VI – UNIT VIBRATIONS (DOM)

Vibrations: - When an elastic body (like shaft, springs etc) which is fixed at one end and is displaced at other end from its equilibrium position by the application of an external force, the body starts to move to and fro (or up and down). Then the body is said to be in vibrations. The vibrations are due to internal elastic forces within the body.

Main Causes Of Vibration

Ans. The main causes of vibration are:

1. Unbalanced centrifugal force in the. System due to faulty design and poor manufacturing.
2. Elastic nature of system.
3. External excitation applied on the system
4. Winds may cause vibration of certain system such as electricity lines, telephone lines etc

Disadvantages of Effects of Vibration

Ans. Disadvantages harmful effects vibration:

1. Vibration causes excessive and unpleasant stresses in the rotating system.
2. Vibration causes rapid wear and tear of machine parts such as gears and bearings.
3. Vibration causes loosening of parts from the machine.
4. Due to vibrations locomotive can leave the track causing accident or heavy loss.
5. Earthquakes are the cause of vibration because of which buildings and other structures (like bridges) may collapse.
6. Proper readings of instruments cannot be taken because of heavy vibrations.
7. Resonance may take place if the frequency of excitation matches with the natural frequency of system causing large amplitudes of vibration thereby resulting in failure of systems e.g. Bridges

How can you eliminate/reduce unnecessary vibrations?

Ans. Unwanted vibrations can be reduced by:

1. Removing external excitation if possible.
2. Using shock absorbers.
3. Dynamic absorbers.
4. Proper balancing of rotating parts.

5. Removing manufacturing defects and material in homogeneities.

6. Resting the system on proper vibration isolators.

What are the advantages of vibration?

Ans. Advantages of vibration

1. Musical Instruments like guitar.

2. In study of earthquake for geological reasons.

3. Vibration is useful for vibration testing equipments.

4. Propagation of sound is due to vibrations.

5. Vibratory conveyors are based on concept of vibration.

6. Pendulum clocks are based on the principle of vibration.

What is the importance of vibration study?

Importance of vibration study. The imp of vibration study is to reduce or eliminate vibration effects over mechanical components by designing them suitably. Proper design and manufacture of parts will reduce. Unbalance in engines which causes excessive and unpleasant stress in rotating system because of vibration, roper design of machine parts will reduce and tear due to vibration and loosening parts. The proper designing and material distribution prevent the locomotive leaving the track due to excessive vibration which may result in accident or heavy loss. Proper designing of structure buildings can prevent the condition of resonance which causes dangerously large oscillations which may result in failure of that part.

Terms Used in Vibratory Motion

(i) Periodic motion: A motion which repeats itself after certain interval of time is called periodic motion.

(ii) Time Period: It is time taken to complete One cycle.

(iii) Frequency: No's of cycles in one sec, S.I. units, the frequency is expressed in hertz (briefly written as Hz)

(iv) Amplitude: Maximum displacement of a vibrating body from mean position is called Amplitude.

(v) Simple Harmonic Motion (S.H.M..) : The motion of a body "to" and "fro" about a fixed point is called S.H.M.

(vi)Resonance: When the frequency of external force is equal to the natural frequency of a vibrating body, the amplitude of vibration becomes excessively large. This is known as "Resonance". At resonance there are chances of machine part or structure to fail due to excessively large amplitude. It is thus important to find natural frequencies of the system in order to avoid resonance.

Explain different methods of vibration analysis ?

Ans. Different methods of vibration analysis are:

Energy method : According to this method total energy of the system remains constant i.e. sum of kinetic energy and potential energy always remains constant.

$$K.E. = \frac{1}{2} m\dot{x}^2$$

$$P.E. = \frac{1}{2} kx^2$$

$$K.E.+P.E. = c$$

$$\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 = c$$

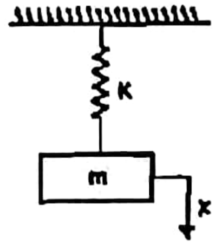


Fig.

