DMM-I, UNIT-III

Riveted Joints, Welded Joints and Bolted Jonts-Different Seals

Riveted joints Introduction:-

The fastenings (i.e. joints) may be classified into the following two groups:

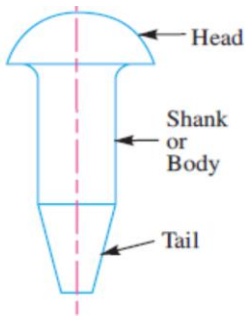
1.Permanent fastenings:- When the machine parts cannot be dismantled without destroying the connecting elements these joints are called as permanent joints.

Examples:- permanent fastenings in order of **strength** are soldered, brazed, welded and riveted joints.

2. Temporary or detachable fastenings:- If the machine parts can be dismantled, without damaging or destroying the connecting elements for maintenance or repair, then that joint is called as temporary joint.

Examples:- Temporary fastenings are screwed, keys, cotters, pins and splined joints.

Rivet:- A rivet is a short cylindrical bar with a head integral to it. The cylindrical portion of the rivet is called **shank** or **body** and lower portion of shank is known as **tail,** as shown in Fig.



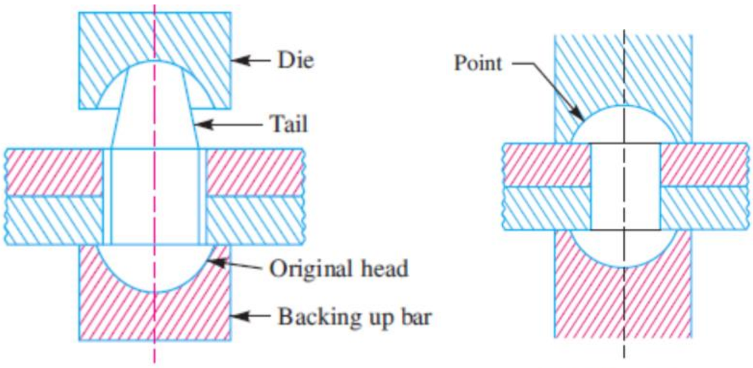
Rivets are used as permanent fastenings that cannot be dismantled without some part or part of the joint being destroyed.

Example: structural work, ship building, bridges, tanks and boiler shells. The riveted joints are widely used for joining light metals.

Riveting:- The process of forming another rivet-head, after the rivet is placed in the holes previously drilled or punched through the plates, is called riveting.

The function of rivets in a joint is to make a connection that has **strength and tightness.** The **strength** is necessary to prevent failure of the joint. The **tightness** is necessary to prevent leakage as in a boiler.

In machine riveting, the die is a part of the hammer which is operated by air, hydraulic or steam pressure.



(*a*) Initial position. (*b*) Final position.

Notes: 1. For steel rivets upto 12 mm diameter the cold riveting process is used. For larger diameter rivets hot riveting process is used.

2. In case of long rivets, only the tail is heated and not the whole shank.

Material of Rivets

1. The material of the rivets must be tough and ductile. They are usually made of low carbon steel, nickel alloy steel and wrought iron. Rivets are also made from non-ferrous materials such as copper, aluminium alloys and brass for anti corrosive properties.

2. According to BIS the rivet material should have tensile strength more than 350 N/mm^2 and elongation not less than 20 percent.

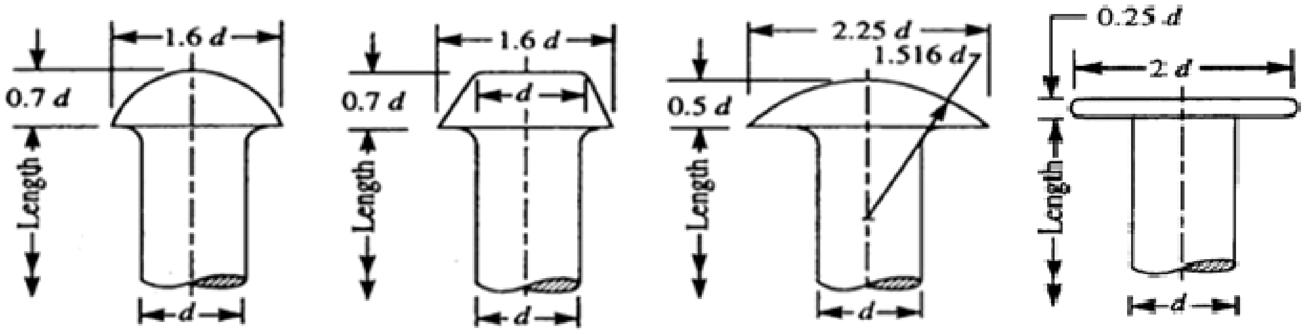
Rivet heads according to Indian standard specification: - (Classify the rivet heads according to Indian standard specification)

According to Indian standard specifications, the rivet heads are classified into the following three types,

i). Rivet heads for general purposes (below 12 mm diameter)

ii). Rivet heads for general purposes (from 12 mm to 48mm diameter) iii). Rivet heads for boiler work (from 12 mm to 48mm diameter)

Ex:- Snap head, pan head, mushroom head, counter sunk, flat counter sunk, round counter sunk, flat head rivets, steeple head, ellipsoid head, conical head, etc.

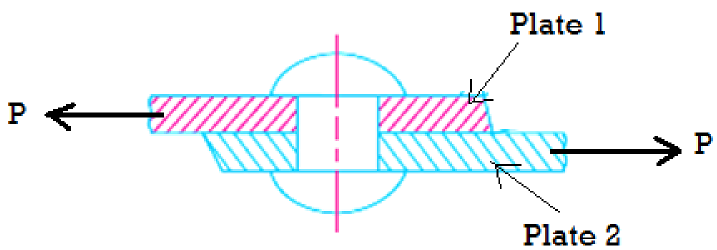


(a) Snap head. (b) Pan head. (c) Mushroom head. (d) Flat head.

Types of Riveted Joints (How do you classify the riveted joints?)

1. According to the joints:-

(a) Lap Joint:-A lap joint is that in which one plate overlaps the other plate and the two plates are then riveted together.



Lap joint

(b). Butt Joint:- A butt joint is that in which the main plates are kept in alignment butting (i.e. touching) each other and a cover plate (i.e. strap) is placed either on one side or on both sides of the main plates. The cover plate is then riveted together with the main plates. Butt joints are of the following two types:

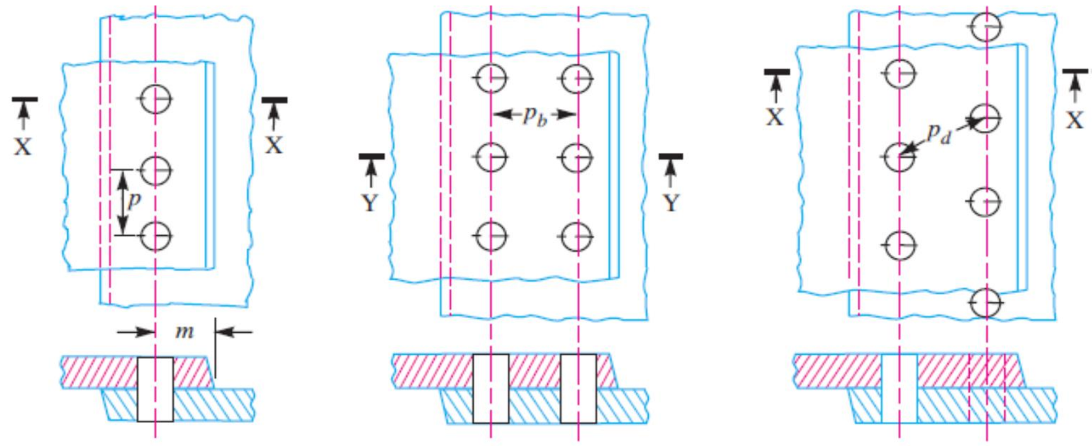
(i) Single strap or single cover butt joint, and (ii) Double strap or double cover butt joint.

2. According to the Riveting:- In addition to the above, following are the types of riveted joints depending upon the number of rows of the rivets.

(a) Single riveted joint, and (b) Double riveted joint. (c) Triple riveted or quadruple riveted (used in boiler shells)

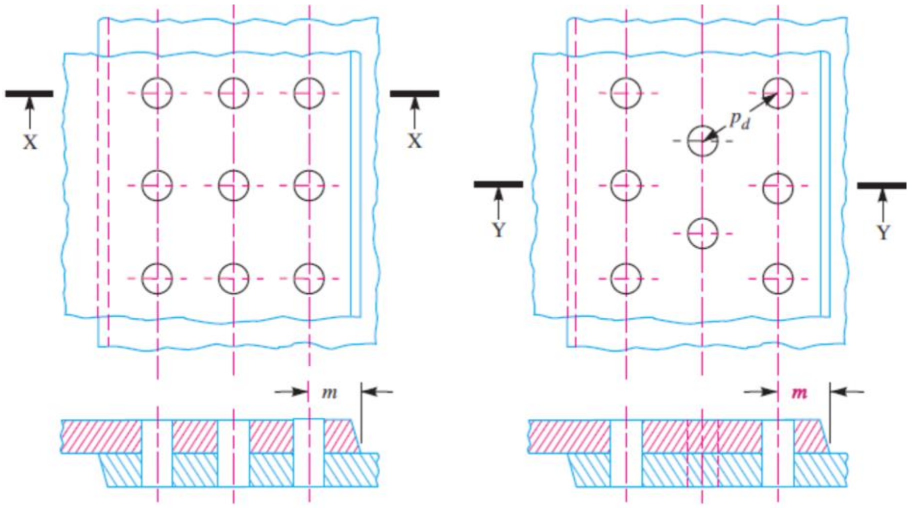
3. According to the arrangement of rivets:-

(i) Chain riveting (ii) Zig-zag riveting



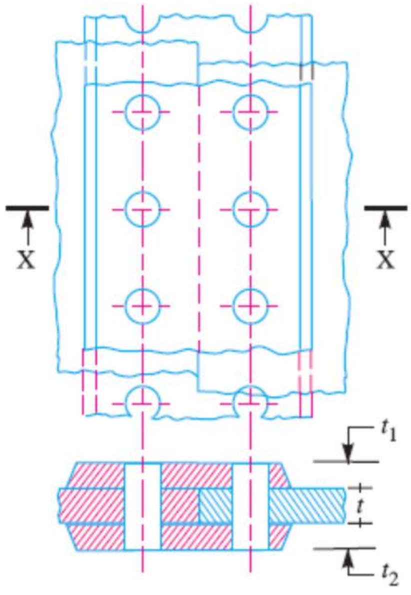
(*a*) Single riveted lap joint. (*b*) Double riveted lap joint (Chain riveting). (*c*) Double riveted lap joint (Zig-zag riveting).

Single and double riveted lap joints



(*a*) Chain riveting. (*b*) Zig-zag riveting.

Triple riveted lap joint.



Single riveted double strap butt joint.

Important Terms Used in Riveted Joints

The following terms in connection with the riveted joints are important.

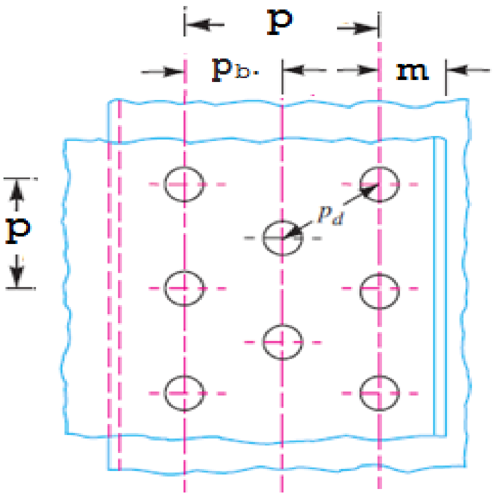
1. Pitch:- It is the distance from the centre of one rivet to the centre of the next rivet measured parallel to the joint as shown in Fig. It is usually denoted by p.

2. Back pitch:- It is the perpendicular distance between the centre lines of the successive rows as shown in Fig. It is usually denoted by p_b.

3. Diagonal pitch:- It is the distance between the centers of the rivets in adjacent rows of zig-zag riveted joint as shown in Fig. It is usually denoted by p_d.

4. Margin or marginal pitch:- It is the distance between the centres of rivet hole to the nearest edge of the plate as shown in Fig. It is usually denoted by m= 1.5d, where d= Diameter of rivet hole.

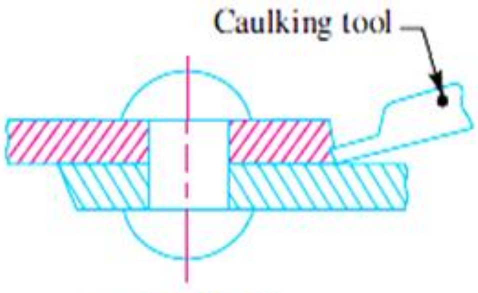
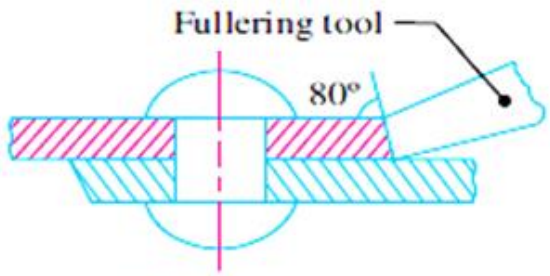
5. Nominal diameter:- It is the standard diameter of the rivet. It is the diameter when the rivet is cold before use in the joint.



Terminology of a riveted joint

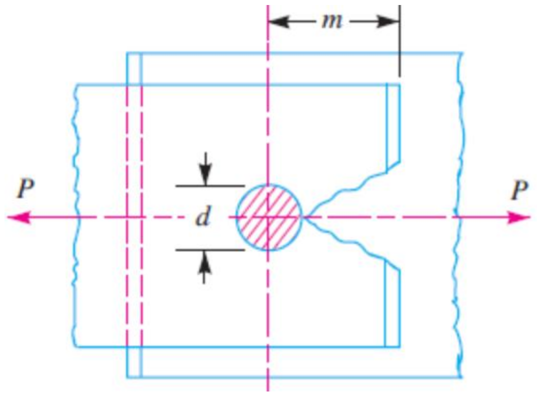
Difference between Caulking and fullering in riveted joints (What is difference between Caulking and fullering? Explain with the help of neat sketches)

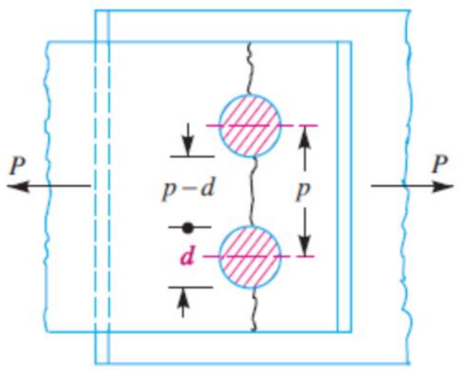
S.NO	**Caulking**	**Fullering**
1	In order to make the joints leak proof or fluid tight in pressure vessels a process known as caulking.	In order to make the joints leak proof or fluid tight in pressure vessels a process known as Fullering
2	This operation is carried out by using narrow blunt tool called Caulking tool.	This operation is carried out by using narrow blunt tool called Caulking tool.
3	The thickness of the tool is about 5mm.	The thickness of the tools equal to the thickness of the plates.
4	Surface finish obtained is less compared to fullering.	It gives clean surface finish.
5	More risk of damaging the plates	Less risk of damaging the plates
6	Caulking tool is used in tight the joint in pressure vessels like steam boilers, air receivers and tanks etc.	Fullering tool is used in tight the joint in pressure vessels like steam boilers, air receivers and tanks etc.

7	 (a) Caulking.	 (b) Fullering.

<u>Failures of a Riveted Joint</u> (Explain various ways in which a riveted joint may fail):- A riveted joint may fail in the following ways:

1. Tearing of the plate at an edge. A joint may fail due to tearing of the plate at an edge as shown in Fig. This can be avoided by keeping the margin, m = 1.5d, where d is the diameter of the rivet hole.





Tearing of the plate at an edge.   Tearing of the plate across the rows of rivets.

2. Tearing of the plate across a row of rivets. Due to the tensile stresses in the main plates, the main plate or cover plates may tear off across a row of rivets as shown in Fig. In such cases, we consider only one pitch length of the plate The resistance offered by the plate against tearing is known as **tearing resistance** or **tearing strength** or **tearing value** of the plate.

Let p = Pitch of the rivets,

d = Diameter of the rivet hole,

t = Thickness of the plate, and

σ_t = Permissible tensile stress for the plate material.

We know that tearing area per pitch length,

$$A_t = (p - d)\ t$$

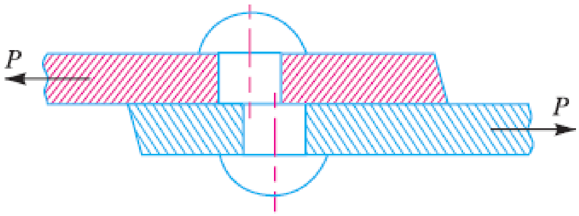
∴ Tearing resistance or pull required to tear off the plate per pitch length,

$$P_t = A_t .\ \sigma_t$$

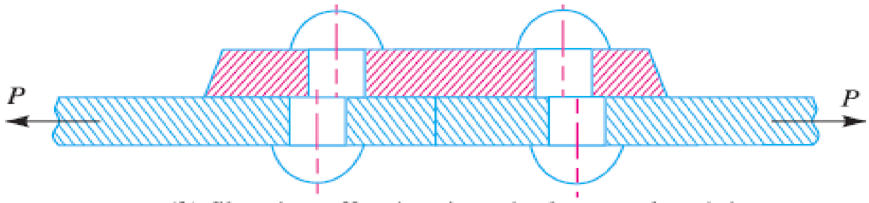
$$= (p - d)\ t.\ \sigma_t$$

The resistance offered by the plate against tearing is known as "tearing strength" of the plate. When the tearing resistance (P_t) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

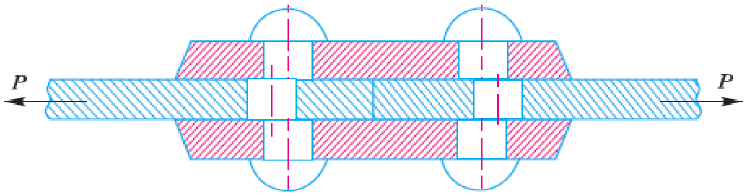
3. Shearing of the rivets. The rivets are sheared off as shown in Fig.



(*a*) Shearing off a rivet in a lap joint.



(*b*) Shearing off a rivet in a single cover butt joint.



Shearing off a rivet in double cover butt joint.

d = Diameter of the rivet hole,

τ = Safe permissible shear stress for the rivet material, and

n = Number of rivets per pitch length.

We know that **shearing area**,

$$A_s = \frac{\pi}{4} \times d^2 \quad \text{...(In single shear)}$$

$$= 2 \times \frac{\pi}{4} \times d^2 \quad \text{...(Theoretically, in double shear)}$$

$$= 1.875 \times \frac{\pi}{4} \times d^2 \quad \text{...(In double shear, according to Indian Boiler Regulations)}$$

Shearing resistance or pull required to shear off the rivet per pitch length,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In single shear)}$$

$$= n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(Theoretically, in double shear)}$$

$$= n \times 1.875 \times \frac{\pi}{4} \times d^2 \times \tau \quad \text{...(In double shear, according to Indian Boiler Regulations)}$$

The resistance offered by the rivet to be sheared-off is known as "shearing resistance or shearing strength" of the rivet. The When the shearing resistance (P_s) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

4. Crushing of the plate or rivets. Sometimes, the rivets do not actually shear off under the tensile stress, but are subjected to crushing. Due to this, the rivet hole becomes of an oval shape and hence the joint becomes loose.

The failure of rivets in such a manner is also known as **bearing failure.** The resistance offered by a rivet to be crushed is known as **crushing resistance** or **crushing strength** or **bearing value** of the rivet.

Let d = Diameter of the rivet hole,

t = Thickness of the plate,

σ_c = Safe permissible crushing stress for the rivet or plate material,

n = Number of rivets per pitch length under crushing.

We know that crushing area per rivet (i.e. projected area per rivet),

$A_c = d.t$

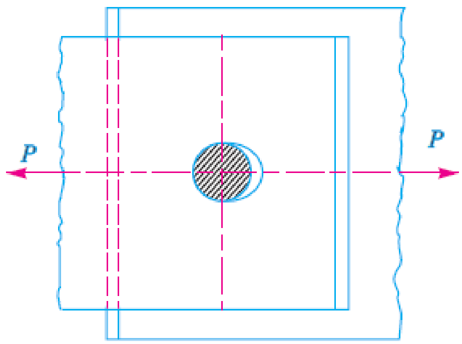
∴ Total crushing area = n.d.t

Crushing resistance or pull required to crush the rivet per pitch length,

$$P_c = n.d.t.\ \sigma_c$$

When the crushing resistance (P_c) is greater than the applied load (P) per pitch length, then this type of failure will not occur.

Note : The number of rivets under shear shall be equal to the number of rivets under crushing.



Crushing of a rivet.

Efficiency of a Riveted Joint

The efficiency of a riveted joint is defined as the ratio of the strength of riveted joint to the strength of the un-riveted or solid plate.

Strength of the riveted joint = Least of P_t, P_s and P_c

Strength of the un-riveted or solid plate per pitch length, $P = p \times t \times \sigma_t$

∴ Efficiency of the riveted joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{p \times t \times \sigma_t}$$

where p = Pitch of the rivets,

t = Thickness of the plate, and

σ_t = Permissible tensile stress of the plate material.

Problem 1).A double riveted lap joint is made between 15 mm thick plates. The rivet diameter and pitch are 25 mm and 75 mm respectively. If the ultimate stresses are 400 MPa in tension,320 MPa in shear and 640 MPa in crushing, find the minimum force per pitch which will rupture the joint. If the above joint is subjected to a load such that the factor of safety is 4, find out the actual stresses developed in the plates and the rivets.

Solution. Given : t = 15 mm ; d = 25 mm ; p = 75 mm ; σ_{tu} = 400 MPa = 400 N/mm^2 ; τ_u = 320 MPa = 320 N/mm^2 ; σ_{cu} = 640 MPa = 640 N/mm^2

Minimum force per pitch which will rupture the joint

The ultimate tearing resistance of the plate per pitch,

$$P_{tu} = (p - d)t \times \sigma_{tu} = (75 - 25)15 \times 400 = 300\,000 \text{ N}$$

Ultimate shearing resistance of the rivets per pitch,

$$P_{su} = n \times \frac{\pi}{4} \times d^2 \times \tau_u = 2 \times \frac{\pi}{4} (25)^2\, 320 = 314\,200 \text{ N} \quad ...(\because n = 2)$$

and ultimate crushing resistance of the rivets per pitch,

$$P_{cu} = n \times d \times t \times \sigma_{cu} = 2 \times 25 \times 15 \times 640 = 480\,000 \text{ N}$$

From above we see that the minimum force per pitch which will rupture the joint is 300 000 N or 300 kN.

Actual stresses produced in the plates and rivets

Since the factor of safety is 4, therefore safe load per pitch length of the joint

$$= 300\,000/4 = 75\,000 \text{ N}$$

Let σ_{ta}, τ_a and σ_{ca} be the actual tearing, shearing and crushing stresses produced with a safe load of 75 000 N in tearing, shearing and crushing.

We know that actual tearing resistance of the plates (P_{ta}),

$$75\,000 = (p - d)\, t \times \sigma_{ta} = (75 - 25)15 \times \sigma_{ta} = 750\, \sigma_{ta}$$

∴ σ_{ta} = 75 000 / 750 = 100 N/mm2 = 100 MPa

Actual shearing resistance of the rivets (P_{sa}),

$$75\,000 = n \times \frac{\pi}{4} \times d^2 \times \tau_a = 2 \times \frac{\pi}{4} (25)^2\, \tau_a = 982\, \tau_a$$

$$\tau_a = 75000 / 982 = 76.4 \text{ N/mm}^2 = 76.4 \text{ MPa} \quad \textbf{Ans.}$$

and actual crushing resistance of the rivets (P_{ca}),

$$75\,000 = n \times d \times t \times \sigma_{ca} = 2 \times 25 \times 15 \times \sigma_{ca} = 750\, \sigma_{ca}$$

$$\sigma_{ca} = 75000 / 750 = 100 \text{ N/mm}^2 = 100 \text{ MPa} \quad \textbf{Ans.}$$

Problem 2). Find the efficiency of the following riveted joints :

1. Single riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 50 mm.

2. Double riveted lap joint of 6 mm plates with 20 mm diameter rivets having a pitch of 65 mm.

Assume Permissible tensile stress in plate = 120 MPa, Permissible shearing stress in rivets = 90 MPa, Permissible crushing stress in rivets = 180 MPa

Solution. Given: t = 6 mm ; d = 20 mm ; σ_t= 120 MPa = 120 N/mm^2 ; τ = 90 MPa = 90 N/mm^2 ; σ_c = 180 MPa = 180 N/mm2

1. Efficiency of the first joint

Pitch, p = 50 mm, the tearing resistance of the plate, shearing and crushing resistances of the rivets.

(i) Tearing resistance of the plate

The tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (50 - 20)\, 6 \times 120 = 21\ 600 \text{ N}$$

(ii) Shearing resistance of the rivet

Since the joint is a single riveted lap joint, therefore the strength of one rivet in single shear is taken. We know that shearing resistance of one rivet,

$$P_s = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4}\ (20)^2\ 90 = 28\ 278 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is a single riveted, therefore strength of one rivet is taken. We know that crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c = 20 \times 6 \times 180 = 21\ 600 \text{ N}$$

∴ Strength of the joint = Least of P_t, P_s and P_c = 21 600 N

We know that strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 50 \times 6 \times 120 = 36\ 000 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t,\ P_s \text{ and } P_c}{P} = \frac{21\ 600}{36\ 000} = 0.60 \text{ or } 60\% \quad \textbf{Ans.}$$

2. Efficiency of the second joint

Pitch, p = 65 mm

(i) Tearing resistance of the plate,

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (65 - 20)\, 6 \times 120 = 32\,400 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted lap joint, therefore strength of two rivets in single shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times \frac{\pi}{4} (20)^2\, 90 = 56\,556 \text{ N}$$

(iii) Crushing resistance of the rivet

Since the joint is double riveted, therefore strength of two rivets is taken. We know that crushing resistance of rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 20 \times 6 \times 180 = 43\,200 \text{ N}$$

∴ Strength of the joint = Least of P_t, P_s and P_c = 32 400 N

We know that the strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 65 \times 6 \times 120 = 46\,800 \text{ N}$$

∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{32\,400}{46\,800} = 0.692 \text{ or } 69.2\%$$

Problem 3). A double riveted double cover butt joint in plates 20 mm thick is made with 25 mm diameter rivets at 100 mm pitch. The permissible stresses are: σ_t = 120 MPa; τ = 100 MPa; σ_c = 150 MPa. Find the efficiency of joint, taking the strength of the rivet in double shear as twice than that of single shear.

