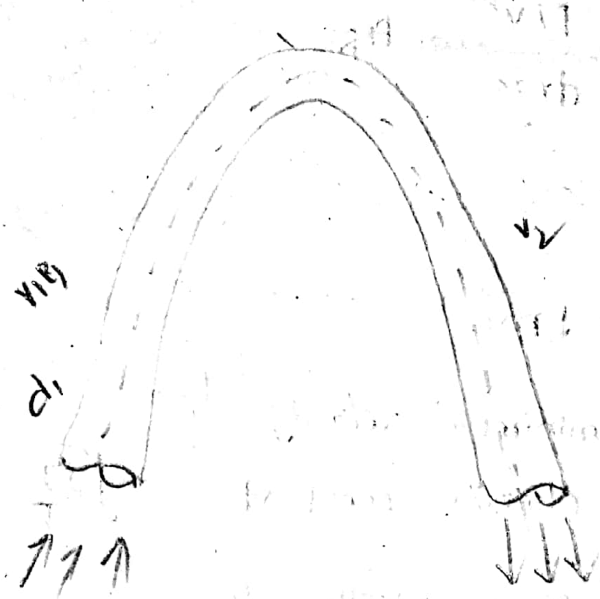
$$H_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]$$

C_c = Co-efficient of friction.

$$h_e = 0.5 \cdot \frac{V_2^2}{2g}$$

Loss of Head due to pipe in bend:-

The velocity of flow changes due to the separation of flow from the bending places

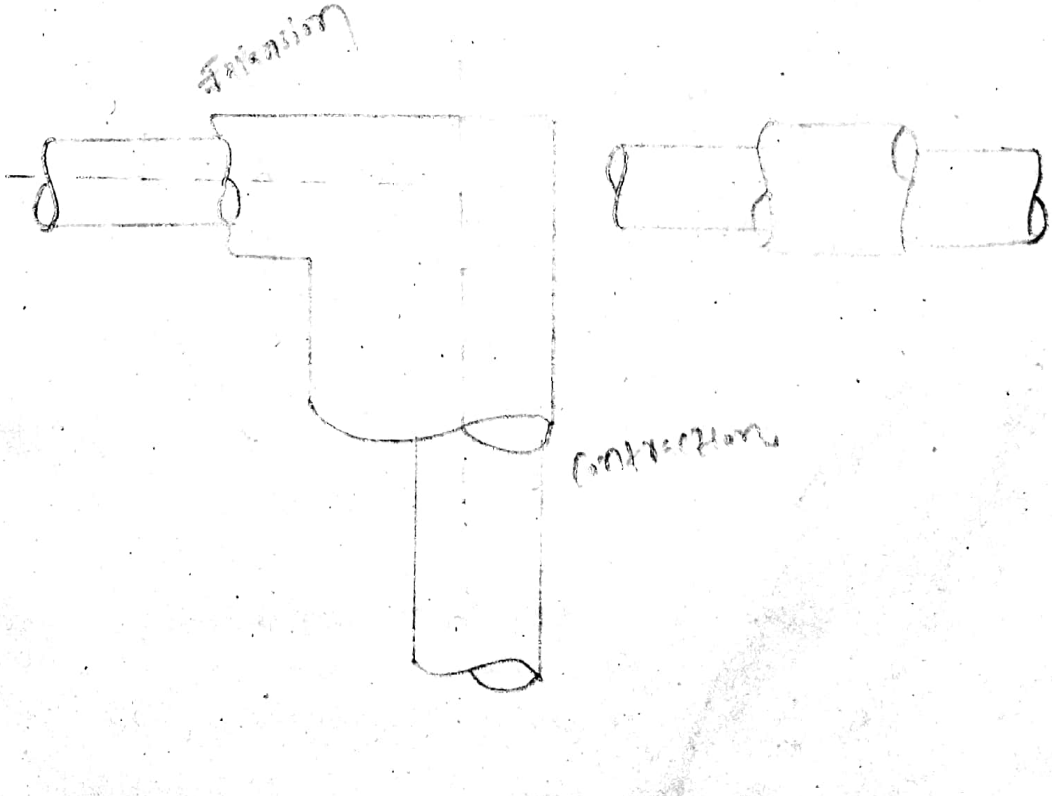


$$H_b = K \cdot \frac{v^2}{2g}$$

K - chang in -

v - velocity

Loos of due to pipe Fitting:-



Darcy Equation:~

F - Friction

wet $\longrightarrow$ $\frac{4FLV^2}{d \times 2g} = h_f$

L - length of pipe.

