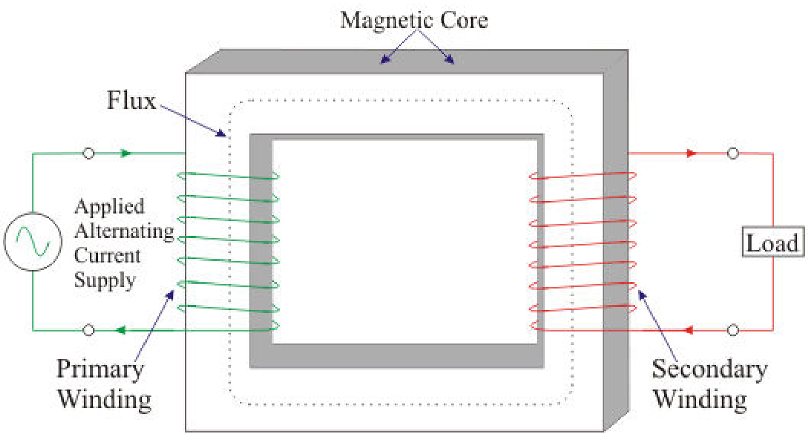
Unit – 3

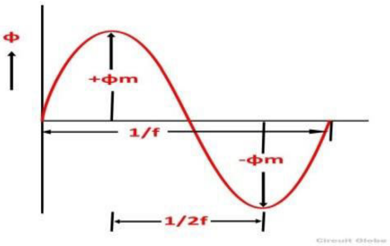
1. Explain the *Working principle* of transformer



Single Phase Transformer

1. The basic working principle of a transformer is mutual induction between two windings linked by common magnetic flux.
2. The primary and secondary coils are electrically separated but magnetically linked to each other.
3. When, primary winding is connected to a source of alternating voltage, alternating magnetic flux is produced around the winding.
4. The core provides magnetic path for the flux, to get linked with the secondary winding. Most of the flux gets linked with the secondary winding which is called as 'useful flux' or main 'flux', and the flux which does not get linked with secondary winding is called as 'leakage flux'.
5. As the flux produced is alternating (the direction of it is continuously changing), EMF gets induced in the secondary winding according to Faraday's law of electromagnetic induction. This induced emf is called 'mutually induced emf', and the frequency of mutually induced emf is same as that of supplied emf. Thus, in a transformer the frequency is same on both sides.
6. If the secondary winding is closed circuit, then mutually induced makes the current flow through it, and hence the electrical energy is transferred from one circuit (primary) to another circuit (secondary).

2. Derive the *EMF Equation* of a Transformer



Let

ϕ_m = Maximum value of flux in Weber
f = Supply frequency in Hz
N_1 = Number of turns in the primary winding
N_2 = Number of turns in the secondary winding
Φ = flux per turn in Weber

As per the faradays laws,

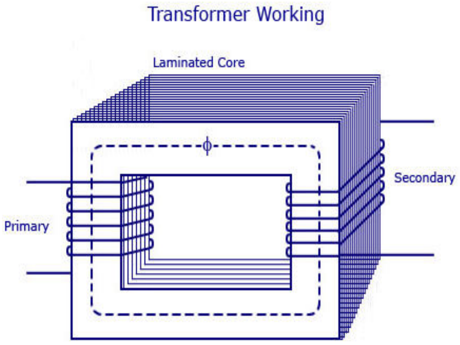
The average value of the emf induced is directly proportional to the rate of change of flux.

- The flux changes from $+\phi_m$ to $-\phi_m$ in half a cycle of 1/2f seconds.
- Flux increases from its zero value to maximum value ϕ_m in one quarter of the cycle i.e. in ¼ of the timeperiod.
- Average rate of change of flux is $\frac{d\phi}{dt} = \frac{\phi_m - 0}{\frac{1}{4f}} = 4\phi_m f$ volts
- Therefore the average e.m.f per turn is $4\phi_m f$
- As $\frac{Rmsvalue}{Averagevalue} = Formfactor = 1.11$ for sinusoidal varying quatities
- Hence, RMS value of e.m.f/turn is $1.11 * 4\phi_m f = 4.44\phi_m f$

- RMS value of e.m.f in the primary & secondary winding. =(e.m.f/turn) * No:of turns
- Therefore Emf induced in primary winding having N_1turns is $E_1 = 4.44\phi_m f N_1$
- Emf induced in secondary winding having N_2 turns is $E_2 = 4.44\phi_m f N_2$

3. Explain the *Construction of Transformer*

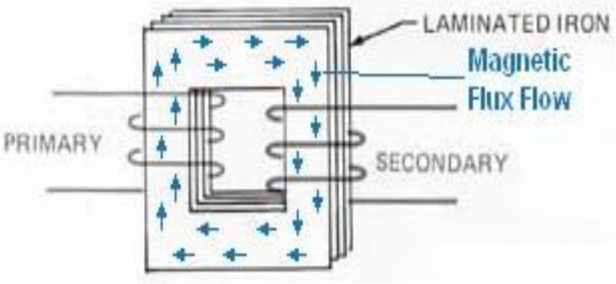
1. The simple construction of a transformer, need two coils having mutual inductance and a laminated steel core.
2. The two coils are insulated from each other and from the steel core.
3. The device will also need some suitable container for the assembled core and windings, a medium with which the core and its windings from its container can be insulated.
4. In order to insulate and to bring out the terminals of the winding from the tank, bushings made of porcelain are used.
5. In all transformers, the core is made of transformer sheet steel laminations assembled to provide a continuous magnetic path with minimum of air-gap included.
6. The steel should have high permeability and low hysteresis loss. For this to happen, the steel should be made of high silicon content and must also be heat treated.
7. By effectively laminating the core, the eddy-current losses can be reduced. The lamination can be done with the help of a light coat of core plate varnish or lay an oxide layer on the surface. For a frequency of 50 Hertz, the thickness of the lamination varies from 0.35mm to 0.5mm for a frequency of 25 Hertz.
8. To reduce the leakage fluxes in the transformer the windings of the primary and secondary coils are interleaved in the core type and sandwiched coils in the shell type.
9. To reduce the volume of the cu wire the core used must be the stepped core or cruciform core.

