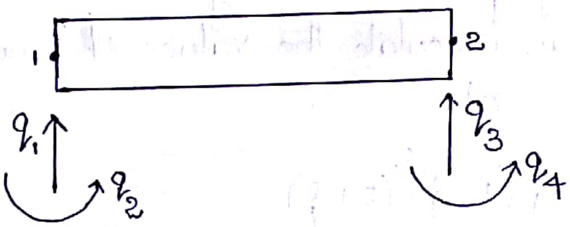
Topic 2 : Beams

* **Beams:**

Beams are slender members that are used for supported transverse loading. Long horizontal members used in buildings and bridges, and shafts supported in bearings are some examples of beams.

In Beam structure, the beams are joined together by welding, so that both forces and moments can be transmitted between the beams.

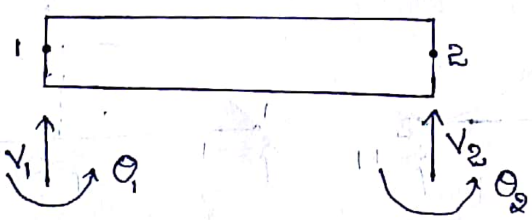


Each node has two degrees of freedom.

Typically, the degrees of freedom of Node 1 are q_1 & q_2. Here q_1 is the transverse displacement and q_2 is slope or rotation.

$$\therefore Q = [q_1, q_2, q_3, q_4, \ldots]^T$$

We can consider the beam in terms of Natural coordinate system, then.



Here, v_1, v_2 = Transverse displacements at Nodes 1 & 2.

θ_1, θ_2 = Angular displacements / slopes.

Where,
$$\theta_1 = \frac{dv_1}{d\xi}$$

$$\theta_2 = \frac{dv_2}{d\xi}$$ in terms of Natural Coordinate system.

Hermite Shape Functions:

$$H_i = a_i + b_i \xi + c_i \xi^2 + d_i \xi^3 \longrightarrow \text{(I)}$$

Where, $i = 1, 2, 3, 4$.

Considering the boundary conditions.

	H_1	H_1'	H_2	H_2'	H_3	H_3'	H_4	H_4'
$\xi = -1$	1	0	0	1	0	0	0	0
$\xi = +1$	0	0	0	0	1	0	0	1

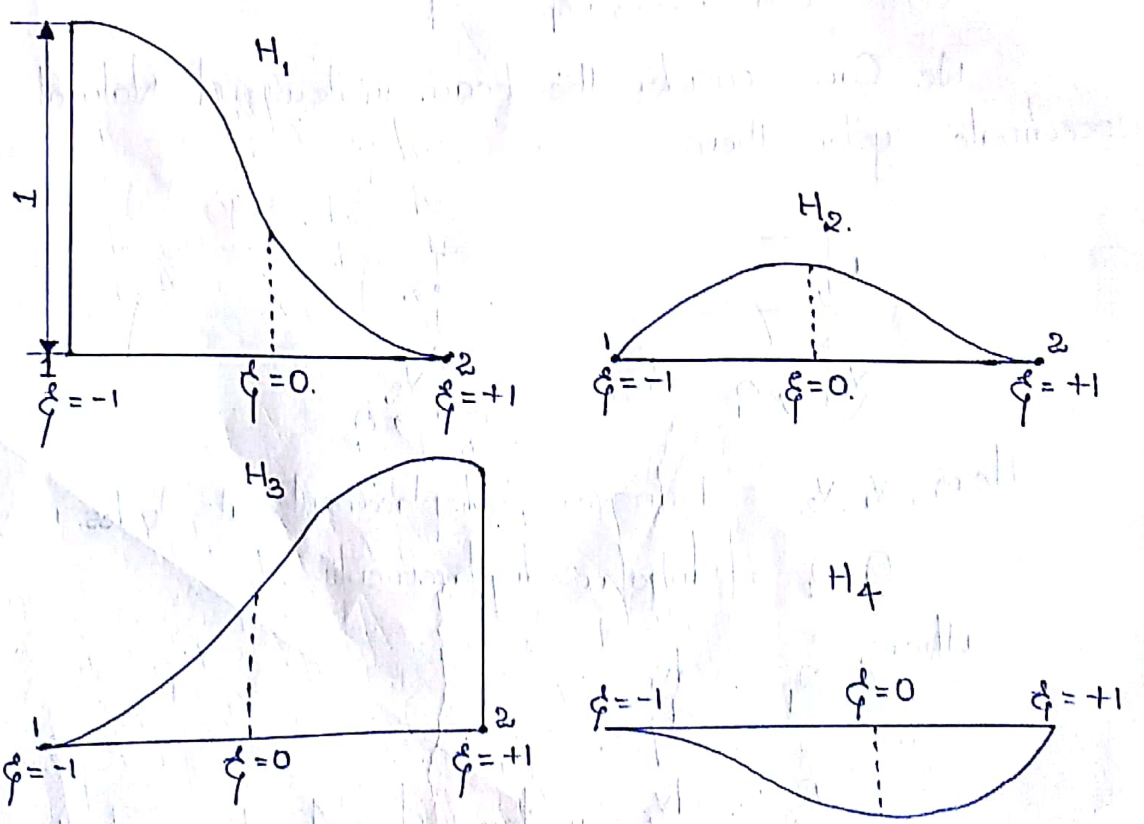
Now by applying the boundary conditions in the above equation (1), we can calculate the values of constants a_i, b_i, c_i and d_i. Then we get:

