UNIT-1

Friction in Machine Elements

Screw Friction

The screws, bolts, studs, nuts etc. are widely used in various machines and structures for temporary fastenings. These fastenings have screw threads, which are made by cutting a continuous helical groove on a cylindrical surface. If the threads are cut on the outer surface of a solid rod, these are known as external threads. But if the threads are cut on the internal surface of a hollow rod, these are known as internal threads. The screw threads are mainly of two types i.e. V-threads and square threads. The V-threads are stronger and offer more frictional resistance to motion than square threads. Moreover, the V-threads have an advantage of preventing the nut from slackening. In general, the V threads are used for the purpose of tightening pieces together e.g. bolts and nuts etc. But the square threads are used in screw jacks, vice screws etc. The following terms are important for the study of screw

Helix. It is the curve traced by a particle, while describing a circular path at a uniform speed and advancing in the axial direction at a uniform rate. In other words, it is the curve traced by a particle while moving along a screw thread.

Pitch. It is the distance from a point of a screw to a corresponding point on the next thread, measured parallel to the axis of the screw

Lead. It is the distance; a screw thread advances axially in one turn.

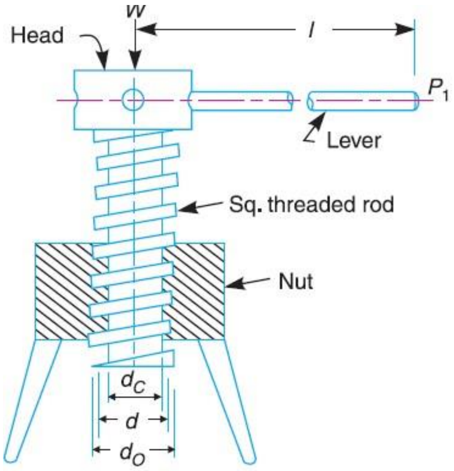
Depth of thread. It is the distance between the top and bottom surfaces of a thread (also known as crest and root of a thread).

Single-threaded screw. If the lead of a screw is equal to its pitch. It is known as single threaded screw.

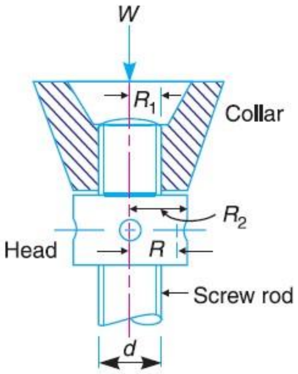
Lead = Pitch × Number of threads

Helix angle. It is the slope or inclination of the thread with the horizontal.

The screw jack is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle. The principle, on which a screw jack works, is similar to that of an inclined plane.



(*a*) Screw jack.



(*b*) Thrust collar.

Fig (*a*) shows a common form of a screw jack, which consists of a square threaded rod (also called screw rod or simply screw) which fits into the inner threads of the nut. The load, to be raised or lowered, is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of the lever for lifting or lowering the load.

Torque Required to Lifting the Load by a Screw Jack :

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Fig (*a*).

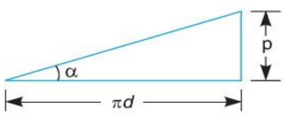
Let p = Pitch of the screw,

d = Mean diameter of the screw, α= Helix angle,

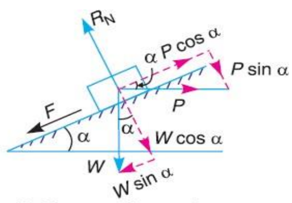
P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut = tan ø, Where ϕ is the friction angle.



(*a*) Development of a screw.



(*b*) Forces acting on the screw.

From the geometry of the Fig(*a*), we find that

$$\tan \alpha = p/\pi d$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown in Fig(*b*).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in Fig(*b*). Resolving the forces along the plane,

$P \cos \alpha = W \sin \alpha + F = W \sin\alpha + \mu.R_N$ **(*i*)**

and resolving the forces perpendicular to the plane,

$R_N = P \sin \alpha + W \cos\alpha$ **(*ii*)** Substituting this value of R_N in equation(*i*),

$P \cos \alpha = W \sin \alpha + \mu (P \sin \alpha + W \cos \alpha)$

$= W \sin \alpha + \mu P \sin \alpha + \mu W \cos \alpha$

$P \cos \alpha - \mu P \sin \alpha = W \sin \alpha + \mu W \cos \alpha$

$P (\cos \alpha - \mu \sin \alpha) = W (\sin \alpha + \mu \cos \alpha)$

$$P = W \times \frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{\sin \alpha + \tan \phi \cos \alpha}{\cos \alpha - \tan \phi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$P = W \times \frac{\sin \alpha \cos \phi + \sin \phi \cos \alpha}{\cos \alpha \cos \phi - \sin \alpha \sin \phi} = W \times \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)}$$

$= W \tan (\alpha + \phi)$

$$T_1 = P \times \frac{d}{2} = W \tan (\alpha + \phi) \frac{d}{2}$$

Torque required to overcoming friction between the screw and nut,

When the axial load is taken up by a thrust collar or a flat surface, as shown in Fig (*b*),so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1.W\left(\frac{R_1 + R_2}{2}\right) = \mu_1.W.R$$

R_1 and R_2 = Outside and inside radii of the collar,

R = Mean radius of the collar, and

μ_1 = Coefficient of friction for the collar.

$$T = T_1 + T_2 = P \times \frac{d}{2} + \mu_1.W.R$$

Total torque required to overcome friction (*i.e.* to rotate the screw),

If an effort P_1 is applied at the end of a lever of arm length *l*, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, *i.e.*

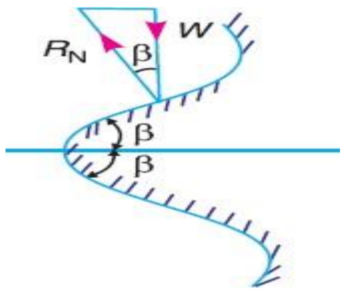
$$T = P \times \frac{d}{2} = P_1.l$$

Friction of a V-thread

The normal reaction in case of a square threaded screw is

$R_N = W \cos\alpha$, where α= Helix angle.

But in case of V-thread (or acme or trapezoidal threads), the normal reaction between the screw and nut is increased because the axial component of this normal reaction must be equal to the axial load.



W, as shown in Fig.

Let 2β= Angle of the V-thread, and

$$R_N = \frac{W}{\cos\beta}$$

frictional force, $F = \mu.R_N = \mu \times \frac{W}{\cos\beta} = \mu_1.W$

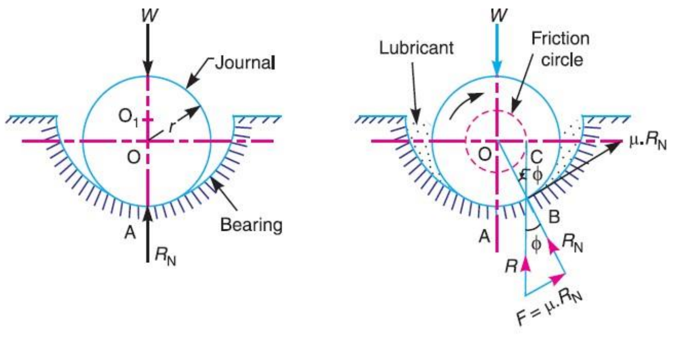
β= Semi-angle of the V-thread.

Friction in Journal Bearing-Friction Circle

A journal bearing forms a turning pair as shown in Fig (*a*). The fixed outer element of a turning pair is

$$\frac{\mu}{\cos\beta} = \mu_1, \text{ known as virtual coefficient of friction.}$$

called a ***bearing*** and that portion of the inner element (*i.e.* shaft) which fits in the bearing is called a ***journal.*** The journal is slightly less in diameter than the bearing, in order to permit the free movement of the journal in a bearing.



(*a*) (*b*)

When the bearing is not lubricated (or the journal is stationary), then there is a line contact between the two elements as shown in Fig (a). The load W on the journal and normal reaction R_N (equal to W) of the bearing acts through the centre. The reaction RN acts vertically upwards at point A. This point A is known as **seat** or **point of pressure.**

Now consider a shaft rotating inside a bearing in clockwise direction as shown in Fig(*b*). The lubricant between the journal and bearing forms a thin layer which gives rise to a greasy friction. Therefore, the reaction *R* does not act vertically upward, but acts at another point of pressure *B*. This is due to the fact that when shaft rotates, a frictional force $F = \mu R_N$ acts at the circumference of the shaft which has a tendency to rotate the shaft in opposite direction of motion and this shifts the point *A* to point *B*. In order that the rotation may be maintained, there must be a couple rotating the shaft.

Let ø= Angle between *R* (resultant of *F* and *R*N) and R_N,

μ = Coefficient of friction between the journal and bearing,

T = Frictional torque in N-m, and

r = Radius of the shaft in meters.

For uniform motion, the resultant force acting on the shaft must be zero and the resultant turning moment on the shaft must be zero. In other words,

$R = W$, and $T = W \times OC = W \times OB \sin\phi = W.r\sin\phi$

Since ø is very small, therefore substituting $\sin\phi = \tan\phi$

$T = W.r\tan\phi = \mu.W.r$ ($\mu = \tan\phi$)

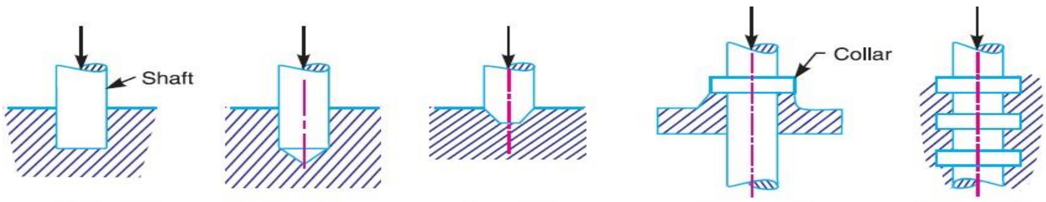
If the shaft rotates with angular velocity ω rad/s, then power wasted in friction,

$P = T\omega = T \times 2\pi N/60$ watts Where N = Speed of the shaft in r.p.m.

Friction of Pivot and Collar Bearing

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust. The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Fig. 10.16 (a) and (b) respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Fig (c).

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Fig (*d*) or several collars along the length of a shaft, as shown in Fig(*e*) in order to reduce the intensity of pressure.



(*a*) Flat pivot. (*b*) Conical pivot. (*c*) Truncated pivot. (*d*) Single flat collar. (*e*) Multiple flat collar.

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed hat

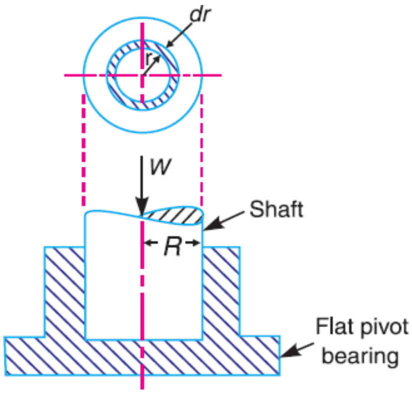
The pressure is uniformly distributed throughout the bearing surface, and

The wear is uniform throughout the bearing surface.

Flat Pivot Bearing:

When a vertical shaft rotates in a flat pivot bearing (known as **foot step bearing**), as shown in Fig., the sliding friction will be along the surface of contact between the shaft and the bearing.

Let W =Load transmitted over the bearing surface,



R =Radius of bearing surface,

p =Intensity of pressure per unit area of bearing Surface between rubbing surfaces, and

μ =Coefficient of friction.

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

Considering uniform pressure

$$p = \frac{W}{\pi R^2}$$

When the pressure is uniformly distributed over the bearing area, then

Consider a ring of radius r and thickness $dr$of the bearing area. Area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring,

$\delta W = p \times A = p \times 2\pi r.dr$ **(*i*)**

Frictional resistance to sliding on the ring acting tangentially at radius r, $F_r = \mu.\delta W = \mu\ p \times 2\pi\ r.dr = 2\pi\mu.p.r.dr$

Frictional torque on the ring,

$T_r = F_r \times r = 2\pi\ \mu\ p\ r.dr \times r = 2\ \pi\ \mu\ pr^2 dr$ **(*ii*)**

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

$$\therefore \text{ Total frictional torque, } T = \int_0^R 2\pi\mu\, p r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$$

$$= 2\pi\mu\, p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad \ldots\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad \ldots(\because\ \omega = 2\pi N/60)$$

N = Speed of shaft in r.p.m.