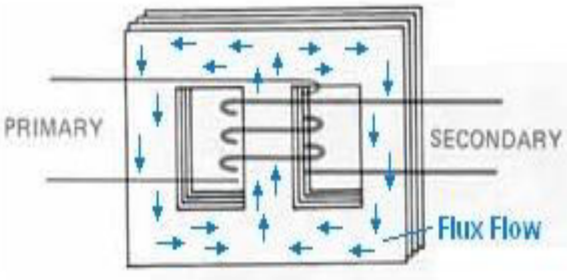


4. Compare and distinguish *the types of transformers*

There are two major types of transformers based on construction. 1. Core type 2. Shell type

S.No	Core type Transformer	Shell type transformer
1	The winding encircles the core	The core encircles the winding
2	The cylindrical type of coils are used	Generally multilayer disc type or sandwiched coils coils are used
3	As windings are distributed, the natural cooling is more effective	As windings are surrounded by the core, the natural cooling does not exists.
4	The coils can be easily removed from the maintenance point of view	For removing any winding for maintenance, a large number of laminations are to be removed. This is difficult.
5	The construction is preferred for low voltage transformers	The construction is used for very high voltage transformers
6	It has a single magnetic core	It has a double magnetic core
7	In a single phase type there are two limbs	In a single phase type the core has three limbs

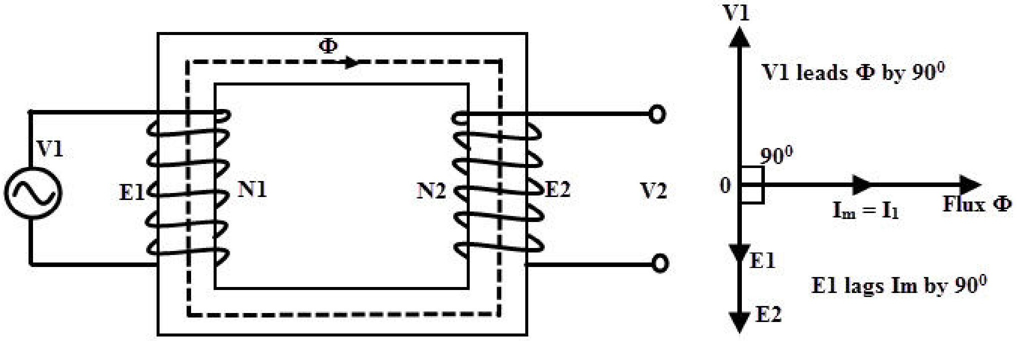




5. Explain the operation of *Transformer on No Load.*

<u>Ideal transformer at No-Load:</u>

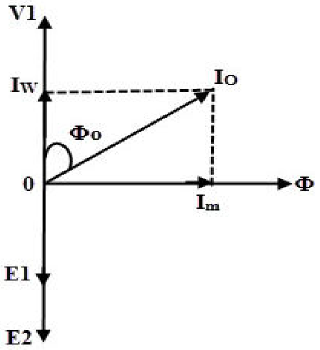
1. The transformer operating at no load, is equivalent to the secondary winding kept open circuited, which means current in the secondary is zero.
2. When primary winding is excited at its rated voltage it draws a current I_m called magnetizing current which is 2 to 10% of the rated current. This generates the magnetic flux in the core by primary mmf N_1I_m
3. As the transformer is ideal, the core loss and cu loss are zero. And the net current taken is to create the mmf or flux of alternating nature.
4. This alternating flux induces the emf's E_1 and E_2 in the coils which lags the flux by 90^0
5. The I_m is inphase to the flux and the applied voltage leads to the I_m by 90^0 being the coil with pure inductive type.
6. Hence, emf's E_1 and E_2 in the coils are inphase to each other and lags the flux by 90^0



Ideal Transformer at No-Load

<u>Transformer at No-Load:</u>

1. The transformer in the practical case draws an additional current I_w to the magnetizing current I_m and total current from the supply mains is I_0 which lags to the applied voltage by an angle Φ_0
2. There are two components of the current in I_0 namely
   i. Active (or) power (or) Watt full component of the current I_w which is in phase to the voltage, and generates the core loss in the transformer
   ii. Reactive (or) Watt less (or) magnetizing component of the current I_m which lags to the voltage by 90^0, and magnetizes the core in the transformer
3. Also, the no-load input power of the transformer is the iron loss (since the cu loss are small at no-load)
4. The no load angle (Φ_0) depends upon the losses in the transformer and is nearly equal to 90^0. So that the power factor is very low and varies from 0.1 to 0.15 lagging.



6.

Working component $I_w = I_0 \cos\varphi_0$

No load current $I_0 = \sqrt{I_w^2 + I_m^2}$

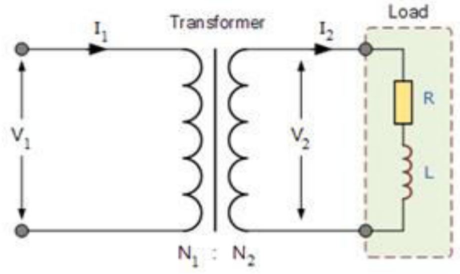
Magnetizing component $I_m = I_0 \sin\varphi_0$

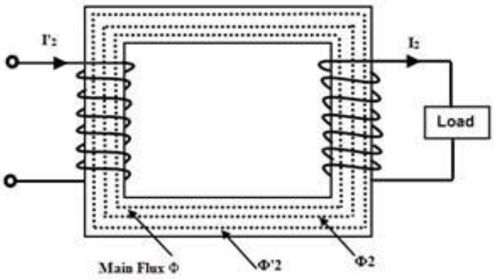
Power factor $\cos\varphi_0 = \frac{I_w}{I_0}$

