UNIT-I INTRODUCTION AND STRESSES IN MACHINE MEMBERS

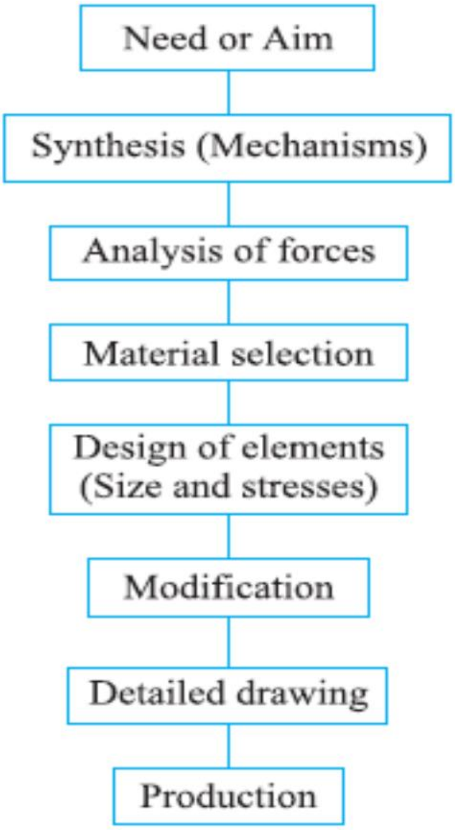
Definition

"Machine design is defined as the use of scientific principles, technical information and imagination in the description of a machine or a mechanical system to perform specific functions with maximum economy and efficiency".

In designing a machine component, it is necessary to have a good knowledge of many subjects such as Mathematics, Engineering Mechanics, Strength of Materials, Theory of Machines, Workshop Processes and Engineering Drawing.

General Procedure in Machine Design***

In designing a machine component, there is no rigid rule. The problem may be attempted in several ways. However, the general procedure to solve a design problem is as follows :



1. *Recognition of need.* First of all, make a complete statement of the problem, indicating the need, aim or purpose for which the machine is to be designed.

2. *Synthesis* (*Mechanisms*). Select the possible mechanism or group of mechanisms which will give the desired motion.

3. *Analysis of forces.* Find the forces acting on each member of the machine and the energy transmitted by each member.

4. *Material selection.* Select the material best suited for each member of the machine.

5. *Design of elements* (*Size and Stresses*). Find the size, find force acting on the member and the permissible stresses for the material used. It should be kept in mind that each member should not deflect or deform than the permissible limit.

6. *Modification.* Modify the size of the member to agree with the past experience and judgment to facilitate manufacture. The modification may also be necessary by consideration of manufacturing to reduce overall cost.

7. *Detailed drawing.* Draw the detailed drawing of each component and the assembly of the machine with complete specification for the manufacturing processes suggested.

8. *Production.* The component as per the drawing is manufactured, assembled in the workshop.

<u>CLASSIFICATION OF DESIGN</u> (How do you classify machine design)

a). Adaptive design:- Adaptive design is one in which designer's work is concerned with the adaptation of the existing design requiring no special skills and knowledge. Example are bicycles and IC engines.

b). Developed design:- Developed design, a high standard of scientific training is essential when the proven existing design are to be modified to their method of manufacture, material, appearance etc. In this case designer starts with an existing design and the final product outcome is remarkably different from the original product.

c). New design:- New design requires a lot of research, technical capability and creativity. A few designers bring out new machines by making use of basic scientific principles.

<u>FACTORS INFLUENCING MACHINE DESIGN</u>:-

What factors should be considered in machine design? (OR) What are the basic requirements of Machine design?

During the design of machine elements the following factors should be considered.

S.No	Factors	Example
1	Type of loading	Static, dynamic, simple, and complicated loads.
2	Size of object	Simple, complicated, small and large.
3	Environment conditions	Pure atmosphere.
4	Material properties	Hard, soft, rigid, elastic and tough etc.
5	Place of employment	Hazardous place, safe place. on road, on water and in air etc
6	Human safety	Foolproof arrangement should be considered.

7	Cost	The design is in such way that the product should be manufactured in lower cost.

GENERAL CONSIDERATIONS IN THE DESIGN OF MACHINE ELEMENTS

What are the General considerations in the design of machine elements?

1.Selection of materials
2. Type of load and stresses
3. Motion of parts
4. Work facilities
5. safety measures
6. Frictional resistance and lubrication
7. cost of construction
8. Use of standard parts

What are the factors to be considered in the selection of materials for the machine members?

1). Properties of materials such as

a). Mechanical properties
b). Electrical properties
c). Physical properties
d). Chemical properties
e). Magnetic properties
f). Thermal properties
g). Acoustic properties
h). Optical properties
i). Metallurgical properties.

2). Easy availability of materials.
3). Cost,
4). Non-hazardous to human being.

MECHANICAL PROPERTIES OF METALS*** (Explain the desirable properties of engineering materials used in mechanical engineering design.)

Mechanical Properties:-"The properties of machine elements which undergo any changes in shape and structure during the application of force on those elements are called as mechanical properties".

1. *Strength*. It is the ability of a material to resist externally applied loads without breaking or yielding.

2. *Stiffness*. It is the ability of a material to resist deformation under loads. The modulus of elasticity is the measure of stiffness.

3. *Elasticity*. It is the property of a material to regain its original shape after deformation when the external forces are removed. steel is more elastic than rubber.

4. *Plasticity*. It is property of a material which retains the deformation produced under load permanently. This property of the material is necessary for forgings, in stamping images on coins and in ornamental work.

5. *Ductility*. It is the property of a material ability to be drawn into a wire by a tensile force. The ductility is usually measured by the terms, percentage elongation and percentage reduction in area.

6. *Malleability*. It is the property of a material ability to be drawn into a thin sheets by a compressive force.(rolled or hammered into thin sheets)

7. Brittleness sudden breaking with minimum distortion . Cast iron is a brittle material.

8. *Toughness*. It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of the material decreases when it is heated.

9. Resilience is defined as the ability of the material to absorb energy when deformed elastically and to release this energy when unloaded. This property is essential for spring materials.

10. *Creep*. The slow and progressive deformation with time. IC engines, boilers,turbines blades, rocket engines, and nuclear reactor components etc are subjected to creep.

11. *Fatigue*. When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of failure of a material is known as **fatigue*. OR

Ability to withstand cyclic stresses.

This property is considered in designing shafts, connecting rods, springs, gears, etc.

12. *Hardness* – resistance to wear, scratching, deformation, machinability etc

The hardness of a metal may be determined by the following tests :

(*a*) Brinell hardness test,

(*b*) Rockwell hardness test,

(*c*) Vickers hardness (also called Diamond Pyramid) test, and

Selection of Materials for Engineering Purposes

The selection of a proper material, for engineering purposes, is one of the most difficult problem for the designer. The best material is one which serve the desired objective at the minimum cost. The following factors should be considered while selecting the material :

1. Availability of the materials,

2. Suitability of the materials for the working conditions in service, and

3. The cost of the materials.

The important properties, which determine the utility of the material are physical, chemical and mechanical properties.