Differentiating w.r.t. x

$$m\dot{x}\ddot{x} + Kx\dot{x} = 0$$

$$m\ddot{x} + Kx = 0$$

Let

$$x = A \sin \omega t$$

$$\ddot{x} = -A\omega^2 \sin \omega t$$

$$\ddot{x} = -\omega^2 x$$

Putting in (I) we get

$$m \times (-\omega^2 x) + Kx = 0$$

$$\omega = \sqrt{\frac{K}{m}} \text{ rad/s}$$

Rayleigh Method : This method is based on the principle that maximum kinetic energy of the term is equal to the maximum potential energy of the system.

$$(KE)_{max} = (PE)_{max}$$

$$\left(\frac{1}{2}m\dot{x}^2\right)_{max} = \left(\frac{1}{2}Kx^2\right)_{max}$$

$$x = A\sin\omega t$$

$$x_{max} = A$$

$$\dot{x} = A\omega\cos\omega t$$

$$(\dot{x})_{max} = A\omega$$

$$\frac{1}{2}m(A\omega)^2 = \frac{1}{2}KA^2$$

$$\omega = \sqrt{\frac{K}{m}} \text{ rad/s}$$

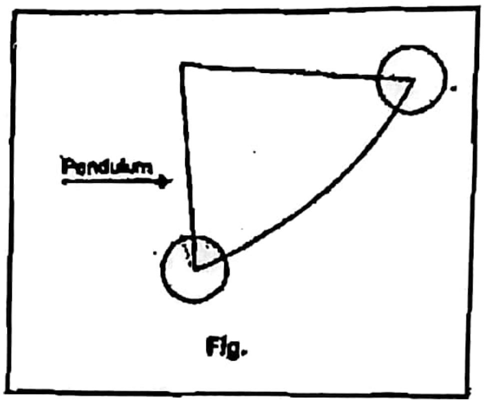


Fig.

According to this method the sum of forces and moments acting on the system is zero if no external force is applied on the system.

Consider fig. I

$$m\ddot{x} + c\dot{x} + Kx = 0 \quad \text{(If no external force is applied)}$$

$$m\ddot{x} + c\dot{x} + Kx = F \quad \text{(If external force F is applied)}$$

What is damping?

Ans. Damping is the resistance offered by a body to the motion of a vibratory system.

Classify different types of damping.

Ans. Types of Damping

1. Viscous 2. Coulomb 3. Structural 4. Non-linear, Slip or interfacial damping 5. Eddy current-damping

Types of Vibratory Motion

The following types of vibratory motion are important from the subject point of view :

1. Free or natural vibrations. When no external force acts on the body, after giving it an initial displacement, then the body is said to be under **free or natural vibrations.** The frequency of the free vibrations is called **free or natural frequency.**

2. Forced vibrations. When the body vibrates under the influence of external force, then the body is said to be under **forced vibrations.** The external force applied to the body is a periodic disturbing force created by unbalance. The vibrations have the same frequency as the applied force.

3. Damped vibrations. When there is a reduction in amplitude over every cycle of vibration, the motion is said to be **damped vibration.** This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the
Motion.

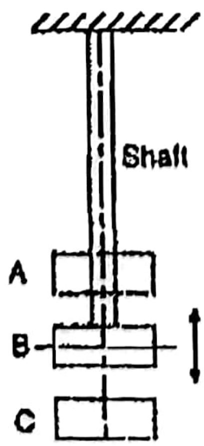
Types of Free Vibrations

The following three types of free vibrations are important from the subject point of view :

1. Longitudinal vibrations, 2. Transverse vibrations, and 3. Torsional vibrations.

Consider a weightless constraint (spring or shaft) whose one end is fixed and the other end carrying a heavy disc, as shown in Fig. 23.1. This system may execute one of the three above mentioned types of vibrations.

1. Longitudinal vibrations: When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Fig(a), then the vibrations are known as **longitudinal vibrations.** In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.



Taking the value of g as 9.81 m/s² and δ in metres.

natural frequency = $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

$$\frac{\text{Stress}}{\text{Strain}} = E \quad \text{or} \quad \frac{W}{A} \times \frac{l}{\delta} = E \quad \text{or} \quad \delta = \frac{W\,l}{E.A}$$

δ = Static deflection *i.e.* extension or compression of the constraint,

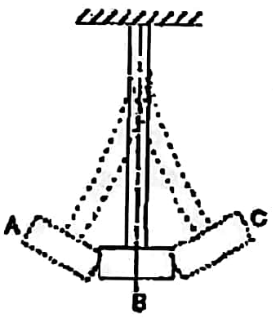
W = Load attached to the free end of constraint.

l = Length of the constraint,

E = Young's modulus for the constraint, and

A = Cross-sectional area of the constraint.

