Chp-2

* Transportation problem :-

1)

	A	B	C	D	supply
1	3	1	7	4	300
2	2	6	5	9	400
3	8	3	3	2	500
Demand	250	350	400	200	1200 / 1200

Sol Supply = Demand

∴ This is balanced problem

(a) North west corner method :-

	A	B	C	D		
1	250 3	1	7	4	3~~0~~0	5~~0~~ → 2
2	2	6	5	9	4~~0~~0	1~~0~~0 → 4
3	8	3	3	2	5~~0~~0	2~~0~~0 → 6
	2~~5~~0	3~~5~~0	4~~0~~0	2~~0~~0		
	1	3~~0~~0	3~~0~~0			
		3	5			

	A	B	C	D
1	250 3	50 1	7	4
2	2	300 6	100 5	9
3	8	3	300 3	200 2

Total cost :-

$(250 \times 3) + (50 \times 1) +$
$(300 \times 6) + (100 \times 5) +$
$(300 \times 3) + (200 \times 2)$

$= 750 + 50 + 1800$
$+ 500 + 900 + 400$

$= 4400$ RS.

b) Least cost method

	A	B	C	D	SUPPLY
1	3	1	7	4	~~300~~ → 1
2	2	6	5 (6)	9	~~400~~ ~~150~~
3	8	3	3 (5)	2	~~500~~ ~~300~~ ~~250~~
De	~~250~~	~~350~~ ~~50~~	~~400~~ ~~150~~	~~200~~	
	(2)	(4)		(3)	

	A	B	C	D
1	3	300 1	7	4
2	250 2	6	150 5	9
3	8	50 3	250 3	200 2

T.C = (300×1) + (250×2) + (150×5) + (50×3) + (250×3) + (200×2)

T.C ⇒ 300 + 500 + 750 + 150 + 750 + 400

T.C ⇒ 2850 RS.

c) Vogel's approximation method :-

	A (1)	B	C	D	SUPPLY			
1	3	300 1 (3)	7	4	~~300~~ → (2)	2	3	
2	250 2	6	5	9	~~400~~ 150	3	1	1
3	8	50 3	3	200 2	~~500~~ 450	1	1	1
Dem	~~250~~	~~350~~ ~~50~~	400	200				
	1	2	2	2				
		2	2	2				
		3	2	7				

c) Vogel's approximate method :-

	A	B	C	D					
1	3	$\overset{300}{1}$	7	4	300	2	3		
2	$\overset{250}{2}$	6	$\overset{150}{5}$	9	~~400~~ 150	3	1	1	1
3	8	$\overset{50}{3}$	$\overset{250}{3}$	$\overset{200}{2}$	~~500~~ 300	1	1	1	0
	250	~~350~~ 50	~~400~~ 150	~~200~~	250				
	1	2	2	2					
		2	2	2					
		3	2	7					
		3	2						
			2						
			2						

$$T.C = (2 \times 250) + (1 \times 300) + (2 \times 200) + (3 \times 50) + (3 \times 250) + (5 \times 150) = 2850$$

2) modi method

	V_1	V_2	V_3	V_4
u_1	$\overset{200}{11}$	$\overset{50}{13}$	17	14
u_2	16	$\overset{175}{18}$	$\overset{125}{14}$	10
u_3	21	24	$\overset{275}{13}$	$\overset{125}{10}$

$$T.C = (11 \times 200) + (13 \times 50) + (18 \times 175) + (14 \times 125) + (13 \times 275) + (10 \times 125)$$

$$= 12575 \text{ Rs}$$

Sol Step-1

allocating cell = 6

$m+n-1 = 3+4-1 = 6$

$\therefore$ allocating cell = $m+n-1$

it is not degenerate type of problem.

Step-2

a) calculate $u_i + v_j$ values for occupied cells.

assume $u_1 = 0$

$u_1 + v_1 = 11 \Rightarrow 0 + v_1 = 11 \Rightarrow v_1 = 11$

$u_1 + v_2 = 13 \Rightarrow 0 + v_2 = 13 \Rightarrow v_2 = 13$

$u_2 + v_2 = 18 \Rightarrow u_2 + 13 = 18 \Rightarrow u_2 = 5$

$u_2 + v_3 = 14 \Rightarrow 5 + v_3 = 14 \Rightarrow v_3 = 9$

$u_3 + v_3 = 13 \Rightarrow u_3 + 9 = 13 \Rightarrow u_3 = 4$

$u_3 + v_4 = 10 \Rightarrow 4 + v_4 = 10 \Rightarrow v_4 = 6$

b) calculate Δ_{ij} values for unoccupied cells:-

$\Delta_{ij} = c_{ij} - (u_i + v_j)$

$\Delta_{13} = c_{13} - (u_1 + v_3) \Rightarrow 17 - (0+9) \Rightarrow 17 - 9 = 8$

$\Delta_{14} = c_{14} - (u_1 + v_4) \Rightarrow 14 - (0+6) \Rightarrow 14 - 6 = 8$

$\Delta_{21} = c_{21} - (u_2 + v_1) \Rightarrow 16 - (5+11) \Rightarrow 16 - 16 = 0$

$\Delta_{24} = c_{24} - (u_2 + v_4) \Rightarrow 10 - (5+6) \Rightarrow 10 - 11 = -1$

$\Delta_{31} = c_{31} - (u_3 + v_1) \Rightarrow 21 - (4+11) = 21 - 15 = 6$

$\Delta_{32} = c_{32} - (u_3 + v_2) \Rightarrow 24 - (4+13) = 24 - 17 = 7$

$\therefore$ $\Delta_{ij} > 0$ & atleast one negitive,

solution is not optimal & not unique

<u>Note</u> :-

<u>case-1</u> :- $\Delta_{ij} > 0$, current solution is optimal & unique.

<u>case-2</u> :- $\Delta_{ij} > 0$, & atleast one zero, current solution is optimal & not unique

<u>case-3</u> :- $\Delta_{ij} > 0$, & atleast one negitive, solution is not optimal & not unique.

<u>step-3</u> :- (looping) :-

$$\begin{array}{cccc} 11^{200} & 13^{50\,-} & 17^{+} & 14 \\ 16 & 18^{175\,+} & 14^{125\,-} & 10 \\ 21 & 24 & 13^{273} & 10^{125} \end{array} \rightarrow \begin{array}{cc} 13 & 17^{50} \\ 18^{225} & 14^{75} \end{array}$$

$$\rightarrow \left.\begin{array}{cccc} 11^{200} & 13 & 17^{50} & 14 \\ 16 & 18^{225} & 14^{75} & 10 \\ 21 & 24 & 13^{273} & 10^{125} \end{array}\right\} \text{~~New M~~}$$

<u>step-1</u>

allocated cells = 6

$m+n-1 \Rightarrow 4+3-1 = 6$

$\therefore 6 = 6$.