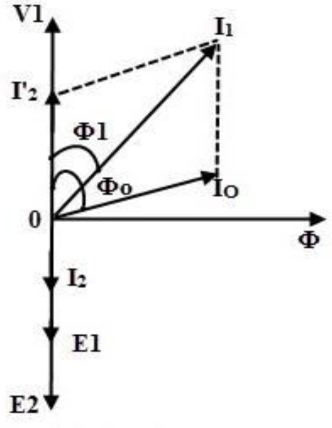
No load power input $P_0 = V_1 I_0 \cos\varphi_0$

6. Explain the operation of Transformer on Load *without leakage impedances* of the coils.

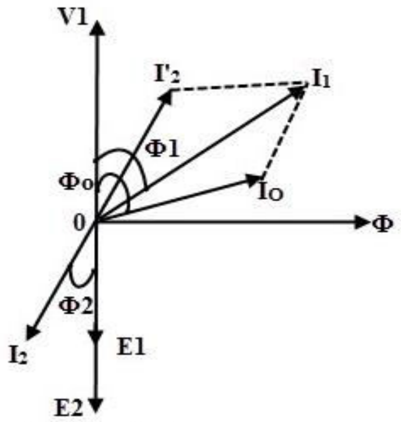
1. When an electrical load is connected to the secondary winding of a transformer a current flows in the secondary winding.
2. This secondary current is due to the induced secondary voltage, set up by the magnetic flux Φ in the core from the primary current(I_0) and the main flux direction is from primary coil to secondary coil (clockwise)



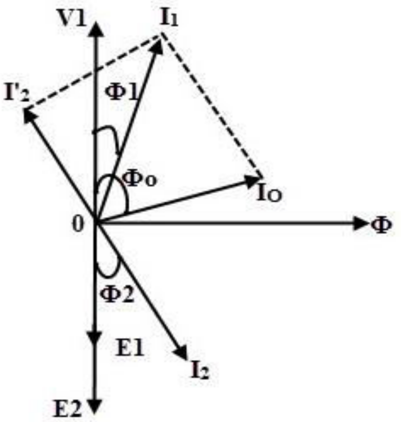




Resistive Load



Inductive Load



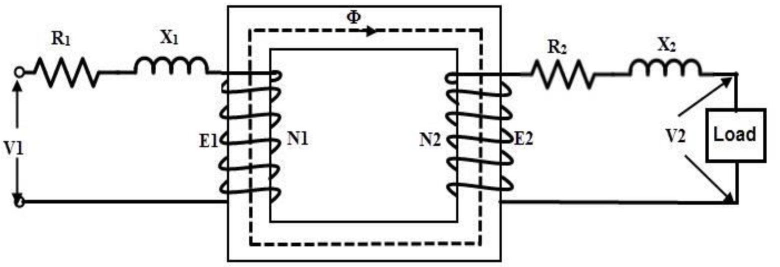
Capacitive Load

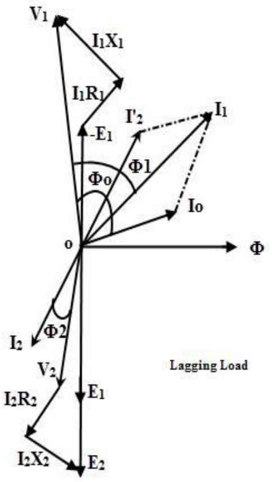
3. The secondary current, I_2 which is determined by the characteristics of the load, creates an secondary or load mmf (N_2I_2) and a secondary magnetic field, Φ_2 is established in the transformer core which flows in the exact opposite direction to the main primary field, Φ_1. i.e Φ_2 is in anti clock wise.
4. These two magnetic fields oppose each other resulting in a combined magnetic field of less magnetic strength than the single field produced by the primary winding alone when the secondary circuit was open circuited.

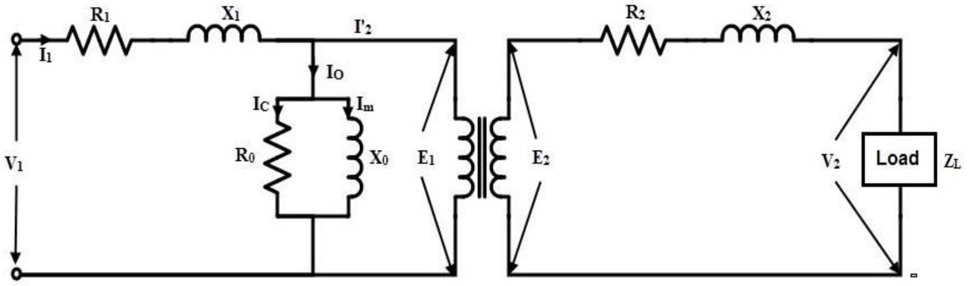
5. This in turn decreases the primary induced emf and leads to the increase in primary current $I_1=I_0+I_2^1$.
6. This additional I_2^1 current is called load component current in the primary and will be in such a way to balance the load mmf by this mmf on the primary
   i.e $N_2I_2=N_1I_2^1$ therefore $I_2^1= I_2K$ where, $K = N_2/N_1$
7. This $N_1I_2^1$ will produce a flux Φ_2^1 equal and opposite to Φ_2. These fluxes will now be cancelled and the net flux in the core will be Φ_1 even under the loading conditions.
8. For lagging load: $I_1^2 = I_0^2 + (I_2^1)^2 + 2I_0I_2^1\cos(\Phi_0\sim \Phi_2)$
9. As the flux remains constant from no-load to load, the iron loss will be same from no-load to load.

7. Explain the operation of transformer on load with *leakage impedances* of the coils

1. Below figure shows the schematic diagram, equivalent circuit and phasor diagram of the transformer with the leakage impedances of the coils.