2. Transverse vibrations: When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Fig(b), then the vibrations are known as **transverse vibrations.** In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.



Taking the value of g as 9.81 m/s² and δ in metres.

natural frequency = $f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}}$.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{9.81}{\delta}} = \frac{0.4985}{\sqrt{\delta}} \text{ Hz}$$

$$\delta = \frac{Wl^3}{3EI} \text{ (in metres)}$$

W = Load at the free end, in newtons.

l = Length of the shaft or beam in metres.

E = Young's modulus for the material of the shaft or beam in N/m², and

I = Moment of inertia of the shaft or beam in m⁴.

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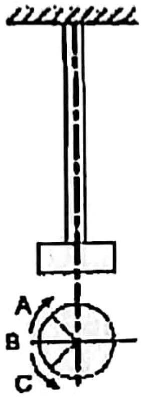
3. Torsional vibrations: When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Fig(c), then the vibrations are known as **torsional vibrations.**

In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.

Frequency (f) = $\frac{1}{2\pi}\sqrt{\frac{q}{I}}$ Where, q= Torsional stiffness = $\frac{T}{\theta} = \frac{G\theta}{L}$



Problem(1) A cantilever shaft 50 mm diameter and 300 mm long has a disc of mass 100 kg at its free end. The Young's modulus for the shaft material is 200 GN/m^2. Determine the frequency of longitudinal and transverse vibrations of the shaft.

Solution. Given : d = 50 mm = 0.05 m ; l = 300 mm = 0.03 m ; m = 100 kg ; E = 200 GN/m^2 = 200 ×109 N/m^2

We know that cross-sectional area of the shaft,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96 \times 10^{-3}\,\text{m}^2$$

and moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.3 \times 10^{-6}\,\text{m}^4$$

Frequency of longitudinal vibration

We know that static deflection of the shaft,

$$\delta = \frac{W.l}{A.E} = \frac{100 \times 9.81 \times 0.3}{1.96 \times 10^{-3} \times 200 \times 10^9} = 0.751 \times 10^{-6}\ \text{m}$$

$\ldots(\because W = m.g)$

Frequency of longitudinal vibration,

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$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.751 \times 10^{-6}}} = 575 \text{ Hz}$$

Frequency of transverse vibration

We know that static deflection of the shaft,

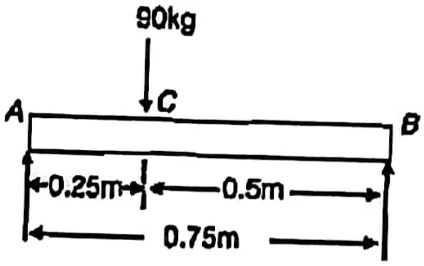
$$\delta = \frac{Wl^3}{3EI} = \frac{100 \times 9.81 \times (0.3)^3}{3 \times 200 \times 10^9 \times 0.3 \times 10^{-6}} = 0.147 \times 10^{-3} \text{ m}$$

Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.147 \times 10^{-3}}} = 41 \text{ Hz}$$

Problem(2) A shaft of length 0.75 m, supported freely at the ends, is carrying a body of mass 90 kg at 0.25 m from one end. Find the natural frequency of transverse vibration. Assume E = 200 GN/m² and shaft diameter = 50 mm.

Solution. Given : l = 0.75 m ; m = 90 kg ; a = AC = 0.25 m ; E = 200 GN/m² = 200 × 109 N/m²; d = 50 mm = 0.05 m



We know that moment of inertia of the shaft.

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 \text{ m}^4$$

$$= 0.307 \times 10^{-6} \text{ m}^4$$

and static deflection at the load point (i.e. at point C),

$$\delta = \frac{Wa^2b^2}{3EIl} = \frac{90\times9.81(0.25)^2(0.5)^2}{3\times200\times10^9\times0.307\times10^{-6}\times0.75} = 0.1\times10^{-3}\ \text{m}$$

$$\ldots (\because b = BC = 0.5\ \text{m})$$

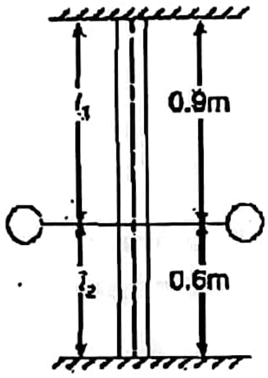
We know that natural frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{0.1\times10^{-3}}} = 49.85\ \text{Hz}$$