$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi)$$

$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1)$$

$$H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$H_4 = \frac{1}{4}(1+\xi)^2(\xi-1)$$



Considering the displacement function in terms of Natural coordinate system.

$$v = H_1 v_1 + H_2 \theta_1 + H_3 v_2 + H_4 \theta_2$$

$$v = H_1 v_1 + H_2 \frac{dv_1}{d\xi} + H_3 v_2 + H_4 \frac{dv_2}{d\xi} \longrightarrow (1)$$

Let Consider the chain rule

$$\frac{dv}{d\xi} = \frac{dv}{dx} \cdot \frac{dx}{d\xi}$$

W.K.T

$$dx = \frac{l_e}{2} d\xi \Rightarrow \frac{dx}{d\xi} = \frac{l_e}{2}.$$

$$\therefore \frac{dv}{d\xi} = \frac{l_e}{2} \cdot \frac{dv}{dx} \text{ — in terms of Global}$$

$\therefore$ Eq. 1 can be written in terms of Global coordinate system as.

$$v = H_1 q_1 + \frac{l_e}{2} H_2 q_2 + H_3 q_3 + H_4 \frac{l_e}{2} q_4$$

where,

$$q_2 = \frac{dv_1}{dx} \qquad q_4 = \frac{dv_2}{dx}.$$

$$v = Hq$$

Where,

$$H = \left[H_1, \frac{l_e}{2} H_2, H_3, \frac{l_e}{2} H_4\right]$$

or

Simply in Matrix form.

$$H = \left[H_1 \quad \frac{l_e}{2} H_2 \quad H_3 \quad \frac{l_e}{2} H_4\right]$$

Now, for Calculation of Stiffness Matrix; We should Consider the Hermite shape functions.

$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi) \qquad H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1) \qquad H_4 = \frac{1}{4}(1+\xi)^2(\xi-1)$$

Now Calculating the Single & double derivations of the Hermite shape functions as shown.