V - velocity

d - dia. of pipe.

dry $\longrightarrow$ $\frac{FLV^2}{d \times 2g} = h_f$

Chezy Formula:~

$V = C\sqrt{mi}$

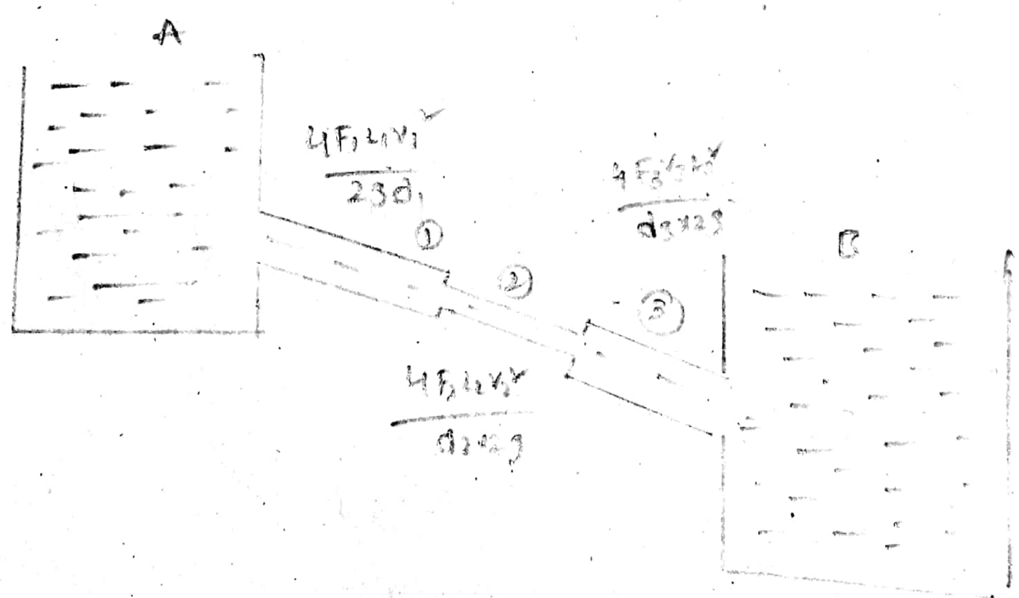
V = minimum velocity

C = Chezy's constant $= \sqrt{\frac{\rho g}{F}}$

m = mean depth $= d/4$.

i = Loss of Head due to Friction $= \sqrt{\frac{h_f}{L}}$

Pipes in Series:~



$$H_F = \frac{4F_1L_1V_1^2}{d_1 \times 2g} + \frac{4F_2L_2V_2^2}{d_2 \times 2g} + \frac{4F_3L_3V_3}{d_3 \times 2g}$$

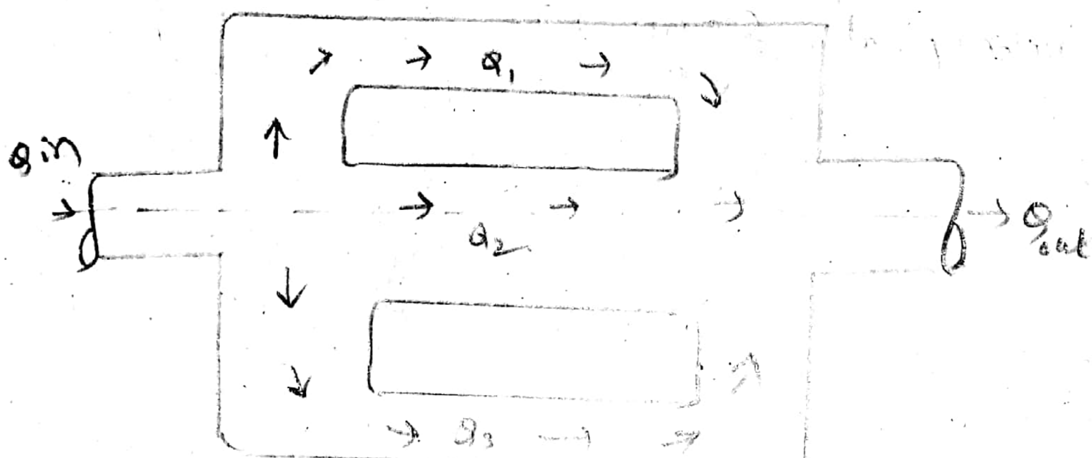
$$= \frac{4F}{2g}\left[\frac{L_1V_1^2}{d_1} + \frac{L_2V_2^2}{d_2} + \frac{L_3V_3^2}{d_3}\cdots\right]$$