Solution. Given: t = 20 mm; d = 25 mm ; p = 100 mm ; σ_t = 120 MPa = 120 N/mm^2 ; τ = 100 MPa = 100 N/mm^2 ; σ_c = 150 MPa = 150 N/mm^2

(i) Tearing resistance of the plate

Tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (100 - 25)\, 20 \times 120 = 180\,000 \text{ N}$$

(ii) Shearing resistance of the rivets

Since the joint is double riveted butt joint, therefore the strength of two rivets in double shear is taken. We know that shearing resistance of the rivets,

$$P_s = n \times 2 \times \frac{\pi}{4} \times d^2 \times \tau = 2 \times 2 \times \frac{\pi}{4} (25)^2\ 100 = 196\ 375\ \text{N}$$

(iii) Crushing resistance of the rivets

Since the joint is double riveted, therefore the strength of two rivets is taken. We know that crushing resistance of the rivets,

$$P_c = n \times d \times t \times \sigma_c = 2 \times 25 \times 20 \times 150 = 150\ 000\ \text{N}$$

∴ Strength of the joint = Least of P_t, P_s and P_c = 150 000 N

Efficiency of the joint

The strength of the unriveted or solid plate,

$$P = p \times t \times \sigma_t = 100 \times 20 \times 120 = 240\ 000\ \text{N}$$

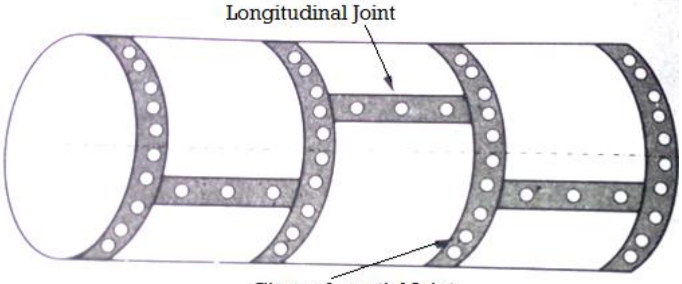
∴ Efficiency of the joint

$$= \frac{\text{Least of } P_t, P_s \text{ and } P_c}{P} = \frac{150\ 000}{240\ 000}$$

$$= 0.625 \text{ or } 62.5\%$$

<u>Riveted joints for boilers and pressure vessels</u>:- (Explain briefly design procedure for circumference lap joint for boiler)

"Pressure vessel is a vessel which stores pressurized fluid in the cylinder. If the fluid is steam, then it is called as boiler." A pressure vessel or boiler has two joints are longitudinal joint and circumferential joint.



Design of Longitudinal Butt Joint for a Boiler

According to Indian Boiler Regulations (I.B.R), the following procedure should be adopted for the design of longitudinal butt joint for a boiler.

1. Thickness of boiler shell. First of all, the thickness of the boiler shell is determined by using the thin cylindrical formula, i.e.

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm as corrosion allowance}$$

t = Thickness of the boiler shell,
P = Steam pressure in boiler,
D = Internal diameter of boiler shell,
σ_t = Permissible tensile stress, and
η_l = Efficiency of the longitudinal joint.

2. Diameter of rivets. By using **Unwin's** empirical formula, i.e. $d = 6\sqrt{t}$ (when t is greater than 8 mm)

3. Pitch of rivets. The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets. It may noted that

(a) The pitch of the rivets should not be less than 2d, which is necessary for the formation of head.

(b) The maximum value of the pitch of rivets for a longitudinal joint of a boiler as per I.B.R. is $p_{max} = C \times t + 41.28$ mm

Where t = Thickness of the shell plate in mm, and
C = Constant.

4.Margin. M=1.5d

<u>Design of Circumferential lap joint:</u>

1. *Thickness of the shell and diameter of rivets*. The thickness of the boiler shell and the
diameter of the rivet will be same as for longitudinal joint.

2. *Number of rivets*. Since it is a lap joint, therefore the rivets will be in single shear.

Shearing resistance of the rivets,

$$P_s = n \times \frac{\pi}{4} \times d^2 \times \tau$$

***3. Pitch of rivets*.**

***4. Number of rows*.**

Problem(4). A pressure vessel has an internal diameter of 1 m and is to be subjected to an internal pressure of 2.75 N/mm2 above the atmospheric pressure. Considering it as a thin cylinder and assuming efficiency of its riveted Joint to be 79% calculate the plate thickness if the tensile stress in the material is not to exceed 88 MPa. Design a longitudinal double riveted double strap butt joint with equal straps for this vessel. The pitch of the rivets in the outer row is to be double the pitch in the inner row and zig-zag riveting is proposed. The maximum allowable shear stress in the rivets is 64 MPa. You may assume that the rivets in double shear are 1.8 times stronger than in single shear and the joint does not fail by crushing. Calculate the efficiency of the joint.

Solution. Given: D = 1 m = 1000 mm ; P = 2.75 N/mm^2 ; η_1 = 79% = 0.79 ; σ_t= 88 MPa = 88 N/mm^2; τ = 64 MPa = 64 N/mm^2

1. Thickness of plate

We know that the thickness of plate,

$$t = \frac{P.D}{2\,\sigma_t \times \eta_l} + 1 \text{ mm} = \frac{2.75 \times 1000}{2 \times 88 \times 0.79} + 1 \text{ mm}$$

$$= 20.8 \text{ say } 21 \text{ mm} \quad \textbf{Ans.}$$

2. Diameter of rivet

Since the thickness of plate is more than 8 mm, therefore diameter of rivet hole,

$$d = 6\sqrt{t} = 6\sqrt{21} = 27.5 \text{ mm}$$

3. Pitch of rivets

Let p = Pitch in the outer row.

The pitch of the rivets is obtained by equating the tearing resistance of the plate to the shearing resistance of the rivets.

We know that the tearing resistance of the plate per pitch length,

$$P_t = (p - d)\, t \times \sigma_t = (p - 28.5)\, 21 \times 88 = 1848\,(p - 28.5)\text{ N} \quad ...(i)$$

Since the pitch in the outer row is twice the pitch of the inner row and the joint is double riveted, therefore for one pitch length there will be three rivets in double shear (i.e. n = 3). It is given that the strength of rivets in double shear is 1.8 times that of single shear, therefore

Shearing strength of the rivets per pitch length,

$$P_s = n \times 1.8 \times \frac{\pi}{4} \times d^2 \times \tau = 3 \times 1.8 \times \frac{\pi}{4}(28.5)^2\, 64 \text{ N}$$

$$= 220\,500 \text{ N} \quad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$1848\,(p - 28.5) = 220\,500$$

$$\therefore \quad p - 28.5 = 220\,500 / 1848 = 119.3$$

$$p = 119.3 + 28.5 = 147.8 \text{ mm}$$

According to I.B.R., the maximum pitch,

$$p_{max} = C \times t + 41.28 \text{ mm}$$

$$\therefore \quad p_{max} = 4.63 \times 21 + 41.28 = 138.5 \text{ say } 140 \text{ mm}$$

Since the value of p_{max} is less than p, therefore we shall adopt the value of

$$p = p_{max} = 140 \text{ mm} \quad \textbf{Ans.}$$

∴ Pitch in the inner row

$$= 140 / 2 = 70 \text{ mm} \quad \textbf{Ans.}$$

4. Distance between the rows of rivets

According to I.B.R.,(Indian Boiler Regulations) the distance between the rows of rivets,

$$p_b = 0.2\, p + 1.15\, d = 0.2 \times 140 + 1.15 \times 28.5 = 61 \text{ mm} \quad \textbf{Ans.}$$

5. Thickness of butt strap

According to I.B.R., the thickness of double butt straps of equal width,

$$t_1 = 0.625\, t \left(\frac{p - d}{p - 2d}\right) = 0.625 \times 21 \left(\frac{140 - 28.5}{140 - 2 \times 28.5}\right) \text{mm}$$

$$= 17.6 \text{ say } 18 \text{ mm} \quad \textbf{Ans.}$$

6. Margin

We know that the margin,

$$m = 1.5\, d = 1.5 \times 28.5 = 43 \text{ mm} \quad \textbf{Ans.}$$

Efficiency of the joint

We know that tearing resistance of the plate,

$$P_t = (p - d)\, t \times \sigma_t = (140 - 28.5)\, 21 \times 88 = 206\ 050 \text{ N}$$

Shearing resistance of the rivets,

$$P_s = n \times 1.8 \times \frac{\pi}{4} \times d^2 \times \tau = 3 \times 1.8 \times \frac{\pi}{4} (28.5)^2\, 64 = 220\ 500 \text{ N}$$

Strength of the solid plate,

$$= p \times t \times \sigma_t = 140 \times 21 \times 88 = 258\ 720 \text{ N}$$

Efficiency of the joint

$$= \frac{\text{Least of } P_t \text{ and } P_s}{\text{Strength of solid plate}} = \frac{206\ 050}{258\ 720} = 0.796 \text{ or } 79.6\% \quad \textbf{Ans.}$$

Since the efficiency of the designed joint is more than the given efficiency, therefore the design is satisfactory.

Riveted Joint for Structural Use–Joints of Uniform Strength (Lozenge Joint):

A riveted joint known as **Lozenge joint** used for roof, bridge work or girders etc. is shown in Fig. 9.19. In such a joint, diamond riveting is employed so that the Joint is made of uniform strength. Fig. shows a triple riveted double strap butt joint.

Let b = Width of the plate,

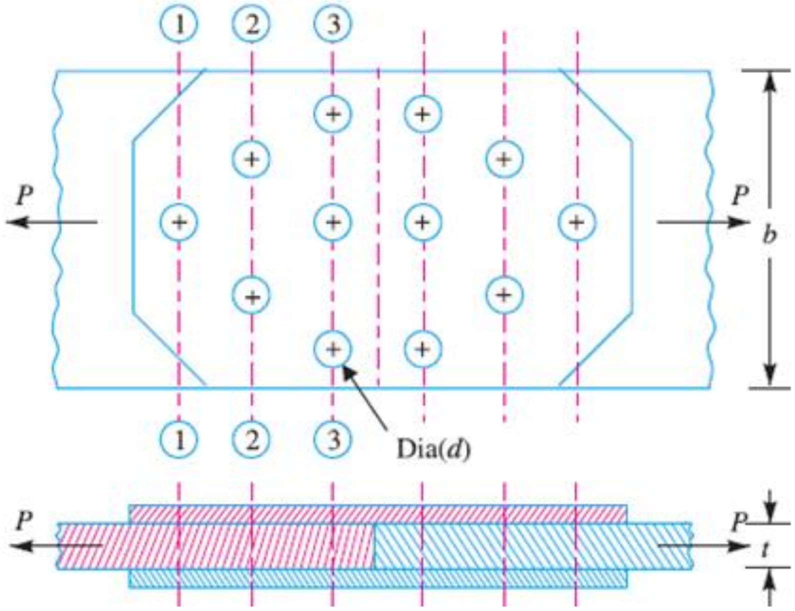
t = Thickness of the plate, and

d = Diameter of the rivet hole.

In designing a Lozenge joint, the following procedure is adopted.

1. Diameter of rivet

The diameter of the rivet hole is obtained by using Un-win's formula, i.e. $d = 6\sqrt{t}$



Riveted joint for structural use.

2. Number of rivets

The number of rivets required for the joint may be obtained by the shearing or crushing resistance of the rivets.

Let P_t = Maximum pull acting on the joint. This is the tearing resistance of the plate at the outer row which has only one rivet. $= (b - d)\, t \times \sigma_t$ and

n = Number of rivets.

Since the joint is double strap butt joint, therefore the rivets are in double shear. It is assumed that resistance of a rivet in double shear is 1.75 times than in single shear in order to allow for possible eccentricity of load and defective workmanship.

∴ Shearing resistance of one rivet,

$$P_s = 1.75 \times \frac{\pi}{4} \times d^2 \times \tau$$

and crushing resistance of one rivet,

$$P_c = d \times t \times \sigma_c$$

∴ Number of rivets required for the joint,

$$n = \frac{P_t}{\text{Least of } P_s \text{ or } P_c}$$

3. From the number of rivets, the number of rows and the number of rivets in each row is decided.

4. Thickness of the butt straps

The thickness of the butt strap, t_1 = 1.25 t, for single cover strap = 0.75 t, for double cover strap

5. Efficiency of the joint

First of all, calculate the resistances along the sections 1-1, 2-2 and 3-3.

At section 1-1, there is only one rivet hole.

∴ Resistance of the joint in tearing along 1-1,

$$P_{t1} = (b - d)\, t \times \sigma_t$$

At section 2-2, there are two rivet holes.

∴ Resistance of the joint in tearing along 2-2,

$P_{t2} = (b - 2d)\, t \times \sigma_t$ + Strength of one rivet in front of section 2-2

(This is due to the fact that for tearing off the plate at section 2-2, the rivet in front of section 2-2 i.e. at section 1-1 must first fracture).

Similarly at section 3-3 there are three rivet holes.

∴ Resistance of the joint in tearing along 3-3,

$P_{t3} = (b - 3d)\, t \times \sigma_t$ + Strength of 3 rivets in front of section 3-3

The least value of Pt1, Pt2, Pt3, Ps or Pc is the strength of the joint.

We know that the strength of unriveted plate,

$$P = b \times t \times \sigma_t$$

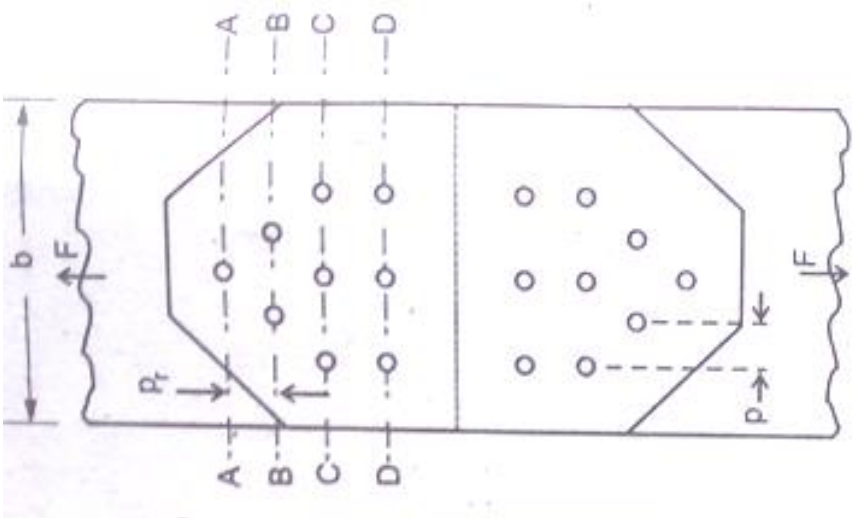
∴ Efficiency of the joint,

$$\eta = \frac{\text{Least of } P_{t1}, P_{t2}, P_{t3}, P_s \text{ or } P_c}{P}$$

Problem 5):-Two MS tie bars for a bridge structure are to be joined by means of a butt joint with double straps. The thickness of the tie bar is 12 mm and carries a load of 400 KN. Design the joint completely taking allowable stresses as 100 MPa in tension, 70 Mpa in shear and 150 MPa in compression.

$F = 400$ kN $\qquad [\sigma_t] = 100\ N/mm^2$

The Allowable stresses are $\qquad [\tau] = 70\ N/mm^2$



Structural joint (Diamond joint)

$$[\sigma_c] = 150 \text{ N/mm}^2$$

$$t = 12 \text{ mm.}$$

Since plate thickness is > 8 mm,

$$d = 6\sqrt{t} = 6\sqrt{12} = 20.78 \text{ mm}$$

$\therefore$ **dia. of rivet = 22 mm**

$\therefore$ Standard rivet hole dia. = 23 mm

Tearing resistance of plate in outer row

$$= (b - d)\, t\, [\sigma_t]$$

$$400 \times 10^3 = (b - 23) \times 12 \times 100$$

$$b - 23 = 333.33 \quad \text{.... (1)}$$

$b = 356.33 \approx$ **357 mm**

Shearing resistance of one

rivet $= 1.875 \times \frac{\pi}{4} d^2 [\tau]$ (Since double shear, double cover) .... (2)

$$= 1.875 \times \frac{\pi}{4} \times 23^2 \times 70$$

$$= 54531.2 \text{ N}$$

Crushing resistance of one rivet

$$= d \cdot t \cdot [\sigma_c] \quad .... (3)$$

$$= 23 \times 12 \times 150$$

$$= 41{,}400 \text{ N}$$

Since crushing resistance is less than shearing resistance.

Therefore, equate 1 & 3 which decides no. of rivets.

$$\therefore n = \frac{\text{tearing resistance}}{\text{crushing resistance}} = \frac{400 \times 10^3}{41{,}400} = 9.66 \simeq 10$$

$n = 10$ rivets

Thickness of strap $= 0.75\, t$

$$= 0.75 \times 12 = 9 \text{ mm}$$

To Find strength of joint at four critical sections. A-A, B-B, C-C, D-D.

Along A-A

Joint has a strength $= (b - d)\, t \cdot [\sigma_t] = (357 - 23) \times 12 \times 100 =$

$$= 400.8 \times 10^3 \text{ N}$$

Along B-B

Joint has a strength $= (b - 2d)\, t\, [\sigma_t]$ + Shearing of rivet (1 rivet) before B-B

$$= (b - 2d)\, t \cdot [\sigma_t] + 1.875 \times \frac{\pi}{4} \times d^2\, [\tau]$$

$$= (357 - 2 \times 23)\ 12\ (100) + 1.875 \times \frac{\pi}{4} \times 23^2 \times 70$$

$$= 373.2 \times 10^3 + 54.531 \times 10^3$$

$$= \mathbf{427.731 \times 10^3\ N}$$

Along C-C

Joint strength $= (b - 3d)\ t\ [\sigma_t]$ + Shearing of rivets (3 rivets) before C-C

$$= (b - 3d)\ t \cdot [\sigma_t] + 3\left(1.875 \times \frac{\pi}{4} \times d^2 \times [\tau]\right)$$

$$= (357 - 3 \times 23) \times 12 \times 100 + 3\left(1.875 \times \frac{\pi}{4} \times 23^2 \times 70\right)$$

$$= 345.6 \times 10^3 + 163.593 \times 10^3$$

$$= \mathbf{509.194 \times 10^3\ N}$$

Along D-D

$$\text{Joint strength} = (b - 3d)\ t \cdot [\sigma_t] + 6\left(1.875 \times \frac{\pi}{4} \times 23^2 \times 70\right)$$

$$= 345.6 \times 10^3 + 327.187 \times 10^3$$

$$= 672.787 \times 10^3$$

$$\text{Shearing resistance of all rivets} = 9 \times 54.531 \times 10^3$$

$$= 490.779 \times 10^3$$

The lowest strength of the joint is along A-A.

$$\eta \text{ of the joint} = \frac{b - d}{b} = \frac{357 - 23}{357} = 0.9356$$

$$= \mathbf{93.56\%}$$

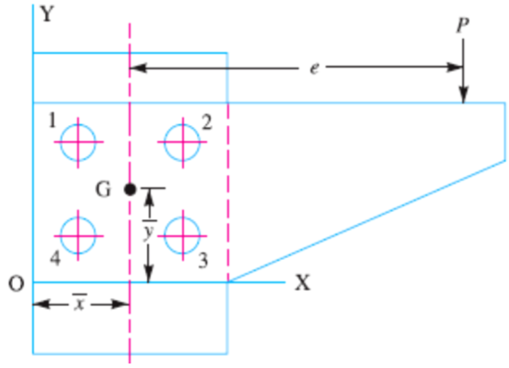
Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system and thus all rivets are not equally loaded, then the joint is said to be an

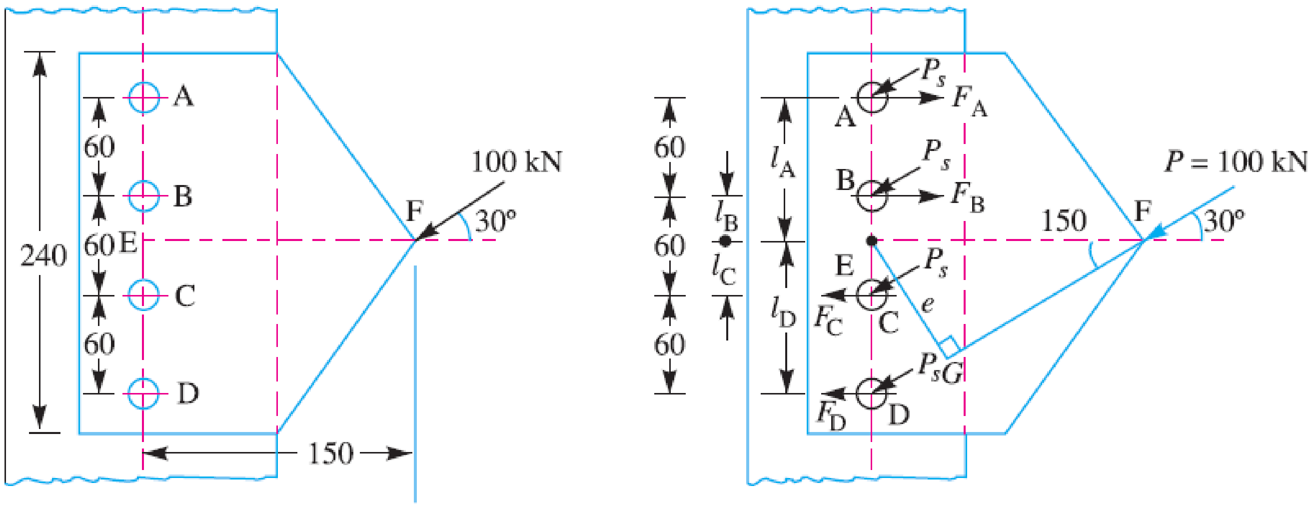
eccentric loaded riveted joint, as shown in Fig. The eccentric loading results in secondary shear caused by the tendency of force to twist the joint about the centre of gravity in addition to direct shear or primary shear.

Let P = Eccentric load on the joint, and

e = Eccentricity of the load i.e. the distance between the line of action of the load and the centroid of the rivet system i.e. G.



Problem 6) A bracket in the form of a plate is fitted to a column by means of four rivets A,B, C and D in the same vertical line, as shown in Fig. 9.33. AB = BC = CD = 60 mm. E is the mid-point of BC. A load of 100 KN is applied to the bracket at a point F which is at a horizontal distance of 150 m from E. The load acts at an angle of 30° to the horizontal. Determine the diameter of the rivets which are made of steel having a yield stress in shear of 240 MPa. Take a factor of safety of 1.5. What would be the thickness of the plate taking an allowable bending stress of 125 MPa for the plate, assuming its total width at section ABCD as 240 mm?



All dimensions in mm.

Solution. Given: n = 4; AB = BC = CD = 60 mm; P = 100 kN = 100 × 10^3 N; EF = 150 mm; θ= 30°; τ_y = 240 MPa = 240 N/mm² ; F.S. = 1.5 ; σ_b = 125 MPa = 125 N/mm² ; b = 240 mm.

Diameter of rivets

Let d = Diameter of rivets.

We know that direct shear load on each rivet,

$$P_s = \frac{P}{n} = \frac{100 \times 10^3}{4} = 25\ 000 \text{ N}$$

The direct shear load on each rivet acts in the direction of 100 kN load (i.e. at 30° to the horizontal) as shown in Fig. 9.34. The centre of gravity of the rivet group lies at E. From Fig. we find that the perpendicular distance from the centre of gravity E to the line of action of the load (or eccentricity of the load) is

$$EG = e = EF \sin 30° = 150 \times \frac{1}{2} = 75 \text{ mm}$$

∴ Turning moment produced by the load P due to eccentricity

$$= P.e = 100 \times 10^3 \times 75 = 7500 \times 10^3 \text{ N-mm}$$

This turning moment is resisted by four bolts, as shown in Fig. Let F_A, F_B, F_C and F_D be the secondary shear load on the rivets, A, B, C, and D placed at distances l_A, l_B, l_C and l_D respectively

from the centre of gravity of the rivet system. From Fig. we find that

$l_A = l_D$ = 60 + 30 = 90 mm and $l_B = l_C$ = 30 mm

$$P \times e = \frac{F_A}{l_A}[(l_A)^2 + (l_B)^2 + (l_C)^2 + (l_D)^2] = \frac{F_A}{l_A}\left[2(l_A)^2 + 2(l_B)^2\right]$$

$$...(\because l_A = l_D \text{ and } l_B = l_C)$$

$$7500 \times 10^3 = \frac{F_A}{90}\left[2(90)^2 + 2(30)^2\right] = 200\ F_A$$

$$F_A = 7500 \times 10^3 / 200 = 37\ 500 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore,

$$F_B = F_A \times \frac{l_B}{l_A} = 37\ 500 \times \frac{30}{90} = 12\ 500 \text{ N}$$

$$F_C = F_A \times \frac{l_C}{l_A} = 37\ 500 \times \frac{30}{90} = 12\ 500 \text{ N}$$

$$F_D = F_A \times \frac{l_D}{l_A} = 37\ 500 \times \frac{90}{90} = 37\ 500 \text{ N}$$

Now let us find the resultant shear load on each rivet.