$$H_1 = \frac{1}{4}(1-\xi)^2(2+\xi)$$

$$= \frac{1}{4}(1+\xi^2-2\xi)(2+\xi)$$

$$= \frac{1}{4}\left[2+\cancel{2\xi^2}-4\xi+\xi+\xi^3-\cancel{2\xi^2}\right]$$

$$H_1 = \frac{1}{4}\left[\xi^3-3\xi+2\right]$$

$$\therefore H_1' = \frac{dH_1}{d\xi} = \frac{1}{4}\left[3\xi^2-3\right]$$

$$H_1'' = \frac{d^2H_1}{d\xi^2} = \frac{1}{4}\left[6\xi\right]$$

Similarly

$$H_2 = \frac{1}{4}(1-\xi)^2(\xi+1)$$

$$= \frac{1}{4}(1+\xi^2-2\xi)(\xi+1)$$

$$= \frac{1}{4}(\xi+\xi^3-2\xi^2+1+\xi^2-2\xi)$$

$$H_2 = \frac{1}{4}(\xi^3-\xi^2-\xi+1)$$

$$\therefore H_2' = \frac{dH_2}{d\xi} = \frac{1}{4}\left[3\xi^2-2\xi-1\right]$$

$$H_2'' = \frac{d^2H_2}{d\xi^2} = \frac{1}{4}\left[6\xi-2\right]$$

$$H_3 = \frac{1}{4}(1+\xi)^2(2-\xi)$$

$$= \frac{1}{4}(1+\xi^2+2\xi)(2-\xi)$$

$$= \frac{1}{4}(2+\cancel{2\xi^2}+4\xi-\xi-\xi^3-\cancel{2\xi^2})$$

$$H_3 = \frac{1}{4}(-\xi^3+3\xi+2)$$

$$\therefore H_3' = \frac{dH_3}{d\xi} = \frac{1}{4}(-3\xi^2+3)$$

$$H_3'' = \frac{d^2H_3}{d\xi^2} = \frac{1}{4}\left[-6\xi\right]$$

& $H_4 = \frac{1}{4}(1+\xi)^2(\xi-1)$

$= \frac{1}{4}(1+\xi^2+2\xi)(\xi-1)$

$= \frac{1}{4}(\xi+\xi^3+2\xi^2-1-\xi^2-2\xi)$

$= \frac{1}{4}(\xi^3+\xi^2-\xi-1)$

$H_4' = \frac{dH_4}{d\xi} = \frac{1}{4}[3\xi^2+2\xi-1]$

$H_4'' = \frac{d^2H_4}{d\xi^2} = \frac{1}{4}[6\xi+2]$

Now Considering the Strain Energy Term

$$U = 1/2 \int EI \frac{d^2v}{dx^2} . dx.$$

By solving we get.

$$U = 1/2\, q^T . K_e q.$$

Where,

$$K_e = \frac{EI}{l_e^3}\begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ -12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix}$$

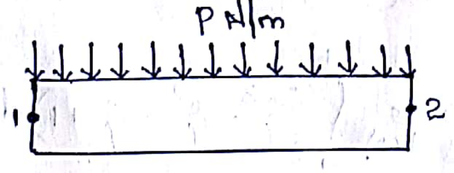
where,

K_e = Element Stiffness Matrix.

l_e = Element Length.

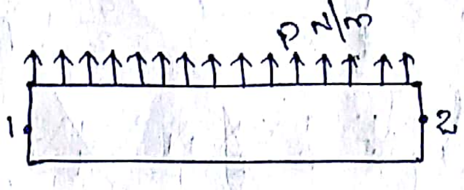
* **Load Vector :**

Let 'p' be the Uniformly distributed load over the length 'L'.



Downward.

or



Upward.

Let

$$\Rightarrow \int P . V^T . dx$$

Where

$$V = Hq \Rightarrow V^T = H^T q^T$$

$$dx = \frac{l_e}{2} d\xi$$

$$\Rightarrow \int_{-1}^{+1} P . H^T q^T . \frac{l_e}{2} d\xi$$

$$\Rightarrow \frac{P l_e}{2} \int_{-1}^{+1} H^T d\xi . q^T$$

Let Consider the Hermite term.

$$\Rightarrow \int_{-1}^{+1} H^T d\xi$$

$$\Rightarrow \int_{-1}^{+1} \left[H_1 \quad \frac{l_e}{2} H_2 \quad H_3 \quad \frac{l_e}{2} H_4 \right]^T d\xi$$

$$\Rightarrow \left[\int_{-1}^{+1} H_1 d\xi \quad \frac{l_e}{2} \int_{-1}^{+1} H_2 d\xi \quad \int_{-1}^{+1} H_3 d\xi \quad \frac{l_e}{2} \int_{-1}^{+1} H_4 d\xi \right]^T$$

