UNIT-VI
BALANCING OF ROTATING MASS

Introduction:- The high-speed engines and other machines are being used frequently. unbalance in these machines arises either due to ecentric rotating or reciprocating masses or geometric centre not coinciding with the mass centre of the rotating components. These masses give rise to dynamic forces that increase the bearing loads and introduce severe stresses in the machine components.

The eccentric rotating or reciprocating mass is called the disturbing mass.

The various causes of unbalance are:

1. Eccentric rotating or reciprocating masses.
2. unsymmetry caused during production process.
3. Non-homogeneity of materials.
4. Elastic deformation during running
5. Faulty mounting resulting in eccentricity
6. misalignment of bearings
7. plastic deformations.

unbalance introduces severe stresses and result in undesirable vibrations in the machines.

By balancing we mean to eliminate either partially or completely the effects due to unbalanced resultant inertia forces and couples to avoid vibration of a machine or device.

(1) What is the necessity of the balancing?

(A) All the machines will be having reciprocating parts or rotating parts or both of them. Now-a-days, machines are run at high speeds which necessitates the complete balancing of rotation as well as reciprocating masses. If the balancing of moving parts is not proper, then it leads to the set up of inertia forces resulting in vibration. These vibrations when occurs at high speed leads to excessive noise, cause wear and tear of the parts. Thus, balancing is done to neutralize the vibrations as far as possible. A perfect balanced machine will have all the resultant forces and couples equal to zero.

5) ** Distinguish the static balance and dynamic balance with appropriate example.

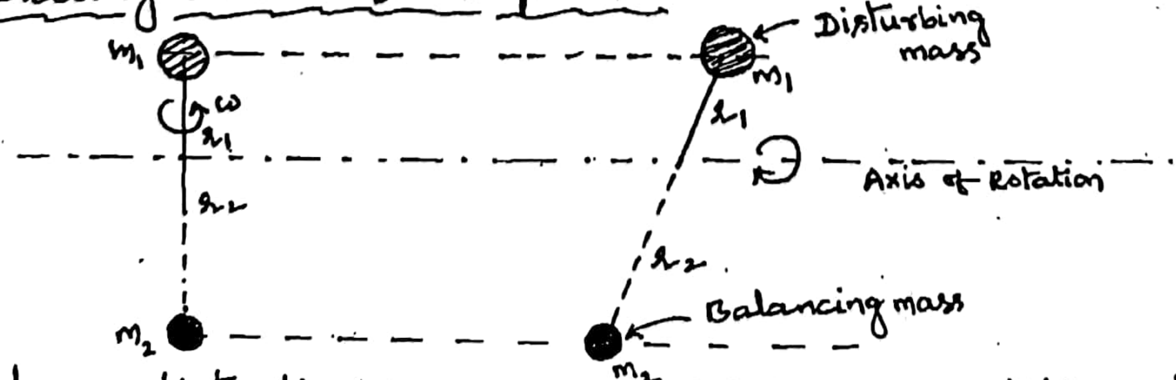
S NO	static balancing	Dynamic balancing
1.	static balancing is considered when the system of masses revolves only in one plane.	dynamic balancing is considered when the system of masses revolves in different plane.
2.	A system is said to be in static balance, if the combined mass centre of the system lies on the axis of rotation.	A system is said to be in dynamic balancing. when there does not exist any resultant centrifugal force as well as resultant couple.
3.	If the conditions for static balancing are met, then the condition for dynamic balance may not be met.	If the conditions for d.b are met, then the condition for static balance are also met.
4.	static balancing machines are used only for the parts whose aerial dimensions are small.	D.b machines can be used for any dimension parts.
5.	On static balancing the parts are acted upon by gravity or centrifugal force	On d.b the parts are acted upon by force as well as couples.
6	It is used for the parts such as gears, impellers etc.	It is used for the longer machines element such as turbine rotors & motor armatures etc.

Balancing of Rotating masses :- some mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses.

The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different plane.

(1) Balancing of a single rotating mass by a single mass rotating in the same plane :-



conider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in figure. Let r_1 be the radius of rotation of the mass m_1.

(i.e distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1).

centrifugal force exerted by the mass m_1 on the shaft

$$F_{C_1} = m_1 . \omega^2 r_1 \quad \text{——— (1)}$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1) such that the centrifugal forces due to the two masses are equal and opposite.

r_2 = Radius of rotation of the balancing mass m_2

∴ centrifugal force due to mass m_2

$$F_{C_2} = m_2 . \omega^2 . r_2 \quad \text{——— (2)}$$

Equating equations (1) & (2)

$$m_1 \cancel{\omega^2} . r_1 = m_2 \cancel{\omega^2} . r_2$$

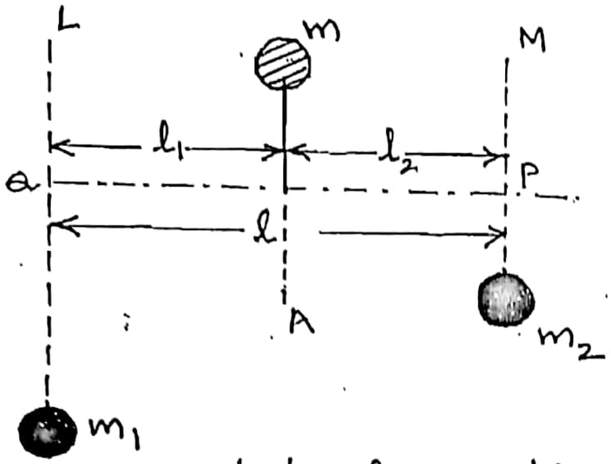
$$\boxed{m_1 r_1 = m_2 r_2}$$

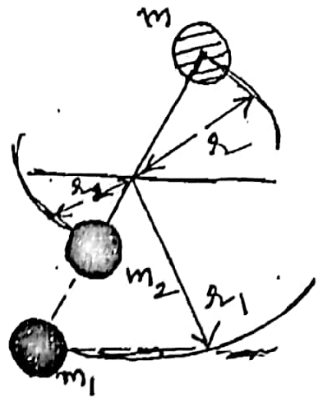
Note:- (1) The product $m_2 r_2$ may be split up in any convinient way. But the radius of rotation of the balancing mass [m_2] is generally made large in order to reduce the balancing mass m_2.

(2) The centrifugal forces are proportional to the product of the mass and radius of rotation of respective masses, because ω^2 is same for each mass.

(2) Balancing of a single rotating mass by two masses rotating in different planes:-

(a) When the plane of the disturbing mass lies in between the planes of the two balancing masses:- consider a disturbing mass m lying in a plane A to be balanced by two rotating masses m_1 and m_2 lying in two different planes L and M as shown in figure. Let r, r_1 and r_2 be the radii of rotation of the masses in planes A, L and M respectively.





Let l_1 = Distance between the planes A and L

l_2 = " " " A and M

l = " " " L and M

centrifugal force exerted by the mass m in the plane A

$$F_c = m \cdot \omega^2 \cdot r$$

Similarly

centrifugal force exerted by the mass m_1 in the plane L

$$F_{c_1} = m_1 \cdot \omega^2 \cdot r_1$$

centrifugal force exerted by the mass m_2 in the plane M

$$F_{c_2} = m_2 \cdot \omega^2 \cdot r_2$$

Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses

$$F_c = F_{c_1} + F_{c_2}$$

$$m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\cancel{\omega^2}[m \cdot r] = [m_1 \cdot r_1 + m_2 r_2]\cancel{\omega^2}$$

$$\therefore m \cdot r = m_1 r_1 + m_2 r_2 \quad \text{——— (1)}$$

Taking moment about 'P'

$$F_{c_1} \times l = F_c \times l_2 \quad \text{or} \quad m_1 \cancel{\omega^2} \cdot r_1 \times l = m \cancel{\omega^2} \cdot r \, l_2$$

$$m_1 r_1 l = m \cdot r \cdot l_2$$

$$m_1 r_1 = m \cdot r \frac{l_2}{l} \quad \text{——— (2)}$$

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Taking moment about Q,

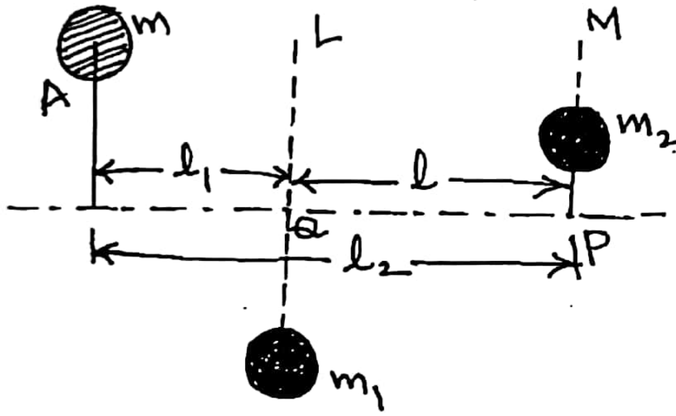
$F_{c_2} \times l = F_c \times l_1$ or $m_2 \cdot \cancel{\omega^2} \cdot r_2\, l = m \cdot \cancel{\omega^2} \cdot r \cdot l_1$

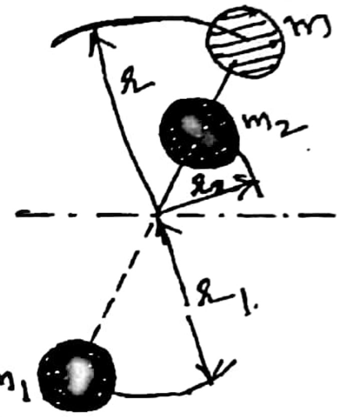
$\therefore \quad m_2 r_2 l = m \cdot r \cdot l_1$

$$m_2 r_2 = m \cdot r \frac{l_1}{l} \qquad \text{———— (3)}$$

Note :- Equation (1) represents the condition for static balance and equations (2) & (3) are dynamic balance.

(ii) When the plane of the disturbing mass lies on one end of the planes of the balancing masses :-





Satisfied in order to balance the system

$$F_c + F_{c_2} = F_{c_1}$$

$$m \cdot \cancel{\omega^2} \cdot r + m_2 \cdot \cancel{\omega^2} \cdot r_2 = m_1 \cdot \cancel{\omega^2} \cdot r_1$$

$$\therefore \quad m \cdot r + m_2 \cdot r_2 = m_1 \cdot r_1 \qquad \text{———— (4)}$$

Taking the moment about P

$F_{c_1} \times l = F_c \times l_2$ or $m_1 \omega^2 \cdot r_1 \times l = m \omega^2 \cdot r \times l_2$

$\therefore \quad m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2$ & $m_1 r_1 = m \cdot r \dfrac{l_2}{l}$ —— (5)

...[Same as equation (2)]

Taking the moment about Q

$F_{c_2} \times l = F_c \times l_1$ or $m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$

$m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1$

$$m_2 \cdot r_2 = m \cdot r \frac{l_1}{l} \qquad \text{———— (6)}$$

[.....Same as equation (3)]

(3) Balancing of several masses rotating in the same plane :-

problem

(1) Four masses m_1, m_2, m_3 and m_4 are 200kg, 300kg, 240kg and 260kg respectively. The corresponding radii of rotation are 0.2m, 0.15m, 0.25m and 0.3m respectively and the angles between successive masses are $45°$, $75°$ and $135°$. Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2m.