Problem(3). **A flywheel is mounted on a vertical shaft as shown in Fig. 23.8. The both ends of the shaft are fixed and its diameter is 50 mm. The flywheel has a mass of 500 kg. Find the natural frequencies of longitudinal and transverse vibrations. Take E = 200 GN/m²**

Solution. Given : d = 50 mm = 0.05 m ; m = 500 kg ; E = 200 GN/m² = 200 × 109 N/m2

We know that cross-sectional area of shaft,



$$A = \frac{\pi}{4}\times d^2 = \frac{\pi}{4}(0.05)^2 = 1.96\times10^{-3}\ \text{m}^2$$

and moment of inertia of shaft,

$$I = \frac{\pi}{64}\times d^4 - \frac{\pi}{64}(0.05)^4 = 0.307\times10^{-6}\ \text{m}^4$$

Natural frequency of longitudinal vibration

m_1 = Mass of flywheel carried by the length l_1.

$m - m_1$ = Mass of flywheel carried by length l_2.

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We know that extension of length l_1

$$= \frac{W_1.l_1}{A.E} = \frac{m_1.g.l_1}{A.E} \quad \text{------------------(i)}$$

Similarly, compression of length l_2

$$= \frac{(W - W_1)\, l_2}{A.E} = \frac{(m - m_1)\, g.l_2}{A.E} \quad \text{--------------(ii)}$$

Since extension of length l1 must be equal to compression of length l2, therefore equating equations (i) and (ii),

$$m_1.l_1 = (m - m_1) l_2$$

$$m_1 \times 0.9 = (500 - m_1) 0.6 = 300 - 0.6 m_1 \quad \text{or} \quad m_1 = 200 \text{ kg}$$

Extension of length l_1,

$$\delta = \frac{m_1.g.l_1}{A.E} = \frac{200 \times 9.81 \times 0.9}{1.96 \times 10^{-3} \times 200 \times 10^9} = 4.5 \times 10^{-6} \text{ m}$$

We know that natural frequency of longitudinal vibration.

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{4.5 \times 10^{-6}}} = 235 \text{ Hz}$$

Natural frequency of transverse vibration

We know that the static deflection for a shaft fixed at both ends and carrying a point load is given by

$$\delta = \frac{W a^3 b^3}{3 E I l^3} = \frac{500 \times 9.81 (0.9)^3 (0.6)^3}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} (1.5)^3} = 1.24 \times 10^{-3} \text{ m}$$

. . . (Substituting $W = m.g$; $a = l_1$, and $b = l_2$)

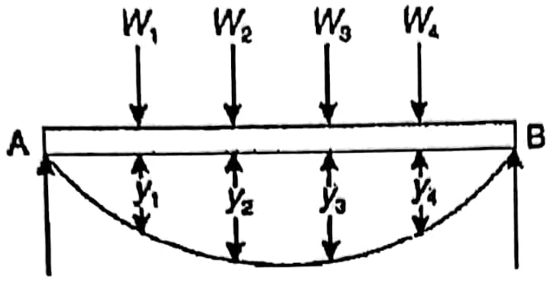
We know that natural frequency of transverse vibration.

$$f_n = \frac{0.4985}{\sqrt{\delta}} = \frac{0.4985}{\sqrt{1.24 \times 10^{-3}}} = 14.24 \text{ Hz}$$

<u>Natural Frequency of Free Transverse Vibrations For a Shaft Subjected to a Number of Point Loads</u>

10.

Consider a shaft AB of negligible mass loaded with point loads W_1, W_2, W_3 and W_4 etc. in newtons, as shown in Fig.. Let m_1, m_2, m_3 and m_4 etc. be the corresponding masses in kg. The natural frequency of such a shaft may be found out by the following two methods :



1. Energy (or Rayleigh's) method

Let y1, y2, y3, y4 etc. be total deflection under loads W1, W2, W3 and W4 etc. as shown in Fig.