a) calculate $U_i + V_j$ values for occupied ce

$U_1 = 0$ assume

$U_1 + V_1 = 11 \Rightarrow 0 + V_1 = 11 \Rightarrow V_1 = 11$

$U_1 + V_3 = 17 \Rightarrow 0 + V_3 = 17 \Rightarrow V_3 = 17$

$U_2 + V_2 = 18 \Rightarrow -3 + V_2 = 18 \Rightarrow V_2 = 21$

$U_2 + V_3 = 14 \Rightarrow U_2 + 17 = 14 \Rightarrow U_2 = -3$

$U_3 + V_3 = 13 \Rightarrow U_3 + 17 = 13 \Rightarrow U_3 = -4$

$U_3 + V_4 = 10 \Rightarrow +4 + 10 = V_4 \Rightarrow V_4 = 14$

b) calculate Δ_{ij} values for unoccupied cells :-

$\Delta_{ij} = C_{ij} - (U_i + V_j)$

$\Delta_{13} = C_{13} - (U_1 + V_3) = 17 - (0 + 17) = 0$

$\Delta_{14} = C_{14} - (U_1 + V_4) = 14 - (0 + (-20)) = 34$

$\Delta_{21} = C_{21} - (U_2 - V_1) = 16 - (-3 + 11) = 8$

$\Delta_{24} = C_{24} - (U_2 - V_4) = 10 - (-3 + -20) = 33$

$\Delta_{31} = C_{31} - (U_3 + V_1) = 21 - (-4 + 11) = 14$

$\Delta_{32} = C_{32} - (U_3 + V_2) = 24 - (-4 + 21) = 7$

$\Delta_{ij} > 0$

current solution is optimal & unique

#

3) stepping stone method :-

sol

	A	B	C
D	100 5	4	3
E	200 8	100 4	3
F	9	100 7	200 5

$T.C = (5 \times 100) + (8 \times 200) + (4 \times 100) + (7 \times 100) + (5 \times 200)$

$T.C = 4200$

First taking unoccupied cells :-

DB = 4 ; DC = 3 ; EC = 3 ; FA = 9

DB :- operstunity cost :

	A	B	C
D	100 5	+ 4	3
E	200 8 +	100 − 4	3
F	9	100 7	200 5

OC = 4 + 8 − 5 − 4

= $3 \geq 0$

DC :- operstunity cost :

	A	B	C
D	100 − 5	4	+ 3
E	200 + 8	100 − 4	3
F	9	+ 100 7	200 − 5

OC = 3 − 5 + 7 − 4 + 8 − 5

= $4 \geq 0$

EC :- oppestunity cost

	A	B	C
D	100 5	4	3
E	200 8	100 − 4	+ 3
F	9	100 7 +	200 − 5

OC = 3 − 4 + 7 − 5

= $1 \geq 0$

FA :- operstunity cost :-

	A	B	C
D	100 5	4	3
E	200 − 8	+ 100 4	3
F	9	100 − 7	200 5

OC = 9 − 7 + 4 − 8

= $-2 \leq 0$

FA is optimality is not reached so we need to improve the current solution

$+9 \rightarrow 0 + 100 = 100$

$-7 \rightarrow 100 - 100 = 0$

$+4 \rightarrow 100 + 100 = 200$

$-8 \rightarrow 200 - 100 = 100$

New table :-

	A	B	C
D	(100) 5	4	3
E	(100) 8	(200) 4	3
F	(100) 9	7	(200) 5

DB . OC :- $4 - 5 + 8 - 4$

$= 3 \geq 0$

DB → 4 ; DC → 3 ; EC → 3 ; FB → 2

	A	B	C
D	(100) 5	4	3
E	(100) 8	(200) 4	3
F	(100) 9	7	(200) 5

DC : OC :- $3 - 5 + 9 - 5$

$= 2 \geq 0$

	A	B	C
D	(100) 5	4	3
E	(100) 8	(200) 4	3
F	(100) 9	7	(200) 5

EC : OC :- $3 - 8 + 9 - 5$

$= -1 \leq 0$

	A	B	C
D	(100) 5	4	3
E	(100) 8	(200) 4	3
F	(100) 9	7	(200) 5

FB : OC :- $7 - 4 + 8 - 9$

$= 2 \geq 0$

EC is optimality is not reached, so we need to improve the current solution

$+3 \rightarrow 0 + 100 = 100$

$-5 \rightarrow 100 - 100 = 0$

$+9 \rightarrow 100 + 100 = 200$

$-8 \rightarrow 200 - 100 = 100$

<u>New</u> <u>table</u> :-

DB = 4 ; DC = 3 ; EA = 8 ; FB = 7

	A	B	C
D	5 (100)	4	3
E	8	4 (200)	3 (100)
F	9 (200)	7	5 (100)

DB : OC :- $5 - 9 + 5 - 3 + 4$

$= 2 \geq 0$

	A	B	C
D	5 (100)	4	3
E	8	4 (200)	3 (100)
F	9 (200)	7	5 (100)

DC : OC :- $3 - 5 + 9 - 5$

$= 2 \geq 0$

	A	B	C
D	5 (100)	4	3
E	8	4 (200)	3 (100)
F	9 (200)	7	5 (100)

EA :- OC :- $8 - 9 + 5 - 3$

$= 1 \geq 0$

	A	B	C
D	5 (100)	4	3
E	8	4 (200)	3 (100)
F	9 (200)	7	5 (100)

FB :- OC :- $7 - 5 + 3 - 4$

$= 1 \geq 0$

$\therefore$ The optimality is reached.

$TTC = (5 \times 100) + (4 \times 200) + (3 \times 100) + (9 \times 200) + (5 \times 100) = 3900$

4) Degenerative transportation problem:-

	D_1	D_2	D_3	D_4	D_5	a_i
O_1	$^{1}\vert 4$	7	$^{1}\vert 3$	8	$^{2}\vert 2$	4
O_2	$^{7}\vert 1$	4	7	3	8	7
O_3	7	$^{3}\vert 2$	$^{6}\vert 4$	7	7	9
O_4	4	8	$^{\varepsilon}\vert 2$	$^{2}\vert 4$	7	2
b_j	8	3	7	2	2	22 / 22

Sol TTC :- $(4\times1) + (1\times7) + (2\times3) + (3\times1) + (6\times4)$
$+ (4\times2) + (2\times2) = 56$

No. of allocations $= 7$

$m+n-1 \Rightarrow 5+4-1 = 8$

no. of allocations $< (m+n-1)$

$\therefore$ It is degenerative transportation problem.

If we add 'ε'

This value is add unoccupied cell with least no. ε with not closed lope at value is 2

a) find $u_i + v_j$ for occupied cells.

assume $u_1 = 0$

$u_1 + v_1 = 4 \Rightarrow 0 + v_1 = 4 \Rightarrow v_1 = 4$

$u_1 + v_3 = 3 \Rightarrow 0 + v_3 = 3 \Rightarrow v_3 = 3$

$u_1 + v_5 = 2 \Rightarrow 0 + v_5 = 2 \Rightarrow v_5 = 2$

$u_2 + v_1 = 1 \Rightarrow u_2 + 4 = 1 \Rightarrow u_2 = -3$

$u_3 + v_2 = 2 \Rightarrow 1 + v_2 = 2 \Rightarrow v_2 = 1$

$u_4 + v_3 = 2 \Rightarrow u_4 + 3 = 2 \Rightarrow u_4 = -1$

$u_3 + v_3 = 4 \Rightarrow u_3 + 3 = 4 \Rightarrow u_3 = 1$

$u_4 + v_4 = 4 \Rightarrow -1 + v_4 = 4 \Rightarrow v_4 = 5$

b) Δ_{ij} for unoccupied cells; $\Delta_{ij} = c_{ij} - (u_i + v_j)$

$\rightarrow d_{12} = c_{12} - (u_1 + v_2) = 7 - (0 + 1) = 6$

$\rightarrow d_{14} = c_{14} - (u_1 + v_4) = 8 - (0 + 5) = 3$

$\rightarrow d_{22} = c_{22} - (u_2 + v_2) = 4 - (-3 + 1) = 6$

$\rightarrow d_{23} = c_{23} - (u_2 + v_3) = 7 - (-3 + 3) = 7$

$\rightarrow d_{24} = c_{24} - (u_2 + v_4) = 3 - (-3 + 5) = 1$

$\rightarrow d_{25} = c_{25} - (u_2 + v_5) = 8 - (-3 + 2) = 9$

$\rightarrow d_{31} = c_{31} - (u_3 + v_1) = 7 - (1 + 4) = 2$

$\rightarrow d_{35} = c_{35} - (u_3 + v_5) = 7 - (1 + 2) = 4$

$\rightarrow d_{41} = c_{41} - (u_4 - v_1) = 4 - (-1 + 4) = 1$

$\rightarrow d_{42} = c_{42} - (u_4 + v_2) = 8 - (-1 + 1) = 8$

$\rightarrow d_{45} = c_{45} - (u_4 - u_5) = 7 - (-1 + 2) = 6$

$\therefore$ minimum transportation cost

$= (4 \times 1) + (3 \times 1) + (2 \times 2) + (1 \times 7) + (2 \times 3)$
$+ (4 \times 6) + (4 \times 2) = 56$