Solution :- Given data :- $m_1 = 200kg$; $m_2 = 300kg$

$M_3 = 240kg$; $m_4 = 260kg$;

$r_1 = 0.2m$; $r_2 = 0.15m$; $r_3 = 0.25m$; $r_4 = 0.3m$

$\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45+75 = 120°$

$\theta_4 = 45+75+135° = 255°$

$r = 0.2m$

Let m = Balancing mass and

θ = The angle which the balancing mass makes with m_1

Since the magnitude of centrifugal forces are proportional to the product of each mass and its radius.

$$\boxed{F_c = m.r}$$

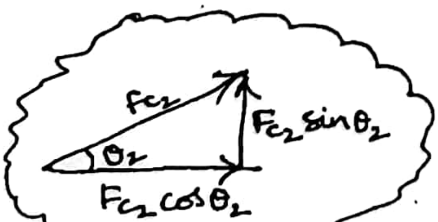
$F_{c_1} = m_1 r_1 \Rightarrow 200 \times 0.2 \Longrightarrow 40kg\text{-}m$

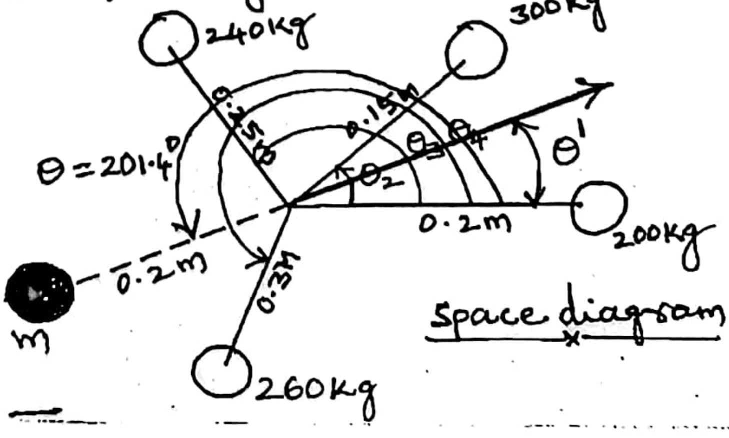
$F_{c_2} = m_2 . r_2 \Rightarrow 300 \times 0.15 \Longrightarrow 45kg\text{-}m$

$F_{c_3} = m_3 . r_3 \Longrightarrow 240 \times 0.25 \Longrightarrow 60kg\text{-}m$

$F_{c_4} = m_4 . r_4 \Longrightarrow 260 \times 0.3 \Longrightarrow 78kg\text{-}m$

1. Analytical method :-





Resolving the horizontal forces $\Sigma H = F_{c_1}\cos\theta_1 + F_{c_2}\cos\theta_2 + F_{c_3}\cos\theta_3 + F_{c_4}\cos\theta_4$.

$$\Sigma H = 40\cos 0^\circ + 45\cos 45^\circ + 60\cos 120^\circ + 78\cos 255^\circ$$
$$= 21.6\ \text{Kg-m}$$

Resolving the vertical forces $\Sigma V = F_{c_1}\sin\theta_1 + F_{c_2}\sin\theta_2 + F_{c_3}\sin\theta_3 + F_{c_4}\sin\theta_4$

$$\Sigma V = 40\sin 0^\circ + 45\sin 45^\circ + 60\sin 120^\circ + 78\sin 255^\circ.$$
$$= 8.5\ \text{Kg-m}$$

$$\therefore \text{Resultant}, R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$
$$= \sqrt{(21.6)^2 + (8.5)^2}$$
$$= 23.2\ \text{kg-m}$$

$$F_c = m.r = \text{Resultant}$$
$$m.r = 23.2$$
$$m = \frac{23.2}{r} \Rightarrow \frac{23.2}{0.2}$$
$$m = 116\ \text{kg}$$

$$\tan\theta' = \frac{\Sigma V}{\Sigma H}$$
$$= \frac{8.5}{21.6} \Rightarrow \theta' = \tan^{-1}(0.3935)$$
$$\theta' = 21.48^\circ$$

Since θ' is the angle of the resultant (R) from the horizontal mass of m_1 (200kg), therefore the angle of the balancing mass from the horizontal mass of m_1.

$$\theta = 180 + 21.48$$
$$\theta = 201.48^\circ$$

(2) Graphical Method :-

(a) First of all, draw the space diagram showing the positions of all the given masses as shown in figure.

(b) Since the centrifugal force of each mass is proportional to the product of the mass and radius,

$F_{c_1} = m_1 r_1 \Rightarrow 200\times 0.2 \Rightarrow 40\text{Kg-m}$

$F_{c_2} = m_2 r_2 \Rightarrow 300\times 0.15 \Rightarrow 45\text{Kg-m}$

$F_{c_3} = m_3 r_3 \Rightarrow 240\times 0.25 \Rightarrow 60\text{Kg-m}$

$F_{c_4} = m_4 r_4 \Rightarrow 260\times 0.3 \Rightarrow 78\text{ Kg-m}$

(c) Now draw the vector diagram with the above values, to some suitable scale, as shown in figure. The closing side of the polygon ae represents the resultant force, By measurement, we find that ae = 23 kg-m.

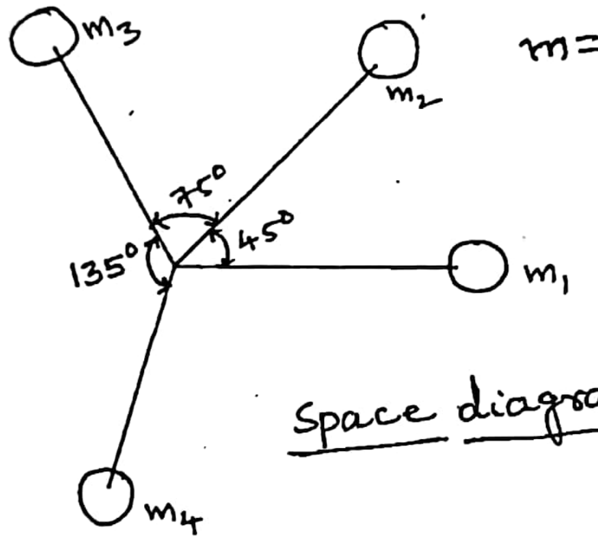
(d) The balancing force is equal to the resultant force, but opposite in direction as shown in figure.

Balancing force $= m.r =$ vector ea

$$m \times 0.2 = 23$$

$$m = \frac{23}{0.2}$$

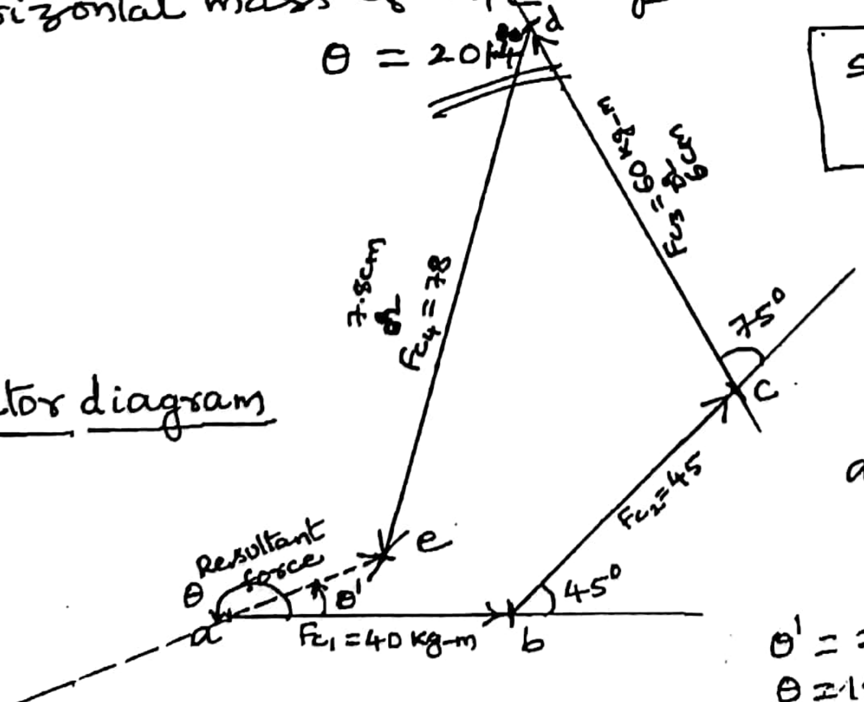
$$m = 115 \text{ kg}$$



Space diagram

By measurement we also find that the angle of inclination of the balancing mass (m) from the horizontal mass of m_1 [200 kg].

$\theta = 201.4°$



Scale:- 1 cm = 10 kg-m

Vector diagram

By measurement

ae = 2.3 cm

= 2.3 × 10 kg-m

= 23 kg-m

$\theta' = 21.4°$

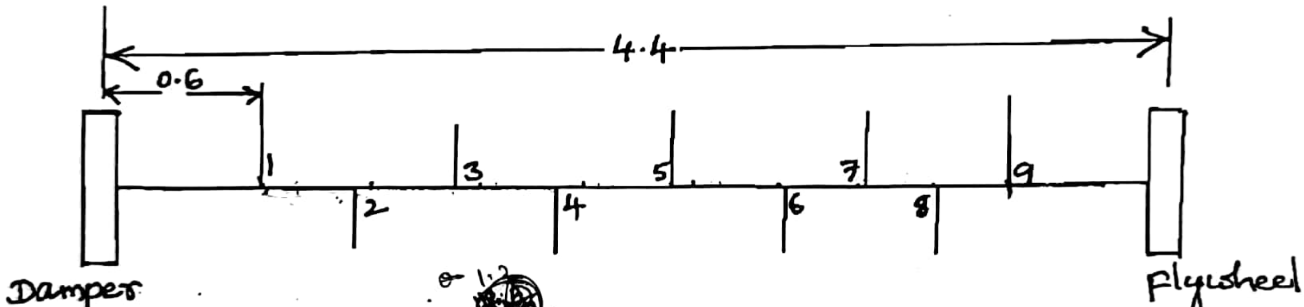
$\theta = 180 + 21.4°$

$= 201.4°$

—: 9 :—

(2) The cranks 2 to 9 of a nine cylinder engine running at 1000 rpm. make 240°, 120°, 160°, 280°, 40°, 80°, 320° and 200° respectively with crank 1, when measured in a counter clockwise direction. The rotating masses for each cylinder are estimated to be 20 kg at 0.5 m radius. The distance between centre lines of cranks is 0.4 m. It is proposed to balance this engine by two masses, one in the damper at a distance of 0.6 m from cylinder one and the other located in the flywheel at a distance of 0.6 m from cylinder Nine. Determine the kg-m magnitudes and the locations of the balancing masses.