Let,

R_1=Resistance of primary coil in Ω *R_2=Resistance of secondary coil in Ω*
X_1=Reactance of primary coil in Ω *X_2=Reactance of secondary coil in Ω*
Z_1=impedance of primary coil in Ω *Z_2=impedance of secondary coil in Ω*
E_1=emf induced in primary coil *E_2=emf induced in secondary coil*
V_1=applied voltage to primary coil *V_2= Load or terminal voltage of transformer*
$I_1Z_1 = I_1(R_1+jX_1)$ = Primary leakage impedance drop
$I_2Z_2 = I_2(R_2+jX_2)$ = Secondary leakage impedance drop

The magnetic core of the transformer is electrically represented with the parallel combination of R_0 and X_0 carrying the currents of I_w and I_m respectively and is placed across the primary coil.

Currents Analysis of the transformer in equivalent circuit

Currents in the transformer at No-load:

$$I_w = \frac{V_1}{R_0} \qquad I_m = \frac{V_1}{X_0} \qquad I_0^2 = I_w^2 + I_m^2 \qquad I_0 = \sqrt{I_w^2 + I_m^2} \qquad \phi_0 = \tan^{-1}\left(\frac{I_m}{I_w}\right)$$

Currents in the transformer with load

$I_1 = (I_0\angle-\phi_0) + (I_2^1\angle\pm\phi_2)$ *Where* $I_2^1 = I_2 \times K$ *and* $K = \frac{N_2}{N_1}$

$I_1 = (I_0\cos\phi_0 + I_2^1\cos\phi_2) + j(I_0\sin(-\phi_0) + I_2^1\sin(\pm\phi_2))$ - for lag and + for lead

Primary phase angle (Φ_1)

$\phi_1 = \tan^{-1}\left(\frac{I_0\sin(-\phi_0) + I_2^1\sin(\pm\phi_2)}{I_0\cos\phi_0 + I_2^1\cos\phi_2}\right)$ *and primary power factor is* $\cos\Phi_1$

Voltages Analysis of the transformer in equivalent circuit

Primary induced emf

$$\boxed{E_1 = (V_1\angle 0) - (I_1\angle\phi_1 * Z_1)}$$

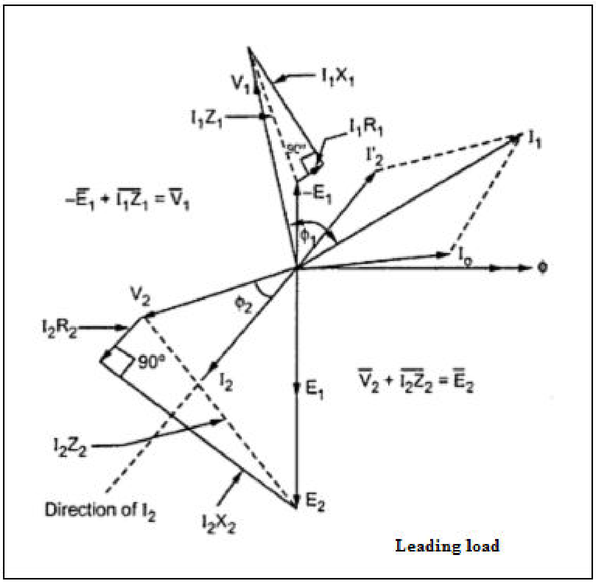
$$E_1 = (V_1 + j0) - \{(I_1\cos\phi_1 + jI_1\sin\phi_1)\times(R_1 + jX_1)\}$$

Using transformation ratio $E_2 = E_1 * K$

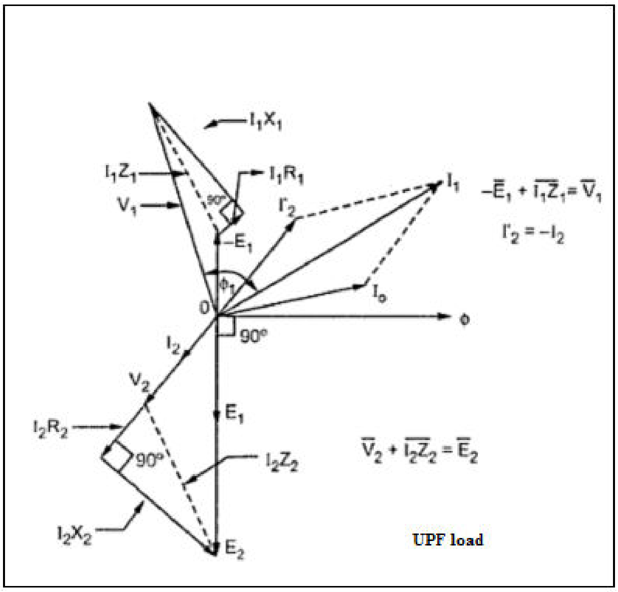
Knowing the E_2 and applying KVL to the secondary loop the load voltage is

$$\boxed{V_2 = E_2 - I_2 Z_2}$$

$$V_2 = E_2\angle\phi' - (I_2\angle\pm\phi_2)Z_2 \qquad V_2 = E_2\angle\phi' - (I_2\angle\pm\phi_2)(R_2 + jX_2)$$



Leading load



UPF load

8. Explain the *equivalent circuits* referred to both primary and secondary of the transformer

The equivalent circuit of the transformer referred to primary is shown in the below figure in which the winding parameters of the secondary are transformed and was referred to primary based on the voltage balancing principle before and after the transformation.