<u>Classification Of Engineering Materials</u> (Enumerate commonly used engineering materials and explain them)

The engineering materials are mainly classified as :

1. Metals and their alloys:- Ex:- iron, steel, copper, aluminum, etc.

The metals may be further classified as :

(*a*) Ferrous metals, and (*b*) Non-ferrous metals.

<u>Ferrous metals</u> containing iron as major constituent, such as cast iron, wrought iron and steel.

<u>Non-ferrous metals</u> containing other than iron such as copper, aluminum, brass, tin, zinc, etc.

2. Non-metals, such as glass, rubber, plastic, fibre,leather and asbestos etc.

1. METALS AND THEIR ALLOYS:-

Cast Iron :-

1. Cast iron is an alloy of iron & carbon, containing more than 2% of carbon

   Typical composition of ordinary cast iron is:

Carbon = 3-4%	Silicon = 1-3%
Manganese = 0.5-1%	Sulphur = up to 0.1%
Phosphorous = up to 0.1%	Iron = Remain.

The different types of cast irons are White cast iron, Malleable cast iron, Grey cast iron, Spheroidal cast iron, chilled cast iron and Alloy cast iron.

Advantages:

- Available in large quantities
- Higher compressive strength,
- Components can be given any complex shape without involving costly machining operations,
- Excellent ability to damp vibrations,
- More resistance to wear even under the conditions of boundary lubrication,
- Mechanical properties of parts do not change between room temperature and 350^0 centigrade

Disadvantages:

- It has poor tensile strength compared to steel
- Cast iron does not offer any plastic deformation before failure
- Cast iron is brittle material
- The machinability of cast iron parts is poor compared to steel.

Applications:

- Machine tool Beds, lathe bed and guide ways
- Automotive components, agricultural components, feed pipe fittings
- Piston, piston rings, cam shafts, pulleys and gears
- Agricultural tractors, implement parts in automotive crank shafts, piston and cylinder heads

Steel:-
It is an alloy of iron and carbon, with carbon content up to a maximum of 1.5%.

Classifications:-
Dead mild steel — up to 0.15% carbon
Low carbon or mild steel — 0.15% to 0.45% carbon (Good ductility, good formability)
Medium carbon steel — 0.45% to 0.8% carbon (High hardness than low carbon steel)
High carbon steel — 0.8% to 1.5% carbon (Good wear and operation resistance)
Applications:-
Carbon 0.1 to 0.2% Tubing, forgings, pressed steel parts, rivits and screws.
Carbon 0.2 to 0.3% Forged and machine parts, structural members, boilers.
Carbon 0.3 to 0.55% Forged and machine parts, automobile bolts, shafts.
Carbon 0.55 to 0.75% Rails, hammers.
Carbon 0.75 to 0.85% Coils and flat springs.
Carbon 0.85 to 0.95% Tools, punches, dies and saws.

Effects of various elemnts in steel
- ***Carbon:-*** Increasing in carbion content upto 0.83% increase the ultimate strength. If the carbon content is going to be more than 0.83% reduces the strength, hardeness, ductility and weldability.
- ***Silicon:-*** Silicon is added in spring steel to increase its toughness.
- ***Manganese:-*** Manganese increases, hardness and ultimate strength increases.
- ***Nickel:-*** Nickel increases strength, hardness and toughness increases.
- ***Chromium:-***Chromium increases hardness & wear resistance, steel containing more than 4% chromium have excellent corrosion resistance

Applications:-
- Stampings, rivets, cams, gears, levers,
- Screws, nets, diet and screw drivers, washers
- Handsaw, springs, chisels and ball bearings

High Speed Tool Steels
These steels are used for cutting metals at a much higher cutting speed than ordinary carbon tool steels. The carbon steel cutting tools do not retain their sharp cutting edges under heavier loads and higher speeds.

Most of the high speed steels contain tungsten as the chief alloying element, but other elements like cobalt, chromium, vanadium, etc. may be present in some proportion. Following are the different types of high speed steels:

1. *18-4-1 High speed steel*. This steel, on an average, contains 18% of tungsten, 4% of chromium and 1% vanadium. It is considered to be one of the best of all purpose tool steels. It is widely used for drills, lathe, planer and shaper tools, milling cutters, reamers, broaches, threading dies, punches, etc.

2. *Molybdenum high speed steel*. This steel, on an average, contains 6% of tungsten, 6% of molybdenum, 4% of chromium and 2% of vanadium. It has excellent toughness and cutting ability. The molybdenum high speed steels are better and cheaper than other types of steels. It is particularly used for drilling and tapping operations.

3. *Super high speed steel*. This steel is also called ***cobalt high speed steel*** because cobalt is added from 2 to 15%, in order to increase the cutting efficiency especially at high temperatures.

This steel, on an average, contains 20% tungsten, 4% chromium, 2% vanadium and 12% cobalt. Since the cost of this steel is more, therefore, it is principally used for heavy cutting operations which impose high pressure and temperatures on the tool.

What is the duralumin?

Duralumin is an Al-Cu-Mg-Mn alloy (94% Al – 4% Cu – 1% Mg – 1%Mn) and it has good correion resistance and strength. It is strong, hard and light weight alloy of aluminium. It is widely used in aircraft construction.

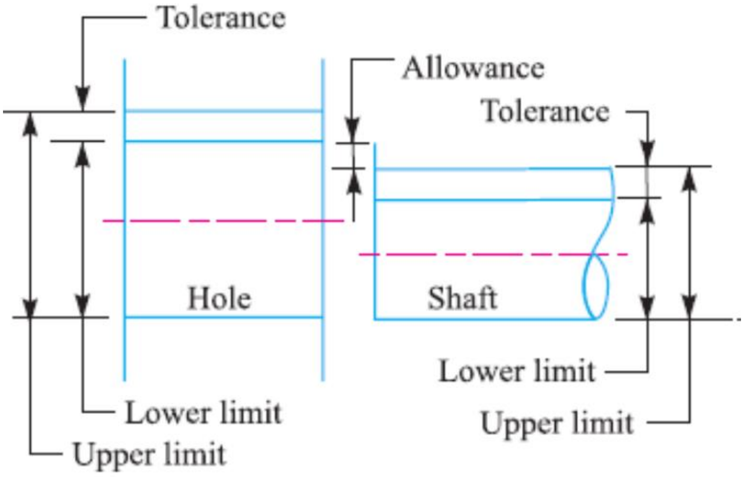
Important Terms used in Limit System

The following terms used in limit system (or interchangeable system) are important from the subject point of view:

1. *Nominal size*. It is the size of a part specified in the drawing as a matter of convenience.