Considering uniform wear

We have already discussed that the rate of wear depends upon the intensity of pressure and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (*i.e.* $p.v.$). Since the velocity of rubbing surfaces increases with the distance (*i.e.* radius r) from the axis of the bearing, therefore for uniform Wear

$$p.r = C \text{ (a constant)} \quad \text{or} \quad p = C / r$$

and the load transmitted to the ring,

$$\delta W = p \times 2\pi r.dr \qquad \text{...[From equation } (i)\text{]}$$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\ dr = 2\pi\mu \times \frac{C}{r} \times r^2\ dr \qquad \text{...}\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\ dr \qquad \text{...}(iii)$$

∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\, \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \text{...}\left(\because\ C = \frac{W}{2\pi R}\right)$$

PROBLEMS

Example 1.A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end foot step bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.

Solution. Given :*D* = 150 mm or *R* = 75 mm = 0.075 m ; *N* = 100 r.p.m or ω= 2 δ × 100/60= 10.47 rad/s ; *W* = 20 kN = 20 × 103 N ; μ = 0.05

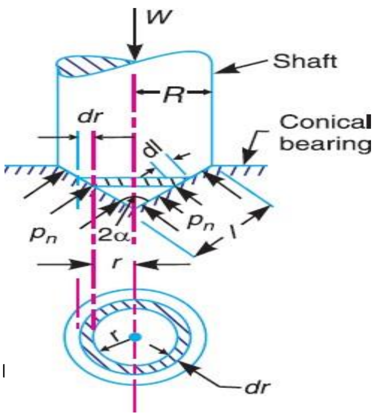
We know that for uniform pressure distribution, the total frictional torque,

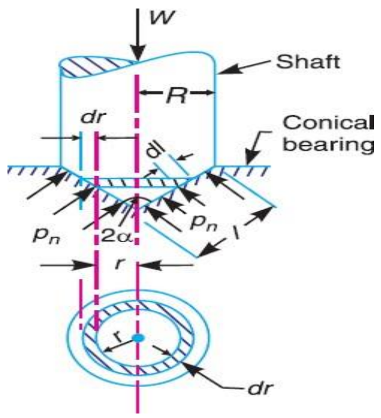
$$T = \frac{2}{3} \times \mu.W.R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50 \text{ N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 50 \times 10.47 = 523.5 \text{ W} \quad \textbf{Ans.}$$

Conical Pivot Bearing





The conical pivot bearing supporting a shaft carrying a load *W* is shown in Fig. Let

P_n = Intensity of pressure normal to the cone,

α = Semi angle of the cone,

μ = Coefficient of friction between the shaft and the bearing,

R = Radius of the shaft.

Consider a small ring of radius *r* and thickness *dr.*

Let *dl* is the length of ring along the cone, such that

$dl = dr \operatorname{cosec} \alpha$

Area of the ring, $A = 2\pi r.dl = 2\pi r.dr \operatorname{cosec} \alpha$ ($dl = dr \operatorname{cosec}\alpha$)

Considering uniform pressure

We know that normal load acting on the ring, δW_n= Normal pressure ×Area

$= p_n \times 2\pi r.dr \operatorname{cosec} \alpha$ vertical load acting on the ring,

δW= Vertical component of $\delta W_n = \delta W_n.\sin\alpha$ Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi\, p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi\, p_n \times \frac{R^2}{2} = \pi R^2.p_n$$

$$p_n = W / \pi R^2$$

$$T_r = F_r \times r = 2\pi\mu.p_n.\operatorname{cosec}\alpha.r.dr \times r = 2\pi\mu.p_n \operatorname{cosec}\alpha.r^2.dr$$

We know that frictional force on the ring acting tangentially at radius *r*,

The vertical load acting on the ring is also given by δW= Vertical component of p_n× Area of the ring

$$= p_n \sin\alpha \times 2\pi r.dr.\text{cosec}\,\alpha = p_n \times 2\pi r.dr$$

Integrating the expression within the limits from 0 to *R* for the total frictional torque on the conical pivot bearing.

$$T = \int_0^R 2\,\pi\,\mu.p_n\ \text{cosec}\ \alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec}\ \alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec}\ \alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec}\ \alpha \qquad ...(i)$$

Total frictional torque:

$$T = \int_0^R 2\,\pi\,\mu.p_n\ \text{cosec}\ \alpha.r^2\ dr = 2\,\pi\mu.p_n.\text{cosec}\ \alpha\left[\frac{r^3}{3}\right]_0^R$$

$$= 2\pi\mu.p_n.\text{cosec}\ \alpha \times \frac{R^3}{3} = \frac{2\pi\ R^3}{3} \times \mu.p_n.\text{cosec}\ \alpha \qquad ...(i)$$

Substituting the value of $p_n$in equation (*i*),

$$T = \frac{2\pi R^3}{3} \times \pi \times \frac{W}{\pi R^2} \times \text{cosec}\ \alpha = \frac{2}{3} \times \mu.W.R.\ \text{cosec}\ \alpha$$

Considering uniform wear

In Fig. let p_r be the normal intensity of pressure at a distance *r* from the central axis. We know that, in case of uniform wear, the intensity of pressure varies inversely with the distance.

$p_r.r = C$ (a constant) or $p_r = C/r$

$$\delta W = p_r \times 2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

The load transmitted to the ring,

Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C\ [r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu.p_r.\text{cosec}\ \alpha.r^2.dr = 2\pi\mu \times \frac{C}{r} \times \text{cosec}\,\alpha.r^2.dr$$

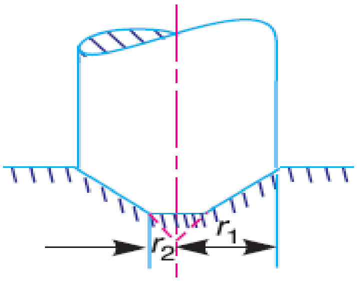
$$= 2\pi\mu.C.\text{cosec}\ \alpha.r.dr$$

Total frictional torque acting on the bearing,

$$T = \pi\mu \times \frac{W}{2\pi R} \times \text{cosec}\ \alpha . R^2 = \frac{1}{2} \times \mu . W . R\ \text{cosec}\ \alpha = \frac{1}{2} \times \mu . W . l$$

Substituting the value of *C*, we have

Trapezoidal or Truncated Conical Pivot Bearing



Area of the bearing surface,

$$A = \pi[(r_1)^2 - (r_2)^2]$$

$\therefore$ Intensity of uniform pressure,

$$p_n = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

If the pivot bearing is not conical, but a frustum of a cone with *r*1 and *r*2, the external and internal radius respectively as shown in Fig, then

Considering uniform pressure

The total torque acting on the bearing is obtained by integrating the value of T_r, within the limits r_1 and r_2.

$$T = \int_{r_2}^{r_1} 2\pi\mu . p_n\ \text{cosec}\ \alpha . r^2 . dr = 2\pi\mu . p_n . \text{cosec}\ \alpha \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu . p_n . \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Total torque acting on the bearing,

$$W = \int_{r_2}^{r_1} 2\pi\ C . dr = 2\pi C [r]_{r_2}^{r_1} = 2\pi C (r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\ \alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Substituting the value of p_n from equation **(*i*),**

Considering uniform wear the load transmitted to the ring, $\delta W = 2\pi C.dr$

Total load transmitted to the ring,

We know that the torque acting on the ring, considering uniform wear, is Total torque acting on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\ \mu.C\ \text{cosec}\ \alpha.r.dr = 2\pi\ \mu.C.\text{cosec}\ \alpha \left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= \pi\ \mu.C.\text{cosec}\ \alpha \left[(r_1)^2 - (r_2)^2\right]$$

Substituting the value of C from equation **(*ii*),** we get

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\ [(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W\ (r_1 + r_2)\ \text{cosec}\ \alpha = \mu.W.R\ \text{cosec}\ \alpha$$

$$R = \text{Mean radius of the bearing} = \frac{r_1 + r_2}{2}$$

PROBLEMS

Example 1.A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution: Given: W = 20 kN = 20 × 103 N ; 2 α= 120º or α= 60º ; p_n= 0.3 N/mm^2 ;N = 200 r.p.m. or ω= 2π× 200/60 = 20.95 rad/s ; μ =0.1

Outer and inner radii of the bearing surface.

Let r_1 and r_2 = Outer and inner radii of the bearing surface, in mm. Since the external diameter is twice the internal diameter, therefore

$r_1 = 2\, r_2$

$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$$(r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3 \quad \text{or} \quad r_2 = 84 \text{ mm } \textbf{Ans.}$$

$$r_1 = 2\, r_2 = 2 \times 84 = 168 \text{ mm } \textbf{Ans.}$$

We know that intensity of normal pressure (p_n),

Power absorbed in friction

$$T = \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{2}{3} \times 0.1 \times 20 \times 10^3 \times \text{cosec } 60^\circ = \left[\frac{(168)^3 - (84)^3}{(168)^2 - (84)^2}\right] \text{N-mm}$$

$$= 301760 \text{ N-mm} = 301.76 \text{ N-m}$$

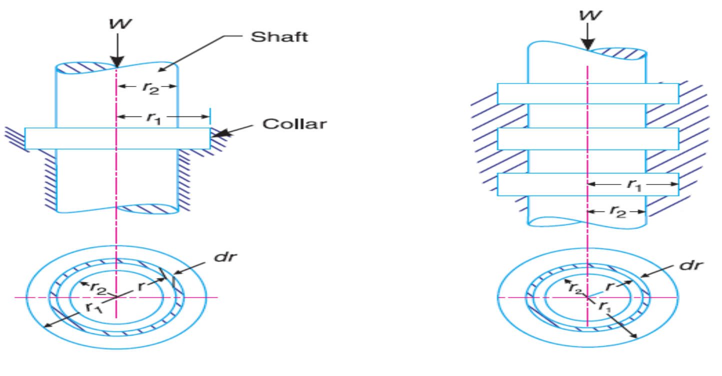
We know that total frictional torque (assuming uniform pressure),

Power absorbed in friction

$P = T.\omega = 301.76 \times 20.95 = 6322 \text{ W} = 6.322 \text{ kW}$

Flat Collar Bearing

We have already discussed that collar bearings are used to take the axial thrust of the rotating hafts. There may be a single collar or multiple collar bearings as shown in Fig.(*a*) and (*b*)respectively. The collar bearings are also known as ***thrust bearings.*** The friction in the collar bearings may be found as discussed below:



(*a*) Single collar bearing (*b*) Multiple collar bearing.

Consider a single flat collar bearing supporting a shaft as shown in Fig(*a*).

Let r_1 = External radius of the collar,

r_2 = Internal radius of the collar.

Area of the bearing surface,

$$A = \pi\,[(r_1)^2 - (r_2)^2]$$

Considering uniform pressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[r_1)^2 - (r_2)^2]} \qquad \ldots(i)$$

The frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu.p.r^2.dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p.r^2.dr = 2\pi\mu.p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu.p\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Substituting the value of p from equation **(*i*),**

Considering uniform wear

The load transmitted on the ring, considering uniform wear is,

$$\delta W = p_r.2\pi r.dr = \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)} \qquad \ldots(ii)$$

Total load transmitted to the collar,

We also know that frictional torque on the ring; we also know that frictional torque on the ring,

$$T_r = \mu.\delta W.r = \mu \times 2\pi C.dr.r = 2\pi\mu.C.r.dr$$

Total frictional torque on the bearing,

$$T = \int_{r_2}^{r_1} 2\pi\mu\, C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2]$$

Substituting the value of *C* from equation **(*ii*),**

$$T = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2] = \frac{1}{2} \times \mu.W(r_1 + r_2)$$

PROBLEMS

Example 1.A thrust shaft of a ship has 6collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the coefficient of friction is 0.12 and speed of the engine90 r.p.m., find the power absorbed in friction at the thrust block, assuming.