Secondary Resistance referred to primary:

$$R_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1}\times\frac{V_2 I_2}{V_2 I_2} = \times\frac{V_1 I_2}{V_2 I_1}\times\frac{V_2}{I_2} = \frac{R_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K}\right) also \frac{V_2}{I_2} = R_2$$

$\therefore R_2^1 = \frac{R_2}{K^2}$ Thus, it is the secondary resistance referred to primary

Secondary Reactance referred to primary:

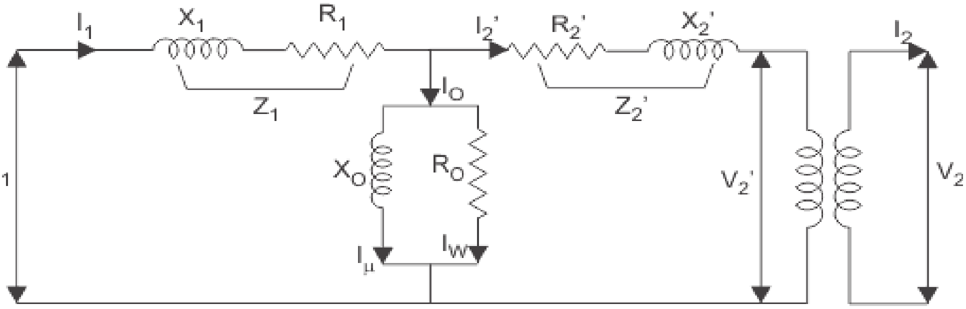
$$X_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1}\times\frac{V_2 I_2}{V_2 I_2} = \times\frac{V_1 I_2}{V_2 I_1}\times\frac{V_2}{I_2} = \frac{X_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K}\right) also \frac{V_2}{I_2} = X_2$$

$\therefore X_2^1 = \frac{X_2}{K^2}$ Thus, it is the secondary reactance referred to primary

Secondary Impedance referred to primary:

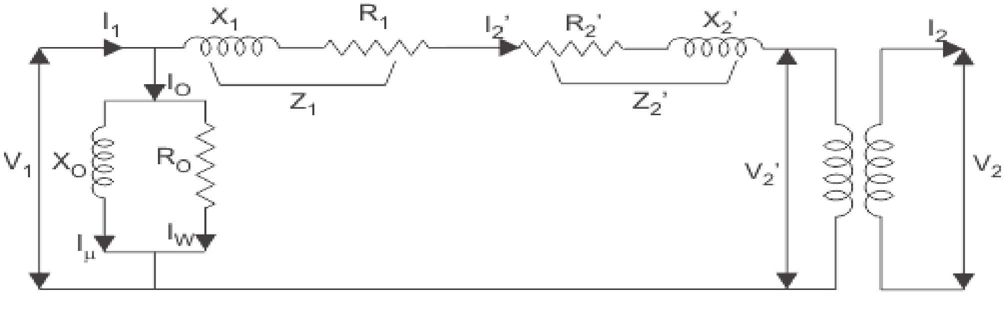
$$Z_2^1 = \frac{V_1}{I_1} = \frac{V_1}{I_1}\times\frac{V_2 I_2}{V_2 I_2} = \times\frac{V_1 I_2}{V_2 I_1}\times\frac{V_2}{I_2} = \frac{Z_2}{K^2} \qquad \left(\because \frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1}{K}\right) also \frac{V_2}{I_2} = Z_2$$

$\therefore Z_2^1 = \frac{Z_2}{K^2}$ Thus, it is the secondary impedance referred to primary



Equivalent Circuit of Transformer referred to Primary

To have simplified calculations the equivalent circuit is modified as bringing the core branch towards the supply voltage instead of having in between the primary and secondary parameters



simplified equivalent circuit of transformer reffered to primary

In this simplified circuit the total resistance, reactance and impedances referred to primary are

$$\therefore R_{eq1} = R_1 + R_2^1 = R_1 + \frac{R_2}{K^2} \qquad \therefore X_{eq1} = X_1 + X_2^1 = X_1 + \frac{X_2}{K^2}$$

$$\therefore Z_{eq1} = Z_1 + Z_2^1 = Z_1 + \frac{Z_2}{K^2}$$

Similarly, the equivalent circuit referred to secondary of the transformer is shown below with their formulas

Primary Resistance referred to secondary:

$$R_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 R_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = R_1$$

$\therefore R_1^1 = R_1 K^2$ Thus, it is the primary resistance referred to secondary

Primary Reactance referred to secondary:

$$X_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 X_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = X_1$$

$\therefore X_1^1 = X_1 K^2$ Thus, it is the primary reactance referred to secondary

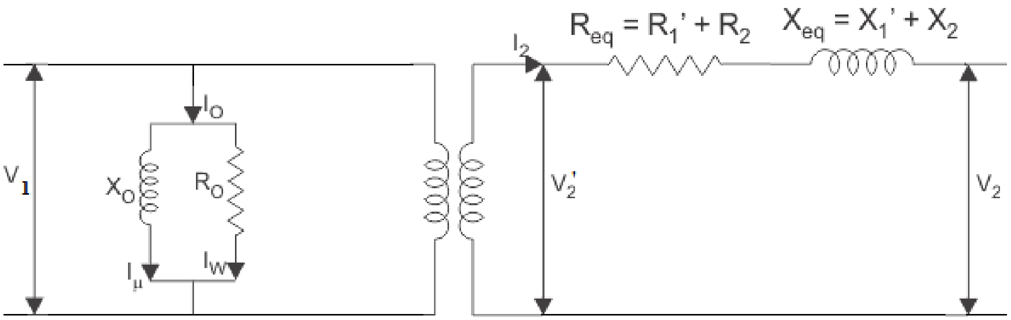
Primary Impedance referred to secondary:

$$Z_1^1 = \frac{V_2}{I_2} = \frac{V_2}{I_2} \times \frac{V_1 I_1}{V_1 I_1} = \times \frac{V_2 I_1}{V_1 I_2} \times \frac{V_1}{I_1} = K^2 Z_1 \qquad \left(\because \frac{V_2}{V_1} = \frac{I_1}{I_2} = K \right) also \frac{V_1}{I_1} = Z_1$$

$\therefore Z_1^1 = Z_1 K^2$ Thus, it is the primary impedance referred to secondary

$$\therefore R_{eq2} = R_2 + R_1^1 = R_2 + R_1 K^2 \qquad \therefore X_{eq2} = X_2 + X_1^1 = X_2 + X_1 K^2$$