2. *Basic size*. It is the size of a part to which all limits of variation (*i.e.* tolerances) are applied to arrive at final dimensioning of the mating parts. The nominal or basic size of a part is often the same.

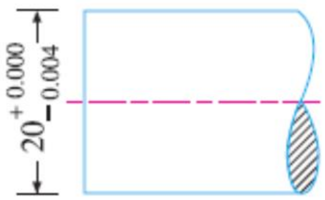
3. *Actual size*. It is the actual measured dimension of the part. The difference between the basic size and the actual size should not exceed a certain limit, otherwise it will interfere with the interchangeability of the mating parts.



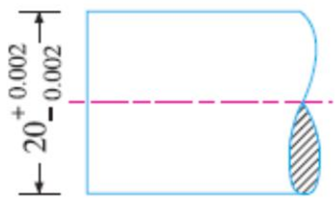
4. *Limits of sizes*. There are two extreme permissible sizes for a dimension of the part as shown in Fig. The largest permissible size for a dimension of the part is called ***upper*** or ***high*** or ***maximum limit*,** whereas the smallest size of the part is known as ***lower*** or ***minimum limit*.**

5. *Allowance*. It is the difference between the basic dimensions of the mating parts. The allowance may be ***positive*** or ***negative***. When the shaft size is less than the hole size, then the allowance is ***positive*** and when the shaft size is greater than the hole size, then the allowance is ***negative*.**

6. *Tolerance*. It is the difference between the upper limit and lower limit of a dimension. In other words, it is the maximum permissible variation in a dimension. The tolerance may be ***unilateral*** or ***bilateral*.** When all the tolerance is allowed on one side of the nominal size, *e.g.* $20^{+0.000}_{-0.004}$, then it is said to be ***unilateral system of tolerance*.** The unilateral system is mostly used in industries as it permits changing the tolerance value while still retaining the same allowance or type of fit. When the tolerance is allowed on both sides of the nominal size, *e.g.* $20^{+0.002}_{-0.002}$, then it is said to be ***bilateral system of tolerance*.** In this case + 0.002 is the upper limit and – 0.002 is the lower limit. The method of assigning unilateral and bilateral tolerance is shown in Fig.(*a*) and (*b*) respectively.

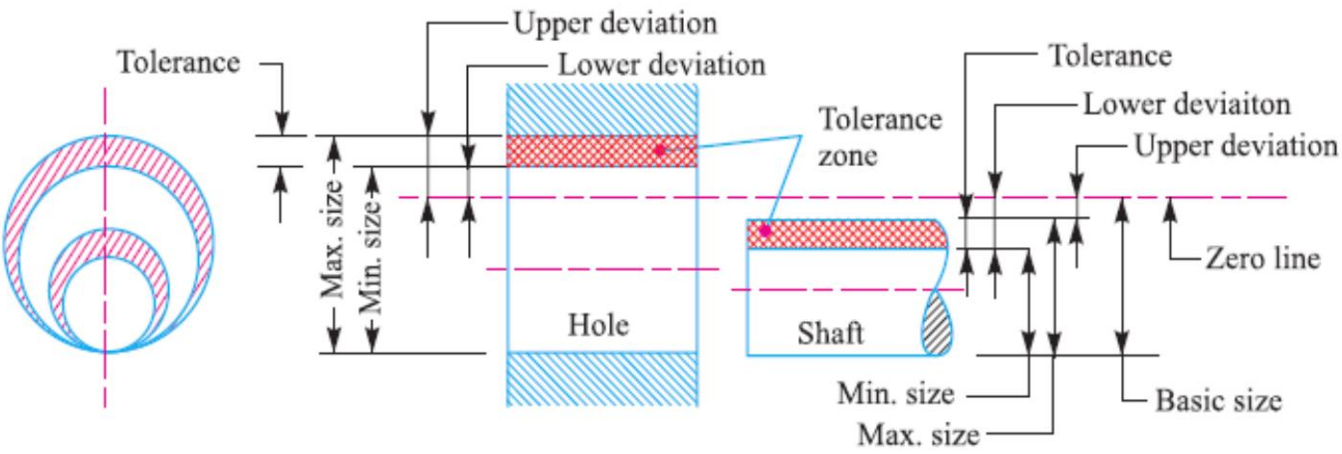


(*a*) Unilateral tolerance.



(*b*) Bilateral tolerance.

7. *Tolerance zone*. It is the zone between the maximum and minimum limit size, as shown in Fig.



8. *Zero line*. It is a straight line corresponding to the basic size. The deviations are measured from this line. The positive and negative deviations are shown above and below the zero line respectively.

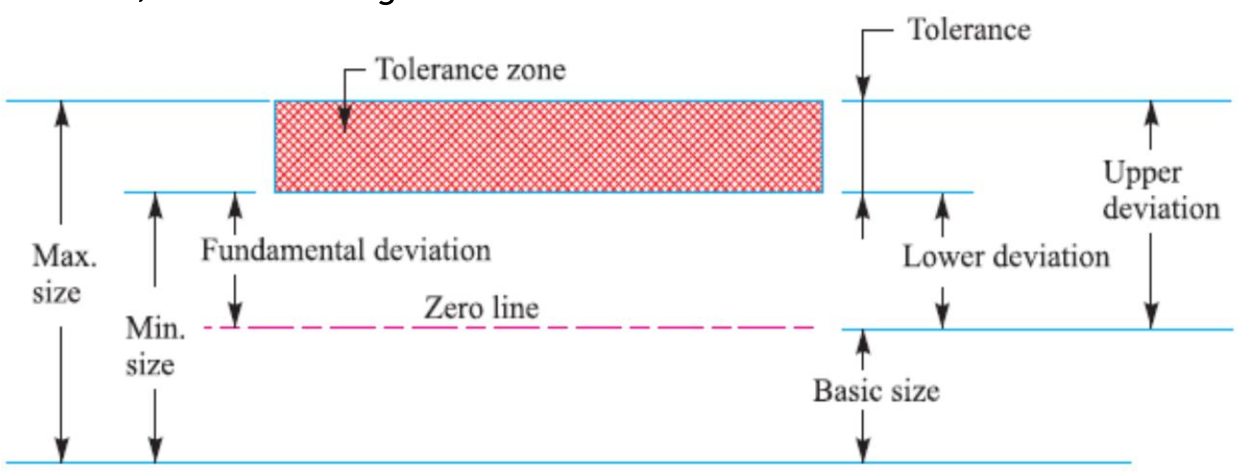
9. *Upper deviation*. It is the algebraic difference between the maximum size and the basic size. The upper deviation of a hole is represented by a symbol *ES* (Ecart Superior) and of a shaft, it is represented by *es*.