1. Uniform pressure

2. Uniform wear.

Solution. Given: n = 6 ; d_1 = 600 mm or r_1 = 300mm ;d_2 = 300 mm or r_2 = 150 mm ;

W = 100 kN= 100 × 10^3 N ;

μ = 0.12 ; *N* = 90 r.p.m. or ω= 2 π× 90/60 = 9.426 rad/s

Power absorbed in friction, assuming uniform pressure

We know that total frictional torque transmitted,

Power absorbed in friction,

P = *T*ω= 2800 × 9.426 = 26 400 W = 26.4 kW

Power absorbed in friction assuming uniform wear

We know that total frictional torque transmitted,

$$T = \frac{1}{2} \times \mu.W(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3\ (300 + 150)\ \text{N-mm}$$

$$= 2700 \times 10^3\ \text{N-mm} = 2700\ \text{N-m}$$

Power absorbed in friction,

P = *T*.ω = 2700 × 9.426 = 25 450 W = 25.45 kW

Example 2.A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diameter is 250 mm. If the intensity of pressure is 0.35 N/mm^2(uniform) and the coefficient of friction is 0.05, estimate power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN. Number of collars required.

Solution. Given: d1 = 400 mm or r_1 = 200 mm ; d_2 = 250 mm or r_2 = 125 mm ; p = 0.35N/mm^2 ; μ = 0.05 ; N = 105 r.p.m or

ω= 2 π× 105/60 = 11 rad/s ; W = 150 kN = 150 × 10^3 N

Power absorbed

We know that for uniform pressure, total frictional torque transmitted

$$T = \frac{2}{3} \times \mu . W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2} \right] = \frac{2}{3} \times 0.05 \times 150 \times 10^3 \left[\frac{(200)^3 - (125)^3}{(200)^2 - (125)^2} \right] \text{N-mm}$$

$$= 5000 \times 248 = 1240 \times 10^3 \text{ N-mm} = 1240 \text{ N-m}$$

Power absorbed,

$P = T\omega$ = 1240 × 11 = 13640 W = 13.64 kW

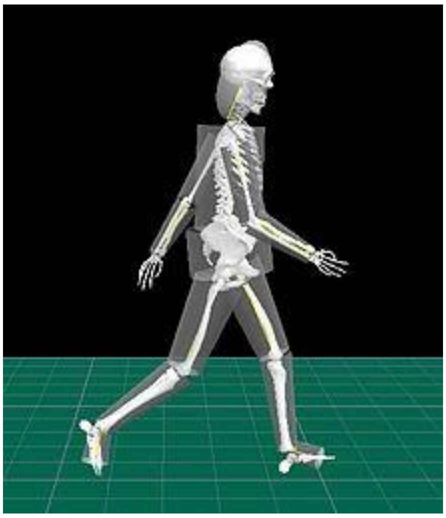
Number of collarsrequired

Let n = Number of collarsrequired.

We know that the intensity of uniform pressure (p),

INDUSTRIAL APPLICATIONS

1. Human body modeled as a system of rigid bodies of geometrical solids. Representative bones were added for better visualization of the walking person.



2. Screw jack

In a ***bottle jack*** the piston is vertical and directly supports a bearing pad that contacts the object being lifted. With a single action piston the lift is somewhat less than twice the collapsed height of the jack, making it suitable only for vehicles with a relatively high clearance. For lifting structures such as houses the hydraulic interconnection of multiple vertical jacks through valves enables the even distribution of forces while enabling close control of the lift.

In a ***floor jack*** a horizontal piston pushes on the short end of a bellcrank , with the long arm providing the vertical motion to a lifting pad, kept horizontal with a horizontal linkage. Floor jacks usually include castors and wheels, allowing compensation for the arc taken by the lifting pad. This mechanism provide a low profile when collapsed, for easy maneuvering underneath the vehicle, while allowing considerable extension

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UNIT 1

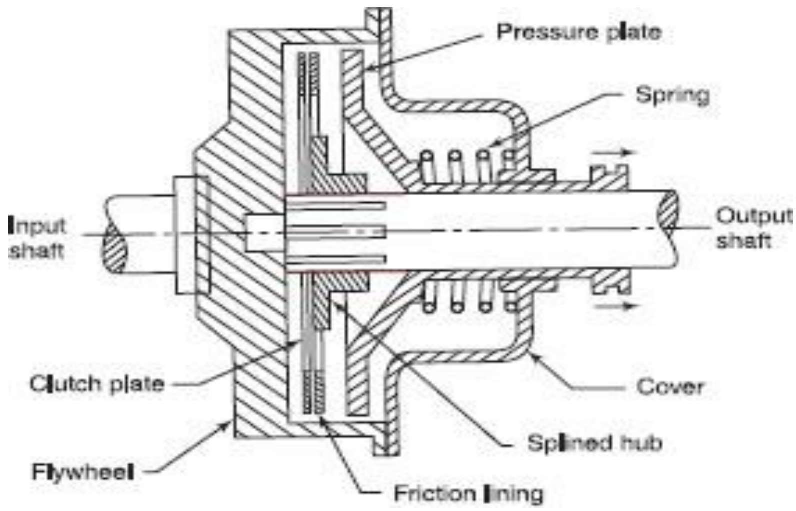
CLUTCHES,BRAKE/DYNAMOMETERS/ TURNING MOMENT DIAGRAM & FLY WHEEL

FRICTION CLUTCHES

A clutch is a device used to transmit the rotary motion of one shaft to another when desired. The axes of the two shafts are coincident. In friction clutches, the connection of the engine shaft to the gear box shaft is affected by friction between two or more rotating concentric surfaces. The surfaces can be pressed firmly against one another when engaged and the clutch tends to rotate as a single unit.

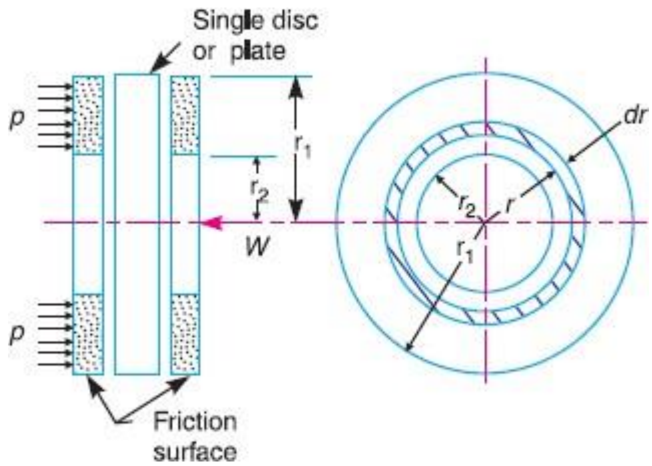
SINGLE PLATE CLUTCH (DISC CLUTCH)

A disc clutch consists of a clutch plate attached to a splined hub which is free to slide axially on splines cut on the driven shaft. The clutch plate is made of steel and has a ring of friction lining on each side. The engine shaft supports a rigidly fixed flywheel. A spring-loaded pressure plate presses the clutch plate firmly against the flywheel when the clutch is engaged. When disengaged, the springs press against a cover attached to the flywheel. Thus, both the flywheel and the pressure plate rotate with the input shaft. The movement of the clutch pedal is transferred to the pressure plate through a thrust bearing. Figure 8.13 shows the pressure plate pulled back by the release levers and the friction linings on the clutch plate are no longer in contact with the pressure plate or the flywheel. The flywheel rotates without driving the clutch plate and thus, the driven shaft.



When the foot is taken off the clutch pedal, the pressure on the thrust bearing is released. As a result, the springs become free to move the pressure plate to bring it in contact with the clutch plate. The clutch plate slides on the splined hub and is tightly gripped between the pressure plate and the fly wheel. The friction between the linings on the clutch plate, and the flywheel on one side and the pressure plate on the other, cause the clutch plate and hence, the driven shaft to rotate. In case the resisting torque on the drive shaft exceeds the torque at the clutch, clutch slip will occur.

Torque transmitted by plate or disc clutch



The following notations are used in the derivation T= Torque transmitted by the clutch

P= intensity of axial pressure

r1&r2=external and internal radii of friction faces

μ= co-efficient of friction

Consider an elemental ring of radius r and thickness dr Friction surface = 2πrdr

Axial force on the dw= pressure *area

= P*2πrdr

Frictional force acting on the ring tangentially at radius r Fr= μdw=μ*p*2πrdr

Frictional torque acting on the ring $T_r=F_r*r=\mu p*2\pi r*dr*r=2\pi\mu pr^2dr$

Considering uniform pressure

When the pressure is uniformly distributed over the entire area of the friction face, then the intensity o pressure,

$P=W/\pi[(r^2-(r)^2]$ (i)

Where *W* = Axial thrust with which the contact or friction surfaces are held together.

We have discussed above that the frictional torque on the elementary ring of radius *r* and thickness *dr* is

$T_r= 2\,\pi\mu.p.r^2dr$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque.

Therefore total frictional torque acting on the friction surface or on the clutch,

$$T = \int_{r_1}^{r_2} 2\pi\mu.p.r^2.dr = 2\,\pi\mu\,p\left[\frac{r_3}{3}\right]_{r_2}^{r_1} = 2\pi\mu\,p\left[\frac{(r_1)^3-(r_2)^3}{3}\right]$$

Substituting the value of *p* from equation (*i*),

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \frac{(r_1)^3 - (r_2)^3}{3}$$

$$= \frac{2}{3} \times \mu.W \left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \mu.W.R$$

R = Mean radius of friction surface

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

2. Considering uniform wear

Let *p* be the normal intensity of pressure at a distance *r* from the axis of the Clutch. Since the intensity of pressure varies inversely with the distance, therefore

$p.r. = C$ (a constant) or $p = C/r$

and the normal force on the ring,

$$\delta W = p.2\pi r.dr = \frac{C}{r} \times 2\pi C.dr = 2\pi C.dr$$

∴ Total force acting on the friction surface,

$$W = \int_{r_2}^{r_1} 2\pi C\, dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that the frictional torque acting on the ring,

$$T_r = 2\pi\mu.p\, r^2.dr = 2\pi\mu \times \frac{C}{r} \times r^2.dr = 2\pi\mu.C.r.dr$$

Total frictional torque on the friction surface,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu.C\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi\mu.C[(r_1)^2 - (r_2)^2] = \pi\mu \times \frac{W}{2\pi(r_1 - r_2)}[(r_1)^2 - (r_2)^2]$$

$$= \frac{1}{2} \times \mu.W(r_1 + r_2) = \mu.W.R$$

R = Mean radius of the friction surface = $(r_1 + r_2)/2$

Multiple plate clutches

In a multi-plate clutch, the number of frictional linings and the metal plates is Increased which increases the capacity of the clutch to transmit torque. Figure8.14 shows a simplified diagram of a multi-plate clutch. The friction rings are splined on their outer circumference and engage with corresponding splines on the flywheel. They are free to slide axially.

The Friction material thus, rotates with the flywheel and the engine shaft. The Number of friction rings depends

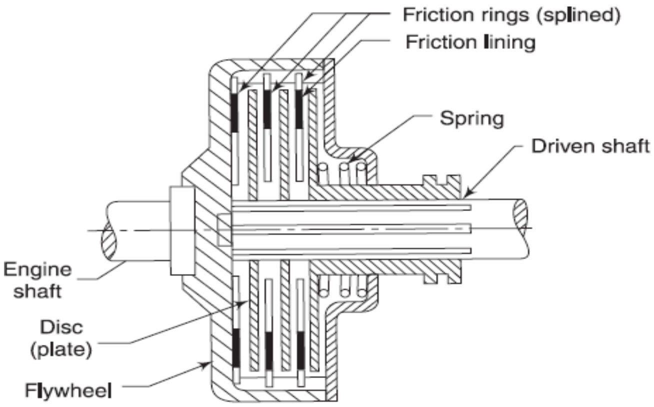


Fig. 8.14

The driven shaft also supports discs on the splines which rotate with the driven shaft and can slide axially. If the actuating force on the pedal is removed, a spring presses the discs into contact with the friction rings and the torque is transmitted between the engine shaft and the driven shaft. If n is the total number of plates both on the driving and the driven members, the number of active surfaces will be $n - 1$.

Let n_1 = Number of discs on the driving shaft, and

n_2 = Number of discs on the driven shaft.

Number of pairs of contact surfaces, $n = n_1 + n_2 - 1$

And total frictional torque acting on the friction surfaces or on the clutch,

$T = n.\mu.W.R$

Where R = Mean radius of the friction surfaces

$$= \frac{2}{3}\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

$$= \frac{r_1 + r_2}{2}$$

PROBLEMS

Example1. Determine the maximum, minimum and average pressure in plate clutch when the axial force is 4 kN. The inside radius of the contact surface is 50 mm and the outside radius is 100 mm. Assume uniform wear.

Solution.

Given: W = 4 kN = 4 × 103 N, r_2 = 50 mm ;r_1 = 100mm

Maximum pressure

Let p_{max} = Maximum pressure.

