Stress: 'σ'

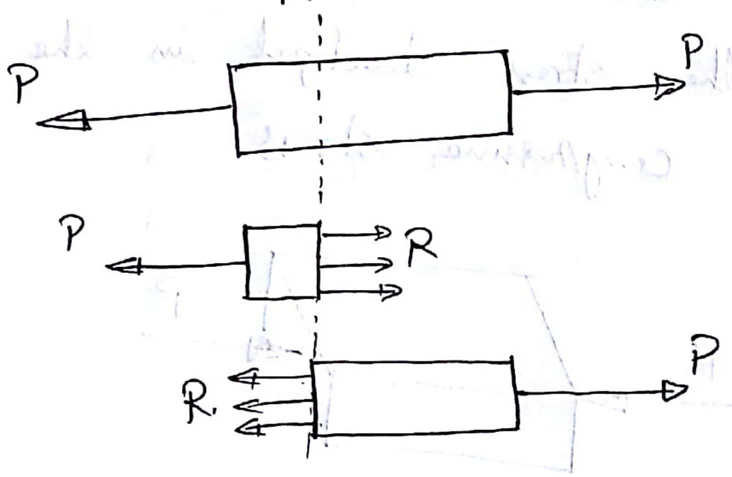
When a material is subjected to an external force, a resisting force is set up within the Component. The internal resistance force per unit area acting on a material is called the stress at a point.

$$\sigma = \frac{P}{A}$$

P is Force applied on the body in newtons (N).

A is ~~Are~~ Cross sectional area before applying load. in m^2.

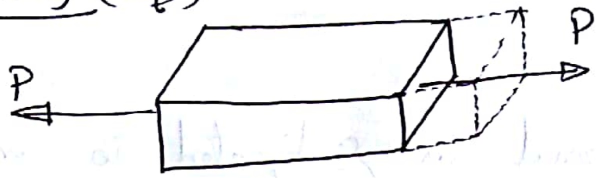
units $\frac{N}{m^2}$ or Pascal.



Types of Stresses

- Tensile Stress
- Compressive Stress
- Shear stress.

Tensile Stress (σ_t)

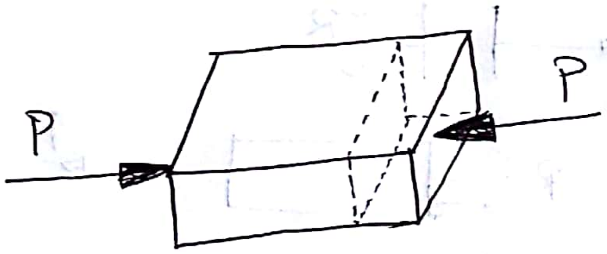


If a member is subjected to external force tensile 'P', then the fibres of the component tend to elongate, the stress developed in the body is called tensile stress.

$$\sigma_t > 0$$

Compressive Stress (σ_c)

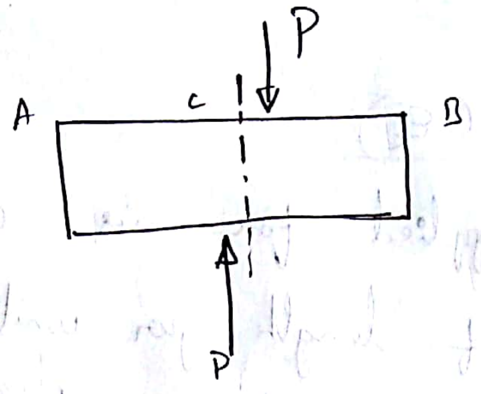
The fibres of the component tend to shorten due to the external compressive force then the stress developed in the body is called compressive stress.

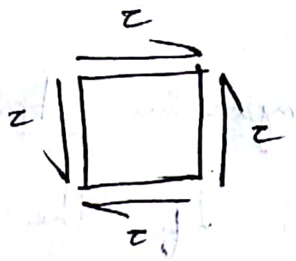


$$\sigma_c < 0$$

Shear Stress (τ)

When forces are transmitted from one part of a body to other, the stresses developed in a plane parallel to the applied force are the shear stress.



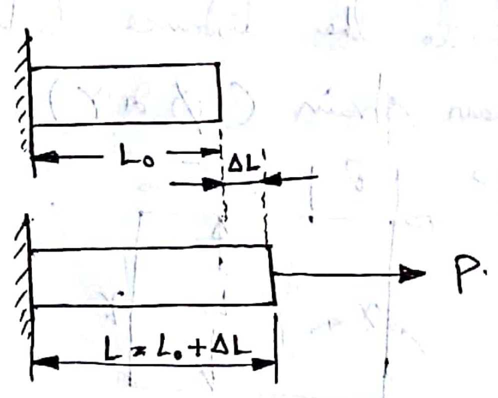


Shear stress acts parallel to plane of interest.

$$\tau = \frac{P}{Area} \quad P \rightarrow \text{shear force.}$$

Strain 'ε'

When an external load is applied on a member, the displacement per unit length is known as strain (dimensionless)



$$\varepsilon = \frac{\Delta L}{L_0}$$

Tensile Strain 'ε_t'

If the applied force is compressive then the increase in length per unit length is known as tensile strain.

$$\varepsilon_t = \frac{\Delta L}{L_0}$$

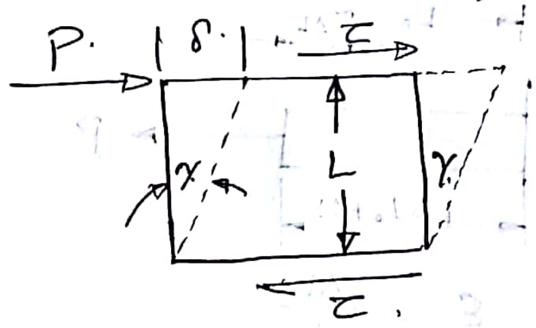
Compressive Strain (ε_c)

If the applied force is compressive then the reduction of length per unit length is known as compressive strain. It is negative.

$$\varepsilon_c = \frac{-\Delta L}{L_0}$$

Shear Strain (γ)

When a force P is applied tangentially to the body in opposite direction, the body tends to shear off across the section, the transversal displacement to the distance between their faces is called shear strain (ϕ or γ)



$$\gamma = \frac{\delta}{L}$$

Hook's law

Hook's law states that when a material is loaded within elastic limit, the stress is proportional to the strain. The ratio of the stress to the corresponding strain is a constant within the elastic limit.

$\sigma = E\varepsilon$ and $\tau = G\phi$.

Modulus of Elasticity or Young's Modulus or Elastic Modulus

The ratio of tensile stress or compressive stress to the corresponding strain is a constant. This ratio is known as young's modulus.

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \text{ or } \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

$$E = \frac{\sigma}{e}.$$

Modulus of Rigidity or Shear Modulus

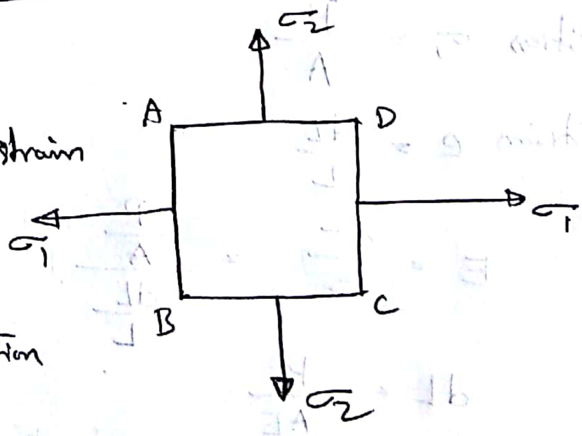
The ratio of shear stress to the corresponding shear strain within the elastic limit, is known as modulus of Rigidity. This is denoted by C or G or N.

$$C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Relationship between stress and strain

Two dimensional

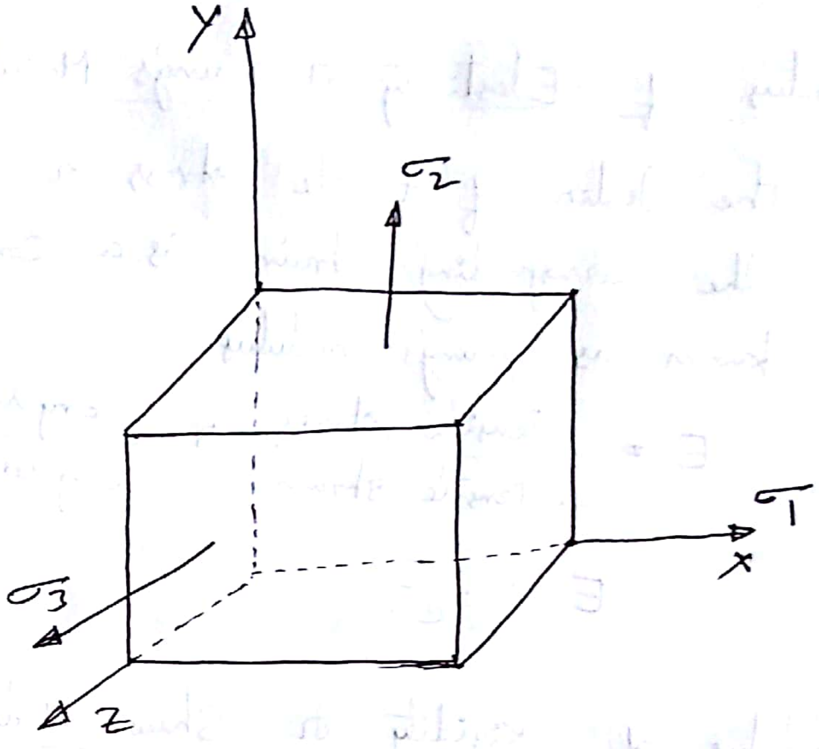
lateral strain = $-\mu \times$ longitudinal strain



$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$
in x-direction

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$
in y-direction

Three - Dimensional Stress system

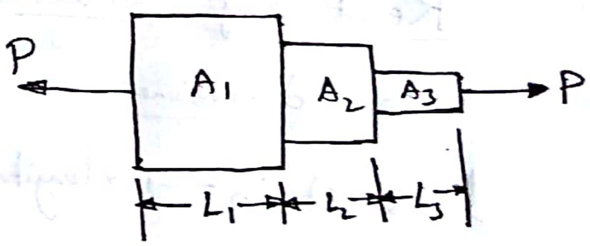


$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E}$$

$$e_3 = \frac{\sigma_3}{E} - \mu\frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

Analysis of bars of Varying sections



Stress $\sigma_1 = \frac{P}{A}$.

