UNIT-III

Bending stresses & shear stresses in beams

Bending stresses - stresses induced by bending moment

shear stresses - stresses induced by shear forces.

In order to determine the practical utility of any beam, it is very necessary to establish a relationship between the radius of curvature to which the beam bends, the bending moment, the bending stress and its cross-sectional dimensions.

Theory of simple bending

Assumptions :

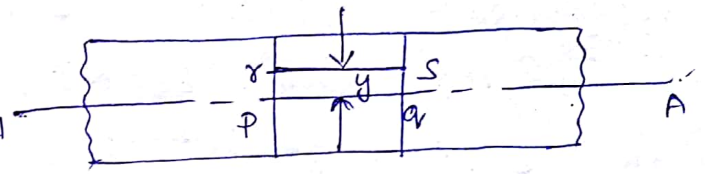
1. The material of the beam is perfectly homogeneous throughout.
2. The stress induced is proportional to the strain and at no place the stress exceeds the elastic limit.
3. The value of modulus of elasticity (E) is same, for the fibres of the beam under compression (or) under tension.
4. The transverse section of the beam, which is plane before bending, remains plane after bending.
5. There is no resultant pull (or) push on the cross-section of the beam.

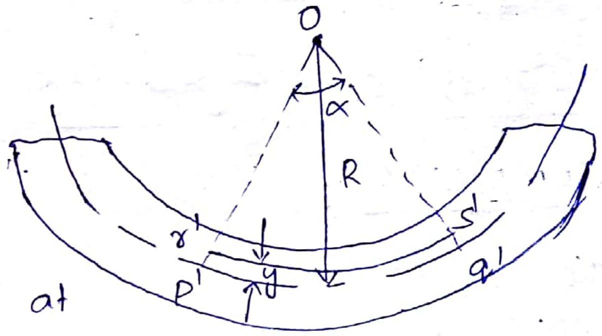
6. The loads are applied in the plane of bending.

7. The radius of curvature of the beam before bending is very large in comparison to its transverse dimensions.

Bending equation :

Fig. shows a longitudinal section of a beam, After bending, the length of a beam will take up a curved shape, the outer radii of the ~~mal~~ beam, the material will be in tension and at the inner radius in compression, and at some radius, there will be no stress.





This layer of the material is the neutral layer (or neutral axis. The neutral layer is then, before bending, the length pq, which after bending becomes p'q'.

consider some layer rs at a distance y from pq, which after bending becomes r's'. let p'q' subtend an angle α at the centre of curvature.

$$r's' = (R-y)\alpha$$

$$p'q' = R\alpha \quad ,$$

Initially, the parallel layers would have equal lengths, so that pq = rs and since there is no stress at the neutral layer, then there is no strain.

$$p'q' = pq.$$

Now, strain in rs = $\frac{rs - r's'}{rs}$

$$rs = pq = p'q'$$

$\therefore$ strain in rs $= \frac{p'q' - r's'}{rs}$

$$= \frac{R\alpha - (R-y)\alpha}{R\alpha}$$

$$= \frac{R\alpha - R\alpha + y\alpha}{R\alpha} = \frac{y}{R} \longrightarrow (1).$$

Now, if the stress in rs is σ, & yong's Modulus is E, then

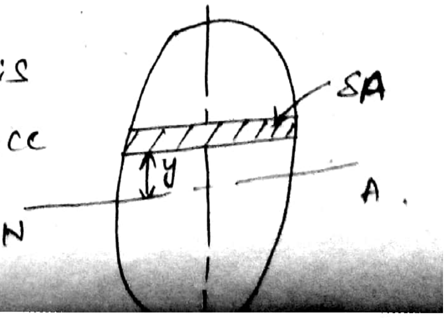
$$\text{strain in } rs = \frac{\sigma}{E} \longrightarrow (2)$$

$$(1) = (2)$$

$$\frac{y}{R} = \frac{\sigma}{E}$$

$$\therefore \frac{\sigma}{y} = \frac{E}{R} \longrightarrow (a)$$

Now, consider the transverse section of the beam. leat us consider a strip which is having area SA, lie at a distance y from the neutral axis.



~~Then, the normal force on this area (δA) =~~

$$\frac{\sigma}{y} = \frac{E}{R}$$

$$\sigma = \frac{E}{R} \times y$$

$$\sigma = \frac{\text{Normal force}}{\text{Area}}$$

Normal force on area δA $= \sigma \times \delta A$

$$= \frac{E}{R} \times y \times \delta A$$

Now, the moment of this force about the neutral axis is = Normal force × ⊥ʳ distance

$$= \frac{E}{R} \times y \times \delta A \times y.$$

$$= \frac{E}{R} y^2 \delta A.$$

This is the resisting moment of the material caused by the stress produced.

Total resisting moment $= \Sigma \frac{E}{R} y^2 \delta A$. (or)

$$= \frac{E}{R} \Sigma y^2 \delta A$$

$\Sigma y^2 \delta A$ - second moment of area about the neutral axis. $= I$

∴ Resisting moment $M = \frac{E}{R} \times I$

$$\frac{M}{I} = \frac{E}{R} \longrightarrow (b)$$

From (a) & (b) $\boxed{\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}}$

This is called the bending equation.

Where, M - Moment of resistance

I - Moment of inertia of the section about neutral axis. (N.A)

E - Young's modulus of elasticity

R - Radius of curvature of N.A

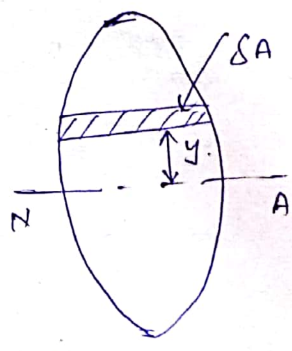
σ - Bending stress.

Position of Neutral axis:

The force acting on a small area δA at a distance 'y' from the neutral axis is given by:

$$\delta F = \sigma . \delta A$$

$$= \frac{E}{R} \times y \times \delta A$$



Total normal force $(F) = \frac{E}{R} \Sigma y \delta A$

$\therefore$ For zero resultant force $\Sigma y . \delta A = 0$.

[No resultant force on the section for equilibrium condition]

Now, $\Sigma y . \delta A$ is the moment of the sectional area about the neutral axis,

Since the moment is zero, the axis must pass through the centre of area.

Hence the neutral axis (on neutral layer), passes through the centre of area.

Section modulus

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{My}{I} = \frac{M}{I/y}$$

$$\sigma = \frac{M}{Z}$$

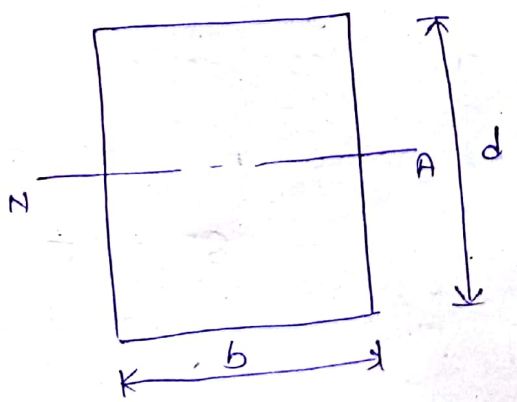
Where Z - section modulus $= \frac{I}{y}$

The strength of the beam section depends mainly on the section modulus.

Section modulus for various sections:

1. Rectangular section:

Consider a rectangular section of width b and depth d. Let the horizontal centroidal axis be neutral axis



Section modulus $Z = \frac{I}{y_{max}}$

$$I = \frac{bd^3}{12}, \quad y_{max} = \frac{d}{2}$$

y - Distance of the max. distant point of the section from the neutral axis.

$$\boxed{\therefore\ Z = \frac{bd^3/12}{d/2} = \frac{bd^2}{6}}$$

Moment of resistance, $M = \sigma \times Z$

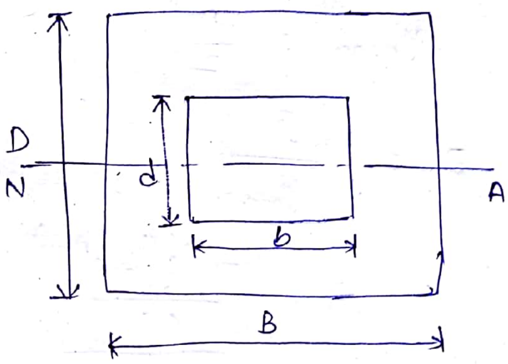
$$= \sigma \times \frac{bd^2}{6}$$

2. Hollow rectangular section

$$Z = \frac{I}{y_{max}}$$

$$I = \frac{BD^3}{12} - \frac{bd^3}{12}$$

$$= \frac{1}{12}\left(BD^3 - bd^3\right)$$



$$y_{max} = D/2$$

$$Z = \frac{\left(BD^3 - bd^3\right)/12}{D/2}$$

$$\boxed{Z = \frac{BD^3 - bd^3}{6D}}$$

Moment of resistance, $M = \sigma Z$

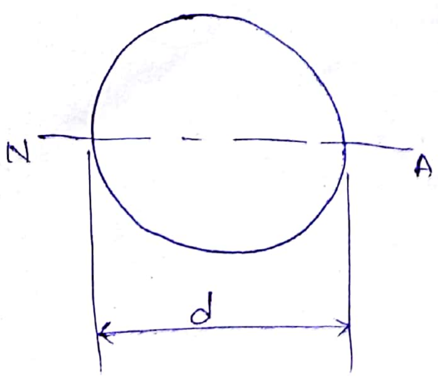
$$\boxed{M = \sigma \times \left(\frac{BD^3 - bd^3}{6D}\right)}$$

3. solid circular section.

$$I = \frac{\pi d^4}{64}$$

$$y_{max} = d/2$$

$$Z = \frac{\pi d^4/64}{d/2} = \frac{\pi d^3}{32}$$



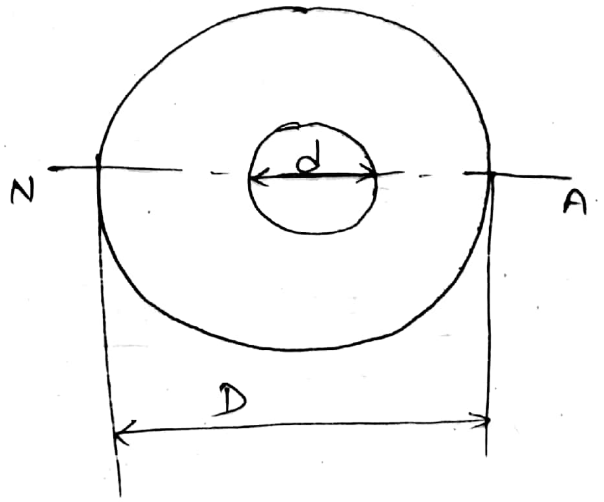
Moment of resistance $= \sigma \times Z$

$$= \sigma \times \frac{\pi d^3}{32}.$$

4. <u>Hollow circular section</u> :

$$I = \frac{\pi}{64}\left(D^4 - d^4\right)$$

$$y_{max} = \frac{D}{2}$$



$$Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{\pi(D^4 - d^4)}{64}}{\frac{D}{2}}$$

$$\boxed{Z = \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right)}$$

Moment of resistance,

$$M = \sigma \times Z = \sigma \times \frac{\pi}{32}\left(\frac{D^4 - d^4}{D}\right).$$

Pblm A 250 mm (depth) x 150 mm (width) rectangular beam is subjected to maximum bending moment of 750 kN-m. (5)

Determine :

1. The maximum stress in the beam.

2. If the value of E for the beam material is 200 GN/m², find out the radius of curvature for that portion of the beam where the bending is maximum.