10. *Lower deviation*. It is the algebraic difference between the minimum size and the basic size. The lower deviation of a hole is represented by a symbol *EI* (Ecart Inferior) and of a shaft, it is represented by *ei*.

11. *Actual deviation*. It is the algebraic difference between an actual size and the corresponding basic size.

12. *Mean deviation*. It is the arithmetical mean between the upper and lower deviations.

13. *Fundamental deviation*. It is one of the two deviations which is conventionally chosen to define the position of the tolerance zone in relation to zero line, as shown in Fig.



Fits

The degree of tightness or looseness between the two mating parts is known as a *fit* of the parts.

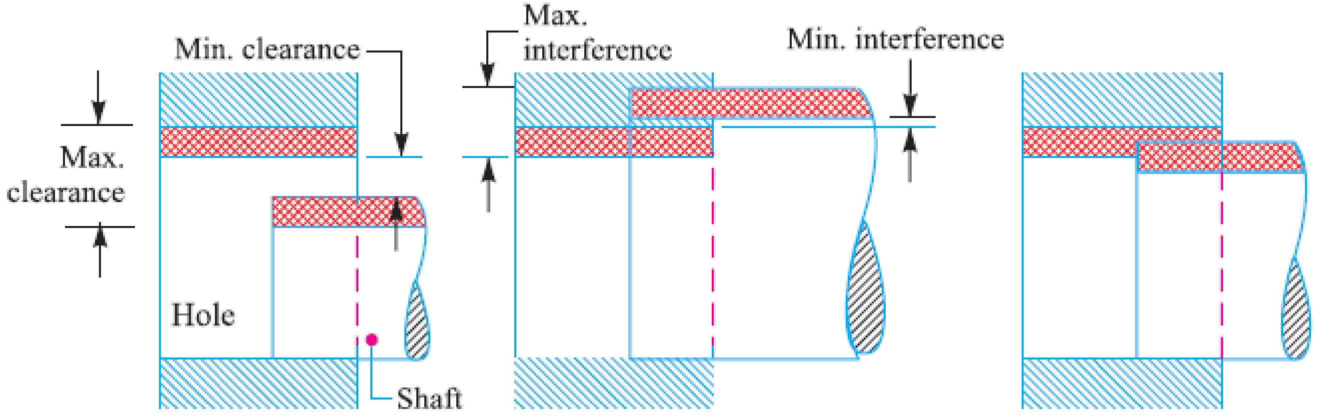
Types of Fits

According to Indian standards, the fits are classified into the following three groups :

1. ***Clearance fit.*** In this type of fit, the size limits for mating parts are so selected that clearance between them always occur, as shown in Fig (*a*). It may be noted that in a clearance fit, the tolerance zone of the hole is entirely above the tolerance zone of the shaft. In a clearance fit, the difference between the minimum size of the hole and the maximum size of the shaft is known as ***minimum clearance*** whereas the difference between the maximum size of the hole and minimum size of the shaft is called ***maximum clearance*** as shown in Fig.(*a*). The clearance fits may be slide fit, easy sliding fit, running fit, slack running fit and loose running fit.

2. ***Interference fit.*** In this type of fit, the size limits for the mating parts are so selected that interference between them always occur, as shown in Fig. 3.5 (*b*). It may be noted that in an interference fit, the tolerance zone of the hole is entirely below the tolerance zone of the shaft. In an interference fit, the difference between the maximum size of the hole and the minimum size of the shaft is known as ***minimum interference*****,** whereas the difference between the minimum size of the hole and the maximum size of the shaft is called ***maximum interference*****,** as shown in Fig.(*b*). The interference fits may be shrink fit, heavy drive fit and light drive fit.

3. ***Transition fit.*** In this type of fit, the size limits for the mating parts are so selected that either a clearance or interference may occur depending upon the actual size of the mating parts, as shown in Fig.(*c*). It may be noted that in a transition fit, the tolerance zones of hole and shaft overlap. The transition fits may be force fit, tight fit and push fit.



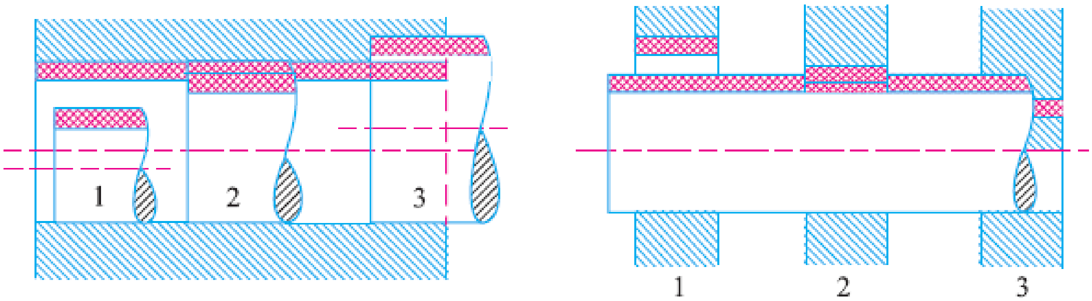
(*a*) Clearance fit. (*b*) Interference fit. (*c*) Transition fit.

Basis of Limit System

The following are two bases of limit system:

1. *Hole basis system*. When the hole is kept as a constant member (*i.e.* when the lower deviation of the hole is zero) and different fits are obtained by varying the shaft size, as shown in Fig.(*a*), then the limit system is said to be on a hole basis.

2. *Shaft basis system*. When the shaft is kept as a constant member (*i.e.* when the upper deviation of the shaft is zero) and different fits are obtained by varying the hole size, as shown in Fig. (*b*), then the limit system is said to be on a shaft basis.



1. Clearance fit. 2. Transition fit. 3. Interference fit.

(*a*) Hole basis system. (*b*) Shaft basis system.

Problem(1):- *A mild steel rod of 12 mm diameter was tested for tensile strength with the gauge length of 60 mm. Following observations were recorded : Final length = 80 mm; Final diameter = 7 mm; Yield load = 3.4 kN and Ultimate load = 6.1 kN. Calculate : 1. yield stress, 2. ultimate tensile stress, 3. percentage reduction in area, and 4. percentage elongation.*

Solution. Given : D = 12 mm ; l = 60 mm ; L = 80 mm ; d = 7 mm ; Wy = 3.4 kN = 3400 N; Wu = 6.1 kN = 6100 N

We know that original area of the rod,

$$A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4}(12)^2 = 113 \text{ mm}^2$$

and final area of the rod,

$$a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(7)^2 = 38.5 \text{ mm}^2$$

1. *Yield stress*

We know that yield stress

$$= \frac{W_y}{A} = \frac{3400}{113} = 30.1 \text{ N/mm}^2 = 30.1 \text{ MPa}$$

2. ***Ultimate tensile stress***

We know the ultimate tensile stress

$$= \frac{W_u}{A} = \frac{6100}{113} = 54 \text{ N/mm}^2 = 54 \text{ MPa}$$

3. ***Percentage reduction in area***

We know that percentage reduction in area

$$= \frac{A - a}{A} = \frac{113 - 38.5}{113} = 0.66 \text{ or } 66\%$$

4. ***Percentage elongation***

We know that percentage elongation

$$= \frac{L - l}{L} = \frac{80 - 60}{80} = 0.25 \text{ or } 25\%$$

Working Stress

When designing machine parts, it is desirable to keep the stress lower than the maximum or ultimate stress at which failure of the material takes place. This stress is known as the ***working stress*** or ***design stress.*** It is also known as ***safe*** or ***allowable stress.***

Note : By failure it is not meant actual breaking of the material. Some machine parts are said to fail when they have plastic deformation set in them, and they no more perform their function satisfactory.