#

* Assignment Problems :-

1) Hungarians Assignment method

	A	B	C	D	E
I	85	75	65	125	75
II	90	78	66	132	78
III	75	66	57	144	69
IV	80	72	60	120	72
V	76	64	56	112	68

Sol Reduced table - 1 :- (Row - Least)

	I	II	III	IV	V
A	~~85~~ 20	~~75~~ 10	~~65~~ 0	~~125~~ 60	~~75~~ 10
B	24	12	0	66	12
C	18	9	0	57	12
D	20	12	0	60	12
E	20	8	0	56	12

Reduced table - 2 :- (column - Least)

	I	II	III	IV	V
A	2	2	$\cancel{0}$	4	$\boxed{0}$
B	6	4	$\boxed{0}$	10	2
C	$\boxed{0}$	1	$\cancel{0}$	1	2
D	2	4	$\cancel{0}$	4	2
E	2	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$	2

Assigning

1) Row with single zero

B

D

2) column with single zero

I

V

II

III

This is not optimum solution since it is having only - 4 - Assignments worker 5 is not assigned to any Job (or) Job D is not assigned into any worker.

* open value -> -2

* Elements under line -> no change

* Elements under PoI -> + 2

New table :-

Job	A	B	C	D	E
I	2	2	2	4	$\boxed{0}$
II	4	2	$\boxed{0}$	8	~~0~~
III	$\boxed{0}$	1	2	1	2
IV	~~0~~	2	~~0~~	2	~~0~~
V	2	$\boxed{0}$	2	~~0~~	2

Row with single zero

1

3

Column with single zero

B

D

Row with single zero after cancelsetion

2

4

- Open value → -1
- Element under line → No change
- Element under POI → +1

<u>New</u> <u>table</u> :-

	A	B	C	D	E
I	2	1	2	3	$\boxed{0}$
II	4	-1	$\boxed{0}$	7	$\cancel{0}$
III	$\cancel{0}$	$\boxed{0}$	2	$\cancel{0}$	2
IV	$\boxed{0}$	1	$\cancel{0}$	-1	$\cancel{0}$
V	3	$\cancel{0}$	3	$\boxed{0}$	3

Row with single zero :-

I

column with single zero :-

No

Row with single zero :-

IV

This is optimum solution

I	II	III	IV	V	
E	C	B	A	D	
75	66	66	80	112	= 399

#

2) unbalanced assignment problem with Restrictions :-

Job (time in hrs)

		A	B	C	D	E
m/c's	m_1	9	11	15	10	11
	m_2	12	9	-	10	9
	m_3	-	11	14	11	7
	m_4	14	8	12	7	8

Sol

	A	B	C	D	E
m_1	9	11	15	10	11
m_2	12	9	-	10	9
m_3	-	11	14	11	7
m_4	14	8	12	7	8
m_5	0	0	0	0	0 → Dummy row.

By adding m_5 row because no. of rows is equal to no. of columns.

Reduce table - 1 :- (Row - least)

	A	B	C	D	E
m_1	$\boxed{0}$	2	6	1	2
m_2	3	$\boxed{0}$	-	1	$\cancel{0}$
m_3	-	4	7	4	$\boxed{0}$
m_4	7	1	5	$\boxed{0}$	1
m_5	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$

Row with single zero
m_1
m_3
m_4
Column with single zero
C
Row with single zero
m_2

This is optimum solution

$9 + 9 + 0 + 7 + 7 = 32$

3) Assignment problem maximization case :- for sales

Territary (in 000's)

sales man		I	II	III	IV
	A	42	35	28	21
	B	30	25	20	15
	C	30	25	20	15
	D	24	20	16	12

Sol

Note :- maximization into minimization

highest value - remaing value.

	I	II	III	IV
A	0	7	14	21
B	12	17	22	27
C	12	17	22	27
D	18	22	26	30

This is minimization problem.

Reduce table - 1

(Row - least)

	I	II	III	IV
A	0	7	14	21
B	0	5	10	15
C	0	5	10	15
D	0	4	8	12

Reduce table - 2

(column - lowest)

	I	II	III	IV
A	$\boxed{0}$	3	6	9
B	~~0~~	1	2	3
C	~~0~~	1	2	3
D	~~0~~	$\boxed{0}$	~~0~~	~~0~~

Assigning :-

1) Row with single zero

A

2) column with single zero

II

This is not a optimum solution.

New table-1 :-

	I	II	III	IV
A	$\boxed{0}$	2	5	8
B	$\cancel{0}$	$\boxed{0}$	1	2
C	$\cancel{0}$	$\cancel{0}$	1	2
D	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

Assigning

1) Row with single zero

A, B

2) column with single zero

3

3) ~~Row with single zero~~

This is not optimum solution

New table-2

	I	II	III	IV
A	$\boxed{0}$	2	4	7
B	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	1
C	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	1
D	2	1	$\cancel{0}$	$\boxed{0}$

assume

Assigning

1) Row with single zero

A

2) column with single zero

IV

This is multiple optimum solution.

I	II	III	IV
A	B	C	D
42	25	20	12 = 99

∴ max. sales = 99000

4) Travelling sales man problem:-

		To			
		A	B	C	D
From	A	-	46	16	40
	B	41	-	50	40
	C	82	32	-	60
	D	40	40	36	-

Sol Reduced table - 1 :-
(Row - Least)

	A	B	C	D
A	-	30	0	24
B	1	-	10	0
C	50	0	-	28
D	4	4	0	-

Reduce table - 2 :-
(column - Least)

	A	B	C	D
A	-	30	$\boxed{0}$	24
B	$\boxed{0}$	-	10	~~0~~
C	49	$\boxed{0}$	-	28
D	3	4	~~0~~	-

Assigning

1) Row with single zero
A, C
2) Column with sing zero
A

This is not optimal solution

New table

	A	B	C	D
A	-	27	$\boxed{0}$	21
B	~~0~~	-	13	$\boxed{0}$
C	49	$\boxed{0}$	-	28
D	$\boxed{0}$	1	~~0~~	-

Assigning

1) Row with single zero
A, C
2) Column with single zero
D
3) Row with single zero
D

This is optimum solution

A	B	C	D
D	C	A	B
40	32	16	40 = 128

A → D → B → C → A
= 128

#

* sequencing :-

1) n - Jobs 2 m/c's

A book binder has one printing press one Binding the m/c & the manuscripts of a number of d/f books. The Time required performing printing & binding operations for each book (or) shown in below. Determine the order in which the book should be prossed in order to minimise the total time required to turn out all the books.