With Consider Minor Losses

$$H_f = 0.5\frac{v^2}{2g}\left[\frac{4F_1L_1V_1^2}{d_1 \times 2g}\right] + 0.5\frac{v^2}{2g}\left[\frac{4F_2L_2V_2^2}{d_2 \times 2g}\right] +$$

$$\frac{v^2}{2g}\left[\frac{4F_3L_3V_3^2}{d_3 \times 2g}\right].$$

Flow through parallel pipes:-



$$Q_{inlet} = Q_{outlet}$$

$$Q_{out} = Q_1 + Q_2 + Q_3$$

Head loss $Q_1 = Q_2 = Q_3$

$$Q_1 + Q_2 + Q_3 = Q_{outlet}$$

$$Hf_1 = \frac{4F_1L_1V_1^2}{d_1 \times 2g}$$

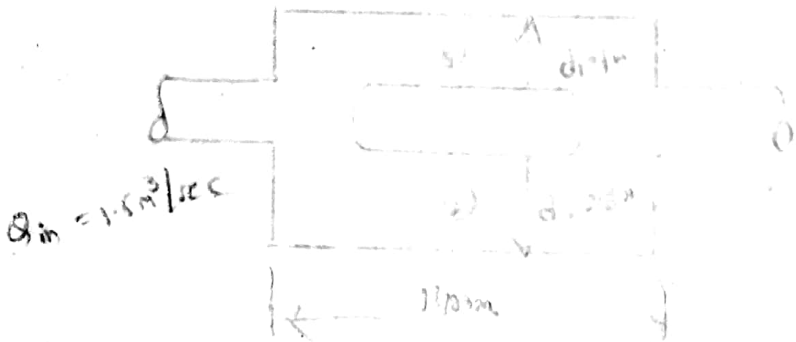
$$Hf_2 = \frac{4F_2L_2V_2^2}{d_1 \times 2g}$$

$$h_{f_3} = \frac{4 f_3 L_3 V_3^2}{d_3 \times 2g}$$

$$F = F_1 = F_2 = F_3$$

A pipe divided into 2 pipes which again forms 1 pipe. The length and diameter of the pipe 1 is 1400 m and 1m and the pipe section 2 1400 m and 2.8 m the rate of flow $1.5 m^3/sec$, if the co-efficient of friction for all sections 0.003. Find the rate of flow section 1 & 2 and also find velocity at section 1 & 2

Sol:



$f = 0.003$

$Q_1 = ?$ $\quad$ $V_1 = ?$

$Q_2 = ?$ $\quad$ $V_2 = ?$

Equivalent pipes

$$\frac{4 F_1 L_1 V_1^2}{d_1 \times 2g} = \frac{4 f_2 L_2 V_2^2}{d_2 \times 2g}$$