Strain $e = \frac{dL}{L}$

$$E = \frac{\sigma}{e} = \frac{\frac{P}{A}}{\frac{dL}{L}}$$

$$dL = \frac{PL}{AE}$$

Total change in length 'dL' $= P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_2} + \frac{L_3}{A_3E_3}\right]$

Factor of Safety

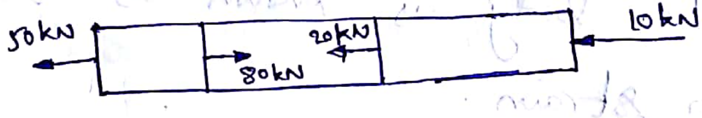
It is defined as the ratio of ultimate tensile stress to the working (or Permissible) stress.

$$\text{Factor of Safety} = \frac{\text{Ultimate stress}}{\text{Permissible stress}}.$$

Principle of Superposition

When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads.

Ex:-



Modular Ratio:

The ratio of $\frac{E_1}{E_2}$ is called the modular ratio of the first material to the second.

Poisson's Ratio:

The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ.

$$\text{Poisson's ratio, } \mu = \frac{\text{Lateral Strain}}{\text{Longitudinal Strain}}$$

The value of 'μ' varies from 0.25 to 0.33 for metals it is 0.45 to 0.50 for rubber.

Longitudinal Strain

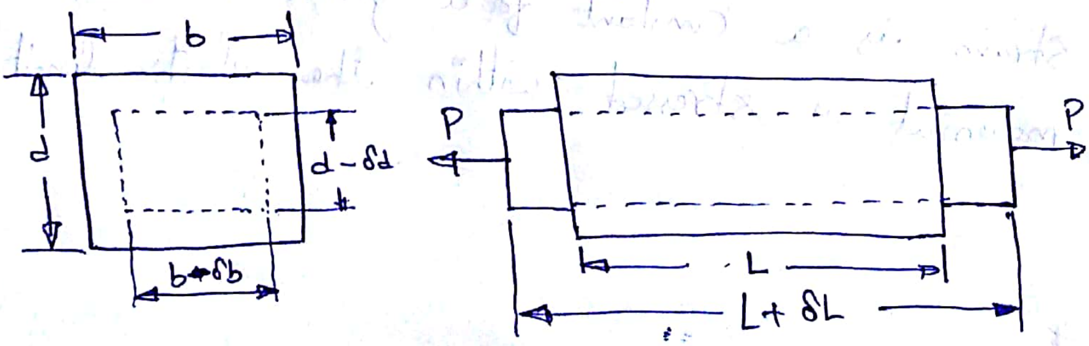
When a body is subjected to an axial tensile or compressive load, there is an axial deformation in the length of the body. The ratio of axial deformation to the original length of the body is known as longitudinal or linear strain.

$$\text{Longitudinal strain} = \frac{\delta L}{L}$$

Lateral Strain

The strain at right angles to the direction of applied load is known as lateral strain

$$\text{Lateral Strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$



Volumetric Strain

The ratio of change in volume to the original volume of a body is called Volumetric strain. It is denoted by e_v,

$$e_v = \frac{\delta V}{V}$$

where δV = change in volume, and
V = Original volume.

Bulk Modulus:-

When a body is subjected to the mutually perpendicular like and equal direct stresses, the ratio of direct stress to the corresponding volumetric strain is found to be constant for a given material when the deformation is within a certain limit. This ratio is known as bulk modulus and is usually denoted by 'K'

$$K = \frac{\text{Direct stress}}{\text{Volumetric Strain}} = \frac{\bar{\sigma}}{\left(\frac{dV}{V}\right)}$$

Principle Plane

The plane, which have no shear stress, are known as principle planes, Hence principal planes are the planes of zero shear stress. These planes carry only normal stresses.

Principal Stresses

The normal stresses, acting on a principal plane, are known as principal stresses.

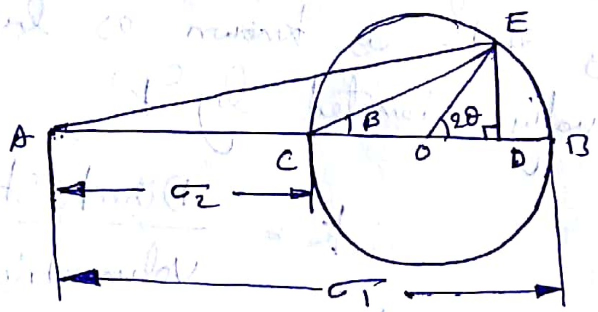
Mohr's Circle

Mohr's circle is a graphical method of finding normal, tangential and resultant stresses on an oblique plane. Mohr's circle will be drawn for the following cases:

i) A body subjected to two mutually perpendicular principal tensile stresses of unequal intensities.

ii) A body subjected to two mutually perpendicular principal stresses which are unequal and unlike (i.e, one is tensile and other is compressive).

iii) A body subjected to two mutually perpendicular principal tensile stresses accompanied by a simple shear stress.



Resilience: The total strain energy stored in a body is called resilience. whenever the straining force is removed from the strained body, the body is capable of doing work. Hence the resilience is also defined as the capacity of a strained body for doing work on the removal of the straining force.

$$U = \frac{\sigma^2}{2E} \times V \qquad \left[V = \text{volume} \atop A \times L \right]$$

Proof Resilience: The maximum strain energy, stored in a body, is known as proof resilience. The strain energy stored in the body will be maximum when the body is stressed upto elastic limit. Hence the proof resilience is the quantity of strain energy stored in a body when strained upto elastic limit.

$$= \frac{\sigma^{*2}}{2E} \times \text{volume}.$$

$\sigma^* =$ stress at the elastic limit.

Modulus of Resilience: It is defined as the proof resilience of a material per unit volume. It is an important property of a material.

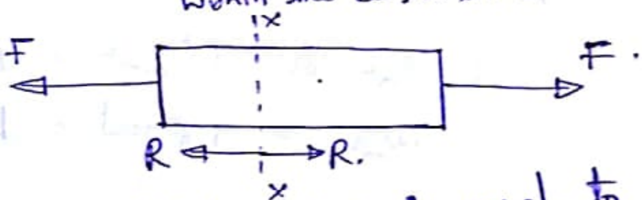
$$\text{Modulus of resilience} = \frac{\text{Proof resilience}}{\text{Volume of the body}}$$

$$= \frac{\sigma^2}{2E}$$

Simple Stresses and Strains

Introduction: When an external force acts on a body, the body tends to undergo some deformation. Due to cohesion between the molecules, the body resists deformation. This resistance by which material of the body opposes the deformation is known as Strength of material.

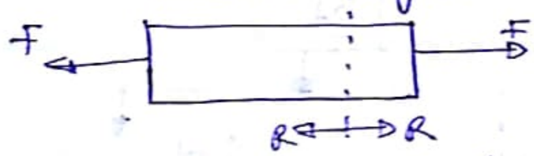
within the elastic limit



→ resisting force is equal to F applied

$\boxed{R = F}$

beyond the elastic limit



→ Resisting force is less than the applied load.

$\boxed{R < F}$

Within elastic stage, the resisting force equals applied load. This resisting force per unit area is called stress or intensity of stress.

Stress:-

Def: The force of resistance per unit area, offered by a body against deformation is known as stress. The external force acting on the body is called the load or force.

Mathematically stress is written as, $\sigma = \frac{P}{A}$

where σ = Stress

P = External force or load, and

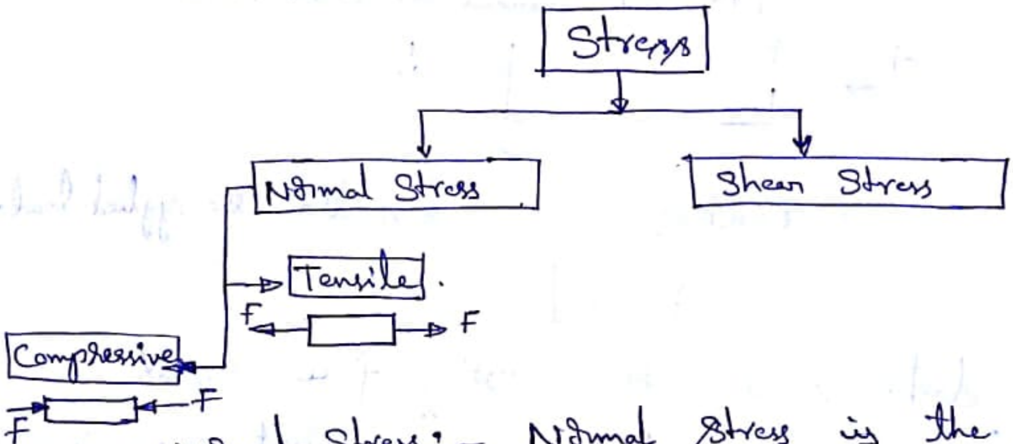
A = Cross - Sectional area

Units of stress: The unit of stress depends upon the unit of load (or force) unit of area.

$$1 \ N/m^2 \text{ or } 1 \ N/mm^2 = 10^6 \ N/m^2$$

$$1 \ N/m^2 = 1 \text{ pascal} = 1 \ Pa = 1 \times 10^{-5} \text{ bar}.$$

Types of stresses

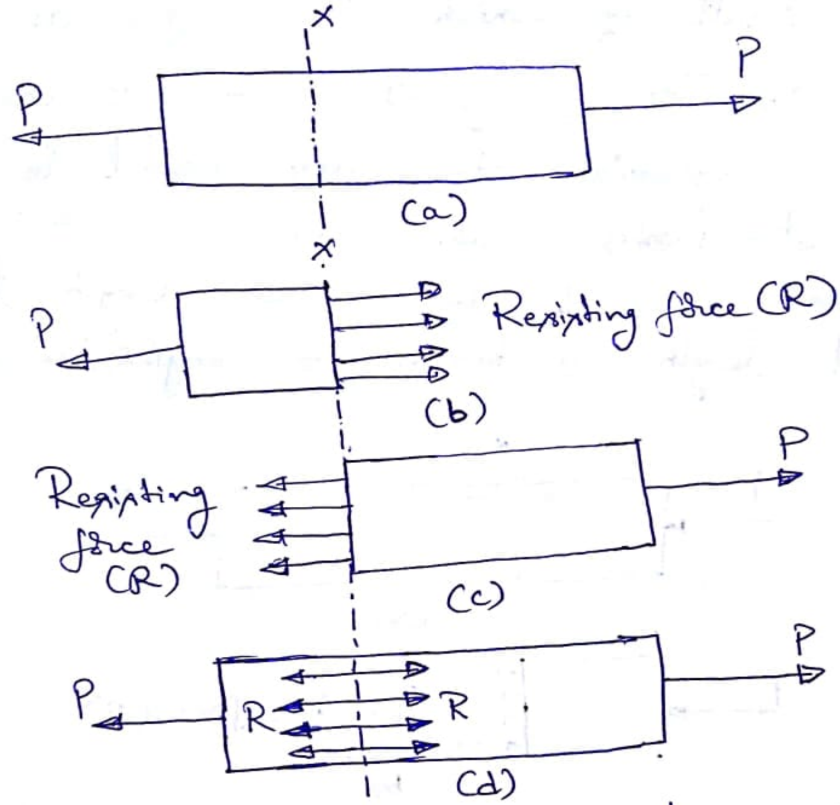


Normal Stress:- Normal stress is the stress which acts in a direction perpendicular to the area. It is represented by σ (sigma).