Now Let

$$\int_{-1}^{+1} H_1 d\xi = \frac{1}{4} \int_{-1}^{+1} (\xi^3 - 3\xi + 2) d\xi$$

$$= \frac{1}{4} \left[\frac{\xi^4}{4} - 3\frac{\xi^2}{2} + 2\xi \right]_{-1}^{+1}$$

$$\Rightarrow \frac{1}{4} \left[\frac{1}{4} - \frac{3}{2} + 2 - \left[\frac{1}{4} - \frac{3}{4} - 2 \right] \right]$$

$$\int_{-1}^{+1} H_1 d\xi = \frac{1}{\cancel{4}} \left[\cancel{4} \right] = 1$$

$$\frac{l_e}{2} \int_{-1}^{+1} H_2 d\xi = \frac{l_e}{4(2)} \int_{-1}^{+1} (\xi^3 - \xi^2 - \xi + 1) d\xi$$

$$= \frac{l_e}{8} \left[\left(\frac{\xi^4}{4} - \frac{\xi^3}{3} - \frac{\xi^2}{2} + \xi \right) \right]_{-1}^{+1}$$

$$= \cancel{\frac{l_e}{8} \left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 - \frac{1}{4} + \frac{1}{3} + \frac{1}{2} + 1 \right]}$$

$$= \frac{l_e}{8}\left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1 - \left[\frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1\right]\right]$$

$$= \frac{l_e}{8}\left[\cancel{\frac{1}{4}} - \frac{1}{3} - \cancel{\frac{1}{2}} + 1 - \cancel{\frac{1}{4}} - \frac{1}{3} + \cancel{\frac{1}{2}} + 1\right]$$

$$= \frac{l_e}{8}\left[\frac{-2}{3} + 2\right]$$

$$\frac{l_e}{2}\int_{-1}^{+1} H_2\, d\xi = \frac{l_e}{8}\left[\frac{-2+6}{3}\right] = \frac{l_e}{\cancel{8}_2}\left[\frac{4}{3}\right] = \frac{l_e}{6}$$

$$\int_{-1}^{+1} H_3\, d\xi = \frac{1}{4}\int_{-1}^{+1}(-\xi^3 + 3\xi + 2)\, d\xi$$

$$= \frac{1}{4}\left[\frac{-\xi^4}{4} + \frac{3\xi^2}{2} + 2\xi\right]_{-1}^{+1}$$

$$= \frac{1}{4}\left[\frac{-1}{4} + \frac{3}{2} + 2 - \left[\frac{-1}{4} + \frac{3}{2} - 2\right]\right]$$

$$= \frac{1}{4}\left[\cancel{\frac{-1}{4}} + \cancel{\frac{3}{2}} + 2 + \cancel{\frac{1}{4}} - \cancel{\frac{3}{2}} + 2\right]$$

$$\int_{-1}^{+1} H_3\, d\xi = 1$$

$$\& \quad \frac{l_e}{2}\int_{-1}^{+1} H_4\, d\xi = \frac{l_e}{8}\int_{-1}^{+1}(\xi^3 + \xi^2 - \xi - 1)\, d\xi$$

$$= \frac{l_e}{8}\left[\frac{\xi^4}{4} + \frac{\xi^3}{3} - \frac{\xi^2}{2} - \xi\right]_{-1}^{+1}$$

$$= \frac{l_e}{8}\left[\frac{1}{4} + \frac{1}{3} - \frac{1}{2} - 1 - \left[\frac{1}{4} - \frac{1}{3} - \frac{1}{2} + 1\right]\right]$$

$$= \frac{l_e}{8}\left[\cancel{\frac{1}{4}} + \frac{1}{3} - \cancel{\frac{1}{2}} - 1 - \cancel{\frac{1}{4}} + \frac{1}{3} + \cancel{\frac{1}{2}} - 1\right]$$

$$= \frac{l_e}{8}\left[\frac{2}{3} - 2\right] \Rightarrow \frac{l_e}{8}\left[\frac{2-6}{3}\right] = \frac{l_e}{\cancel{8}_2}\left[\frac{-\cancel{4}}{3}\right]$$

$$\frac{l_e}{2}\int_{-1}^{+1} H_4\, d\xi = \frac{-l_e}{6}$$

$$\therefore \quad \Rightarrow \frac{Pl_e}{2}\begin{bmatrix} 1 & \frac{l_e}{6} & 1 & \frac{-l_e}{6}\end{bmatrix}^T q^T$$