3. The value of the longitudinal stress at a distance of 65 mm from the top surface of the beam.

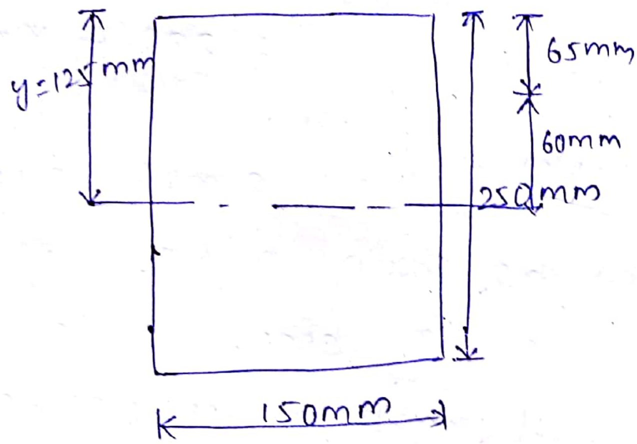
Sol Given

$b = 150mm = 0.15m$

$d = 250mm = 0.25m$

$M = 750 \ kN\text{-}m$

$E = 200 \ GN/m^2$



1. $\frac{M}{I} = \frac{\sigma}{y}$ $\quad \left[y = \frac{d}{2} \right]$

$\sigma = \frac{M}{I} \times y$

$I = \frac{bd^3}{12} = \frac{0.15 \times 0.25^3}{12} = 0.0001953 \ m^4$

$\sigma = \frac{750 \times 10^3 \times 0.125}{0.0001953} = 4.8 \times 10^8 \ N/m^2$

$\sigma = 480 \ MN/m^2$

2. $\frac{M}{I} = \frac{E}{R}$, we get

$$R = \frac{EI}{M}$$

$$R = \frac{200 \times 10^9 \times 0.0001953}{750 \times 10^3} = 52.08 \text{ m}$$

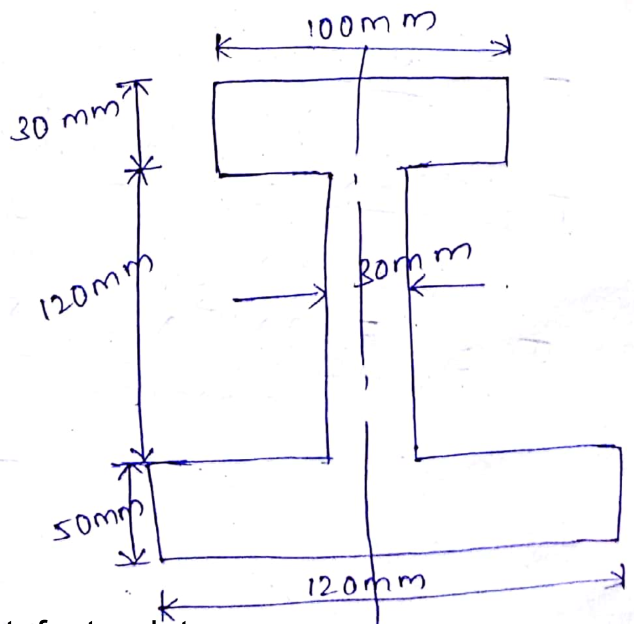
3. $$\sigma_t = \frac{M x . y_t}{I}$$

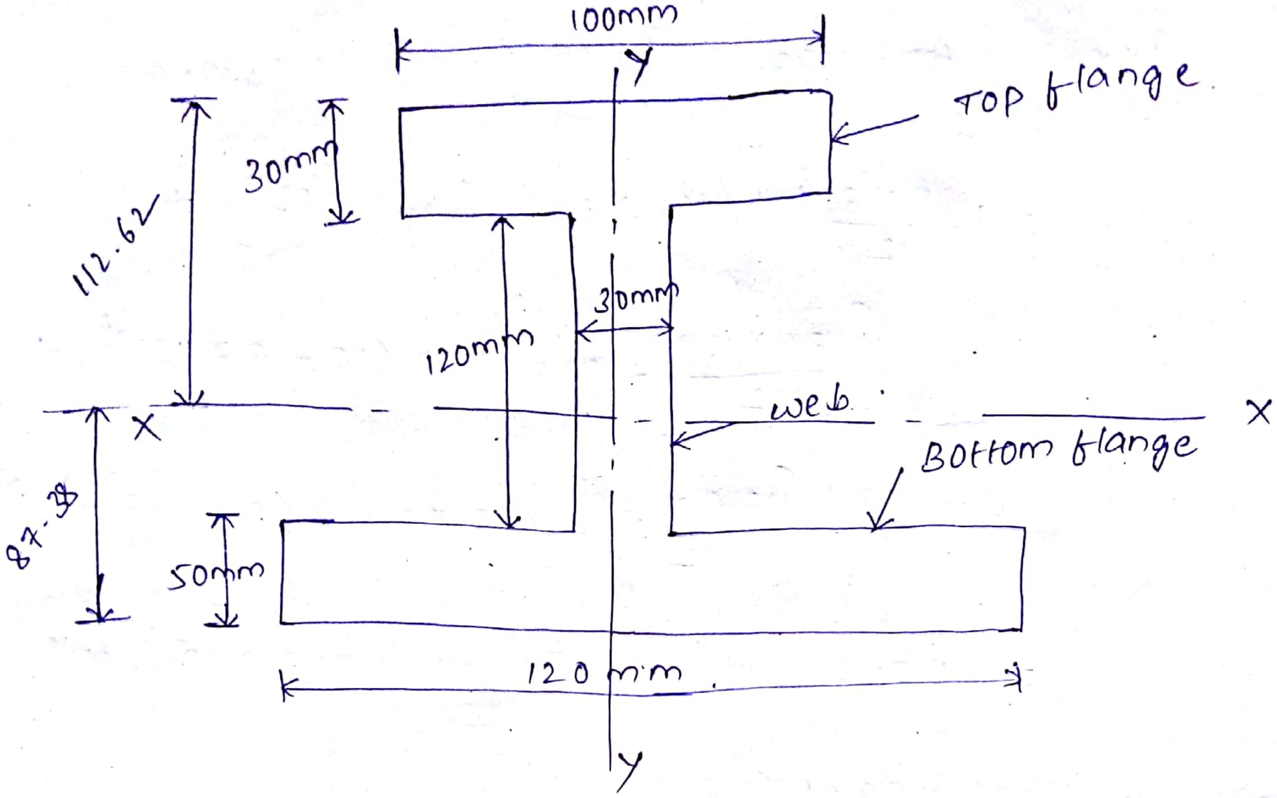
$y_t = 125 - 65 = 60\text{mm}.$

$$= \frac{750 \times 10^3 \times (60 \times 10^{-3})}{0.0001953} \times 10^{-6}$$

$$= 230.4 \;\cancel{\times 10^{6}}\; \text{MN/m}^2$$

Pblm. A beam simply supported at ends and having c/s as shown in fig. is loaded with a UDL over whole of its span. If the beam is 8m long, find the UDL. if maximum permissible bending stress in tension is limited to 30 MN/m² and in compression to 45 MN/m². What are the actual maximum bending stresses setup in the section.





Component	Area (A) mm^2	centroidal distance from bottom edge 'y' (mm)	Ay (mm^3)
Top flange	$100 \times 30 = 3000$	$200 - \frac{30}{2} = 185mm$	$3000 \times 185 = 555000$
web	$120 \times 30 = 3600$	$50 + \frac{120}{2} = 110mm$	396000
Bottom flange	$120 \times 50 = 6000$	$\frac{50}{2} = 25mm$	150000
	$\Sigma A = 12600$		$\Sigma Ay = 1101000$

Distance of the centroidal axis XX from the bottom edge.

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{1101000}{12600} = 87.38 \text{ mm.}$$

$I_{xx} = I_{top\,flange} + I_{web} + I_{bottom\,flange}$.

$$= \left[\frac{100\times30^3}{12} + 100\times30\,(112.62-15)^2\right] +$$

$$\left[\frac{30\times120^3}{12} + 30\times120\times(110-87.38)^2\right] +$$

$$\left[\frac{120\times50^3}{12} + 120\times50\times(87.38-25)^2\right]$$

$$= 5957\times10^4\ mm^4 \text{ (or) } 5957\times10^{-8}\ m^4.$$

Maximum bending moment

$$M = \frac{wL^2}{8} = \frac{w\times8^2}{8} = 8w. \qquad (w\text{-UDL}).$$

For tension side of the I-section

$$\frac{M}{I} = \frac{\sigma_t}{y_t}$$

$$M = \frac{\sigma_t}{y_t}\times I$$

$$= \frac{5957\times10^{-8}\times30\times10^6}{87.38\times10^{-3}}$$

$$= 20452\ N\text{-}m = 20.452\ kN\text{-}m.$$

For compression side of the I-section.

$$M = \frac{I\times\sigma_c}{y_c} = \frac{5957\times10^{-8}\times45\times10^6}{112.62\times10^{-3}}$$

$$= 23.8026\ kN\text{-}m$$

Moment of resistance,

$M = 20.452$ kN-m (∵ min. of the above two values)

$8w = 20.452$

$w = \frac{20.452}{8} = 2.556$ kN/m

Actual max. stress in the top-most fibres of the beam.

$= \frac{M}{I} \times y_c$

$= \frac{20.452 \times 10^3 \times 112.62 \times 10^{-3}}{5957 \times 10^{-8}}$

$= 38.6 \times 10^6$ N/m^2 $= 38.6$ MN/m^2 (comp.)

Actual max. stress in the bottom most fibres of the beam $= 30$ MN/m^2 (tensile).

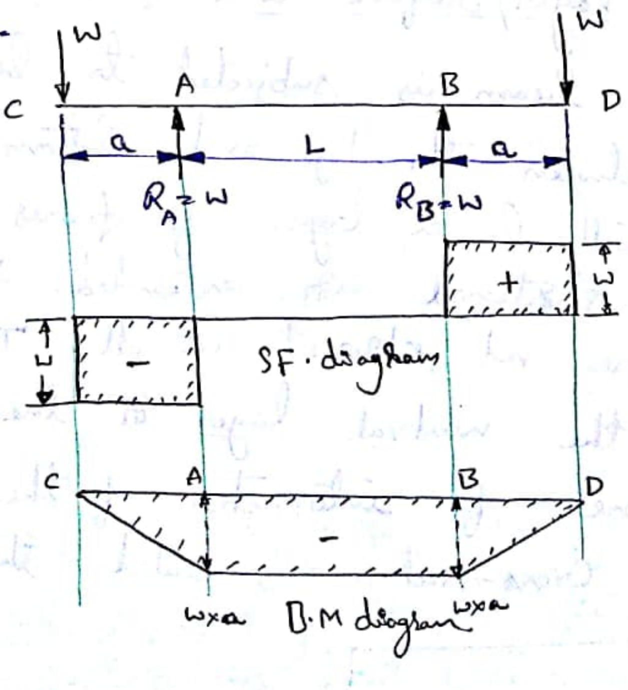
Bending Stresses in Beams

Stresses introduced by bending moment are called bending stresses.
Stresses introduced by shear force are called shear stresses:

☆ Pure bending or Simple bending

If a length of a beam is subjected to a constant bending moment and no shear force, then the stresses will set up in that length of the beam due to bending moment only and that length of the beam is said to be in pure bending or simple bending. The stresses set up in that length of beam are known as bending stresses.