From Fig. 9.34, we find that angle between F_A and $P_s = \theta_A = 150°$

Angle between F_B and $P_s = \theta_B = 150°$

Angle between F_C and $P_s = \theta_C = 30°$

Angle between F_D and $P_s = \theta_D = 30°$

Resultant load on rivet A,

$$R_A = \sqrt{(P_s)^2 + (F_A)^2 + 2 P_s \times F_A \times \cos \theta_A}$$

$$= \sqrt{(25\ 000)^2 + (37\ 500)^2 + 2 \times 25\ 000 \times 37\ 500 \times \cos 150°}$$

$$= \sqrt{625 \times 10^6 + 1406 \times 10^6 - 1623.8 \times 10^6} = 15\ 492 \text{ N}$$

Resultant shear load on rivet B,

$$R_B = \sqrt{(P_s)^2 + (F_B)^2 + 2 P_s \times F_B \times \cos \theta_B}$$

$$= \sqrt{(25\ 000)^2 + (12\ 500)^2 + 2 \times 25\ 000 \times 12\ 500 \times \cos 150°}$$

$$= \sqrt{625 \times 10^6 + 156.25 \times 10^6 - 541.25 \times 10^6} = 15\ 492 \text{ N}$$

Resultant shear load on rivet C,

$$R_C = \sqrt{(P_s)^2 + (F_C)^2 + 2 P_s \times F_C \times \cos \theta_C}$$

$$= \sqrt{(25\ 000)^2 + (12\ 500)^2 + 2 \times 25\ 000 \times 12\ 500 \times \cos 30°}$$

$$= \sqrt{625 \times 10^6 + 156.25 \times 10^6 + 541.25 \times 10^6} = 36\ 366 \text{ N}$$

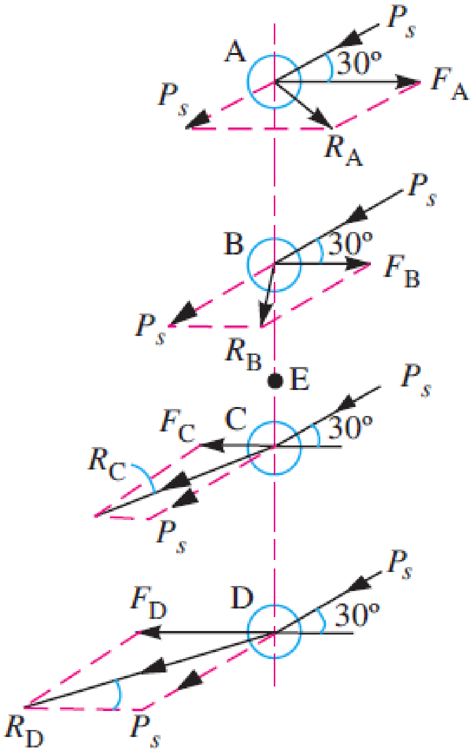
and resultant shear load on rivet D,

$$R_D = \sqrt{(P_s)^2 + (F_D)^2 + 2P_s \times F_D \times \cos\theta_D}$$

$$= \sqrt{(25\,000)^2 + (37\,500)^2 + 2 \times 25\,000 \times 37\,500 \times \cos 30°}$$

$$= \sqrt{625 \times 10^6 + 1406 \times 10^6 + 1623.8 \times 10^6} = 60\,455 \text{ N}$$

The resultant shear load on each rivet may be determined graphically as shown in Fig. From above we see that the maximum resultant shear load is on rivet D. We know that maximum resultant shear load (R_D),



$$60\,455 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times \frac{\tau_y}{F.S.}$$

$$= \frac{\pi}{4} \times d^2 \times \frac{240}{1.5} = 125.7\, d^2$$

$$d^2 = 60\,455 / 125.7 = 481$$

$$d = 21.9 \text{ mm}$$

Thickness of the plate

Let t = Thickness of the plate in mm,

σ_b = Allowable bending stress for the plate = 125 MPa = 125 N/mm^2 ...(Given)

b = Width of the plate = 240 mm ...(Given)

Consider the weakest section of the plate (i.e. the section where it receives four rivet holes of diameter 23.5 mm and thickness t mm) as shown in Fig. We know that moment of inertia of the plate about X-X,

I_{XX} = M.I. of solid plate about X-X – M.I. of 4 rivet holes about X-X

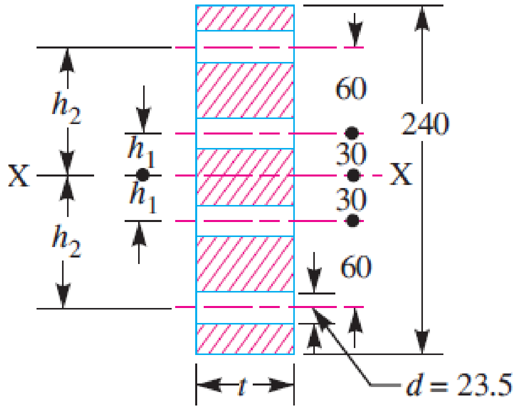
$$= \frac{1}{12} \times t\,(240)^3 - \left[4 \times \frac{1}{12} \times t\,(23.5)^3 + 2 \times t \times 23.5\,(30^2 + 90^2)\right]$$

$$= 1152 \times 10^3\, t - [4326\, t + 423 \times 10^3\, t] = 724\,674\, t \text{ mm}^4$$

Bending moment,

$$M = P \times e = 100 \times 10^3 \times 75$$

$$= 7500 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Distance of neutral axis (X–X) from the top most fibre of the plate,

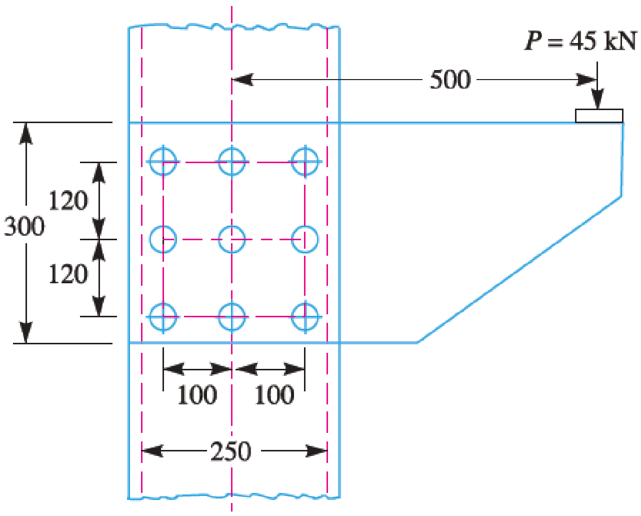
$$y = \frac{b}{2} = \frac{240}{2} = 120 \text{ mm}$$

We know that $$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\frac{7500 \times 10^3}{724\,674\, t} = \frac{125}{120}$$

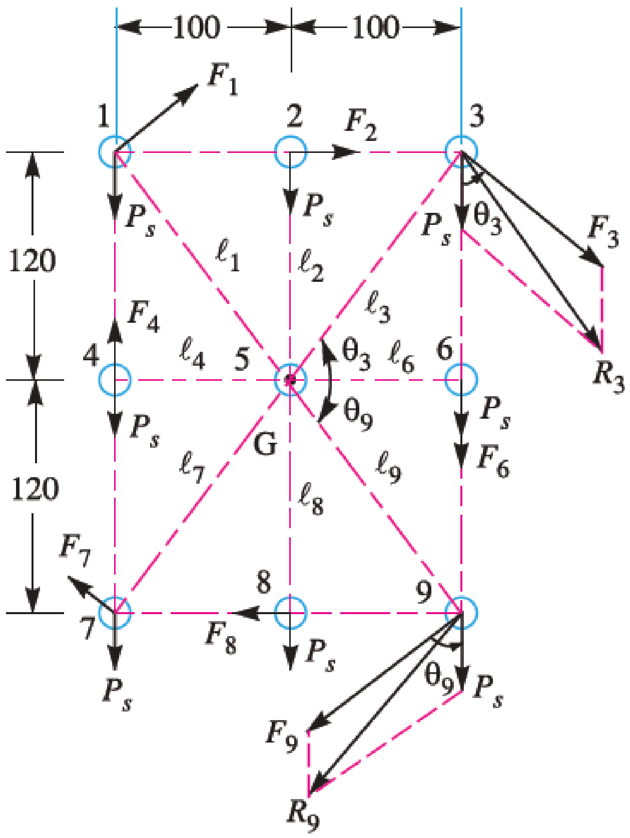
$$\frac{10.35}{t} = 1.04 \quad \text{or} \quad t = \frac{10.35}{1.04} = 9.95 \text{ say } 10 \text{ mm}$$

Problem7). The bracket as shown in Fig. is to carry a load of 45 kN. Determine the size of the rivet if the shear stress is not to exceed 40 MPa. Assume all rivets of the same size.



Solution:-

Given : $P = 45\text{ kN} = 45 \times 10^3\text{ N}$; $\tau = 40\text{ MPa} = 40\text{ N/mm}^2$; $e = 500\text{ mm}$; $n = 9$



Let us find the centre of gravity of the rivet system.

Since all the rivets are of same size and placed symmetrically, therefore the centre of gravity of The rivet system lies at *G* (rivet 5) as shown in Fig.

We know that direct shear load on each rivet,

$$P_s = P / n = 45 \times 10^3 / 9 = 5000 \text{ N}$$

The direct shear load acts parallel to the direction of load *P*, *i.e.* vertically downward as shown
in the figure.
Turning moment produced by the load *P* due to eccentricity *e*

$$= P.e = 45 \times 10^3 \times 500 = 22.5 \times 10^6 \text{ N-mm}$$

This turning moment tends to rotate the joint about the centre of gravity (*G*) of the rivet system in a clockwise direction. Due to this turning moment, secondary shear load on each rivet is produced. It may be noted that rivet 5 does not resist any moment.
Let F_1, F_2, F_3, F_4, F_6, F_7, F_8 and F_9 be the secondary shear load on rivets 1, 2, 3, 4, 6, 7, 8 and 9 at distances l_1, l_2, l_3, l_4, l_6, l_7, l_8 and l_9 from the centre of gravity (*G*) of the rivet system as shown in Fig. From the symmetry of the figure, we find that

$$l_1 = l_3 = l_7 = l_9 = \sqrt{(100)^2 + (120)^2} = 156.2 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moments of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_6)^2 + (l_7)^2 + (l_8)^2 + (l_9)^2\right]$$

$$= \frac{F_1}{l_1}\left[4(l_1)^2 + 2(l_2)^2 + 2(l_4)^2\right] \quad \text{....}(\because l_1 = l_3 = l_7 = l_9;\ l_2 = l_8 \text{ and } l_4 = l_6)$$

$$45 \times 10^3 \times 500 = \frac{F_1}{156.2}\left[4(156.2)^2 + 2(120)^2 + 2(100)^2\right] = 973.2\ F_1$$

$$F_1 = 45 \times 10^3 \times 500 / 973.2 = 23\ 120 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity (*G*), therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = F_8 = 23\ 120 \times \frac{120}{156.2} = 17\ 762 \text{ N} \qquad \text{...}(\because l_2 = l_8)$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = F_7 = F_9 = 23\ 120 \text{ N} \qquad \text{...}(\because l_3 = l_7 = l_9 = l_1)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = F_6 = 23\ 120 \times \frac{100}{156.2} = 14\ 800 \text{ N} \qquad \text{...}(\because l_4 = l_6)$$

The secondary shear loads acts perpendicular to the line joining the centre of rivet and the centre of gravity of the rivet system, as shown in Fig. And their direction is clockwise. By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 6 and 9 are heavily loaded. Let us now find the angle between the direct and secondary shear loads for these rivets. From the geometry of the figure, we find that

$$\cos\theta_3 = \cos\theta_9 = \frac{100}{l_3} = \frac{100}{156.2} = 0.64$$

Resultant shear load on rivets 3 and 9,

$$R_3 = R_9 = \sqrt{(P_s)^2 + (F_3)^3 + 2\,P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(5000)^2 + (23\,120)^2 + 2 \times 5000 \times 23\,120 \times 0.64} = 26\,600 \text{ N}$$

$$...(\because F_3 = F_9 \text{ and } \cos\theta_3 = \cos\theta_9)$$

and resultant shear load on rivet 6,

$$R_6 = P_s + F_6 = 5000 + 14\,800 = 19\,800 \text{ N}$$

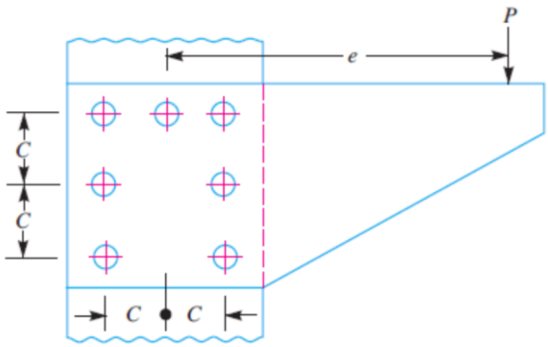
The resultant shear load ($R3$ or $R9$) may be determined graphically as shown in Fig. From above we see that the maximum resultant shear load is on rivets 3 and 9.

If d is the diameter of the rivet hole, then maximum resultant shear load (R_3),

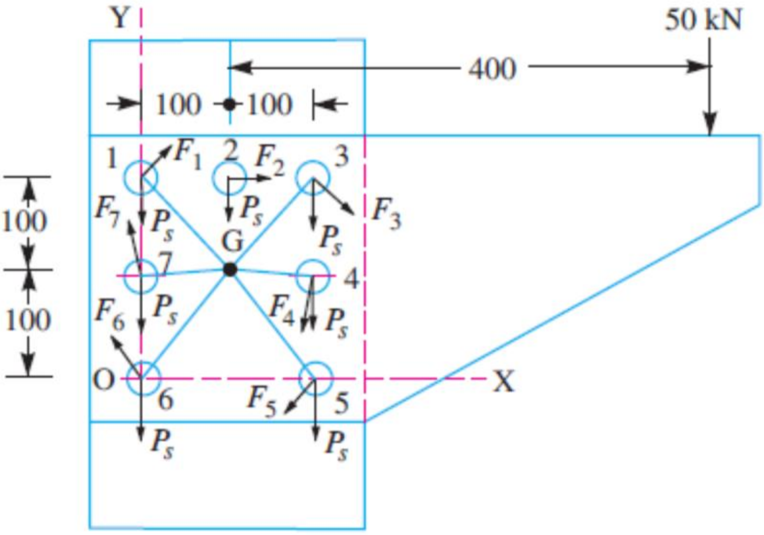
$$26\,600 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 40 = 31.42\,d^2$$

$$d^2 = 26\,600 / 31.42 = 846 \quad \text{or} \quad d = 29 \text{ mm}$$

Problem 8). An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 KN; rivet spacing, C = 100 mm; load arm, e = 400 mm. Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.



Solution. Given: t = 25 mm; P = 50 kN = 50 × 103 N ; e = 400 mm ; n = 7 ; τ = 65 MPa = 65 N/mm^2 ; σ_C= 120 MPa = 120 N/mm^2



First of all, let us find the centre of gravity (G) of the rivet system.

Let **x** = Distance of centre of gravity from OY,

y = Distance of centre of gravity from OX,

$x_1, x_2, x_3...$ = Distances of centre of gravity of each rivet from OY, and

$y_1, y_2, y_3...$ = Distances of centre of gravity of each rivet from OX.

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$

$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \qquad ...(\because x_1 = x_6 = x_7 = 0)$$

$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$

$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm} \qquad ...(\because y_5 = y_6 = 0)$$

∴ the centre of gravity (G) of the rivet system lies at a distance of 100 mm from OY and 114.3 mm from OX, as shown in Fig.

We know that direct shear load on each rivet,

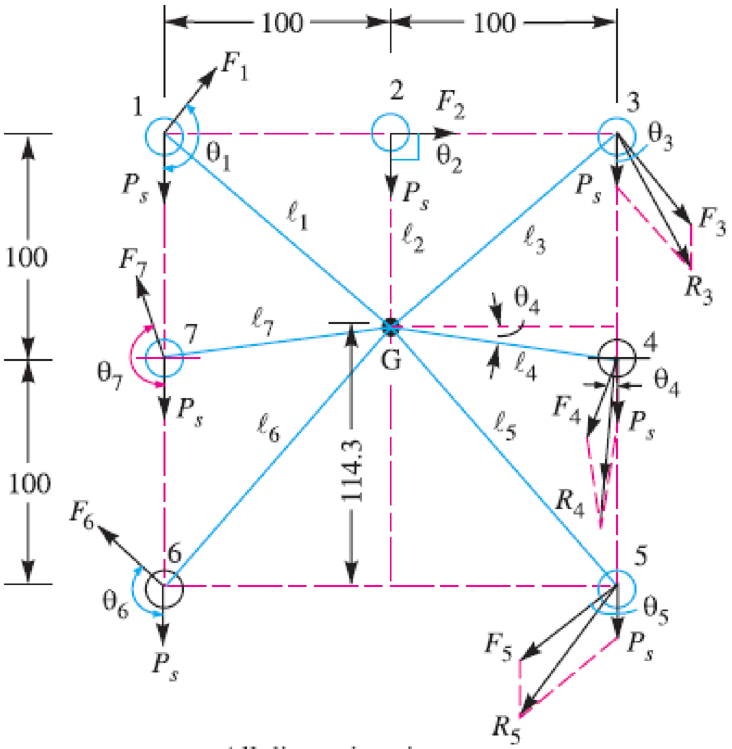
$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

The direct shear load acts parallel to the direction of load P i.e. vertically downward as shown in Fig.

Turning moment produced by the load P due to eccentricity (e)

= P × e = 50 × 103 × 400 = 20 × 106 N-mm

This turning moment is resisted by seven rivets as shown in Fig.



All dimensions in mm.

Let F_1, F_2, F_3, F_4, F_5, F_6 and F_7 be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l1, l2, l3, l4, l5, l6 and l7 respectively from the centre of gravity of the rivet system as shown in Fig.

From the geometry of the figure, we find that

$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right]$$

$$\ldots(\because l_1 = l_3;\ l_4 = l_7 \text{ and } l_5 = l_6)$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

$$20 \times 10^6 \times 131.7 = F_1(34\,690 + 7345 + 20\,402 + 46\,208) = 108\,645\,F_1$$

$$F_1 = 20 \times 10^6 \times 131.7 / 108\,645 = 24\,244 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 24\ 244 \text{ N} \qquad ...(\because l_1 = l_3)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24\ 244 \times \frac{152}{131.7} = 27\ 981 \text{ N}$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27\ 981 \text{ N} \qquad ...(\because l_6 = l_5)$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = 18\ 593 \text{ N} \qquad ...(\because l_7 = l_4)$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig. we find that

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\ 244)^2 + 2 \times 7143 \times 24\ 244 \times 0.76} = 30\ 033 \text{ N}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\ P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\ 593)^2 + 2 \times 7143 \times 18\ 593 \times 0.99} = 25\ 684 \text{ N}$$

resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_5)^2 + (F_5)^2 + 2\ P_5 \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\ 981)^2 + 2 \times 7143 \times 27\ 981 \times 0.658} = 33\ 121\ \text{N}$$

The resultant shear load may be determined graphically, as shown in Fig. From above we see that the maximum resultant shear load is on rivet 5. If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33\ 121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51\ d^2$$

$$d^2 = 33\ 121 / 51 = 649.4 \quad \text{or} \quad d = 25.5\ \text{mm}$$

From Table, we see that according to IS: 1929–1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\ 121}{25.5 \times 25}$$

$$= 51.95\ \text{N/mm}^2 = 51.95\ \text{MPa}$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.

Welded Joints

Introduction

A welded joint is a permanent joint which is obtained by the fusion of the edges of the two parts to be joined together, with or without the application of pressure and a filler material. The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding.

Advantages and Disadvantages of Welded Joints over Riveted Joints*

***Advantages of* welded joints over riveted joints**

1. The efficiency of wlded joint is more compared to riveted joints.

2. The welded structure is lighter than the riveted structure.

3. The pipes which are in circular shape makes difficult for riveting but they can be easily welded.

4. The strength of the welded joint is more compared to riveted joint.

5. The process of welding takes less time than the riveting.

***Disadvantages of* welded joints over riveted joints**

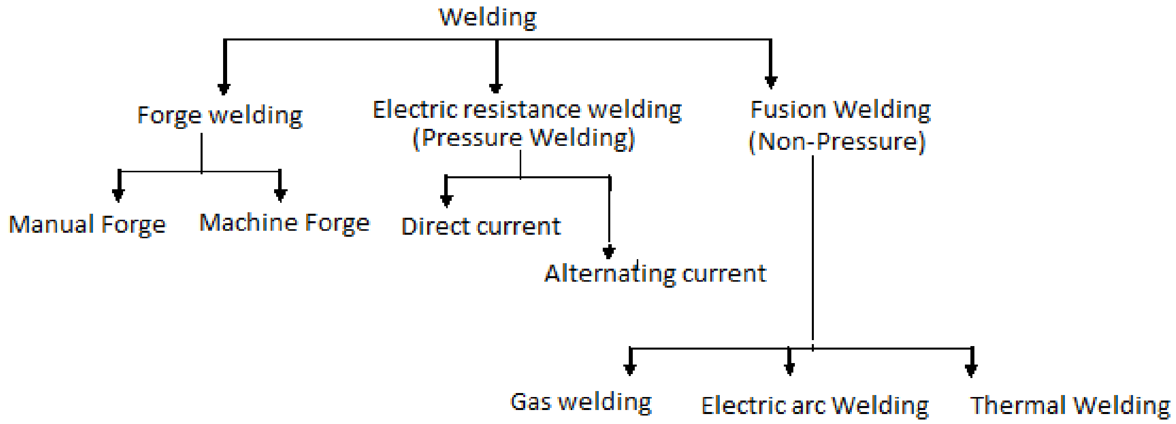
1. The inspection of welding work is more difficult than riveting work.

2. Welding work is dangerous for human eyes.

3. It requires a highly skilled labour and supervision.

4. Additional stresses may develop in the members due to uneven heating and cooling during fabrication.

Classification of Welding

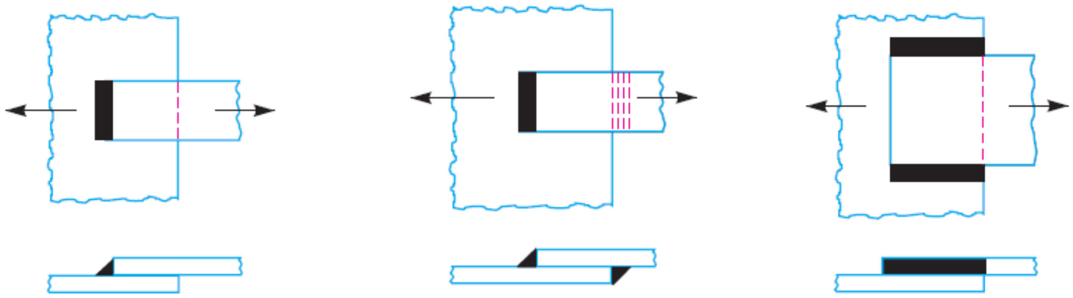


Welding Processes

The welding processes may be broadly classified into the following two groups:

1. Welding processes that use heat alone *e.g*. fusion welding.

2. Welding processes that use a combination of heat and pressure *e.g*. forge welding.



(*a*) Single transverse. (*b*) Double transverse. (*c*) Parallel fillet.

Types of Welded Joints

Following two types of welded joints are important:

1. Lap joint or fillet joint, and **2.** Butt joint.

Lap Joint

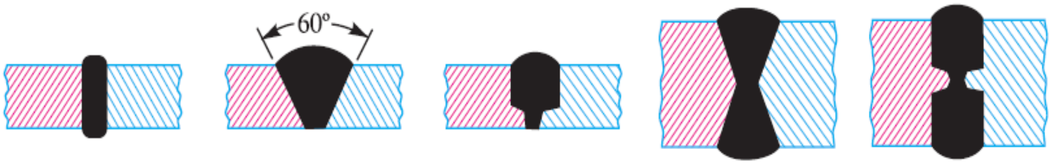
The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be

1. Single transverse fillet, **2.** Double transverse fillet, and **3.** Parallel fillet joints.

The fillet joints are shown in Fig. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.

Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Fig. In butt welds, the plate edges do not require beveling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be beveled to V or U-groove on both sides.



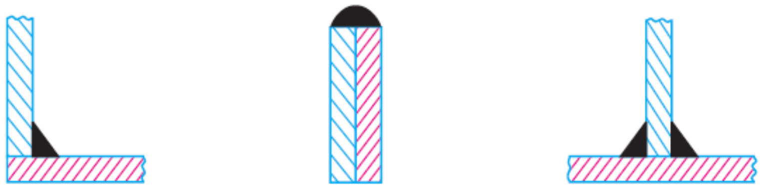
(*a*) Square butt joint. (*b*) Single *V*-butt joint. (*c*) Single *U*-butt joint. (*d*) Double *V*-butt joint. (*e*) Double *U*-butt joint.

The butt joints may be

1. Square butt joint, **2.** Single V-butt joint **3.** Single U-butt joint,

4. Double V-butt joint, and **5.** Double U-butt joint.

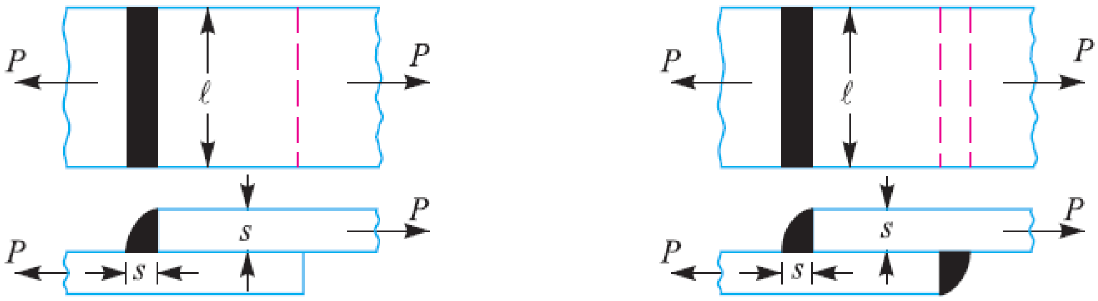
These joints are shown in Fig. The other types of welded joints are corner joint; edge joint and T-joint as shown in Fig.



(*a*) Corner joint. (*b*) Edge joint. (*c*) *T*-joint.

Strength of Transverse Fillet Welded Joints

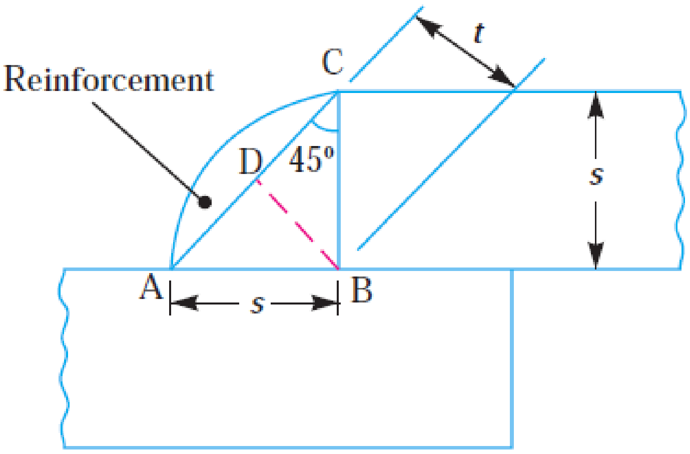
We have already discussed that the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. (*a*) and (*b*) respectively.



(*a*) Single transverse fillet weld. (*b*) Double transverse fillet weld.

Transverse fillet welds.