$$\Rightarrow \begin{bmatrix} \frac{Pl_e}{2} & \frac{Pl_e^2}{12} & \frac{Pl_e}{2} & \frac{-Pl_e^2}{12}\end{bmatrix}^T q^T$$

$$\Rightarrow f_e q^T$$

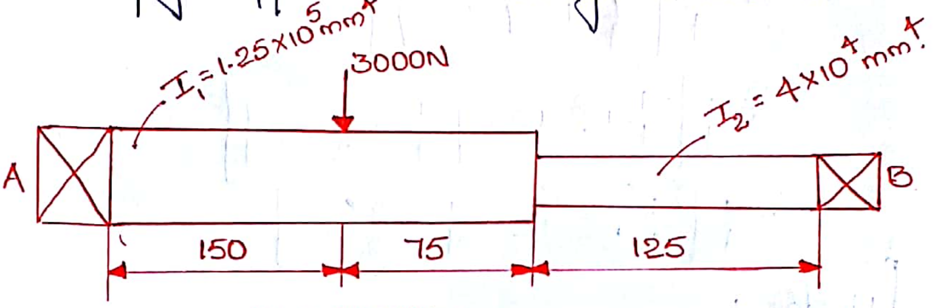
$$\therefore f_e = \left[\frac{Pl_e}{2} \quad \frac{Pl_e^2}{12} \quad \frac{Pl_e}{2} \quad -\frac{Pl_e^2}{12}\right]^T \text{ for Upward dist.}$$

$$\& \; f_e = \left[-\frac{Pl_e}{2}, \; -\frac{Pl_e^2}{12} \quad -\frac{Pl_e}{2} \quad \frac{Pl_e^2}{12}\right]^T \text{. for downward dist.}$$

Problems:

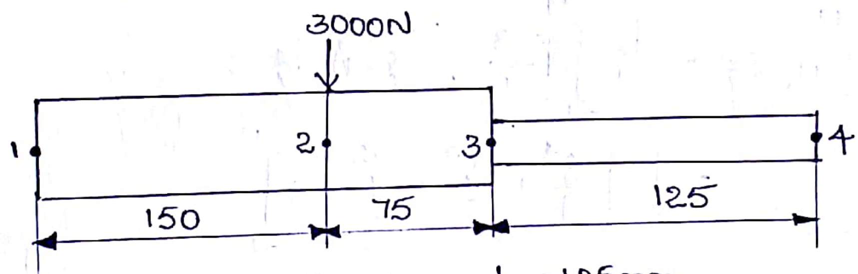
Model 1:

1. Find the deflection at the load and the slopes at the ends for the steel shaft shown in fig. Consider the shaft to be simply supported at bearings A and B.



E = 200GPas.

Sol: Given data :



$L_1 = 150 \quad L_2 = 75 \quad L_3 = 125mm$

$$E = E_1 = E_2 = E_3 = 200GPas = 200\times10^9 N/m^2$$
$$= 200\times10^9\times10^{-6} N/mm^2$$
$$= 2\times10^5 N/mm^2$$

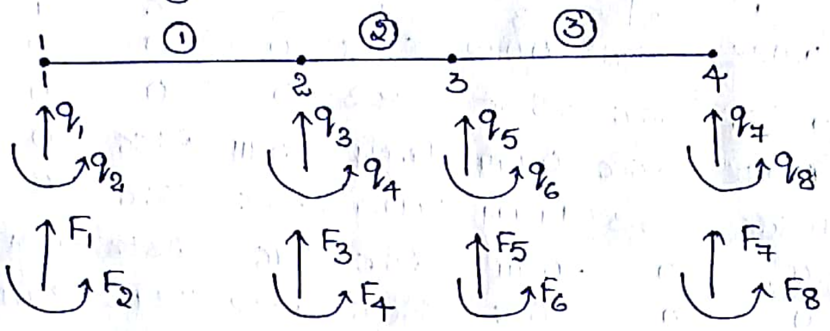
$I_1 = 1.25\times10^5 mm^4$

$I_2 = 1.25\times10^5 mm^4$

$I_3 = 4\times10^4 mm^4$

$P = 3000N.$

Step 1 : Discretization of Domain



Element	Nodes	
①	1	2
②	2	3
③	3	4

Step 2 : Calculating Element Stiffness Matrix.