Ex:-

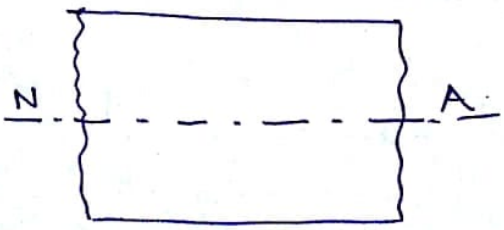


✯ Assumptions – oct/nov-2017, Regular

1) The material of the beam is homogeneous and isotropic.
2) The value of young's modulus of elasticity is the same in tension and compression.
3) The transverse sections which were plane before bending, remain plane after bending also.
4) the beam is initially straight and all longitudinal filaments bend into circular arcs with a common centre of curvature
5) The radius of curvature is large compared with the dimensions of the cross-section.
6) Each layer of the beam is free to expand & contract, independently of the layer, above & below it.

✯ Neutral layer/Surface and Neutral axis oct/nov-2017, Reg

when a beam is subjected to bending, at a level between the top and bottom of the beam there will be a layer of fibres which are neither shortened nor extended. Fibres in this layer are not stressed at all. This layer is called the neutral layer & the neutral surface. The line of intersection of the neutral surface on a cross-section is called the neutral axis.



Theory of Simple Bending (or) Expression for bending stress

Consider a beam subjected to pure bending (fig 'a'). The part of length δx will be deformed as shown in fig 'b'.

Let R = Radius of neutral layer $N'N'$

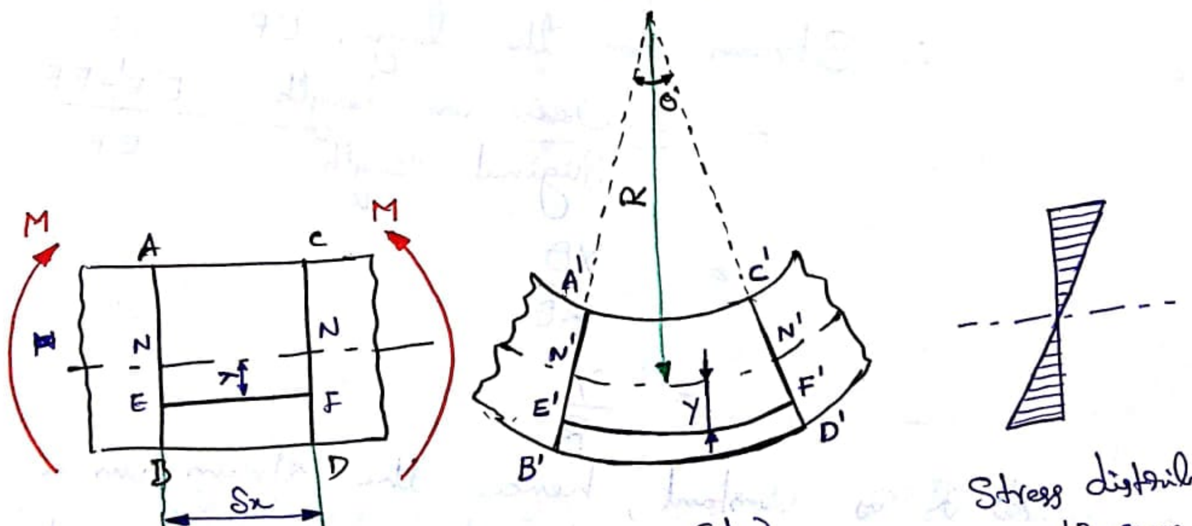
θ = Angle subtended at 'O' by $A'B'$ and $C'D'$

NN = neutral layer.

EF = Imaginary layer at a distance 'y' from NN with same length.

$E'F'$ = Imaginary layer after bending

δx = length of the neutral layer.



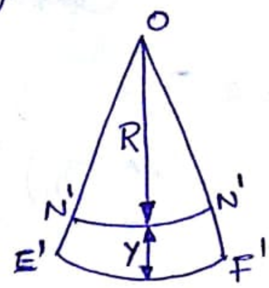
(a) (b) Stress distribution diagram

Strain variation along the depth of beam :-

The length of layer EF is equal to neutral layer NN

$$NN = EF = \delta x$$

After bending, the length of neutral layer N'N' will remain unchanged due to no stresses acting at neutral layer. But length of layer E'F' will increase.



Now from figure 'b',

$$N'N' = R \times \theta$$

$$E'F' = (R+y) \times \theta$$

But $N'N' = NN = \delta x$

Hence $\delta x = R \times \theta$

$\therefore$ Increase in the length of the layer EF

$$= E'F' - EF$$

$$= (R+y)\theta - R \times \theta$$

$$= y\theta$$

$\therefore$ Strain in the layer EF

$$= \frac{\text{Increase in length}}{\text{Original length}} = \frac{E'F' - EF}{EF}$$

$$= \frac{y\theta}{R\theta}$$

$$= \frac{y}{R}$$

As R is constant, hence the strain in a layer is proportional to its distance from the neutral axis

Stress Variation :-

Let f = Stress in the layer EF (f or σ)
E = Young's Modulus of the beam

$$E = \frac{\text{Stress in the layer EF}}{\text{Strain in the layer EF}}$$

$$E = \frac{f}{\left(\frac{y}{R}\right)}$$

$$f = \frac{E}{R} \cdot y$$

Hence the stress intensity in any fibre or layer is proportional to the distance of the fibre from the neutral layer.

$$\boxed{\frac{f}{y} = \frac{E}{R}}$$

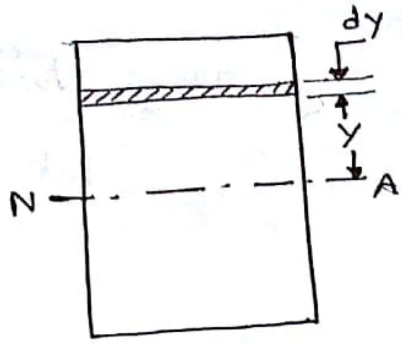
E & R are Constant.

Neutral Axis : (NA)

The neutral axis of any transverse section of a beam is defined as the line of intersection of the neutral layer with the transverse section (NA).

The beam is subjected to pure sagging moment, then the stresses will be compressive at any point above the neutral axis and tensile below the neutral axis. There is no stress at the neutral axis.

The stresses at a distance 'y' from the neutral axis is given by equation



$$f = \frac{E}{R} \times y$$

Let NA be the neutral axis of the section. Consider a small layer at a distance y from the neutral axis, Let dA = Area of the layer

Now the force on the layer.

$= $ Stress on layer $\times$ Area of layer.

$= f \times dA$

$= \frac{E}{R} \times y \times dA$

Total force on the beam section is obtained by integrating the above equation.

$\therefore$ Total force on the beam section

$$= \int \frac{E}{R} \times y \times dA$$

$$= \frac{E}{R} \int y \, dA$$

But for pure bending, there is no force on the section of the beam (or force is zero).

$$\therefore \frac{E}{R} \int y \, dA = 0$$

$$\boxed{\int y \, dA = 0}$$

$y \times dA$ represents the moment of area dA about neutral axis.

$\int y \times dA$ represents the moment of entire area of the section about neutral axis.

The centroidal axis of a section gives the position of neutral axis

Moment of Resistance:

Due to pure bending, the layers above NA are subjected to compressive stresses and the layers below NA are subjected to tensile stresses. Due to these stresses, the forces will be acting on the layers. these forces will have moment about the NA. The total moment of these forces about the NA for a section is known as moment of resistance of that section.

The forces on the layer at a distance 'y' from neutral axis is given by equation as

Force on layer $= \frac{E}{R} \times y \times dA$

Moment of this force about NA

$= \text{Force on layer} \times y$

$= \frac{E}{R} \times y \times dA \times y$

$= \frac{E}{R} \times y^2 \times dA$

Total moment of the forces on the section of the beam (or moment of resistance)

$$= \int \frac{E}{R} \times y^2 \times dA = \frac{E}{R} \int y^2 \times dA$$

Let M = External moment applied on the beam section.

External bending moment should be equal to moment of resistance to make the system equilibrium

$$M = \frac{E}{R} \int y^2 dA.$$

But $\int y^2 dA$ represents the moment of inertia of the area of the section about the neutral axis.

Let this moment of inertia be I.

$$M = \frac{E}{R} I$$

$$\boxed{\frac{M}{I} = \frac{E}{R}}$$

But, we know that $\frac{f}{y} = \frac{E}{R}$.

then $$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$ (bending equation)

Section Modulus :-

Section modulus is defined as the ratio of moment of inertia of a section about the neutral axis to the distance of the outermost layer from the neutral axis. It is denoted by the symbol Z.

$$Z = \frac{I}{Y_{max}}$$

The stress is maximum at the greatest distance from the neutral axis. is given by

$$f_{max} = \frac{M}{I} \cdot y_{max}$$

Let Y_{max} be the distance of the most distant point of the section from the neutral axis. Let f_{max} be the stress at this distance,

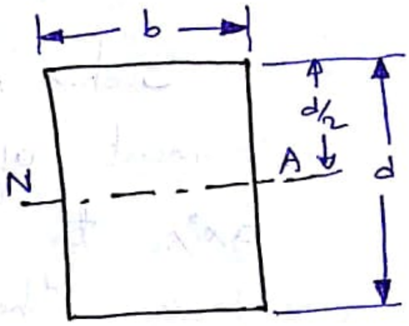
$$M = f_{max} \cdot \frac{I}{y_{max}} \qquad \boxed{Z = \frac{I}{y_{max}}}$$

$$\boxed{M = f_{max} \cdot Z}$$

M is the maximum bending moment (or moment of resistance offered by the section). Hence moment of resistance offered by the section is maximum, when section modulus Z is maximum. Hence section modulus represent the strength of the section.

Section Modulus for various shapes or beam Sections

1) Rectangular Section



Moment of inertia of a Rectangular section about an axis through its C.G (or through NA) is given by,

$$I = \frac{bd^3}{12}$$

Distance of outermost layer from N.A is given by

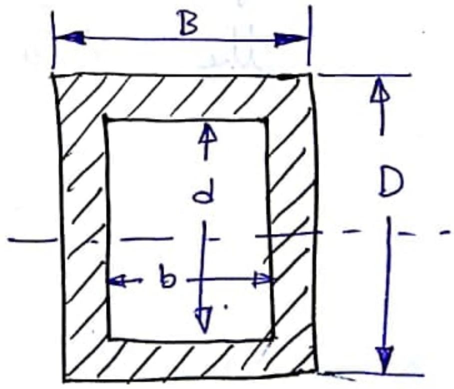
$$y_{max} = \frac{d}{2}$$

$\therefore$ Section modulus is given by

$$Z = \frac{I}{y_{max}} = \frac{bd^3}{6(\cancel{12} \times \frac{d}{\cancel{2}})} = \frac{bd^2}{6}$$

$$\boxed{Z = \frac{bd^2}{6}}$$

2) Hollow Rectangular Section



Here $I = \frac{BD^3}{12} - \frac{bd^3}{12}$

$$= \frac{1}{12}(BD^3 - bd^3)$$

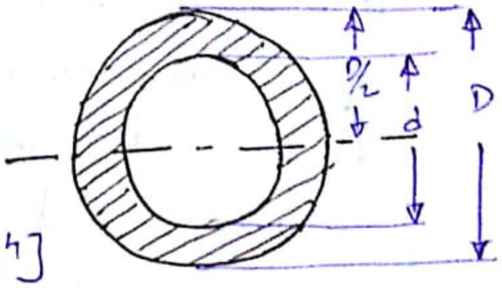
$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{1}{12}(BD^3 - bd^3)}{D/2} = \frac{1}{6D}[BD^3 - bd^3]$$