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle *ABC* with hypotenuse *AC* making equal angles with other two sides *AB* and *BC*. The enlarged view of the fillet is shown in Fig. The length of each side is known as ***leg*** or ***size of the weld*** and the perpendicular distance of the hypotenuse from the intersection of legs (*i.e. BD*) is known as ***throat thickness.*** The minimum area of the weld is obtained at the throat *BD*, which is given by the product of the throat thickness and length of weld.



t = Throat thickness (BD),

s = Leg or size of weld,

= Thickness of plate, and

l = Length of weld,

we find that the throat thickness,

$$t = s \times \sin 45° = 0.707\, s$$

Minimum area of the weld or throat area,

$$A = \text{Throat thickness} \times \text{Length of weld}$$

$$= t \times l = 0.707\, s \times l$$

If σ_t is the allowable tensile stress for the weld metal, then the tensile strength of the joint for single fillet weld,

$$P = \text{Throat area} \times \text{Allowable tensile stress} = 0.707\, s \times l \times \sigma_t$$

and tensile strength of the joint for double fillet weld,

$$P = 2 \times 0.707\, s \times l \times \sigma_t = 1.414\, s \times l \times \sigma_t$$

Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint as shown in Fig.(*a*). We have already discussed in the previous article, that the minimum area of weld or the throat area,

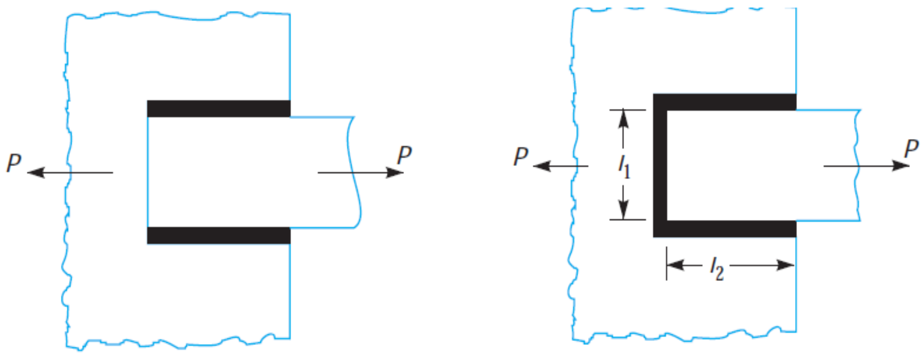
$$A = 0.707\, s \times l$$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld,

$$P = \text{Throat area} \times \text{Allowable shear stress} = 0.707\, s \times l \times \tau$$

and shear strength of the joint for double parallel fillet weld,

$$P = 2 \times 0.707 \times s \times l \times \tau = 1.414\, s \times l \times \tau$$



(a) Double parallel fillet weld.

(b) Combination of transverse and parallel fillet weld.

Special Cases of Fillet Welded Joints

The following cases of fillet welded joints are important from the subject point of view.

1. ***Circular fillet weld subjected to torsion.*** Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig.

Let d = Diameter of rod,

r = Radius of rod,

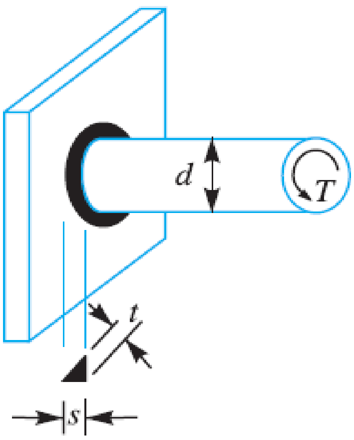
T = Torque acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

J = Polar moment of inertia of the

$$\text{weld section} = \frac{\pi t d^3}{4}$$



Circular fillet weld subjected to torsion

We know that shear stress for the material,

$$\tau = \frac{T.r}{J} = \frac{T \times d/2}{J}$$

$$= \frac{T \times d/2}{\pi t d^3/4} = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

∴ Length of throat, $t = s \sin 45° = 0.707\, s$

and maximum shear stress,

$$\tau_{max} = \frac{2T}{\pi \times 0.707\, s \times d^2} = \frac{2.83\, T}{\pi\, s\, d^2}$$

2. *Circular fillet weld subjected to bending moment.* Consider a circular rod connected to a rigid plate by a fillet weld as shown in Fig.

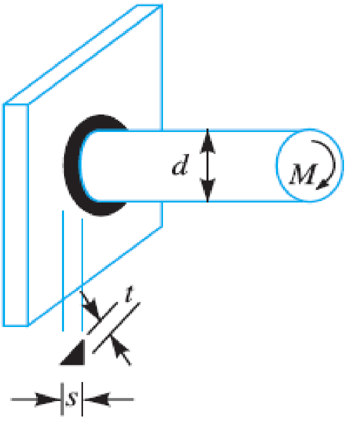
Let d = Diameter of rod,

M = Bending moment acting on the rod,

s = Size (or leg) of weld,

t = Throat thickness,

Z = Section modulus of the weld section



Circular fillet weld subjected to bending moment.

$$= \frac{\pi t d^2}{4}$$

We know that the bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{M}{\pi t d^2/4} = \frac{4M}{\pi t d^2}$$

This bending stress occurs in a horizontal plane along a leg of the fillet weld. The maximum bending stress occurs on the throat of the weld which is inclined at 45° to the horizontal plane.

∴ Length of throat, $t = s \sin 45° = 0.707\ s$

and maximum bending stress,

$$\sigma_{b(max)} = \frac{4\ M}{\pi \times 0.707\ s \times d^2} = \frac{5.66\ M}{\pi\ s\ d^2}$$

Problem(1):-A plate 100 mm wide and 10 mm thick is to be welded to another plate by means of double parallel fillets. The plates are subjected to a static load of 80 kN. Find the length of weld if the permissible shear stress in the weld does not exceed 55 MPa.

Given data: Width = 100 mm ; Thickness = 10 mm ; P = 80 kN = 80 × 10^3 N ;

τ = 55 MPa = 55 N/mm^2

Let l =Length of weld, and

s = Size of weld = Plate thickness = 10 mm ... (Given)

We know that maximum load which the plates can carry for double parallel fillet weld (P),

$$80 \times 103 = 1.414 \times s \times l \times \tau = 1.414 \times 10 \times l \times 55 = 778\ l$$

$$\therefore \quad l = 80 \times 103 / 778 = 103 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l = 103 + 12.5 = 115.5 \text{ mm}$$

Problem(2):-A plate 100 mm wide and 12.5 mm thick is to be welded to another plate by means of parallel fillet welds. The plates are subjected to a load of 50 kN. Find the length of the weld so that the maximum stress does not exceed 56 MPa. Consider the joint first under static loading and then under fatigue loading.

Given data: Width = 100 mm ; Thickness = 12.5 mm ; P = 50 kN = 50 × 10^3N ;

τ = 56 MPa = 56 N/mm^2

Length of weld for static loading

Let l = Length of weld, and

s = Size of weld = Plate thickness = 12.5 mm ... (Given)

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

$$50 \times 103 = 1.414\ s \times l \times \tau$$

$$= 1.414 \times 12.5 \times l \times 56 = 990\ l$$

$$\therefore \quad l = 50 \times 103 / 990 = 50.5 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$l = 50.5 + 12.5 = 63$ mm

Length of weld for fatigue loading

From Table we find that the stress concentration factor for parallel fillet welding is 2.7.

∴ Permissible shear stress,

$\tau = 56 / 2.7 = 20.74$ N/mm^2

We know that the maximum load which the plates can carry for double parallel fillet welds (P),

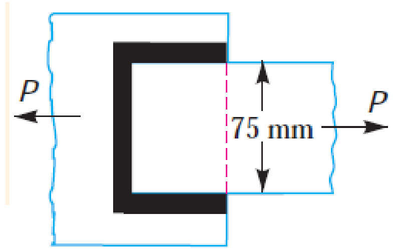
$50 \times 103 = 1.414\, s \times l \times \tau = 1.414 \times 12.5 \times l \times 20.74 = 367\, l$

∴ $l = 50 \times 103 / 367 = 136.2$ mm

Adding 12.5 for starting and stopping of weld run, we have

$l = 136.2 + 12.5 = 148.7$ mm

Problem(3):-A plate 75 mm wide and 12.5 mm thick is joined with another plate by a single transverse weld and a double parallel fillet weld as shown in Fig. The maximum tensile and shear stresses are 70 MPa and 56 MPa respectively. Find the length of each parallel fillet weld, if the joint is subjected to both static and fatigue loading.



Given data : Width = 75 mm ; Thickness = 12.5 mm ; $\sigma_t = 70$ MPa = 70 N/mm^2 ; $\tau = 56$ MPa = 56 N/mm^2.

The effective length of weld (l_1) for the transverse weld may be obtained by subtracting 12.5 mm from the width of the plate.

∴ $l_1 = 75 - 12.5 = 62.5$ mm

Length of each parallel fillet for static loading

Let l_2 = Length of each parallel fillet.

We know that the maximum load which the plate can carry is

P = Area × Stress = $75 \times 12.5 \times 70 = 65\,625$ N

Load carried by single transverse weld,

$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70 = 38\,664$ N

and the load carried by double parallel fillet weld,

$P_2 = 1.414\, s \times l2 \times \tau = 1.414 \times 12.5 \times l2 \times 56 = 990\, l_2$ N

∴ Load carried by the joint (P),

$65\,625 = P_1 + P_2 = 38\,664 + 990\, l_2$ or $l_2 = 27.2$ mm

Adding 12.5 mm for starting and stopping of weld run, we have

$l_2 = 27.2 + 12.5 = 39.7$ say 40 mm

Length of each parallel fillet for fatigue loading

From Table, we find that the stress concentration factor for transverse welds is 1.5 and for parallel fillet welds is 2.7.

∴ Permissible tensile stress,

$$\sigma_t = 70 / 1.5 = 46.7 \text{ N/mm}^2$$

and permissible shear stress,

$$\tau = 56 / 2.7 = 20.74 \text{ N/mm}^2$$

Load carried by single transverse weld,

$$P_1 = 0.707\, s \times l_1 \times \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7 = 25\,795 \text{ N}$$

and load carried by double parallel fillet weld,

$$P_2 = 1.414\, s \times l2 \times \tau = 1.414 \times 12.5\, l2 \times 20.74 = 366\, l_2 \text{ N}$$

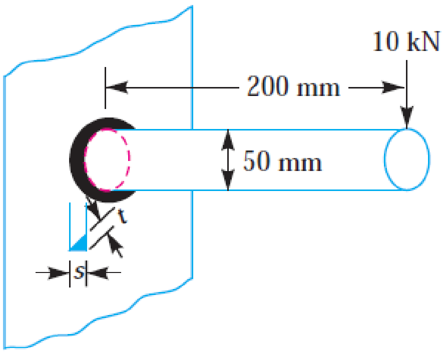
∴ Load carried by the joint (P),

$$65\,625 = P_1 + P_2 = 25\,795 + 366\, l_2 \text{ or } l_2 = 108.8 \text{ mm}$$

Adding 12.5 mm for starting and stopping of weld run, we have

$$l_2 = 108.8 + 12.5 = 121.3 \text{ mm}$$

<u>Problem(4)</u>:- *A 50 mm diameter solid shaft is welded to a flat plate as shown in Fig. If the size of the weld is 15 mm, find the maximum normal and shear stress in the weld.*



Given data : $D = 50$ mm ; $s = 15$ mm ; $P = 10$ kN $= 10\,000$ N ; $e = 200$ mm

Let t = Throat thickness.

The joint, as shown in Fig. is subjected to direct shear stress and the bending stress. We know that the throat area for a circular fillet weld,

$$A = t \times \pi D = 0.707\, s \times \pi D$$
$$= 0.707 \times 15 \times \pi \times 50$$
$$= 1666 \text{ mm2}$$

∴ Direct shear stress,

$$\tau = \frac{P}{A} = \frac{10\,000}{1666} = 6 \text{ N/mm}^2 = 6 \text{ MPa}$$

We know that bending moment,

$$M = P \times e = 10\,000 \times 200 = 2 \times 10^6 \text{ N-mm}$$

From Table , we find that for a circular section, section modulus,

$$Z = \frac{\pi\, t\, D^2}{4} = \frac{\pi \times 0.707\, s \times D^2}{4} = \frac{\pi \times 0.707 \times 15\,(50)^2}{4} = 20\,825 \text{ mm}^3$$

∴ Bending stress,

$$\sigma_b = \frac{M}{Z} = \frac{2 \times 10^6}{20\,825} = 96 \text{ N/mm}^2 = 96\,\text{MPa}$$

Maximum normal stress

We know that the maximum normal stress,

$$\sigma_{t(max)} = \frac{1}{2}\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2} \times 96 + \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2}$$

$$= 48 + 48.4 = 96.4 \text{ MPa}$$ **Ans.**

Maximum shear stress

We know that the maximum shear stress,

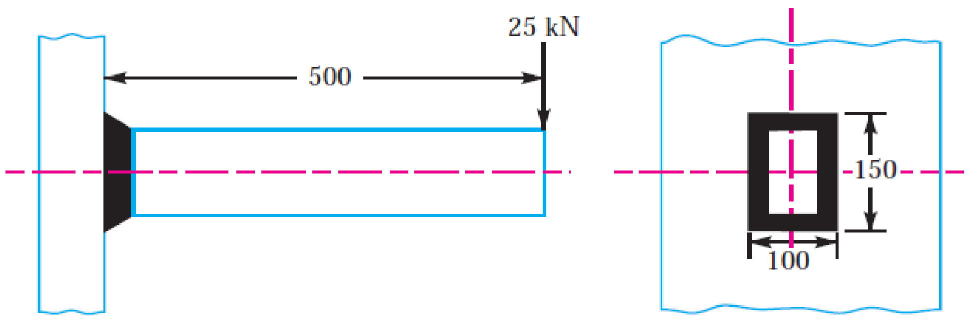
$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{(96)^2 + 4 \times 6^2} = 48.4 \text{ MPa}$$ **Ans.**

Problem(5):- A rectangular cross-section bar is welded to a support by means of fillet welds as shown in Fig. Determine the size of the welds, if the permissible shear stress in the weld is limited to 75 MPa.

Solution. Given : P = 25 kN = 25 × 10^3N ; τ_{max} = 75 MPa = 75N/mm^2 ; l = 100 mm ; b = 150 mm ; e = 500 mm

Let s = Size of the weld, and

t = Throat thickness.



All dimensions in mm

The joint, as shown in Fig. is subjected to direct shear stress and the bending stress. We know that the throat area for a rectangular fillet weld,

$$A = t\,(2b + 2l) = 0.707\, s\,(2b + 2l)$$

$$= 0.707s\,(2 \times 150 + 2 \times 100) = 353.5\, s \text{ mm}^2 \qquad \ldots (\,t = 0.707s)$$

$$\therefore \text{ Direct shear stress, } \tau = \frac{P}{A} = \frac{25 \times 10^3}{353.5\, s} = \frac{70.72}{s} \text{ N/mm}^2$$

We know that bending moment,

$$M = P \times e = 25 \times 10^3 \times 500 = 12.5 \times 10^6 \text{ N-mm}$$

From Table we find that for a rectangular section, section modulus,

$$Z = t\left(b.l + \frac{b^2}{3}\right) = 0.707\, s\left[150 \times 100 + \frac{(150)^2}{3}\right] = 15\,907.5\, s \text{ mm}^3$$

$$\therefore \text{ Bending stress, } \quad \sigma_b = \frac{M}{Z} = \frac{12.5 \times 10^6}{15\,907.5\, s} = \frac{785.8}{s} \text{ N/mm}^2$$

We know that maximum shear stress (τ_{max}),

$$75 = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\,\tau^2} = \frac{1}{2}\sqrt{\left(\frac{785.8}{s}\right)^2 + 4\left(\frac{70.72}{s}\right)^2} = \frac{399.2}{s}$$

$s = 399.2 / 75 = 5.32$ mm **Ans.**

BOLTED JOINTS

Introduction:-In thread fastening process, joining of two parts is by means of nuts and bolts. Thread is formed on the surface of a cylindrical rod or cylindrical hole by machining a helical groove.

A screw made by cutting a single helical groove on the cylinder is known as ***single threaded*** (or single-start) screw and if a second thread is cut in the space between the grooves of the first, a ***double threaded*** (or double-start) screw is formed. Similarly, triple and quadruple (*i.e.* multiple-start) threads may be formed. The helical grooves may be cut either ***right hand*** or ***left hand*****.**

Advantages and Disadvantages of Screwed Joints*:-

Advantages

1. Threaded joints can be easily assembled and disassembled.
2. Screwed joints are highly reliable in operation.
3. Threaded joints are self-locking.
4. Threaded joints are standardised.

5.Screws are relatively cheap to produce.

Disadvantages

1. Due to vibrations during operating conditions of machine parts, threaded joints may get loosen.
2. The stress concentration is high near the threaded portion of the parts, which causes fatigue failure.

Important Terms Used in Screw Threads

The following terms used in screw threads, as shown in Fig. are important from the subject point of view :

1. *Major diameter (or) Nominal diameter (D_o)*. It is the largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as ***outside*** or ***nominal diameter*****.**

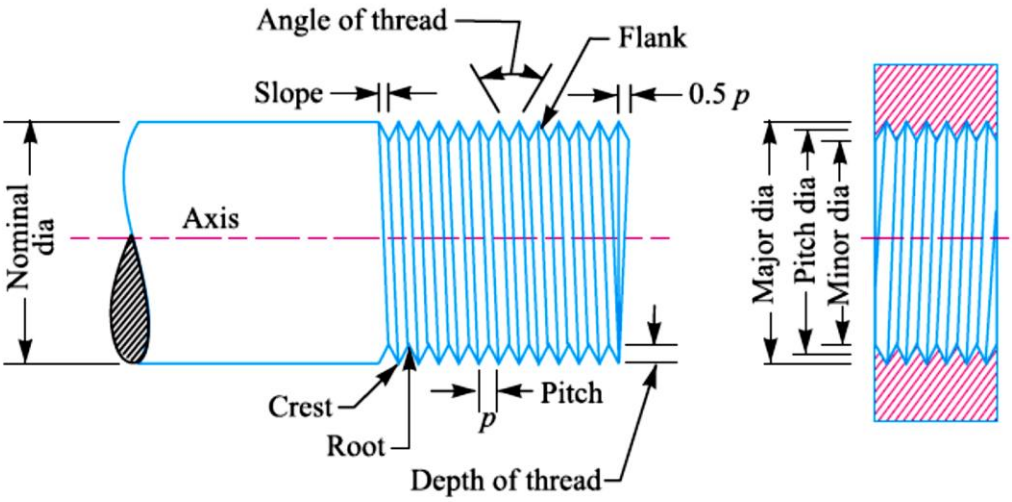
2. *Minor diameter (or) Core diameter (or) root diameter (d_c)*. It is the smallest diameter of an external or internal screw thread. It is also known as ***core*** or ***root diameter*****.**

3. *Pitch diameter (or) Effective diameter (d_p)*. It is defined as an imaginary cylinder diameter, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads.

4. *Pitch*. It is an axial distance between two similar points on adjacent threads. It is measured in millimeters. Mathematically,

$$\text{Pitch} = \frac{1}{\text{No. of threads per unit length of screw}}$$

5. *Lead*. It is defined as the axial distance which a screw thread advances in one rotation of the nut. Lead is equal to the pitch in case of single start threads, it is twice the pitch in double start, thrice the pitch in triple start and so on.



Terms used in screw threads.

6. *Crest*. It is the top surface generated by the two adjacent flanks of the thread.

7. *Root*. It is the bottom surface generated by the two adjacent flanks of the thread.

8. *Depth of thread*. It is the perpendicular distance between the crest and root.

9. *Flank*. The inclined surface, which joins the crest and root.

10. *Angle of thread*. The inclined angle between two flank surfaces.

11. *Slope.* It is defined as the half the pitch of the thread.

Common Types of Screw Fastenings:-

Threaded fastenings are classified according to their shape and the basic application.

The basic types of threaded fastings are follows.

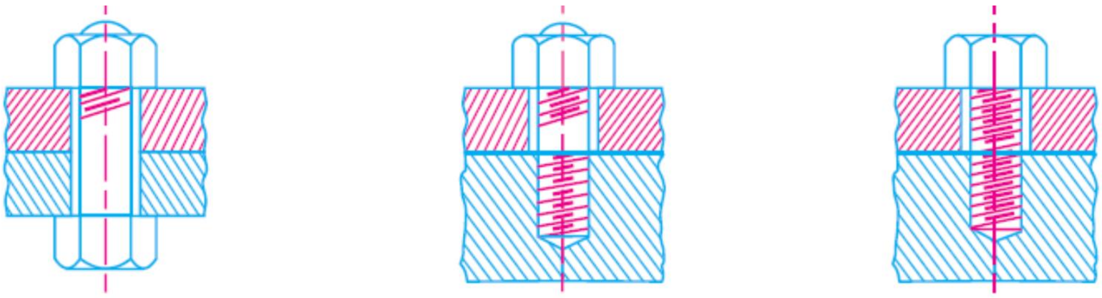
(a) Through bolt (b) Tap bolt (c) Studs (d) Cap screws and (e) Machine screws (f) Set screws.

1. *Through bolts*. It is as shown in fig(a). The bolt consists of a cylindrical shank, which is passed through drilled holes in the two parts to be joined together. The bolt consists of threads for the nut at one end and hexagonal or square head at the other end. The threaded portion of the bolt is screwed into the nut.

Machining finish is not required for through bolts. According to the usage, the through bolts are known as machine bolts, carriage bolts, automobile bolts, etc.

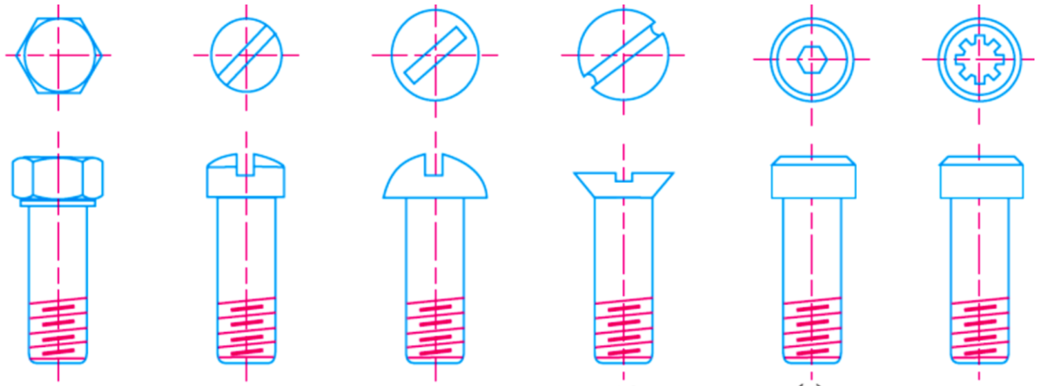
2. *Tap bolts.* A tap bolt or screw differs from a bolt. It is screwed into a tapped hole of one of the parts to be fastened without the nut, as shown in Fig (*b*).

3. *Studs*. A stud shown in fig (c) consists of threaded portion at both ends. One end of the stud is screwed into a tapped hole of the parts to be fastened and the other end is tightened by means of a nut. Studs are normally used for fixing various kinds of covers of engine and pump cylinders, valves etc.



(*a*) Through bolt. (*b*) Tap bolt. (*c*) Stud.

4.*Cap screws.* The cap screws are similar to tap bolts except smaller in size. The cap screws are available according to their shapes of head as shown in fig.



(*a*) (*b*) (*c*) (*d*) (*e*) (*f*)

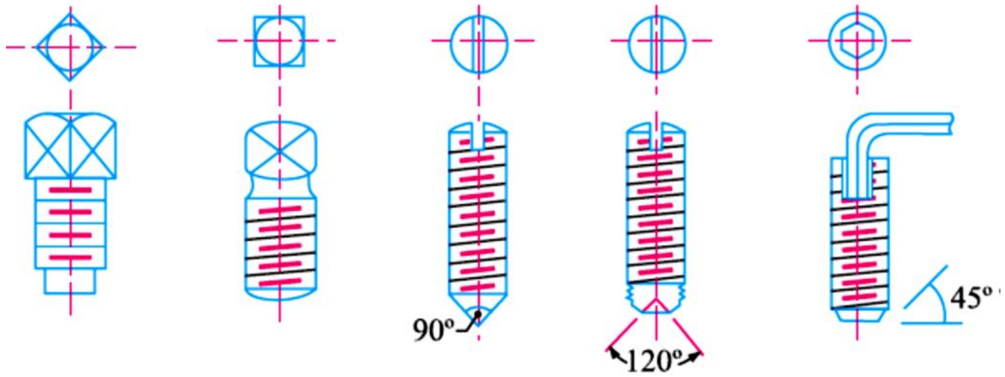
(*a*) Hexagonal head; (*b*) Fillister head; (*c*) Round head; (*d*) Flat head; (*e*) Hexagonal socket; (*f*) Fluted socket.

Types of cap screws.

5. *Machine screws*. These are similar to cap screws with the head slotted for a screw driver. These are generally used with a nut.

6. *Set screws*. The set screws are shown in Fig. These are used to prevent relative motion between the two parts. Set screws can also be used instead of key. Which prevent relative motion between the shaft and the hub.

The threaded portion of the set screw passes through the tapped hole in one of the parts to be joined and the end of the screw (also called point) presses against the other part.



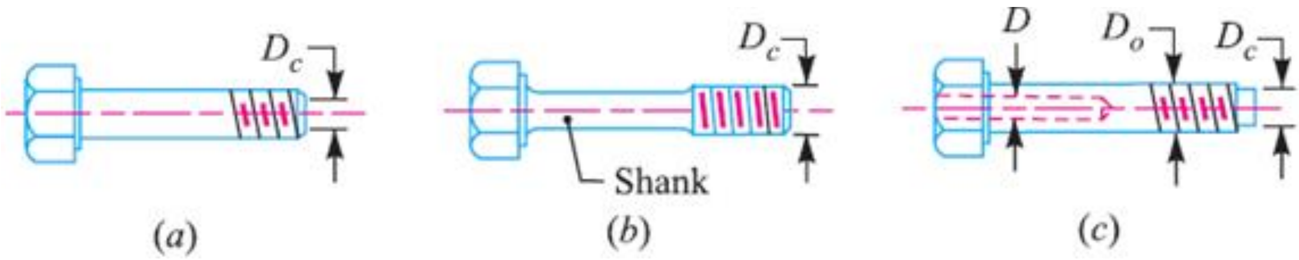
Bolts of Uniform Strength (Discuss about the bolts of uniform strength)

In engineering applications, the bolts are subjected to shock and impact loads. Ex: cylinder head bolts of an IC engine, connecting rod bolts. In such cases, bolts should be designed based on resilience. Resiliance is the ability of the material to absorb energy when deformed elastically and to release this energy when unloaded.

The following fig(a) represents an ordenary bolt. The major diameter of the thread(D_0) and shank diameter are equal. In this case stress concentration is more. Reduce the stress concentration the following two methods are considered.

The diameter of the shank is reduced to core diameter of the threads as shown in figure(b).

The cross-sectional area of the shank is reduced by drilling a hole as shown in figure(c).



Bolts of uniform strength.

When the bolt is subjected to a tensile force, the stress in the shank and the threaded portion are equal in the first method. Similarly, in the second method, to find diameter of the hole D.