Factor of Safety

It is defined, in general, as the **ratio of the maximum stress to the working stress.** Mathematically,

$$\text{Factor of safety} = \frac{\text{Maximum stress}}{\text{Working or design stress}}$$

In case of ductile materials *e.g.* mild steel, where the yield point is clearly defined, the factor of safety is based upon the yield point stress. In such cases,

$$\text{Factor of safety} = \frac{\text{Yield point stress}}{\text{Working or design stress}}$$

In case of brittle materials *e.g.* cast iron, the yield point is not well defined as for ductile materials. Therefore, the factor of safety for brittle materials is based on ultimate stress.

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working or design stress}}$$

This relation may also be used for ductile materials.

Poisson's Ratio

It has been found experimentally that when a body is stressed within elastic limit, the lateral strain bears a constant ratio to the linear strain, Mathematically,

$$\frac{\text{Lateral strain}}{\text{Linear strain}} = \text{Constant}$$

This constant is known as **Poisson's ratio** and is denoted by $1/m$ or μ.

Problem(2):- A shaft is transmitting 97.5 kW at 180 r.p.m. If the allowable shear stress in the material is 60 MPa, find the suitable diameter for the shaft. The shaft is not to twist more that 1° in a length of 3 metres. Take C = 80 GPa.

Solution. Given : $P = 97.5$ kW $= 97.5 \times 10^3$ W ; $N = 180$ r.p.m. ; $\tau = 60$ MPa = 60 N/mm2 ; $\theta = 1° = \pi / 180 = 0.0174$ rad ; $l = 3$ m = 3000 mm ; $C = 80$ GPa $= 80 \times 10^9$ N/m^2 $= 80 \times 10^3$ N/mm^2

Let T = Torque transmitted by the shaft in N-m, and

d = Diameter of the shaft in mm.

The power transmitted by the shaft (P),

$$97.5 \times 10^3 = \frac{2\,\pi\,N.T}{60} = \frac{2\pi \times 180 \times T}{60} = 18.852\,T$$

$$T = 97.5 \times 10^3/18.852 = 5172 \text{ N-m} = 5172 \times 10^3 \text{ N-mm}$$

Now let us find the diameter of the shaft based on the strength and stiffness.

1. ***Considering strength of the shaft***

We know that the torque transmitted (T),

$$5172 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\,d^3$$

$$d^3 = 5172 \times 10^3/11.78 = 439 \times 10^3 \quad \text{or} \quad d = 76 \text{ mm}$$

2. *Considering stiffness of the shaft*

Polar moment of inertia of the shaft,

$$J = \frac{\pi}{32} \times d^4 = 0.0982\, d^4$$

$$\frac{T}{J} = \frac{C.\theta}{l}$$

$$\frac{5172 \times 10^3}{0.0982\, d^4} = \frac{80 \times 10^3 \times 0.0174}{3000} \quad \text{or} \quad \frac{52.7 \times 10^6}{d^4} = 0.464$$

$$d^4 = 52.7 \times 10^6/0.464 = 113.6 \times 10^6 \quad \text{or} \quad d = 103 \text{ mm}$$

Taking larger of the two values, we shall provide *d* = 103 say 105 mm**.**

<u>THEORIES OF FAILURE</u>

Describe various types of theories of failure (OR) state and explain various theories of failure under static loading (OR) Explain briefly the various theories of failure

A machine member is subjected to combined loading, i.e bending and torsion. It is not possible to decide which combination of normal stress (bending stress due to bending or shear stress due to torsion) causes the failure of the member.

In order to predict the failure under combined loads, failure of the member.

1. Maximum principal (or normal) stress theory (also known as Rankine's theory).

2. Maximum shear stress theory (also known as Guest's or Tresca's theory).

3. Maximum principal (or normal) strain theory (also known as Saint Venant theory).

4. Maximum strain energy theory (also known as Haigh's theory).

5. Maximum distortion energy theory (also known as Hencky and Von Mises theory).

<u>1.Maximum Principal or Normal Stress Theory (Rankine's Theory)</u>

According to this theory, the failure occurs whenever the maximum principal stress induced in the machine component becomes equal to the strength.

The maximum principal or normal stress (σ_{t1}) in a bi-axial stress system is given by

$$\sigma_{t1} = \frac{\sigma_{yt}}{F.S.}, \text{ for ductile materials}$$

$$= \frac{\sigma_u}{F.S.}, \text{ for brittle materials}$$

σ_{yt} = Yield point stress in tension as determined from simple tension test, and

σ_u = Ultimate stress.

This theory is best suited for brittle materials.

2.Maximum Shear Stress Theory (Guest's or Tresca's Theory)

According to this theory, the failure occurs whenever the maximum shear stress induced in the component becomes equal to the maximum shear stress in a tension of test specimen. When the specimen begins to yield.

$$\tau_{max} = \tau_{yt}/F.S. \quad \ldots(i)$$

τ_{max} = Maximum shear stress in a bi-axial stress system,

τ_{yt} = Shear stress at yield point as determined from simple tension test, and

$F.S.$ = Factor of safety.

Since the shear stress at yield point in a simple tension test is equal to one-half the yield stress in tension, therefore the equation **(*i*)** may be written as

$$\tau_{max} = \frac{\sigma_{yt}}{2 \times F.S.}$$

This theory is best suited for ductile materials.

3.Maximum Principal Strain Theory (Saint Venant's Theory)

According to this theory, the failure occurs whenever the maximum strain in the component becomes equal to the strain in the tension test specimen when yielding begins.

The maximum principal (or normal) strain in a bi-axial stress system is given by

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m\,.\,E}$$

∴ According to the above theory,

$$\varepsilon_{max} = \frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{m\,.\,E} = \varepsilon = \frac{\sigma_{yt}}{E \times F.S.} \quad \ldots(i)$$

σ_{t1} and σ_{t2} = Maximum and minimum principal stresses in a bi-axial stress system,

ε = Strain at yield point as determined from simple tension test,

$1/m$ = Poisson's ratio,

E = Young's modulus, and

$F.S.$ = Factor of safety.