$$\boxed{Z = \frac{1}{6D}[BD^3 - bd^3]}$$

3. Circular Section



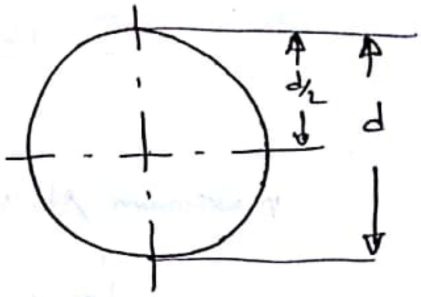
$$I = \frac{\pi}{64}\left[D^4 - d^4\right]$$

$$y_{max} = \frac{D}{2}$$

$$Z = \frac{I}{y_{max}} = \frac{\frac{\pi}{64}\left[D^4 - d^4\right]}{\left(\frac{D}{2}\right)}$$

$$\boxed{Z = \frac{\pi}{32D}\left[D^4 - d^4\right]}$$

4) Circular Section



$$I = \frac{\pi d^4}{64}.$$

$$y_{max} = \frac{d}{2}$$

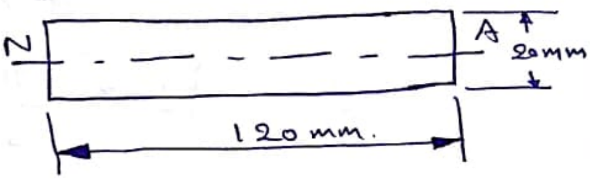
$$\text{Section modulus } Z = \frac{I}{y_{max}}$$

$$= \frac{\frac{\pi d^4}{64}}{d/2}$$

$$\boxed{Z = \frac{\pi d^3}{32}}$$

Prob.1:- A steel plate is bent into a circular arc of radius 10 metres. If the plate section be 120mm. wide and 20mm thick, find the maximum stress induced and the bending moment which can produce this stress. Take $E = 2\times10^5\ N/mm^2$

Sol:-



Moment of inertia of the section about the neutral axis

$$I = \frac{bd^3}{12}$$

$$= \frac{120\times20^3}{12} = 80000\ mm^4$$

$$\left[\frac{M}{I} = \frac{f}{y} = \frac{E}{R}\right]$$

maximum stress

$$f_{max} = \frac{E}{R}\, y_{max}$$

$$= \frac{2\times10^5}{10\times10^3}\times\frac{20}{2}\ N/mm^2$$

$$= 200\ N/mm^2$$

Bending Moment.

$$M = \frac{E}{R}\, I$$

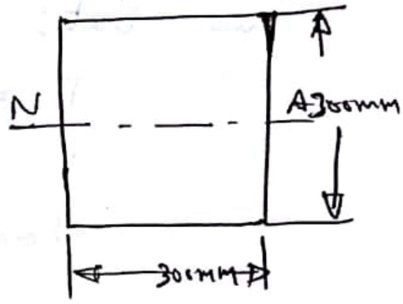
$$= \frac{2\times10^5}{10\times10^3}\times8\times10^4$$

$$= 16\times10^5\ Nmm$$

$$= 1600\ Nm$$

Prob 2 :- A Rectangular beam 300mm deep is simply supported over a span of 4m. Determine the UDL per meter which the beam may carry if the bending stress should not exceed 120 N/mm^2 Take $I = 8 \times 10^6\ mm^4$

Section Modulus. $Z = \frac{I}{y_{max}}$



given that :-

$f_{max} = 120\ N/mm^2$

$I = 8 \times 10^6\ mm^4$.

$l = 4m$.

$b \& d = 300mm$.

$$Z = \frac{8 \times 10^6}{150} = 5.33 \times 10^4\ mm^3$$

bending moment $= f \times Z$

$= 120 \times 5.33 \times 10^4$

Moment of resistance $= 6.396 \times 10^6\ Nmm$.

Maximum bending moment for a simply supported beam with UDL. $= \frac{wl^2}{8}$

$$\frac{wl^2}{8} = 6.396 \times 10^6\ Nmm$$

$$\frac{w \times (4 \times 1000)^2}{8} = 6.396 \times 10^6$$

$$w = 3.198\ N/mm$$

Prob 3: The moment of inertia of a beam section 500mm deep is 69.49×10^7 mm^4. Find the longest span over which a beam of this section, when simply supported, could carry a uniformly distributed load of 50 kN/m. The flange stress in the material is not to exceed 110 N/mm^2.

Given that:

$$I = 69.49 \times 10^7 \text{ mm}^4$$

$$w = 50 \text{ kN/m}$$

$$f_{max} = 110 \text{ N/mm}^2$$

$$M = f \times Z.$$

$$= 110 \times \frac{69.49 \times 10^7}{250}$$

$$= 30.57 \times 10^7 \text{ Nmm}.$$

for a simply supported beam with UDL, the maximum bending moment $= \frac{wl^2}{8}$

Equating max bending moment to the moment of resistance

$$\frac{wl^2}{8} = 30.57 \times 10^7 \text{ Nmm}.$$

$$l^2 = \frac{30.576 \times 10^7 \times 8}{\frac{50\cancel{000}}{1\cancel{000}}}$$

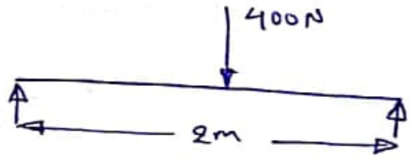
$$l^2 = 48.922 \times 10^6$$

$$l = 6994.3 \text{ mm}$$

Say $l = 7$ m

Prob 4:- A Square beam 20mm × 20mm in section and 2m long is supported at the ends. The beam fails when a point load of 400N is applied at the centre of the beam. What uniformly distributed load per metre length will break a cantilever of the same material 40mm wide, 60mm deep and 3m long?

Given:



Depth of beam, $d = 20mm$

Width of beam, $b = 20mm$.

Length of beam, $L = 2m$.

Point load, $W = 400N$

Maximum bending Moment for a simply supported beam carrying a point load.

$$M = \frac{WL}{4} = \frac{400 \times 2}{4} = 200\,Nm$$

$$M = 200000\ N\,mm.$$

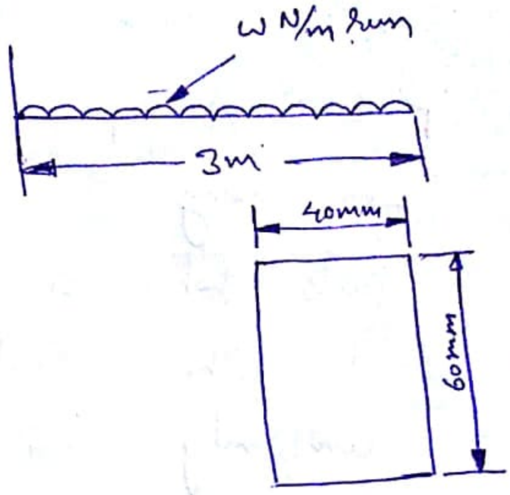
Moment of resistance $M = f_{max} Z$.

$$Z = \frac{bd^2}{6} = \frac{20 \times 20^2}{6} = \frac{4000}{3}\ mm^3$$

$$f_{max} = \frac{200000}{\left(\frac{4000}{3}\right)} = \frac{200000 \times 3}{4000}$$

$$= 150\ N/mm^2$$

Cantilever



Let w = UDL/m run

σ_{max} or f_{max} = 150 N/mm²

width of Cantilever, b = 40 mm

Depth of cantilever, d = 60mm

Length of cantilever, L = 3m

$$Z = \frac{bd^2}{6} = \frac{40 \times 60^2}{6} = 24000 \text{ mm}^3$$

Maximum B.M for a cantilever.

$$= \frac{wL^2}{2} = \frac{w \times 3^2}{2} = 4.5w \text{ Nm.}$$

$$= 4.5 \times 1000 w \text{ Nmm}$$

$$M = 4.5 \times 1000 w \text{ Nmm}$$

Now using equation, we get

$$M = f_{max} Z.$$

$$4.5 \times 1000 w = 150 \times 24000$$

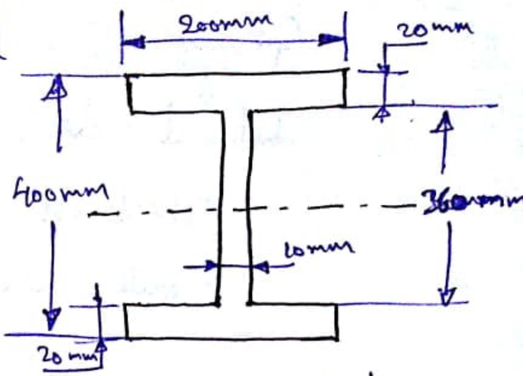
$$w = \frac{150 \times 24000}{4.5 \times 1000}$$

$$w = 800 \text{ N/m}$$

Prob 5:- A rolled steel joist of I section has the dimensions as shown in fig. this beam of I section carries a udl of 40 kN/m run on a span of 10m, calculate the maximum stress produced due to bending.

Given:- udl 'w' = 40 kN/m = 40000 N/m

span 'L' = 10m



Moment of inertia about the neutral axis

$$= \frac{200 \times 400^3}{12} - \frac{(200-10) \times 360^3}{12}$$

$$= 327946666 \text{ mm}^4$$

Maximum B.M is given by for a simply supported beam with UDL.

$$M = \frac{wL^2}{8} = \frac{40000 \times 10^2}{8}$$

$$= 500000 \text{ Nm}$$

$$= 500000 \times 1000 \text{ Nmm}$$

$$= 5 \times 10^8 \text{ Nmm}$$

Now using the relation,

$$\frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} \times y$$

$$f_{max} = \frac{5 \times 10^8}{327946666} \times 200$$

$$\boxed{f_{max} = 304.92 \text{ N/mm}^2}$$

Prob-6: A water main of 500mm internal diameter and 20 mm thick is running full. The water main is of cast iron and is supported at two points 10m apart. Find the maximum stress in the metal. The cast iron and water weigh 72000 N/m³ and 10000 N/m³ respectively.

Given

Internal dia, $D_i = 500mm = 0.5m$

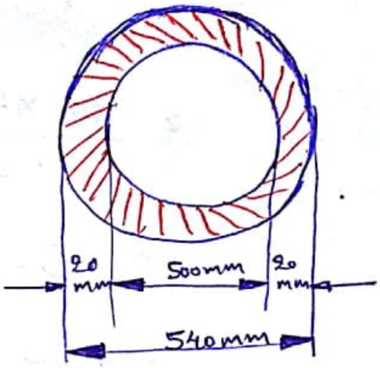
Thickness of pipe $t = 20mm$

$\therefore$ outer dia $D_o = D_i + 2 \times t$

$= 500 + 2 \times 20$

$= 540mm$

$= 5.4 m$



Weight density of cast iron $= 72000 \; N/m^3$

weight density of water $= 10000 \; N/m^3$

Internal area of pipe $= \frac{\pi}{4} D_i^2 = \frac{\pi}{4} \times 0.5^2$

$= 0.1960 \; m^2$

This is also equal to the area of water section.