Since the intensity of pressure is maximum at the inner radius (r_2), therefore

$p_{max} \times r2 = C$ or $C = 50\, p_{max}$

We know that the total force on the contact surface (W),

4 × 103 = 2 π C ($r_1 - r_2$) = 2 π× 50 p_{max} (100 – 50) = 15 710 p_{max}

P_{max} = 4 × 103/15 710 = 0.2546 N/mm^2

Minimum pressure

Let p_{min} = Minimum pressure.

Since the intensity of pressure is minimum at the outer radius (r_1), Therefore $P_{min} \times r_1 = C$ or $C = 100\, p_{min}$

We know that the total force on the contact surface (W),

4 × 103 = 2 π C ($r_1 - r_2$) = 2π × 100 p_{min} (100 – 50) = 31 420 p_{min}

P_{min} = 4 × 103/31 420 = 0.1273 N/mm^2

Average pressure

We know that average pressure,

$$p_{av} = \frac{\text{Total normal force on contact surface}}{\text{Cross-sectional area of contact surfaces}}$$

$$= \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{4 \times 10^3}{\pi[(100)^2 - (50)^2]} = 0.17 \text{ N/mm}^2$$

Example2. A single plate clutch, with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surfaceis not to exceed 0.1 N/mm2.Ifthe coefficient of friction is 0.3, determine the power transmitted by a clutch at a speed 2500r.p.m.

Solution.Given: d_1 = 300 mm or r_1 = 150 mm ;d_2 = 200 mm or r_2 = 100 mm ;

p = 0.1 N/mm^2; μ = 0.3 ;N= 2500 r.p.m. or ω= 2π× 2500/60 = 261.8 rad/s

Since the intensity of pressure (p) is maximum at the inner radius (r_2), therefore for uniform

$p.r_2 = C$ or $C = 0.1 \times 100 = 10$ N/mm

We know that the axial thrust,

$W = 2\,\pi\,C\,(r_1 - r_2) = 2\,\pi \times 10\,(150 - 100) = 3142$ N

$$R = \frac{r_1 + r_2}{2} = \frac{150 + 100}{2} = 125 \text{ mm} = 0.125 \text{ m}$$

Mean radius of the friction surfaces for uniform wear,

We know that torque transmitted,

$T = n.\mu.W.R = 2 \times 0.3 \times 3142 \times 0.125 = 235.65$ N-m

Power transmitted by a clutch,

$P = T*\omega = 235.65 \times 261.8 = 61\,693$ W = 61.693 kW

CONE CLUTCH

A cone clutch, as shown in Fig. 10.24, was extensively used in automobiles but now-a-days it has been replaced completely by the disc clutch

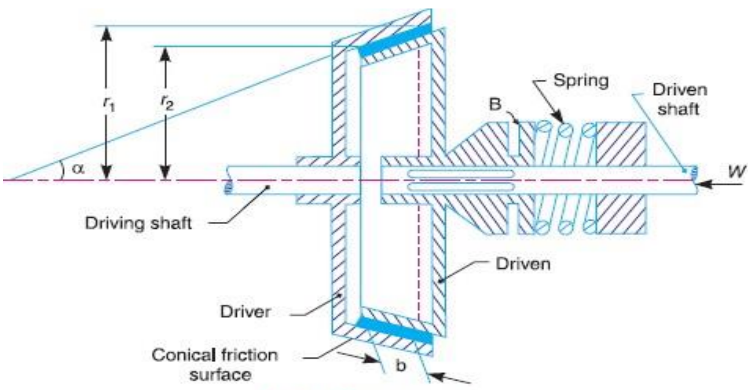


Fig. 10.24. Cone clutch.

It consists of one pair of friction surface only. In a cone clutch, the driver is keyed to the driving shaft by a sunk key and has an inside conical surface or face which exactly fits into the outside conical surface of the driven.

The driven member resting on the feather key in the driven shaft, maybe shifted along the shaft by a forked lever provided at *B*, in order to engage the clutch by bringing the two conical surfaces in contact. Due to the frictional resistance set up at this contact surface, the torque is transmitted from one shaft to another. In some cases, a spring is placed around the driven shaft in contact with the hub of the driven. This spring holds the clutch faces in contact and maintains the pressure between them, and the forked lever is used only for disengagement of the clutch. The contact surfaces of the clutch may be metal to metal contact, but more often the driven member is lined with some material like wood, leather, cork or asbestos etc. The material of the clutch faces (*i.e.* contact surfaces) depends upon the allowable normal pressure and the coefficient of friction.

Consider a pair of friction surface as shown in Fig. Since the area of contact of a pair of friction surface is a frustum of a cone, therefore the torque transmitted by the cone clutch maybe determined in the similar manner as discussed.

Let p_n = Intensity of pressure with which the conical friction surfaces are held together (*i.e.* normal pressure between contact surfaces),

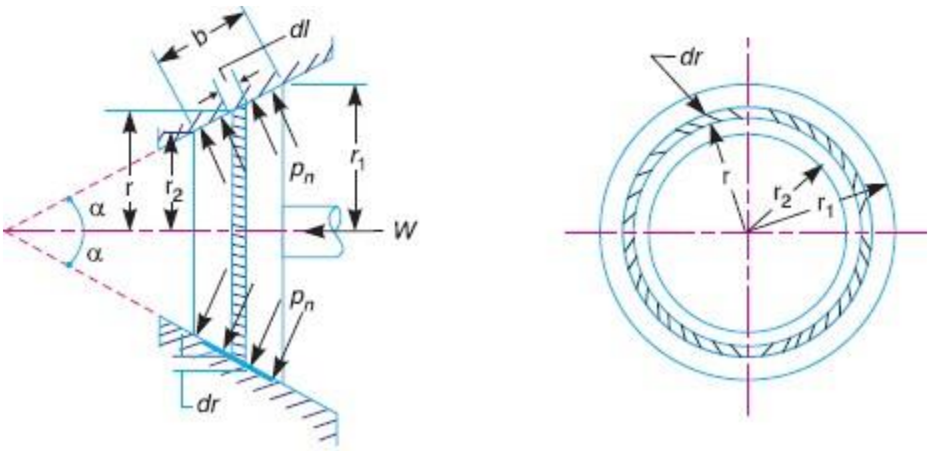
r_1 and r_2 = Outer and inner radius of friction surfaces respectively

R = Mean radius of the friction surface=$(r_1 + r_2)/2$

α= Semi angle of the cone (also called face angle of the cone) or the angle of the friction surface with the axis of the clutch,

μ = Coefficient of friction between contact surfaces, and

b = Width of the contact surfaces (also known as face width or clutch face).



(*a*) (*b*)

Consider a small ring of radius *r* and thickness *dr*, as shown in Fig. 10.25 (*b*).

Let *dl* is length of ring of the friction surface, such that

$dl = dr.\text{cose}\ \alpha$ Area of the ring= $A = 2\pi\ r.dl = 2\pi r.dr \text{cosec}\ \alpha$

We shall consider the following two cases :

When there is a uniform pressure and when there is a uniform wear.

Considering uniform pressure

We know that normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_n \times 2\ \pi r.dr.\text{cosec}\ \alpha$ The axial load acting on the ring,

δW= Horizontal component of δW_n (*i.e.* in the direction of *W*)

$= \delta W_n \times \sin\alpha = p_n \times 2\pi\ r.dr.\ \text{cosec}\ \alpha \times \sin\alpha = 2\pi \times p_n .r.dr$

Total axial load transmitted to the clutch or the axial spring force required,

$$W = \int_{r_2}^{r_1} 2\pi p_n.r.dr = 2\pi p_n \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi p_n \left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \pi p_n \left[(r_1)^2 - (r_2)^2\right]$$

$$p_n = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

We know that frictional force on the ring acting tangentially at radius *r*, $F_r = \mu.\delta W_n = \mu.p_n \times 2\pi\, r.dr.\text{cosec}\, \alpha$ Frictional torque acting on the ring,

Integrating this expression within the limits from r_2 to r_1 for the total frictional torque on the clutch

∴ Total frictional torque,

$$T = \int_{r_2}^{r_1} 2\pi\mu.p_n.\text{cosec}\,\alpha.r^2.dr = 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu\, p_n.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

Substituting the value of p_n from equation (*i*), we get

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]} \times \text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W.\text{cosec}\,\alpha\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Considering uniform wear

In Fig., let $p_r$be the normal intensity of pressure at a distance *r* from the axis of the clutch. We know that, in case of uniform wear, the intensity of pressure varies inversely with thedistance.

$P_r.r = C$ (a constant) or $p_r = C / r$

We know that the normal load acting on the ring,

δW_n = Normal pressure × Area of ring = $p_r \times 2\pi r.dr \text{cosec}\alpha$ The axial load acting on the ring ,

$\delta W = \delta W_n \times \sin\alpha = p_r.2\pi\, r.dr.\text{cosec}\,\alpha.\sin\alpha = p_r \times 2\pi\, r.dr$

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

∴ Total axial load transmitted to the clutch,

$$W = \int_{r_2}^{r_1} 2\pi C.dr = 2\pi C[r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

$$C = \frac{W}{2\pi(r_1 - r_2)}$$

We know that frictional force acting on the ring,

$F_r = \mu.\delta W_n = \mu.p_r \times 2\pi r \times dr\text{cosec}\alpha$ Frictional torque acting on the ring,

$$= \mu \times \frac{C}{r} \times 2\pi r^2.dr.\text{cosec}\ \alpha = 2\pi\mu.C\ \text{cosec}\ \alpha \times r\ dr$$

∴ Total frictional torque acting on the clutch,

$$T = \int_{r_2}^{r_1} 2\pi\mu.C.\text{cosec}\ \alpha.r\,dr = 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{r^2}{2}\right]_{r_2}^{r_1}$$

$$= 2\pi\mu.C.\text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

Substituting the value of C from equation (*i*), we have

$$T = 2\pi\mu \times \frac{W}{2\pi(r_1 - r_2)} \times \text{cosec}\ \alpha\left[\frac{(r_1)^2 - (r_2)^2}{2}\right]$$

$$= \mu.W\ \text{cosec}\ \alpha\left(\frac{r_1 + r_2}{2}\right) = \mu.W.R\ \text{cosec}\ \alpha$$

$$R = \frac{r_1 + r_2}{2} = \text{Mean radius of friction surface}$$

PROBLEMS

Example 1.An engine developing 45 kW at 1000 r.p.m. is fitted with a cone clutch built inside the flywheel. The cone has a face angle of 12.5º and a maximum mean diameter of 500 mm. The coefficient of friction is 0.2. The normal pressure on the clutch face is not to exceed 0.1 N/mm 2. Determine: 1. the axial spring force necessary to engage to clutch, and 2. the face widthrequired.

Solution.Given :P = 45 kW = 45 × 103 W ; N = 1000 r.p.m. or ω= 2π× 1000/60 = 104.7rad/s ; α= 12.5º ; D = 500 mm or R = 250 mm = 0.25 m ; μ = 0.2 ;

p_n=0.1N/mm^2

Axial spring force necessary to engage theclutch

First of all, let us find the torque (T) developed by the clutch and the normal load (W_n) acting on the friction surface.

We know that power developed by the clutch (P),

45 × 103 = $T\omega$= T × 104.7 or T = 45 × 10^3/104.7 = 430 N-m

We also know that the torque developed by the clutch (T), 430 = μ.W_n.R = 0.2 × W_n× 0.25 = 0.05 W_n

W_n = 430/0.05 = 8600 N

Axial spring force necessary to engage the clutch,

W_e = W_n (sin α+ μ cosα)

= 8600 (sin 12.5º + 0.2 cos 12.5º) = 3540 N

Face widthrequired

Let b = Face widthrequired

We know that normal load acting on the friction surface (W_n), 8600 = $p_n \times 2\pi R.b$= 0.1 × 2π× 250 × b = 157 b

b = 8600/157 = 54.7 mm

Example 2. A conical friction clutch is used to transmit 90 kW at 1500 r.p.m. The semi cone angle is 20º and the coefficient of friction is 0.2. If the mean diameter of the bearing surface is375 mm and the intensity of normal pressure is not to exceed 0.25 N/mm2, find the dimensions of the conical bearing surface and the axial load required.