Tensile Stress:- The stress induced in a body, when subjected to two equal and opposite pulls, as a result of which there is an increase in length, is known as tensile stress.

→ The tensile stress acts normal to the area and it pulls on the area

→ The ratio of increase in length to the original length is known as tensile strain.



Where P = Pull (or force) acting on the body
A = Cross-sectional area of the body
L = Original length of the body.
dL = Increase in length
σ = Stress induced in the body, and
e = Strain (ie tensile strain)

Tensile Stress(σ) = $\dfrac{\text{Resisting force (R)}}{\text{Cross-sectional area(A)}} = \dfrac{\text{Tensile load}}{A}$

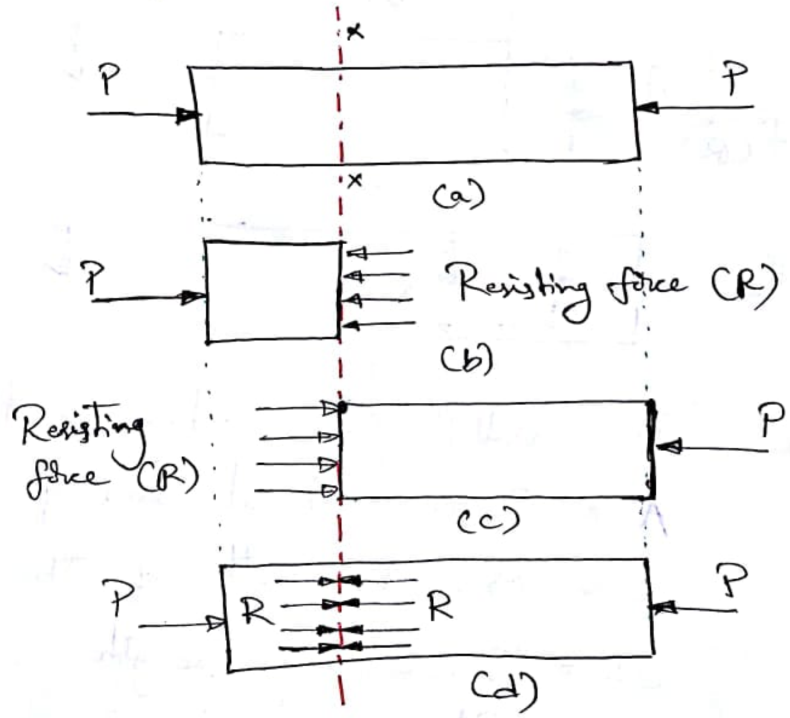
$$\sigma = P/A$$

Tensile strain (e) = $\dfrac{\text{Increase in length}}{\text{Original length}} = \dfrac{dL}{L}$

Compressive Stress: The stress induced in a body, when subjected to two equal and opposite pushes, as a result of which there is a decrease in length of the body, is known as compressive stress.

→ The compressive stress acts normal to the area and it pushes on the area.

→ The ratio of decrease in length to the original length is known as compressive strain.



$$\text{Compressive Stress}(\sigma) = \frac{\text{Resisting Force (R)}}{\text{Area (A)}} = \frac{\text{Push (P)}}{\text{Area(A)}}$$

$$= \frac{P}{A}$$

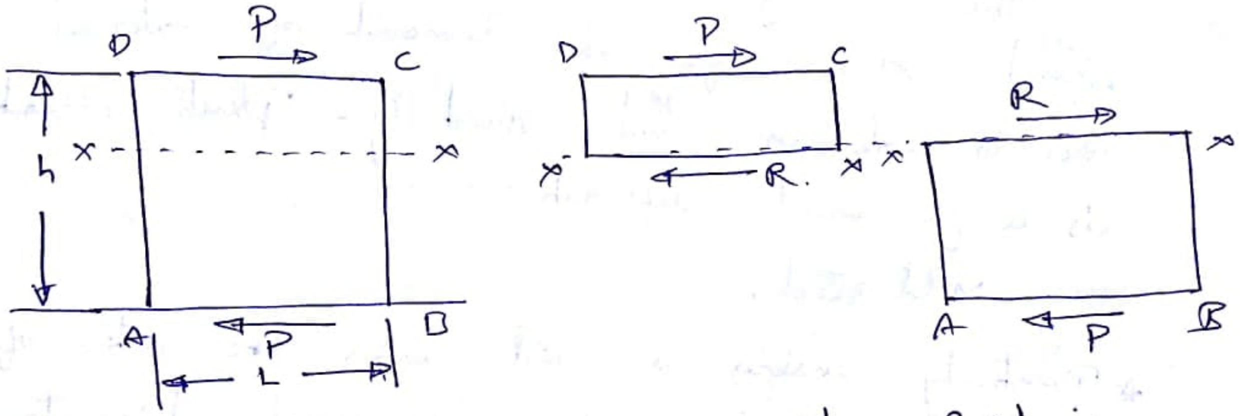
$$\text{Compressive Strain (e)} = \frac{\text{Decrease in length}}{\text{Original length}}$$

$$= \frac{dL}{L}$$

⑤

Shear Stress: the stress induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as shown in fig below, as a result of which the body tends to shear off across the section, is known as shear stress.

→ the corresponding strain is known as shear strain



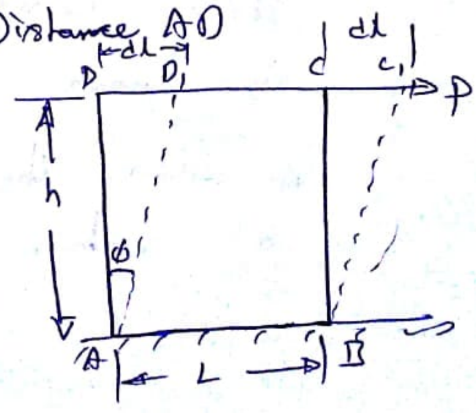
$$\text{Shear stress } (\tau) = \frac{\text{Shear resistance}}{\text{Shear area}}$$

$$= \frac{R}{A} \quad \left(\begin{array}{l} R = P \text{ and} \\ A = L \times 1 \end{array}\right).$$

$$= \frac{P}{L \times 1}$$

$$\text{Shear Strain } (\phi) = \frac{\text{Transversal displacement}}{\text{Distance } AD}$$

$$\phi = \frac{DD_1}{AD} = \frac{dl}{h}$$



Elasticity:

The ability of a material to resume its normal shape after the removal of external force or load is called elasticity. Elastic deformation is a non-permanent deformation.

ex: Rubber, steel → within the elastic limit.

Plasticity:

The ability of a material to change its normal shape after the removal of external force & load is called plasticity. Plastic deformation is a permanent deformation.

ex:- Mild steel,

* plasticity enables a solid under the action of external forces to undergo permanent deformation without rupture.

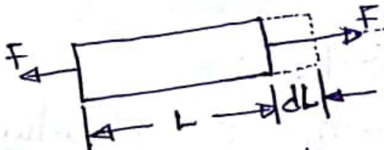
Elastic limit

The limiting value of force upto and within which, the deformation completely disappears on the removal of the force. The value of stress corresponding to this limiting force is known as the elastic limit of the material.

* when the resistance or stress is within a certain limit is called elastic limit.

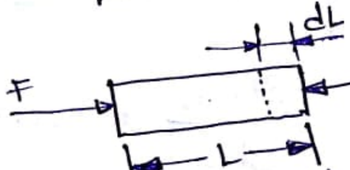
Strain: when a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to the original dimension is known as strain. Strain is dimensionless.

1) Tensile strain:



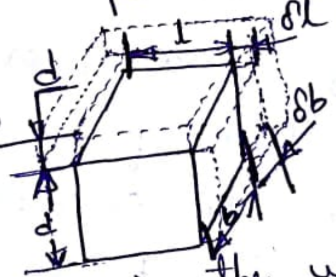
$\varepsilon_T = \frac{dL}{L}$ (+ve)

2) Compressive strain:



$\frac{dL}{L}$ (-ve)

3) Volumetric strain:



$V = lbd$

$\delta V = \delta l\, bd + \delta b\, ld + \delta d\, lb$

$\frac{\delta V}{V} = \frac{\delta l}{l} + \frac{\delta b}{b} + \frac{\delta d}{d}$

$\varepsilon_v = \varepsilon_x + \varepsilon_y + \varepsilon_z$

. The volumetric strain is the unit change in volume, ie the change in volume divided by the original volume.

4) Shear strain is defined as change of angle of side faces that are originally perpendicular to each other.

Hooke's Law

Hooke's law states that when a material is loaded within elastic limit, the stress is proportional to the strain produced by the stress. This means the ratio of the stress to the corresponding strain is a constant within the elastic limit. This constant is known as Modulus of Elasticity or Modulus of Rigidity or Elastic Modulsi.

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$$

Modulus of Elasticity or Young's Modulus

The ratio of tensile stress or compressive stress to the corresponding strain is a constant. This ratio is known as Young's Modulus or Modulus of Elasticity and is denoted by E.

$$E = \frac{\text{Tensile stress}}{\text{Tensile strain}} \text{ or } \frac{\text{Compressive stress}}{\text{Compressive strain}}$$

$$E = \frac{\sigma}{e}$$

Modulus of Rigidity or Shear Modulus

The ratio of shear stress to the corresponding shear strain ~~etc~~ within the elastic limit is known as Modulus of Rigidity or shear Modulus. This is denoted by C or G or N.

$$C = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

Factor of Safety

It is defined as the ratio of ultimate tensile stress to the working (or permissible) stress.

$$F \cdot S = \frac{\text{Ultimate Stress}}{\text{Permissible stress}}$$

Working Stress:

The working stress or allowable stress is the maximum safe stress a material may withstand.

Two dimensional stress System

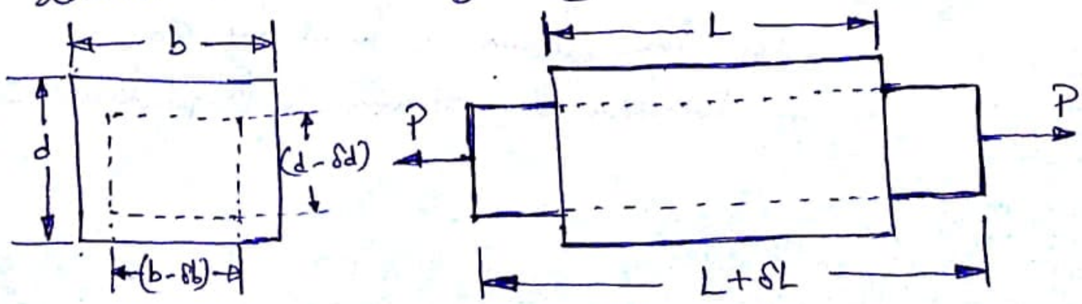
Longitudinal Strain: When a body is subjected to an axial tensile load, there is an increase in the length of the body.