Book	1	2	3	4	5	6
m_1 processing time(hrs)	30	120	50	20	90	110
m_2 processing time(hrs)	80	100	90	60	30	10

sol Job sequence chart :-

seq :-	1st	2nd	3rd	4th	5th	6th
Job No :-	4	1	3	2	5	6

	machine -1		machine - 2		
Job.No:-	Time in	Time out	Time in	Time out	Idel
4	0 (0+20)	20	20 (20+60)	80	20
1	20 (20+30)	50	80 (80+80)	160	—
3	50 (50+50)	100	160 (160+90)	250	—
2	100 (100+120)	220	250 (250+100)	350	—
5	220 (220+90)	310	350 (350+30)	380	—
6	310 (310+110)	420	420 (420+10)	430	40

min total time elapsed = 430

idle time on m_1 = 10 hrs

idle time on m_2 = 60 hrs

2) N - Jobs 3 M/c's :-

Job	A	B	C	D	E
machine - A	8	10	6	7	11
machine - B	5	6	2	3	4
machine - C	4	9	8	6	5

Sol)

	A	B	C	D	E
m_1	13	16	8	10	15
m_2	9	15	10	9	9

Job - sequence chart :-

Seq. No :- 1^{st} 2^{nd} 3^{rd} 4^{th} 5^{th}

Job No :- C B E A D

Job name	M - A Time in	M - A Time out	M - B Time in	M - B Time out	Idle	M - C Time in	M - C Time out	Idle
C	0 (0+6)	6	6 (6+2)	8	6	8 (8+8)	16	8
B	6 (6+10)	16	16 (16+6)	22	8	22 (22+9)	31	6
E	16 (16+11)	27	27 (27+4)	31	5	31 (31+5)	36	-
A	27 (27+8)	35	35 (35+5)	40	4	40 (40+4)	44	4
D	35 (35+7)	42	42 (42+3)	45	2	45 (45+6)	51	1
					Total 25 + 6 = 31			19

minimum total time elapsed = 51

idle time on A = 9

idle time on B = 31

idle time on C = 19

#

3) 2 - Jobs - m - machines

use the graphical method to minimize the time needed to process the following Jobs on each m/c. find which Job would be done first & also calculated the total time elapsed to complete the Job?

Job - 1	m/c	-	A	B	C	D	E
	seq time	-	3	4	2	6	2

Job - 2	m/c	-	B	C	A	D	E
	seq time	-	5	4	3	2	6

This is Job shop scheduling.

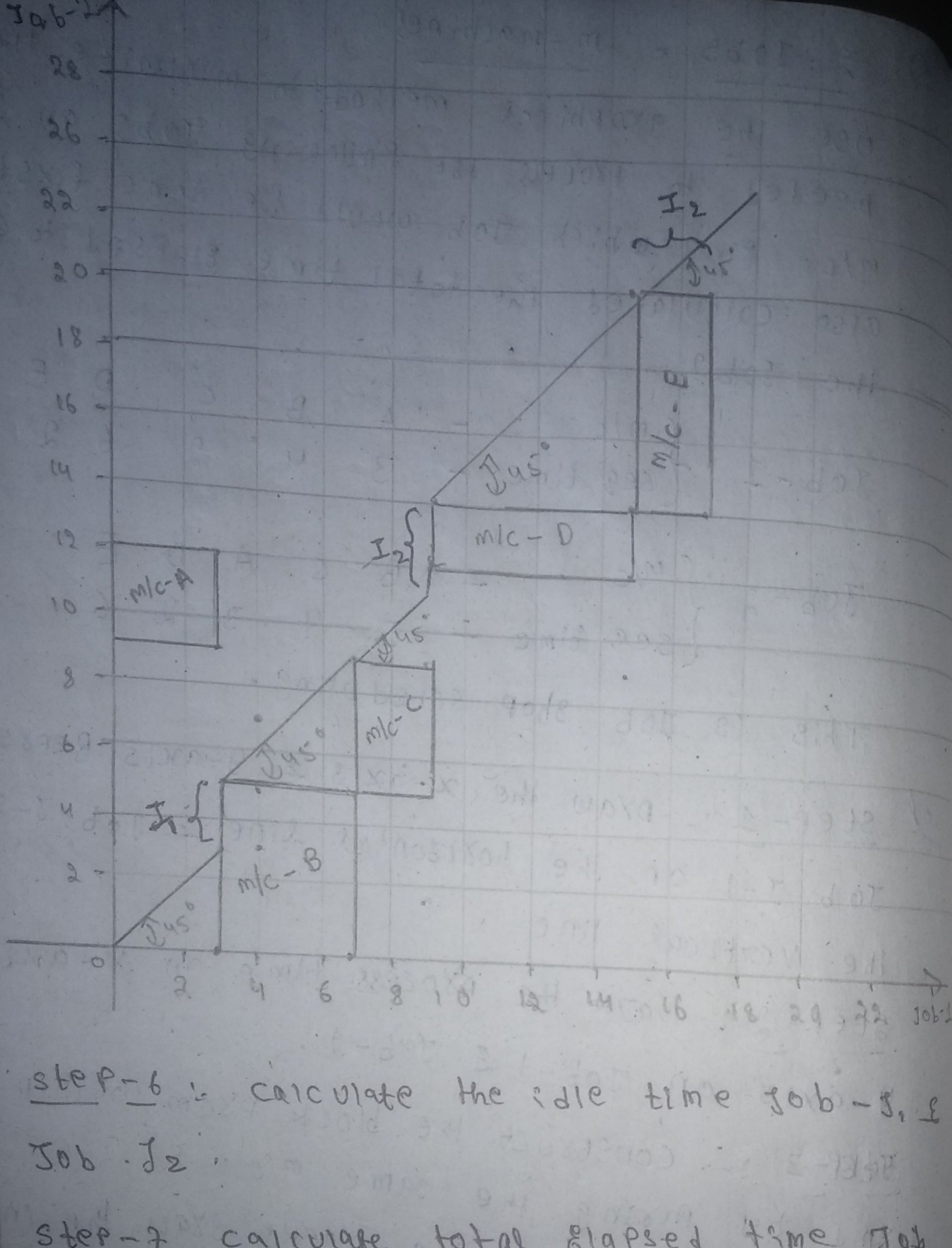
sol step-1 :- Draw the x-axis & y-axis Representing Job J-1 on the horizontal line & Job J-2 on the vertical line.

step-2 :- mark the process time on x-axis & y-axis on Job-1 & Job-2.

step-3 :- construct the block from origin to the end by paring the same m/c.

step-4 :- After drawing blocks draw the inclined line of 45° to the horizontal starting from in time of the individual m/c & ending at out time of the same machine.

step-5 :- After drawing inclined planes the vertical lines indicated idle times of the Job-1. Horizontal line indicates idle time for the Job J_2.



step-6 : calculate the idle time job - J_1 & Job J_2.

step-7 calculate total Elapsed time Job J_1 & Job J_2.

Idel time for J_1 = 2 + 3 = 5

Idle time for J_2 = 2

#

N - Jobs - M machines :-

Job	M_1	M_2	M_3	M_4	M_5	
A	7	5	2	3	9	= 17 & 19
B	6	6	4	5	10	= 21 & 25
C	5	4	5	6	8	= 20 & 23
D	8	3	3	2	6	= 16 & 14

Conditions:-

→ min of machine $M_1 \geq$ max of machine M_2 --- M_{m-1}.

$5 \geq 6$ (x) wrong

→ min of machine $M_2 \geq$ max of machine M_2 --- M_m

$6 \geq 6$ (✓) correct

In this two conditions if any one is satisfy Then we apply Johnson's alogrithm.

let x_1, x_2 be two machines.