$$k_1 = \frac{2\times10^5 \times 1.25\times10^5}{(150)^3}\begin{bmatrix} 12 & 6(150) & -12 & 6(150) \\ 6(150) & 4(150)^2 & -6(150) & 2(150)^2 \\ -12 & -6(150) & 12 & -6(150) \\ 6(150) & 2(150)^2 & -6(150) & 4(150)^2 \end{bmatrix}$$

$$= 10^6 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \left[\begin{array}{cccc} 0.088 & 6.666 & -0.088 & 6.666 \\ 6.666 & 666.6 & -6.66 & 333.33 \\ -0.088 & -6.666 & 0.088 & -6.666 \\ 6.666 & 333.3 & -6.66 & 666.6 \end{array}\right] \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$k_2 = \frac{2\times10^5 \times 1.25\times10^5}{(75)^3}\begin{bmatrix} 12 & 6(75) & -12 & 6(75) \\ 6(75) & 4(75)^2 & -6(75) & 2(75)^2 \\ -12 & -6(75) & 12 & -6(75) \\ 6(75) & 2(75)^2 & -6(75) & 4(75)^2 \end{bmatrix}$$

$$= 10^6 \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cccc} 0.711 & 26.66 & -0.711 & 26.6 \\ 26.66 & 1333.3 & -26.66 & 666.6 \\ -0.711 & -26.66 & 0.711 & -26.6 \\ 26.66 & 666.6 & -26.66 & 1333.3 \end{array}\right] \end{array} \begin{array}{c} \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$$k_3 = 10^6 \begin{array}{c} \begin{array}{cccc} 5 & 6 & 7 & 8 \end{array} \\ \left[\begin{array}{cccc} 0.0491 & 3.072 & -0.0491 & 3.072 \\ 3.072 & 256 & -3.072 & 128 \\ -0.0491 & -3.072 & 0.0491 & -3.072 \\ 3.072 & 128 & -3.072 & 256 \end{array}\right] \end{array} \begin{array}{c} \\ 5 \\ 6 \\ 7 \\ 8 \end{array}$$

Step 3 : Calculating Global Stiffness Matrix.

$$
K = 10^6 \begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
0.088 & 6.666 & -0.088 & 6.666 & 0 & 0 & 0 & 0 \\
6.666 & 666.6 & -6.66 & 333.3 & 0 & 0 & 0 & 0 \\
-0.088 & -6.666 & 0.199 & 19.994 & -0.111 & 26.66 & 0 & 0 \\
6.666 & 333.3 & 19.994 & 1999.9 & -26.6 & 666.6 & 0 & 0 \\
0 & 0 & -0.111 & -26.6 & 0.16 & -23.58 & -0.049 & 3.012 \\
0 & 0 & 26.6 & 666.6 & -23.58 & 1589.3 & -3.072 & 128 \\
0 & 0 & 0 & 0 & -0.0491 & -3.012 & 0.049 & -3.012 \\
0 & 0 & 0 & 0 & 3.012 & 128 & -3.012 & 256
\end{bmatrix}
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
$$

Step 4 : Calculating Global load Vector.

$$
F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \\ F_7 \\ F_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -3000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

Step 5 : Considering Finite Element Equation.

$$F = KU$$

$$
\begin{bmatrix} 0 \\ 0 \\ -3000 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 10^6
\begin{array}{c}
\begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\begin{bmatrix}
 & & & & & & & \\
 & & & & & & & \\
 & & & & & & & \\
 & & & K_e & & & & \\
 & & & & & & & \\
 & & & & & & & \\
 & & & & & & & \\
 & & & & & & &
\end{bmatrix}
\end{array}
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array}
\begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \end{bmatrix}
$$

Step 6 : Applying Boundary Conditions.