Solution.Given: P = 90 kW = 90 × 10^3 W ;N = 1500 r.p.m. or ω = 2 π× 1500/60 = 156rad/s ; α = 20º ; μ = 0.2 ; D = 375 mm or R = 187.5 mm ; p_n = 0.25 N/mm^2

Dimensions of the conical bearing surface

Let r_1 and r_2 = External and internal radii of the bearing surface respectively,

b = Width of the bearing surface in mm, and

T = Torque transmitted.

We know that power transmitted (P), 90 × 103 = Tω = T × 156

T = 90 × 103/156 = 577 N-m = 577 × 10^3 N-mm

The torque transmitted (T),

577 × 103 = 2 π μ $p_n.R^2.b$ = 2π × 0.2 × 0.25 $(187.5)^2 b$ = 11 046 b

b = 577 × 10^3/11 046 = 52.2 mm

We know that $r_1 + r_2 = 2R$ = 2 × 187.5 =375 mm.............. i

$r1 – r2 = b \sin α$= 52.2 sin 20º =18 mm...................... ii

From equations (*i*) and (*ii*),

r_1 = 196.5 mm, and r_2 = 178.5 mm

Axial load required

Since in case of friction clutch, uniform wear is considered and the intensity of pressure is maximum at the minimum contact surface radius ($r2$), therefore

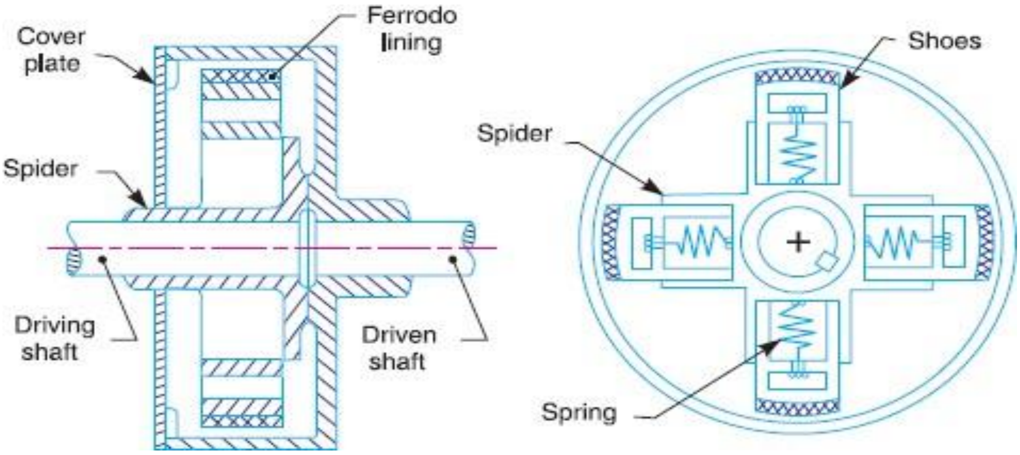
$p_n.r_2 = C$ (a constant) or C = 0.25 × 178.5 = 44.6 N/mm

We know that the axial load required,

$W = 2\pi C (r_1 - r_2)$ = 2π × 44.6 (196.5 – 178.5) = 5045 N

Centrifugal Clutch

The centrifugal clutches are usually incorporated into the motor pulleys. It consists of a number of shoes on the inside of a rim of the pulley, as shown in Fig. 10.28. The outer surface of the shoes is covered with



a friction material. These shoes, which can move radially in guides, are held

Against the boss (or spider) on the driving shaft by means of springs. The springs exert a radially inward force which is assumed constant. The mass of the shoe, when revolving, causes it to exert a radially outward force (*i.e.* centrifugal force). The magnitude of this centrifugal force depends upon the speed at which the shoe is revolving. A little consideration will show that when the centrifugal force is less than the spring force, the shoe remains in the same position as when the driving shaft was stationary, but when the centrifugal force is equal to the spring force, the shoe is just floating. When the centrifugal force exceeds the spring force, the shoe moves outward and comes into contact with the driven member and presses against it. The force with which the shoe presses against the driven member is the difference of the centrifugal force and the spring force. The increase of speed causes the shoe to press harder and enables more torque to be transmitted. In order to determine the mass and size of the shoes, the following procedure is adopted:

Mass of the shoes

Consider one shoe of a centrifugal clutch as shown in Fig Let m = Mass of eachshoe,

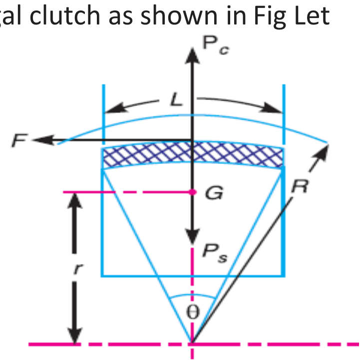


Fig. 10.29. Forces on a shoe of centrifugal clutch.

n = Number of shoes,

r = Distance of centre of gravity of the shoe from the centre of the spider,

R = Inside radius of the pulley rim,

N = Running speed of the pulley in r.p.m.,

ω = Angular running speed of the pulley in rad/s

= 2πN/60 rad/s,

ω_1 = Angular speed at which the engagement begins to take place, and

$\propto$ = Coefficient of friction between the shoe and rim.

We know that the centrifugal force acting on each shoe at the running speed,

*$P_c = m\,.\omega^2.r$

and the inward force on each shoe exerted by the spring at the speed at which engagement begins to take place,

$P_s = m\,(\omega_1)^2 r$

∴ The net outward radial force (*i.e.* centrifugal force) with which

The shoe presses against the rim at the running speed = $P_c - P_s$

The frictional force acting tangentially on each shoe, F = $\propto\ (P_c - P_s)$

∴ Frictional torque acting on each shoe, = $F \times R = \propto\ (P_c - P_s)\ R$

Total frictional torque transmitted,

$T = \propto\ (P_c - P_s)\ R \times n = n.F.R$

From this expression, the mass of the shoes (m) may be evaluated. Size of the shoes

Let l = Contact length of theshoes, b = Width of theshoes,

R = Contact radius of the shoes. It is same as the inside radius of the rim of the pulley.

θ = Angle subtended by the shoes at the centre of the spider in radians.

p = Intensity of pressure exerted on the shoe. In order to ensure reason-able life, the intensity of pressure may be taken as 0.1 N/mm^2.

We know that θ = l/Rrador $l = \theta.R$

∴ Area of contact of the shoe, $A = l.b$

The force with which the shoe presses against the rim

$A \times p = l.b.p$

Since the force with which the shoe presses against the rim at the running speed is ($P_c - P_s$), therefore

$$l.b.p = P_c - P_s$$

PROBLEMS

Example 1.

A centrifugal clutch is to transmit 15 kW at 900 r.p.m. The shoes are four in number. The speed at which the engagement begins is 3/4th of the running speed. The inside radius of the pulley rim is 150 mm and the centre of gravity of the shoe lies at 120 mm from the centre of the spider. The shoes are lined with Ferrodo for which the coefficient of friction may be taken as 0.25. Determine: 1. Mass of the shoes, and 2. Size of the shoes, if angle subtended by the shoes at the centre of the spider is 60º and the pressure exerted on the shoes is $0.1 N/mm^2$.

Solution.:

Given: P = 15 kW = 15×10^3 W; N = 900 r.p.m. or ω = 25 × 900/60 = 94.26 rad/s ; n = 4 ; R = 150 mm = 0.15 m ; r = 120 mm = 0.12 m ; ∝ = 0.25

Since the speed at which the engagement begins (*i.e.* ω_1) is 3/4th of the running speed (*i.e.* ω), therefore

$$\omega_1 = \frac{3}{4}\,\omega = \frac{3}{4} \times 94.26 = 70.7 \text{ rad/s}$$

Let T = Torque transmitted at the running speed.

We know that power transmitted (P),

$= T.\omega = T \times 94.26$ or $T = 15 \times 10^3/94.26$ =

15 × 10 =159 N-m

Mass of the shoes

Let m = Mass of the shoes in kg.

We know that the centrifugal force acting on each shoe,

$P_c = m.\omega^2.r = m\,(94.26)^2 \times 0.12 = 1066\,m$ N

the inward force on each shoe exerted by the spring *i.e*. the centrifugal force at the engagement speed ω_1,

$P_s = m\,(\omega_1)^2 r = m\,(70.7)^2 \times 0.12 = 600\,m$ N

∴ Frictional force acting tangentially on each shoe,

F= $\propto\,(P_c - P_s) = 0.25\,(1066\,m - 600\,m)\;116.5\,m$ N

We know that the torque transmitted (T),

$159 = n.F.R = 4 \times 116.5\,m \times 0.15 = 70\,m$ or $m = 2.27$ kg

Size of theshoes

Let *l* = Contact length of shoes in mm,

b = Width of the shoes in mm,

θ Angle subtended by the shoes at the centre of the spider in radians

= 60º = π/3 rad, and

p = Pressure exerted on the shoes in N/mm^2 = 0.1 N/mm^2

$$\text{We know that} \qquad l = \theta . R = \frac{\pi}{3} \times 150 = 157.1 \text{ mm}$$

$$l.b.p = P_c - P_s = 1066\, m - 600\, m = 466\, m$$

$$\therefore \quad 157.1 \times b \times 0.1 = 466 \times 2.27 = 1058$$

b = 1058/157.1 × 0.1 = 67.3 mm

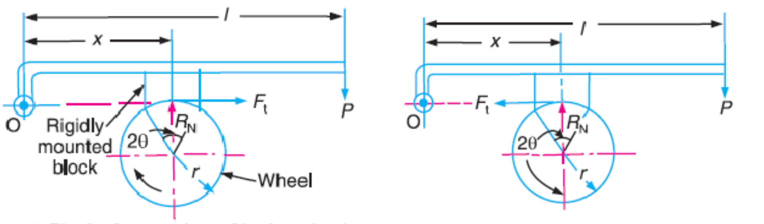
BRAKES AND DYNAMOMETERS

A *brake* is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine. In the process of performing this function, the brake absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators etc

Single Block or Shoe Brake

A single block or shoe brake is shown in Fig. It consists of a block or shoe which is pressed against the rim of a revolving brake wheel drum. The block is made of a softer material than the rim of the wheel. This type of a brake is commonly used on railway trains and tram cars. The friction between the block and the wheel causes a tangential braking force to act on the wheel, which retard the rotation of the wheel. The block is pressed against the wheel by a force applied to one end of a lever to which the block is rigidly fixed as shown in Fig. 19.1. The other end of the lever is pivoted on a fixed fulcrum *O*

Let *P* = Force applied at the end of thelever



(*a*) Clockwise rotation of brake wheel (*b*) Anticlockwise rotation of brake wheel.

R_N= Normal force pressing the brake block on the wheel,

r = Radius of the wheel,

2θ = Angle of contact surface of the block,

μ = Coefficient of friction, and

F_t = Tangential braking force or the frictional force acting at the contact surface of the block and the wheel

If the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. In such cases, tangential braking force on the wheel,

$F_t = \mu.R_N$...(*i*)

The braking torque, $T_B = F_t.r = \mu.R_N.r$ ...(*ii*)

Let us now consider the following three cases:

Case1. When the line of action of tangential braking force (*Ft*) passes through the fulcrum *O* of the lever, and the brake wheel rotates clockwise as shown in Fig. 19.1(*a*), then for equilibrium, taking moments about the fulcrum.

$$R_N \times x = P \times l \text{ or } R_N = \frac{P \times l}{x}$$

Braking torque,

$$T_B = \mu.R_N.r = \mu \times \frac{P.l}{x} \times r = \frac{\mu.P.l.r}{x}$$

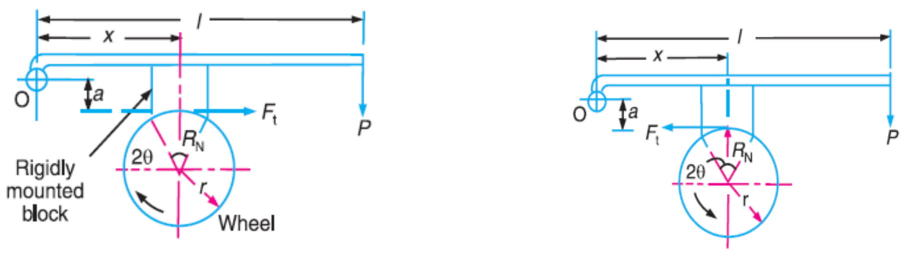
O, wehave

It may be noted that when the brake wheel rotates anticlockwise as shown in Fig. 19.1 (*b*), then the braking torque is same, *i.e.*

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x}$$

Case2. When the line of action of the tangential braking force (*Ft*) passes through a distance '*a*' below the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

When the brake wheel rotates anticlockwise, as shown in Fig. 19.2 (*b*), then for equilibrium,



(*a*) Clockwise rotation of brake wheel. (*b*) Anticlockwise rotation of brake wheel.