Given data:- $N = 1000$ rpm
$m = 20$ kg
$r = 0.5$ m



Taking moment about damper, we get

cylinder No:	L	θ	cosθ	sinθ	Lcosθ	Lsinθ
1	0.6	0	1	0	0.6	0
2	1.0	240	−0.5	−0.866	−0.5	−0.866
3	1.4	120	−0.5	0.866	−0.7	1.2124
4	1.8	160	−0.939	0.342	−1.6902	0.6156
5	2.2	280	0.173	−0.984	0.3806	−2.1648
6	2.6	40	0.766	0.642	1.9916	1.6692
7	3.0	80	0.173	0.984	0.519	2.952
8	3.4	320	0.766	−0.642	2.6	−2.1828
9	3.8	200	−0.939	−0.342	−3.568	−1.2996
					−0.367	−0.064

magnitude of the correcting unbalance in the flywheel plane is at a distance of 4.4m from damper.

$$\therefore \text{Magnitude} = \frac{m.r\sqrt{[\Sigma L\cos\theta]^2 + [\Sigma L\sin\theta]^2}}{4.4}$$

$$= \frac{20\times0.5\sqrt{(-0.367)^2 + (-0.064)^2}}{4.4}$$

$$(m)_{damper} = 0.846\ kg\text{-}m$$

$$\theta_{damper} = \tan^{-1}\left[\frac{\Sigma L\sin\theta}{\Sigma L\cos\theta}\right]$$

$$= \tan^{-1}\left[\frac{-0.064}{-0.367}\right]$$

$$\theta_{damper} = 9.89^\circ$$

Taking moment about the flywheel, we get

cylinder NO	L	θ	cosθ	sinθ	Lcosθ	Lsinθ
1	3.8	0	1	0	3.8	0
2	3.4	240	−0.5	−0.866	−1.7	−2.9444
3	3.0	120	−0.5	0.866	−1.5	2.598
4	2.6	160	−0.939	0.342	−2.4414	0.8892
5	2.2	280	0.173	−0.984	0.3806	−2.1648
6	1.8	40	0.766	0.642	1.3788	1.1556
7	1.4	80	0.173	0.984	0.2422	1.3776
8	1.0	320	0.766	−0.642	0.766	−0.642
9	0.6	200	−0.939	−0.342	−0.5634	−0.2052
					0.3628	0.0644

magnitude of correcting unbalance in the damper plane at a distance of 4.4 from flywheel is given by

$$\text{magnitude} = \frac{m r\sqrt{[\Sigma L\cos\theta]^2 + [\Sigma L\sin\theta]^2}}{4.4}$$

$$= \frac{20\times0.5\sqrt{(0.3628)^2 + (0.0644)^2}}{4.4}$$

mass of flywheel $= 0.837\ kg\text{-}m$

$$\theta_{Flywheel} = \tan^{-1}\left[\frac{\Sigma L\sin\theta}{\Sigma L\cos\theta}\right] \Longrightarrow \tan^{-1}\left[\frac{0.0644}{0.3628}\right] \Longrightarrow 10.06^\circ$$

2011

③ Three masses of 8kg, 12kg and 15kg attached at radial distances of 80mm, 100mm, and 60mm respectively to a disc on a shaft are in complete balance. Determine the angular positions of the masses 12kg and 15kg relative to 8kg mass.

Given data:- $m_1 = 8kg$; $m_2 = 12kg$; $m_3 = 15kg$

$r_1 = 80mm \Rightarrow 0.08m$

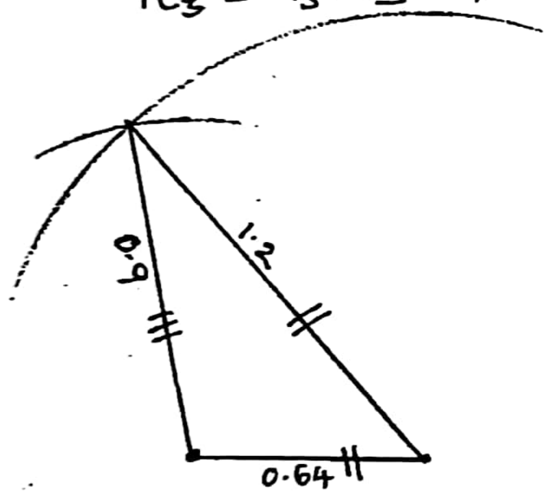
$r_2 = 100mm \Rightarrow 0.1m$

$r_3 = 60mm \Rightarrow 0.06m$

$F_{c_1} = m_1 \cdot r_1 \Rightarrow 8 \times 0.08 \Rightarrow 0.64$ kg-m

$F_{c_2} = m_2 \cdot r_2 \Rightarrow 12 \times 0.1 \Rightarrow 1.2$ kg-m

$F_{c_3} = m_3 \cdot r_3 \Rightarrow 15 \times 0.06 \Rightarrow 0.9$ kg-m

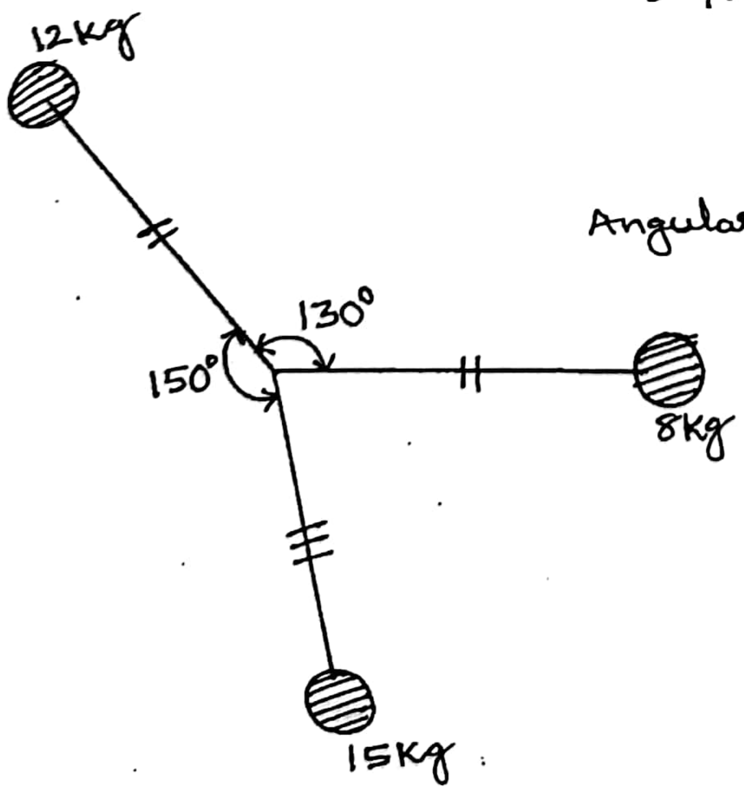


Scale :- 1kg-m = 5cm

0.64 kg-m = 3.2 cm

1.2 kg-m = 6cm

0.9 kg-m = 4.5 cm



By measurement :-

Angular position from 8kg to 12kg = 130°

Angular position from 8kg to 15kg = 130+150
= 280°

(4) Balancing of several masses rotating in different planes:- In this balance, the following two conditions must be satisfied.

(1) The forces in the reference plane must balance, i.e the resultant force must be zero.

(2) The couples about the reference plane must balance i.e the resultant couple must be zero.

Problems

M (1) A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, Find their magnitudes and angular positions.

Sol

Given data:-

$m_A = 200 kg$; $m_B = 300 kg$; $m_C = 400 kg$; $m_D = 200 kg$

$r_A = 80 mm \Rightarrow 0.08 m$; $r_B = 70 mm \Rightarrow 0.07 m$

$r_C = 60 mm \Rightarrow 0.06 m$; $r_D = 80 mm \Rightarrow 0.08 m$

$r_X = r_Y = 100 mm \Rightarrow 0.1 m$.

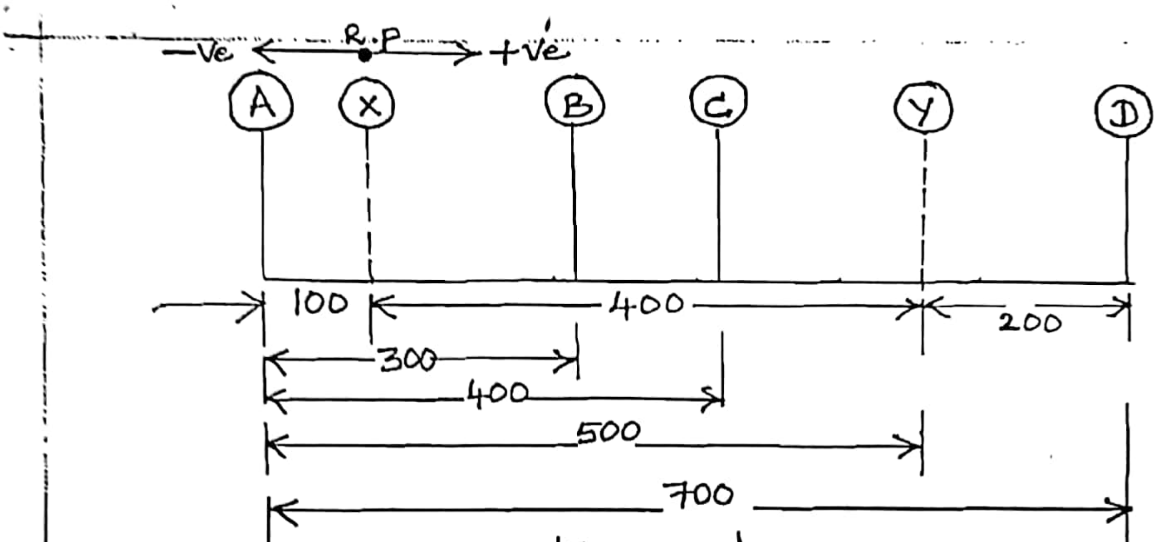
Let m_X = Balancing mass placed in plane X and

m_Y = " " " " Y.

The position of planes and angular position of the masses [assuming the mass A as horizontal] are shown in figure.

Assume the plane X as the reference plane [R.P]. The distances of the planes to the right of plane X are taken as '+ve' while the distances of the planes to the left of plane X are taken as '-ve'.