$$\frac{V_1^2}{d_1} = \frac{V_2^2}{d_2}$$

$$\frac{V_1^2}{1} = \frac{V_2^2}{2.8}$$

$$V_1 = \frac{\sqrt{V_2^2}}{\sqrt{2.8}} \Rightarrow V_1 = \frac{V_2}{1.67}$$

$$Q_1 = A_1 \times V_1$$

$$= \frac{\pi}{4} d_1^2 \times \frac{V_2}{1.67}$$

$$= \frac{\pi}{4} \times 1 \times \frac{V_2}{1.67}$$

$$= 0.47 V_2$$

$$Q_2 = A_2 \times V_2$$

$$= \frac{\pi}{4} d_2^2 \times V_2$$

$$= \frac{\pi}{4} \times (2.8)^2 \times V_2$$

$$= 6.65 V_2$$

$$Q_{inlet} = Q_1 + Q_2$$

$$1.5 = 0.47 V_2 + 6.65 \Rightarrow V_2 = 0.226.$$

$$V_1 = \frac{0.226}{1.67} = 0.135$$

$$Q_1 = \frac{\pi}{4} (1)^2 \times 0.135 = 0.106$$

$$Q_2 = \frac{\pi}{4} (2.8)^2 \times 0.226 = 1.391$$

$$Q_{out} = Q_1 + Q_2$$

$$= 0.106 + 1.391 = 1.497$$

$$Q_{in} = Q_{out} \Rightarrow 1.5 = 1.497$$

$$\text{Head loss} = 1.5 - 1.497 = 0.003.$$

<u>Flow</u> <u>through</u> <u>branched</u> <u>pipes</u> :-

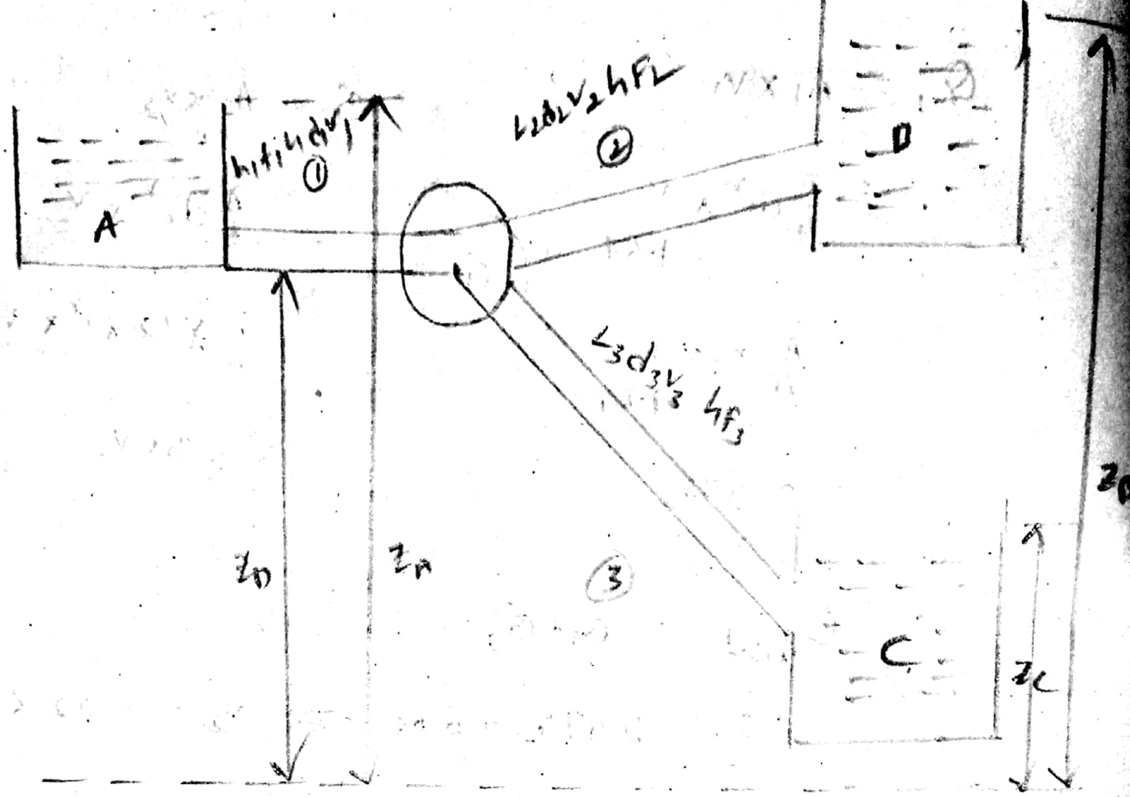
$$F = F_1 = F_2 = F_3$$

Flow Conditions

(i) A to D $\quad Z_A = Z_D + hf_1 + \frac{P_D}{\rho g}$

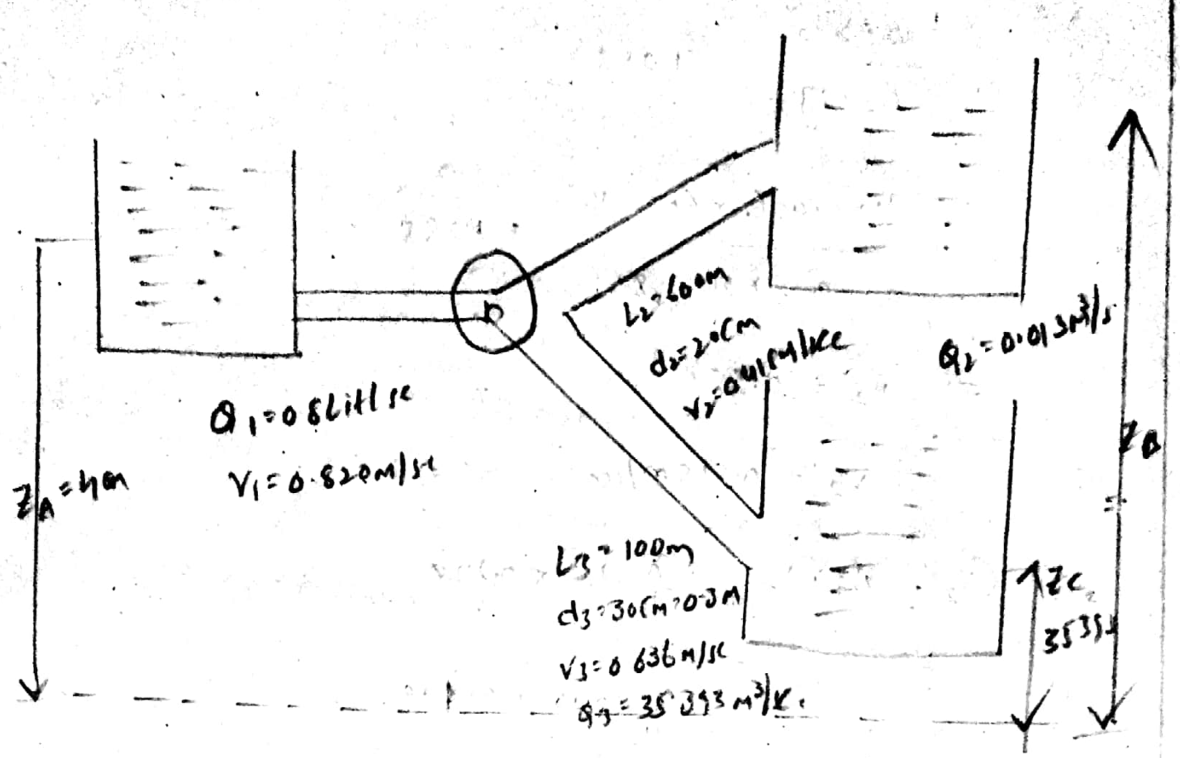
(ii) B to D $\quad Z_D + Hf_2 = Z_B + P_D/\rho g$