(*a*), then for equilibrium, taking moments about the fulcrum *O*, Case 3.When the lie of action of the tangential braking force (*Ft*) passes through a distance '*a*' above the fulcrum *O*, and the brake wheel rotates clockwise as shown in Fig.

(*a*), then fore equilibrium, taking moments about the fulcrum *O*, wehave

$$R_N.x = P.l + F_t.a = P.l + \mu.R_N.a$$

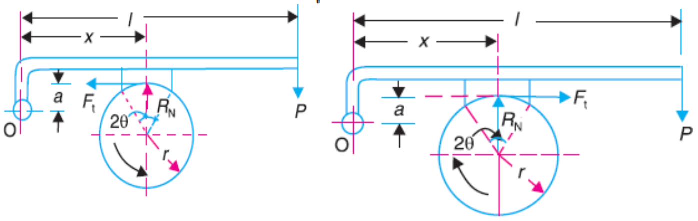
$$R_N(x - \mu.a) = P.l \qquad \text{or} \qquad R_N = \frac{P.l}{x - \mu.a}$$

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

$R_N.x = P.l + Ft.a = P.l + \mu.R_N.a$

$R_N(x - \mu.a) = P.l$

$$R_N = \frac{P.l}{x - \mu.a}$$



(*b*) Anticlockwise rotation of brake wheel. (*a*) Clockwise rotation of brake wheel.

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x - \mu.a}$$

When the brake wheel rotates anticlockwise as shown in Fig. 19.3 (*b*), then for equilibrium, taking moments about the fulcrum *O*, we have

$R_N \times x + Ft \times a = P.l \; R_N \times x + \mu.R_N \times a = P.l$

$$T_B = \mu.R_N.r = \frac{\mu.P.l.r}{x + \mu.a}$$

Pivoted Block or Shoe Brake : We have discussed in the previous article that when the angle of contact is less than 60°, then it may be assumed that the normal pressure between the block and the wheel is uniform. But when the angle of contact is greater than 60°, then the unit pressure normal to the surface of contact is less at the ends than at the centre.

Instead of being rigidly attached to the lever. This gives uniform wear of the brake lining in the direction of the applied force. The braking torque or a pivoted block

$$T_B = F_t \times r = \mu'.R_N.r$$

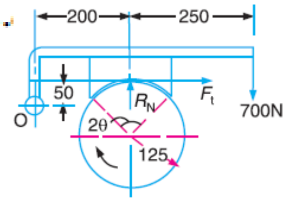
$$\mu' = \text{Equivalent coefficient of friction} = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta},$$

μ = Actual coefficient of friction.

PROBLEMS

Example1. A single block brake is shown in Fig. 19.5.The diameter of the drum is 250 mm and the angle of contact is90°. If the operating force of 700 N is applied at the end of alever and the coefficient of friction

$$R_N = \frac{P.l}{x + \mu.}$$



All dimensions in mm.

Fig. 19.5

between the drum and the lining is0.35,Determine the torque that may be transmitted by the

Solution. Given: d = 250 mm or r = 125 mm ; 2θ = 90° = π / 2 rad ; P = 700 N ; μ = 0.35

Since the angle of contact is greater than 60°, therefore equivalent coefficient of friction,

$$\mu' = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta} = \frac{4 \times 0.35 \times \sin 45^\circ}{\pi/2 + \sin 90^\circ} = 0.385$$

R_N = Normal force pressing the block to the brake drum, and

F_t = Tangential braking force = $\mu'.R_N$

Taking moments about the fulcrum O, we have

$$700(250 + 200) + F_t \times 50 = R_N \times 200 = \frac{F_t}{\mu'} \times 200 = \frac{F_t}{0.385} \times 200 = 520\, F_t$$

520 F_t – 50F_t = 700 × 450 or F_t = 700 × 450/470 = 670 N

We know that torque transmitted by the blockbrake,

T_B = F_t ×r = 670 × 125 = 8 3750 N-mm =83.75N-m

Example 2.A bicycle and rider of mass 100 kg are travelling at the rate of 16 km/h on a level road. A brake is applied to the rear wheel which is 0.9 m in diameter and this is the only resistance acting. How far will the bicycle travel and how many turns will it make before it comes to rest? The pressure applied on the brake is 100 N and μ = 0.05.

Solution.Given: m = 100 kg, v = 16 km / h = 4.44 m / s ;D = 0.9 m ; R

N = 100 N; μ = 0.05

Distance travelled by the bicycle before it comes to rest

Let x = Distance travelled (in meters) by the bicycle before it comes to rest.

We know that tangential braking force acting at the point of contact of the brake wheel,

$F_t = \mu . R_N = 0.05 \times 100 = 5$ N

$= F_t \times x = 5 \times x = 5x$ N-m   (i)

We know that kinetic energy of the bicycle

$$= \frac{m.v^2}{2} = \frac{100(4.44)^2}{2}$$
$$= 986 \text{ N-m} \qquad \ldots (ii)$$

In order to bring the bicycle to rest, the work done against friction must be equal to kinetic energy of the bicycle. Therefore equating equations (*i*) and (*ii*),

$5x = 986$ or $x = 986/5 = 197.2$ m

Number of revolutions made by the bicycle before it comes to rest

Let N = Required number of revolutions.

We know that distance travelled by the bicycle (x), $197.2 = \pi DN = \pi \times 0.9N = 2.83N$
$N = 197.2 / 2.83 = 70$

Double Block or Shoe Brake

When a single block brake is applied to a rolling wheel, an additional load is thrown on the shaft bearings due to the normal force (RN). This produces bending of the shaft. In order to overcome this drawback, a double block or shoe brake, as shown in Fig. 19.9, is used. It consists of two brake blocks applied at the opposite ends of a diameter of the wheel which eliminate or reduces the unbalanced force on the shaft. The brake is set by a spring which pulls the upper ends of the brake arms together. When a force P is applied to the bell crank lever, the spring is compressed and the brake is released. This type of brake is often used on electric cranes and the force P is produced by an electromagnet or solenoid.

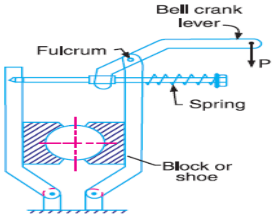


Fig. 19.9. Double block or shoe brake.

In a double block brake, the braking action is doubled by the use of two blocks and these blocks may be operated practically by the same force which will operate one. In case of double block or shoe brake, the braking torque is given by

$T_B = (F_{t1} + F_{t2})\, r$

Where F_{t1} and F_{t2} are the braking forces on the two blocks.

Internal Expanding Brake

An internal expanding brake consists of two shoes $S1$ and $S2$ as shown in Fig.
19.24. The outer surface of the shoes are lined with some friction material (usually with Ferodo) to increase the coefficient of friction and to prevent wearing away of the metal. Each shoe is pivoted at one end about a fixed fulcrum $O1$ and $O2$ and made to contact a cam at the other end. When the cam rotates, the shoes are pushed outwards against the rim of the drum. The friction between the shoes and the drum produces the braking torque and hence reduces the speed of the drum. The shoes are normally held in off position by a spring as shown in Fig. 19.24. The drum encloses the entire mechanism to keep out dust and moisture. This type of brake is commonly used in motor cars and light trucks.

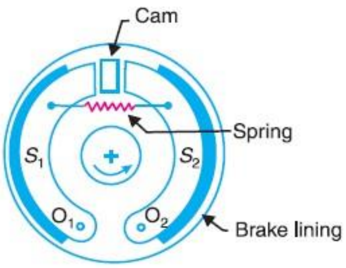


Fig. 19.24. Internal expanding brake.

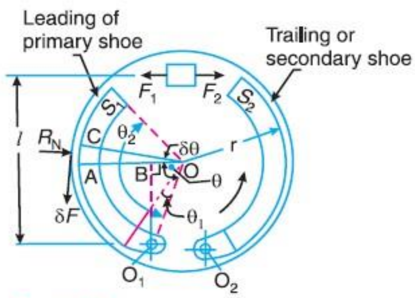


Fig. 19.25. Forces on an internal expanding brake.

We shall now consider the forces acting on such a brake, when the drum rotates in the anticlockwise direction as shown in Fig. 19.25. It may be noted that for the anticlockwise direction, the left hand shoe is known as *leading* or *primary shoe* while the right hand shoe is known as *trailing* or *secondaryshoe*.

Let r = Internal radius of the wheelrim,

b = Width of the brake lining,

p_1 = Maximum intensity of normal pressure,

p_N = Normal pressure,

F_1 = Force exerted by the cam on the leading shoe, and

F_2 = Force exerted by the cam on the trailing shoe.

Consider a small element of the brake lining

AC subtending an angle δθ at the centre. Let *OA* makes an angle θ with OO_1 as shown in Fig. 19.25. It is assumed that the pressure distribution on the shoe is nearly uniform, however the friction lining wears out more at the free end. Since the shoe turns about*O*1, therefore the rate of wear of the shoe lining at *A* will be proportional to the radial displacement of that point. The rate of wear of the shoe lining varies directly as the perpendicular distance from $O_1$to *OA*, *i.e.*

O_1B. From the geometry of the figure,

$O_1B = OO_1 \sin\theta$

Normal pressure at *A*,

$$p_N \propto \sin\theta \quad \text{or} \quad p_N = p_1 \sin\theta$$

∴ Normal force acting on the element,

$$\delta R_N = \text{Normal pressure} \times \text{Area of the element}$$

$$= p_N (b.r.\delta\theta) = p_1 \sin\theta (b.r.\delta\theta)$$

and total braking torque about *O* for whole of one shoe,

$$T_B = \mu\, p_1 b\, r^2 \int_{\theta_1}^{\theta_2} \sin\theta\, d\theta = \mu\, p_1 b\, r^2 [-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= \mu\, p_1 b r^2 (\cos\theta_1 - \cos\theta_2)$$)

Moment of normal force δR_N of the element about the fulcrum O_1,

$$\delta M_N = \delta R_N \times O_1B = \delta R_N (OO_1 \sin\theta)$$

$$= p_1 \sin\theta (b.r.\delta\theta)(OO_1 \sin\theta) = p_1 \sin^2\theta (b.r.\delta\theta) OO_1$$

∴ Total moment of normal forces about the fulcrum O_1,

$$= p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \frac{1}{2}(1 - \cos 2\theta)\, d\theta \qquad \ldots\left[\because \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta - \frac{\sin 2\theta}{2}\right]_{\theta_1}^{\theta_2}$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[\theta_2 - \frac{\sin 2\theta_2}{2} - \theta_1 + \frac{\sin 2\theta_1}{2}\right]$$

$$= \frac{1}{2} p_1.b.r.OO_1 \left[(\theta_2 - \theta_1) + \frac{1}{2}(\sin 2\theta_1 - \sin 2\theta_2)\right]$$

$$M_N = \int_{\theta_1}^{\theta_2} p_1 \sin^2\theta (b.r.\delta\theta) OO_1 = p_1.b.r.OO_1 \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta$$

Now for leading shoe, taking moments about the fulcrum O_1,

$$F_1 \times l = M_N - M_F$$

and for trailing shoe, taking moments about the fulcrum O_2,

$$F_2 \times l = M_N + M_F$$

$$\ldots \left(\because AB = r - OO_1 \cos\theta\right)$$

$$= \mu\, p_1 \sin\theta (b.r.\delta\theta)\,(r - OO_1 \cos\theta)$$

$$= \mu . p_1 .b.r (r \sin\theta - OO_1 \sin\theta \cos\theta)\delta\theta$$

$$= \mu . p_1 .b.r \left(r\sin\theta - \frac{OO_1}{2} \sin 2\theta \right) \delta\theta \quad \ldots (\because 2\sin\theta\cos\theta = \sin 2\theta)$$

$\therefore$ Total moment of frictional force about the fulcrum O_1,

$$M_F = \mu\, p_1 b\, r \int_{\theta_1}^{\theta_2} \left(r\sin\theta - \frac{OO_1}{2}\sin 2\theta \right) d\theta$$

$$= \mu\, p_1 b\, r \left[-r\cos\theta + \frac{OO_1}{4}\cos 2\theta \right]_{\theta_1}^{\theta_2}$$

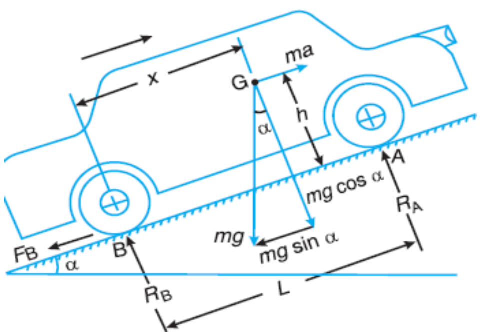
$$= \mu\, p_1 b\, r \left[-r\cos\theta_2 + \frac{OO_1}{4}\cos 2\theta_2 + r\cos\theta_1 - \frac{OO_1}{4}\cos 2\theta_1 \right]$$

$$= \mu\, p_1 b\, r \left[r(\cos\theta_1 - \cos\theta_2) + \frac{OO_1}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]$$

Braking of a Vehicle

In a four wheeled moving vehicle, the brakes may be applied to the rear wheels only, the front wheels only, and all the four wheels. In all the above mentioned three types of braking, it is required to determine the retardation of the vehicle when brakes are applied. Since the vehicle retards, therefore it is a problem of dynamics. But it may be reduced to an equivalent problem of statics by including the inertia force in the system of forces actually applied to the vehicle. The inertia force is equal and opposite to the braking force causing retardation.