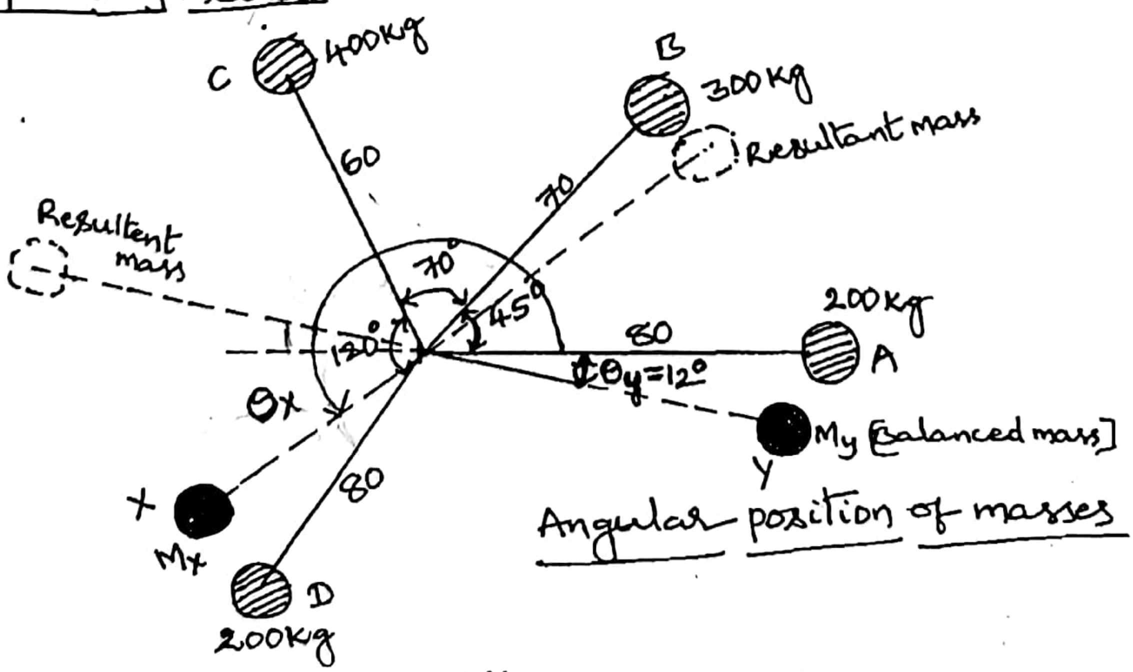
The data may be tabulated as shown in table.



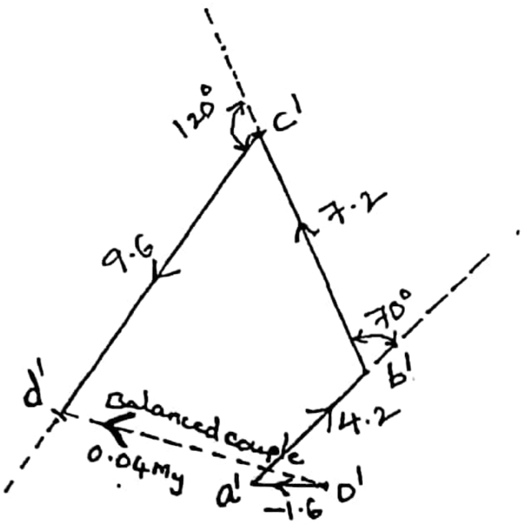
position of planes

plane	mass (m) kgs	Radius (r) metres	centrifugal force ÷ ω^2 (m×r) kg-m	Distance from X-plane (l) metres	couple ÷ ω^2 (m.r.l) kg-m^2
A	200	0.08	16	-0.1	-1.6
X [R.P]	m_x	0.1	$0.1 m_x$	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_y	0.1	$0.1 m_y$	0.4	$0.04 m_y$
D	200	0.08	16	0.6	9.6

Graphically method:-



Angular position of masses



Couple polygon

Scale :- 1cm = 1 Kg-m²

By measurement :-

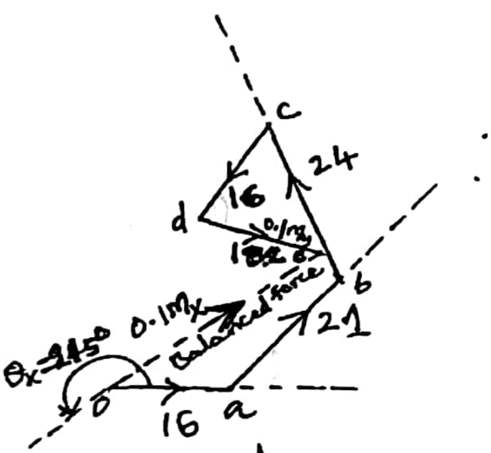
$d'o' = 0.04 m_y$

$7.3 = 0.04 m_y$

$m_y = \frac{7.3}{0.04}$

$m_y = 182.5$ kg

$\theta_y = 12^\circ$ ← Angle d'o'a' (from mass A clockwise)



Force diagram

By measurement of

$oe = 0.1 M_x$

$35.5 = 0.1 m_x$

$m_x = \frac{35.5}{0.1}$

$m_x = 355$ kg

$\theta_x = 215^\circ$ (from mass A Anticlock)

1. First of all draw the couple polygon from the data given table. take some suitable scale.
2. Next draw the force diagram from the data given table, take some suitable scale.

-A
$m_x = X$
B
C
$m_y = Y$
D

Reference books :-

Theory of machines
1. R.S Kurmi & JK. Gupta — S.chand
2. SS. Rattan — MC GrawHill pub.
3. V.P Singh — Dhanpat Rai pub.
4. P.L Ballaney — KP public
5. Sadhu singh — pearson pub.

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(2) A, B, C and D are four masses carried by a rotating shaft at radii 100mm, 150mm, 150mm and 200mm respectively. The planes in which masses rotate are spaced at 500mm apart and the magnitude of the masses B, C and D are 9kg, 5kg and 4kg respectively. Find the required mass A and the relative angular settings of the 4 masses so that the shaft shall be in complete balance.

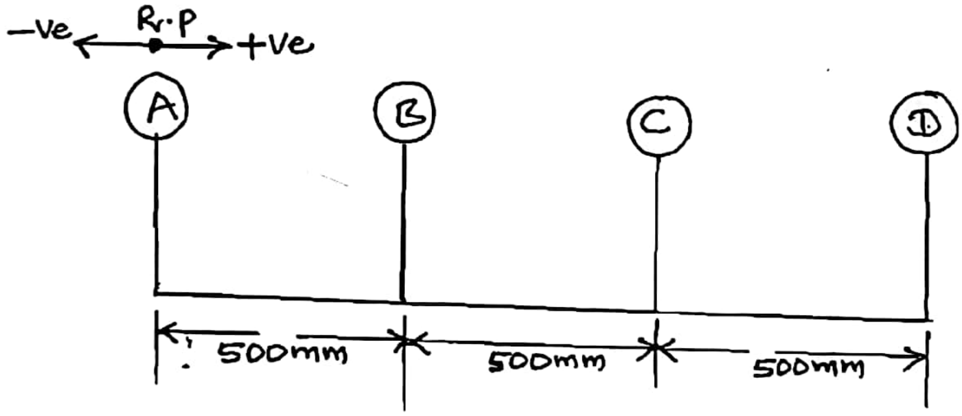
Given data:- $r_A = 100mm \Rightarrow 0.1m$

$r_B = 150mm \Rightarrow 0.15m$

$r_C = 150mm \Rightarrow 0.15m$

$r_D = 200mm \Rightarrow 0.2m$

$m_B = 9kg$; $m_C = 5kg$; $m_D = 4kg$.



Assuming plane of mass 'A' as reference plane and tabulating the data.

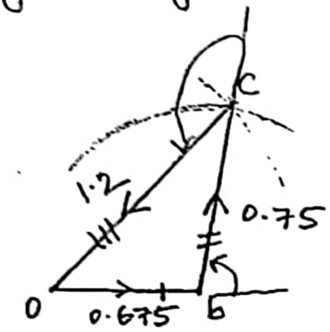
plane	Mass (m) kgs	Radius (r) metres	centrifugal force $\div \omega^2$ (m.r) kg-m	Distance from plane A (l)	couple $\div \omega^2$ m.r.l (kg-m²)
A [R.P]	m_A	0.1	$0.1m_A$	0	0
B	9	0.15	1.35	0.5	0.675
C	5	0.15	0.75	1.0	0.75
D	4	0.2	0.8	1.5	1.2

1) First draw the couple polygon.

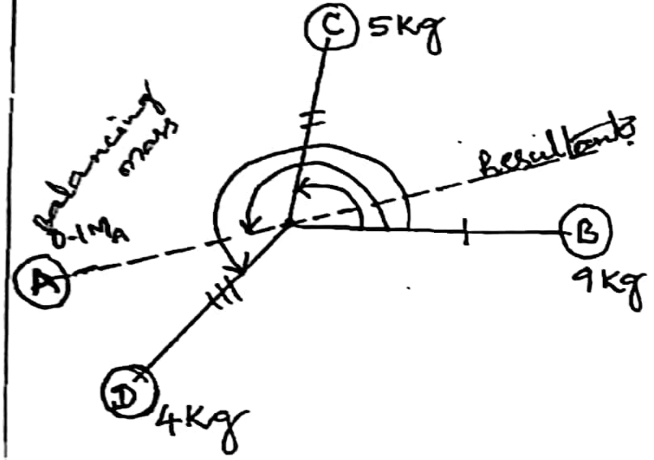
2). Angular position of masses diagram

3). Force polygon diagram.

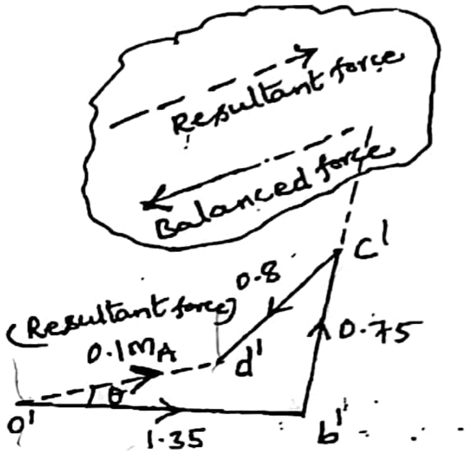
Scale :- 1 Kg-m = 3 cm (Force polygon)
1 Kg-m^2 = 3 cm (couple polygon)



(1) couple polygon



(2) Angular position of masses



(3) Force polygon

By measurement:-

$0.1\, m_A = 0.86$

$m_A = 8.6$ kg

By measurement:-

Resultant (θ) = 12^0

Balancing (θ) = 12+180 = 192^0

Angle from B to C in anticlockwise = 80^0

Angle from B to A " " = 195^0.