The longitudinal strain is also defined as the deformation of the body per unit length in the direction of the applied load.

$$\text{longitudinal strain} = \frac{\delta L}{L}$$

Lateral Strain: the strain at right angles to the direction of applied load is known as lateral strain.

$$\text{lateral strain} = \frac{\delta b}{b} \text{ or } \frac{\delta d}{d}$$



Poisson's Ratio: The ratio of lateral strain to the longitudinal strain is a constant for a given material, when the material is stressed within the elastic limit. This ratio is called Poisson's ratio and it is generally denoted by μ

$$\text{Poisson's ratio } \mu = \frac{\text{Lateral strain}}{\text{longitudinal strain}}$$

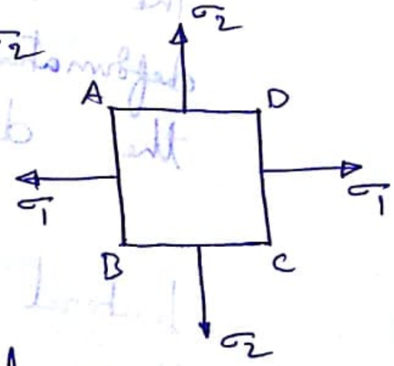
lateral strain = $\mu \times$ Longitudinal strain

As lateral strain is opposite in sign to longitudinal strain, hence algebraically, lateral strain is written as

Lateral strain = $-\mu \times$ Longitudinal strain

Relationship between stress and Strain

Consider a two-dimensional figure ABCD, subjected to two mutually perpendicular stresses σ_1 and σ_2



σ_1 = Normal stress in x-direction

σ_2 = Normal stress in y-direction

Consider the strain Produced by σ_1

in x direction the strain is longitudinal $\frac{\sigma_1}{E}$

in y direction the strain is lateral $-\mu \frac{\sigma_1}{E}$

Consider the strain Produced by σ_2

in y direction the strain is longitudinal $\frac{\sigma_2}{E}$

in x direction the strain is lateral $-\mu \frac{\sigma_2}{E}$

e_1 = Total strain in x- direction

e_2 = Total strain in y- direction.

Now total strain in the direction of x due to stresses σ_1 and σ_2 = $\frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$

Similarly in y- direction

$$= \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

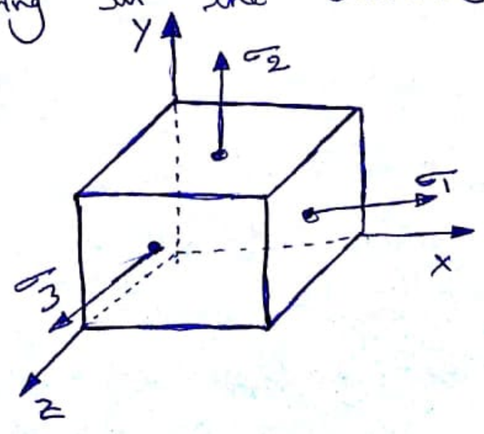
$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$$

$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$$

The above two equations give the stress and strain relationship for the two-dimensional stress system. In the above equations, tensile stress is taken to be positive whereas the compressive stress negative.

For Three-Dimensional Stress System:

A three-dimensional body subjected to three orthogonal normal stresses $\sigma_1, \sigma_2, \sigma_3$ acting in the directions of x, y and z respectively



Consider the strain produced by σ_1 stress in x direction is longitudinal $\frac{\sigma_1}{E}$

in y & z direction the strain is lateral $= -\mu \frac{\sigma_2}{E}$
& $-\mu \frac{\sigma_3}{E}$

Total strain in x - direction

$$e_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} \quad \text{——— (i)}$$

Similarly total strain in the direction of y due to stresses σ_1, σ_2 and σ_3

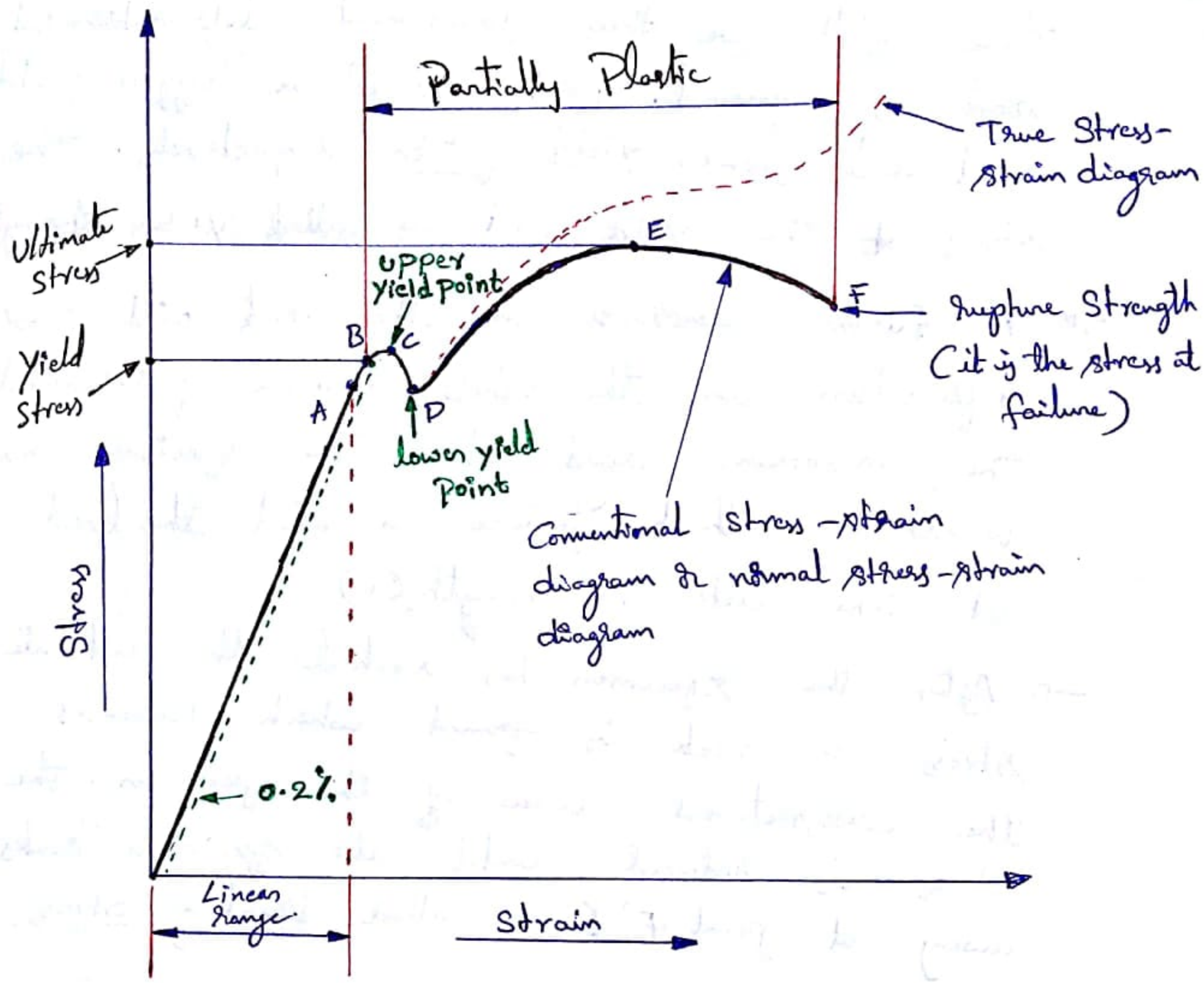
$$e_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} \quad \text{——— (ii)}$$

and the total strain in the direction of z due to stresses σ_1, σ_2 and σ_3

$$e_3 = \frac{\sigma_3}{E} - \mu \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} \quad \text{——— (iii)}$$

The above three equations give the stress and strain relationship for the three orthogonal normal stress system.

Tensile Test Curve for Mild Steel. (Stress-Strain Curve)



→ So it is evident from the graph that the strain is proportional to strain 'δ' elongation is proportional to the load giving a straight line relationship. This law of proportionality is valid upto a point A (proportionality limit) : 'A' (upto).

→ For a short period of time beyond the point A, the material may still be elastic in the sense that the deformations are completely recovered when the load is removed. The limiting point B is termed as Elastic limit.

→ Beyond the elastic limit plastic deformation occurs and strains are not totally recoverable. There will be thus permanent deformation when load is removed. Point 'C' & 'D' are upper yield point and lower yield points respectively. The stress at the yield point is called yield strength.

→ A Further increase in the load will cause deformation in the whole volume of the metal. The maximum load which the specimen can withstand without failure is called the load at the ultimate strength (E)

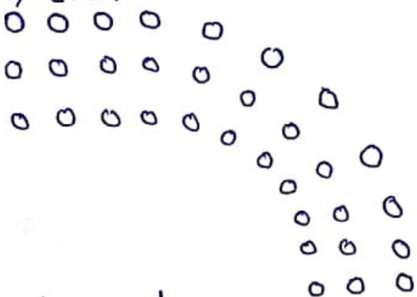
→ After the specimen has reached the ultimate stress a neck is formed which decreases the cross sectional area of the specimen. The stress is reduced until the specimen breaks away at point 'F' & is called Breaking stress

Stress: Stress is a force applied over an area for a pressure force

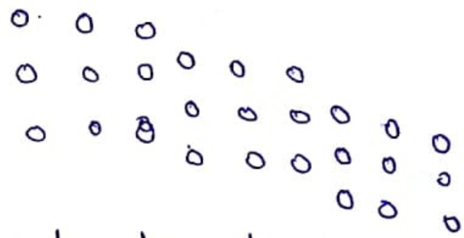
Strain: Strain is the change in length in the direction of load.