$x_1 = m_1 + m_2 + \ldots M_{m-1}$

$x_2 = m_2 + m_3 + \ldots M_m$

	A	B	C	D
$M_1 \to x_1$	17	21	20	16
$M_2 \to x_2$	19	25	23	14

sequency chart :-

sequency No. :- 1, 2, 3, 4

Job No. :- A, B, C, D

Idle time = total Elapsed time - (sum of corresponding machines)

Job	machine - 1		machine - 2		machine - 3		machine - 4		machine - 5	
	Time in	Time out	Time in	Time out	Time in	Time out	Time in	Time out	Time in	Time out
A	0	(0+7) 7	7	(7+5) 12	12	(12+2) 14	14	(14+3) 17	17	(17+9) 26
B	7	(7+5) 12	12	(12+4) 16	16	(16+5) 21	21	(21+6) 27	27	(27+8) 35
C	12	(12+6) 18	18	(18+6) 24	24	(24+4) 28	28	(28+5) 33	35	(35+10) 45
D	18	(18+9) 26	26	(26+3) 29	29	(29+3) 32	33	(33+2) 35	45	(45+6) 51

Idle time on m/c = Total Elapsed time - sum of processing times for corresponding machi

Idle time on M_1 = 51 - 26 = 25 hrs

" " " M_2 = 51 - 19 = 33 hrs

" " " M_3 = 51 - 14 = 37 hrs

" " " M_4 = 51 - 16 = 35 hrs

" " " M_5 = 51 - 33 = 18 hrs

UNIT 2

TRANSPORTATION PROBLEM AND SEQUENCING

Marketed by:

SIA GROUP

PART-A

SHORT QUESTIONS WITH SOLUTIONS

Q1. Discuss transportation problem with an example.

Ans:

Transportation Problem: Transportation problem is a special type of linear programming problem in which goods or products are transferred from sources to destination for minimizing the total cost of transportation. The main aim of the transportation problem is to reduce the transportation cost and fulfill the needs of the individuals located at the destinations. Transportation model includes many activities such as transporting the product from plants to warehouses, warehouses to wholesalers, wholesalers to retailers and retailers to customers. The transportation model is effectively used in scheduling, production, investments, plant location, inventory control, employment scheduling, personnel assignment, product mix problems and so on.

Example: A spare part manufacturing firm has m factories situated in m different cities. The total supply potential of the finished product is utilized by 'n' retailer in different places of the country, then the problem of transportation is to find the transportation pattern that reduces the total cost of transporting the spare parts from various factory locations to various retail dealers.

Q2. Write briefly about degeneracy in transportation problem.

Ans: A solution to transportation problem is said to be a degenerate one when the number of occupied cells i.e., positive allocations, is less than $(m + n - 1)$ where m is the number of rows and n is the number of columns. In such cases, the current solution cannot be improved because it is not possible to draw a closed path for every occupied cell and when using MODI method for finding the optimal solution, u_i, v_j values cannot be computed.

Thus, degeneracy has to be removed to improve the current solution. The degeneracy in the transportation problems may occur at two stages:

(a) Degeneracy at the initial solution stage.

(b) Degeneracy during testing of the optimal solution.

Q3. What do you understand by assignment problem?

Ans: The assignment problem is a particular case of the transportation problem in which the objective is to assign a number of tasks (jobs, origins or sources) to an equal number of facilities (machines, persons or destinations) at a minimum cost (or maximum profit). Suppose, we have 'n' jobs to be performed on 'm' machines (one job to one machine) and our objective is to assign the jobs to the machines at the minimum cost (or maximum profit) under the assumption that each machine can perform one job but with varying degree of efficiencies.

The assignment problem can be in the form of $m \times n$ matrix (C_{ij}) called a cost matrix or effectiveness matrix where C_{ij} is the cost of assigning i^{th} machine to the j^{th} job.

Q4. Distinguish between assignment and transportation problem. Model Paper-III, Q1(ii)

Ans: Assignment problem may be regarded as a special case of transportation model in which the facilities represent the sources and the jobs represent the destinations. The supply amount at each source and the demand amount at each destination exactly equals 1. The cost of transporting (assigning) facility i to job j is c_{ij}. The resulting transportation model is represented below.

Differences Between AP and TP

1. The cost matrix has to be a square matrix in AP whereas, it can be square or non-square matrix in TP.
2. AP is always having a degenerate solution whereas, TP may or may not have degenerate solution.

3. The supply and demand of each resource and destination respectively are taken to be one for AP whereas, in TP they can take any value.

4. The optimal solution for AP would always be such that there will be only one assignment in a given row or column of the matrix whereas in TP, there can be more than one assignment in a given row or column.

Q5. What is un-balanced AP?

Model Paper-II, Q1(i)

Ans: An assignment problem is said to be unbalanced when the number of facilities is not exactly equal to the number of jobs i.e., when the assignment matrix is not a square one. The Hungarian method is used for obtaining optimal solution to AP requires the matrix to be a square one i.e., number of rows = number of columns. If not, the AP is an unbalanced and has to be converted to a balanced one as given below.

- If the number of rows < number of columns, add dummy row(s) with zero cost element in that row(s).
- If the number of rows > number of columns, add dummy column(s) with zero cost element in that column(s).

The cost matrix thus obtained with dummy row(s) or column(s) can be now solved by the Hungarian method in usual manner.

Q6. Distinguish between a travelling salesman problem and a assignment problem.

Ans:

Model Paper-I, Q1(i)

	Travelling Salesman Problem		Assignment Problem
1.	Objective of travelling salesman problem is to minimize the transportation cost in transporting various quantities of single homogeneously commodity.	1.	Objective of an assignment problem is to maximize overall profit or minimize overall cost while assigning number of operations and equal number of operators.
2.	The travelling salesman problem may be symmetrical and asymmetrical form.	2.	The assignment problem is asymmetrical form.
3.	It needs to formulate into sequencing form.	3.	It need not to formulate into sequencing form.
4.	There are n cities, then $(n-1)!$ routes can be generated.	4.	There are n persons n jobs then allocation $n!$.

Q7. What are the various assumptions of sequencing?

Ans:

Assumptions

1. The processing time of each job on each machine duration needs to be predetermined and constant.
2. Any job started for processing must be completed at any cost.
3. A job must be operated only on the basis of predetermined processing order.
4. On one machine only one job has to be processed.
5. It takes very less time for the jobs to get transferred from one machine to another and hence the value can be taken as equal to zero or negligible.
6. An operations must be completed, before its succeeding operation starts.
7. The sequence of processing the machines once prescribed should not be changed i.e., needs to follow no passing rule.
8. Only one machine of each type is available.

Q8. Briefly explain:

(a) Total elapsed time

(b) Idle time.

Ans:

Model Paper-IV, Q1(i)

(a) **Total Elapsed Time:** Time required between starting the first job in the optimum sequence on machine X and completing the last job in the optimum sequence on machine Y.

(b) **Idle Time on Machine X:** It is defined as the time when the last job on the optimum sequence is completed on machine Y.

(or)

Time when the last job on the optimum sequence is completed on machine X.

To find idle time on machine-1 for N-jobs.

Idle time on machine-1 is given by,

$$\begin{bmatrix}\text{Total elapsed time}\end{bmatrix} - \begin{bmatrix}\text{Time when the last job in a} \\ \text{sequence finishes on} \\ \text{machine } M_1\end{bmatrix}$$

To determine idle time on machine-2 for N-jobs.

Idle time on machine-2 is given by,

$$\begin{bmatrix}\text{Time at which the first} \\ \text{job in a sequence starts on} \\ \text{machine } M_2\end{bmatrix} + \sum_{k=2}^{n}\begin{bmatrix}\text{Time when the } k^{th} \text{ job} \\ \text{in a sequence} \\ \text{starts on machine } M_2\end{bmatrix} - \begin{bmatrix}\text{Time when the } (k-1)^{th} \\ \text{job in a sequence finishes} \\ \text{on machine } M_2\end{bmatrix}.$$

Total elapsed time = Time when the n^{th} job in a sequence finishes on machine M_2

$$= \sum_{k=1}^{n} M_{2k} + \sum_{k=1}^{n} I_{2k}$$

Where,

M_{2k} = Time consumed to process k^{th} job on machine M_2.

I_{2k} = Time in which machine M_2 remains idle before starting work in k^{th} job and after processing $(k-1)^{th}$ job.

Q9. What are the various steps involved in the processing of 'n' jobs through 'm' Machines?