In this method, an axial hole is drilled through the head as far as the thread portion such that the area of the shank becomes equal to the root area of the thread.

Let D = Diameter of the hole.

D_o = Outer diameter of the thread, and

D_c = Root or core diameter of the thread.

$$\therefore \quad \frac{\pi}{4} D^2 = \frac{\pi}{4}\left[(D_o)^2 - (D_c)^2\right]$$

$$D^2 = (D_o)^2 - (D_c)^2$$

$$\therefore \quad D = \sqrt{(D_o)^2 - (D_c)^2}$$

But drilled holes results in stress concentration and also machining of long hole is difficult. Hence bolt with reduced shank diameter (first method) is generally used.

<u>Designation of Screw Threads</u> (Thread standards)

According to Indian standards, IS : 4218, 1967, the designation of screw threads are of two types.

(a). Coarse series threads- designated by the letter 'M' followed by the nominal or major diameter in mm. Ex: M12

(b) Fine series threads- designated by the letter 'M' followed by the nominal or major diameter and pitch, both represented in mm. Ex: M12 X 12.5

$$D_o = D_c / 0.84$$

<u>Design of a Nut</u>

When a bolt and nut is made of mild steel, then the effective height of nut is made equal to the nominal diameter of the bolt. If the nut is made of weaker material than the bolt, then the height of nut should be larger, such as 1.5 *d* for gun metal, 2 *d* for cast iron and 2.5 *d* for aluminium alloys (where *d* is the nominal diameter of the bolt).

<u>Problem(1)</u>:- *A steam engine of effective diameter 300 mm is subjected to a steam pressure of 1.5 N/mm². The cylinder head is connected by 8 bolts having yield point 330 MPa and endurance limit at 240 MPa. The bolts are tightened with an initial preload of 1.5 times the steam load. A soft copper gasket is used to make the joint leak-proof. Assuming a factor of safety 2, find the size of bolt required. The stiffness factor for copper gasket may be taken as 0.5.*

Solution. Given : D = 300 mm ; p = 1.5 N/mm² ; n = 8 ; σ_y = 330 MPa = 330 N/mm²; σ_e = 240 MPa = 240 N/mm² ; $P_1 = 1.5\, P_2$; $F.S.$ = 2 ; K = 0.5

We know that steam load acting on the cylinder head,

$$P_2 = \frac{\pi}{4}(D)^2\, p = \frac{\pi}{4}(300)^2\, 1.5 = 106\,040 \text{ N}$$

∴ Initial pre-load,

$$P_1 = 1.5\, P_2 = 1.5 \times 106\,040 = 159\,060 \text{ N}$$

We know that the resultant load (or the maximum load) on the cylinder head,

$P_{max} = P_1 + K.P_2 = 159\ 060 + 0.5 \times 106\ 040 = 212\ 080$ N

This load is shared by 8 bolts, therefore maximum load on each bolt,

$P_{max} = 212\ 080 / 8 = 26\ 510$ N

and minimum load on each bolt,

$P_{min} = P_1 / n = 159\ 060/8 = 19\ 882$ N

We know that mean or average load on the bolt,

$$P_m = \frac{P_{max} + P_{min}}{2} = \frac{26\ 510 + 19\ 882}{2} = 23\ 196 \text{ N}$$

and the variable load on the bolt,

$$P_v = \frac{P_{max} - P_{min}}{2} = \frac{26\ 510 - 19\ 882}{2} = 3314 \text{ N}$$

Let d_c = Core diameter of the bolt in mm.

∴ Stress area of the bolt,

$$A_s = \frac{\pi}{4}(d_c)^2 = 0.7854\ (d_c)^2 \text{ mm}^2$$

We know that mean or average stress on the bolt,

$$\sigma_m = \frac{P_m}{A_s} = \frac{23\ 196}{0.7854\ (d_c)^2} = \frac{29\ 534}{(d_c)^2} \text{ N/mm}^2$$

and variable stress on the bolt,

$$\sigma_v = \frac{P_v}{A_s} = \frac{3314}{0.7854\ (d_c)^2} = \frac{4220}{(d_c)^2} \text{ N/mm}^2$$

According to *Soderberg's formula, the variable stress,

$$\sigma_v = \sigma_e \left(\frac{1}{F.S} - \frac{\sigma_m}{\sigma_y}\right)$$

$$\frac{4220}{(d_c)^2} = 240\left(\frac{1}{2} - \frac{29\ 534}{(d_c)^2\ 330}\right) = 120 - \frac{21\ 480}{(d_c)^2}$$

or $$\frac{4220}{(d_c)^2} + \frac{21\ 480}{(d_c)^2} = 120 \quad \text{or} \quad \frac{25\ 700}{(d_c)^2} = 120$$

∴ $$(d_c)^2 = 25\ 700 / 120 = 214 \quad \text{or} \quad d_c = 14.6 \text{ mm}$$

From Table (coarse series), the standard core diameter is d_c = 14.933 mm and the corresponding size of the bolt is M18.

Bolted Joints under Eccentric Loading:-

There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc. The eccentric load may be

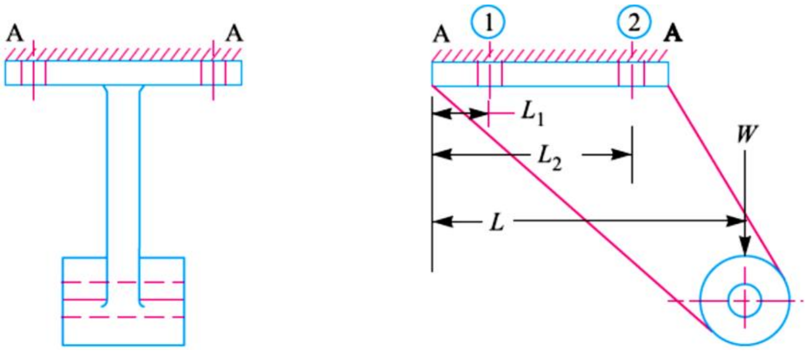
1. Parallel to the axis of the bolts,

2. Perpendicular to the axis of the bolts, and

3. In the plane containing the bolts.

Eccentric Load Acting Parallel to the Axis of Bolts:-

Consider a bracket having a rectangular base bolted to a wall by means of four bolts as shown in Fig. A little consideration will show that each bolt is subjected to a direct tensile load of



Eccentric load acting parallel to the axis of bolts.

$$W_{t1} = \frac{W}{n}$$ where n is the number of bolts

Further the load W tends to rotate the bracket about the edge A-A. Due to this, each bolt is stretched by an amount that depends upon its distance from the tilting edge. Since the stress is a function of *elongation, therefore each bolt will experience a different load which also depends upon the distance from the tilting edge. For convenience, all the bolts are made of same size. In case the flange is heavy, it may be considered as a rigid body.

Let w be the load in a bolt per unit distance due to the turning effect of the bracket and let W_1 and W_2 be the loads on each of the bolts at distances L_1 and L_2 from the tilting edge.

* We know that elongation is proportional to strain which in turn is proportional to stress within elastic limits.

∴ Load on each bolt at distance L_1,

$$W_1 = w.L_1$$

and moment of this load about the tilting edge

$$= w\,.L_1 \times L_1 = w\,(L_1)^2$$

Similarly, load on each bolt at distance $L2$,

$$W_2 = w.L_2$$

and moment of this load about the tilting edge

$$= w.L_2 \times L_2 = w\,(L_2)^2$$

∴ Total moment of the load on the bolts about the tilting edge

$= 2w\,(L_1)^2 + 2w\,(L_2)^2$ ....***(i)***

... (There are two bolts each at distance of L_1 and L_2)

Also the moment due to load W about the tilting edge

$= W.L$ ....***(ii)***

From equations ***(i)*** and ***(ii)***, we have

$$WL = 2w\,(L_1)^2 + 2w\,(L_2)^2$$

$$w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} \quad \text{...(iii)}$$

It may be noted that the most heavily loaded bolts are those which are situated at the greatest distance from the tilting edge. In the case discussed above, the bolts at distance L_2 are heavily loaded.

∴ Tensile load on each bolt at distance L_2,

$$W_{t2} = W_2 = w.L_2 = \frac{W.L.L_2}{2[(L_1)^2 + (L_2)^2]} \quad \text{... [From equation (iii)]}$$

and the total tensile load on the most heavily loaded bolt,

$$Wt = Wt1 + Wt2 \quad \text{.................(iv)}$$

If d_c is the core diameter of the bolt and σ_t is the tensile stress for the bolt material, then total tensile load,

$$W_t = \frac{\pi}{4}\,(d_c)^2\,\sigma_t \quad \text{....................(v)}$$

From equations ***(iv)*** and ***(v)***, the value of d_c may be obtained.

<u>Problem (2):-</u> A bracket, as shown in Fig above, supports a load of 30 kN. Determine the size of bolts, if the maximum allowable tensile stress in the bolt material is 60 MPa. The distances are : L_1 = 80 mm, L_2 = 250 mm, and L = 500 mm.

Given data: W = 30 kN ; σ_t = 60 MPa = 60 N/mm2 ; L_1 = 80 mm ; L_2 = 250 mm ;

L = 500 mm

We know that the direct tensile load carried by each bolt,

$$W_{t1} = \frac{W}{n} = \frac{30}{4} = 7.5 \text{ kN}$$

and load in a bolt per unit distance,

$$w = \frac{W.L}{2\,[(L_1)^2 + (L_2)^2]} = \frac{30 \times 500}{2\,[(80)^2 + (250)^2]} = 0.109 \text{ kN/mm}$$

Since the heavily loaded bolt is at a distance of $L2$ mm from the tilting edge, therefore load on the heavily loaded bolt,

$W_{t2} = w.L_2 = 0.109 \times 250 = 27.25$ kN

∴ Maximum tensile load on the heavily loaded bolt,

$W_t = W_{t1} + W_{t2} = 7.5 + 27.25 = 34.75$ kN $= 34\,750$ N

Let d_c = Core diameter of the bolts.

We know that the maximum tensile load on the bolt (W_t),

$$34\,750 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(d_c)^2\,60 = 47\,(d_c)^2$$

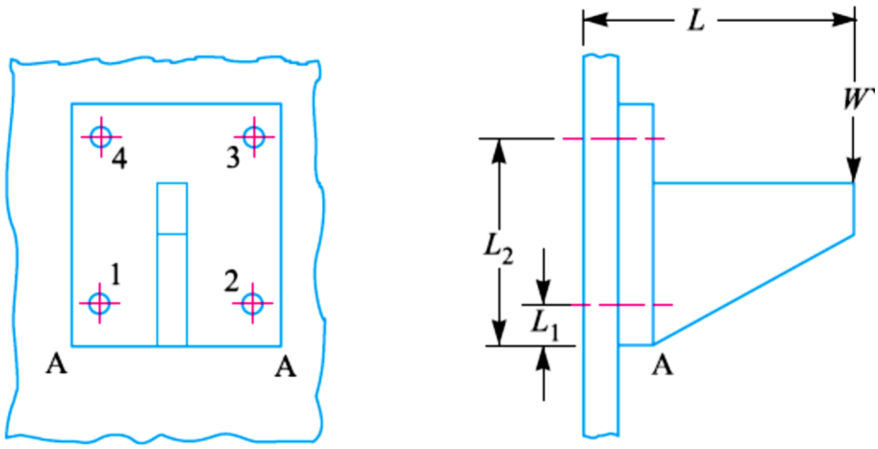
$(d_c)^2 = 34\,750 / 47 = 740$ or $d_c = 27.2$ mm

From Table (coarse series), we find that the standard core diameter of the bolt is 28.706 mm and the corresponding size of the bolt is M 33.

Eccentric Load Acting Perpendicular to the Axis of Bolts

(Explain the method of determining the size of the bolt when the bracket carries an eccentric load perpendicular to the arms of the bolt.)

A wall bracket carrying an eccentric load perpendicular to the axis of the bolts is shown in Fig.



Eccentric load perpendicular to the axis of bolts.

In this case, the bolts are subjected to direct shearing load which is equally shared by all the bolts. Therefore direct shear load on each bolts,

$W_s = W/n$, where n is number of bolts.

The eccentric load W tries to tilt the bracket in the clockwise direction about the edge A-A.

The maximum tensile load on a heavily loaded bolt (W_t) In this case, bolts 3 and 4 are heavily loaded.

∴ Maximum tensile load on bolt 3 or 4,

$$W_{t2} = W_t = \frac{W.L.L_2}{2\,[(L_1)^2 + (L_2)^2]}$$

When the bolts are subjected to shear as well as tensile loads, then the equivalent loads may be determined by the following relations :

Equivalent tensile load,

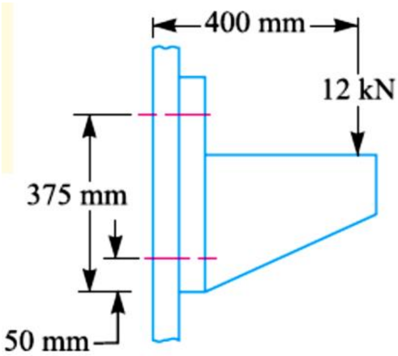
$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

and equivalent shear load,

$$W_{se} = \frac{1}{2}\left[\sqrt{(W_t)^2 + 4(W_s)^2}\right]$$

Knowing the value of equivalent loads, the size of the bolt may be determined for the given allowable stresses.

Problem(3):-*For supporting the travelling crane in a workshop, the brackets are fixed on steel columns as shown in Fig. The maximum load that comes on the bracket is 12 kN acting vertically at a distance of 400 mm from the face of the column. The vertical face of the bracket is secured to a column by four bolts, in two rows (two in each row) at a distance of 50 mm from the lower edge of the bracket. Determine the size of the bolts if the permissible value of the tensile stress for the bolt material is 84 MPa. Also find the cross-section of the arm of the bracket which is rectangular.*



Solution. Given : W = 12 kN = 12 × 10³ N ; L = 400 mm ; L_1 = 50 mm ; L_2 = 375 mm

σ_t = 84 MPa = 84 N/mm² ; n = 4

We know that direct shear load on each bolt,

$$W_s = \frac{W}{n} = \frac{12}{4} = 3 \text{ kN}$$

Since the load W will try to tilt the bracket in the clockwise direction about the lower edge, therefore the bolts will be subjected to tensile load due to turning moment. The maximum loaded bolts are 3 and 4 (See Fig.), because they lie at the greatest distance from the tilting edge A–A (*i.e.* lower edge).

We know that maximum tensile load carried by bolts 3 and 4,

$$W_t = \frac{W.L.L_2}{2\,[(L_1)^2 + (L_2)^2]} = \frac{12 \times 400 \times 375}{2\,[(50)^2 + (375)^2]} = 6.29 \text{ kN}$$

Since the bolts are subjected to shear load as well as tensile load, therefore equivalent tensile load,

$$W_{te} = \frac{1}{2}\left[W_t + \sqrt{(W_t)^2 + 4(W_s)^2}\right] = \frac{1}{2}\left[6.29 + \sqrt{(6.29)^2 + 4 \times 3^2}\right] \text{ kN}$$

$$= \frac{1}{2}(6.29 + 8.69) = 7.49 \text{ kN} = 7490 \text{ N}$$

Size of the bolt

Let d_c = Core diameter of the bolt.

We know that the equivalent tensile load (W_{te}),

$$7490 = \frac{\pi}{4}(d_c)^2 \sigma_t = \frac{\pi}{4}(d_c)^2 84 = 66\,(d_c)^2$$

$$(d_c)^2 = 7490 / 66 = 113.5 \qquad \text{or} \qquad d_c = 10.65 \text{ mm}$$

From Table(coarse series), the standard core diameter is 11.546 mm and the corresponding size of the bolt is M 14.

Cross-section of the arm of the bracket

Let t and b = Thickness and depth of arm of the bracket respectively.

∴ Section modulus,

$$Z = \frac{1}{6}\,t.b^2$$

Assume that the arm of the bracket extends upto the face of the steel column. This assumption gives stronger section for the arm of the bracket.

∴ Maximum bending moment on the bracket,

$$M = 12 \times 10^3 \times 400 = 4.8 \times 10^6 \text{ N-mm}$$

We know that the bending (tensile) stress (σ_t),

$$84 = \frac{M}{Z} = \frac{4.8 \times 10^6 \times 6}{t.b^2} = \frac{28.8 \times 10^6}{t.b^2}$$

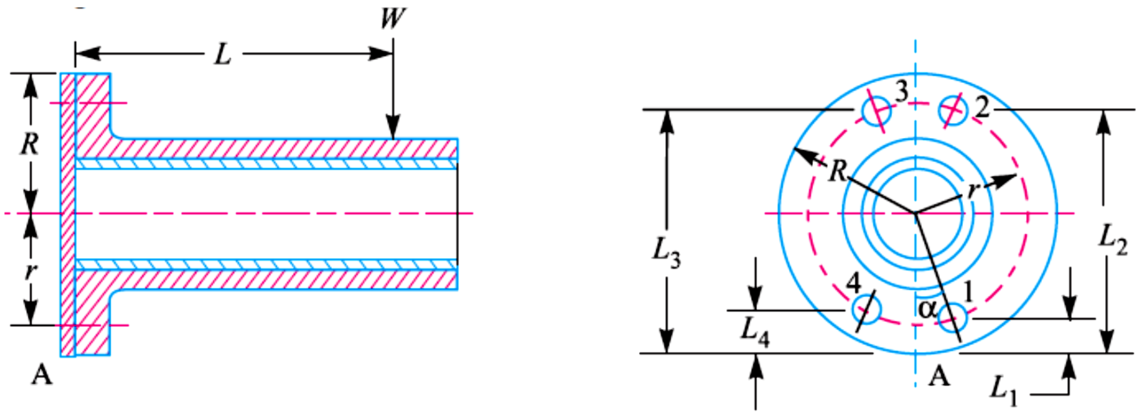
$$t.b^2 = 28.8 \times 10^6 / 84 = 343 \times 10^3 \qquad \text{or} \qquad t = 343 \times 10^3 / b^2$$

Assuming depth of arm of the bracket, b = 250 mm, we have

$$t = 343 \times 10^3 / (250)^2 = 5.5 \text{ mm}$$

Eccentric Load on a Bracket with Circular Base

Sometimes the base of a bracket is made circular as in case of a flanged bearing of a heavy machine tool and pillar crane etc. Consider a round flange bearing of a machine tool having four bolts as shown in Fig.



(*a*) (*b*)

Eccentric load on a bracket with circular base.

Let R = Radius of the column flange,

r = Radius of the bolt pitch circle,

w = Load per bolt per unit distance from the tilting edge,

L = Distance of the load from the tilting edge, and

L_1, L_2, L_3, and L_4 = Distance of bolt centers from the tilting edge A.

As discussed in the previous article, equating the external moment $W \times L$ to the sum of the resisting moments of all the bolts, we have,

$$W.L = w\,[(L_1)^2 + (L_2)^2 + (L_3)^2 + (L_4)^2]$$

$$w = \frac{W.L}{(L_1)^2 + (L_2)^2 + (L_3)^2 + (L_4)^2} \qquad ...(i)$$

Now from the geometry of the Fig.(*b*), we find that

$L_1 = R - r\cos\alpha$ $\quad L_2 = R + r\sin\alpha$ $\quad L_3 = R + r\cos\alpha$ and $L_4 = R - r\sin\alpha$

Substituting these values in equation (*i*), we get

$$w = \frac{W.L}{4\,R^2 + 2\,r^2}$$

$$\therefore \text{ Load in the bolt situated at 1} = w.L_1 = \frac{W.L.L_1}{4\,R^2 + 2\,r^2} = \frac{W.L\,(R - r\cos\alpha)}{4\,R^2 + 2\,r^2}$$

This load will be maximum when $\cos\alpha$ is minimum *i.e.* when $\cos\alpha = -1$ or $\alpha =$ 180°.

Maximum load in a bolt

$$= \frac{W.L\,(R + r)}{4\,R^2 + 2\,r^2}$$

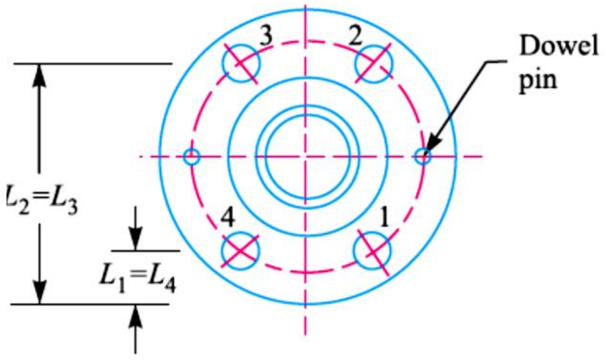
In general, if there are n number of bolts, then load in a bolt

$$= \frac{2W.L\,(R - r\cos\alpha)}{n\,(2R^2 + r^2)}$$

and maximum load in a bolt,

$$W_t = \frac{2\,W.L\,(R + r)}{n\,(2R^2 + r^2)}$$

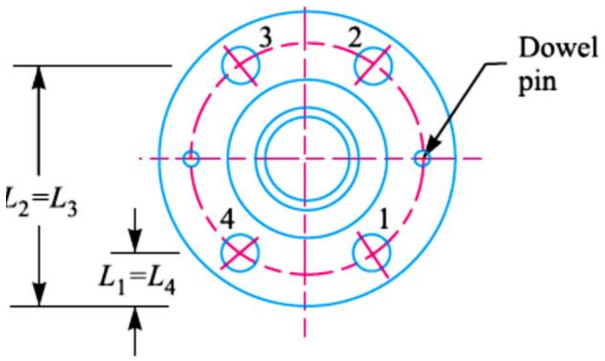
The above relation is used when the direction of the load *W* changes with relation to the bolts as in the case of pillar crane. But if the direction of load is fixed, then the maximum load on the bolts may be reduced by locating the bolts in such a way that two of them are equally stressed as shown in Fig. In such a case, maximum load is given by



$$W_t = \frac{2\,W.L}{n}\left[\frac{R + r\cos\left(\frac{180}{n}\right)}{2R^2 + r^2}\right]$$

Knowing the value of maximum load, we can determine the size of the bolt.

Problem(4):- *A flanged bearing, as shown in Fig. is fastened to a frame by means of four bolts spaced equally on 500 mm bolt circle. The diameter of bearing flange is 650 mm and a load of 400 kN acts at a distance of 250 mm from the frame. Determine the size of the bolts, taking safe tensile stress as 60 MPa for the material of the bolts.*



Solution. Given : $n = 4$; $d = 500$ mm or $r = 250$ mm ; $D = 650$ mm or $R = 325$ mm ; $W = 400$ kN $= 400 \times 10^3$ N ; $L = 250$ mm ; $\sigma_t = 60$ MPa $= 60$ N/mm^2

Let d_c= Core diameter of the bolts.

We know that when the bolts are equally spaced, the maximum load on the bolt,

$$W_t = \frac{2W.L}{n}\left[\frac{R + r\cos\left(\frac{180}{n}\right)}{2R^2 + r^2}\right]$$

$$= \frac{2 \times 400 \times 10^3 \times 250}{4}\left[\frac{325 + 250\cos\left(\frac{180}{4}\right)}{2\,(325)^2 + (250)^2}\right] = 91\,643 \text{ N}$$

We also know that maximum load on the bolt (W_t),

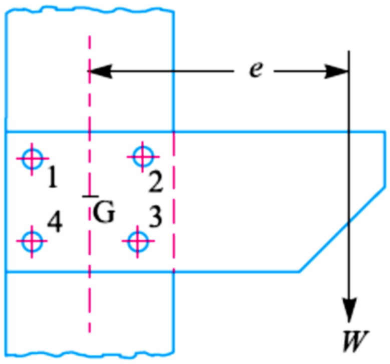
$$91\,643 = \frac{\pi}{4}(d_c)^2\,\sigma_t = \frac{\pi}{4}(d_c)^2\,60 = 47.13\,(d_c)^2$$

$$(d_c)^2 = 91\,643 / 47.13 = 1945 \qquad \text{or} \qquad d_c = 44 \text{ mm}$$

From Table, we find that the standard core diameter of the bolt is 45.795 mm and corresponding size of the bolt is M 52. **Ans.**

Eccentric Load Acting in the Plane Containing the Bolts

When the eccentric load acts in the plane containing the bolts, as shown in Fig. then the same procedure may be followed as discussed for eccentric loaded riveted joints.



Gaskets:- A gasket is a device used to create and maintain a barrier against the transfer of fluid across the mating surfaces of a mechanical assembly. It is used in static joints, such as cylinder block and cylinder head. There are two types of gaskets – metalic and non metalic.metalic gaskets consist of sheets of lead, copper or aluminium. Nonmetalic gaskets are made of asbestos, cork, rubber or plastics. Metalic gaskets are used for hightemperature and high pressure applications.

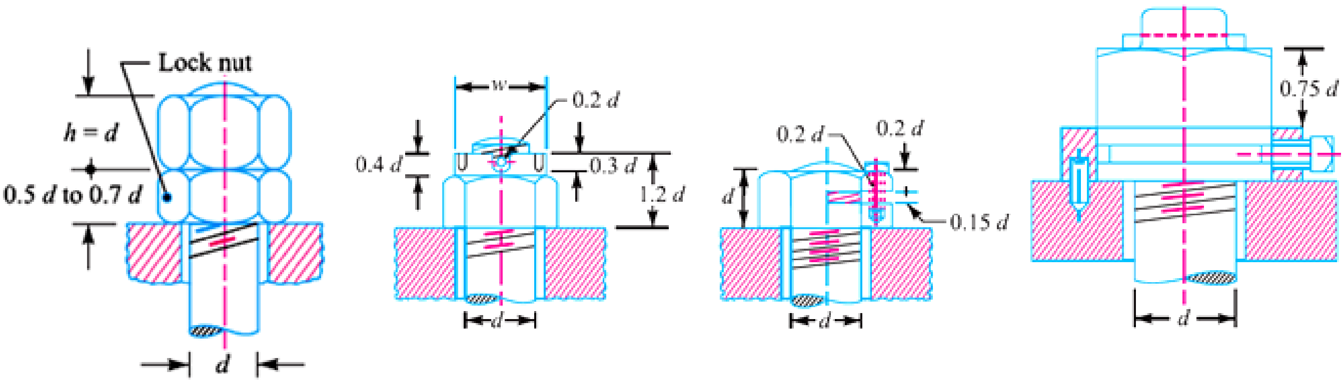
Locking Devices:- Ordinary thread fastenings, generally, remain tight under static loads, but many of these fastenings become loose under the action of variable loads or when machine is subjected to vibrations. The loosening of fastening is very dangerous and must be prevented. In order to prevent this, a large number of locking devices are available, some of which are discussed below :

1. *Jam nut or lock nut.* A most common locking device is a jam, lock or check nut. It has about one-half to two-third thickness of the standard nut. The thin lock nut is first tightened down with ordinary force, and then the upper nut (*i.e.* thicker nut) is tightened down upon it, as shown in Fig.(*a*). The upper nut is then held tightly while the lower one is slackened back against it.