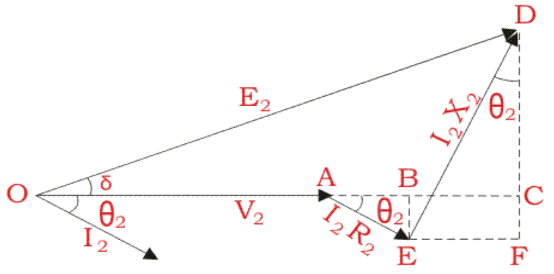
$$\therefore Z_{eq2} = Z_2 + Z_1^1 = Z_2 + Z_1 K^2$$



Approximate Equivalent Circuit of Transformer referred to Secondary

9. Derive the expression for *voltage regulation and efficiency* of the transformer

Definition of voltage regulation : Voltage regulation is defined as the percentage change in the output voltage from no-load to full-load expressed in full load voltage.



$OC = OA + AB + BC$

$Here,\ OA = V_2$

$Here,\ AB = AE\cos\theta_2 = I_2R_2\cos\theta_2$

$and,\ BC = DE\sin\theta_2 = I_2X_2\sin\theta_2$

Derivation of voltage regulation for the lagging power factor load,

assuming the angle between OC and OD as very small, and neglected it, OD is nearly equal to OC ($E_2 > V_2$)

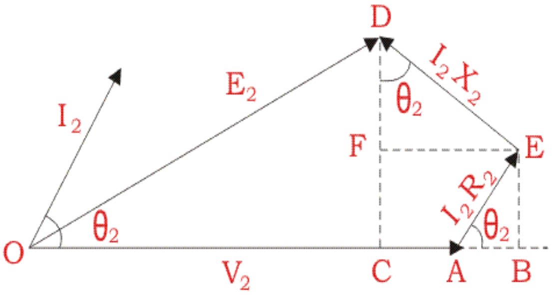
$$E_2 = OC = OA + AB + BC, \quad E_2 = OC = V_2 + I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi$$

Thus, the % voltage regulation is

$$\frac{E_2 - V_2}{V_2} * 100 = \frac{V_2 + I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi - V_2}{\mathrm{V}_2} \times 100 = \frac{I_2 R_{eq2} \cos\phi + I_2 X_{eq2} \sin\phi}{\mathrm{V}_2} \times 100$$

Derivation of voltage regulation for the leading power factor load,

Similarly, from the phasor diagram of the leading pf load , ($E_2 < V_2$)



Here

$EF = DE \sin\theta = I_2X_2 \sin\theta$

$AB = AE \cos\theta = I_2R_2 \cos\theta$

$OA = V_2$ and $OD = E_2$

assuming the angle between OA and OD as very small, and neglected it, OD is nearly equal to OC ($E_2 < V_2$)

$V_2 - E_2 = OA - OC = CA = CB - AB$, thus $V_2 = E_2 + CB - AB$

Thus, the % voltage regulation is

$$\frac{E_2 - V_2}{V_2} * 100 = \frac{E_2 - \mathrm{E}_2 - \mathrm{CB} + \mathrm{AB}}{\mathrm{V}_2} \times 100 = \frac{I_2 R_{eq2} \cos\phi - I_2 X_{eq2} \sin\phi}{\mathrm{V}_2} \times 100$$

Therefore,

$$\%regulation = \frac{I_2 R_{eq2} \cos\phi \pm I_2 X_{eq2} \sin\phi}{\mathrm{V}_2} \times 100$$ (+) for lagging pf and (-) for leading pf

10. Discuss the losses and efficiency in the transformer

Transformer is a static device, i.e. it doesn’t have any parts, so no mechanical losses exist in the transformer and only electrical losses are observed.

So there are two primary types of losses in the transformer:

1. Copper losses
2. Iron losses

Other than these, some small amount of power losses in the form of ‘stray losses’ are also observed, which are produced due to the leakage of magnetic flux.

Copper losses

1. These losses occur in the windings of the transformer when heat is dissipated due to the current passing through the windings and the internal resistance offered by the windings.
2. So these are also known as ohmic losses or I^2R losses, where 'I' is the current passing through the windings and R is the internal resistance of the windings.
3. These losses are present both in the primary and secondary windings of the transformer and depend upon the load attached across the secondary windings since the current varies with the variation in the load, so these are *variable losses.*

Iron losses or Core Losses

1. These losses occur in the core of the transformer and are generated due to the variations in the flux.
2. These losses depend upon the magnetic properties of the materials which are present in the core, so they are also known as iron losses, as the core of the Transformer is made up of iron. And since they do not change like the load, so these losses are also *constant losses.*

There are two types of Iron losses in the transformer:

1. Eddy Current losses
2. Hysteresis Loss

Eddy Current Losses

1. When an alternating current is supplied to the primary windings of the transformer, it generates an alternating magnetic flux in the winding which is then induced in the secondary winding also through Faraday's law of electromagnetic induction, and is then transferred to the externally connected load.
2. During this process, the other conduction materials of which the core is composed of; also gets linked with this flux and an emf is induced.
3. But this magnetic flux does not contribute anything towards the externally connected load or the output power and is dissipated in the form of heat energy.
4. So such losses are called Eddy Current losses and are mathematically expressed as:

$$P_e = K_e\, f^2\, K_f^2\, B_m^2$$

Where;

K_e = Constant of Eddy Current

K_f^2 = Form Constant

B_m = Strength of Magnetic Field

Hysteresis Loss

1. Hysteresis loss is defined as the electrical energy which is required to realign the domains of the ferromagnetic material which is present in the core of the transformer.
2. These domains loose their alignment when an alternating current is supplied to the primary windings of the transformer and the emf is induced in the ferromagnetic material of the core which disturbs the alignment of the domains and afterwards they do not realign properly.
3. For their proper realignment, some external energy supply, usually in the form of current is required. This extra energy is known as Hysteresis loss.