iii) C to D $Z_C + H_{f_3} = Z_D + \frac{P_D}{\rho g}$



Three Reservoiers Connected by pipe System Find the discharge for the reservoyer B and C. if the flow as A is 58 lit/sec. The Length of pipe A to D 1200m. & diameter 30 cm the length of pipes D to B and D to C is 600m and 800m and the diameters of DB and DC 20cm and 30cm. The Height of Reservoyer From A reference of datum line 40m and Reservoyer B is 30m and also Find the datum line of C. and velocities at all Section. Co-efficient of Friction 0.006.

Sol:



(i) Fluid Flow through A to D.

$$Z_A = Z_D + H_{F_1} + \frac{P_D}{\rho g}$$

$$H_{F_1} = \frac{4F_1L_1V_1^2}{d_1 \times 2g} = \frac{4 \times 0.006 \times 1200 \times V_1^2}{0.3 \times 2 \times 9.81} = 3.288$$

Discharge $Q = 0.058\ m^3/sec$

$$Q = A_1 \times V_1$$

$$0.058 = \frac{\pi}{4}(0.3)^2 \times V_1$$

$$V_1 = 0.820\ m/sec.$$

$$40 = Z_D + 3.288 + \frac{P_D}{\rho g}$$

$$Z_D + \frac{P_D}{\rho g} = 40 - 3.288 = 36.712$$

(ii) Fluid flow through B to D

$$Z_B + H_{F_2} = Z_D + \frac{P_D}{\rho g}$$

$$38 + H_{F_2} = 36.7$$

$$H_{F_2} = 36.712 - 38$$

$$H_{F_2} = 1.288$$

$$\frac{4F_2 L_2 V_2^2}{d_2 \times 2g} = 1.288$$

$$\Rightarrow \frac{4 \times 0.006 \times 600 \times V_2^2}{0.2 \times 2 \times 9.81} = 1.288$$

$$\Rightarrow V_2^2 = 0.350$$

$$V_2 = 0.518 \text{ m/sec}$$

$$Q_2 = A_2 \times V_2 = \frac{\pi}{4} d_2^2 \times 0.418$$

$$= \frac{\pi}{4} (0.2)^2 \times 0.418$$

$$Q_2 = 0.018 \text{ m}^3\text{/sec.}$$

iii) fluid through C to D

$$Z_C + HF_3 = Z_D + \frac{P_D}{\rho g}$$

$$Z_C + \frac{4F_3 L_3 V_3^2}{d_3 \times 2g} = 36.7$$

$$Z_C = 36.7 - \frac{4 \times 0.006 \times 800 \times V_3^2}{(0.3) \times 2 \times 9.81}$$

We know $Q_A = Q_B + Q_C$

$$Q_C = Q_A - Q_B = 0.058 - 0.018$$

$$Q_C = 0.04$$

$$Q_C = A_3 \times V_3$$

$$V_3 = \frac{0.045}{\frac{\pi}{4} d_3^2} = 0.565 \text{ m/sec.}$$

$$Z_C = 35.671.$$

A Fluid through a three reservoirs A, B & C are Connected by a pipe Here 1000 m, 900 m, 700 m and the diameters of the pipe 300mm, 180mm & 150mm respectively where the velocities at all pipes 1·08 m/sec, 1·816 m/sec & 2·37 m/sec. Find the datum lines at Z_B & Z_C. take co-efficient of friction 0·005. The datum line of $Z_A = 60m$.