We know that maximum potential energy

$$= \frac{1}{2} \times m_1 . g . y_1 + \frac{1}{2} \times m_2 . g . y_2 + \frac{1}{2} m_3 . g . y_3 + \frac{1}{2} \times m_4 . g . y_4 + ...$$

$$= \frac{1}{2} \Sigma m . g . y$$

and maximum kinetic energy

$$= \frac{1}{2} \times m_1 (\omega y_1)^2 + \frac{1}{2} \times m_2 (\omega . y_2)^2 + \frac{1}{2} \times m_3 (\omega y_3)^2 + \frac{1}{2} \times m_4 (\omega . y_4)^2 + ...$$

$$= \frac{1}{2} \times \omega^2 \left[m_1 (y_1)^2 + m_2 (y_2)^2 + m_3 (y_3)^2 + m_4 (y_4)^2 + ... \right]$$

$$= \frac{1}{2} \times \omega^2 \Sigma m . y^2$$ . . . (where ω = Circular frequency of vibration)

Equating the maximum kinetic energy to the maximum potential energy, we have

$$\frac{1}{2} \times \omega^2 \Sigma m . y^2 = \frac{1}{2} \Sigma m . g . y$$

$$\omega^2 = \frac{\Sigma m \, g \, y}{\Sigma m . y^2} = \frac{g \, \Sigma m \, y}{\Sigma m . y^2} \quad \text{or} \quad \omega = \sqrt{\frac{g \, \Sigma m \, y}{\Sigma m . y^2}}$$

Natural frequency of transverse vibration,

$$f_n = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{g \, \Sigma m \, y}{\Sigma m . y^2}}$$

2. Dunkerley's method

The natural frequency of transverse vibration for a shaft carrying a number of point loads and uniformly distributed load is obtained from Dunkerley's empirical formula. According to this

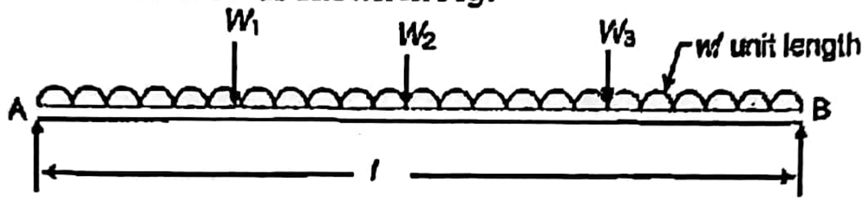
$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^2} + \dots + \frac{1}{(f_{ns})^2}$$

f_n = Natural frequency of transverse vibration of the shaft carrying point loads and uniformly distributed load.

f_{n1}, f_{n2}, f_{n3}, etc. = Natural frequency of transverse vibration of each point load.

f_{ns} = Natural frequency of transverse vibration of the uniformly distributed load (or due to the mass of the shaft).

Now, consider a shaft AB loaded as shown in Fig.



Shaft carrying a number of point loads and a uniformly distributed load.

Let δ_1, δ_2, δ_3 etc. = Static deflection due to the load W_1, W_2, W_3 etc. when considered separately.

δ_s = Static deflection due to the uniformly distributed load or due to the mass of the shaft.

We know that natural frequency of transverse vibration due to load W_1,

$$f_{n1} = \frac{0.4985}{\sqrt{\delta_1}} \text{ Hz}$$

Similarly, natural frequency of transverse vibration due to load W_2,

$$f_{n2} = \frac{0.4985}{\sqrt{\delta_2}} \text{ Hz}$$

and, natural frequency of transverse vibration due to load W_3,

$$f_{n3} = \frac{0.4985}{\sqrt{\delta_1}} \text{ Hz}$$

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Also natural frequency of transverse vibration due to uniformly distributed load or weight of the shaft,

$$f_{ns} = \frac{0.5615}{\sqrt{\delta_S}} \text{ Hz}$$

Therefore, according to Dunkerley's empirical formula, the natural frequency of the whole system,

$$\frac{1}{(f_n)^2} = \frac{1}{(f_{n1})^2} + \frac{1}{(f_{n2})^2} + \frac{1}{(f_{n3})^3} + \ldots + \frac{1}{(f_{ns})^2}$$

$$= \frac{\delta_1}{(0.4985)^2} + \frac{\delta_2}{(0.4985)^2} + \frac{\delta_3}{(0.4985)^2} + \ldots + \frac{\delta_S}{(0.5615)^2}$$

$$= \frac{1}{(0.4985)^2}\left[\delta_1 + \delta_2 + \delta_3 + \ldots + \frac{\delta_S}{1.27}\right]$$

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \ldots + \frac{\delta_S}{1.27}}} \text{ Hz}$$

Notes : 1. When there is no uniformly distributed load or mass of the shaft is negligible, then $\delta_s = 0$.