From equation (*i*), we may write that

$$\sigma_{t1} - \frac{\sigma_{t2}}{m} = \frac{\sigma_{yt}}{F.S.}$$

This theory is not used, in general, because it only gives reliable results in particular cases.

4.Maximum Strain Energy Theory (Haigh's Theory)

According to this theory, the failure occurs when strain energy stored per unit volume of the stressed element becomes equal to the strain energy stored per unit volume in the tension test specimen at the yield point.

We know that strain energy per unit volume in a bi-axial stress system,

$$U_1 = \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right]$$

and limiting strain energy per unit volume for yielding as determined from simple tension test,

$$U_2 = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

According to the above theory, $U_1 = U_2$.

$$\therefore \quad \frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory may be used for ductile materials.

5.Maximum Distortion Energy Theory (Hencky and Von Mises Theory)

According to this theory, the failure occurs when the strain energy of distortion per unit volume of the component becomes equal to the strain energy of distortion per unit volume of the tension test specimen.

Mathematically, the maximum distortion energy theory for yielding is expressed as

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

This theory is mostly used for ductile materials in place of maximum strain energy theory.

Note: The maximum distortion energy is the difference between the total strain energy and the strain energy due to uniform stress.

Problem(3):- *The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Find the diameter of bolt required according to 1. Maximum principal stress theory; 2. Maximum shear stress theory; 3. Maximum principal strain theory; 4. Maximum strain energy theory; and 5. Maximum distortion energy theory. Take permissible tensile stress at elastic limit = 100 MPa and poisson's ratio = 0.3*

Given data : P_{t1} = 10 kN ; P_s = 5 kN ; $\sigma_{t(el)}$ = 100 MPa = 100 N/mm^2 ; $1/m$ = 0.3

Let d = Diameter of the bolt in mm.

∴ Cross-sectional area of the bolt,

$$A = \frac{\pi}{4} \times d^2 = 0.7854\, d^2 \text{ mm}^2$$

We know that axial tensile stress,

$$\sigma_1 = \frac{P_{t1}}{A} = \frac{10}{0.7854\, d^2} = \frac{12.73}{d^2} \text{ kN/mm}^2$$

and transverse shear stress,

$$\tau = \frac{P_s}{A} = \frac{5}{0.7854\, d^2} = \frac{6.365}{d^2} \text{ kN/mm}^2$$

1. *According to maximum principal stress theory*

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{12.73}{2\,d^2} + \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} + \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 + \frac{1}{2}\sqrt{4+4}\right] = \frac{15.365}{d^2} \text{ kN/mm}^2 = \frac{15\,365}{d^2} \text{ N/mm}^2$$

According to maximum principal stress theory,

$$\sigma_{t1} = \sigma_{t(el)} \quad \text{or} \quad \frac{15\,365}{d^2} = 100$$

$$\therefore \qquad d^2 = 15\,365/100 = 153.65 \quad \text{or} \quad d = 12.4 \text{ mm}$$ **Ans.**

2. *According to maximum shear stress theory*

We know that maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad ...(\because \sigma_2 = 0)$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right] = \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{9}{d^2} \text{ kN/mm}^2 = \frac{9000}{d^2} \text{ N/mm}^2$$

According to maximum shear stress theory,

$$\tau_{max} = \frac{\sigma_{t(el)}}{2} \quad \text{or} \quad \frac{9000}{d^2} = \frac{100}{2} = 50$$

$$\therefore \quad d^2 = 9000 / 50 = 180 \quad \text{or} \quad d = 13.42 \text{ mm } \textbf{Ans.}$$

3. ***According to maximum principal strain theory***

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{15\,365}{d^2}$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{12.73}{2\,d^2} - \frac{1}{2}\left[\sqrt{\left(\frac{12.73}{d^2}\right)^2 + 4\left(\frac{6.365}{d^2}\right)^2}\right]$$

$$= \frac{6.365}{d^2} - \frac{1}{2} \times \frac{6.365}{d^2}\left[\sqrt{4+4}\right]$$

$$= \frac{6.365}{d^2}\left[1 - \sqrt{2}\right] = \frac{-2.635}{d^2} \text{ kN/mm}^2$$

$$= \frac{-2635}{d^2} \text{ N/mm}^2$$

We know that according to maximum principal strain theory,

$$\frac{\sigma_{t1}}{E} - \frac{\sigma_{t2}}{mE} = \frac{\sigma_{t(el)}}{E} \quad \text{or} \quad \sigma_{t1} - \frac{\sigma_{t2}}{m} = \sigma_{t(el)}$$

$$\therefore \quad \frac{15\,365}{d^2} + \frac{2635 \times 0.3}{d^2} = 100 \quad \text{or} \quad \frac{16\,156}{d^2} = 100$$

$$d^2 = 16\,156 / 100 = 161.56 \quad \text{or} \quad d = 12.7 \text{ mm } \textbf{Ans.}$$

4. ***According to maximum strain energy theory***

We know that according to maximum strain energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1} \times \sigma_{t2}}{m} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} \times 0.3 = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{24.3 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{2430}{d^4} = 1 \quad \text{or} \quad \frac{26\,724}{d^4} = 1$$

$$d^4 = 26\,724 \quad \text{or} \quad d = 12.78 \text{ mm} \quad \textbf{Ans.}$$

5. *According to maximum distortion energy theory*

According to maximum distortion energy theory,

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - 2\sigma_{t1} \times \sigma_{t2} = [\sigma_{t(el)}]^2$$

$$\left[\frac{15\,365}{d^2}\right]^2 + \left[\frac{-2635}{d^2}\right]^2 - 2 \times \frac{15\,365}{d^2} \times \frac{-2635}{d^2} = (100)^2$$

$$\frac{236 \times 10^6}{d^4} + \frac{6.94 \times 10^6}{d^4} + \frac{80.97 \times 10^6}{d^4} = 10 \times 10^3$$

$$\frac{23\,600}{d^4} + \frac{694}{d^4} + \frac{8097}{d^4} = 1 \quad \text{or} \quad \frac{32\,391}{d^4} = 1$$

$$d^4 = 32\,391 \quad \text{or} \quad d = 13.4 \text{ mm} \quad \textbf{Ans.}$$

Problem(4):-*A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of bending moment 10 kN-m and a torsional moment 30 kN-m. Determine the diameter of the shaft using two different theories of failure, and assuming a factor of safety of 2. Take E = 210GPa and poisson's ratio = 0.25.*

Given data : σ_{yt} = 700 MPa = 700 N/mm^2 ; M = 10 kN-m = 10 × 10^6 N-mm ; T = 30 kN-m = 30 × 10^6 N-mm ; $F.S.$ = 2 ; E = 210 GPa = 210 × 10^3 N/mm^2 ; 1/m = 0.25

Let d = Diameter of the shaft in mm.