Elastic deformation is a non-permanent deformation

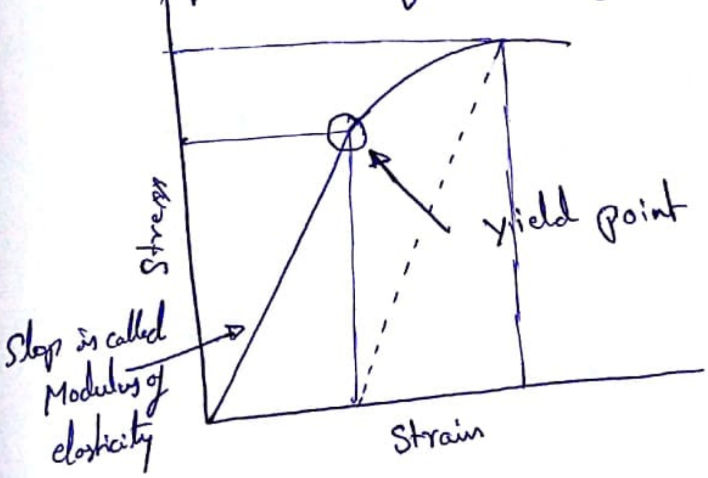
Ex: Rubber, steel



Ex: Al, MS

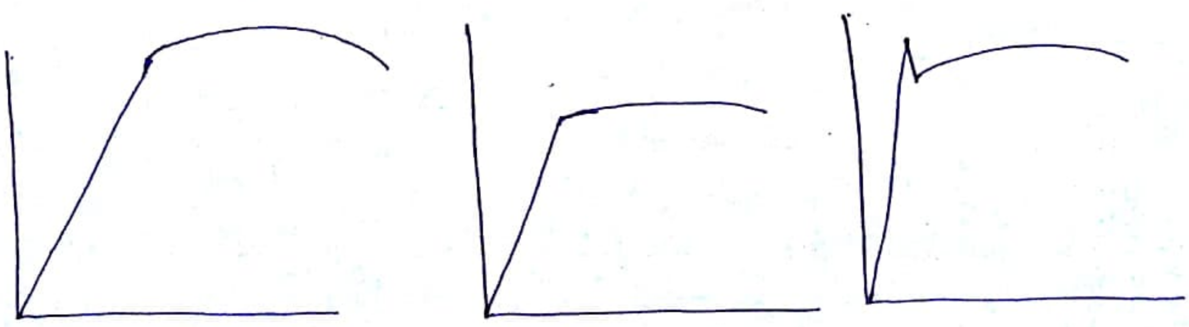


Plastic deformation is a permanent deformation.



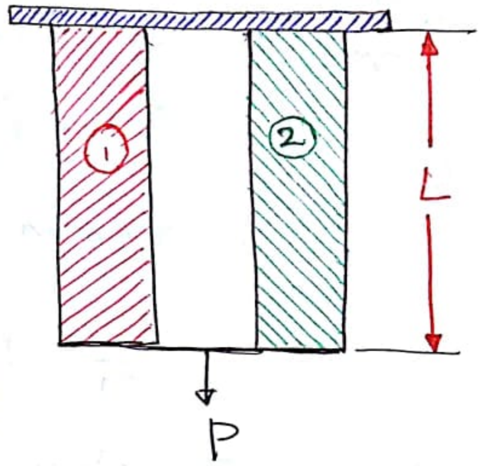
Stress = E x strain

Aluminum　　Galvanized Steel　　Steel



Analysis of Bars of Composite Sections

A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with with each other and behaving as one unit for extension or compression when subjected to an axial tensile or compressive loads, is called a composite bar.



Note:

1) The extension or compression in each bar is equal. Hence the deformation per unit length ie., strain in each bar is equal.
2) The total external load on the composite bar is equal to the sum of the loads carried by each different material.

Let: P = Total load on the composite bar,
L = Length of composite bar and also length of bars of different materials
A_1 = Area of cross-section of bar 1,
A_2 = Area of cross-section of bar 2,
E_1 = Youngs modulus of bar 1,
E_2 = Youngs modulus of bar 2,
P_1 = Load shared by bar 1,
P_2 = Load shared by bar 2,
σ_1 = Stress induced in bar 1, and
σ_2 = Stress induced in bar 2.

Now the total load on the composite bar is equal to the load carried by the two bars.

$$P = P_1 + P_2 \quad \text{——— (i)}$$

The stress in bar, 1, $= \dfrac{\text{Load carried by bar 1}}{\text{Area of cross-section of bar 1}}$

$$\sigma_1 = \frac{P_1}{A_1} \quad \text{——— (ii)}$$

The stress in bar 2,

$$\sigma_2 = \frac{P_2}{A_2} \quad \text{——— (iii)}$$

$$P_1 = \sigma_1 A_1 \, , \quad P_2 = \sigma_2 A_2$$

Substituting above P_1 & P_2 in eq (i)

$$\boxed{P = \sigma_1 A_1 + \sigma_2 A_2} \quad \text{——— (iv)}$$

Since the ends of the two bars are rigidly connected, each bar will change in length by the same amount. Also the length of each bar is same and hence the ratio of change in length to the original length will be same for each bar.

But strain in bar 1, $= \dfrac{\text{Stress in bar 1}}{\text{Youngs modulus of bar 1}} = \dfrac{\sigma_1}{E_1}$

similarly strain in bar 2, $= \dfrac{\sigma_2}{E_2}$

But strain in bar 1 = strain in bar 2.

$$= \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad \text{——— (v)}$$

$$\boxed{\sigma_1 = \frac{E_1}{E_2}\sigma_2}$$

Modular Ratio: The ratio of $\frac{E_1}{E_2}$ is called the modular ratio of the first material to the second.

Thermal Stresses

Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are setup in a body, when the temperature of the body is raised & lowered and the body is not allowed to expand & contract freely. But if the body is allowed to expand & contract freely, no stresses will be set up in the body.

Consider a body which is heated to a certain temperature.

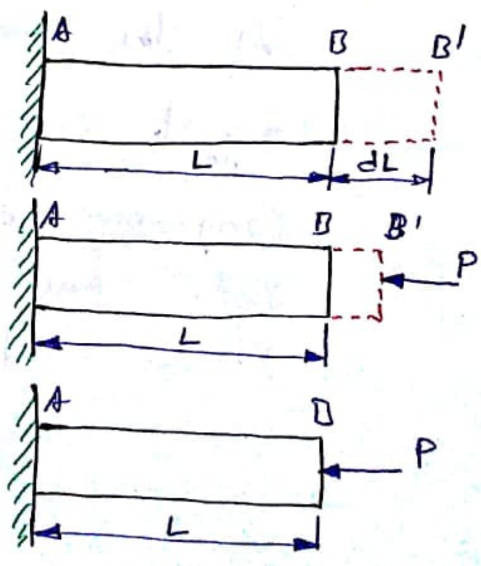
Let, L = Original

T = Rise in temperature

E = Young's Modulus

α = Co-efficient of linear expansion

dL = Extension of rod due to rise of temperature



If the rod is free to expand, then extension of the rod is given by

$dL = \alpha \cdot T \cdot L$

AB = Original length

BB' = increase in length due to temperature raise

Now suppose that an external compressive load, P is applied at B' so that the rod is decreased in its length from $(L+\alpha TL)$ to L

then Compressive strain = $\dfrac{\text{Decrease in length}}{\text{Original length}}$

$$= \frac{\alpha \cdot T \cdot L}{L+\alpha TL}$$

$$= \frac{\alpha TL}{L} = \alpha T$$

$$\boxed{\text{Compressive strain} = \alpha T}$$

But $\dfrac{\text{stress}}{\text{strain}} = E$

stress = strain × E

$$\boxed{\text{stress} = \alpha . T . E}$$

Load or thrust on the rod = stress × Area

$= \alpha . T . E \times A$

$$\boxed{P = \alpha . T . E . A}$$

If the ends of the body are fixed to rigid supports, so that its expansion is prevented, then compressive stress and strain will be setup in the rod. these stresses and strains are known as thermal stresses and thermal strain.

Thermal strain, $e = \dfrac{\text{Extension prevented}}{\text{Original length}}$

$$\frac{dL}{L} = \frac{\alpha T L}{L} = \alpha T$$

And thermal stress, $\sigma = \alpha T E$

Thermal stress is known as temperature stress and thermal strain is also known as temperature strain.

Stress and strain when the supports yield.

If the supports yield by an amount equal to 'δ', then the actual expansion.

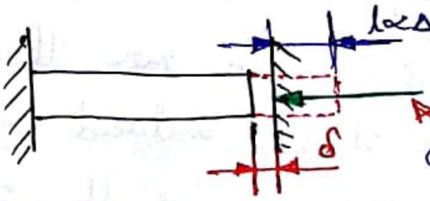
$$= \text{Expansion due to rise in temp} - \delta$$

$$= \alpha \cdot T \cdot L - \delta.$$

Thermal strain, $\boxed{e = \dfrac{\alpha \cdot T \cdot L - \delta}{L}}$

thermal stress $\boxed{\sigma = \dfrac{\alpha \cdot T \cdot L - \delta}{L} \times E}$

This can also be explain when the supports yield



$\boxed{\Delta t = T}$

Change in length due to mechanical load

Δl = change in length due to mechanical load + change in length due to thermal load.

$$\delta = \frac{-Pl}{AE} + l\alpha\Delta t$$

$$\frac{\sigma L}{E} = L\alpha\Delta t - \delta$$

$$\sigma = \left(\frac{L\alpha\Delta t - \delta}{L}\right)E$$

$$\boxed{\sigma = \left(\alpha\Delta t - \frac{\delta}{L}\right)E}$$

Thermal Stresses in Composite bars

Let us consider a composite bar consisting of two members (brass & steel). Let the composite bar be heated through some temperature. If the members are free to expand then no stresses will be induced in the members. But the two members are rigidly fixed and hence the composite bar as a whole will expand by the same amount. As the coefficient of linear expansion of brass is more than that of the steel, the brass will expand more than the steel. Hence the free expansion of brass will be more than that of the steel. But both the members are not free to expand, and hence the expansion of the composite bar, as a whole, will be less than that of the brass, but more than that of the steel. Hence the stress induced in the brass will be compressive whereas the stress in steel will be tensile and load or force on the brass will be compressive whereas on the steel the load will be tensile

Let A_b = Area of Cross-section of brass bar.

σ_b = Stress in brass

e_b = Strain in brass

α_b = Coefficient of linear expansion for brass

E_b = Young's modulus for copper.

A_s, σ_s, e_s and α_s = Corresponding values of area, stress, strain and Co-efficient of linear expansion for steel,

and E_s = Young's modulus for steel

δ = actual expansion of the composite bar.

Now load on the brass = stress in brass × Area of brass

$$= \sigma_b \times A_b$$

And load on the steel = $\sigma_s \times A_s$

For the equilibrium of the system, Compression in Copper should be equal to tension in the steel.

or load on the brass = load on the steel.

$$\boxed{\sigma_b \times A_b = \sigma_s \times A_s}$$

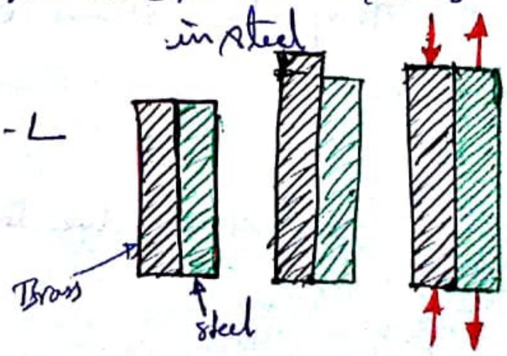
Also we know that actual expansion of steel

= Actual expansion of brass. — (i)

But actual expansion of steel =

free expansion of steel + Expansion due to tensile stress in steel

$$= \alpha_s \cdot T \cdot L + \frac{\sigma_s}{E_s} \cdot L$$



And actual expansion of Copper

= free expansion of Copper − Contraction due to Compressive stress induced in brass.

$$= \alpha_b \cdot T \cdot L - \frac{\sigma_b}{E_b} L$$

Substituting these values in equation (i), we get

$$\alpha_s \times T \times L + \frac{\sigma_s}{E_s} \times L = \alpha_b \times T \times L - \frac{\sigma_b}{E_b} \times L$$

$$\boxed{\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_b T - \frac{\sigma_b}{E_b}}$$

Hoop Stress:-

Thin steel ~~tyre~~ tyre of internal diameter d. Such a tyre can be shrunk on to a wheel of slightly bigger diameter D. The steel tyre is heated so that its diameter exceeds D. In this stage the steel tyre is slipped on to the wheel. If now the tyre be cooled it is prevented from assuming its original diameter 'd'. Hence it will grip the wheel.