$\therefore$ Area of water section $= 0.196 \; m^2$

Outer area of pipe $= \frac{\pi}{4} D_o^2 = \frac{\pi}{4} \times 0.54^2 \; m^2$

$\therefore$ Area of pipe section $= \frac{\pi}{4} D_o^2 - \frac{\pi}{4} D_i^2$

$= \frac{\pi}{4} [0.54^2 - 0.5^2]$

$= 0.0327 \; m^2$

Moment of inertia of pipe section about neutral axis,

$$I = \frac{\pi}{64} [D_o^4 - D_i^4]$$

$$= \frac{\pi}{64} [540^4 - 500^4]$$

$$= 1.105 \times 10^9 \; mm^4$$

Let us now find the weight of pipe and weight of water for one metre length.

Weight of the pipe for one metre run

$= $ weight density of cast iron $\times$ volume of pipe

$= 72000 \times [\text{Area of pipe section} \times \text{length}]$

$= 72000 \times [0.0327 \times 1$

$= 2354\,N$

Weight of the water for one metre length

$= 10000 \times [\text{Area of water section} \times \text{length}]$

$= 10000 \times 0.196 \times 1$

$= 1960\,N$

$\therefore$ Total weight on the pipe for one metre run

$= 2354 + 1960$

$= 4314\,N$

Hence the above weight is the UDL

$\therefore$ Maximum bending moment due to UDL

$$M = \frac{w \times L^2}{8}$$

$$= \frac{4314 \times 10^2}{8}$$

$$= 53925\ Nm$$

Now using $\frac{M}{I} = \frac{f}{y}$

$$f = \frac{M}{I} \times y$$

The stress is maximum, when y is maximum.

But maximum value of .

$$y = \frac{D_o}{2} = \frac{540}{2}$$

$$y_{max} = 270 \text{ mm}.$$

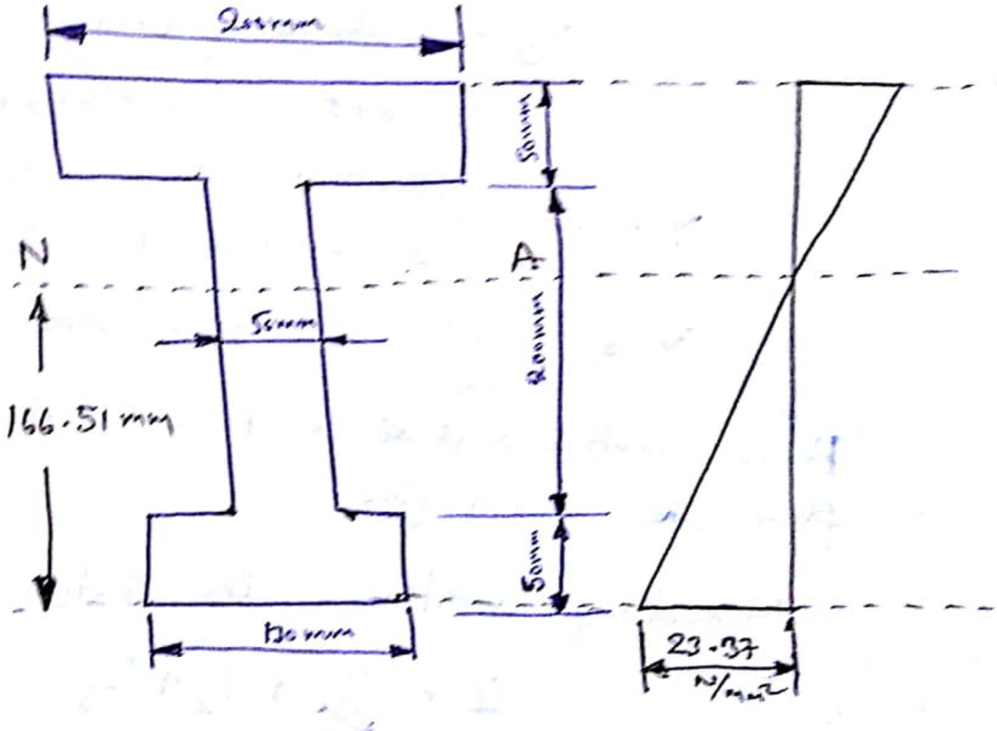
$\therefore$ Maximum stress,

$$\sigma_{max} \text{ or } f_{max} = \frac{M}{I} \times y_{max}$$

$$= \frac{53925 \times 10^3}{1.105 \times 10^9} \times 270$$

$$= 13.18 \text{ N/mm}^2$$

Prob 7: A cast iron bracket subject to bending has the cross-section of I-form with unequal flanges. The dimension of the section are shown in the figure. Find the position of the neutral axis and moment of inertia of the section about the neutral axis. If the maximum bending moment on the section is 40 MN mm, determine the maximum bending stress. What is the nature of the stress.



Given that:

Maximum bending Moment $M = 40$ MN mm.

$= 40 \times 10^6$ Nmm

Let $\bar{y}$ is the distance of the C.G from the bottom face. The section is symmetrical about y-axis and hence $\bar{y}$ is only to be calculated.

Then, $\bar{Y} = \dfrac{A_1Y_1 + A_2Y_2 + A_3Y_3}{(A_1 + A_2 + A_3)}$

where A_1 = Area of bottom flange = $130 \times 50 = 6500 \, mm^2$

Y_1 = Distance of C.G of A_1 from bottom face.

$= \dfrac{50}{2} = 25 \, mm$

A_2 = Area of web = $200 \times 50 = 10000 \, mm^2$

Y_2 = Distance of C.G of A_2 from bottom face.

$= 50 + \dfrac{200}{2} = 150 \, mm$

A_3 = Area of top flange = $200 \times 50 = 10000 \, mm^2$

Y_3 = Distance of C.G of A_3 from bottom face

$= 50 + 200 + \dfrac{50}{2} = 275 \, mm$

$$\bar{Y} = \frac{6500 \times 25 + 10000 \times 150 + 10000 \times 275}{6500 + 10000 + 10000}$$

$$\bar{Y} = \frac{4412500}{26500} = \underline{166.51 \, mm}$$

Hence neutral axis is at a distance of 166.51 mm from the bottom face.

Moment of inertia of the section about the NA.

$$I = I_1 + I_2 + I_3$$

where I_1 = MOI of bottom flange about NA.

= MOI of bottom flange about an axis passing through its C.G. + $A_1 \times$ (Distance of its C.G from NA)^2

$$= \frac{130 \times 50^3}{12} + 6500 \times (166.51 - 25)^2$$

$= 131517186.6 \, mm^4$

3_{12}

Similarly I_2 = MOI of web about NA

$$= \frac{50 \times 200^3}{12} + A_2 \cdot (166.51 - Y_2)^2$$

$$= \frac{50 \times 200^3}{12} + 10{,}000\,(166.51 - 150)^2$$

$$= 33605913.43 \text{ mm}^4$$

I_3 = MOI of top flange about NA.

$$= \frac{200 \times 50^3}{12} + A_3\,(Y_3 - 166.51)^2$$

$$= \frac{200 \times 50^3}{12} + 10000\,(275 - 166.51)^2$$

$$= 119784134.3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3$$

$$= 131517186.6 + 33605913.43 + 119784134.3$$

$$I = 284907234.9 \text{ mm}^4$$

Now distance of C.G from the upper top fibre

$$= 300 - \bar{Y} = 300 - 166.51 = 133.49 \text{mm}$$

and the distance of C.G. from the bottom fibre.

$$= \bar{Y} = 166.51 \text{ mm}$$

Hence we shall take the value of $Y = 166.51$ mm for maximum bending stress.

Now using the equation

$$\frac{M}{I} = \frac{\sigma}{Y}$$

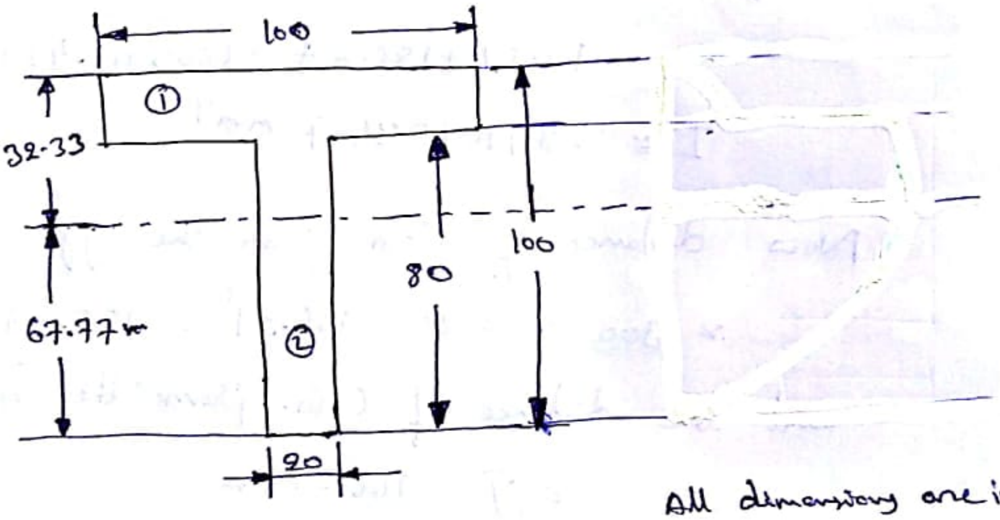
$$\sigma = \frac{M}{I} \times y$$

$$= \frac{40 \times 10^6}{2849072\text{3}4.9} \times 166.51$$

$$= 23.377 \ N/mm^2$$

$\therefore$ Maximum bending stress $= 23.377 \ N/mm^2$

Prob 8:- A Cast iron beam is of T- section as shown in figure - the beam is simply supported on a span of 8m. The beam carries a uniformly distributed load of 1.5 kN/m length on the entire span. Determine the maximum tensile and maximum compressive stresses



All dimensions are in mm.

Given.

length, $L = 8m$

UDI $w = 1.5 kN/m = 1500 \ N/m$.

To find the position of the N.A. the C.G of the section is to be calculated first. The C.G. will be lying on the Y-Y axis.

Let $\bar{y}$ = Distance of the C.G of the section from the bottom.

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{(100\times 20)\times\left(80+\frac{20}{2}\right) + 80\times 20\times\frac{80}{2}}{(100\times 2) + (80\times 20)}$$

$$= \frac{244000}{3600} = 67.77\,mm$$

∴ NA lies at a distance of 67.77mm from the bottom face or $100-67.77 = 32.23$ mm from the top face.

Now moment of inertia of the section about NA is given by

$$I = I_1 + I_2$$

where I_1 = M.O.I of top flange about N.A

= M.O.I of top flange about its C.G + $A_1 \times$ (distance of its C.G from N.A)^2

$$= \frac{100\times 20^3}{12} + (100\times 20)\times(32.23-10)^2$$

$$= 66666.7 + 988345.8$$

$$= 1055012.5\ mm^4.$$

I_2 = M.O.I of web about N.A.

$$= \frac{20\times 80^3}{12} + (80\times 20)\times(67.77-40)^2$$

$$= 2087209.9\ mm^4.$$

$I = I_1 + I_2$

$= 1055012.5 + 2087209.9$

$= 3142222.4 \text{ mm}^4$

For a simply supported beam, the maximum tensile stress will be at the extreme bottom fibre and maximum compressive stress will be at the extreme top fibre.