2. *Castle nut.* It consists of a hexagonal portion with a cylindrical upper part which is slotted in line with the centre of each face, as shown in Fig. The split pin passes through two slots in the nut and a hole in the bolt, so that a positive lock is obtained unless the pin shears. It is extensively used on jobs subjected to sudden shocks and considerable vibration such as in automobile industry.

3. *Sawn nut.* It has a slot sawed about half way through, as shown in Fig. After the nut is screwed down, the small screw is tightened which produces more friction between the nut and the bolt. This prevents the loosening of nut.

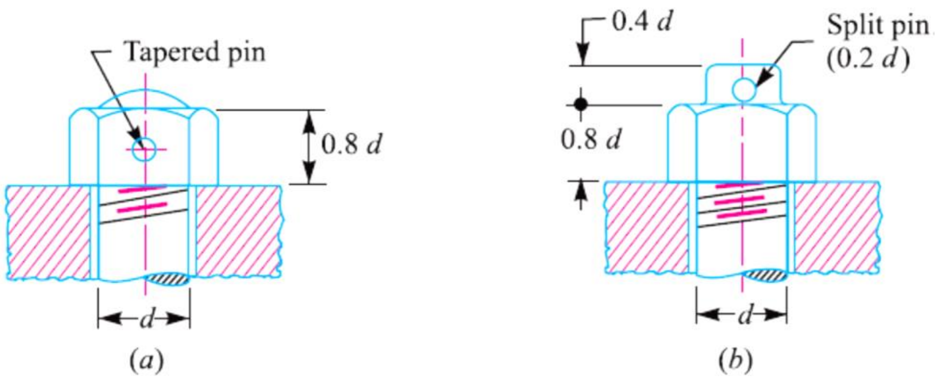
4. *Penn, ring or grooved nut.* It has a upper portion hexagonal and a lower part cylindrical as shown in Fig. It is largely used where bolts pass through connected pieces reasonably near their edges such as in marine type connecting rod ends.



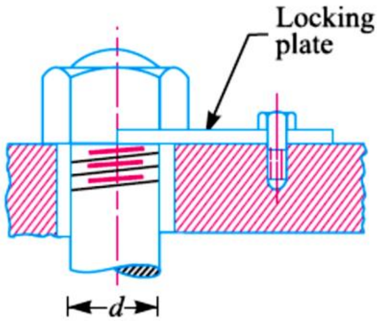
Jam nut or lock nut. Castle nut. Sawn nut. Penn, ring or grooved nut.

5. *Locking with pin.* The nuts may be locked by means of a taper pin or cotter pin passing through the middle of the nut as shown in Fig.(*a*). But a split pin is often driven through the bolt above the nut, as shown in Fig(*b*).

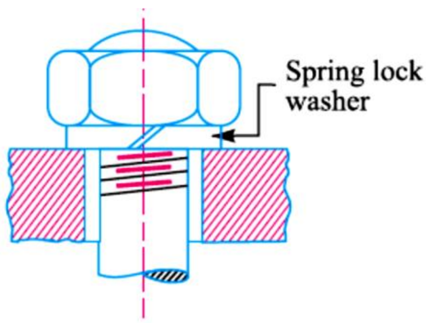
6. *Locking with plate.* A form of stop plate or locking plate is shown in Fig. The nut can be adjusted and subsequently locked through angular intervals of 30° by using these plates.



Locking with pin.



Locking with plate.



Locking with washer.

7. *Spring lock washer*. A spring lock washer is shown in above Fig. As the nut tightens the washer against the piece below, one edge of the washer is caused to dig itself into that piece, thus increasing the resistance so that the nut will not loosen so easily. There are many kinds of spring lock washers manufactured, some of which are fairly effective.

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DMM-I UNIT-III
KEYS, COTTERS, KNUCKLE JOINTS

Introduction:- (What is the key? State its functions)

"A key is a device which is used for connecting two machine parts (Ex. Shaft with pulley gear or crank) for preventing relative motion of rotation with respect to each other." In other words, key is used to transmit torque from a shaft to a gear or pulley.

The primary function of key is to prevent the relative rotation between the shaft and mating member. The secondary function is to prevent the relative movements in the axial direction of the shaft. Thus, the connected parts act as a single unit.

It is always inserted parallel to the axis of the shaft. Keys are used as temporary fastenings and are subjected to considerable crushing and shearing stresses.

Keys are generally made from cold rolled mild steel. Keys are designed based on diameter of shaft.

Types of Keys (Explain different types of keys with sketches)

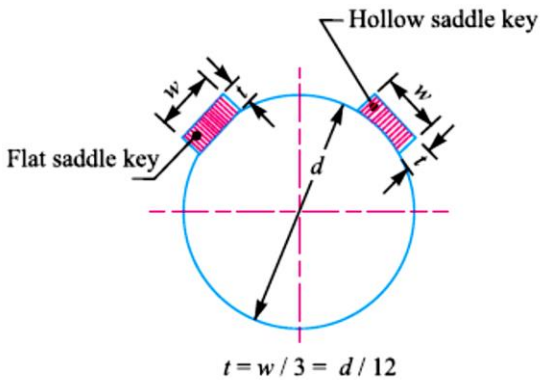
The following types of keys are important:

1. Saddle keys, **2.**Sunk keys, **3.**Gib head tapered key, 4. Feather key, 5. Woodruff key, 6. Round key or Pin, and 7.Splines.

1. Saddle keys:- The saddle keys are of the following two types:

a).Flat saddle key, and b).Hollow saddle key.

A ***flat saddle key :*** Bottom of the key is flat and the shaft is flattened to match as shown in fig. It is likely to slip round the shaft under load. Therefore it is used for comparatively light loads.

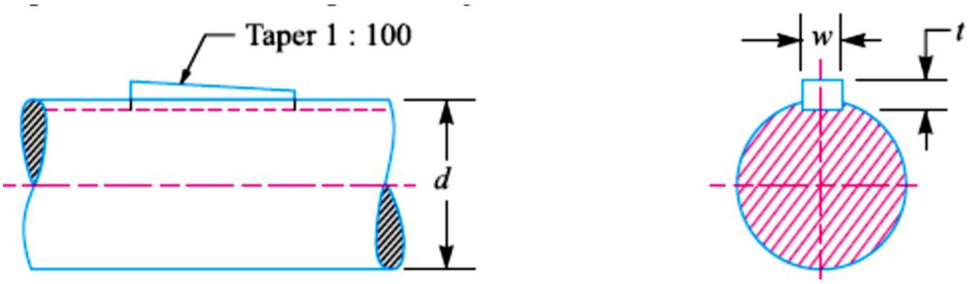


Saddle key.

A ***hollow saddle key*** : Bottom of the key is machined to have a curved surface.

2. Sunk keys:-The sunk keys are provided half in the keyway of the shaft and half in the keyway of the hub of the pulley.

Rectangular sunk key. A rectangular sunk key is shown in Fig.



Rectangular sunk key

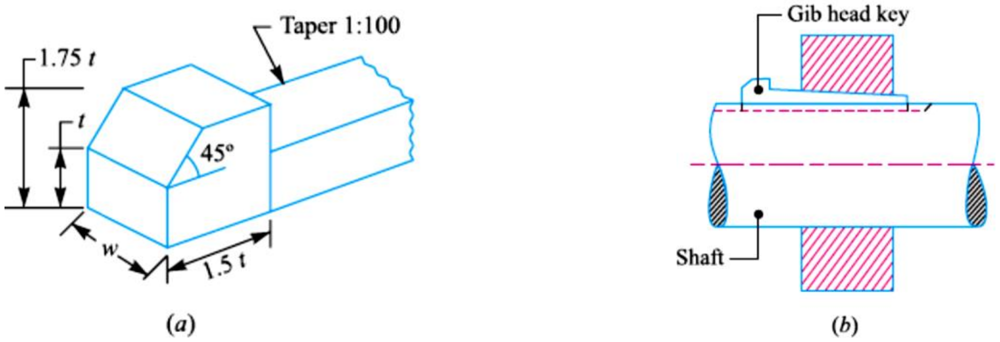
The usual proportions of this key are :

Width of key, $w = d / 4$; and thickness of key, $t = 2w / 3 = d / 6$

Where d = Diameter of the shaft or diameter of the hole in the hub.

The key has taper 1 in 100 on the top side only.

3.*Gib-head tapered key*. It is a rectangular sunk key with a head at one end known as ***gib head*.** It is usually provided to facilitate the removal of key. A gib head key is shown in Fig(*a*) and its use in shown in Fig(*b*).

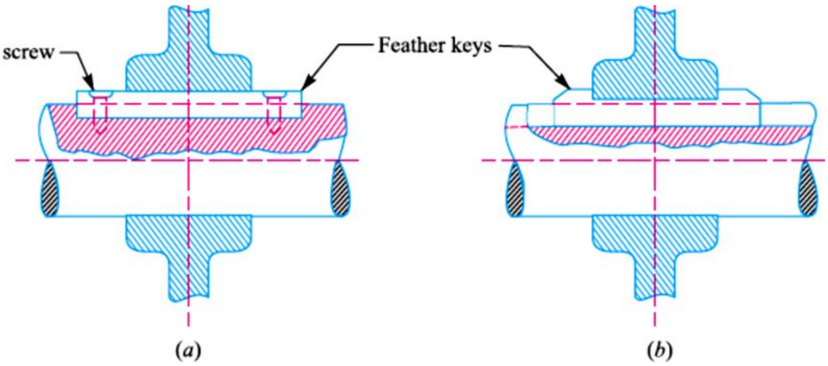


Gib-head key.

The usual proportions of the gib head key are :

Width, $w = d / 4$; and thickness at large end, $t = 2w / 3 = d / 6$

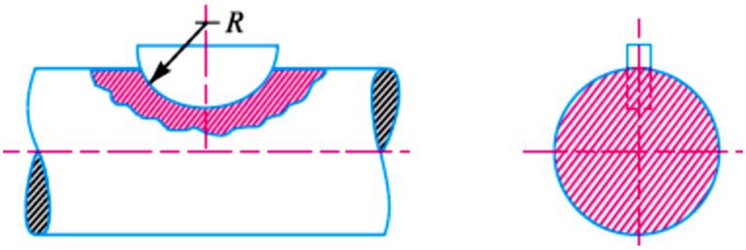
4.*Feather key*. A key attached to one member of a pair and which permits relative axial movement is known as ***feather key*.**



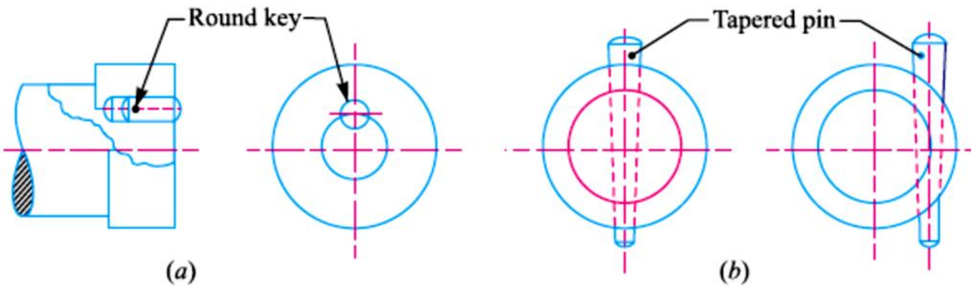
Feather key.

The feather key may be screwed to the shaft as shown in Fig(*a*) or it may have double gib heads as shown in Fig.(*b*). The various proportions of a feather key are same as that of rectangular sunk key and gib head key.

5.Woodruff key. The woodruff key is an easily adjustable key. It is a piece from a cylindrical disc having segmental cross-section in front view as shown in Fig. A woodruff key is capable of tilting in a recess milled out in the shaft by a cutter having the same curvature as the disc from which the key is made. This key is largely used in machine tool and automobile construction.



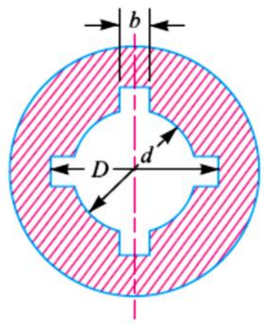
6. Round key or Pin:- The round keys, as shown in Fig (*a*), are circular in section and fit into holes drilled partly in the shaft and partly in the hub. They have the advantage that their keyways may be drilled and reamed after the mating parts have been assembled. Round keys are usually low power drives.



Round keys.

Sometimes the tapered pin, as shown in Fig(*b*), is held in place by the friction between the pin and the reamed tapered holes.

7.Splines:- Splines are multiple parallel keys integral with the shaft or the hub Such shafts are known as *splined shafts* as shown in Fig. These shafts usually have four, six, ten or sixteen splines. The splined shafts are relatively stronger than shafts having a single keyway.



$D = 1.25\ d$ and $b = 0.25\ D$

The splined shafts are used for automobile transmission and sliding gear transmissions. By using splined shafts, we obtain axial movement as well as positive drive.

Forces acting on a Sunk Key

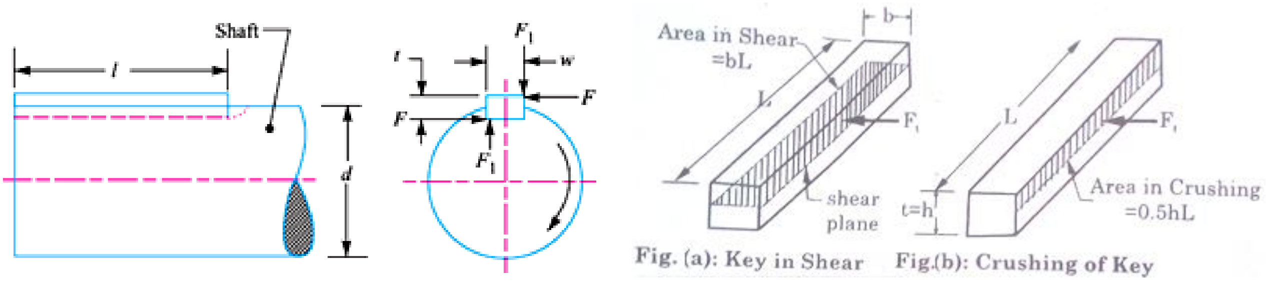
When a key is used in transmitting torque from a shaft to a rotor or hub, the following two types of forces act on the key :

1. Forces (F_1) due to fit of the key in its keyway, as in a tight fitting straight key or in a tapered key driven in place. These forces produce compressive stresses in the key which are difficult to determine in magnitude.

2. Forces (F) due to the torque transmitted by the shaft. These forces produce shearing and compressive (or crushing) stresses in the key.

The distribution of the forces along the length of the key is not uniform because the forces are concentrated near the torque-input end. The non-uniformity of distribution is caused by the twisting of the shaft within the hub.

The forces acting on a key for a clockwise torque being transmitted from a shaft to a hub are shown in Fig.



Forces acting on a Sunk Key

Strength of a Sunk Key

A key connecting the shaft and hub is shown in Fig.

Let T = Torque transmitted by the shaft,

F = Tangential force acting at the circumference of the shaft,

d = Diameter of shaft, l = Length of key, w = Width of key.

t = Thickness of key, and

τ and σ_c = Shear and crushing stresses for the material of key.

Due to the power transmitted by the shaft, the key may fail due to shearing or crushing.

Considering shearing of the key, the tangential shearing force acting at the circumference of the shaft,

F = Area resisting shearing × Shear stress = $l \times w \times \tau$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times w \times \tau \times \frac{d}{2} \quad \text{...............}(i)$$

Considering crushing of the key, the tangential crushing force acting at the circumference of the shaft,

$$F = \text{Area resisting crushing} \times \text{Crushing stress} = l \times \frac{t}{2} \times \sigma_c$$

$\therefore$ Torque transmitted by the shaft,

$$T = F \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} \quad \text{...............}(ii)$$

The key is equally strong in shearing and crushing, if

$$l \times w \times \tau \times \frac{d}{2} = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2}$$ ....[Equating equations ***(i)*** and ***(ii)***]

$$\frac{w}{t} = \frac{\sigma_c}{2\tau} \quad \text{..................}(iii)$$

The permissible crushing stress for the usual key material is at least twice the permissible shearing stress. Therefore from equation ***(iii)***, we have $w = t$. In other words, a square key is equally strong in shearing and crushing.

In order to find the length of the key to transmit full power of the shaft, the shearing strength of the key is equal to the torsional shear strength of the shaft.

We know that the shearing strength of key,

$$T = l \times w \times \tau \times \frac{d}{2} \quad \text{..................}(iv)$$

and torsional shear strength of the shaft,

$$T = \frac{\pi}{16} \times \tau_1 \times d^3 \quad \text{..................}(v)$$

...(Taking τ_1 = Shear stress for the shaft material)

From equations ***(iv)*** and ***(v)***, we have

$$l \times w \times \tau \times \frac{d}{2} = \frac{\pi}{16} \times \tau_1 \times d^3$$

$$l = \frac{\pi}{8} \times \frac{\tau_1 d^2}{w \times \tau} = \frac{\pi d}{2} \times \frac{\tau_1}{\tau} = 1.571\, d \times \frac{\tau_1}{\tau}$$ ... (Taking $w = d/4$) ......***(vi)***

When the key material is same as that of the shaft, then $\tau = \tau_1$

$l = 1.571\, d$ .... [From equation ***(vi)***]

Problem(1):- *Design the rectangular key for a shaft of 50 mm diameter. The shearing and crushing stresses for the key material are 42 MPa and 70 MPa. Assume* Width of key, w = 16 mm,and thickness of key, t = 10 mm

Given data : d = 50 mm ; τ = 42 MPa = 42 N/mm^2 ; σ_c = 70 MPa = 70 N/mm^2 ,

w = 16 mm, t = 10 mm

The rectangular key is designed as discussed below:

The length of key is obtained by considering the key in shearing and crushing.

Let l = Length of key.

Considering shearing of the key. We know that shearing strength (or torque transmitted) of the key,

$$T = l \times w \times \tau \times \frac{d}{2} = l \times 16 \times 42 \times \frac{50}{2} = 16\,800\, l \text{ N-mm} \qquad \text{.................}(i)$$

and torsional shearing strength (or torque transmitted) of the shaft,

$$T = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42\,(50)^3 = 1.03 \times 10^6 \text{ N-mm} \qquad \text{...............}(ii)$$

From equations ***(i)*** and ***(ii)***, we have

$l = 1.03 \times 10^6 / 16\,800 = 61.31$ mm

Now considering crushing of the key. We know that crushing strength (or torque transmitted) of the key,

$$T = l \times \frac{t}{2} \times \sigma_c \times \frac{d}{2} = l \times \frac{10}{2} \times 70 \times \frac{50}{2} = 8750\, l \text{ N-mm} \qquad \text{................}(iii)$$

From equations ***(ii)*** and ***(iii)*****,** we have

$l = 1.03 \times 10^6 / 8750 = 117.7$ mm

Taking larger of the two values, we have length of key,

l = 117.7 say 120 mm **Ans.**

<u>Problem(2):-</u> *A 45 mm diameter shaft is made of steel with a yield strength of 400 MPa. A parallel key of size 14 mm wide and 9 mm thick made of steel with a yield strength of 340 MPa is to be used. Find the required length of key, if the shaft is loaded to transmit the maximum permissible torque. Use maximum shear stress theory and assume a factor of safety of 2.*

Given data: d = 45 mm; σ_{yt} for shaft = 400 MPa = 400 N/mm^2 ; w = 14 mm ;

t = 9 mm; σ_{yt} for key = 340 MPa = 340 N/mm^2; $F.S.$ = 2

Let l = Length of key.

According to maximum shear stress theory, the maximum shear stress for the shaft,

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{400}{2 \times 2} = 100 \text{ N/mm}^2$$

and maximum shear stress for the key,

$$\tau_k = \frac{\sigma_{yt}}{2 \times F.S.} = \frac{340}{2 \times 2} = 85 \text{ N/mm}^2$$

We know that the maximum torque transmitted by the shaft and key,

$$T = \frac{\pi}{16} \times \tau_{max} \times d^3 = \frac{\pi}{16} \times 100\,(45)^3 = 1.8 \times 10^6 \text{ N-mm}$$

First of all, let us consider the failure of key due to shearing. We know that the maximum torque transmitted (T),

$$1.8 \times 10^6 = l \times w \times \tau_k \times \frac{d}{2} = l \times 14 \times 85 \times \frac{45}{2} = 26\,775\, l$$

$$l = 1.8 \times 10^6 / 26\,775 = 67.2 \text{ mm}$$

Now considering the failure of key due to crushing. We know that the maximum torque transmitted by the shaft and key (T),

$$1.8 \times 10^6 = l \times \frac{t}{2} \times \sigma_{ck} \times \frac{d}{2} = l \times \frac{9}{2} \times \frac{340}{2} \times \frac{45}{2} = 17\,213\, l \qquad \ldots \left(\text{Taking } \sigma_{ck} = \frac{\sigma_{yt}}{F.S.}\right)$$

$$l = 1.8 \times 10^6 / 17\,213 = 104.6 \text{ mm}$$

Taking the larger of the two values, we have

l = 104.6 say 105 mm **Ans.**

Effect of Keyways:-

A little consideration will show that the keyway cut into the shaft reduces the load carrying capacity of the shaft. This is due to the stress concentration near the corners of the keyway and reduction in the cross-sectional area of the shaft.

It other words, the torsional strength of the shaft is reduced. The following relation for the weakening effect of the keyway is based on the experimental results by H.F. Moore.

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right)$$

Where e = Shaft strength factor. It is the ratio of the strength of the shaft with Keyway to the strength of the same shaft without keyway,

w = Width of keyway, d = Diameter of shaft, and

$$h = \text{Depth of keyway} = \frac{\text{Thickness of key } (t)}{2}$$

It is usually assumed that the strength of the keyed shaft is 75% of the solid shaft, which is somewhat higher than the value obtained by the above relation.

In case the keyway is too long and the key is of sliding type, then the angle of twist is increased in the ratio k_θ as given by the following relation:

$$k_\theta = 1 + 0.4\left(\frac{w}{d}\right) + 0.7\left(\frac{h}{d}\right)$$

Where k_θ = Reduction factor for angular twist.

Problem(3):- *A 15 kW, 960 r.p.m. motor has a mild steel shaft of 40 mm diameter and the extension being 75 mm. The permissible shear and crushing stresses for the mild steel key are 56 MPa and 112 MPa. Design the keyway in the motor shaft extension. Check the shear strength of the key against the normal strength of the shaft.*

Given data : P = 15 kW = 15 × 10^3 W ; N = 960 r.p.m. ; d = 40 mm ; l = 75 mm ;
τ = 56 MPa = 56 N/mm^2 ; σ_c = 112 MPa = 112 N/mm^2

We know that the torque transmitted by the motor,

$$T = \frac{P \times 60}{2\,\pi N} = \frac{15 \times 10^3 \times 60}{2\,\pi \times 960} = 149 \text{ N-m} = 149 \times 10^3 \text{ N-mm}$$

Let w = Width of keyway or key.

Considering the key in shearing. We know that the torque transmitted (T),

$$149 \times 10^3 = l \times w \times \tau \times \frac{d}{2} = 75 \times w \times 56 \times \frac{40}{2} = 84 \times 10^3\, w$$

This width of keyway is too small. The width of keyway should be at least d / 4.

$$w = \frac{d}{4} = \frac{40}{4} = 10 \text{ mm } \textbf{Ans.}$$

Since $\sigma_c = 2\tau$, therefore a square key of w = 10 mm and t = 10 mm is adopted.

According to H.F. Moore, the shaft strength factor,

$$e = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{h}{d}\right) = 1 - 0.2\left(\frac{w}{d}\right) - 1.1\left(\frac{t}{2d}\right) \qquad \ldots (\because h = t/2)$$

Strength of the shaft with keyway,

$$= \frac{\pi}{16} \times \tau \times d^3 \times e = \frac{\pi}{16} \times 56\,(40)^3\;0.8125 = 571\,844 \text{ N}$$

And shear strength of the key

$$= l \times w \times \tau \times \frac{d}{2} = 75 \times 10 \times 56 \times \frac{40}{2} = 840\,000 \text{ N}$$

$$\therefore \quad \frac{\text{Shear strength of the key}}{\text{Normal strength of the shaft}} = \frac{840\,000}{571\,844} = 1.47 \textbf{ Ans.}$$

COTTER JOINTS

Introduction

A cotter is a flat wedge shaped piece of rectangular cross-section and its width is tapered (either on one side or both sides) from one end to another for an easy adjustment. The taper varies from 1 in 48 to 1 in 24.It is usually made of mild steel or wrought iron.

A cotter joint is a temporary fastening and it is used to connect rigidly two co-axial rods or bars which are subjected to axial tensile or compressive forces.

It is usually used in connecting a piston rod to the crosshead of a reciprocating steam engine, a piston rod and its extension as a tail or pump rod, strap end of connecting rod.

Difference between keys and cotters:- Keys are usually driven parallel to the axis of the shafts which are subjected to torsional or twisting stresses.

Cotters are normally driven at right angle to the axes of connected parts, which are subjected to tensile or compressive stresses.

Why a single taper is provide in cotter and not an both sides?

The taper is provided to ensure the tightness of the joint and to facilitate easy withdrawal of cotter from the joint. In cotter, the taper is provided only on one side because machining a taper on two sides of a machine part is more difficult. In addition, there is no specific advantage in providing taper on both sides.

Types of Cotter Joints

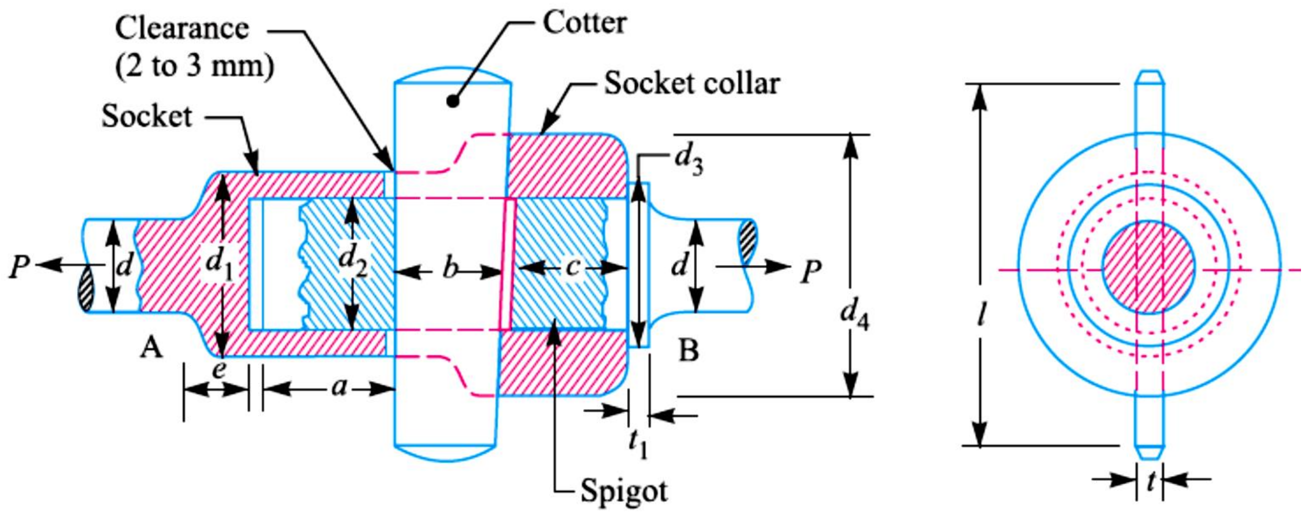
Following are the three commonly used cotter joints to connect two rods by a cotter :

1. Socket and spigot cotter joint, **2.** Sleeve and cotter joint, and **3.** Gib and cotter joint.

Socket and Spigot Cotter Joint

In a socket and spigot cotter joint, one end of the rods (say *A*) is provided with a socket type of end as shown in Fig. and the other end of the other rod (say *B*) is inserted into a socket. The end of the rod which goes into a socket is also called ***spigot***.