Ans:

Processing of 'n' Jobs through 'M' Machines: For processing 'N' jobs through 'M' machines, the first step is to change the 'M' machine problem into 2 machine problem and then scheduling has to to be done by using Johnson and Bellman methods.

Step 1: Assume machine X to be the first machine and machine M as the last machine.

Step 2: The machines which are in between X and M constitute machine Y (second machine). The minimum of X_i or the minimum of 'M_i' or both should be more than the maximum of Y_i which is the maximum element given under all the middle machines.

Step 3: In this step, the values of hypothetical machines G and H are computed as follows,

$G = X +$ Sum of all elements of middle machine except the last machine

$H =$ Sum of all elements of middle machines + Element of last machine

These 'G' and, 'H' are referred to as "fictitious machines".

Step 4: In this step, the optimal sequence is determined by considering the values of machine G and H. The total elapsed time is computed by considering all the machines in the given order.

Step 7: Proceed to cell [4, 4] the magnitude of the allocation is given by,

$$x_{44} = \text{Min}(a_4, b_4 - x_{34})$$
$$= \text{Min}(30, 10 - 5)$$
$$= \text{Min}[30, 5] = 5.$$

	P_1	P_2	P_3	P_4	P_5	Availability
W_1	[20] 10	[5] 2	3	15	9	
W_2	5	[15] 10	[15] 15	2	4	
W_3	15	5	[15] 14	[5] 7	15	
W_4	20	15	13	[5]	[25] 8	25 ~~30~~
				~~5~~	~~25~~	

∴ The proposed solution is feasible, as all the availability and demand constraints are fully satisfied.

Transportation cost $= 10 \times 20 + 2 \times 5 + 10 \times 15 + 15 \times 15 + 14 \times 15 + 7 \times 5 + 0 \times 5 + 8 \times 25 =$ ₹ 1030.

(B) Least Cost Method (LCM)

Q16. Explain least cost method for obtaining an initial basic feasible solution of a transportation problem.

Ans:

Lowest Cost Entry Method (Matrix Minima Method): The main aim of the transportation problems is to minimize the total transportation cost so we should always try to transport from only those cells or routes where the total cost of transportation is lowest. The least cost method of transportation problem for obtaining the initial basic feasible solution gives importance to the minimum unit cost transportation. The least cost method is explained in detail with the help of following example.

Example

					Available
	•(19)	•(30)	•(50)	7(10)	7
	2(70)	•(30)	7(40)	•(60)	9
	3(40)	8(8)	•(70)	7(20)	18
Requirements	5	8	7	14	

We should begin from lowest cost entry (8) which is in the cell (3, 2) and now allocate as much possible i.e., $x_{32} = 8$. Similarly, move on to the next lowest cost i.e., (10) which lies in cell (1, 4) then, allocate $x_{14} = 7$.

No allocation can be made to next lowest cost (19) as it lies in cell (1, 1) and amount from factory F_1 is already used in the cell (1, 4). Then next least cost is (20) in the cell (3, 4), it is possible to allocate $x_{34} = 7$ to complete 7 units in column 4, which are required. Next lowest cost is (30) in cells (2, 2) and (1, 2). Since, requirement of column 2 is already exhausted no allocation is possible. Hence, in this way the required feasible solution is obtained.

This feasible solution gives lower transportation cost, i.e.,

$$= 2(70) + 3(40) + 8(8) + 7(40) + 7(10) + 7(20)$$
$$= 140 + 120 + 64 + 280 + 70 + 140 = 814$$

(C) VAM Method

Q17. Briefly explain the Vogel's approximation method.

Model Paper-II, Q2(a) | May-13, Set-2, Q2(a)

OR

Explain Vogel's approximation method for obtaining an initial basic feasible solution of a transportation problem with example.

Ans: VAM or Vogel's Approximation Method is also known as penalty or regret method. It is basically a heuristic method. Allocation is done on the basis of opportunity cost (penalty) that would have incurred if allocation in certain cells with minimum costs were missed.

The procedure of VAM is as follows,

Step 1: Compute the penalties for each row and column by taking the difference between the smallest and next smallest unit transportation cost in that particular row or column can be determined.

Step 2: Choose the row or column with the largest penalty and allocate as much as possible in the cell having the least cost in the selected row or column.

If a tie occurs in the penalties, select the one which has the minimum cost.

If there is a tie in the minimum cost also, select that row/column which will have maximum possible allocation.

Step 3: Adjust the supply and demand for the allocation made and eliminate (strike out) the row or column in which either supply or demand is exhausted. If both are exhausted, eliminate both the rows and columns.

Step 4: Recompute the penalties for the remaining rows and columns (uncrossed rows and columns).

Step 5: Repeat the procedure until the entire available supply at various sources and entire demand at various destinations are satisfied.

Example

Solve the following transportation problem.

	To			Supply
	4	5	7	25
From	7	7	3	20
	7	3	5	40
Demand	20	25	20	

Solution:

	D_1	D_2	D_3	Supply
O_1	4	5	7	25
O_2	7	7	3	20
O_3	7	3	5	40
Demand	20	25	20	85 / 65

As the demand ≠ supply, the given transportation problem is unbalanced. Hence we have to create a dummy destination D_4 with a demand of 20 unit so as to make demand = supply.

Obtaining IBFS by VAM method,

	D_1	D_2	D_3	D_4	Supply	Penalty
O_1	4	5	7	0 [20]	25 5	4 ←
O_2	7	7	3	0	20	3
O_3	7	3	5	0	40	3
Demand	20	25	20	20	85 / 85	
Penalty	3	2	2	0		

	D_1	D_2	D_3	Supply	Penalty
O_1	4	5	7	5	1
O_2	7	7	3 [20]	20	4 ←
O_3	7	3	5	40	2
Demand	20	25	20	65 / 65	
Penalty	3	2	2		

	D_1	D_2	Supply	Penalty
O_1	4	5	5	1
O_2	7	3 [25]	40 15	4 ←
Demand	20	25	45 / 45	
Penalty	3	2		

	D_1	Supply
O_1	4 [5]	5
O_2	7 [15]	15
	20	

	D_1	D_2	D_3	D_4	
O_1	4 [5]	5	7	0 [20]	25
O_2	7	7	3 [20]	0	20
O_3	7 [15]	3 [25]	5	0	40
	20	25	20	20	

Transportation cost by VAM is,

$= 4\times 5 + 0\times 20 + 3\times 20 + 7\times 15 + 3\times 25$

$= 260$

PROBLEMS

Q18. Obtain the VAM starting solution for the following transportation problem. And solve it,

Depots	Customer				Supply
	1	2	3	4	
1	18	16	8	11	100
2	14	14	8	10	125
3	19	15	16	15	70
4	8	12	19	11	80
Demand	55	130	95	95	375

Solution: May-12, Set-2, Q

Depots ↓	Customer				Supply
	1	2	3	4	
1	18	16	8	11	100
2	14	14	8	10	125
3	19	15	16	15	70
4	8	12	19	11	80
Demand	55	130	95	95	375

Total supply = 100 + 125 + 70 + 80 = 375

Total demand = 55 + 130 + 95 + 95 = 375

Total supply is equal to total demand.

Hence, the given transportation is balanced model.