Now, consider a vehicle moving up an inclined plane, as shown in Fig.



Let α= Angle of inclination of the plane to the horizontal,

m = Mass of the vehicle in kg (such that its weight is *m.g*newtons),

h = Height of the C.G. of the vehicle above the road surface in metres,

x = Perpendicular distance of C.G. from the rear axle in metres,

L = Distance between the centres of the rear and front wheels of the vehicle in metres,

R_A = Total normal reaction between the ground and the front wheels in newtons,

R_B = Total normal reaction between the ground and the rear wheels in newtons,

μ = Coefficient of friction between the tyres and road surface, and

a = Retardation of the vehicle in m/s2.

We shall now consider the above mentioned three cases of braking, one by one. In all these cases, the braking force acts in the opposite direction to the direction of motion of the vehicle.

When the brakes are applied to the rear wheels only

It is a common way of braking the vehicle in which the braking force acts at the rear wheels only.

Let F_B =Total braking force(in newtons) acting at the rear wheels due to the application of the brakes. Its maximum value isμ.R_B.

The various forces acting on the vehicle are shown in Fig. For the equilibrium of the vehicle, the forces acting on the vehicle must be in equilibrium.

Resolving the forces parallel to the plane,

$F_B + m.g.\sin\alpha = m.a$. . . (*i*)

Resolving the forces perpendicular to the plane,

$R_A + R_B = m\,g\cos\alpha$………(*ii*)

Taking moments about *G*, the centre of gravity of the vehicle

$F_B \times h + R_B \times x = R_A(L - x)$ (*iii*)

Substituting the value of $F_B = \mu.R_B$, and $R_A = m.g\cos\alpha - R_B$ [from equation (*ii*)] in the above expression, we have

$\mu.R_B \times h + R_B \times x = (m.g\cos\alpha - R_B)(L - x)$ $R_B(L + \mu.h) = m.g\cos\alpha(L - x)$

$$R_B = \frac{m.g\cos\alpha(L-x)}{L+\mu.h}$$

and

$$R_A = m.g\cos\alpha - R_B = m.g\cos\alpha - \frac{m.g\cos\alpha(L-x)}{L+\mu.h}$$

$$= \frac{m.g\cos\alpha(x+\mu.h)}{L+\mu.h}$$

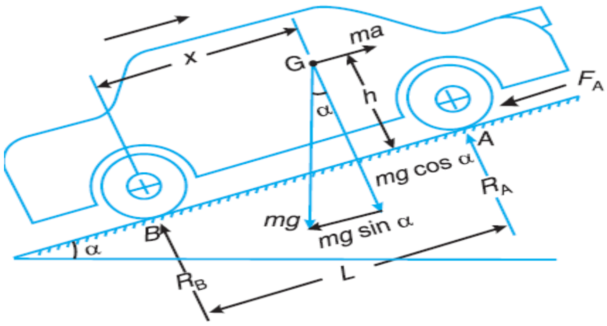
We know from equation (*i*),

$$a = \frac{F_B + m.g\sin\alpha}{m} = \frac{F_B}{m} + g\sin\alpha = \frac{\mu.R_B}{m} + g\sin\alpha$$

$$= \frac{\mu.g\cos\alpha(L-x)}{L+\mu.h} + g\sin\alpha \quad \text{. . . (Substituting the value of } R_B\text{)}$$

When the brakes are applied to front wheelsonly

It is a very rare way of braking the vehicle, in which the braking force acts at the front wheels only.



Let F_A = Total braking force (in newtons) acting at the front wheels due to the application of brakes. Its maximum value is $\mu.R_A$.

The various forces acting on the vehicle are shown in Fig. Resolving the forces parallel to theplane,

$F_A + m.g\sin\alpha = m.a$ . . . (*i*)

Resolving the forces perpendicular to the plane,

$R_A + R_B = m.g\cos\alpha$ . . . (*ii*)

Taking moments about *G*, the centre of gravity of the vehicle,

$F_A \times h + R_B \times x = R_A (L - x)$

Substituting the value of $F_A = \mu.R_A$ and $R_B = m.g\cos\alpha - R_A$ [from equation (*ii*)] in the above expression, we have

$\mu.R_A \times h + (m.g\cos\alpha - R_A)\, x = R_A (L - x)$ $\mu.R_A \times h + m.g\cos\alpha \times x = R_A \times L$

$$\therefore \qquad R_A = \frac{m.g\cos\alpha \times x}{L - \mu.h}$$

and

$$R_B = m.g\cos\alpha - R_A = m.g\cos\alpha - \frac{m.g\cos\alpha \times x}{L - \mu.h}$$

$$= m.g\cos\alpha\left(1 - \frac{x}{L - \mu.h}\right) = m.g\cos\alpha\left(\frac{L - \mu.h - x}{L - \mu.h}\right)$$

We know from equation (*i*),

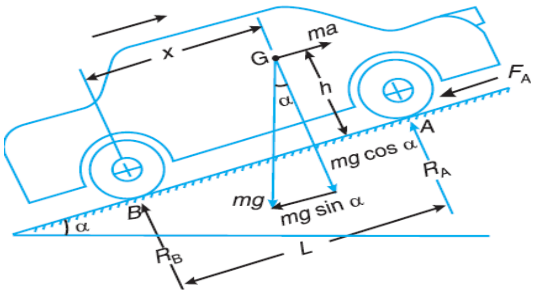
$$a = \frac{F_A + m.g\sin\alpha}{m} = \frac{\mu.R_A + m.g\sin\alpha}{m}$$

$$= \frac{\mu.m.g\cos\alpha \times x}{(L - \mu.h)m} + \frac{m.g\sin\alpha}{m} \qquad \text{. . . (Substituting the value of } R_A)$$

$$= \frac{\mu.g\cos\alpha \times x}{L - \mu.h} + g\sin\alpha$$

When the brakes are applied to all the fourwheels

This is the most common way of braking the vehicle, in which the braking force acts on both the rear and front wheels.



Let F_A = Braking force provided by the front wheels = μ.R_A,and

F_B = Braking force provided by the rear wheels = μ.R_B.

Little consideration will show that when the brakes are applied to all the four wheels, the braking distance (i.e. the distance in which the vehicle is brought to rest after applying the brakes) will be the least. It is due to this reason that the brakes are applied to all the four wheels. The various forces acting on the vehicle are shown in fig.

Resolving the forces parallel to the plane, $F_A+F_B+m.g\sin\alpha=m.a$.......(i)

Resolving forces vertical to the plane

$R_A + R_B = m.g\cos\alpha$. . .(*ii*)

Taking moments about *G*, the centre of gravity of the vehicle, $(F_A + F_B)\,h + R_B \times x = R_A(L - x)$.......(iii)

Substituting the value of $F_A = \mu.R_A$, $F_B = \mu.R_B$ and $R_B = m.g\cos\alpha - R_A$

[From equation (*ii*)] in the above expression,

$\mu\,(R_A + R_B)\,h + (m\,g\cos\alpha - R_A)\,x = R_A(L - x)$

$\mu\,(R_A + mg\cos\alpha - R_A)h + (mg\cos\alpha - R_A)x = R_A(L - x)$ $\mu.m.g\cos\alpha \times h + m.g\cos\alpha \times x = R_A \times L$

$$R_A = \frac{m.g\cos\alpha(\mu.h + x)}{L}$$

$$R_B = m.g\cos\alpha - R_A = m.g\cos\alpha - \frac{mg\cos\alpha(\mu.h + x)}{L}$$

$$= m.g\cos\alpha\left[1 - \frac{\mu.h + x}{L}\right] = m.g\cos\alpha\left(\frac{L - \mu.h - x}{L}\right)$$

Now from equation (i), $\mu.R_A + \mu\,R_B + m.g\sin\alpha = m.a$

$\mu(R_A + R_B) + m.g\sin\alpha = m.a$

$\mu.m.g.\cos\alpha + m.g\sin\alpha = m.a$ . . . [From equation (ii)]

$a = g(\mu.\cos\alpha + \sin\alpha)$

PROBLEMS

Example 1. A car moving on a level road at a speed 50 km/h has a wheel base 2.8metres, distance of C.G. from ground level 600 mm, and the distance of C.G. from rear wheels 1.2metres. Find the distance travelled by the car before coming to rest when brakes are applied,To the rear wheels, To the front wheels, and To all the four wheels. The coefficient of friction between the tyres and the road may be taken as0.6.

Solution.

Given :u = 50 km/h = 13.89 m/s ; L = 2.8 m ; h = 600 mm = 0.6 m ; x = 1.2 m ; μ = 0.6 Let s = Distance travelled by the car before coming to rest.

When brakes are applied to the rearwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g(L-x)}{L+\mu.h} = \frac{0.6\times9.81(2.8-1.2)}{2.8+0.6\times0.6} = 2.98 \text{ m/s}^2$$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times2.98} = 32.4 \text{ m}$$

When brakes are applied to the frontwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$$a = \frac{\mu.g.x}{L-\mu.h} = \frac{0.6\times9.18\times1.2}{2.8-0.6\times0.6} = 2.9 \text{ m/s}^2$$

We know that for uniform retardation,

When the brakes are applied to all the fourwheels

Since the vehicle moves on a level road, therefore retardation of the car,

$a = g.\mu = 9.81\times0.6 = 5.886 \text{ m/s}_2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(13.89)^2}{2\times5.886} = 16.4 \text{ m}$$

Example2. A vehicle moving on a rough plane inclined at 10° with the horizontal at a speed of 36 km/h has a wheel base 1.8 metres. The centre of gravity of the vehicle is 0.8 metre from the rear wheels and

0.9 metre above the inclined plane. Find the distance travelled by the vehicle before coming to rest and the time taken to do so when

The vehicle moves up the plane,and

The vehicle moves down theplane.

The brakes are applied to all the four wheels and the coefficient of friction is 0.5.

Solution.

Given : α= 10°; *u* = 36 km / h = 10 m / s ; *L* = 1.8 m ; *x* = 0.8 m ; *h* = 0.9 m ; μ = 0.5 Let *s* = Distance travelled by the vehicle before coming to rest,and

t = Time taken by the vehicle in coming to rest.

When the vehicle moves up the plane and brakes are applied to all the four wheel

Since the vehicle moves up the inclined plane, therefore retardation of the vehicle,

$a = g\ (\mu \cos\alpha + \sin\alpha)$
$= 9.81\ (0.5\cos 10° + \sin 10°) = 9.81(0.5\times0.9848 + 0.1736) = 6.53\ m/s^2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2\times6.53} = 7.657 \text{ m}$$

and final velocity of the vehicle (*v*),

$0 = u + a.t = 10 - 6.53t$ (Minus sign due toretardation)

$t = 10 / 6.53 = 1.53$

When the vehicle moves down the plane and brakes are applied to all the four wheels

Since the vehicle moves down the inclined plane, therefore retardation of the vehicle,

$a = g\ (\mu \cos\alpha - \sin\alpha)$

$= 9.81(0.5\cos 10° - \sin 10°) = 9.81(0.5\times0.9848 - 0.1736) = 3.13\ m/s^2$

We know that for uniform retardation,

$$s = \frac{u^2}{2a} = \frac{(10)^2}{2\times3.13} = 16 \text{ m}$$

and final velocity of the vehicle (*v*),

$0 = u + a.t = 10 - 3.13t$ . . . (Minus sign due toretardation)

$t = 10/3.13 = 3.2$ s

DYNAMOMETER

A dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Types of Dynamometers

Absorptiondynamometers,

Transmissiondynamometers.