   Mathematically, they can be defined as;

$$P_h = K_h\, B_m^{1.6}\, f\, V$$

- The **Efficiency** of the transformer is defined as the ratio of power output to the input power.

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{\text{output power}}{\text{output power + losses}}$$

$$\eta = \frac{\text{output power}}{\text{output power + iron losses + copper losses}}$$

$$\eta = \frac{V_2 I_2 \text{Cos}\varphi_2}{V_2 I_2 \text{Cos}\varphi_2 + P_i + P_c}$$

Where,

V_2 = Secondary terminal voltage

I_2 = Full load secondary current in A

Cosϕ_2 = power factor of the load

P_i = Iron losses = hysteresis losses + eddy current loss

Pc = Full load copper losses = $I_2^2 R_{eq}$

Also, the efficiency at any amount of load(x) is given by

$$\eta = \frac{outputinwatts}{inputinwatts} = \frac{xVA\cos\phi}{xVA\cos\phi + W_i + x^2 W_{FLCu}} \times 100$$

Condition for maximum efficiency in the transformer:

$$\eta = \frac{outputinwatts}{inputinwatts} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + W_i + I_2^2 r_{e2}} = \frac{1}{1 + \frac{W_i}{V_2 I_2 \cos\phi} + \frac{I_2^2 r_{e2}}{V_2 I_2 \cos\phi}} = \frac{1}{1 + \frac{W_i}{V_2 I_2 \cos\phi} + \frac{I_2 r_{e2}}{V_2 \cos\phi}}$$

To get the maximum efficiency the denominator must be small, therefore condition to be the denominator minimum is

$$\frac{d\left(1 + \frac{W_i}{V_2 I_2 \cos\phi} + \frac{I_2 r_{e2}}{V_2 \cos\phi}\right)}{dI_2} = 0$$

$$\frac{d\left(1 + \frac{W_i}{V_2 I_2 \cos\phi} + \frac{I_2 r_{e2}}{V_2 \cos\phi}\right)}{dI_2} = 0 + \left((-)\frac{W_i}{V_2 I_2^2 \cos\phi}\right) + \left(\frac{r_{e2}}{V_2 \cos\phi}\right) = 0$$

$$\frac{r_{e2}}{V_2\cos\phi}=\frac{W_i}{V_2 I_2^2\cos\phi} \qquad r_{e2}=\frac{W_i}{I_2^2} \qquad I_2^2 r_{e2}=W_i$$

Therefore the condition for obtaining the maximum efficiency is the variable loss $\left(I_2^2 r_{e2}\right)$ must be equal to the constant loss W_i.

Also, the load current at which the maximum efficiency occurs is $I_{2\max}=\sqrt{\left(\frac{W_i}{r_{e2}}\right)}$

Multiplying both sides with 1000 $*V_2$

$$1000*\mathrm{V}_2*I_{2\max}=1000*\mathrm{V}_2*\sqrt{\left(\frac{W_i}{r_{e2}}\right)} \qquad \text{Load KVA}_{\max}=1000*\mathrm{V}_2*\sqrt{\left(\frac{W_i}{r_{e2}}\right)}$$

$$\text{Load KVA}_{\max}=1000*\mathrm{V}_2*\frac{\mathrm{I}_{2\text{Fullload}}}{\mathrm{I}_{2\text{Fullload}}}\sqrt{\left(\frac{W_i}{r_{e2}}\right)}$$

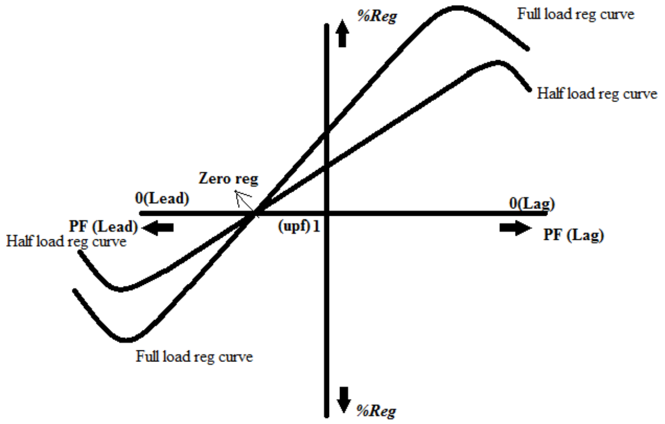
$$\text{Load KVA}_{\max}=1000*\mathrm{V}_2*\mathrm{I}_{2\text{Fullload}}\sqrt{\left(\frac{W_i}{\mathrm{I}^2_{2\text{Fullload}}r_{e2}}\right)}$$

$$\text{Load KVA}_{\max}=\text{Full load KVA}\sqrt{\left(\frac{W_i}{\mathrm{I}^2_{2\text{Fullload}}r_{e2}}\right)}$$

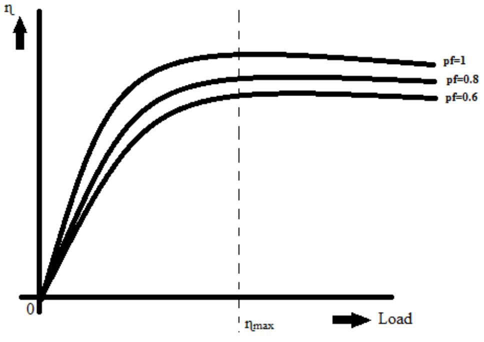
$$\text{The Load KVA at which maximum efficiency}=\text{Full load KVA}\sqrt{\left(\frac{W_i}{\mathrm{W}_{\text{cuFullload}}}\right)}$$

$$\boxed{\text{The Load KVA at which maximum efficiency}=\text{Full load KVA}\sqrt{\frac{W_i}{\mathrm{W}_{\text{cuFullload}}}}}$$

Variation of voltage regulation and efficiency with respect to load and load powerfactors



Regulation curves w.r.t load pf and amount of loads



Varaiation of efficiency with respect to load and load power factor

11. Explain OC and SC tests on a single phase transformer

Ans: Purpose of conducting OC and SC tests is to find

i) Equivalent circuit parameters ii) Efficiency iii) Regulation

Open Circuit Test:

1. The OC test is performed on LV side at rated voltage and HV side is kept opened.
2. As the test is conducted on LV side the meters selected will be at low range values like smaller voltmeter, smaller ammeter and low pf wattmeter
3. As the no-load current is quite small about 2 to 5% of the rated current, the ammeter required here will be smaller range even after on LV side which are designed for higher current values.
4. The voltmeter, ammeter and the wattmeter readings V_0, I_0 and W_0 respectively are noted by applying rated voltage on LV side.
5. The wattmeter will record the core loss because of noload input power.