A rectangular hole is made in the socket and spigot. *A* cotter is then driven tightly through a hole in order to make the temporary connection between the two rods. The load is usually acting axially, but it changes its direction and hence the cotter joint must be designed to carry both the tensile and compressive loads. The compressive load is taken up by the collar on the spigot.



Design of Socket and Spigot Cotter Joint

The socket and spigot cotter joint is shown in Fig.

Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of socket,

d_2 = Diameter of spigot or inside diameter of socket,
d_3 = Outside diameter of spigot collar,
t_1 = Thickness of spigot collar,
d_4 = Diameter of socket collar,
c = Thickness of socket collar,
b = Mean width of cotter,
t = Thickness of cotter,
l = Length of cotter,
a = Distance from the end of the slot to the end of rod,
σ_t = Permissible tensile stress for the rods material,
τ = Permissible shear stress for the cotter material, and
σ_c = Permissible crushing stress for the cotter material.

1. *Failure of the rods in tension*

The rods may fail in tension due to the tensile load P. We know that
Area resisting tearing

$$= \frac{\pi}{4} \times d^2$$

∴ Tearing strength of the rods,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

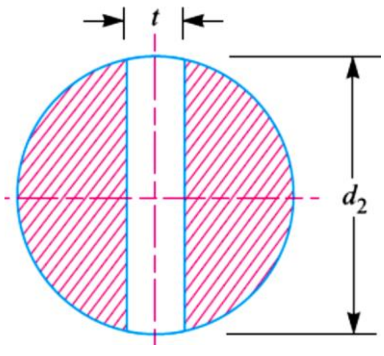
Equating this to load (P), we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be determined.

2. *Failure of spigot in tension across the weakest section (or slot)*

Since the weakest section of the spigot is that section which has a slot in it for the cotter, as shown in Fig.



therefore Area resisting tearing of the spigot across the slot

$$= \frac{\pi}{4} (d_2)^2 - d_2 \times t$$

and tearing strength of the spigot across the slot

$$= \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

Equating this to load (P), we have

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

From this equation, the diameter of spigot or inside diameter of socket (d_2) may be determined.

Note : In actual practice, the thickness of cotter is usually taken as $d_2 / 4$.

3. *Failure of the rod or cotter in crushing*

We know that the area that resists crushing of a rod or cotter = $d_2 \times t$

∴ Crushing strength = $d_2 \times t \times \sigma_c$

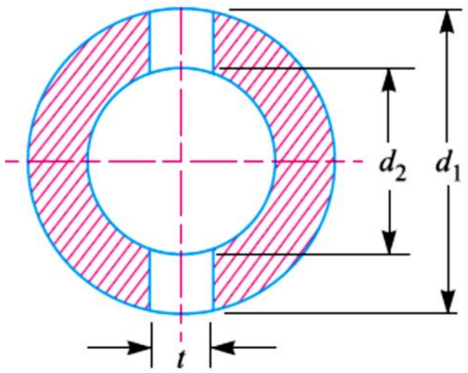
Equating this to load (P), we have

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

4. *Failure of the socket in tension across the slot*

We know that the resisting area of the socket across the slot, as shown in Fig.



$$= \frac{\pi}{4}\left[(d_1)^2 - (d_2)^2\right] - (d_1 - d_2)\,t$$

∴ Tearing strength of the socket across the slot

$$= \left\{\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right\}\sigma_t$$

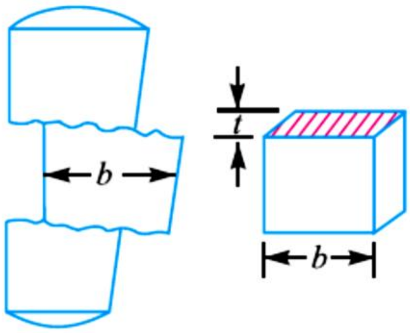
Equating this to load (P), we have

$$P = \left\{\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right\}\sigma_t$$

From this equation, outside diameter of socket (d_1) may be determined.

5. *Failure of cotter in shear*

Considering the failure of cotter in shear as shown in Fig.



Since the cotter is in double shear, therefore shearing area of the cotter

$$= 2\, b \times t$$

and shearing strength of the cotter

$$= 2\, b \times t \times \tau$$

Equating this to load (P), we have

$$P = 2\, b \times t \times \tau$$

From this equation, width of cotter (b) is determined.

6. *Failure of the socket collar in crushing*

Considering the failure of socket collar in crushing as shown in Fig.

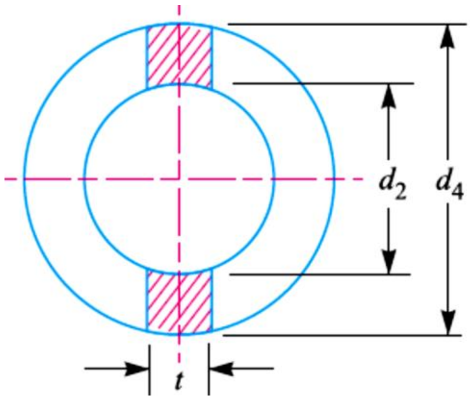
We know that area that resists crushing of socket collar = $(d_4 - d_2)\, t$

and crushing strength $= (d_4 - d_2)\, t \times \sigma_c$

Equating this to load (P), we have

$$P = (d_4 - d_2)\, t \times \sigma_c$$

From this equation, the diameter of socket collar (d_4) may be obtained.



7. *Failure of socket end in shearing*

Since the socket end is in double shear, therefore area that resists shearing of socket collar = $2\,(d_4 - d_2)\, c$

and shearing strength of socket collar

$$= 2\,(d_4 - d_2)\, c \times \tau$$

Equating this to load (P), we have

$$P = 2\,(d_4 - d_2)\, c \times \tau$$

From this equation, the thickness of socket collar (c) may be obtained.

8. *Failure of rod end in shear*

Since the rod end is in double shear, therefore the area resisting shear of the rod end = $2\, a \times d_2$

and shear strength of the rod end = $2\, a \times d_2 \times \tau$

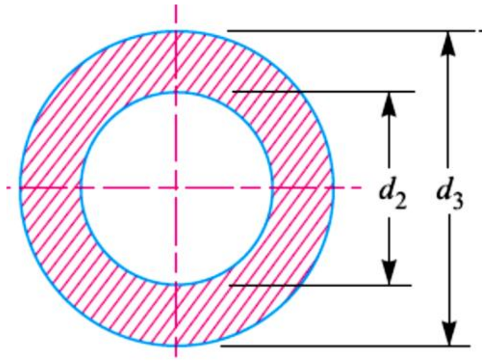
Equating this to load (P), we have

$$P = 2\, a \times d_2 \times \tau$$

From this equation, the distance from the end of the slot to the end of the rod (a) may be obtained.

9. *Failure of spigot collar in crushing*

Considering the failure of the spigot collar in crushing as shown in Fig.



We know that area that resists crushing of the collar

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]$$

and crushing strength of the collar

$$= \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

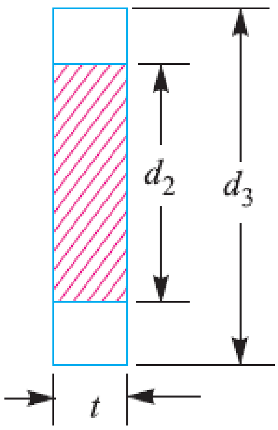
Equating this to load (P), we have

$$P = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c$$

From this equation, the diameter of the spigot collar (d_3) may be obtained.

10. *Failure of the spigot collar in shearing*

Considering the failure of the spigot collar in shearing as shown in Fig.



We know that area that resists shearing of the collar

$= \pi\, d_2 \times t_1$

and shearing strength of the collar,

$= \pi\, d_2 \times t_1 \times \tau$

Equating this to load (*P*) we have

$P = \pi\, d_2 \times t_1 \times \tau$

From this equation, the thickness of spigot collar (t_1) may be obtained.

<u>Problem(1):-</u> *Design and draw a cotter joint to support a load varying from 30 kN in compression to 30 kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically.*

Tensile stress = compressive stress = 50 MPa ; shear stress = 35 MPa and crushing stress = 90 MPa.

Given data : P = 30 kN = 30 × 10^3 N ; σ_t = 50 MPa = 50 N / mm^2 ; τ = 35 MPa = 35 N / mm^2 ; σ_c = 90 MPa = 90 N/mm2

The cotter joint is shown in Fig. The joint is designed as discussed below :

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rod in tension. We know that load (*P*),

$$30 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 50 = 39.3\, d^2$$

∴ $d^2 = 30 \times 10^3 / 39.3 = 763$ or d = 27.6 say 28 mm **Ans.**

2. *Diameter of spigot and thickness of cotter*

Let d_2 = Diameter of spigot or inside diameter of socket, and

t = Thickness of cotter. It may be taken as d_2 / 4.

Considering the failure of spigot in tension across the weakest section. We know that load (*P*),

$$30 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right]50 = 26.8\,(d_2)^2$$

∴ $(d_2)^2 = 30 \times 10^3 / 26.8 = 1119.4$ or d_2 = 33.4 say 34 mm

and thickness of cotter, $t = \frac{d_2}{4} = \frac{34}{4} = 8.5$ mm

Let us now check the induced crushing stress. We know that load (*P*),

$$30 \times 10^3 = d_2 \times t \times \sigma_c = 34 \times 8.5 \times \sigma_c = 289\,\sigma_c$$

$$\sigma_c = 30 \times 10^3 / 289 = 103.8 \text{ N/mm}^2$$

Since this value of σ_c is more than the given value of σ_c = 90 N/mm2, therefore the dimensions d_2 = 34 mm and t = 8.5 mm are not safe. Now let us find the values of d_2 and t by substituting the value of σ_c = 90 N/mm^2 in the above expression,

$$30 \times 10^3 = d_2 \times \frac{d_2}{4} \times 90 = 22.5\,(d_2)^2$$

∴ $(d_2)^2 = 30 \times 10^3 / 22.5 = 1333$ or $d_2 = 36.5$ say 40 mm **Ans.**

and $t = d_2 / 4 = 40 / 4 = 10$ mm **Ans.**

3. *Outside diameter of socket*

Let d_1 = Outside diameter of socket.

Consider the failure of the socket in tension across the slot. We know that load (P)

$$30 \times 10^3 = \left[\frac{\pi}{4}\left\{(d_1)^2 - (d_2)^2\right\} - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}\left\{(d_1)^2 - (40)^2\right\} - (d_1 - 40)\,10\right]50$$

$30 \times 10^3/50 = 0.7854\,(d_1)^2 - 1256.6 - 10\,d_1 + 400$

or $(d_1)^2 - 12.7\,d_1 - 1854.6 = 0$

∴ $d_1 = 49.9$ say 50 mm **Ans.** ....(Taking +ve sign)

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of the cotter in shear. Since the cotter is in double shear, therefore load (P),

$30 \times 10^3 = 2\,b \times t \times \tau = 2\,b \times 10 \times 35 = 700\,b$

∴ $b = 30 \times 10^3 / 700 = 43$ mm **Ans.**

5. *Diameter of socket collar*

Let $d4$ = Diameter of socket collar.

Considering the failure of the socket collar and cotter in crushing. We know that load (P),

$30 \times 10^3 = (d_4 - d_2)\,t \times \sigma_c = (d_4 - 40)10 \times 90 = (d_4 - 40)\,900$

∴ $d_4 - 40 = 30 \times 10^3 / 900 = 33.3$ or $d_4 = 33.3 + 40 = 73.3$ say 75 mm **Ans.**

6. *Thickness of socket collar*

Let c = Thickness of socket collar.

Considering the failure of the socket end in shearing. Since the socket end is in double shear, therefore load (P),

$30 \times 10^3 = 2(d_4 - d_2)\,c \times \tau = 2\,(75 - 40)\,c \times 35 = 2450\,c$

∴ $c = 30 \times 10^3 / 2450 = 12$ mm **Ans.**

7. *Distance from the end of the slot to the end of the rod*

Let a = Distance from the end of slot to the end of the rod.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$30 \times 10^3 = 2\,a \times d_2 \times \tau = 2a \times 40 \times 35 = 2800\,a$

∴ $a = 30 \times 10^3 / 2800 = 10.7$ say 11 mm **Ans.**

8. *Diameter of spigot collar*

Let d_3 = Diameter of spigot collar.

Considering the failure of spigot collar in crushing. We know that load (P),

$$30 \times 10^3 = \frac{\pi}{4}\left[(d_3)^2 - (d_2)^2\right]\sigma_c = \frac{\pi}{4}\left[(d_3)^2 - (40)^2\right]90$$

$$(d_3)^2 - (40)^2 = \frac{30 \times 10^3 \times 4}{90 \times \pi} = 424$$

∴ $(d_3)^2 = 424 + (40)2 = 2024$ or $d3 = 45$ mm **Ans.**

9. *Thickness of spigot collar*

Let t_1 = Thickness of spigot collar.

Considering the failure of spigot collar in shearing. We know that load (P),

$30 \times 10^3 = \pi d2 \times t_1 \times \; = \pi \times 40 \times t_1 \times 35 = 4400\, t_1$

∴ $t_1 = 30 \times 10^3 / 4400 = 6.8$ say 8 mm **Ans.**

10. The length of cotter (l) is taken as 4 d.

∴ $l = 4\,d = 4 \times 28 = 112$ mm **Ans.**

11. The dimension e is taken as 1.2 d.

∴ $e = 1.2 \times 28 = 33.6$ say 34 mm **Ans.**

<u>Sleeve and Cotter Joint</u>:(Write about working principle of sleeve and cotter joint)

Sleeve and cotter joint is type of joint, which is used to connect two similar coaxial cylindrical rods or two tie rods or two pipes or two tubes. These cylindrical rods are connected together by a common sleeve and two wedge shaped tapered cotters. Appropriate slots are cut in the sleeve and in the cylindrical rods. The cotters are assembled into these slots.

Further,

- ❖ In this type of joint, a sleeve or muff is used over the two rods and then two cotters are driven in the holes.
- ❖ The taper of cotter is usually 1 in 24.
- ❖ The taper sides of the two cotter should face each other.

<u>The various proportions for the sleeve and cotter joint in terms of the diameter of rod (d) are as follows</u> :

Outside diameter of sleeve, $d_1 = 2.5\,d$

Diameter of enlarged end of rod, d_2 = Inside diameter of sleeve = $1.25\,d$

Length of sleeve, $L = 8\,d$

Thickness of cotter, $t = d_2/4$ or $0.31\,d$

Width of cotter, $b = 1.25\,d$

Length of cotter, $l = 4\,d$

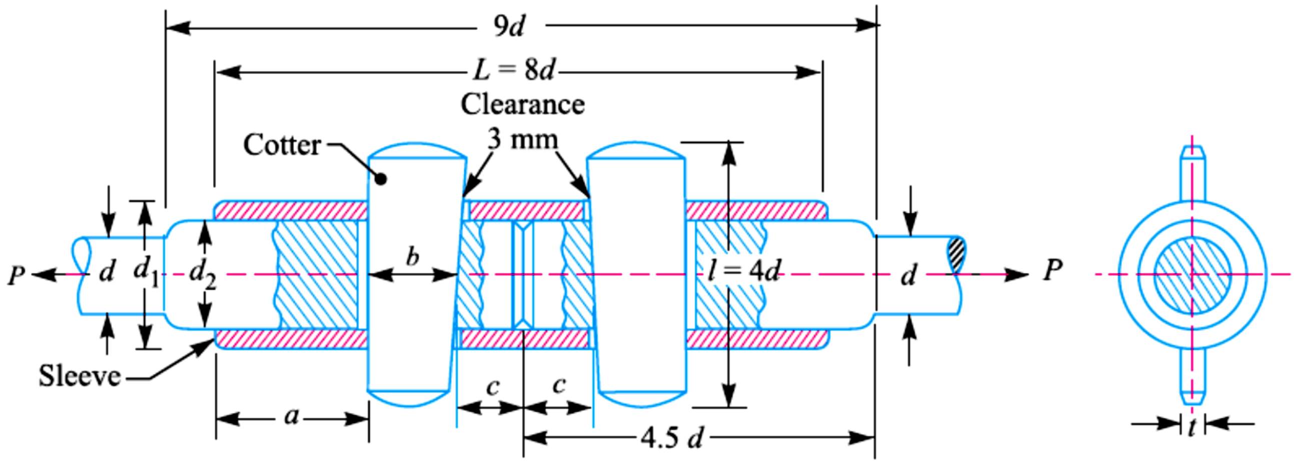
Distance of the rod end (a) from the beginning to the cotter hole (inside the sleeve end)

= Distance of the rod end (c) from its end to the cotter hole

= 1.25 d

Design of Sleeve and Cotter Joint

The sleeve and cotter joint is shown in above Fig.



Let P = Load carried by the rods,

d = Diameter of the rods,

d_1 = Outside diameter of sleeve,

d_2 = Diameter of the enlarged end of rod,

t = Thickness of cotter,

l = Length of cotter,

b = Width of cotter,

a = Distance of the rod end from the beginning to the cotter hole (inside the sleeve end),

c = Distance of the rod end from its end to the cotter hole,

σ_t , τ and σ_c = Permissible tensile, shear and crushing stresses respectively for the material of the rods and cotter.

1. *Failure of the rods in tension*

The rods may fail in tension due to the tensile load P. We know that

$$\text{Area resisting tearing} = \frac{\pi}{4} \times d^2$$

∴ Tearing strength of the rods

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to load (P), we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rods (d) may be obtained.

2. *Failure of the rod in tension across the weakest section (i.e. slot)*

Since the weakest section is that section of the rod which has a slot in it for the cotter, therefore area resisting tearing of the rod across the slot

$$= \frac{\pi}{4}(d_2)^2 - d_2 \times t$$

and tearing strength of the rod across the slot

$$= \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

Equating this to load (P), we have

$$P = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t$$

From this equation, the diameter of enlarged end of the rod (d_2) may be obtained.

Note: The thickness of cotter is usually taken as $d_2 / 4$.

3. *Failure of the rod or cotter in crushing*

We know that the area that resists crushing of a rod or cotter

$$= d_2 \times t$$

$\therefore$ Crushing strength $= d_2 \times t \times \sigma_c$

Equating this to load (P), we have

$$P = d_2 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

4. *Failure of sleeve in tension across the slot*

We know that the resisting area of sleeve across the slot

$$= \frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t$$

Tearing strength of the sleeve across the slot

$$= \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

Equating this to load (P), we have

$$P = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

From this equation, the outside diameter of sleeve (d_1) may be obtained.

5. *Failure of cotter in shear*

Since the cotter is in double shear, therefore shearing area of the cotter

$$= 2b \times t$$

and shear strength of the cotter $= 2b \times t \times \tau$

Equating this to load (P), we have

$$P = 2b \times t \times \tau$$

From this equation, width of cotter (b) may be determined.

6. *Failure of rod end in shear*

Since the rod end is in double shear, therefore area resisting shear of the rod end $= 2\, a \times d_2$

and shear strength of the rod end $= 2\, a \times d_2 \times \tau$

Equating this to load (P), we have $P = 2\, a \times d_2 \times \tau$

From this equation, distance (a) may be determined.

7. *Failure of sleeve end in shear*

Since the sleeve end is in double shear, therefore the area resisting shear of the sleeve end $= 2\,(d_1 - d_2)\, c$

and shear strength of the sleeve end $= 2\,(d_1 - d_2)\, c \times \tau$

Equating this to load (P), we have $P = 2\,(d_1 - d_2)\, c \times \tau$

From this equation, distance (c) may be determined.

Problem(2):-*Design a sleeve and cotter joint to resist a tensile load of 60 kN. All parts of the joint are made of the same material with the following allowable stresses :* σ_t *= 60 MPa ;* τ *= 70 MPa ; and* σ_c *= 125 MPa.*

Given data : $P = 60$ kN $= 60 \times 10^3$ N ; $\sigma_t = 60$ MPa $= 60$ N/mm^2 ;

$\tau = 70$ MPa $= 70$ N/mm^2 ; $\sigma_c = 125$ MPa $= 125$ N/mm^2

1. *Diameter of the rods*

Let d = Diameter of the rods.

Considering the failure of the rods in tension. We know that load (P),

$$60 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 60 = 47.13\, d^2$$

$\therefore$ $d^2 = 60 \times 10^3 / 47.13 = 1273$ or $d = 35.7$ say 36 mm **Ans.**

2. *Diameter of enlarged end of rod and thickness of cotter*

Let d_2 = Diameter of enlarged end of rod, and

t = Thickness of cotter. It may be taken as $d_2 / 4$.

Considering the failure of the rod in tension across the weakest section (*i.e.* slot). We know that load (P),

$$60 \times 10^3 = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times t\right]\sigma_t = \left[\frac{\pi}{4}(d_2)^2 - d_2 \times \frac{d_2}{4}\right] 60 = 32.13\,(d_2)^2$$

$\therefore$ $(d_2)^2 = 60 \times 10^3 / 32.13 = 1867$ or $d_2 = 43.2$ say 44 mm **Ans.**

and thickness of cotter,

$$t = \frac{d_2}{4} = \frac{44}{4} = 11 \text{ mm } \textbf{Ans.}$$

Let us now check the induced crushing stress in the rod or cotter. We know that load (P), $60 \times 10^3 = d_2 \times t \times \sigma_c = 44 \times 11 \times \sigma_c = 484\, \sigma_c$

$\therefore$ $\sigma_c = 60 \times 10^3 / 484 = 124$ N/mm^2

Since the induced crushing stress is less than the given value of 125 N/mm², therefore the dimensions d_2 and t are within safe limits.

3. *Outside diameter of sleeve*

Let d_1 = Outside diameter of sleeve.

Considering the failure of sleeve in tension across the slot. We know that load (P)

$$60 \times 10^3 = \left[\frac{\pi}{4}[(d_1)^2 - (d_2)^2] - (d_1 - d_2)\,t\right]\sigma_t$$

$$= \left[\frac{\pi}{4}[(d_1)^2 - (44)^2] - (d_1 - 44)\,11\right]60$$

$$60 \times 10^3 / 60 = 0.7854\,(d_1)^2 - 1520.7 - 11\,d_1 + 484$$

or $(d_1)^2 - 14\,d_1 - 2593 = 0$

$$d_1 = \frac{14 \pm \sqrt{(14)^2 + 4 \times 2593}}{2} = \frac{14 \pm 102.8}{2}$$

d_1= 58.4 say 60 mm **Ans.** ....(Taking +ve sign)

4. *Width of cotter*

Let b = Width of cotter.

Considering the failure of cotter in shear. Since the cotter is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,b \times t \times \tau = 2 \times b \times 11 \times 70 = 1540\,b$$

∴ $b = 60 \times 10^3 / 1540 = 38.96$ say 40 mm **Ans.**

5. *Distance of the rod from the beginning to the cotter hole (inside the sleeve end)*

Let a = Required distance.

Considering the failure of the rod end in shear. Since the rod end is in double shear, therefore load (P),

$$60 \times 10^3 = 2\,a \times d_2 \times \tau = 2\,a \times 44 \times 70 = 6160\,a$$

∴ $a = 60 \times 10^3 / 6160 = 9.74$ say 10 mm **Ans.**

6. *Distance of the rod end from its end to the cotter hole*

Let c = Required distance.

Considering the failure of the sleeve end in shear. Since the sleeve end is in double shear, therefore load (P),

$60 \times 10^3 = 2\,(d_1 - d_2)\,c \times \tau = 2\,(60 - 44)\,c \times 70 = 2240\,c$

∴ $c = 60 \times 10^3 / 2240 = 26.78$ say 28 mm **Ans.**

<u>Gib and Cotter Joint:</u> (Describe the purpose of gib in cotter joint. What are the applications of cotter joints?)

Gib and cotter joint is used for joining two square rods. One end of the rod is forged in the shape of a fork while the other rod is pushed into the fork. Slots are provided in the fork and the rod to accommodate the gib and cotter while assembling the parts. The gib is inserted first so that the straight surface touches the slot of the fork and then cotter is hammered into the rest of the slot. Care should be taken to ensure that the tapered side of the gib and cotter should be in contact face to face with each other.

Applications of cotter joints:-

- Used to connect bicycle pedal to sprocket wheel
- Used to connect piston rod in cross head
- Used to connect piston rod with its extension
- Used to connect two halves of a flywheel
- Used for joining tail rod with piston rod of a wet air pump.

Design of Gib and Cotter Joint for Square Rods:- (What are the design procedure for gib and cotter joint for square rods)

Consider a gib and cotter joint for square rods as shown in Fig. The rods may be subjected to a tensile or compressive load. All components of the joint are assumed to be of the same material.