Angle from B to D " " = 80+145 = 225^0

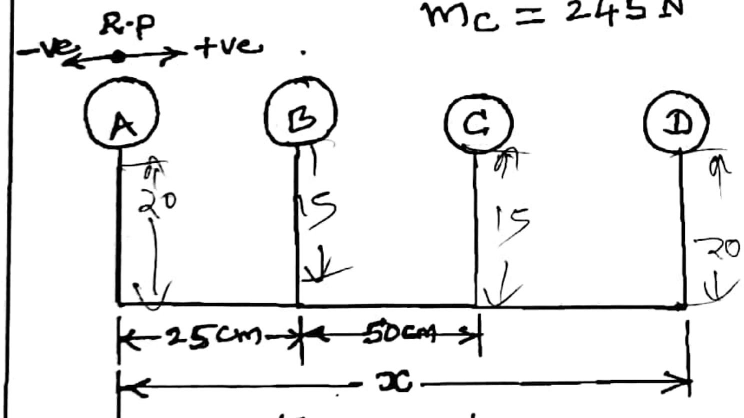
(B) A shaft carries four masses A, B, C and D placed in parallel planes perpendicular to the shaft axis and in this order along the shaft. The masses of B and C are 353 N and 245 N respectively and both are assumed to be concentrated at a radius of 15 cm, while the masses in planes A and D are both at a radius of 20 cm. The angle between the radii of B and C is 100° and that between B and A is 190°, both angles being measured in the same sense. The planes containing A and B are 25 cm apart and those containing B and C are 50 cm apart. If the shaft is to be in complete dynamic balance, determine (i) Masses of A and D (ii) distance between the planes containing C and D (iii) angular position of the mass D.

Given data:- $m_B = 353N$ $\qquad$ $r_B = 15cm \Rightarrow 0.15m$

$m_C = 245N$ $\qquad$ $r_C = 15cm \Rightarrow 0.15m$

$r_A = 20cm \Rightarrow 0.2m$

$r_D = 20cm \Rightarrow 0.2m$

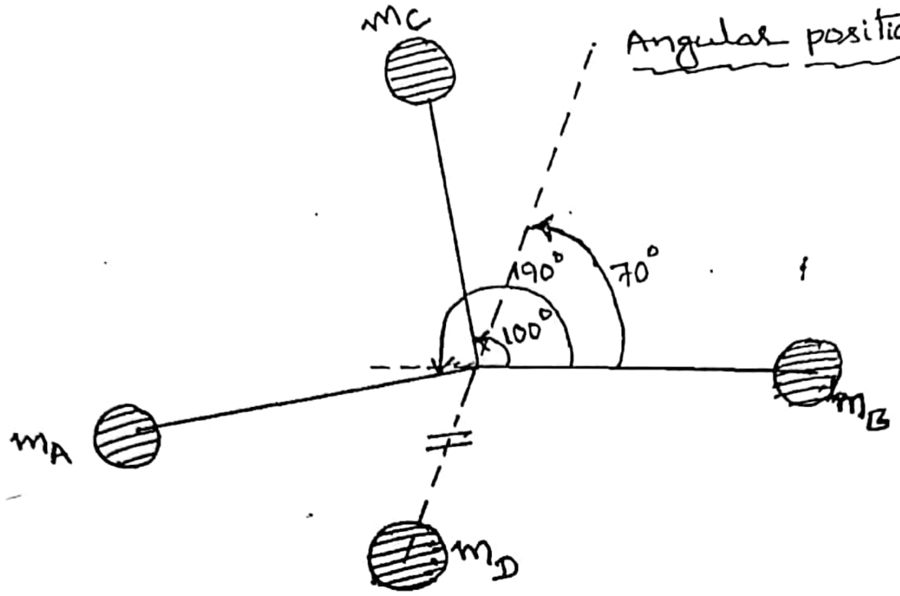


position of planes

Assuming plane A is the Reference plane.

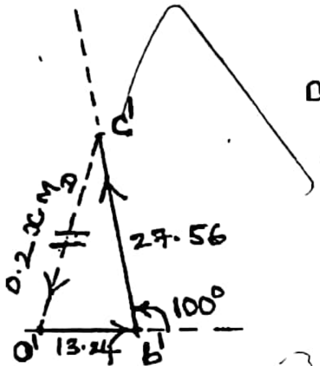
plane	mass (m) Newtons	radius (eccentricity) (r) metre	Centrifugal force ÷ ω^2 (m.r)	Distance from plane A l (metres)	couple ÷ ω^2 (m.r.l) (N-m)
A	m_A	0.2	$0.2m_A$	0	0
B	353N	0.15	52.95	0.25	13.24
C	245N	0.15	36.75	0.75	27.56
D	m_D	0.2	$0.2m_D$	x	$0.2xm_D$

Angular position of masses

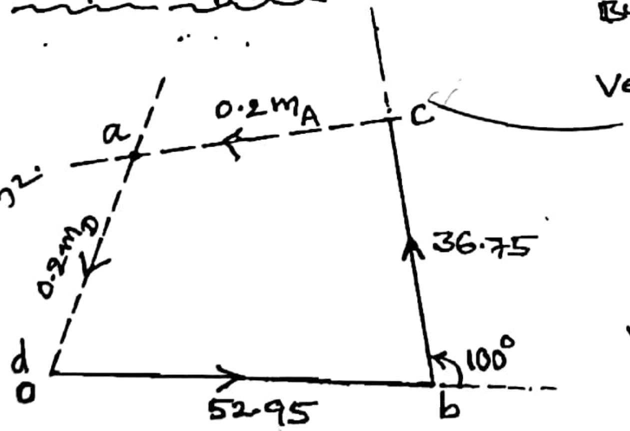


Scale:-
$\frac{1\,cm}{10}$ = [illegible] N-m²

By measurement:-
Angle $b'o'c' = 70°$



Couple polygon



scale
$\frac{1\,cm}{10}$ = [illegible] N-m²

By measurement:-

Vector ac = 3.8 × 10 ← scale

$0.2 m_A = 38$

$m_A = \frac{38}{0.2}$

$m_A = 190\,N$

Vector oa = 3.3 × 10 ← scale

$0.2 m_D = 33$

$m_D = 165\,N$

From couple polygon Vector $o'c' = 2.9 \times 10$

$0.2\, x\, m_D = 29$

$x = \frac{29}{0.2 \times 165} \Rightarrow 0.878\,m = 87.8\,cm$

Distances from mass C to mass D = 87.8 − 75
= 12.8 cm

Angle of mass D from mass B in anticlockwise = 70 + 180
= 250°

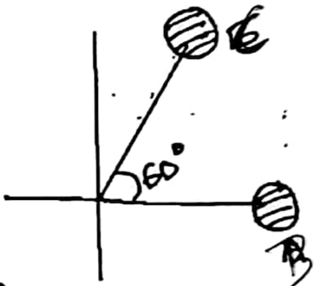
(4) A shaft 3m span between the bearings carries two masses of 5kg and 10 kg acting at the extremities of the arms 0.45m and 0.6m long respectively. The planes in which the masses rotate are 1.2m and 2.4m respectively from the left hand bearing and the angle between the arms is 60°. If the speed of rotation is 100 rpm. Find the displacing force on the two bearings of the machine. If the masses are balanced by two additional rotating masses acting at a radius 0.3 and placed 0.3m from each bearing, Estimate the magnitude of the two balanced masses and the angles at which they may be set with respect to the two arms.

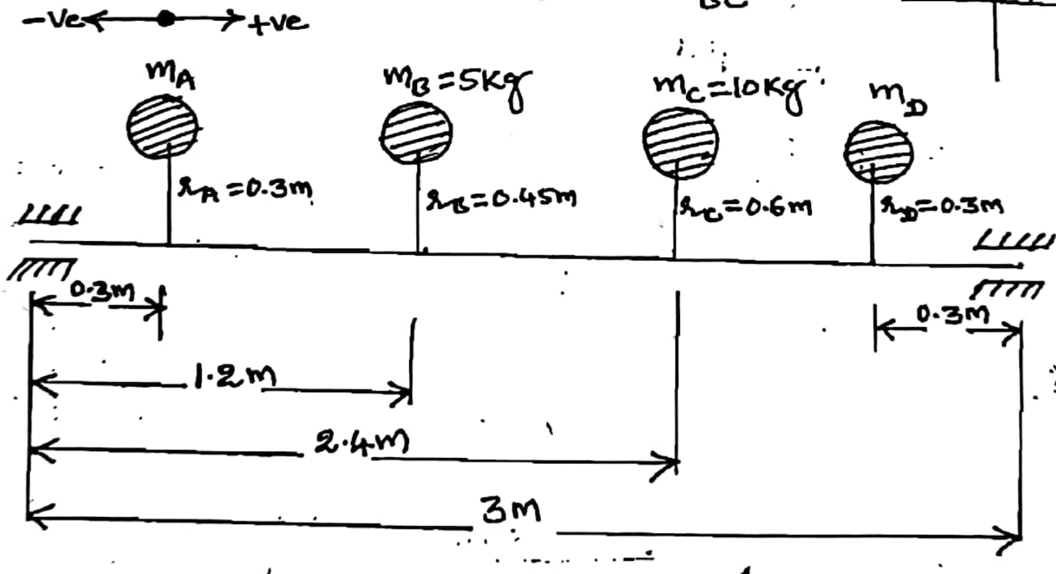
Sol Given data :- $m_B = 5kg$; $m_C = 10kg$