Sol:-

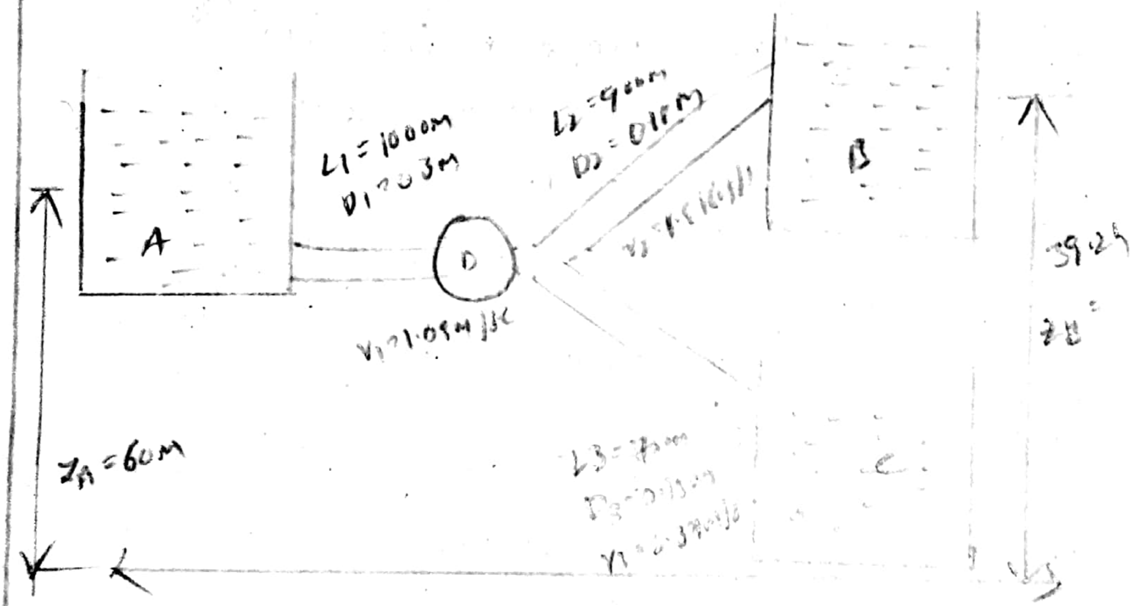
$L_A = 1000m$ $D_A = 300mm = 0.3m$

$L_B = 900m$ $D_B = 180mm = 0.18m$

$L_C = 700m$ $D_C = 150mm = 0.15m$

$Z_A = 60m$

$V_A = 1.08 m/sec$, $V_B = 1.816 m/se$, $V_C = 2.37 m/sec.$



(i) Fluid through A to D :-

$$Z_A = Z_D + HF_1 + \frac{P_D}{\rho g}$$

$$60 = Z_D + 3.96 + \frac{P_D}{\rho g}$$

$$HF_1 = \frac{4 F_1 L_1 V_1^2}{d_1 \times 2g}$$

$$= \frac{4 \times 0.005 \times (1.08)^2 \times 1000}{(0.3) \times 2 \times 9.81} = 3.96$$

$$z_D + \frac{P_D}{\rho g} = 60 - 3.46 = 56.04$$

(ii) Fluid through B to D :-

$$z_B + H_{F_2} = z_D + \frac{P_D}{\rho g}$$

$$H_{F_2} = \frac{4 \times 0.005 \times 900 \times 1.8(1)^2}{0.18 \times 2 \times 9.81} = 16.80$$

$$z_B + 16.80 = 56.04$$

$$z_B = 39.24m$$

Fluid through C to D :-

$$z_C + H_{F_3} = z_D + \frac{P_D}{\rho g}$$

$$H_{F_3} = \frac{4 \times 0.005 \times 700 \times (2.37)^2}{0.15 \times 2 \times 9.81}$$

$$H_{F_3} = 26.71$$

$$z_C = 56.04 - 26.71$$

$$z_C = 29.33$$

$$Q_A = A_1 \times V_1 = \frac{\pi}{4} d^2 \times 1.08$$

$$= \frac{\pi}{4}(0.3)^2 \times 1.08 = 0.076 \ m^3/sec$$

$$Q_2 = A_2 \times V_2 = \frac{\pi}{4} \times (0.18)^2 \times 1.816$$

$$= 0.046 \ m^3/sec.$$

$$Q_3 = A_3 \times V_3 = \frac{\pi}{4}(0.15)^2 \times 2.37 = 0.041 \ m^3/sec.$$

A pipe line section 1 is 60 cm diameter is devided into two pipes of section 2 & 40cm & section 3 is 30cm respectively if the rate of flow in main pipe is 1.5 m³/sec. the mean velocity at section three is 7.5 m/sec. Find the rate of flow at section 2 & find the velocities at section 1 and 2 & 3.