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3 + \ldots}} \text{ Hz}$$

2. The value of $\delta_1 \cdot \delta_2 \cdot \delta_3$ etc. for a simply supported shaft may be obtained from the relation

$$\delta = \frac{Wa^2b^2}{3EIl}$$

δ = Static deflection due to load W,

a and b = Distances of the load from the ends,

E = Young's modulus for the material of the shaft.

I = Moment of inertia of the shaft, and

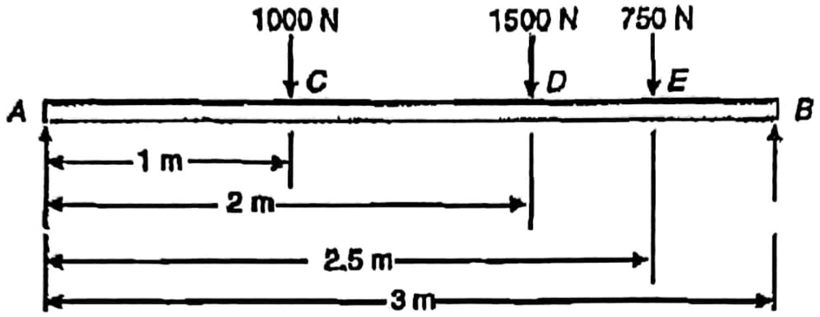
l = Total length of the shaft.

Problem(4). A shaft 50 mm diameter and 3 metres long is simply supported at the ends and carries three loads of 1000 N, 1500 N and 750 N at 1 m, 2 m and 2.5 m from

the left support. The Young's modulus for shaft material is 200 GN/m^2. Find the frequency of transverse vibration.

Solution. Given : d = 50 mm = 0.05 m ; l = 3 m, W_1 = 1000 N ; W_2 = 1500 N ; W_3 = 750 N; E = 200 GN/m^2 = 200 × 109 N/m^2

The shaft carrying the loads is shown in Fig.



We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64}(0.05)^4 = 0.307 \times 10^{-6}\ m^4$$

and the static deflection due to a point load W,

$$\delta = \frac{Wa^2b^2}{3EIl}$$

Static deflection due to a load of 1000 N,

$$\delta_1 = \frac{1000 \times 1^2 \times 2^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 7.24 \times 10^{-3}\ m$$

... (Here a = 1 m, and b = 2 m)

Similarly, static deflection due to a load of 1500 N,

$$\delta_2 = \frac{1500 \times 2^2 \times 1^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 10.86 \times 10^{-3}\ m$$

... (Here a = 2 m, and b = 1 m)

and static deflection due to a load of 750 N,

$$\delta_3 = \frac{750(2.5)^2(0.5)^2}{3 \times 200 \times 10^9 \times 0.307 \times 10^{-6} \times 3} = 2.12 \times 10^{-3}\ m$$

... (Here a = 2.5 m, and b = 0.5 m)

We know that frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta_1 + \delta_2 + \delta_3}} = \frac{0.4985}{\sqrt{7.24 \times 10^{-3} + 10.86 \times 10^{-3} + 2.12 \times 10^{-3}}}$$

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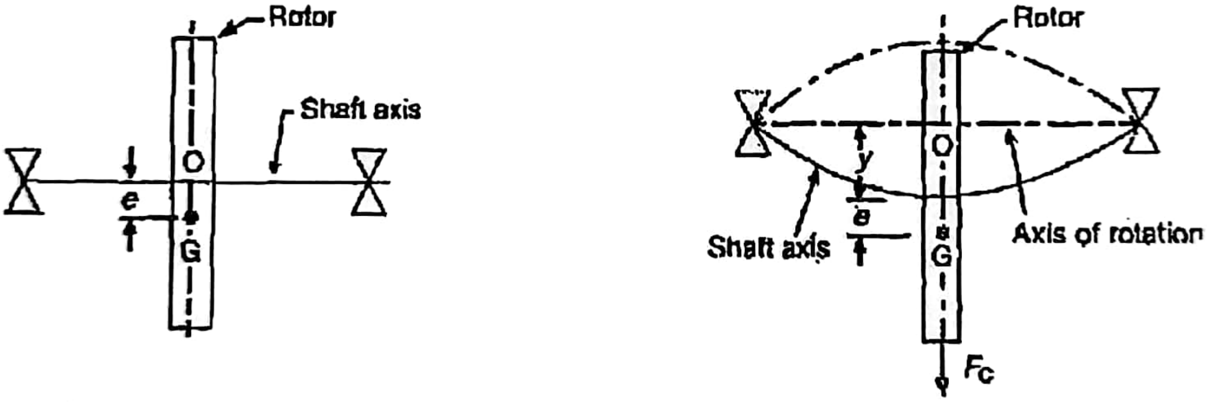
$$= \frac{0.4985}{0.1422} = 3.5 \text{ Hz}.$$