Let P = Load carried by the rods,

x = Each side of the rod,

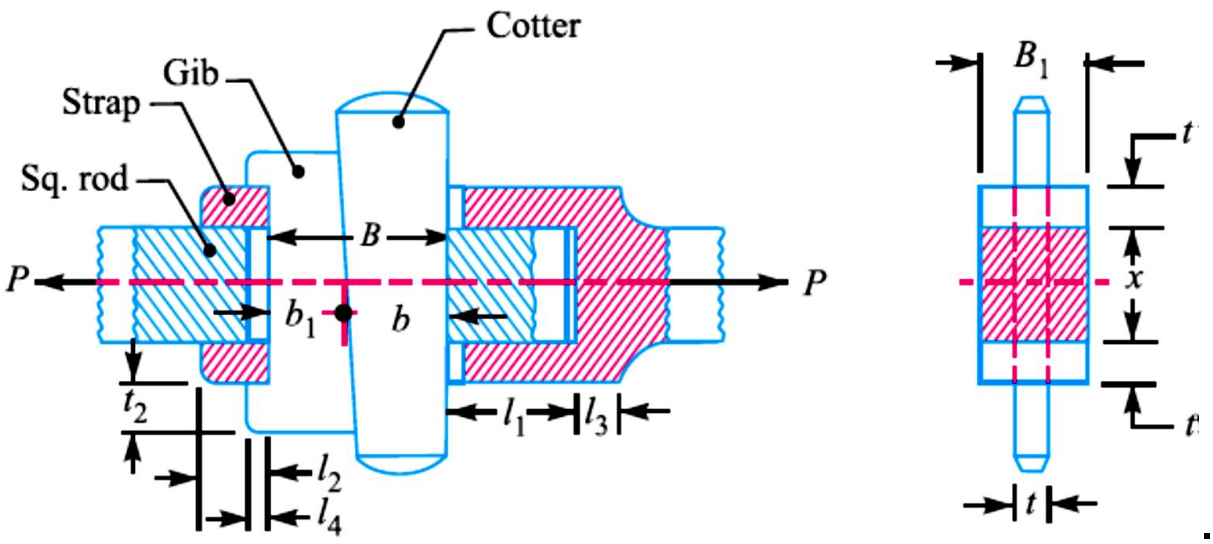
B = Total width of gib and cotter,

B_1 = Width of the strap,

t = Thickness of cotter,

t_1 = Thickness of the strap, and

σ_t , τ and σ_c = Permissible tensile, shear and crushing stresses.



Gib and cotter joint for square rods.

1. *Failure of the rod in tension*

The rod may fail in tension due to the tensile load *P*. We know that

Area resisting tearing = $x \times x = x^2$

∴ Tearing strength of the rod = $x^2 \times \sigma_t$

Equating this to the load (*P*), we have $P = x^2 \times \sigma_t$

From this equation, the side of the square rod (*x*) may be determined. The other dimensions are fixed as under :

Width of strap, B_1 = Side of the square rod = *x*

Thickness of cotter,

$$t = \frac{1}{4} \text{ width of strap} = \frac{B_1}{4}$$

Thickness of gib = Thickness of cotter (*t*)

Height (t_2) and length of gib head (l_4) = Thickness of cotter (*t*)

2. *Failure of the gib and cotter in shearing*

Since the gib and cotter are in double shear, therefore,

Area resisting failure = $2\,B \times t$

and resisting strength = $2\,B \times t \times \tau$

Equating this to the load (*P*), we have $P = 2B \times t \times \tau$

From this equation, the width of gib and cotter (*B*) may be obtained. In the joint, as shown in Fig. one gib is used, the proportions of which are

Width of gib, $b1 = 0.55\,B$; and width of cotter, $b = 0.45\,B$

In case two gibs are used, then

Width of each gib = $0.3\,B$; and width of cotter = $0.4\,B$

3. *Failure of the strap end in tension at the location of gib and cotter*

Area resisting failure = $2\,[B_1 \times t_1 - t_1 \times t] = 2\,[x \times t_1 - t_1 \times t]$ ... ($B_1 = x$)

∴ Resisting strength = $2\,[\,x \times t_1 - t_1 \times t]\,\sigma_t$

Equating this to the load (*P*), we have

$$P = 2\,[x \times t_1 - t_1 \times t]\,\sigma_t$$

From this equation, the thickness of strap (t_1) may be determined.

4. *Failure of the strap or gib in crushing*

The strap or gib (at the strap hole) may fail due to crushing.

Area resisting failure = $2\,t_1 \times t$

∴ Resisting strength = $2\,t_1 \times t \times \sigma_c$

Equating this to the load (*P*), we have

$$P = 2\,t1 \times t \times \sigma_c$$

From this equation, the induced crushing stress may be checked.

5. *Failure of the rod end in shearing*

Since the rod is in double shear, therefore

Area resisting failure = 2 $l_1 \times x$

∴ Resisting strength = 2 $l_1 \times x \times \tau$

Equating this to the load (P), we have

$$P = 2\, l_1 \times x \times \tau$$

From this equation, the dimension l_1 may be determined.

6. *Failure of the strap end in shearing*

Since the length of rod (l_2) is in double shearing, therefore

Area resisting failure = $2 \times 2\, l_2 \times t_1$

∴ Resisting strength = $2 \times 2\, l_2 \times t_1 \times \tau$

Equating this to the load (P), we have

$$P = 2 \times 2\, l_2 \times t_1 \times \tau$$

From this equation, the length of rod (l_2) may be determined. The length l_3 of the strap end is proportioned as 2/3 rd of side of the rod. The clearance is usually kept 3 mm. The length of cotter is generally taken as 4 times the side of the rod.

Problem(3):- *Design a gib and cotter joint as shown in Fig. to carry a maximum load of 35 kN. Assuming that the gib, cotter and rod are of same material and have the following allowable stresses :*

σ_t = *20 MPa ;* τ = *15 MPa ; and* σ_c = *50 MPa*

Given data : P = 35 kN = 35 000 N ; σ_t = 20 MPa = 20 N/mm^2 ; τ = 15 MPa = 15 N/mm^2 ; σ_c = 50 MPa = 50 N/mm^2

1. *Side of the square rod*

Let x = Each side of the square rod.

Considering the failure of the rod in tension. We know that load (P),

$35\,000 = x^2 \times \sigma_t = x^2 \times 20 = 20\, x^2$

∴ $x^2 = 35\,000 / 20 = 1750$ or $x = 41.8$ say 42 mm **Ans.**

Other dimensions are fixed as follows :

Width of strap, $B_1 = x$ = 42 mm **Ans.**

Thickness of cotter,

$$t = \frac{B_1}{4} = \frac{42}{4} = 10.5 \text{ say } 12 \text{ mm}$$ **Ans.**

Thickness of gib = Thickness of cotter = 12 mm **Ans.**

Height (t_2) and length of gib head (l_4) = Thickness of cotter = 12 mm **Ans.**

2. *Width of gib and cotter*

Let B = Width of gib and cotter.

Consider the failure of the gib and cotter in double shear. We know that load (P),

$35\,000 = 2\, B \times t \times \tau = 2\, B \times 12 \times 15 = 360\, B$

∴ $B = 35\,000 / 360 = 97.2$ say 100 mm **Ans.**

Since one gib is used, therefore

Width of gib, $b_1 = 0.55\,B = 0.55 \times 100 = 55$ mm **Ans.**
and width of cotter, $b = 0.45\,B = 0.45 \times 100 = 45$ mm **Ans.**

3. *Thickness of strap*

Let t_1 = Thickness of strap.

Considering the failure of the strap end in tension at the location of the gib and cotter. We know that load (P),

$$35\,000 = 2\,(x \times t_1 - t_1 \times t)\,\sigma_t = 2\,(42 \times t_1 - t_1 \times 12)\,20 = 1200\,t_1$$

$\therefore$ $t_1 = 35\,000 / 1200 = 29.1$ say 30 mm **Ans.**

Now the induced crushing stress may be checked by considering the failure of the strap or gib in crushing. We know that load (P),

$$35\,000 = 2\,t_1 \times t \times \sigma_c = 2 \times 30 \times 12 \times \sigma_c = 720\,\sigma_c$$

$$\therefore \quad \sigma_c = 35\,000 / 720 = 48.6 \text{ N/mm}^2$$

Since the induced crushing stress is less than the given crushing stress, therefore the joint is safe.

4. *Length (l_1) of the rod*

Considering the failure of the rod end in shearing. Since the rod is in double shear, therefore load (P),

$$35\,000 = 2\,l_1 \times x \times \tau = 2\,l_1 \times 42 \times 15 = 1260\,l_1$$

$\therefore$ $l_1 = 35\,000 / 1260 = 27.7$ say 28 mm **Ans.**

5. *Length (l_2) of the rod*

Considering the failure of the strap end in shearing. Since the length of the rod ($l2$) is in double shear, therefore load (P),

$$35\,000 = 2 \times 2\,l_2 \times t_1 \times \tau = 2 \times 2\,l_2 \times 30 \times 15 = 1800\,l_2$$

$\therefore$ $l_2 = 35\,000 / 1800 = 19.4$ say 20 mm **Ans.**

Length (l_3) of the strap end

$$= \frac{2}{3} \times x = \frac{2}{3} \times 42 = 28 \text{ mm}$$ **Ans.**

and length of cotter = $4\,x = 4 \times 42 = 168$ mm **Ans.**

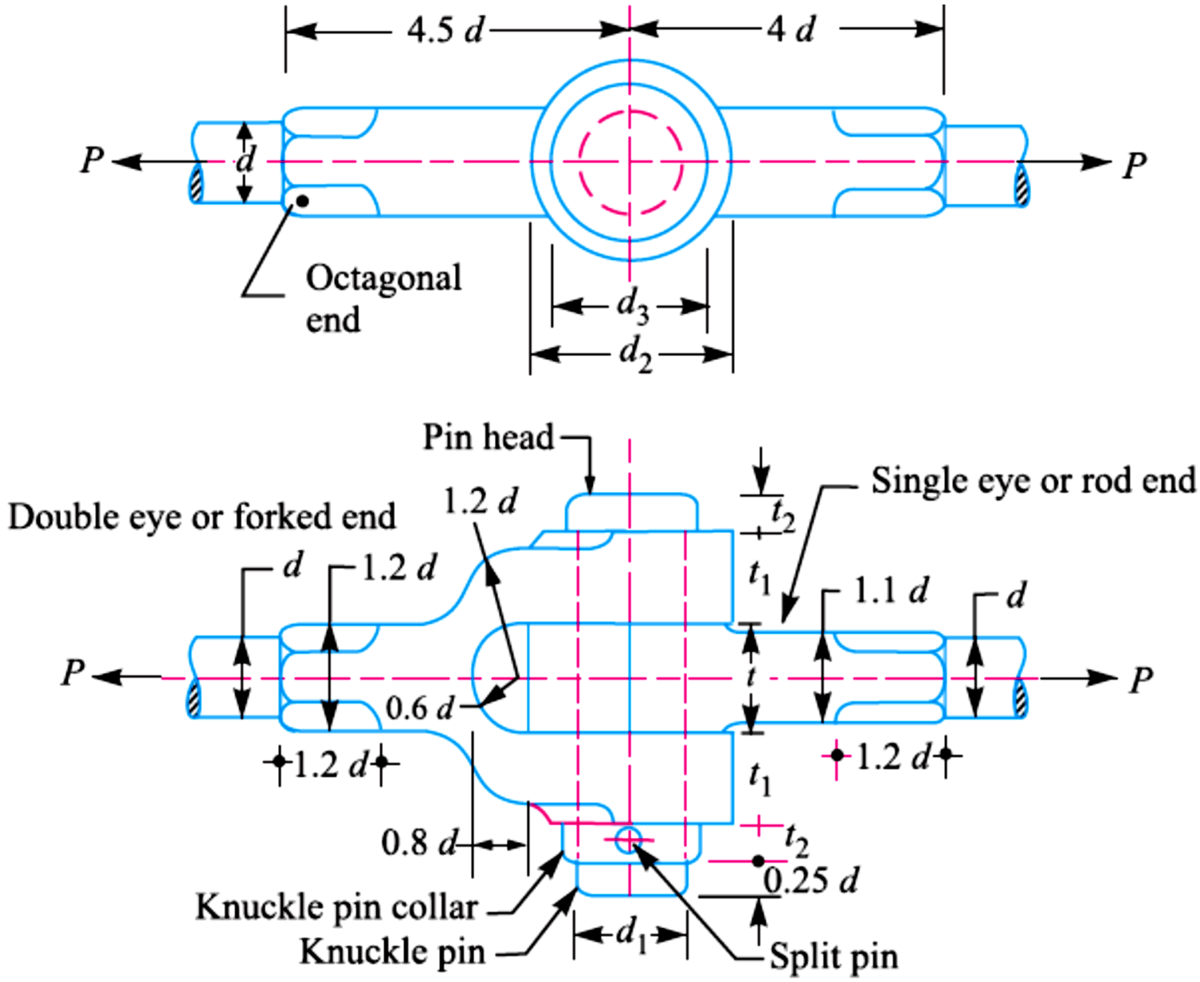
<u>Knuckle Joint</u>:- **(what is the Knuckle joint?)**

A knuckle joint is used to connect two rods which are under the action of tensile loads. However, if the joint is guided, the rods may support a compressive load. A knuckle joint may be readily disconnected for adjustments or repairs.

Its use may be found in the link of a cycle chain, tie rod joint for roof truss, valve rod joint with eccentric rod, pump rod joint, tension link in bridge structure and lever and rod connections of various types.

In knuckle joint is shown in Fig, one end of one of the rods is made into an eye and the end of the other rod is formed into a fork with an eye in each of the fork leg. The knuckle pin passes through both the eye hole and the fork holes and may be secured by means of a collar and taper pin or spilt pin. The knuckle pin may be prevented from rotating in the fork by means of a small stop, pin, peg. The material used for the joint may be steel or wrought iron.

Dimensions of Various Parts of the Knuckle Joint:-



Knuckle joint

If d is the diameter of rod, then diameter of pin,

$d_1 = d$

Outer diameter of eye, $d_2 = 2\,d$

Diameter of knuckle pin head and collar, $d_3 = 1.5\,d$

Thickness of single eye or rod end, $t = 1.25\,d$

Thickness of fork, $t_1 = 0.75\,d$

Thickness of pin head, $t_2 = 0.5\,d$

Other dimensions of the joint are shown in Fig.

Methods of Failure of Knuckle Joint:-

Consider a knuckle joint as shown in Fig.

Let P = Tensile load acting on the rod,

d = Diameter of the rod,

d_1 = Diameter of the pin,

d_2 = Outer diameter of eye,

t = Thickness of single eye,

t_1 = Thickness of fork.

σ_t , τ and σ_c = Permissible stresses for the joint material in tension, shear and crushing respectively.

In determining the strength of the joint for the various methods of failure, it is assumed that

1. There is no stress concentration, and

2. The load is uniformly distributed over each part of the joint.

1. *Failure of the solid rod in tension*

Since the rods are subjected to direct tensile load, therefore tensile strength of the rod,

$$= \frac{\pi}{4} \times d^2 \times \sigma_t$$

Equating this to the load (P) acting on the rod, we have

$$P = \frac{\pi}{4} \times d^2 \times \sigma_t$$

From this equation, diameter of the rod (d) is obtained.

2. *Failure of the knuckle pin in shear*

Since the pin is in double shear, therefore cross-sectional area of the pin under shearing

$$= 2 \times \frac{\pi}{4} (d_1)^2$$

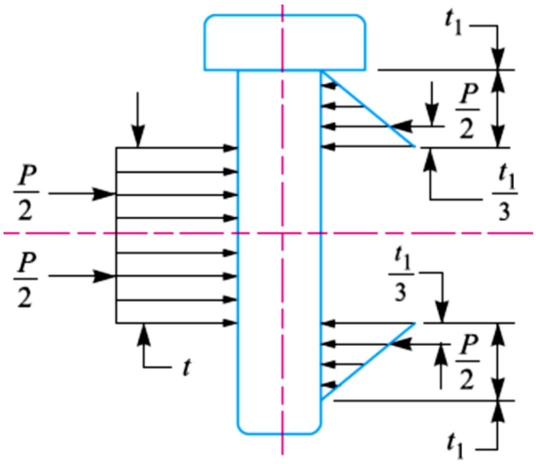
and the shear strength of the pin

$$= 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

Equating this to the load (P) acting on the rod, we have

$$P = 2 \times \frac{\pi}{4} (d_1)^2 \ \tau$$

From this equation, diameter of the knuckle pin (d_1) is obtained.



$$M = \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2}\right) - \frac{P}{2} \times \frac{t}{4}$$

$$= \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{2} - \frac{t}{4}\right) \qquad = \frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)$$

and section modulus, $Z = \frac{\pi}{32}(d_1)^3$

Maximum bending (tensile) stress,

$$\sigma_t = \frac{M}{Z} = \frac{\frac{P}{2}\left(\frac{t_1}{3} + \frac{t}{4}\right)}{\frac{\pi}{32}(d_1)^3}$$

From this expression, the value of d_1 may be obtained.

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may tear off due to the tensile load. We know that area resisting tearing $= (d_2 - d_1)\, t$

$\therefore$ Tearing strength of single eye or rod end $= (d_2 - d_1)\, t \times \sigma_t$

Equating this to the load (P) we have $P = (d_2 - d_1)\, t \times \sigma_t$

From this equation, the induced tensile stress (σ_t) for the single eye or rod end may be checked. In case the induced tensile stress is more than the allowable working stress, then increase the outer diameter of the eye (d_2).

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to tensile load. We know that area resisting shearing $= (d_2 - d_1)\, t$

$\therefore$ Shearing strength of single eye or rod end $= (d_2 - d_1)\, t \times \tau$

Equating this to the load (P), we have $P = (d_2 - d_1)\, t \times \tau$

From this equation, the induced shear stress (τ) for the single eye or rod end may be checked.

5. *Failure of the single eye or rod end in crushing*

The single eye or pin may fail in crushing due to the tensile load. We know that area resisting crushing = $d_1 \times t$

$\therefore$ Crushing strength of single eye or rod end $= d_1 \times t \times \sigma_c$

Equating this to the load (P), we have $P = d_1 \times t \times \sigma_c$

From this equation, the induced crushing stress (σ_c) for the single eye or pin may be checked. In case the induced crushing stress in more than the allowable working stress, then increase the thickness of the single eye (t).

6. *Failure of the forked end in tension*

The forked end or double eye may fail in tension due to the tensile load. We know that area resisting tearing $= (d_2 - d_1) \times 2\, t_1$

$\therefore$ Tearing strength of the forked end $= (d_2 - d_1) \times 2\, t_1 \times \sigma_t$

Equating this to the load (P), we have $P = (d_2 - d_1) \times 2t_1 \times \sigma_t$

From this equation, the induced tensile stress for the forked end may be checked.

7. *Failure of the forked end in shear*

The forked end may fail in shearing due to the tensile load. We know that area resisting shearing $= (d_2 - d_1) \times 2t_1$

$\therefore$ Shearing strength of the forked end $= (d_2 - d_1) \times 2t_1 \times \tau$

Equating this to the load (P), we have $P = (d_2 - d_1) \times 2t_1 \times \tau$

From this equation, the induced shear stress for the forked end may be checked. In case, the induced shear stress is more than the allowable working stress, then thickness of the fork (t_1) is increased.

8. *Failure of the forked end in crushing*

The forked end or pin may fail in crushing due to the tensile load. We know that area resisting crushing = $d_1 \times 2\, t_1$

$\therefore$ Crushing strength of the forked end $= d_1 \times 2\, t_1 \times \sigma_c$

Equating this to the load (P), we have $P = d_1 \times 2\, t_1 \times \sigma_c$

From this equation, the induced crushing stress for the forked end may be checked.

<u>Problem(4):-</u> *Design a knuckle joint to transmit 150 kN. The design stresses may be taken as 75 MPa in tension, 60 MPa in shear and 150 MPa in crushing.*

Given data : P = 150 kN = 150×10^3 N ; σ_t = 75 MPa = 75 N/mm^2 ; τ = 60 MPa = 60 N/mm^2 ; σ_c = 150 MPa = 150 N/mm^2

The knuckle joint is shown in Fig. The joint is designed by considering the various methods of failure as discussed below :

1. *Failure of the solid rod in tension*

Let d = Diameter of the rod.

We know that the load transmitted (P),

$$150 \times 10^3 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 75 = 59\,d^2$$

∴ $d^2 = 150 \times 10^3 / 59 = 2540$ or $d = 50.4$ say 52 mm **Ans.**

Now the various dimensions are fixed as follows :

Diameter of knuckle pin, $d_1 = d = 52$ mm

Outer diameter of eye, $d_2 = 2\,d = 2 \times 52 = 104$ mm

Diameter of knuckle pin head and collar, $d_3 = 1.5\,d = 1.5 \times 52 = 78$ mm

Thickness of single eye or rod end, $t = 1.25\,d = 1.25 \times 52 = 65$ mm

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 52 = 39$ say 40 mm

Thickness of pin head, $t_2 = 0.5\,d = 0.5 \times 52 = 26$ mm

2. *Failure of the knuckle pin in shear*

Since the knuckle pin is in double shear, therefore load (P),

$$150 \times 10^3 = 2 \times \frac{\pi}{4} \times (d_1)^2\,\tau = 2 \times \frac{\pi}{4} \times (52)^2\,\tau = 4248\,\tau$$

$\tau = 150 \times 10^3 / 4248 = 35.3$ N/mm^2 = 35.3 MPa

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may fail in tension due to the load. We know that load (P), $150 \times 10^3 = (d_2 - d_1)\,t \times \sigma_t = (104 - 52)\,65 \times \sigma_t = 3380\,\sigma_t$

∴ $\sigma_t = 150 \times 10^3 / 3380 = 44.4$ N / mm^2 = 44.4 MPa

4. *Failure of the single eye or rod end in shearing*

The single eye or rod end may fail in shearing due to the load. We know that load (P), $150 \times 10^3 = (d_2 - d_1)\,t \times \tau = (104 - 52)\,65 \times \tau = 3380\,\tau$

$\tau = 150 \times 10^3 / 3380 = 44.4$ N/mm^2 = 44.4 MPa

5. *Failure of the single eye or rod end in crushing*

The single eye or rod end may fail in crushing due to the load. We know that load (P), $150 \times 10^3 = d_1 \times t \times \sigma_c = 52 \times 65 \times \sigma_c = 3380\,\sigma_c$

∴ σ *c* = 150 × 103 / 3380 = 44.4 N/mm2 = 44.4 MPa

$$\sigma_c = 150 \times 10^3 / 3380 = 44.4\ \text{N/mm}^2 = 44.4\ \text{MPa}$$

6. *Failure of the forked end in tension*

The forked end may fail in tension due to the load. We know that load (P),

$150 \times 10^3 = (d_2 - d_1)\,2\,t_1 \times \sigma_t = (104 - 52)\,2 \times 40 \times \sigma_t = 4160\,\sigma_t$

∴ $\sigma_t = 150 \times 10^3 / 4160 = 36$ N/mm^2 = 36 MPa

7. *Failure of the forked end in shear*

The forked end may fail in shearing due to the load. We know that load (P),

$150 \times 10^3 = (d_2 - d_1)\,2\,t_1 \times \tau = (104 - 52)\,2 \times 40 \times \tau = 4160\,\tau$

$\tau = 150 \times 10^3 / 4160 = 36$ N/mm^2 = 36 MPa

8. *Failure of the forked end in crushing*

The forked end may fail in crushing due to the load. We know that load (*P*),

$$150 \times 10^3 = d_1 \times 2\, t_1 \times \sigma_c = 52 \times 2 \times 40 \times \sigma_c = 4160\, \sigma_c$$

$$\therefore \quad \sigma_c = 150 \times 10^3 / 4180 = 36 \text{ N/mm}^2 = 36 \text{ MPa}$$

From above, we see that the induced stresses are less than the given design stresses, therefore the **joint is safe.**

Problem(6):- *Design a knuckle joint for a tie rod of a circular section to sustain a maximum pull of 70 kN. The ultimate strength of the material of the rod against tearing is 420 MPa. The ultimate tensile and shearing strength of the pin material are 510 MPa and 396 MPa respectively. Determine the tie rod section and pin section. Take factor of safety = 6.*

Given data : P = 70 kN = 70 000 N ; σ_{tu} for rod = 420 MPa ; σ_{tu} for pin = 510 MPa ; τ_u = 396 MPa ; *F.S.* = 6

We know that the permissible tensile stress for the rod material,

$$\sigma_t = \frac{\sigma_{tu} \text{ for rod}}{F.S.} = \frac{420}{6} = 70 \text{ MPa} = 70 \text{ N/mm}^2$$

and permissible shear stress for the pin material,

$$\tau = \frac{\tau_u}{F.S.} = \frac{396}{6} = 66 \text{ MPa} = 66 \text{ N/mm}^2$$

We shall now consider the various methods of failure of the joint as discussed below:

1. *Failure of the rod in tension*

Let d = Diameter of the rod.

We know that the load (*P*),

$$70\,000 = \frac{\pi}{4} \times d^2 \times \sigma_t = \frac{\pi}{4} \times d^2 \times 70 = 55\, d^2$$

$\therefore$ $d^2 = 70\,000 / 55 = 1273$ or d = 35.7 say 36 mm **Ans.**

The other dimensions of the joint are fixed as given below :

Diameter of the knuckle pin, $d_1 = d = 36$ mm

Outer diameter of the eye, $d_2 = 2\,d = 2 \times 36 = 72$ mm

Diameter of knuckle pin head and collar, $d_3 = 1.5\,d = 1.5 \times 36 = 54$ mm

Thickness of single eye or rod end, $t = 1.25\,d = 1.25 \times 36 = 45$ mm

Thickness of fork, $t_1 = 0.75\,d = 0.75 \times 36 = 27$ mm

Now we shall check for the induced stresses as discussed below :

2. *Failure of the knuckle pin in shear*

Since the knuckle pin is in double shear, therefore load (*P*),

$$70\ 000 = 2 \times \frac{\pi}{4}(d_1)^2 \tau = 2 \times \frac{\pi}{4}(36)^2 \tau = 2036\tau$$

$$\tau = 70\ 000 / 2036 = 34.4 \text{ N/mm}^2$$

3. *Failure of the single eye or rod end in tension*

The single eye or rod end may fail in tension due to load. We know that load (*P*),

$$70\ 000 = (d_2 - d_1)\ t \times \sigma_t = (72 - 36)\ 45\ \sigma_t = 1620\ \sigma_t$$

$$\therefore \quad \sigma_t = 70\ 000 / 1620 = 43.2 \text{ N/mm}^2$$

4. *Failure of the forked end in tension*

The forked end may fail in tension due to the load. We know that load (*P*),

$$70\ 000 = (d_2 - d_1)\ 2\ t_1 \times \sigma_t = (72 - 36) \times 2 \times 27 \times \sigma_t = 1944\ \sigma_t$$

$$\therefore \quad \sigma_t = 70\ 000 / 1944 = 36 \text{ N/mm}^2$$

From above we see that the induced stresses are less than given permissible stresses, therefore **the joint is safe.**

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SHAFTS

Introduction:-A shaft is a rotating machine element which transmit power from one place to another. Shaft are subjected to tensile, bending or torsional stresses or to a combination of these stresses.

A transmitting shaft is circular is cross section, which supports transmission elements like pulleys, gears and sprockets.

The design of transmission shaft consists of determining the corret shaft diameter based on 1). Strength, 2). Rigidity and Stiffness.

Axle: It is a non rotating shaft, which supports the rotating components of the machine. It does not transmit a useful torque. Axle is subjected to only bending.

S*pindle: It* is a short rotating shaft in case of drilling machine, lathe spindles.

Line shaft or transmission shaft: It is a comparatively long shaft which is driven by a motor. The line shaft transmits motion to various machines through counter shafts. The counter shaft is an intermediate shaft placed between the line shaft and various driven machines.