Maximum bending moment is given by

$$M = \frac{w \times L^2}{8} = \frac{1500 \times 8^2}{8} = 12000 \text{ Nm}.$$

$$= 12000 \times 1000 = 12000000 \text{ Nmm}$$

Now using the relation

$$\frac{M}{I} = \frac{\sigma}{y} \text{ or } \sigma = \frac{M}{I} \times y$$

i) for maximum tensile stress,

y = Distance of extreme bottom fibre from N.A

$= 67.77 \text{mm}.$

$$\sigma = \frac{12000000}{3142222.4} \times 67.77 = 258.81 \text{ N/mm}^2$$

ii) For maximum compressive stress,

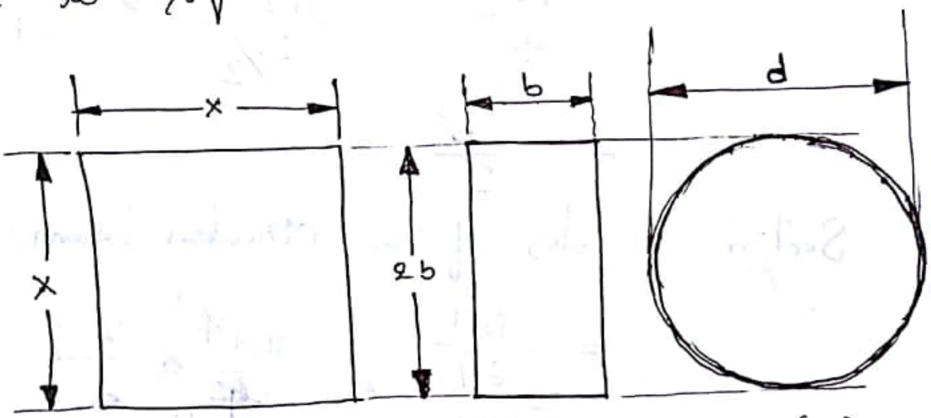
y = Distance of extreme top fibre from N.A = 32.23mm

$$\sigma = \frac{M}{I} \times y = \frac{12000000}{3142222.4} \times 32.23$$

$$= 123.08 \text{ N/mm}^2$$

Prob 8:- Three beams have the same length, same allowable bending stress and the same bending moment. The cross-section of the beams are a square, rectangle with depth twice the width and a circle. Find the ratios of weights of the circular and the rectangular beams with respect to square beams.

Given:



(a) (b) (c)

Let:- x = side of a square beam
b = Width of rectangular beam
$2b$ = Depth of the rectangular beam
d = Diameter of a circular section.

the moment of resistance of a beam is given by

$$M = \sigma \times Z$$

where Z = section Modulus.

As all the three beams have the same allowable bending stress (σ), and same bending moment (M), therefore the section modulus (Z) of the three beams must be equal.

Section modulus of a square beam

$$= \frac{I}{y} = \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{x^{\cancel{4}3}}{\cancel{12}_6} \times \frac{\cancel{2}}{\cancel{x}} = \frac{x^3}{6}$$

Section modulus of a rectangular beam

$$= \frac{\frac{bd^3}{12}}{\frac{d}{2}} = \frac{\frac{b \times (2b)^3}{12}}{\frac{2b}{2}} = \frac{\cancel{b}\,\cancel{8}^{2}b^3}{\cancel{12}\,3} \times \frac{\cancel{2}}{\cancel{2b}}$$

$$= \frac{2b^3}{3}$$

Section modulus of a circular beam

$$= \frac{\frac{\pi d^4}{64}}{\frac{d}{2}} = \frac{\pi d^{\cancel{4}3}}{\cancel{64}_{32}} \times \frac{\cancel{2}}{\cancel{d}}$$

$$= \frac{\pi d^3}{32}$$

Equating the section modulus of a square beam with that of a ~~Sd~~ rectangular beam, we get

$$\frac{x^3}{6} = \frac{2}{3} b^3$$

$$b^3 = \frac{3x^3}{12} = \frac{x^3}{4} = 0.25x^3$$

$$b = (0.25)^{1/3} x$$

$$b = 0.63x \quad \text{——— (i)}$$

Equating the section modulus of a square beam with that of a circular beam, we get

$$\frac{x^3}{6} = \frac{\pi d^3}{32}$$

$$d^3 = \frac{32x^3}{6\pi}.$$

$$d = \left(\frac{32}{6\pi}\right)^{1/3} x$$

$$d = 1.1927x \quad \text{——— (ii)}$$

The weights of the beams are proportional to their cross-sectional areas. Hence

$$\frac{\text{weight of rectangular beam}}{\text{weight of square beam}} = \frac{\text{Area of rectangular beam}}{\text{Area of square beam}}.$$

$$= \frac{b \times 2b}{x \times x} = \frac{0.63x \times 2 \times 0.63x}{x \times x}$$

$$= 0.7938$$

and $$\frac{\text{weight of circular beam}}{\text{weight of square beam}} = \frac{\text{Area of circular beam}}{\text{Area of square beam}}$$

$$= \frac{\frac{\pi d^2}{4}}{x^2} = \frac{\pi d^2}{4x^2}.$$

$$= \frac{\pi \times (1.1927x)^2}{4x^2} \quad (\because d = 1.1927x)$$

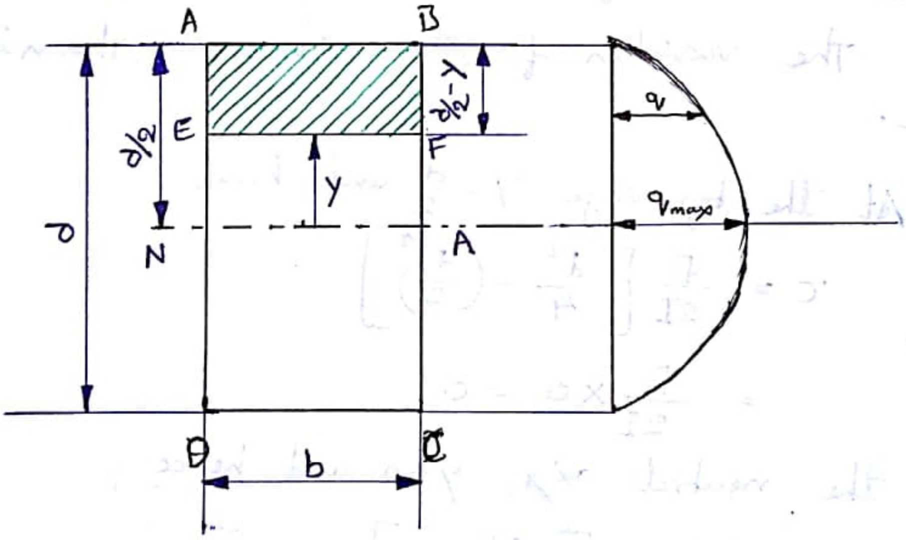
$$= \underline{\underline{1.1172}}$$

Shear stress distribution for different Sections

Rectangular Section:-

A rectangular section of a beam of width b and depth d. Let F is the shear force acting at the section. Consider a level EF at a distance y from the neutral axis.

The shear stress at this level is given by equation as $\tau = F \cdot \frac{A\bar{y}}{Ib}$ ——— ①



(a) (b)

where A = Area of the section above y (ie, shaded area ABFE)

$= \left(\frac{d}{2} - y\right) \times b$

$\bar{y}$ = Distance of the C.G of area A from neutral axis

$= y + \frac{1}{2}\left(\frac{d}{2} - y\right) = y + \frac{d}{4} - \frac{y}{2}$

$= \frac{1}{2}\left(y + \frac{d}{2}\right)$

b = Actual width of the section at the level EF

I = MOI of the whole section about N.A.

Substituting these values in the above equation, we get

$$\tau = \frac{F\left(\frac{d}{2}-y\right)\times b\times\frac{1}{2}\left(y+\frac{d}{2}\right)}{b\times I}$$

$$= \frac{F}{2I}\left(\frac{d^2}{4}-y^2\right) \quad \text{------ (2)}$$

From the above equation, we see that τ increases as y decreases.

Also the variation of τ with respect to y is a parabola. The variation of shear stress is shown in the figure.

At the top edge, $y = \frac{d}{2}$ and hence.

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4}-\left(\frac{d}{2}\right)^2\right]$$

$$= \frac{F}{2I}\times 0 = 0$$

At the neutral axis, y = 0 and hence.

$$\tau = \frac{F}{2I}\left[\frac{d^2}{4}-0\right]$$

$$= \frac{F}{2I}\frac{d^2}{4} \qquad \left(I = \frac{bd^3}{12}\right)$$

$$= \frac{Fd^2}{8I} = \frac{Fd^2}{8\times\frac{bd^3}{12}}$$

$$= \frac{12\times F}{8bd} = 1.5\frac{F}{bd} \quad \text{------ (3)}$$

Now average shear stress, $\tau_{avg} = \frac{\text{Shear force}}{\text{Area of section}} = \frac{F}{b \times d}$

Substituting the above in the equation (i), we get

$$\tau = 1.5 \times \tau_{avg} \quad \text{------} \quad (4)$$

The above equation gives the shear stress at the neutral axis where $y = 0$. This stress is also the maximum shear stress.

$$\tau_{max} = 1.5\, \tau_{avg} \quad \text{------} \quad (5)$$

From the equation (1) $\tau = \frac{A\bar{y}}{Ib}$.

$A\bar{y}$ can also be calculated as given below:

$A\bar{y}$ = Moment of shaded area about NA.

Consider a strip of thickness dy at a distance y from NA. Let dA is the area of this strip.

Then dA = Area of strip = $b \times dy$

The moment of the shaded area about N.A is obtained by integrating the above equation between the limits y to $d/2$

∴ Moment of shaded area about N.A.

$$= \int_{y}^{d/2} y \times b \times dy$$

$$= b\int_y^{d/2} y \times dy$$

$$= b\left[\frac{y^2}{2}\right]_y^{d/2} = \frac{b}{2}\left[\left(\frac{d}{2}\right)^2 - y^2\right]$$

$$= b/_2\left[\frac{d^2}{4} - y^2\right]$$

But moment of shaded area about N.A is also equal to $A\bar{y}$.

$$A\bar{y} = \frac{b}{2}\left[\frac{d^2}{4} - y^2\right]$$

Substituting the value of $A\bar{y}$ in equation (1)

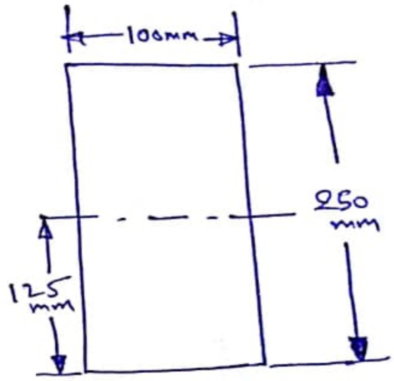
$$\tau = F \cdot \frac{A\bar{y}}{Ib}$$

$$= \frac{F \times \frac{b}{2}\left[\frac{d^2}{4} - y^2\right]}{I \times b}$$

$$\boxed{\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)}$$

Prob:- A Rectangular beam 100mm wide and 250mm deep is subjected to a maximum shear force of 50kN. Determine:

i) Average shear stress ii) Maximum shear stress, and
iii) Shear stress at a distance of 25mm above the neutral axis.