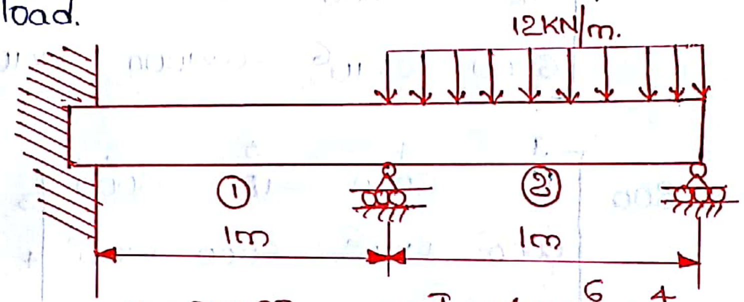
Since Node 1 and Node 2 are fixed i.e., $q_1 = q_2 = q_7 = q_8 = 0$. By using Elimination approach, Eliminate 1, 2, 7, & 8 rows and Columns. Then the final Matrix is.

$$\begin{bmatrix} -3000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = 10^6 \begin{bmatrix} 0.799 & 19.994 & -0.711 & 26.66 \\ 19.994 & 1999.9 & -26.6 & 666.6 \\ -0.711 & -26.6 & 0.76 & -23.58 \\ 26.6 & 666.6 & -23.58 & 1589.3 \end{bmatrix} \begin{bmatrix} q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix}$$

By solving the simultaneous Equations, we get the values of q_3 ; q_4, q_5 & q_6.

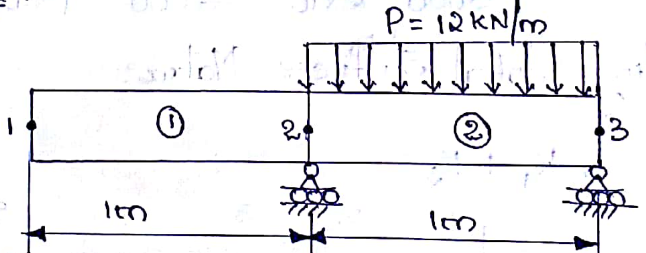
Model 2 :

2. For the beam and loading shown in Fig. Determine
   (a) The slopes at 2 & 3.
   (b) The Vertical deflection at the Midpoint of the distributed load.



$E = 200 GPa$ $\quad$ $I = 4 \times 10^6 \, mm^4$.

Sol:- Given data :-



$E = 200 \times 10^9 \, N/m^2$

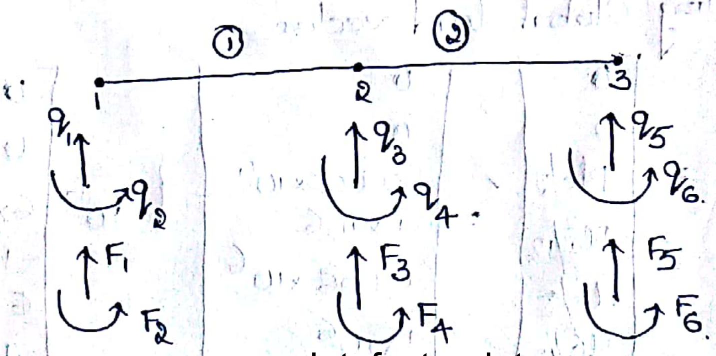
$= 200 \times 10^9 \times 10^{-6} \, N/mm^2$

$\therefore E = 2 \times 10^5 \, N/mm^2$.

$L_1 = L_2 = 1m = 1000 \, mm$.

$P = 12 KN/m = 12 N$.

Step 1 :- Discretization of Domain.



Elements	Nodes	
①	1	2
②	2	3

Step 2: Calculating element stiffness Matrix.