Hence a tensile stress is induced circumferentially along the tyre. Such a stress is called a hoop stress

Temperature strain 'e' = $\frac{\text{Contraction Prevented}}{\text{Original length}} = \frac{\pi D - \pi d}{\pi d}$

$$e = \frac{D-d}{d}.$$

Hoop stress due to fall of temperature $\boxed{P = eE = \left(\frac{D-d}{d}\right)E}$

Resilience:- when a body is loaded, with in elastic limit, it changes its dimensions and on the removal of the load, it regains its original dimensions. So long as it remains loaded, it has stored energy itself. This energy which is absorbed in a body, when strained with in elastic limit is known as strain energy. The strain energy always capable of doing some work.

The strain energy stored in a body due to external loading. with in elastic limit is known as Resilience.

The max energy which can be stored in a body upto. the elastic limit called proof resilience.

The proof resilience per unit volume of a material is known as modulus of resilience

Let 'P' be the load causing the deformation. Let 'σ' be the stress in the member when the full extension δ has taken place.

$$\therefore \text{Stress} \quad \sigma = \frac{R}{A}$$

$$R = \sigma A = P$$

$\therefore$ Workdone against the resistance, on the member = strain energy stored by the member

= Average resistance $\times$ displacement

$$= \frac{R}{2}\delta.$$

but $e = \frac{\delta}{l}$ and $R = \sigma A$

$\therefore$ Strain energy stored by the member

$$= \frac{\sigma A}{2} \cdot e l$$

$$= \frac{1}{2} \sigma e (Al)$$

$= \frac{1}{2} \times$ stress $\times$ strain $\times$ volume of the member.

$$e = \frac{P}{E}$$

$\therefore$ Strain energy stored by the member

$$= \frac{1}{2} \times \sigma \times \frac{\sigma}{E} \times Al$$

$$= \frac{\sigma^2}{2E} Al$$

$\therefore$ Strain energy stored by the member per unit volume $= \frac{P^2}{2E}$ or $\boxed{\frac{\sigma^2}{2E}}$

Prob:1 A rod 10mm × 10mm cross-section is carrying an axial tensile load 10kN. What is the tensile stress developed in the rod.

$$\sigma = \frac{P}{A}$$

$$= \frac{10\,kN}{10 \times 10\ mm^2} = 100\ N/mm^2$$

$$= 100\,MPa.$$

Prob:2 A rod 100mm in original length. When we apply an axial tensile load 10kN the final length of the rod after application of the load is 100.1mm. So in this rod tensile strain is developed and is given by.

$$\varepsilon_t = \frac{\Delta L}{L_0} = \frac{L - L_0}{L_0}$$

$$= \frac{100.1\,mm - 100\,mm}{100\,mm}$$

$$= \frac{0.1\,mm}{100\,mm} = 0.001 \quad (\text{Tensile})$$

Prob:3 A block 100mm × 100mm base and 10mm height. When we apply a tangential force 10kN to the upper it is displaced 1mm relative to lower face. Then the direct shear stress in the element.

$$\tau = \frac{10\,kN}{100\,mm \times 100\,mm} = \frac{10 \times 10^3\,N}{100\,mm \times 100\,mm} = 1\,N/mm^2$$

$$= 1\,MPa.$$

And shear strain in the element $(\gamma) = \frac{1\,mm}{10\,mm} = 0.1$

Prob: 4 An elastic rod 25 mm in dia, 200mm long extends by 0.25 mm under a tensile load of 40 kN. Find the stress, strain and elastic modulus for the material of the rod.

Given data:

diameter of the rod(d) = 25mm

length of the rod(l) = 200 mm

change in length(δl) = 0.25 mm.

load P = 40kN

1) Stress $\sigma = \frac{P}{A}$

$$A = \frac{\pi \times 25^2}{4} = 490.87 \text{ mm}^2$$

$$= \frac{40 \times 10^3}{490.87} = 81.49 \text{ N/mm}^2$$

2) Strain $e = \frac{\delta l}{l} = \frac{0.25}{200} = 0.00125$

3) Elastic modulus $E = \frac{\sigma}{e}$

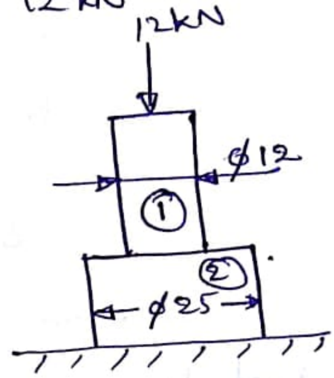
$$= \frac{81.49}{0.00125}$$

$$= 65192 \text{ N/mm}^2$$

$$= 0.65 \times 10^5 \text{ N/mm}^2$$

Analysis of bars of varying sections

Prob: 5 Find the maximum and minimum stresses produced in the stepped bar shown in Fig. due to an axially applied applied compressive load of 12 kN



Area of the upper part ①

$$A_1 = \frac{\pi}{4}(12)^2 = 113.10\,mm^2$$

Area of the lower part ②

$$A_2 = \frac{\pi}{4}(25)^2 = 490.87\,mm^2$$

Maximum stress

$$f_{max} = \frac{P}{A_1} = \frac{12\times10^3}{113.10} = 106.10\ N/mm^2$$

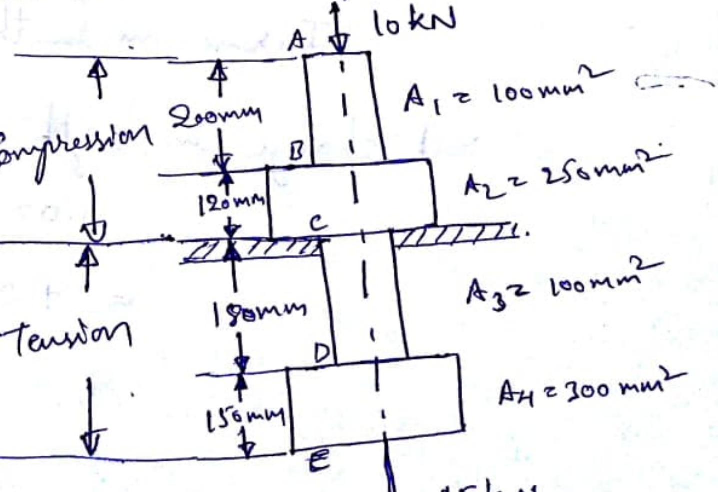
Minimum stress

$$f_{min} = \frac{P}{A_2} = \frac{12\times10^3}{490.87} = 24.45\ N/mm^2$$

Prob: 6 Fig shows a loaded rod of steel in equilibrium. It is supported on a stop at C. Find the change in the length of the rod. Take modulus of elasticity of steel equal to $2\times10^5\ N/mm^2$.

→ free hanging bodies are subjected to tensile loading

→ If a body is supported other side and the force acting into the material then it is subjected to compressive loading



Part AB

Compressive stress $\sigma_c = \frac{10 \times 10^3}{100} = 100\ N/mm^2$

$$E = \frac{\sigma}{e} \qquad \left[e = \frac{\delta L}{L}\right]$$

$$E = \frac{\sigma}{\frac{\delta L}{L}}$$

$$\delta L = \frac{\sigma}{E} \times L$$

Decrease in length $= \frac{100}{2 \times 10^5} \times 200 = 0.10\text{mm} (-)$

Part BC

Compressive stress $= \frac{10 \times 10^3}{250} = 40\ N/mm^2$

Decrease in length $= \frac{40}{2 \times 10^5} \times 120 = 0.024\ \text{mm} (-)$

Part CD

Tensile stress $= \frac{15 \times 10^3}{100} = 150\ N/mm^2$

increase in length $= \frac{150}{2 \times 10^5} \times 180 = 0.135\text{mm} (+)$

Part DE

Tensile stress $= \frac{15 \times 10^3}{300} = 50\ N/mm^2$

Increase in length $= \frac{50}{2 \times 10^5} \times 150 = 0.0375\text{mm} (+)$

Net change in length =

$$0.0375 + 0.1350 - 0.0240 - 0.100$$

$$= +0.0485\ \text{mm}\ (\text{Increase in length})$$

Prob 7: Find the minimum diameter of a steel wire, which is used to raise a load of 4000 N if the stress in the rod is not to exceed 95 MN/m²

Given data

$P = 4000\,N$

$\sigma = 95\ MN/m^2$

$= 95\ \frac{N}{mm^2}$

$A = \frac{\pi}{4} D^2$

$\sigma = \frac{P}{A}$

$95 = \frac{4000 \times 4}{\pi \times D^2}$

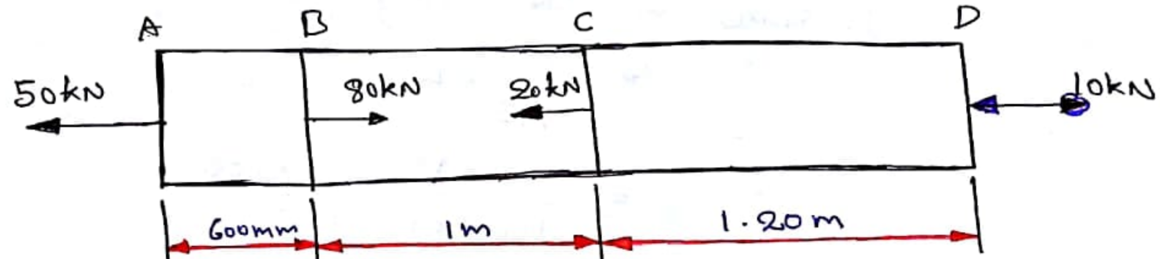
$D^2 = \frac{4000 \times 4}{\pi \times 95} = 53.61\,mm^2$

$D = \underline{7.32}\ mm$

Principle of Superposition :-

When a number of loads are acting on a body the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads.

Prob No:8 A brass bar, having cross-sectional area of 1000 mm², is subjected to axial force as shown in the figure.



Find the total elongation of the bar.