First of all, let us find the maximum and minimum principal stresses.

We know that section modulus of the shaft

$$Z = \frac{\pi}{32} \times d^3 = 0.0982\, d^3 \text{ mm}^3$$

$\therefore$ Bending (tensile) stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{10 \times 10^6}{0.0982\, d^3} = \frac{101.8 \times 10^6}{d^3} \text{ N/mm}^2$$

and shear stress due to torsional moment,

$$\tau = \frac{16\, T}{\pi\, d^3} = \frac{16 \times 30 \times 10^6}{\pi\, d^3} = \frac{152.8 \times 10^6}{d^3} \text{ N/mm}^2$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1 + \sigma_2}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{101.8 \times 10^6}{2d^3} + \frac{1}{2}\left[\sqrt{\left(\frac{101.8 \times 10^6}{d^3}\right)^2 + 4\left(\frac{152.8 \times 10^6}{d^3}\right)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{1}{2} \times \frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{50.9 \times 10^6}{d^3} + \frac{161 \times 10^6}{d^3} = \frac{211.9 \times 10^6}{d^3} \text{ N/mm}^2$$

and minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1 + \sigma_2}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1 - \sigma_2)^2 + 4\,\tau^2}\right]$$

$$= \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] \qquad \ldots(\because \sigma_2 = 0)$$

$$= \frac{50.9 \times 10^6}{d^3} - \frac{161 \times 10^6}{d^3} = \frac{-110.1 \times 10^6}{d^3} \text{ N/mm}^2$$

Let us now find out the diameter of shaft (*d*) by considering the maximum shear stress theory and maximum strain energy theory.

1. ***According to maximum shear stress theory***

We also know that according to maximum shear stress theory,

$$\tau_{max} = \tfrac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{1}{2}\left[\sqrt{\left(\frac{101.8\times10^6}{d^3}\right)^2 + 4\left(\frac{152.8\times10^6}{d^3}\right)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\left[\sqrt{(101.8)^2 + 4\,(152.8)^2}\right]$$

$$= \frac{1}{2}\times\frac{10^6}{d^3}\times 322 = \frac{161\times10^6}{d^3}\ \text{N/mm}^2$$

2. ***According to maximum strain energy theory***

We know that according to maximum strain energy theory,

$$\frac{1}{2E}\left[(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m}\right] = \frac{1}{2E}\left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \frac{2\,\sigma_{t1}\times\sigma_{t2}}{m} = \left(\frac{\sigma_{yt}}{F.S.}\right)^2$$

$$\left[\frac{211.9\times10^6}{d^3}\right]^2 + \left[\frac{-110.1\times10^6}{d^3}\right]^2 - 2\times\frac{211.9\times10^6}{d^3}\times\frac{-110.1\times10^6}{d^3}\times 0.25 = \left(\frac{700}{2}\right)^2$$

$$\frac{44\ 902\times10^{12}}{d^6} + \frac{12\ 122\times10^{12}}{d^6} + \frac{11\ 665\times10^{12}}{d^6} = 122\ 500$$

$$\frac{68\ 689\times10^{12}}{d^6} = 122\ 500$$

$$d^6 = 68\ 689\times10^{12}/122\ 500 = 0.5607\times10^{12}$$

$$d = 90.8\ \text{mm}$$

Problem(5):- *A mild steel shaft of 50 mm diameter is subjected to a bending moment of 2000N-m and a torque T. If the yield point of the steel in tension is 200 MPa, find the maximum value of this torque without causing yielding of the shaft according to 1. the maximum principal stress; 2. the maximum shear stress; and 3. the maximum distortion strain energy theory of yielding*.

Given data : d = 50 mm ; M = 2000 N-m = 2 × 10^6 N-mm ; σ_{yt} = 200 MPa = 200 N/mm²

Let T = Maximum torque without causing yielding of the shaft, in N-mm.

1. *According to maximum principal stress theory*

We know that section modulus of the shaft,

$$Z = \frac{\pi}{32} \times d^3 = \frac{\pi}{32}(50)^3 = 12\,273 \text{ mm}^3$$

∴ Bending stress due to the bending moment,

$$\sigma_1 = \frac{M}{Z} = \frac{2 \times 10^6}{12\,273} = 163 \text{ N/mm}^2$$

and shear stress due to the torque,

$$\tau = \frac{16\,T}{\pi\,d^3} = \frac{16\,T}{\pi\,(50)^3} = 0.0407 \times 10^{-3}\,T \text{ N/mm}^2$$

$$\ldots \left[\because T = \frac{\pi}{16} \times \tau \times d^3\right]$$

We know that maximum principal stress,

$$\sigma_{t1} = \frac{\sigma_1}{2} + \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} + \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} \text{ N/mm}^2$$

Minimum principal stress,

$$\sigma_{t2} = \frac{\sigma_1}{2} - \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right]$$

$$= \frac{163}{2} - \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= 81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} \text{ N/mm}^2$$

and maximum shear stress,

$$\tau_{max} = \frac{1}{2}\left[\sqrt{(\sigma_1)^2 + 4\,\tau^2}\right] = \frac{1}{2}\left[\sqrt{(163)^2 + 4\,(0.0407 \times 10^{-3}\,T)^2}\right]$$

$$= \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\ \text{N/mm}^2$$

We know that according to maximum principal stress theory,
...(Taking *F.S.* = 1)

$$\sigma_{t1} = \sigma_{yt}$$

$$\therefore\ 81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} = 200$$

$$6642.5 + 1.65 + 10^{-9}\,T^2 = (200 - 81.5)^2 = 14\,042$$

$$T^2 = \frac{14\,042 - 6642.5}{1.65 \times 10^{-9}} = 4485 \times 10^9$$

$$T = 2118 \times 10^3\ \text{N-mm} = 2118\ \text{N-m}$$

2. ***According to maximum shear stress theory***

We know that according to maximum shear stress theory,

$$\tau_{max} = \tau_{yt} = \frac{\sigma_{yt}}{2}$$

$$\therefore\ \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2} = \frac{200}{2} = 100$$

$$6642.5 + 1.65 \times 10^{-9}\,T^2 = (100)^2 = 10\,000$$

$$T^2 = \frac{10\,000 - 6642.5}{1.65 \times 10^{-9}} = 2035 \times 10^9$$

$$T = 1426 \times 10^3\ \text{N-mm} = 1426\ \text{N-m}$$

3. ***According to maximum distortion strain energy theory***

We know that according to maximum distortion strain energy theory

$$(\sigma_{t1})^2 + (\sigma_{t2})^2 - \sigma_{t1} \times \sigma_{t2} = (\sigma_{yt})^2$$

$$\left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2 + \left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]^2$$