$$K_1 = \frac{2\times10^5 \times 4\times10^6}{1000^3}\begin{bmatrix} 12 & 6\times1000 & -12 & 6\times1000 \\ 6\times1000 & 4\times1000^2 & -6\times1000 & 2\times1000^2 \\ -12 & -6\times1000 & 12 & -6\times1000 \\ 6\times1000 & 2\times1000^2 & -6\times1000 & 4\times1000^2 \end{bmatrix}$$

$$= 800 \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 12 & 6000 & -12 & 6000 \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 \\ -12 & -6000 & 12 & -6000 \\ 6000 & 2\times10^6 & -6\times1000 & 4\times10^6 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$$K_2 = 800 \begin{array}{c} \begin{array}{cccc} 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 12 & 6000 & -12 & 6000 \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 \\ -12 & -6000 & 12 & -6000 \\ 6000 & 2\times10^6 & -6000 & 4\times10^6 \end{bmatrix} \end{array} \begin{array}{c} 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Step 3: Calculating Global stiffness Matrix:

$$K = K_1 + K_2$$

$$= 800 \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 12 & 6000 & -12 & 6000 & 0 & 0 \\ 6000 & 4\times10^6 & -6000 & 2\times10^6 & 0 & 0 \\ -12 & -6000 & 24 & 0 & -12 & 6000 \\ 6000 & 2\times10^6 & 0 & 8\times10^6 & -6000 & 2\times10^6 \\ 0 & 0 & -12 & -6000 & 12 & -6000 \\ 0 & 0 & 6000 & 2\times10^6 & -6000 & 4\times10^6 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

Step 4: Calculating Global load vector.

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -Pl_e/2 \\ -Pl_e^2/12 \\ Pl_e/2 \\ Pl_e^2/12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -0.006\times10^6 \\ -1\times10^6 \\ -0.006\times10^6 \\ 1\times10^6 \end{bmatrix} = 10^6 \begin{bmatrix} 0 \\ 0 \\ 6\times10^{-3} \\ -1 \\ 6\times10^{-3} \\ 1 \end{bmatrix}$$

Step5:- Considering Finite Element Equation.

$F = KU$ or $F = KQ$

$$10^6\begin{bmatrix}0\\0\\6\times10^{-3}\\-1\\6\times10^{3}\\1\end{bmatrix} = 800\begin{bmatrix}12 & 6000 & -12 & 6000 & 0 & 0\\6000 & 4\times10^6 & -6000 & 2\times10^6 & 0 & 0\\-12 & -6000 & 24 & 0 & -12 & 6000\\6000 & 2\times10^6 & 0 & 8\times10^6 & -6000 & 2\times10^6\\0 & 0 & -12 & -6000 & 12 & -6000\\0 & 0 & 6000 & 2\times10^6 & -6000 & 4\times10^6\end{bmatrix}\begin{bmatrix}q_1\\q_2\\q_3\\q_4\\q_5\\q_6\end{bmatrix}$$

Step6:- Applying Boundary Conditions.

Since there is a Roller support at Nodes 2 and 3, therefore $q_3=0$ & $q_5=0$ & $q_1=0, q_2=0$ since fixed

So Eliminating 1, 2, 3, & 5 rows and columns using Elimination Approach, we get.

$$10^6\begin{bmatrix}-1\\1\end{bmatrix} = 800\begin{bmatrix}8\times10^6 & 2\times10^6\\2\times10^6 & 4\times10^6\end{bmatrix}\begin{bmatrix}q_4\\q_6\end{bmatrix}$$

$$q_4 = -2.67\times10^{-4}$$

$$q_6 = 4.46\times10^{-4}.$$

(b) $v(\xi) = H_1q_1 + \frac{l_e}{2}H_2q_2 + H_3q_3 + \frac{l_e}{2}H_4q_4$

Here Since the Uniformly distributed load is for Element ②

$$v(\xi) = H_1q_3 + \frac{l_e}{2}H_2q_4 + H_3q_5 + \frac{l_e}{2}H_4q_6.$$

Here $q_3 = q_5 = 0$.

$$v(\xi) = \frac{l_e}{2}H_2q_4 + \frac{l_e}{2}H_4q_6.$$

At $\xi=0$, $H_2 = \frac{1}{4}(1-\xi)^2(\xi+1) = 1/4$ & $H_4 = 1/2(1+\xi)^2(\xi-1) = -1/4$

$$v(\xi=0) = \frac{l_e}{2}(1/4)q_4 + l_e/2(-1/4)q_6$$

$$= \frac{1000}{2}\times1/4(-2.67\times10^{-4}) + \frac{1000}{2}(-1/4)(4.46\times10^{-4})$$

$$= -0.0893\,mm.$$