Take $E = 1.05 \times 10^5$ N/mm²

Given Data:

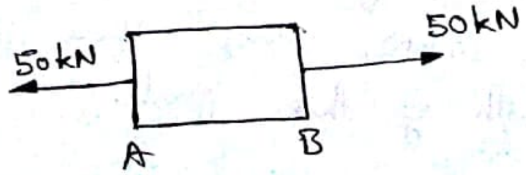
Area 'A' = 1000 mm²

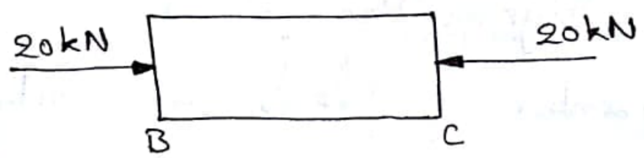
Value of 'E' = 1.05×10^5 N/mm²

dL = Total elongation of the bar.

The force of 80kN acting at B is split up into three forces of 50kN, 20kN, and 10kN.

Then





Part AB This part is subjected to a tensile load of 50 kN. Hence there will be increase in length of this part.

Increase in the length of AB

$$= \frac{P_1}{AE} \times L_1$$

$$= \frac{50 \times 1000}{1000 \times 1.05 \times 10^5} \times 600$$

$$= 0.2857 \text{ mm}$$

Part BC This part is subjected to a compressive load of 20kN or 20,000N. Hence there will be decrease in length of this part.

∴ Decrease in the length of BC.

$$= \frac{P_2}{AE} \times L_2 = \frac{20000}{1000 \times 1.05 \times 10^5} \times 1000$$

$$= 0.1904 \text{ mm.}$$

Part BD this part is subjected to a compressive load of 10 kN, Hence there will be decrease in length of this part.

Decrease in the length of BD

$$= \frac{P_3}{AE} \times L_3 = \frac{10000}{1000 \times 1.05 \times 10^5} = 2200$$

$= 0.2095$ mm.

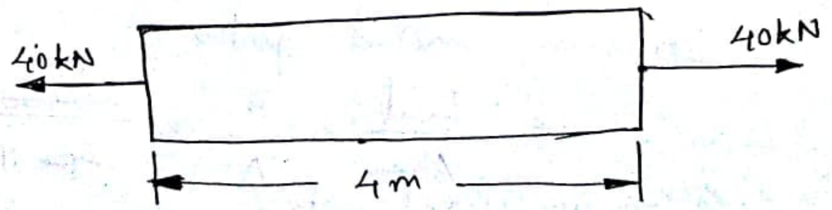
$\therefore$ Total elongation $= 0.2857 - 0.1904 - 0.2095$

$= -0.1142$ mm

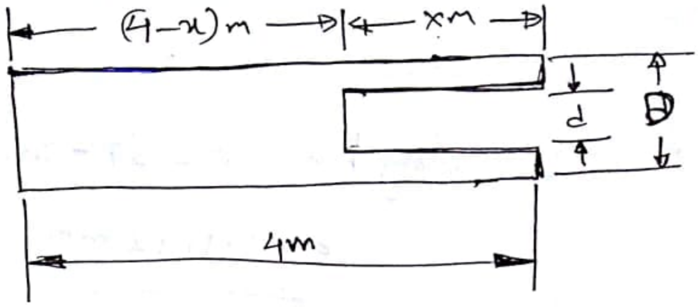
Negative sign shows, that there will be decrease in length of the bar.

Prob: 9 A tensile load of 40 kN is acting on a rod of diameter 40mm and of length 4m. A bore of diameter 20mm is made centrally on the rod. To what length the rod should be bored so that the ~~length~~ total extension will increase 30% under the same tensile load. Take $E = 2 \times 10^5$ N/mm²

Given data
P (tensile load) = 40kN
D (diameter of the rod) = 40mm.
L (length of the rod) = 4m.
$E = 2 \times 10^5$ N/mm²
d (bore dia) = 20mm



Area of Rod, $A = \frac{\pi}{4}(40^2) = 400\pi \ mm^2$



Area of bore, $a = \frac{\pi}{4} \times 20^2 = 100\pi \ mm^2$

Total extension after bore = 1.3 × Extension before bore

change in length of the bar without bore

$$\delta l = \frac{P}{AE} \times L$$

$$= \frac{40000 \times 4000}{400\pi \times 2 \times 10^5}$$

$$= \frac{2}{\pi} \ mm$$

Extension after the bar is made

$$= 1.3 \times \delta l.$$

$$= 1.3 \times \frac{2}{\pi}$$

$$= \frac{2.6}{\pi} \ mm$$

Stress in unbored portion

$$= \frac{Load}{Area} = \frac{P}{A} = \frac{\overset{100}{\cancel{40000}}}{\cancel{400}\pi} = \frac{100}{\pi} \ N/mm^2$$

Stress in bored portion

$$= \frac{P}{A-a} = \frac{40000}{(400\pi - 100\pi)} = \frac{40000}{300\pi}$$

Extension of unbored portion

$$\delta l = \frac{\text{Stress}}{E} \times \text{length of unbored portion}$$

$$= \frac{100}{\pi \times 2 \times 10^5} \times (4-x) \times 1000$$

$$= \frac{(4-x)}{2\pi} \text{ mm.}$$

Extension of bored portion

$$= \frac{\text{Stress}}{E} \times \text{length of bored portion}$$

$$= \frac{4000}{300\pi \times 2 \times 10^5} \times 1000x = \frac{4x}{6\pi} \text{ mm.}$$

Total extension after the bore is made

$$= \frac{4-x}{2\pi} + \frac{4x}{6\pi}$$

$$\frac{2.6}{\pi} = \frac{4-x}{2\pi} + \frac{4x}{6\pi}$$

$$\frac{2.6}{\cancel{\pi}} = \frac{3(4-x) + 4x}{6\cancel{\pi}}$$

$$6 \times 2.6 = 12 - 3x + 4x$$

$$x = 15.6 - 12$$

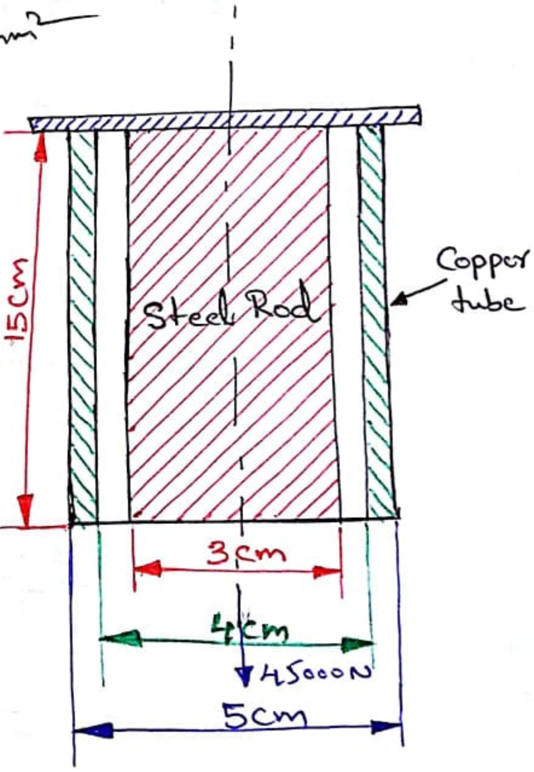
$$x = 3.6 \text{ meters}$$

Rod should be bored upto a length of 3.6m

Prob:10, A steel rod of 3cm diameter is enclosed centrally in a hollow copper tube of external diameter 5cm and internal diameter 4cm. The composite bar is then subjected to an axial pull of 45000N. If the length of each bar is equal to 15cm, determine: i) The stresses in the rod and tube, and ii) Load carried by each bar.

Take E for steel = 2.1×10^5 N/mm²

Copper = 1.1×10^5 N/mm²



Sol: Diameter of steel rod = 3cm = 30mm.

Area of steel rod $A_s = \frac{\pi}{4}(30)^2$

$A_s = 706.86$ mm²

External diameter of copper tube = 5cm = 50mm.

Internal diameter of copper tube = 4cm = 40mm

Area of copper tube,

$A_c = \frac{\pi}{4}[50^2 - 40^2]$ mm²

$= 706.86$ mm²

Axial pull on composite bar, P = 4500 N

Length of each bar, L = 15cm.

Young's modulus for steel, $E_s = 2.1 \times 10^5$ N/mm²

" Copper $E_c = 1.1 \times 10^5$ N/mm²

i) Stress in the rod and tube.

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c$$

$$= \frac{2.1 \times 10^5}{1.1 \times 10^5} \times \sigma_c$$

$$\sigma_s = 1.909\, \sigma_c$$

load on steel + load on Copper = Total load.

$$P_s + P_c = P$$

$$\sigma_s A_s + \sigma_c A_c = 45000N.$$

$$\sigma_s \times 706.86 + \sigma_c \times 706.86 = 45000N$$

$$1.909\, \sigma_c \times 706.86 + \sigma_c \times 706.86$$

$$2056.25\, \sigma_c = 45000N$$

$$\sigma_c = \frac{45000}{2056.25}$$

$$\boxed{\sigma_c = 21.88\ N/mm^2}$$

$\sigma_s = 1.909\, \sigma_c$; $\sigma_s = 1.909 \times 21.88$

$$\boxed{\sigma_s = 41.77\ N/mm^2}$$

ii) Load carried by each bar.:

Load = stress × Area

Load carried by steel bar = $P_s = \sigma_s \times A_s$

$P_s = 41.77 \times 706.86$ $\boxed{P_s = 29525.5N}$

Load carried by Copper tube = $P_c = 45000 - P_s$

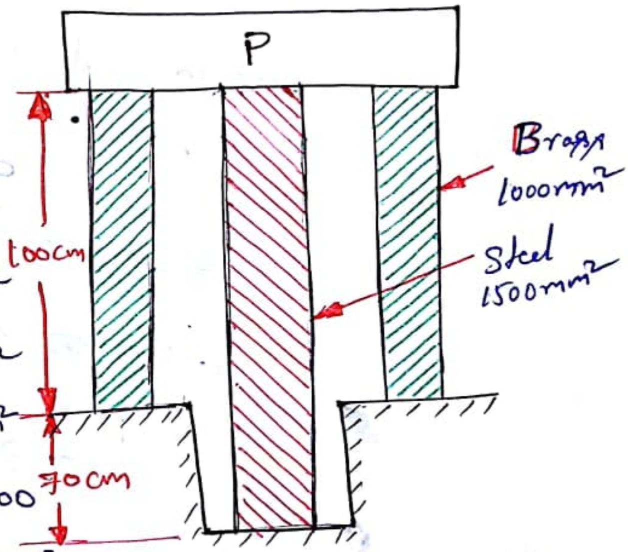
$$\boxed{P_c = 15474.5\ N}$$

Prob 11: Two brass rods and one steel rod together supported a load as shown in figure. If the stresses in brass and steel are not to exceed 60 N/mm² and 120 N/mm², find the safe load that can be supported.