$r_B = 0.45m$; $r_C = 0.6m$

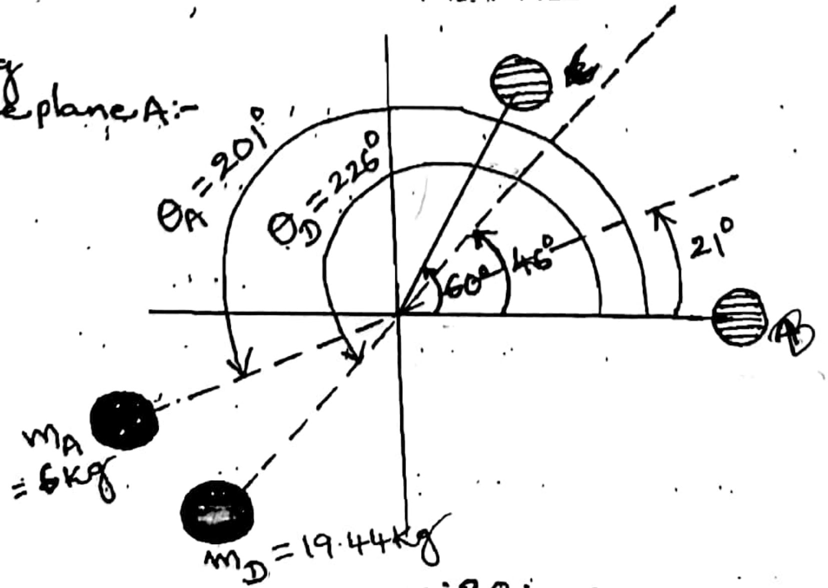
$r_A = 0.3m$; $r_D = 0.3m$

$N = 100 rpm$; $\theta_{BC} = 60°$

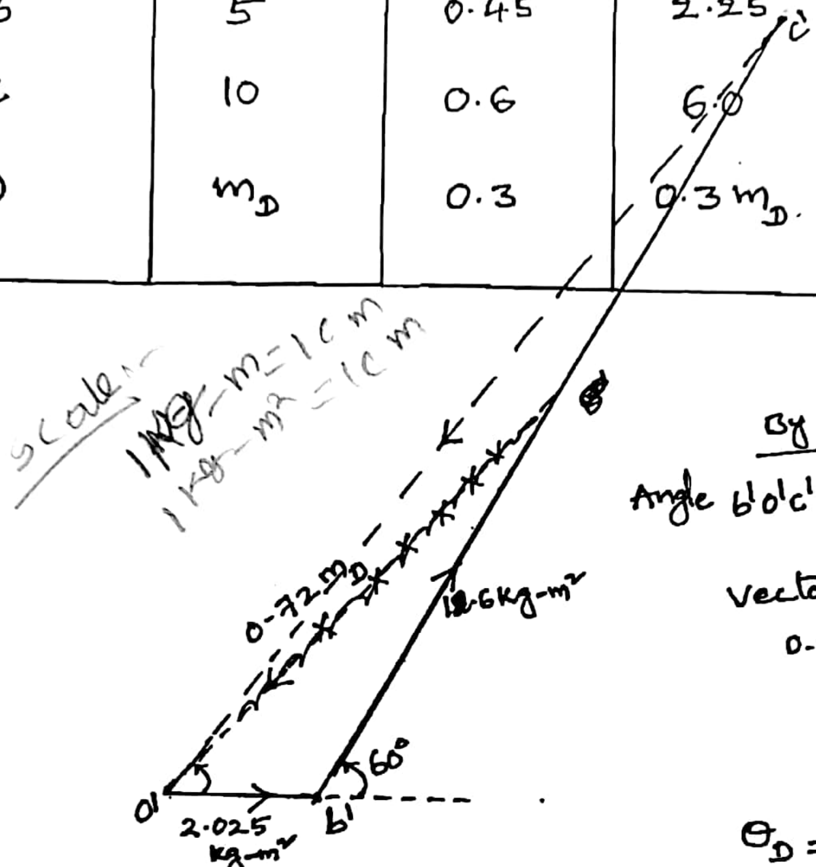




Assuming Reference plane A:-



plane	mass(m) kg	radius(r) m	centrifugal force ÷ ω^2 (m.r) kg-m	Distance from plane A (l) metres	couple ÷ ω^2 (m.r.l) kg-m²
A [R.P]	m_A	0.3	$0.3\,m_A$	0	0
B	5	0.45	2.25	0.9	2.025
C	10	0.6	6.0	2.1	~~12.6~~ 12.6
D	m_D	0.3	$0.3\,m_D$	2.4	$0.72\,m_D$



Couple polygon

By measurement:-

Angle $b'o'c' = 46^\circ$

vector $c'o'$ = 14

$0.72 m_D = 14$

$m_D = \frac{14}{0.72}$

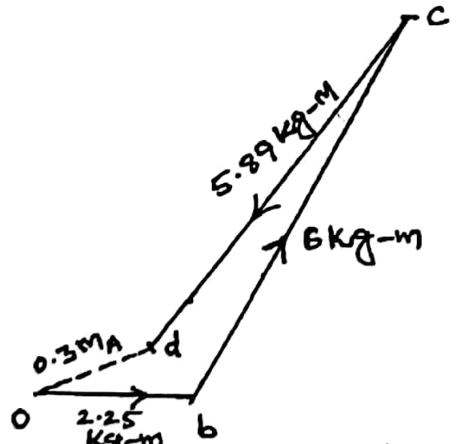
$m_D = 19.44$ kg

$\theta_D = 180 + 46^\circ = 226^\circ$ from mass B

centrifugal force of plane D = $0.3 m_D$

$= 0.3 \times 19.44 \Rightarrow 5.83$ kg-m

~~Angle $oc'b' = 8^\circ$~~



centrifugal force polygon

By measurement:-

vector od = 1.7

$0.3 m_A = 1.7$

$m_A = \frac{1.7}{0.3}$

$m_A = 5.67$ kg

$m_A \approx 6$ kg

Angle $bod = 21^\circ$

$\theta_A = 180 + 21$

$\theta_A = 201^\circ$ from mass B

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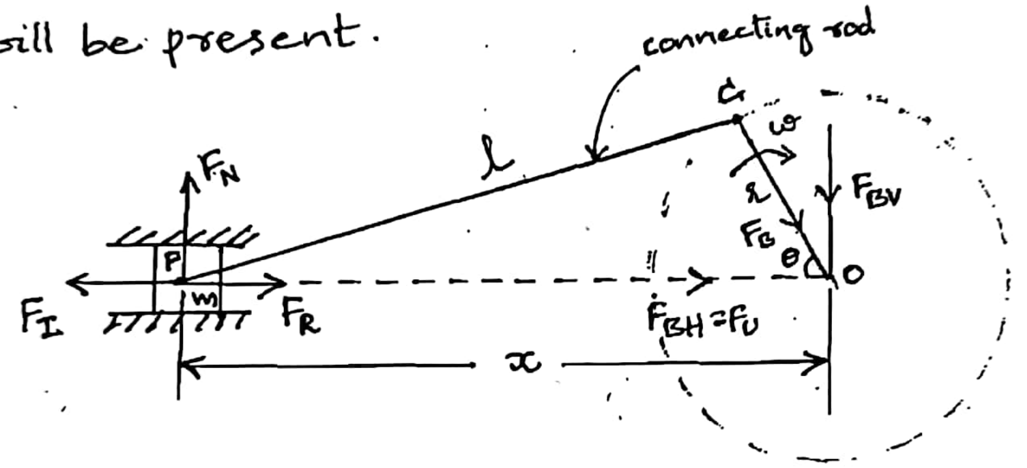
UNIT – VII

Balancing of Reciprocating Masses

Introduction :- The various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalanced force or shaking force. Thus if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.



Reciprocating engine mechanism

Let F_R = Force required to accelerate the reciprocating parts.

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or Normal force acting on the cross-head guides, and

F_B = Force acting on the crankshaft bearing or main bearing.

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (i.e F_{BH}) acting along the line of reciprocating is also equal and opposite to F_I. This force $F_{BH} = F_U$ is an unbalanced force (or) shaking force and required to be properly balanced.

The force on the sides of the cylinder walls (F_N) and the vertical component of F_B [i.e F_{BV}] are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ & $F_{BV} \times x$.

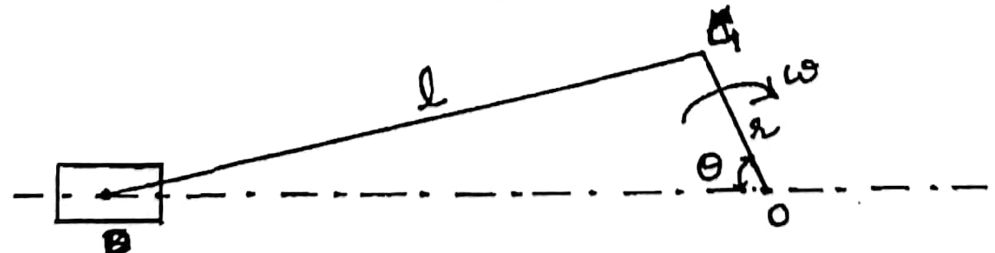
Thus the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

Primary and secondary unbalanced forces of Reciprocating Masses :-

(OR)

Prove that maximum secondary unbalanced forces is $\frac{1}{n}$ times maximum primary unbalanced for 'n' cylinder reciprocating engine.

(A) consider a horizontal reciprocating engine mechanism as shown in the figure.



Let l = length of connecting rod

r = Radius of crank

ω = Angular speed of the crank

m = mass of the reciprocating parts of the engine

θ = Angle of inclination of the crank with line of stroke.

of the crank.

$$F_s = \frac{m\omega^2 r}{n}$$

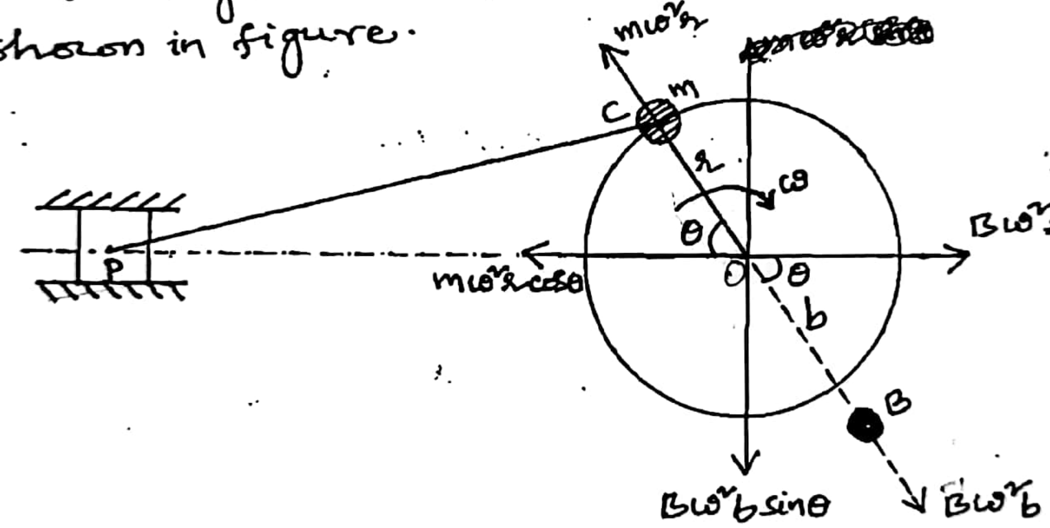
$$= \frac{1}{n}(m\omega^2 r)$$

$$\boxed{F_s = \frac{1}{n}[F_p]}$$

∴ Maximum secondary unbalanced forces is $\frac{1}{n}$ times maximum primary unbalanced force.

partial balancing of unbalanced primary force in a reciprocating engine:-

The primary unbalanced ($m\omega^2 r\cos\theta$) may be considered as the component of the centrifugal force produced by a rotating mass m placed at the crank radius 'r' as shown in figure.