Obtaining IBFS by VAM method,

Depots ↓	Customer				Supply	RP_1
	1	2	3	4		
1	18	16	8	11	100	3
2	14	14	8	10	125	2
3	19	15	16	15	70	0
4	8 [55]	12	19	11	25 ~~80~~	3
Demand	~~55~~	130	95	95	375	
CP_1	6 ↑	2	0	1		

Depots	2	3	4	Supply	RP_2
1	16	8 [95]	11	5 ~~100~~	3 ←
2	14	8	10	125	2
3	15	16	15	70	0
4	12	19	11	25	1
Demand	130	~~95~~	95	375	
CP_2	2	0	1		

Depots	2	4	Supply	RP_3
1	~~16~~	~~11~~ [5]	5 ~~100~~	~~5~~ ←
2	14	10	125	4
3	15	15	70	0
4	12	11	25 ~~80~~	1
Demand	130	~~95~~ 90	375	
CP_3	2	1		

Depots	2	4	Supply	RP.
2	14	10 $\boxed{90}$	~~125~~ 35	4 ←
3	15	15	70	0
4	12	11	~~80~~ 25	1
Demand	130	~~95~~ 90	375	
CP.	2	1		

Depots	2	Supply
2	14	~~125~~ 35
3	15	70
~~4~~	~~12~~ $\boxed{25}$	~~80~~ 25
Demand	~~130~~ 105	375

Depots	2	Supply
~~2~~	14 $\boxed{35}$	~~125~~ ~~35~~
3	15	70
	~~105~~ 70	

Depots	2	Supply
3	15 $\boxed{70}$	70
Demand	~~105~~ ~~70~~ ~~130~~	375

Depots	1	2	3	4	Supply
1	18	16	8 $\boxed{95}$	11 $\boxed{5}$	100
2	14	14 $\boxed{35}$	8	10 $\boxed{90}$	125
3	19	15 $\boxed{70}$	16	15	70
4	8 $\boxed{55}$	12 $\boxed{25}$	19	11	80
Demand	55	130	95	95	375

Transportation cost by VAM is,

$= (8 \times 95) + (11 \times 5) + (14 \times 35) + (10 \times 90) +$

$(15 \times 70) + (8 \times 55) + (12 \times 25)$

$= ₹\ 3995.$

Q19. Transwell transportation company has agreed to transport the products from factory A and B to market places C,D and E. The unit transportation cost (in hundred of rupees), supply and demand are given in the table below.

Factory/Market	C	D	E	Supply
A	3	7	9	30
B	4	6	2	10
Demand	10	27	3	

What is the maximum profit for the transportation company from the trans-shipment of the products?

Solution:

	C	D	E	Supply
A	3	7	9	30
B	4	6	2	10
Demand	10	27	3	40/40

Maximization Case: Since the given matrix is a profit matrix, the objective is to maximize profit. To solve the problem convert the given matrix to a maximization case. This is done by subtracting all the cell entries from the largest profit element (i.e.) 9.

Example: For cell(1,1) = 9 – 3 = 6

Thus, the new matrix of minimization type is,

	C	D	E	Supply
A	6	2	0	30
B	5	3	7	10
Demand	10	27	3	40/40

To Find Initial Basic Feasible Solution by VAM

				Supply
	6	2	0	30
	5	3	7	10
Demand	10	27	3	40 / 40

				Supply	Penalty
	6	2	$\boxed{3}$ 0	30	2
	5	3	7	10	2
Demand	10	27	3		
Penalty	2	1	7 ↑		

		Supply	Penalty
6	(27) 2	27	4 ←
5	3	10	2
Demand 10	27		
Penalty 1	1		

	Supply
(10) 5	10
Demand 10	

6	(27) 2	(3) 0	30
(10) 5	3	7	10
10	27	3	40 / 40

$\therefore$ The total profit is $= \sum p_i x_i$

Where,

p_{ij} = Profit (refer original matrix)

$= (27 \times 7) + (3 \times 9) + (10 \times 4)$

$= 189 + 27 + 40$

= Rs.256

2.3 OPTIMAL SOLUTION – MODI METHOD AND STEPPING STONE METHOD

Q20. Explain MODI method for obtaining optimal solution.

Ans:

MODI Method (u-v Method) Algorithm: MODI method or u-v method is an optimal solution technique for Transportation Problem (TP). The algorithm of MODI method is as follows,

(a) Determine an Initial Basic Feasible Solution (IBFS) using any one of three methods viz., NWCM, LCM, VAM.

(b) Ensure that the number of occupied cells is exactly equal to $(m + n - 1)$ where m is number of rows and n is number of columns.

(If number of occupied cells $< (m + n - 1)$ it is called degeneracy and separate procedure has to be adopted to solve).

(c) Determine a set of numbers for each row and each column. To compute u_i and v_j form from the equations $C_{ij} = u_i + v_j$ for each of the occupied cell. Assign zero to one of the variables (u_i or v_j) and solve the equations to get the set of numbers.

(d) Compute the opportunity cost (improvement index) using $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for each of the unoccupied cell.

(e) Check the sign of each opportunity cost. If the opportunity costs of all unoccupied cells is ≥ 0 and negative and then the current solution is the optimal solution. If not, then it implies that the current solution can be improved i.e., total transportation cost can be reduced and go to step 'f'.

(f) Select the unoccupied cell with the largest negative opportunity cost.

(g) Trace a closed path (loop) for the chosen unoccupied cell.

(h) Start with a plus (+) sign at the unoccupied cell and assign alternate plus (+) and minus (–) signs on each corner of the closed path.

(i) Determine the minimum number of units that should be shipped to this unoccupied cell. The smallest cell with a negative position is chosen. The quantity is added to all the cells on the path marked with plus (+) sign and subtracted from those cells marked with minus (–) sign.

(j) Repeat the whole procedure until an optimum solution is attained i.e., when all opportunity costs are either positive or zero.

PROBLEMS

Q21. A company has three plants X, Y, Z and each producing 50, 100, 150 units of a similar products. There are five warehouses W_1, W_2, W_3, W_4 and W_5 having demand of 100, 70, 50, 40 and 40 units respectively. The cost of transporting the products from plants to warehouses is given in the following matrix,

	W_1	W_2	W_3	W_4	W_5
X	20	28	32	55	70
Y	48	36	40	44	25
Z	35	55	22	45	48

Determine the transportation schedule so that the cost is minimized.

Model Paper-I, Q2(b) | May-13, Set-1, Q2(b)

Solution:

Let us solve using VAM for IBFS and stepping stone method for optimal solution.

Vogel's Approximation Method (VAM)

Tableau-I

	W_1	W_2	W_3	W_4	W_5	Supply	Penalty
P_1	20	28	32	55	70	50	8
P_2	48	36	40	44	25 (40)	~~100~~ 60	11
P_3	35	55	22	45	48	150	13
Demand	100	70	50	40	~~40~~ 0		
Penalty	15	8	10	1	23↑		

...obtained by Vogel's method.

2.5 DEGENERACY IN TRANSPORTATION PROBLEM

Q31. When do you say a solution to a transportation problem is degenerate? Model Paper-III, Q6(b)

OR

What is degeneracy in transportation problem? May-13, Set-1, Q2(a)

Ans:

Degeneracy in Transportation Problem: A solution to transportation problem is said to be a degenerate one when the number of occupied cells i.e., positive allocations, is less than $(m + n - 1)$ where m is the number of rows and n is the number of columns. In such cases, the current solution cannot be improved because it is not possible to draw a closed path for every occupied cell and when using MODI method for finding the optimal solution, u_i, v_j values cannot be computed.

Thus, degeneracy has to be removed to improve the current solution. The degeneracy in the transportation problems may occur at two stages:

(a) Degeneracy at the initial solution stage.

(b) Degeneracy during testing of the optimal solution.

To resolve degeneracy, artificial quantity, denoted by Greek letter ε (epsilon) is used in one or more of the unoccupied cells so that the number of occupied cells is $m + n - 1$. The ε is placed in the unoccupied cell with the lowest transportation cost.

Once ε is placed in an unoccupied cell, that cell becomes an occupied cell. ε will remain until degeneracy is removed or a final solution is arrived at whichever occurs first. The TP is now solved in the usual manner.