Critical or Whirling Speed of a Shaft

In actual practice, a rotating shaft carries different mountings and accessories in the form of gears, pulleys, etc. When the gears or pulleys are put on the shaft, the centre of gravity of the pulley or gear does not coincide with the centre line of the bearings or with the axis of the shaft, when the shaft is stationary. This means that the centre of gravity of the pulley or gear is at a certain distance from the axis of rotation and due to this, the shaft is subjected to centrifugal force. This force will bent the shaft which will further increase the distance of centre of gravity of the pulley or gear from the axis of rotation. This correspondingly increases the value of centrifugal force, which further increases the distance of centre of gravity from the axis of rotation. This effect is cumulative and ultimately the shaft fails. The bending of shaft not only depends upon the value of eccentricity (distance between centre of gravity of the pulley and the axis of rotation) but also depends upon the speed at which the shaft rotates.

The speed at which the shaft runs so that the additional deflection of the shaft from the axis of rotation becomes infinite, is known as critical or whirling speed.



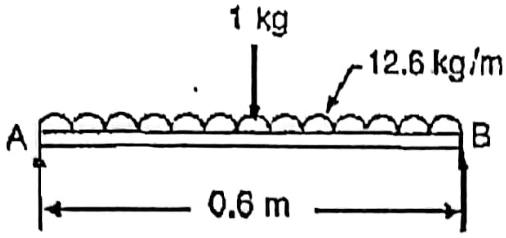
(a) When shaft is stationary. (b) When shaft is rotating.

Critical or whirling speed of a shaft.

Problem(5). Calculate the whirling speed of a shaft 20 mm diameter and 0.6 m long carrying a mass of 1 kg at its mid-point. The density of the shaft material is 40 Mg/m^3, and Young's modulus is 200 GN/m^2. Assume the shaft to be freely supported.

Given data : $d = 20$ mm $= 0.02$ m ; $l = 0.6$ m ; $m_1 = 1$ kg ; $\rho = 40$ Mg/m^3 $= 40 \times 10^6$ g/m^3 $= 40 \times 10^3$ kg/m^3 ; $E = 200$ GN/m^2 $= 200 \times 10^9$ N/m^2

The shaft is shown in Fig.



We know that moment of inertia of the shaft,

$$I = \frac{\pi}{64} \times d^4 = \frac{\pi}{64} (0.02)^4 \text{ m}^4$$

$$= 7.855 \times 10^{-9} \text{ m}^4$$

Since the density of shaft material is 40×10^3 kg/m^3, therefore mass of the shaft per metre length,

$$m_S = \text{Area} \times \text{length} \times \text{density} = \frac{\pi}{4} (0.02)^2 \times 1 \times 40 \times 10^3 = 12.6 \text{ kg/m}$$

We know that static deflection due to 1 kg of mass at the centre,

$$\delta = \frac{Wl^3}{48EI} = \frac{1 \times 9.81(0.6)^3}{48 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 28 \times 10^{-6} \text{ m}$$

and static deflection due to mass of the shaft,

$$\delta_S = \frac{5wl^4}{384EI} = \frac{5 \times 12.6 \times 9.81(0.6)^4}{384 \times 200 \times 10^9 \times 7.855 \times 10^{-9}} = 0.133 \times 10^{-3} \text{ m}$$

Frequency of transverse vibration,

$$f_n = \frac{0.4985}{\sqrt{\delta + \frac{\delta_S}{1.27}}} + \frac{0.4985}{\sqrt{28 \times 10^{-6} + \frac{0.133 \times 10^{-3}}{1.27}}}$$

$$= \frac{0.4985}{11.52 \times 10^{-3}} = 43.3 \text{ Hz}$$

N_c = Whirling speed of a shaft.

We know that whirling speed of a shaft in r.p.s. is equal to the frequency of transverse vibration in Hz , therefore

N_c = 43.3 r.p.s. = 43.3 × 60 = 2598 r.p.m.

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