The primary force acts from O to P along the line of stroke. Hence, balancing of primary force is considered as equivalent to the balancing of mass m rotating at the crank radius r. This is balanced by having a mass B at a radius b, placed diametrically opposite to the crank pin C.

centrifugal force due to mass B $= B\cdot\omega^2\cdot b$

Resolving the horizontal forces

$$m\cancel{\omega^2} r\,\cancel{\cos\theta} = B\cdot\cancel{\omega^2}\cdot b\,\cancel{\cos\theta}$$

$$mr = Bb$$

Resolving the vertical forces, when $\theta = 90°$, $m\omega^2 r\sin 90 = B\omega^2 b\sin 90$

-: 4 :-

$mr = Bb$

Adding fraction 'c' of the reciprocating masses is balanced

$cmr = Bb$

$\therefore$ unbalanced force along the line of stroke.

$= m\omega^2 r\cos\theta - B\omega^2 b\cos\theta$

$= m\omega^2 r\cos\theta - Bb\,\omega^2\cos\theta \quad (\because Bb = c.mr)$

$= m\omega^2 r\cos\theta - cmr\,\omega^2\cos\theta$

$= (1-c)\, m\omega^2 r\cos\theta$

and unbalanced force along the perpendicular to the line of stroke

$= B\omega^2 . b\sin\theta \quad (\because Bb = cmr)$

$= cm\omega^2 r\sin\theta$

$\therefore$ Resultant unbalanced force at any instant.

$= \sqrt{((1-c)m\omega^2 . r\cos\theta)^2 + (cm\omega^2 . r\sin\theta)^2}$

$= m.\omega^2 . r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$

Note:- If the balancing mass is required to balance the revolving masses as well as reciprocating masses, then

$B.b = m_1 r + cmr$

$= (m_1 + cm)\, r$

where m_1 = magnitude of the revolving masses

m = magnitude of the reciprocating masses

① A single cylinder reciprocating engine has speed 240 rpm. stroke 300 mm, mass of reciprocating parts 50 kg, mass of revolving parts at 150 mm radius 37 kg. If two-third of the reciprocating parts and all the revolving parts are to be balanced, find 1. The balance mass required at a radius of 400 mm, and 2. The residual unbalanced force when the crank has rotated 60° from inner dead centre.

Sol Given data:- $N = 240$ rpm or $\omega = \frac{2\pi \times 240}{60} \Rightarrow 25.14$ rad/sec

stroke $= 300$ mm $= 0.3$ m ; $m = 50$ kg ; $m_1 = 37$ kg ;

$r = 150$ mm $= 0.15$ m ; $c = \frac{2}{3}$

1). Balance mass required :-

Let B = Balance mass required and

b = Radius of rotation of the balance mass = 400 mm ⇒ 0.4 m

we know that,

$$B.b = \left(m_1 + c\,m\right) r$$

$$B \times 0.4 = \left(37 + \tfrac{2}{3} \times 50\right) 0.15 \quad \cancel{\Rightarrow 10.58}$$

$$B = \frac{10.55}{0.4} \Rightarrow B = 26.34 \text{ kg}$$

2). Residual unbalanced force :-

Let θ = Crank angle from inner dead centre $= 60°$

we know that residual unbalanced force

$$= m\omega^2 r\sqrt{(1-c)^2\cos^2\theta + c^2\sin^2\theta}$$

$$= 50(25.14)^2(0.15)\sqrt{\left[1-\tfrac{2}{3}\right]^2\cos^2 60° + \left[\tfrac{2}{3}\right]^2\sin^2 60°}$$

$$= 4740 \times 0.601$$

$$= 2849\ N$$

Effect of partial balancing of Reciprocating parts of two cylinder locomotives :- The reciprocating parts are only partially balanced. Due to this partial balancing of the reciprocating parts, there is an unbalanced primary force along the line of stroke and also an unbalanced primary force perpendicular to the line of stroke. The effect of an unbalanced primary force along the line of stroke is to produce;
1. variation in tractive force along the line ~~of stroke~~ and 2. swaying couple.

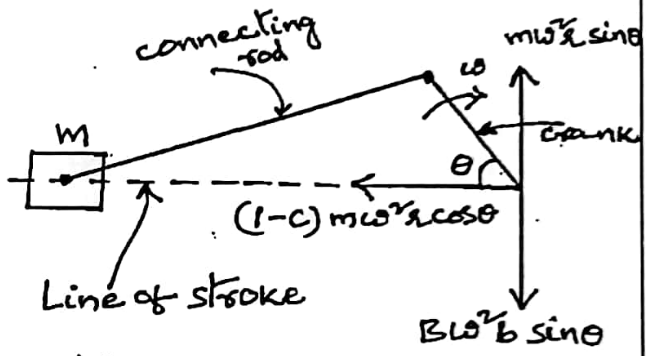
The effect of an unbalanced primary force perpendicular to the line of stroke is to produce variation in pressure on the rails, which results in hammering action on the rails. "The maximum magnitude of the unbalanced force along the perpendicular to the line of stroke is known as a hammer blow".

Variation of Tractive force:- "The resultant unbalanced force due to the two cylinders, along the line of stroke, is known as tractive force".

Let the crank for the first cylinder be inclined at an angle 'θ' with the line of stroke as shown in figure. Since the crank for the second cylinder is at right angle to the first crank, therefore the angle of inclination for the second crank will be $(90°+\theta)$.

Let m = mass of the reciprocating parts per cylinder

c = Fraction of the reciprocating parts to be balanced.



Variation of Tractive force

unbalanced force along the line of stroke for cylinder 1 $= (1-c)\, m\omega^2 r\cos\theta$

unbalanced force along the line of stroke for cylinder 2 $= (1-c)\, m\omega^2 r\cos(90°+\theta)$

$\therefore$ As per definition, the tractive force,

F_T = Resultant unbalanced force along the line of stroke.

$$= (1-c)\, m\omega^2 r\cos\theta + (1-c)\, m\omega^2 r\cos(90°+\theta)$$

$$= (1-c)\, m\omega^2 r\cos\theta + (1-c)\, m\omega^2 r\sin\theta$$

$$F_T = (1-c)\, m\omega^2 r\,[\cos\theta - \sin\theta]$$

The tractive force is maximum or minimum when $(\cos\theta - \sin\theta)$ is maximum or minimum. For $(\cos\theta - \sin\theta)$ to be maximum or minimum,

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0 \implies -\sin\theta - \cos\theta = 0$$

$$-\sin\theta = \cos\theta$$

$\tan\theta = -1$ or $\theta = 135^\circ$ or 315°

Thus, the tractive force is maximum or minimum, when $\theta = 135^\circ$ or 315°.

∴ Maximum and minimum value of the tractive force or the variation in tractive force.

$$= \pm (1-c)\, m\omega^2 r\,[\cos 135^\circ - \sin 135^\circ]$$

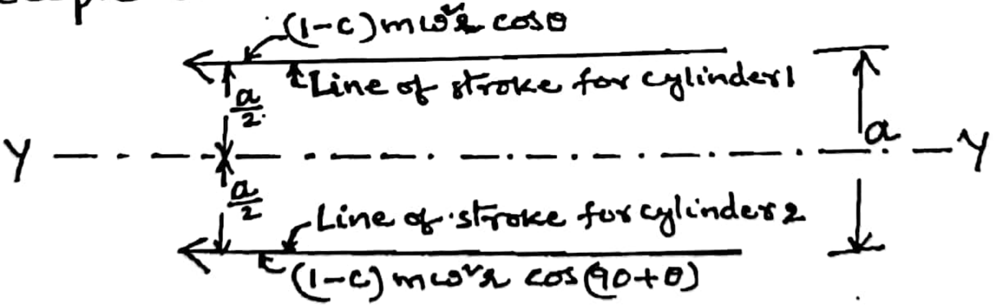
$$F_T = \pm \sqrt{2}\,(1-c)\, m\omega^2 r$$

(4) Swaying couple :- The unbalanced forces along the line of stroke for the two cylinders constitute a couple about the centre line YY between the cylinders as shown in figure.//

This couple has swaying effect about a vertical axis and tends to sway the engine alternately in clockwise and anticlockwise directions. Hence the couple is known as "swaying couple".

(or)

"The unbalanced forces acting at a distance between the line of stroke of two cylinders, constitute a couple in the horizontal direction. This couple is known as swaying couple."



Let a = Distance b/w the centre lines of the two cylinders

$$\therefore \text{swaying couple} = (1-c)\,m\omega^2 r\cos\theta \times \frac{a}{2} - \left[(1-c)\,m\omega^2 r\cos(90+\theta)\right]\left(\frac{a}{2}\right)$$

$[\because \cos(90+\theta) = \sin\theta]$

$$= (1-c)\,m\omega^2 r\,\frac{a}{2} \times [\cos\theta - \sin\theta]$$

The swaying couple is maximum or minimum, when

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) \quad \text{or} \quad -\sin\theta + \cos\theta = 0$$

$$-\sin\theta = -\cos\theta \implies \tan\theta = 1$$

$$\theta = 45^\circ \text{ or } 225^\circ$$

Thus, the swaying couple is maximum or minimum when $\theta = 45^\circ$ or 225°.

$\therefore$ Maximum and minimum value of the swaying

$$\text{couple} = \pm(1-c)\,m\omega^2 r \times \frac{a}{2}\left[\cos 45^\circ + \sin 45^\circ\right]$$

$$= \pm\frac{a}{\sqrt{2}}(1-c)\,m\omega^2 r.$$

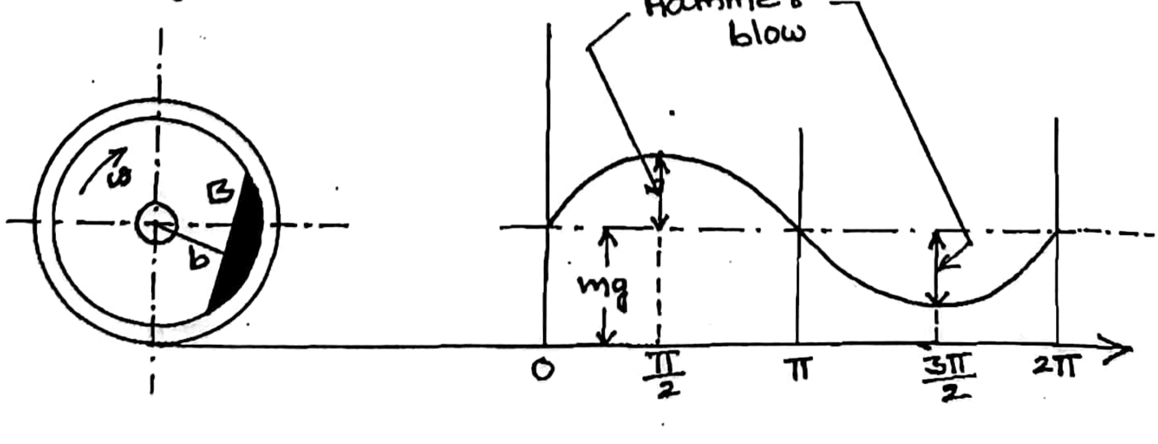
Note:- $c = \frac{2}{3}$ to $\frac{3}{4}$ $\longrightarrow$ two cylinder locomotives with two pairs of coupled wheels.