Take: $E_{steel} = 2 \times 10^5$ N/mm²

$E_{brass} = 1 \times 10^5$ N/mm²

The cross sectional area of steel rod is 1500 mm² and of each brass rod is 1000 mm².



Given:

Stress in brass, $\sigma_b = 60$ N/mm²

Stress in steel, $\sigma_s = 120$ N/mm²

E for steel, $E_s = 2 \times 10^5$ N/mm²

E for brass, $E_b = 1 \times 10^5$ N/mm²

Area of steel rod, $A_s = 1500$ mm²

Area of two brass rods, $A_b = 2 \times 1000$

$= 2000$ mm²

Length of steel rod, $L_s = 170$ mm

Length of brass rods, $L_b = 100$ mm

We know that decrease in the length of steel rod should be equal to the decrease in length of brass rods.

But decrease in length of steel rods

= Strain in steel rod × Length of steel rod.

$= e_s \times L_s$ where e_s is strain in steel

Similarly decrease in length of brass rods

= Strain in brass rods × Length of brass rods

= $e_b \times L_b$ where e_b is strain in brass rod

Equating the decrease in length of steel rods to the decrease in length of brass rods, we get

$$e_s L_s = e_b L_b \quad \text{(or)} \quad \frac{e_s}{e_b} = \frac{L_b}{L_s} = \frac{100}{170}$$

But Stress in steel = Strain in steel × E_s

(or) $\sigma_s = e_s \times E_s$ ——— (i)

Similarly stress in brass is given by

$\sigma_b = e_b \times E_b$ ——— (ii)

Dividing equation (i) by equation (ii), we get

$$\frac{\sigma_s}{\sigma_b} = \frac{e_s \times E_s}{e_b \times E_b} = \frac{100}{170} \times \frac{2\times10^5}{1\times10^5} = 1.176$$

Suppose steel is permitted to reach its safe stress of 2×10^5 N/mm², the corresponding stress in brass will be

$$\sigma_b = \frac{\sigma_s}{1.176} = \frac{2\times10^5}{1.176} = 1.7\times10^5 \text{ N/mm}^2$$

1.7×10^5 N/mm² which exceeds the safe stress of 1×10^5 N/mm² for brass. Therefore let brass be allowed to reach its safe stress of 1×10^5 N/mm². Then corresponding stress in steel will be 1.176×10^5 N/mm² which is less than 2×10^5 N/mm²

Total load = P = Load on steel + Load on copper

$= \sigma_s \times A_s + \sigma_b \times A_b$

$= 1.176 \times 10^5 \times 1500 + 1 \times 10^5 \times 2000$

$= 3764 \times 10^5 N$ or $376.4 \times 10^6 N$

$= 376.4$ MN.

Prob 12:

A steel rod of 3cm diameter and 5m long is connected to two grips and the rod is maintained at a temperature of 95°C. Determine the stresses and pull exerted when the temperature falls to 30°C, if

i) the ends do not yield, and

ii) the ends yield by 0.12 cm.

Take $E = 2 \times 10^5$ MN/m² and $\alpha = 12 \times 10^{-6}/°C$

Given:

Dia of the rod, $d = 3cm = 30$ mm

∴ Area of the rod, $A = \frac{\pi}{4} \times 30^2 = 225\pi$ mm²

Length of the rod, $L = 5m = 5000$ mm,

Initial temperature, $T_1 = 95°C$

Final temperature, $T_2 = 30°C$

∴ Fall in temperature $= 95 - 30 = 65°C$

Modulus of elasticity $E = 2 \times 10^5$ MN/m²

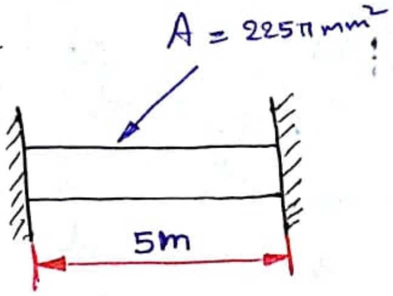
$= 2 \times 10^5 \times 10^6$ N/m²

$= 2 \times 10^5$ N/mm²

Coefficient of linear expansion $\alpha = 12 \times 10^{-6}/°C$

Pg

i) When the ends do not yield



The stress is given by

$$= \alpha T E$$

$$= 12 \times 10^{-6} \times 65 \times 2 \times 10^{5} \ N/mm^2$$

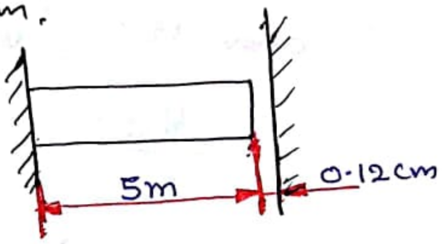
$$= 156 \ N/mm^2 \ (\text{tensile}).$$

Pull in the rod = stress × Area

$$= 156 \times 225\pi$$

$$= 110269.9 \ N$$

ii) When the ends yield by 0.12cm.



$$\delta = 0.12 \ cm = 1.2 mm$$

The stress when the ends yield is given by equation

$$= \frac{\alpha TEL - \delta E}{L} = \left(\frac{\alpha TL - \delta}{L}\right) E$$

$$= \frac{(12 \times 10^{-6} \times 16 \times 5000 - 1.2)}{5000} \times 2 \times 10^{5} \ N/mm^2$$

$$= \frac{(3.9 - 1.2)}{5000} \times 2 \times 10^{5} = 108 \ N/mm^2$$

Pull in the rod = stress × Area.

$$= 108 \times 225\pi$$

$$= \underline{76340.7 \ N}$$

Prob: 13 A steel rod of 20mm diameter passes centrally through a copper tube of 50mm external diameter and 40mm internal diameter. The tube is closed at each end by rigid plates of negligible thickness. The nuts are tightened lightly home on the projecting parts of the rod. If the temp of the assembly is raised by 50°C, calculate the stresses developed in copper and steel. Take E for steel and copper as 200 GN/m² and 100 GN/m² and α for steel and copper as 12×10^{-6} per °C and 18×10^{-6} per °C.

Given that :-

Dia of steel rod = 20mm.

∴ Area of steel rod, $A_s = \frac{\pi}{4} \times 20^2 = 100\pi \ mm^2$

Area of copper tube, $A_c = \frac{\pi}{4}(50^2 - 40^2) \ mm^2 = 225\pi \ mm^2$

Rise of temperature, $T = 50°C$

$1 \ N/mm^2 = 10^6 \ N/m^2$

E for steel, $E_s = 200 \ GN/m^2$

$= 200 \times 10^9 \ N/m^2$

$= 200 \times 10^3 \ N/mm^2$

E for copper, $E_c = 100 \ GN/m^2$

$= 100 \times 10^3 \ N/mm^2$

$\alpha_s = 12 \times 10^{-6}$ per °C

$\alpha_c = 18 \times 10^{-6}$ per °C

As α for copper is more than that of steel, hence the free expansion of copper will be more than that of steel where is a rise in temperature. But the ends of the rod and the tube is fixed to the rigid plates and the nuts are tightened on the projected parts of the rod. Hence the two members are not free to expand.

The tube and rod will expand by the same amount. The free expansion of the copper tube is more than the common expansion, whereas free expansion of the steel rod will be less than the common expansion.

Let σ_s = Tensile stress in steel.

σ_c = Compressive stress in copper.

For the equilibrium of the system;

Tensile load on steel = Compressive load on copper.

or $\boxed{\sigma_s \cdot A_s = \sigma_c A_c}$

$$\sigma_s = \frac{A_c}{A_s} \times \sigma_c$$

$$= \frac{225\pi}{100\pi} \times \sigma_c = 2.25\sigma_c \quad -(i)$$

We know that the copper tube and steel rod will actually expand by the same amount.

$\therefore$ Actual expansion of steel = Actual expansion of Copper.

actual expansion of steel = $\alpha_s T L + \frac{\sigma_s}{E_s} L$

actual expansion of Copper = $\alpha_c T L - \frac{\sigma_c}{E_c} L$

then

$$\alpha_s T + \frac{\sigma_s}{E_s} = \alpha_c T - \frac{\sigma_c}{E_c}$$

$$12 \times 10^{-6} \times 50 + \frac{2.25 \sigma_c}{200 \times 10^3} = 18 \times 10^{-6} \times 50 - \frac{\sigma_c}{100 \times 10^3}$$

$$2.125 \sigma_c = 30$$

$$\sigma_c = \frac{30}{2.125}$$

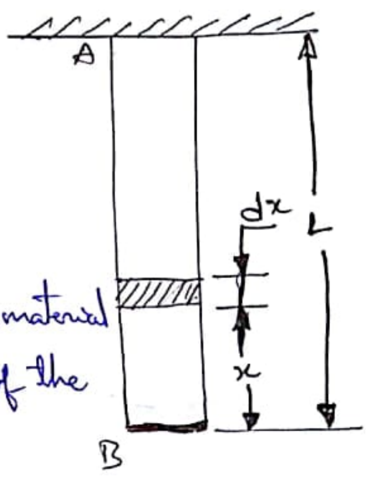
$$\boxed{\sigma_c = 14.117 \ N/mm^2}$$

Substituting σ_c in equation (i) we get

$$\sigma_s = 2.25 \times 14.117$$

$$\boxed{\sigma_s = 31.76 \ N/mm^2}$$

Prob 2-13 Elongation of a bar due to its own weight



A bar AB fixed at end A and hanging freely under its own weight

Let L = Length of bar,
A = Area of cross-section,
E = Young's modulus for the bar material
ω = weight per unit volume of the bar material.

Consider a small strip of thickness dx at a distance x from the lower end.

Weight of the bar for a length of x is given by,

P = Specific weight × volume of bar upto length x

$= \omega \times A \times x$

This means that on the strip, a weight of $\omega \times A \times x$ is acting in the downward direction. Due to this weight, there will be some increase in the length of element. But length of the element is dx.

Now stress on the element

$$= \frac{\text{weight acting on element}}{\text{Area of Cross-section}} = \frac{\omega \times A \times x}{A} = \omega \times x$$

The above equation shows that stress due to self weight in a bar is not uniform. It depends on x. The stress increases with the increase of x

Strain in the element $= \frac{\text{Stress}}{E} = \frac{w \times x}{E}$

$\therefore$ Elongation of the element

$= \text{Strain} \times \text{length of element}$

$= \frac{w \times x}{E} \times dx$

Total elongation of the bar is obtained by integrating the above equation between limits zero and L.

$$\therefore \quad \delta L = \int_0^L \frac{w \times x}{E} dx$$

$$= \frac{w}{E} \int_0^L x\, dx$$

$$= \frac{w}{E} \left[\frac{x^2}{2}\right]_0^L$$

$$= \frac{w}{E} \frac{L^2}{2} \qquad \boxed{\text{where } W = w \times L}$$

$$= \frac{wL^2}{2E} =$$

$$\boxed{\delta l = \frac{WL}{2E}}$$