Sol:- Given:

Width, $b = 100mm$

Depth, $d = 250mm$

Maximum Shear force, $F = 50{,}000N$

i) Average shear stress is given by

$$\tau_{avg} = \frac{F}{area} = \frac{50{,}000}{b \times d}$$

$$= \frac{50{,}000}{100 \times 250} = 2\ N/mm^2$$

ii) Maximum shear stress is given by equation.

$$\tau_{max} = 1.5 \times \tau_{avg}$$

$$= 1.5 \times 2 = 3\ N/mm^2$$

iii) The shear stress at a distance y from N.A is given by equation

$$\tau = \frac{F}{2I}\left(\frac{d^2}{4} - y^2\right)$$

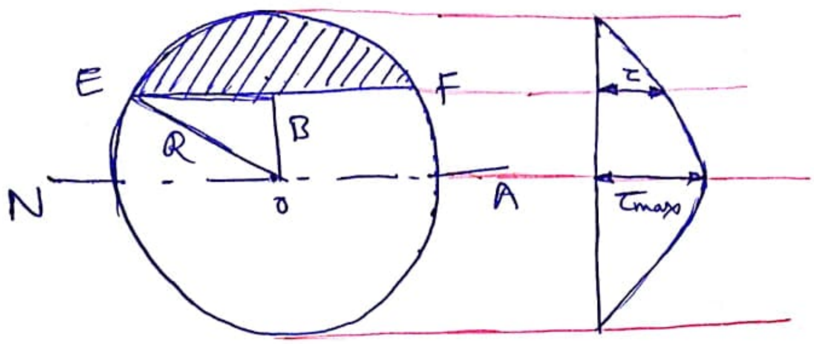
$$= \frac{50000}{2I}\left(\frac{250^2}{4} - 25^2\right)$$

$$= \frac{50000}{2 \times \frac{bd^3}{12}}\left(\frac{62500}{4} - 625\right)$$

$$= \frac{50000 \times 12}{2 \times 100 \times 250^3} \times 15000\ N/mm^2$$

$$\boxed{\tau = 2.88\ N/mm^2}$$

Circular Section.



Let R is the radius of the circular section
& F is the shear force acting on the section.
Consider a level EF at a distance y from the neutral axis
The shear stress at this level is given by equation

$$\tau = \frac{F \times A \times \bar{y}}{I \times b} \quad \text{——— (1)}$$

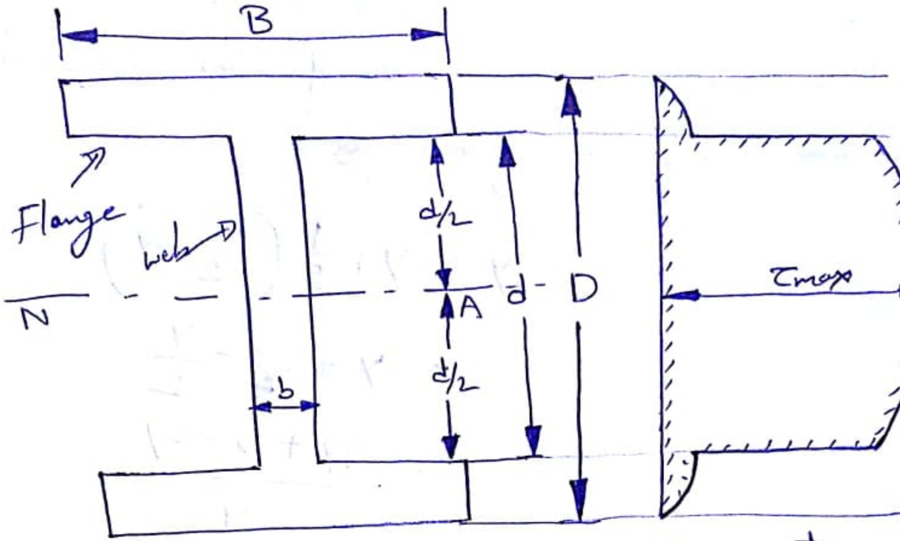
where $A\bar{y}$ = Moment of the shaded area about the neutral axis,

I = Moment of inertia of the whole circular section

b = width of the beam at the level EF

Consider a strip of thickness dy at a distance y from N.A. Let dA is the area of strip

Shear Stress distribution over I-Section



Let B = Overall width of the section
D = overall depth of the section
b = Thickness of the web, and
d = Depth of web.

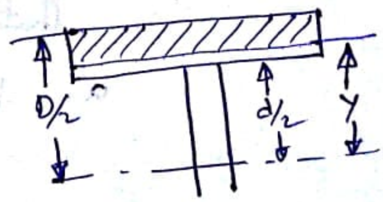
The shear stress at a distance y from the N-A is given by equation

$$\tau = F \times \frac{A\bar{y}}{I \times b} \quad \text{——— (1)}$$

In this case the shear stress distribution in the web and shear stress distribution in the flange are to be calculated separately.

i) Shear Stress distribution in the flange.

Consider a section at a distance y from N-A in the flange.



width of the section = B

Shaded area of flange, $A = B\left(\frac{D}{2} - y\right)$

Distance of the C.G of the shaded area from neutral axis is given as.

$$\bar{y} = y + \frac{1}{2}\left(\frac{D}{2} - y\right)$$

$$= y + \frac{D}{4} - \frac{y}{2}$$

$$= \frac{4y + D - 2y}{4}$$

$$= \frac{1}{2}\left(\frac{D}{2} - y\right)$$

Hence shear stress in the flange becomes,

$$\tau = \frac{F \times A\bar{y}}{I \times B}$$

$$= \frac{F \times B\left(\frac{D}{2} - y\right) \times \frac{1}{2}\left(\frac{D}{2} + y\right)}{IB}$$

$$= \frac{F}{2I}\left[\left(\frac{D}{2}\right)^2 - y^2\right]$$

$$\tau = \frac{F}{2I}\left(\frac{D^2}{2} - y^2\right)$$

Hence, the variation of shear stress (τ) with respect to y in the flange is parabolic. It is clear that with increase of y, shear stress decreases.

(a) For the upper edge of the flange,

$$y = \frac{D}{2}$$

Hence shear stress, $\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{D}{2}\right)^2\right] = 0$

(b) For the lower edge of the flange,

$$y = \frac{d}{2}$$

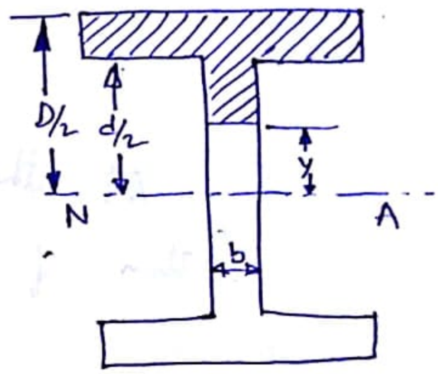
Hence $$\tau = \frac{F}{2I}\left[\frac{D^2}{4} - \left(\frac{d}{2}\right)^2\right]$$

$$= \frac{F}{2I}\left[\frac{D^2}{4} - \frac{d^2}{4}\right]$$

$$= \frac{F}{8I}(D^2 - d^2)$$

ii) Shear stress distribution in the web

Consider a section at a distance y in the web from the N.A as shown in figure.



Width of the section $= b$

Here $A\bar{Y}$ is made up of two parts.

ie moment of the flange area about N.A plus moment of the shaded area of the web about the N.A.

$$= B\left(\frac{D}{2} - \frac{d}{2}\right) \times \frac{1}{2}\left(\frac{D}{2} + \frac{d}{2}\right) + b\left(\frac{d}{2} - y\right) \times \frac{1}{2}\left(\frac{d}{2} + y\right)$$

$$= \frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$$

Hence the shear stress in the web becomes as

$$\tau = \frac{F \times A\bar{Y}}{I \times b}$$

$$\tau = \frac{F}{I \times b} \times \left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - y^2\right)\right]$$

It is clear that variation of τ with respect to y is parabolic. Also with the increase of y, τ decreases.

At the neutral axis, $y = 0$ and hence shear stress is maximum.

$$\therefore \tau_{max} = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2} \times \frac{d^2}{4}\right]$$

$$= \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

At the junction of top of the web and bottom of flange,

$$y = d/2$$

Hence shear stress is given by,

$$\tau = \frac{F}{I \times b}\left[\frac{B}{8}(D^2 - d^2) + \frac{b}{2}\left(\frac{d^2}{4} - \left(\frac{d}{2}\right)^2\right)\right]$$

$$= \frac{F \times B \times (D^2 - d^2)}{8 I \times b}$$

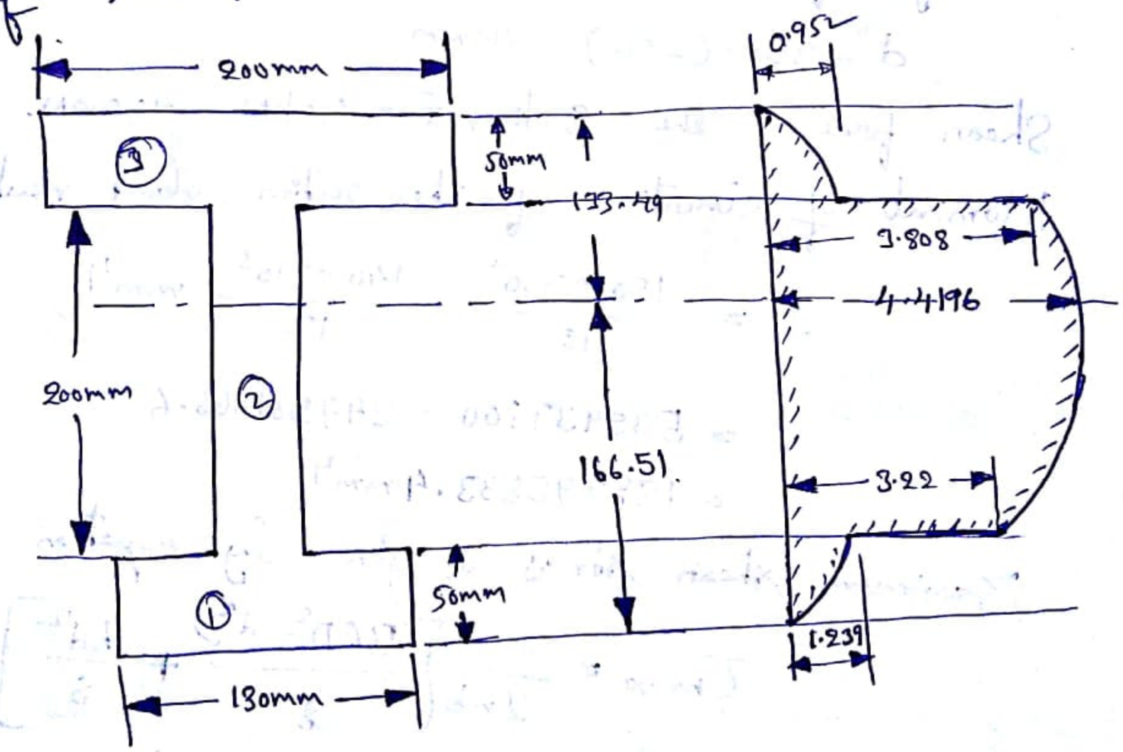
The shear stress at the junction of the flange and web changes abruptly. From these two equations it is clear that the stress at the junction changes abruptly from $\frac{F}{8I}(D^2 - d^2)$ to

$$\frac{B}{b} \times \frac{F}{8I}(D^2 - d^2)$$

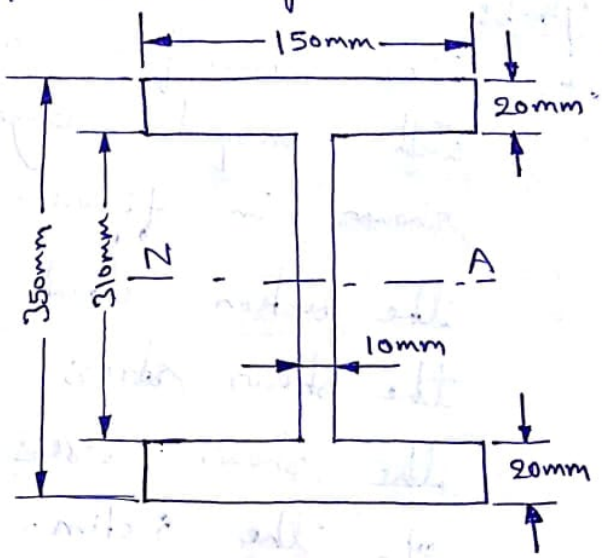
$$= 0.000021234 \left[\frac{150}{8} (122500 - 96100) + 120125 \right]$$

$$= \underline{\underline{13.06 \text{ N/mm}^2}}$$

Prob:- The shear force acting on a beam at an I-section with unequal flanges is 50kN. The section is shown in figure. The moment of inertia of the section about N.A is 2.849×10^4. Calculate the shear stress at the N.A and also draw the shear stress distribution over the depth of the section.