$c = \frac{2}{5}$ $\longrightarrow$ Four cylinder locomotives with three or more pairs of coupled wheels.

(3) Hammer Blow:- "The maximum value of the unbalanced force perpendicular to the line of stroke is called as hammer blow."

With very high speed this unbalanced force may be very harmful causing the lifting of the wheels from the rails and hitting on it. The effect of hammer blow is to cause the variation in pressure between the wheel and the rail. The

The variation is shown in figure for one revolution of wheel.



Let $W = mg$ the dead load acting on each wheel. This load acts in the downward direction.

Let B be the balancing mass of reciprocating parts acting at a radius b.

So vertical unbalance force $= B\omega^2 b \sin\theta$

The maximum value of this unbalanced force

$$= B\omega^2 b \quad [\text{when } \theta = 90°, 270°]$$

The force which will try to keep the rail down

$mg \pm B\omega^2 b$

$$= mg - B\omega^2 . b \quad [\text{at } \theta = 90°]$$

If $mg - B\omega^2 . b$ is negative, the wheel will not be lifted from the rails. So the limiting condition in order that the wheel does not lift from rails can be obtained.

$$mg = B\omega^2 . b$$

$$\boxed{\omega = \sqrt{\frac{m.g}{B.b}}}$$ ⟹ permissible value of angular speed of wheel.

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Mechanical Engg.
K.H.I.T

* Distinguish balancing of in-Line engines and radial engines with appropriate examples.

Balance of In-Line engine	Balance of Radial Engines
1. primary force is balanced by arranging or attaching the equal masses to their respective crank pins and treating the problem as one of the revolving masses.	primary force is balanced by placing the two masses each being half of unbalanced mass at direct crank pin and other at reverse crank pin, cranks makes equal and opposite angles with line of stroke.
2. Secondary force is balanced by providing equal mass at the imaginary crank of length $\frac{r}{4n}$ and revolving at a speed of twice that of actual crank speed.	secondary force is balanced by replacing mass 'm' of reciprocating parts by two equal masses $\left[\frac{m}{2}\right]$ one mass is kept at direct crank pin and the other at reverse crank pin. crank ~~pin~~ makes equal and opposite angles with line of stroke.
3. For a multicylinder in-line engines in which number of cranks are not less than four, the primary forces are completely balanced by suitable arranging crank angles.	For a multi cylinder engines in which number of cylinders are not less than four, the secondary direct and reverse cranks is well balanced.

—:||:—

(1) An inside cylinder locomotive has its cylinder centre lines 0.7m apart and has a stroke of 0.6m. The rotating masses per cylinder are equivalent to 150Kg at the crank pin, and the reciprocating masses per cylinder to 180Kg. The wheel centre lines are 1.5m apart. The cranks are at right angles.

The whole of the rotating and $\frac{2}{3}$ of the reciprocating masses are to be balanced by masses placed at a radius of 0.6m, Find the magnitude and direction of the balancing masses.

Find the fluctuation in rail pressure under one wheel, variation of tractive effort and the magnitude of swaying couple at a crank speed of 300 rpm.

Sol Given data:- $a = 0.7m$; $l_B = l_C = 0.6m$

$r_B = r_C = 0.3M$; $m_1 = 150Kg$; $m_2 = 180Kg$; $c = \frac{2}{3}$,

$r_A = r_D = 0.6m$; $N = 300\,rpm$ & $\omega = \frac{2\pi(300)}{60}$

$= 31.42\,rad/sec$

we know that the equivalent mass of the rotating parts to be balanced per cylinder at the crank pin,

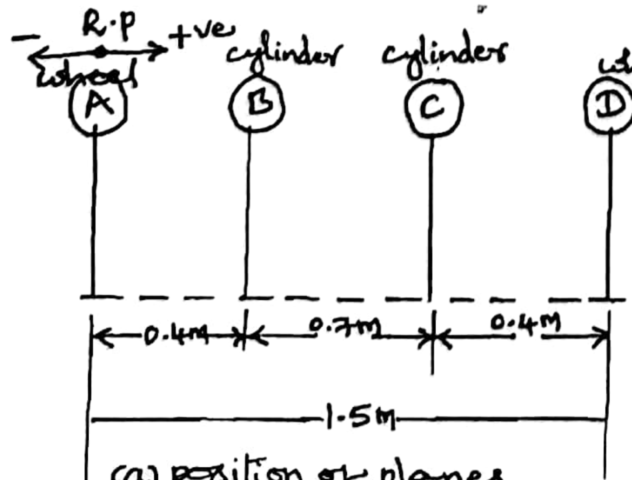
$$m = m_B = m_C = m_1 + cm_2$$

$$= 150 + \frac{2}{3} \times 180 = 270Kg$$

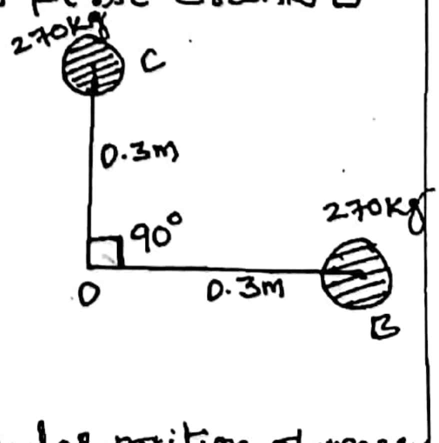
Magnitude and direction of the balancing masses:-

Let m_A and m_D = magnitude of the balancing masses.

θ_A and θ_B = Angular position of the balancing masses m_A and m_D from the first crank B.

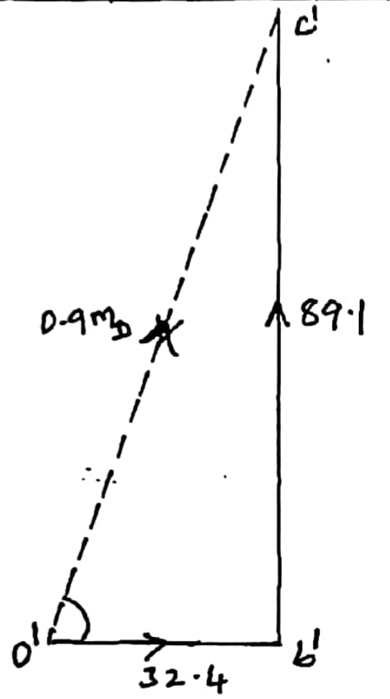


(a) position of planes



(b) Angular position of masses

plane	mass (m) kg	Radius (r) metre	Centrifugal force ÷ ω^2 (m × r)	Distance from R.P (l) metre	couple ÷ ω^2 (m.r.l) kg-m²
A (R.P)	m_A	0.6	$0.6m_A$	0	0
B	270	0.3	81	0.4	32.4
C	270	0.3	81	1.1	89.1
D	m_D	0.6	$0.6m_D$	1.5	$0.9m_D$



(c) Couple polygon

by measurement :-

vector $o'c' = 94.5$ Kg-m²

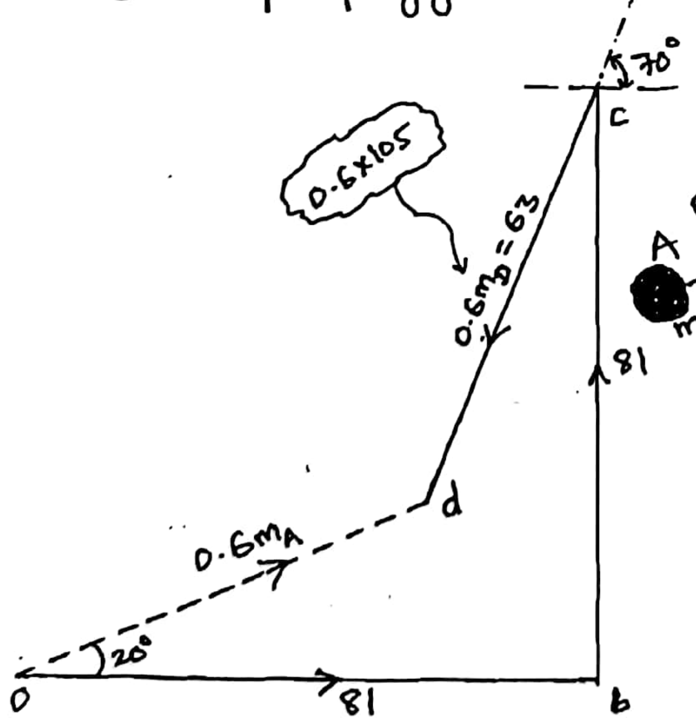
$0.9m_D = 94.5$

$m_D = \frac{94.5}{0.9} \Rightarrow 105$ kg

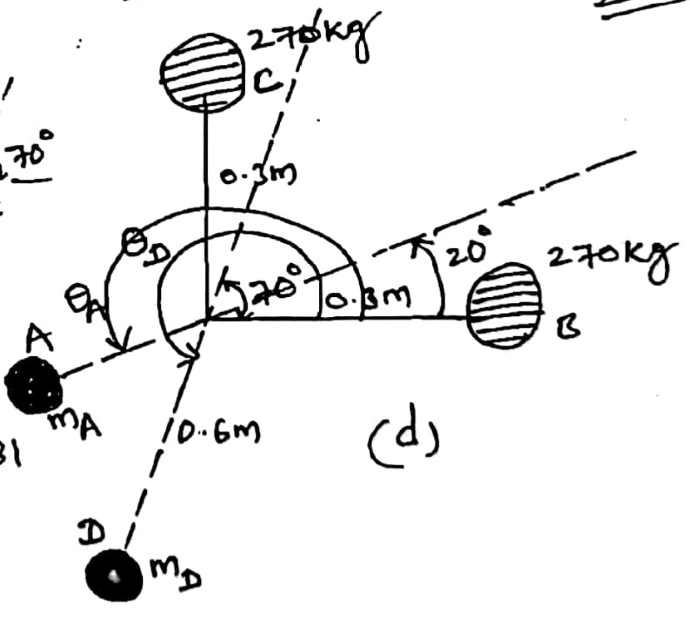
by measurement

Angle $b'o'c' = 70°$

$\theta_D = 250°$ from mass B Anticlockwise



(e) Force polygon



By measurement :-

vector od = 63 kg-m

$0.6m_A = 63$

$m_A = 105$ kg

$\angle bod = 20°$

$\theta_A = 200°$ From mass B (ACW).