During the solution stage, degeneracy occurs, when the inclusion of the unoccupied cell with maximum negative opportunity cost results in vacating of two or more occupied cells simultaneously. Whatever be the reason for occurrence of degeneracy, the method to resolve it is by allocating ε to the unoccupied cells.

ε to Unoccupied Cells

(i) Degeneracy at initial solution stage:

ε is allocated to unoccupied cell with least transportation cost.

(ii) Degeneracy during optimal stage:

ε is allocated to unoccupied cell which recently becomes unoccupied (i.e., vacated).

Q46. Explain the difference between transportation problem and an assignment problem.

Ans:

Model Paper-IV, Q2(a) | Nov./Dec.-12, Set-4, Q2(a)

	Transportation Problem or Allocation Problem		Assignment Problem
1.	The problem deals with estimating the minimum total transportation cost (or maximum profit) and the number of units to be transported from various origins to various destinations.	1.	The problem deals with estimating minimum total assignment cost (or maximum profit) and the assignment schedule of various resources to various jobs.
2.	Supply at any source can take any positive value.	2.	Supply at any source can take the value of 1 only.
3.	Demand at any destination can take any positive value.	3.	Demand at any job can take the value of 1 only.
4.	Balanced TP is the one where total supply is equal to total demand.	4.	Balanced AP is the one where resources are equal to jobs (i.e., number of rows = number of columns)
5.	Degeneracy may be seen at the initial stage or during the testing of optimal solution.	5.	As assignment problem is always found to be degenerate.
6.	The optimal schedule may consist of one or more assignments for each row and each column.	6.	The optimal schedule will definitely consist of only of only one assignment in a given row or column.
7.	Standard TP will have the objective of cost minimization.	7.	Standard AP will have the objective of cost or or distance or time minimization.
8.	Transhipment problem is an extension of TP.	8.	Crew assignment and travelling salesman problem are extensions of AP.

PROBLEMS

... in the sequence finishes.

Q58. Consider 3 machine and 5 job flow shop problem as shown in the table. Determine the optimal sequence of jobs that minimize the total elapsed time. Processing time on machines is in hours.

Job	Machine 1	Machine 2	Machine 3
1	11	10	12
2	13	8	20
3	15	6	15
4	12	7	19
5	20	9	7

Table

Solution:

Johnson's Rule (Extension)

Step-1: Check if either or both the following conditions satisfied.

Condition

Min $M_1 \geq$ Max M_2

Minimum processing time on machine, $M_1 = 11$

Maximum processing time on machine, $M_2 = 10$

$11 > 10$

$\therefore$ Min $M_1 >$ Max M_2

Condition 1 is satisfied.

Since, condition-1 has satisfied no need to check condition 2.

Step-2: Creating two fictitious machines G and H with processing times as follows,

$G = M_1 + M_2$

$H = M_2 + M_3$

Jobs	Machines	
	G	H
1	11 + 10	10 + 12
2	13 + 8	8 + 20
3	15 + 6	6 + 15
4	12 + 7	7 + 19
5	20 + 9	9 + 7

New processing time table (in minutes)

Machine/Jobs	1	2	3	4	5
G	21	21	21	19	29
H	22	28	21	26	16

Step 3: Applying Johnson's rule of processing 'n' jobs through 2 machines.

The minimum time is 16 under machine H, so it has to be sequenced at extreme right.

				5

By following the above procedure, we get the sequence as,

4	1	2	3	5

Computation of Times

Job	Machine M_1				Machine M_2				Machine M_3			Idle
	In Time	Process Time	Out Time	Idle Time	In Time	Process Time	Out Time	Idle Time	In Time	Process Time	Out Time	
4	0	12	12	0	12	7	19	12	19	19	38	19
1	12	11	23	0	23	10	33	4	38	12	50	0
2	23	13	36	0	36	8	44	3	50	20	70	0
3	36	15	51	0	51	6	57	7	70	15	85	0
5	51	20	71	0+21	71	9	80	14+12	85	7	92	0
Total				21				52				19

Therefore, total elapsed time = 92 hours

Idle time for machine, M_1 = 21 minutes (92 – 71)

Idle time for machine, M_2 = 52 minutes

Idle time for Machine, M_3 = 19 minutes.

Q61. Explain the graphical method to solve two job M machines sequencing problem with given technological ordering for each job. What are the limitations of the above method?

Nov./Dec.-12, Set-2, Q2(a)

Ans:

Processing of Two Jobs on 'N' Machines: This type of sequencing problem deals with two jobs to be processed through 'N' machines with main objective of determining optimum solution or sequence that minimizes total elapsed time.

Algorithm/Method to Solve these Types of Sequencing Problems

Step 1: Draw two mutually perpendicular lines (i.e., X and Y axes) with horizontal line representing processing time for job 1, while 2nd job remains idle and vertical line represents processing time for job 2 while job 1 remains idle.

Step 2: Mark the processing time for jobs 1 and 2 on horizontal and vertical lines respectively according to the given problem.

Step 3: Construction of various blocks with respect to each machine considering the time at which it will be used for job 1 and job 2.

Example

Let us consider a machine-A,

Job 1 on machine-A = X-axis ⇒ 0 to 3 hrs.

Job 2 on machine-A = Y-axis ⇒ 4 to 7 hrs.

Now a block can be obtained by just drawing parallel lines to Y-axis through 0 and 3 and parallel lines to X-axis through 4 and 7 as shown in figure (1). The intersection of all four lines result in a block.

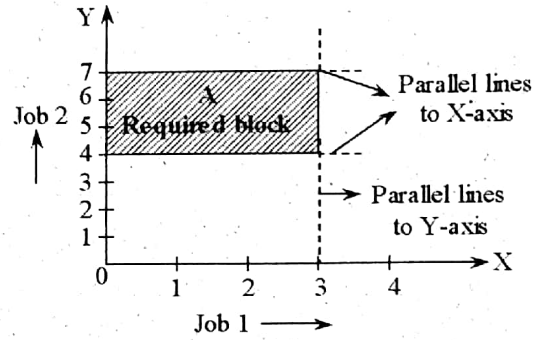


Figure (1)

Blocks must be drawn from origin to end point.

Step 4

After constructing the blocks, draw a line inclined at 45° to horizontal starting from in-time of individual machine and ending at out-time of same machine as shown in figure (2). This indicates the duration during which both jobs are in process.

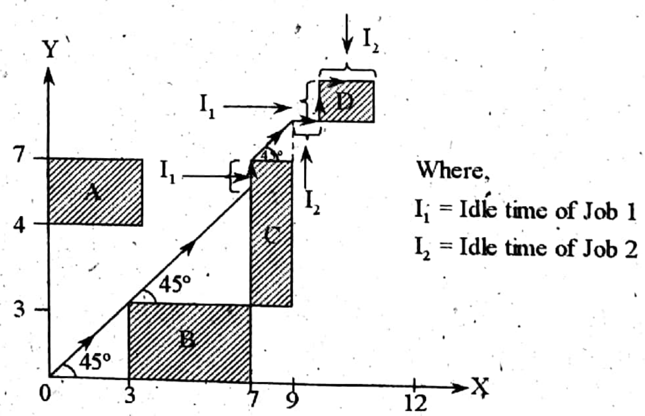


Figure (2)

Step 5

In above step, in the figure (2) after drawing 45° inclined line, vertical lines indicate the idle time of job 1 (i.e., vertical lines which are not bounded by 45° line).

Step 6

Horizontal lines indicates the idle time of job 2.

Step 7

Total elapsed time is given by,

Total processing time of job 1 or job 2 + Idle time of job 1 or job 2.

Limitations

1. It is a complex method and is solved using algorithm.
2. Idle time of both the jobs should be presented in graph.
3. The processing time of job 1 and job 2 should not exceed a diagonal region of 45 degree.
4. The processing time of both the machines should be presented in a given technological order of the problem.