An I-section beam 350mm × 150 mm has a web thickness of 10mm and a flange thickness of 20mm. If the shear force acting on the section is 40kN, find the maximum shear stress developed in the I-section



Given:

Overall depth, D = 350mm

Overall width, B = 150mm

Web thickness, b = 10mm

Flange thickness, = 20mm

∴ Depth of web,

$$d = 350 - (2 \times 20) = 310\,mm.$$

Shear force on the section, F = 40kN = 40,000N.

Moment of inertia of the section about neutral axis,

$$I = \frac{150 \times 350^3}{12} - \frac{140 \times 310^3}{12} \; mm^4$$

$$= 535937500 - 347561666.6$$

$$= 188375833.4\,mm^4$$

Maximum shear stress is given by equation

$$\tau_{max} = \frac{F}{I \times b}\left[\frac{B(D^2 - d^2)}{8} + \frac{bd^2}{8}\right]$$

$$= \frac{40000}{188375833.4 \times 10}\left[\frac{150(350^2 - 310^2)}{8} + \frac{10 \times 310^2}{8}\right]$$

$$= 13.06 \; N/mm^2$$

8.3.4. T-Section. The shear stress distribution over a T-section is obtained in the same manner as over an I-section. But in this case the position of neutral axis (*i.e.*, position of C.G.) is to be obtained first, as the section is not symmetrical about *x-x* axis. The shear stress distribution diagram will also not be symmetrical.

Problem 8.9. *The shear force acting on a section of a beam is 50 kN. The section of the beam is of T-shaped of dimensions 100 mm × 100 mm × 20 mm as shown in Fig. 8.12. The moment of inertia about the horizontal neutral axis is 314.221 × 10^4 mm^4. Calculate the shear stress at the neutral axis and at the junction of the web and the flange.*

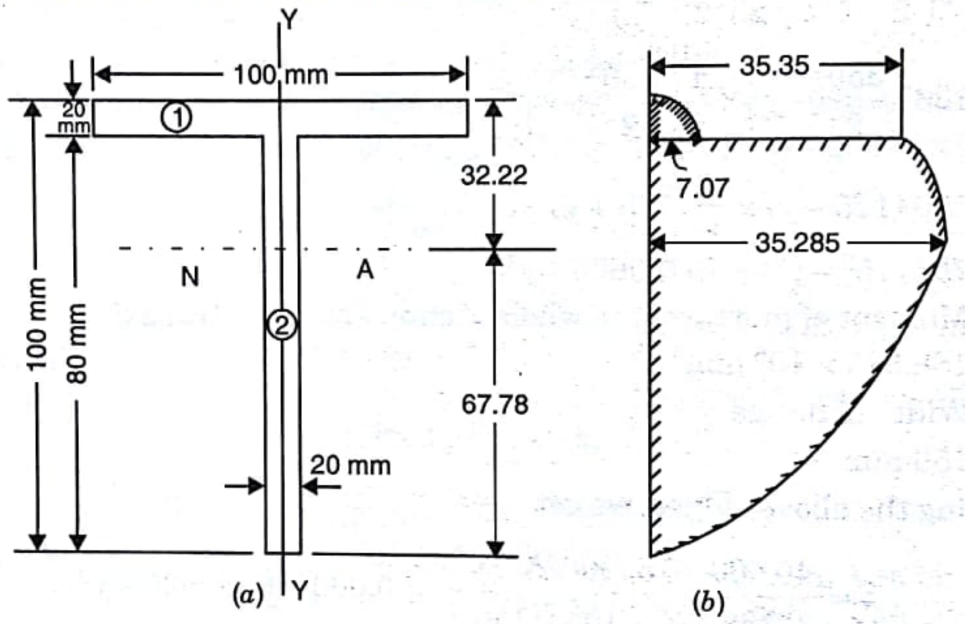


Fig. 8.12

Sol. Given :

Shear force, $F = 50 \text{ kN} = 50000 \text{ N}$

Moment of inertia about N.A.,

$$I = 314.221 \times 10^4 \text{ mm}^4.$$

First calculate the position of neutral axis. This can be obtained if we know the position of C.G. of given T-section. The given section is symmetrical about the axis *Y-Y* and hence the C.G. of the section will lie on *Y-Y* axis.

Let y^* = Distance of the C.G. of the section from the top of the flange.

Then $$y^* = \frac{A_1 y_1 + A_2 y_2}{(A_1 + A_2)}$$

$$= \frac{(100 \times 20) \times 10 + (20 \times 80) \times \left(20 + \frac{80}{2}\right)}{(100 \times 10) + (10 \times 90)}$$

$$= \frac{20000 + 96000}{2000 + 1600} = 32.22.$$

Hence, neutral axis will be at a distance of 32.22 mm from the top of the flange as shown in Fig. 8.12 (*a*).

Shear stress distribution in the flange

Now the shear stress at the top edge of the flange, and bottom of the web is zero.

Shear stress in the flange just at the junction of the flange and web is given by,

$$\tau = \frac{F \times A\bar{y}}{I \times b}$$

where $A = 100 \times 20 = 2000 \text{ mm}^2$

$\bar{y}$ = Distance of C.G. of the area of flange from N.A.

$$= 32.22 - \frac{20}{2} = 22.22 \text{ mm}$$

b = Width of flange = 100 mm

$$\therefore \quad \tau = \frac{50000 \times 2000 \times 22.22}{314.221 \times 10^4 \times 100} = 7.07 \text{ N/mm}^2.$$

Shear stress distribution in the web

The shear stress in the web just at the junction of the web and flange will suddenly increase from 7.07 N/mm^2 to $7.07 \times \frac{100}{20} = 35.35$ N/mm^2. The shear stress will be maximum at N.A. Hence shear stress at the N.A. is given by

$$\tau = \frac{F \times A\bar{y}}{I \times b}$$

where $A\bar{y}$ = Moment of the above N.A. about N.A.

= Moment of area of flange about N.A. + Moment of area of web about N.A.

$$= 20 \times 100 \times (32.22 - 10) + 20 \times (32.22 - 10) \times \frac{22.22}{2}$$

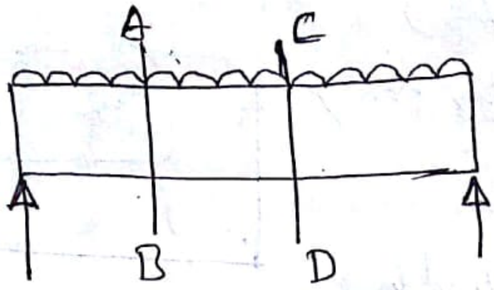
$$= 44440 + 4937.28 = 49377.284 \text{ mm}^2$$

$b = 20$ mm

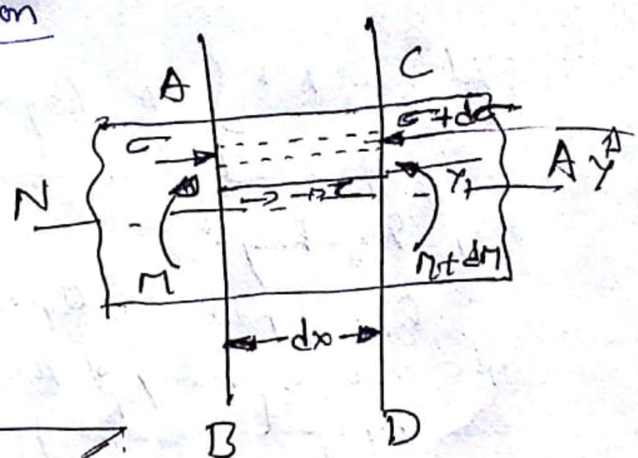
$$\therefore \quad \tau = \frac{50000 \times 49377.284}{314.221 \times 10^4 \times 20} = 39.285 \text{ N/mm}^2$$

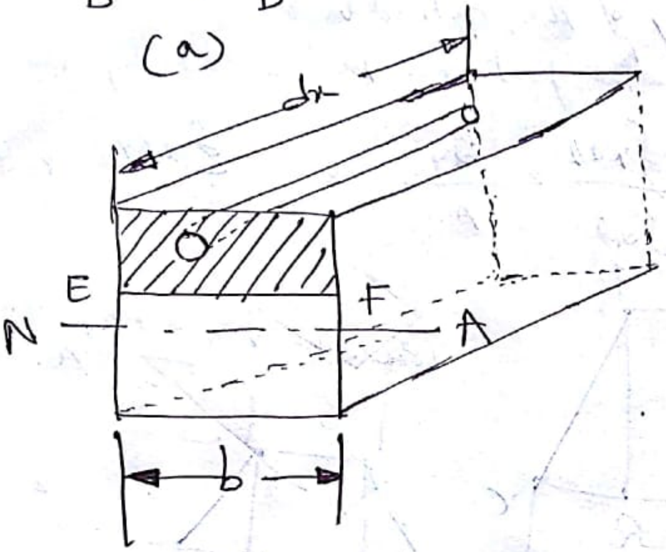
Now the shear stress distribution diagram can be drawn as shown in Fig. 8.12 (*b*).

Shear Stress at a Section



(a)





$$\sigma = \frac{M}{I} y.$$

Bending stress is a function of bending moment and the distance y from NA.

starting $\sigma = \frac{M}{I} \times y$

at the end of the cylinder $\sigma + d\sigma = \frac{(M+dM)}{I} \times y$

Force on the end of the element cylinder on the section AB

$= \text{stress} \times \text{Area of elemental cylinder}$

$= \sigma \times dA$

$= \frac{M}{I} \times y \times dA$

similarly, $(\sigma + d\sigma)\, dA$

$= \frac{(M+dM)}{I} \times y \times dA$

Net unbalanced force on the elemental cylinder

$$= \frac{(M+dM)}{I} \times y \times dA - \frac{M}{I} \times y \times dA$$

$$= \frac{dM}{I} \times y \times dA$$

A = Area of the section above the level EF

$\bar{y}$ = distance of the C.G of the area A from the NA

Shear Resistance at the level EF & shear force

= Total unbalanced force

$$= \frac{dM}{I} \times A \times \bar{y} \quad \text{——— (i)}$$

Let τ = Intensity of horizontal shear at the level EF

b = width of beam at the level EF

∴ Area on which τ is acting.

$= b \times dx$

Shear force due to τ

= shear stress × shear area

$$= \tau \times b \times dx \quad \text{——— (ii)}$$

Equating the two value of shear force given by equation (i & ii).

we get,

$$\tau \times b \times dx = \frac{dM}{I} \times A \times \bar{y}$$

$$\tau = \frac{dM}{I} \times \frac{A \times \bar{y}}{b \times dx}$$

$$\boxed{\frac{dM}{dx} = F.}$$

shear force

$$\boxed{\tau = F \times \frac{A\bar{y}}{I \times b}}$$