$$- \left[81.5 + \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right]\left[81.5 - \sqrt{6642.5 + 1.65 \times 10^{-9}\,T^2}\right] = (200)^2$$

$$2\left[(81.5)^2 + 6642.5 + 1.65\times10^{-9}\,T^2\right] - \left[(81.5)^2 - 6642.5 + 1.65\times10^{-9}\,T^2\right] = (200)^2$$

$$(81.5)^2 + 3\times6642.5 + 3\times1.65\times10^{-9}\,T^2 = (200)^2$$

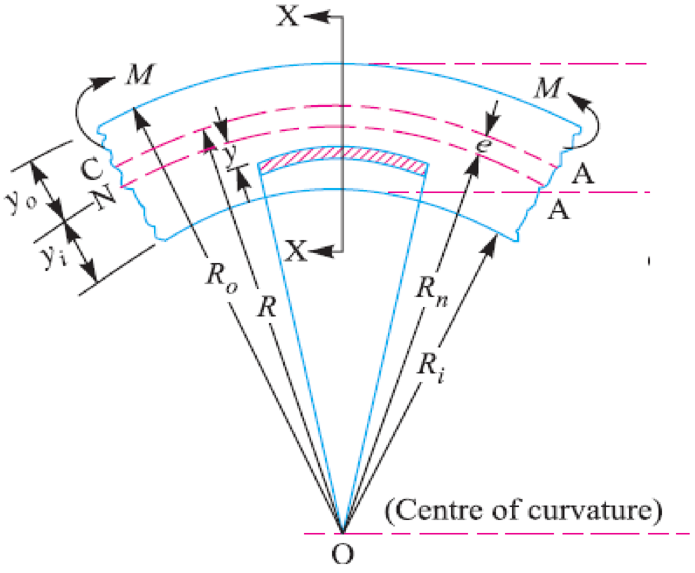
$$26\,570 + 4.95\times10^{-9}\,T^2 = 40\,000$$

$$T^2 = \frac{40\,000 - 26\,570}{4.95\times10^{-9}} = 2713\times10^9$$

$$T = 1647\times10^3 \text{ N-mm} = 1647 \text{ N-m}$$

S. No	Table	Equation
1	Stress	$\sigma = \frac{P}{A}$ $P = Force\ or\ load\ acting\ on\ a\ boby$ $A = Cross\ sectional\ Area\ of\ the\ body$
2	Strain	$\varepsilon = \frac{\delta l}{l}$ $\delta l = Change\ in\ length\ of\ body$ $l = original\ length\ of\ the\ body$
3	Young Modulus or Modulus of Elasticity (E)	$\sigma\ \alpha\ \varepsilon$ $\sigma = E.\varepsilon......E = \frac{\sigma}{\varepsilon}$ $E = \frac{P.l}{A.\delta l}.....\delta l = \frac{P.l}{A.E}$
4	Shear Stress	$\tau = \frac{P}{A}$ $For\ single\ Shear,\ \ A = \frac{\pi}{4}.d^2$ $For\ Double\ Shear, A = 2.\frac{\pi}{4}.d^2$
5	Shear Modulus or Modulus of Rigidity	$\tau\ \alpha\ \phi$ $\tau = C\ \ \phi$ $\phi = Shear\ \ Strain$ $C = Modulus\ of\ Rigidity$
6	Factor of Safety	$F.S = \frac{Yeild\ Point\ stress}{Working\ or\ Design\ stress}$ $F.S = \frac{Ultimate\ stress}{Working\ or\ Design\ stress}$

7	Impact stress	$$\sigma_i = \frac{W}{A}\left[1+\sqrt{1+\frac{2.h.A.E}{W.l}}\right]$$
8	Torsional shear Stress	$$\frac{\tau}{r} = \frac{T}{J} = \frac{G.\theta}{l}$$
	where τ = Torsional shear stress induced at the outer surface of the shaft or maximum shear stress, r = Radius of the shaft, T = Torque or twisting moment, J = Second moment of area of the section about its polar axis or polar moment of inertia, C = Modulus of rigidity for the shaft material, l = Length of the shaft, and θ = Angle of twist in radians on a length l.	
9	For Solid Shaft of Diameter d	$$\frac{T}{J} = \frac{\tau}{r} \quad \text{or} \quad T = \tau \times \frac{J}{r}$$ $$J = I_{XX} + I_{YY} = \frac{\pi}{64} \times d^4 + \frac{\pi}{64} \times d^4 = \frac{\pi}{32} \times d^4$$ $$T = \tau \times \frac{\pi}{32} \times d^4 \times \frac{2}{d} = \frac{\pi}{16} \times \tau \times d^3$$
10	For Hollow Shaft of Diameter d	$$J = \frac{\pi}{32}\,[(d_o)^4 - (d_i)^4] \text{ and } r = \frac{d_o}{2}$$ $$T = \tau \times \frac{\pi}{32}\,[(d_o)^4 - (d_4)^4] \times \frac{2}{d_o} = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right]$$ $$= \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \quad \ldots \left(\text{Substituting, } k = \frac{d_i}{d_o}\right)$$
11	Power transmitted by the shaft	$$P = \frac{2\,\pi\,N.T}{60} = T.\omega \quad \ldots \left(\because\ \omega = \frac{2\,\pi\,N}{60}\right)$$ where T = Torque transmitted in N-m, and ω = Angular speed in rad/s.
12	Bending Stress in straight Beam	$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$ where M = Bending moment acting at the given section, σ = Bending stress, I = Moment of inertia of the cross-section about the neutral axis, y = Distance from the neutral axis to the extreme fibre, E = Young's modulus of the material of the beam, and R = Radius of curvature of the beam.

13	Bending Stress in straight Beam	$\sigma = \frac{M}{I} \times y = \frac{M}{I/y} = \frac{M}{Z}$ The ratio *I/y* is known as ***section modulus*** and is denoted by *Z*.
14	Bending stress in Curved Beams	 $e = R - R_n,\ Y_i = R_n - R_i,\ Y_o = R_o - R_n,$
15	Resultant Bending Stress	$\sigma_{Ri} = \sigma_t + \sigma_{bi}$ $\sigma_{Ro} = \sigma_t + \sigma_{bo}$
16	Axial Direct Stress	$\sigma_t = \frac{W}{A} ...A - Area\ of\ Cross\ Section$
17	Maximum Bending stress at the inside fibre	$\sigma_{bi} = \frac{M \cdot y_i}{A \cdot e \cdot R_i}$ where y_i = Distance from the neutral axis to the inside fibre = $R_n - R_i$, and R_i = Radius of curvature of the inside fibre.
18	Maximum Bending stress at the outside fibre	$\sigma_{bo} = \frac{M \cdot y_o}{A \cdot e \cdot R_o}$ where y_o = Distance from the neutral axis to the outside fibre = $R_o - R_n$, and R_o = Radius of curvature of the outside fibre.