In the *absorption dynamometers*, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and is transformed into heat, during the process of measurement. But in the *transmission dynamometers*, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through the dynamometer to some other machines where the power developed is suitably measured.

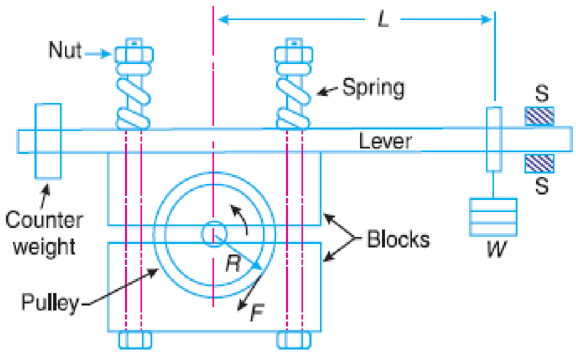
Classification of Absorption Dynamometers

Prony brakedynamometer, 2.Rope brakedynamometer.

rony brakedynamometer

A simplest form of an absorption type dynamometer is a prony brake dynamometer, as shown in Fig. It consists of two wooden blocks placed around a pulley fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two bolts and nuts, as shown in Fig.

A helical spring is provided between the nut and the upper block to adjust the pressure on the pulley to control its speed. The upper block has a long lever attached to it and carries a weight *W* at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of thelever.



When the brake is to be put in operation, the long end of the lever is loaded with suitable weights *W* and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight *W* must balance the moment of the frictional resistance between the blocks and the pulley.

Let *W* = Weight at the outer end of the lever innewtons,

L = Horizontal distance of the weight *W* from the centre of the pulley inmetres,

F = Frictional resistance between the blocks and the pulley in newtons,

R = Radius of the pulley in metres, and

N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$T = W.L = F.R$ N-m

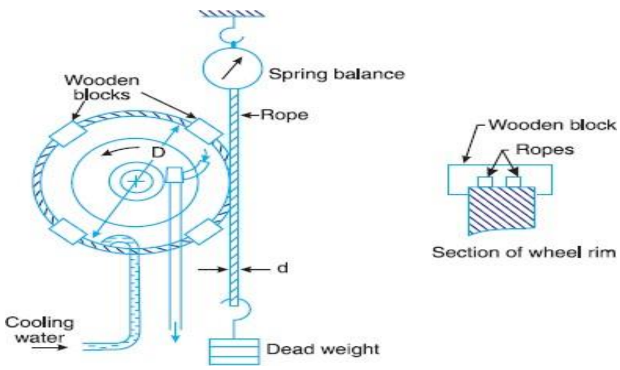
Work done in one revolution Work done in one revolution
= Torque × Angle turned in radians
= $T \times 2\pi$ N-m
_ Work done per minute

= $T \times 2\pi N$ N-m
We know that brake power of the engine

$$B.P. = \frac{\text{Work done per min.}}{60} = \frac{T \times 2\pi N}{60} = \frac{W.L \times 2\pi N}{60} \text{ watts}$$

Rope Brake Dynamometer

It is another form of absorption type dynamometer which is most commonly used for measuring the brake power of the engine. It consists of one, two or more ropes wound around the flywheel or rim of a pulley fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance while the lower end of the ropes is kept in position by applying a dead weight as shown in Fig.19.32. In



order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.

Let *W* = Dead load innewtons,
S = Spring balance reading in newtons, *D* = Diameter of the wheel in metres, *d* = diameter of rope in metres, and
N = Speed of the engine shaft in r.p.m.

Net load on the brake= (*W* – *S*) N

We know that distance moved in one revolution= π (*D*+ *d*) m Work done per revolution= (*W* − *S*) _ (*D* + *d*)N-m
Work done per minute= (*W* − *S*) π (*D* + *d*) *N*N-m

Brake power of the engine,

$$B.P = \frac{\text{Work done per min}}{60} = \frac{(W-S)\,\pi\,(D+d)N}{60} \text{ watts}$$

,
If the diameter of the rope (*d*) is neglected, then brake power of the engine

$$B.P. = \frac{(W-S)\pi D\, N}{60} \text{ watts}$$

Example 1.In a laboratory experiment, the following data were recorded with rope brake: Diameter of the flywheel 1.2 m; diameter of the rope12.5 mm; speed of the engine 200 r.p.m.; dead load on the brake 600 N; spring balance reading 150 N. Calculate the brake power of the engine.

Solution.Given : D = 1.2 m ; d = 12.5 mm
= 0.0125 m ; N = 200 r.p.m ; W = 600 N ; S = 150 N
We know that brake power of the engine,

$$B.P. = \frac{(W-S)\,\pi\,(D+d)N}{60} = \frac{(600-150)\,\pi\,(1.2+0.0125)200}{60} = 5715 \text{ W}$$

Classification of Transmission Dynamometers

Epicyclic-traindynamometer,

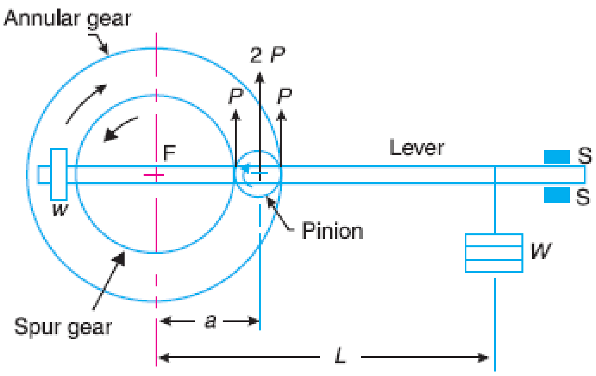
Belt transmission dynamometer,and

Torsiondynamometer

Epicyclic-train Dynamometer

An epicyclic-train dynamometer, as shown in Fig. 19.33, consists of a simple epicyclic train of gears, *i.e*. a spur gear, an annular gear (a gear having internal teeth) and a pinion. The spur gear is keyed to the engine shaft (*i.e*. driving shaft) and rotates in anticlockwise direction. The annular gear is also keyed to the driving shaft and rotates in clockwise direction. The pinion or the intermediate gear meshes with both the spur and annular gears. The pinion revolves freely on a lever which is pivoted to the common axis of the driving and driven shafts. A weight *w* is placed at the smaller end of the lever in order to keep it in position. A little consideration will show that if the

friction of the pinon which the pinion rotates is neglected, then the tangential effort *P* exerted by the spur gear on the pinion and the tangential reaction of the annular gear on the pinion areequal.



Since these efforts act in the upward direction as shown, therefore total upward force on the lever acting through the axis of the pinion is 2*P*. This force tends to rotate the lever about its fulcrum and it is balanced by a dead weight *W* at the end of the lever. The stops *S*, *S* are provided to control the movement of the lever.

$2P \times a = W.L$ or $P = W.L/2a$

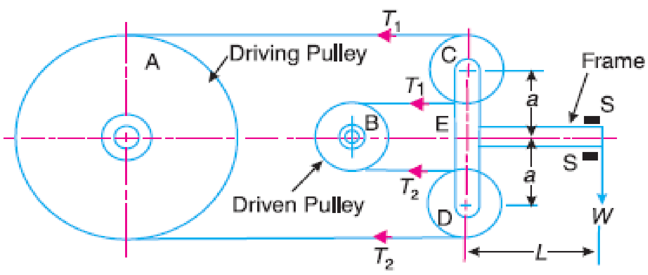
R = Pitch circle radius of the spur gear in metres, and

N = Speed of the engine shaft in r.p.m. Torque transmitted, $T = P.R$

Belt Transmission Dynamometer-Froude or Throneycroft Transmission Dynamometer

power transmitted $= \dfrac{T \times 2\pi N}{60} = \dfrac{P.R \times 2\pi N}{60}$ watts

When the belt is transmitting power from one pulley to another, the tangential effort on the driven pulley is equal to the difference between the tensions in the tight and slack sides of the belt. A belt dynamometer is introduced to measure directly the difference between the tensions of the belt, while it isrunning.



A belt transmission dynamometer, as shown in Fig, is called a Froude or Throneycroft transmission dynamometer. It consists of a pulley *A*(called driving pulley) which is rigidly fixed to the shaft of an engine whose power is required to be measured. There is another pulley *B* (called driven pulley) mounted on another shaft to which the power from pulley *A* is transmitted. The pulleys *A* and *B* are connected by means of a continuous belt passing round the two loose pulleys *C* and *D* which are mounted on a *T*-shaped frame. The frame is pivoted at *E* and its movement is controlled by two stops *S*,*S*. Since the tension in the tight side of the belt (*T*1) is greater than the tension in the slack side of the belt (*T*2), therefore the total force acting on the pulley *C* (*i.e*. 2*T*1) is greater than the total force acting on the pulley *D* (*i.e*. 2*T*2). It is thus obvious that the frame causes movement about *E* in the anticlockwise direction. In order to balance it, a weight *W* is applied at a distance *L* from *E* on the frame as shown inFig.

Now taking moments about the pivot *E*, neglecting friction, $2T_1 \times a = 2T_2 \times a + W L$

$$T_1 - T_2 = \frac{W.L}{2a}$$

Let *D* = diameter of the pulley *A* inmetres,

N = Speed of the engine shaft in r.p.m.

Work done in one revolution = $(T_1 - T_2)\pi D$ N-m work done per minute = $(T_1 - T_2)\pi DN$ N-m

$$\therefore \text{ Brake power of the engine, B.P.} = \frac{(T_1 - T_2)\pi DN}{60} \text{ watts}$$

Torsion Dynamometer

A torsion dynamometer is used for measuring large powers particularly the power transmitted along the propeller shaft of a turbine or motor vessel. A little consideration will show that when the power is being transmitted, then the driving end of the shaft twists through a small angle relative to the driven end of the shaft. The amount of twist depends upon many factors such as torque acting on the shaft (*T*), length of the shaft (*l*), diameter of the shaft (*D*) and modulus of rigidity (*C*) of the material of the shaft. We know that the torsion equationis

$$\frac{T}{J} = \frac{C.\theta}{l}$$

where θ = Angle of twist in radians, and

J = Polar moment of inertia of the shaft.

For a solid shaft of diameter D, the polar moment of inertia

$$J = \frac{\pi}{32} \times D^4$$

and for a hollow shaft of external diameter D and internal diameter d, the polar moment of inertia,

$$J = \frac{\pi}{32}(D^4 - d^4)$$

From the above torsion equation,

$$T = \frac{C.J}{l} \times \theta = k.\theta$$

Where $k = C.J/l$ is a constant for a particular shaft. Thus, the torque acting on the shaft is proportional to the angle of twist. This means that if the angle of twist is measured by some means, then the torque and hence the power transmitted may be determined. We know that the power transmitted P= 2πNT/60watts,

Where N is the speed in r.p.m.

PROBLEMS

Example 1. A torsion dynamometer is fitted to a propeller shaft of a marine engine. It is found that the shaft twists 2° in a length of 20 metres at 120 r.p.m. If the shaft is hollow with 400 mm external diameter and 300 mm internal diameter, find the power of the engine. Take modulus of rigidity for the shaft material as 80 GPa.

Solution.

Given : _ = 2° = 2 × _ /180 = 0.035 rad ; l = 20 m ; N = 120 r.p.m. ; D = 400 mm= 0.4 m ;

d = 300 mm = 0.3 m ;C = 80 GPa = 80 × 109 N/m^2

We know that polar moment of inertia of the shaft

$$J = \frac{\pi}{32}(D^4 - d^4) = \frac{\pi}{32}\left[(0.4)^4 - (0.3)^4\right] = 0.0017\text{m}^4$$

and torque applied to the shaft,

$$T = \frac{C.J}{l} \times \theta = \frac{80 \times 10^9 \times 0.0017}{20} \times 0.035 = 238 \times 10^3 \text{ N-m}$$

We know that power of the engine,

$$P = \frac{T \times 2\pi N}{60} = \frac{238 \times 10^3 \times 2\pi \times 120}{60} = 2990 \times 10^3 \text{ W} = 2990 \text{ kW}$$