Sol:- Given data:-

at section 1:-

Diameters $D_1 = 60\ cm = 0.6m.$

$Q_1 = 1.5\ m^3/sec$

$V_1 = ?$

at section 2:-

$D_2 = 40\ cm = 0.4m$

$Q_2 = ?$

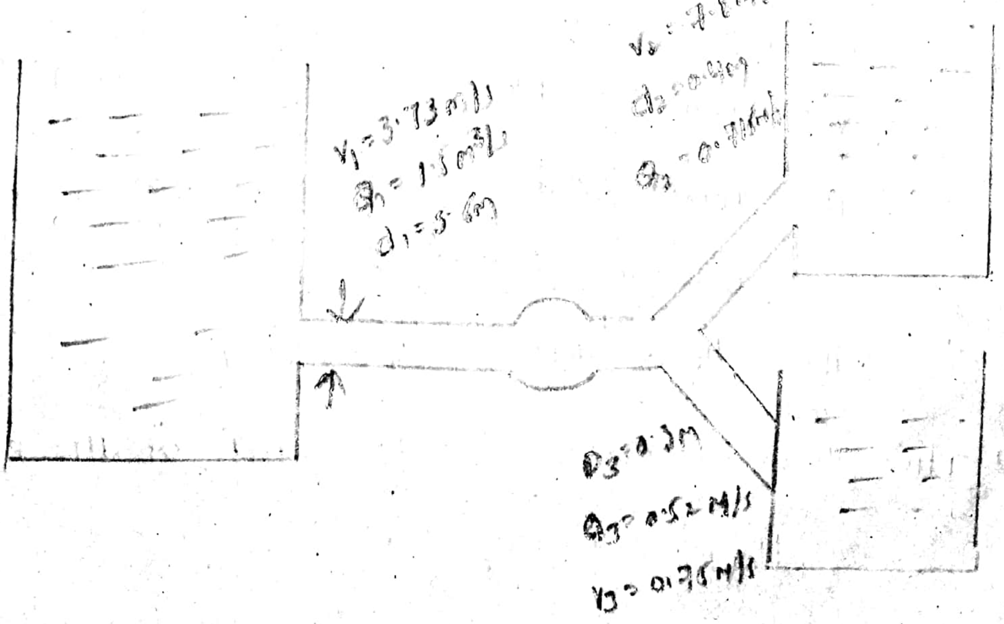
$V_2 = ?$

at section 3:-

$D_3 = 30cm = 0.3m$

$Q_3 = 0.525\ m^3/sec$

$V_3 = 7.5\ m/sec.$



$A_1 = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.6)^2 = 0.4 m^2$

$Q_1 = A_1 \times V_1 \Rightarrow V_1 = \frac{Q_1}{A_1} = \frac{1.5}{0.4} = 3.75$ m/sec.

$A_3 = \frac{\pi}{4} d_3^2 \Rightarrow \frac{\pi}{4}(0.3)^2 = 0.070 m^2$

$Q_3 = A_3 \times V_3 = 0.070 \times 7.5 = 0.525 m^3/sec.$

$Q_1 = Q_2 + Q_3$

$Q_2 = Q_1 - Q_3 = 1.5 - 0.525 = 0.975 m^3/sec.$

$Q_2 = A_2 \times V_2$

$V_2 = \frac{Q_2}{A_2} = \frac{0.975}{0.125} = 7.8$

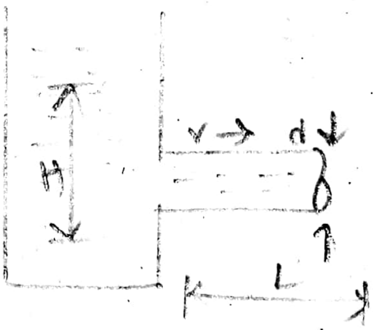
$A_2 = \frac{\pi}{4} d_2^2$

$= \frac{\pi}{4}(0.4)^2$

$= 0.125 m^2$

Power Transmitted through Pipes:-

The Power is Transmitted through pipe by Flowing water depends upon weight of liquid Flowing through the pipe and total head avail-able at the end of the pipe.



where L = length of pipe

d = dia "

v = velocity " Flow

H = Total Head available at inlet of pipe

H_f = Head loss due to Friction

i) The Head available at the outlet of the pipe

$H_{out} = H_i - H_f$

(ii) Weight of water Flow through pipe

$$W = \rho \times g \times \text{discharge}$$
$$= \rho \times g \times Q_{out}$$
$$= \rho \times g \times A \times V$$
$$= \rho \times g \times \frac{\pi}{4} d^2 \times V.$$

(iii) Power Transmitted at the outlet of the pipe

P = weight of water × Head of outlets.

$$P = \rho \times g \times \frac{\pi}{4} d^2 \times V \times (H_i - H_f)$$

$$P = \rho \times g \times \frac{\pi}{4} d^2 \times V \times \left(H_i - \frac{4FLV^2}{d \times 2g}\right) W$$

(iv) Efficiency of Power Transmission.

$$\eta_p = \frac{\text{Power available at the outlet of pipe}}{\text{Power Supplied at the inlet of pipe}}$$

$$= \frac{\text{weight of water} \times H_{out}}{\text{weight of water} \times H\,\text{inlet}}$$

$$= \frac{W \times H_{out}}{W \times H_{in}} = \frac{W \times (H_i - H_f)}{W \times H_i} = \frac{H_i - H_f}{H_i}$$

(v) Max. efficiency of Power Transmission :-

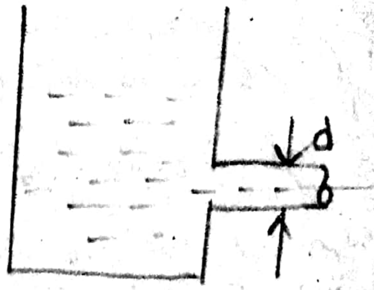
$$H_i = 3H_F$$
$$H_F = \frac{H_i}{3}$$

A pipe diameter 280mm and length 4000m is used to the power Transmission by water The total head of the inlet of the pipe is 600m. Find the Max. Power available at the outlet of the pipe. Take

Co-efficient of Friction 0·006

Sol:-

Max. Power $H_f = \frac{H_i}{3} = \frac{600}{3} = 200$



Head loss due to Friction

$$H_f = \frac{4FLV^2}{2gd}$$

$$200 = \frac{4 \times 0.006 \times 4000 \times v^2}{0.28 \times 2 \times 9.81}$$

$$V = 3.38 \text{ m/sec.}$$

Discharge $Q = A \times V$

$$= \frac{\pi}{4} d^2 \times V$$

$$= \frac{\pi}{4} (0.28)^2 \times 3.38 = 0.208 \text{ m}^3/\text{s.}$$

Head available at outlet $H_{out} = H_i - H_f$

$$= H_i - \frac{H_f}{3}$$

$$= 600 - 200$$

$$= 400 \text{ m.}$$

Power Transmitted at the outlet of the pipe

P = Weight of water X Head at outlet

$$P = \rho \times g \times Q \times 400$$

$$P = \rho \times g \times A \times V \times 400$$

$$P = \rho \times g \times \frac{\pi}{4} d^2 \times V \times 400$$

$$P = 1000 \times 9.81 \times \frac{\pi}{4} (0.28)^2 \times 3.38 \times 400$$

$$P = 816,679.479 \text{ W}$$

$$P = 816.679 \text{ KW.}$$

A pipe line of length 3000 m is used for power Transmission through a pipe is 110.362 KW & water head at inlet 500m. and loss of head due to Friction 100m if the dia. of the pipe is 130mm. Find the efficiency & velocity and also find discharge take co-efficient of friction 0.006.

Sol: Given data:-

$L = 3000 m$

$P = 110.362 KW$

$H_{in} = 500 m$

$H_F = 100 m$

$D = 130 mm = 0.13 m$

$F = 0.006$

$\eta_p = ? \; ; \; v = ?$

$$\text{Efficiency } \eta_p = \frac{H_i - H_F}{H_i} = \frac{500-100}{500} = \frac{400}{500} = 0.8 = 80\%$$

P = weight of water × H_{out}

$$P = \frac{\rho \times g \times Q \times H_{out}}{1000}$$

$$110.362 = \frac{1000 \times 9.81 \times Q \times (500-100)}{1000}$$

$Q = 0.028 \, m^3/se = 28$ lit

$Q = A \times V \Rightarrow 0.028 = \frac{\pi}{4}(0.13)^2 \times V$

$V = 2.10$ m/s.