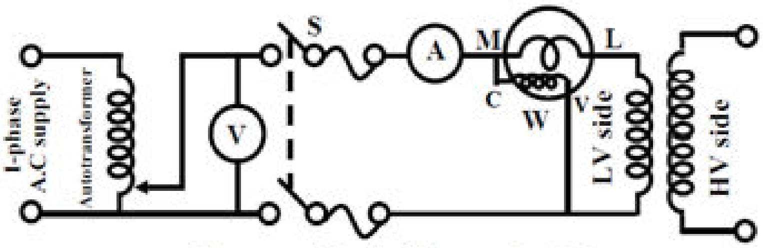


Figure : Circuit diagram for O.C test

Calculations from OC test readings:

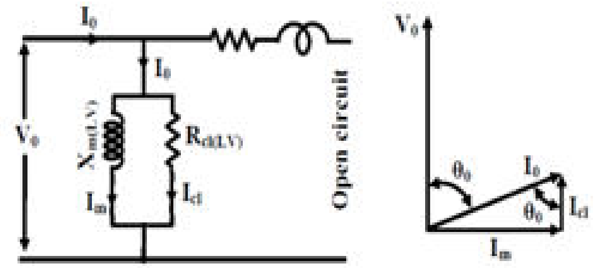
R_0, X_0 and Iron loss are calculated from the OC test results as

Core resistance $R_0 = \frac{V_0}{I_w} = \frac{V_0}{I_0 \cos\phi_0}$

Magnetizing reactance $X_0 = \frac{V_0}{I_m} = \frac{V_0}{I_0 \sin\phi_0}$

Where $\cos\phi_0 = \frac{P_0}{V_0 I_0}$

and iron loss $W_i = P_0$ (No load input power)



(a) Equivalent circuit under O.C test (b) phasor diagram

Figure 2.2: Equivalent circuit & phasor diagram during O.C test

Short Circuit Test:

1. The SC test is performed on HV side at rated current and LV side is kept Shorted.
2. As the test is conducted on HV side the meters selected will be at low range values like smaller voltmeter, smaller ammeter and unity pf wattmeter

3. As the voltage required to circulate the short circuit rated current is very small about 10 to 15% of the rated HV voltage, so the voltmeter required here will be smaller range even the test is conducted on HV side.
4. The voltmeter, ammeter and the wattmeter readings V_{sc}, I_{sc} and W_{sc} respectively are noted by passing rated current on HV side.
5. The wattmeter will record thecopper loss corresponding to the I_{sc}.

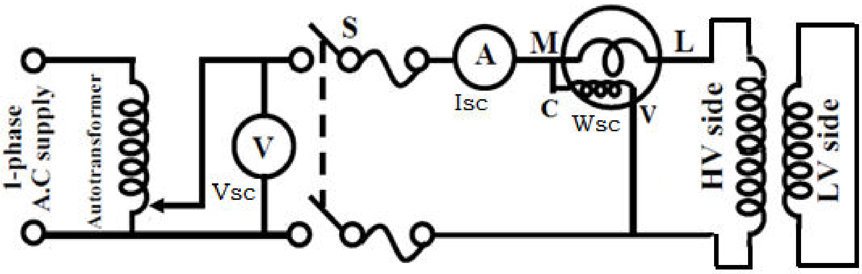


Figure 2.3 **: Circuit diagram for S.C test**

Calculations from SC test readings:

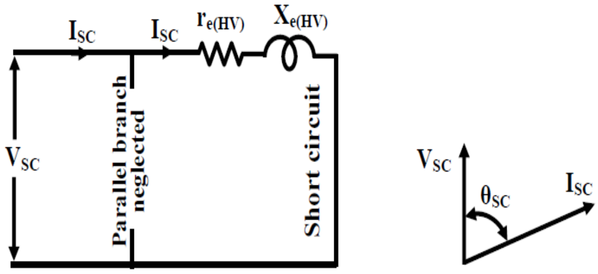
$r_{e(HV)}$, $x_{e(HV)}$ and cu loss are calculated from the SC test results as

Equivalent resistance referred to HV side is

$$R_{sc} = \frac{P_{sc}}{I_{sc}^2} = r_{e(HV)}$$

Equivalent impedance referred to HV side is

$$Z_{sc} = \frac{V_{sc}}{I_{sc}} = z_{e(HV)}$$



(a) Equivalent circuit under S.C test (b) phasor diagram

Figure 2.4: Equivalent circuit & phasor diagram during S.C test

Equivalent reactance referred to HV side is $X_{sc} = \sqrt{Z_{sc}^2 - R_{sc}^2} = x_{e(HV)}$

The culoss is equal to the wattmeter reading W_{sc}

- Thus, the approximate equivalent circuit of the transformer can be drawn by the calculated values of R_0 and X_0 on LV side and $r_{e(HV)}$ and $x_{e(HV)}$ *on* HV side.
- The efficiency at any load is calculated from the losses W_i and W_{cufl} as

$$\eta_x = \frac{xVA\cos\phi}{xVA\cos\phi + W_i + x^2 W_{FLCu}} \times 100$$

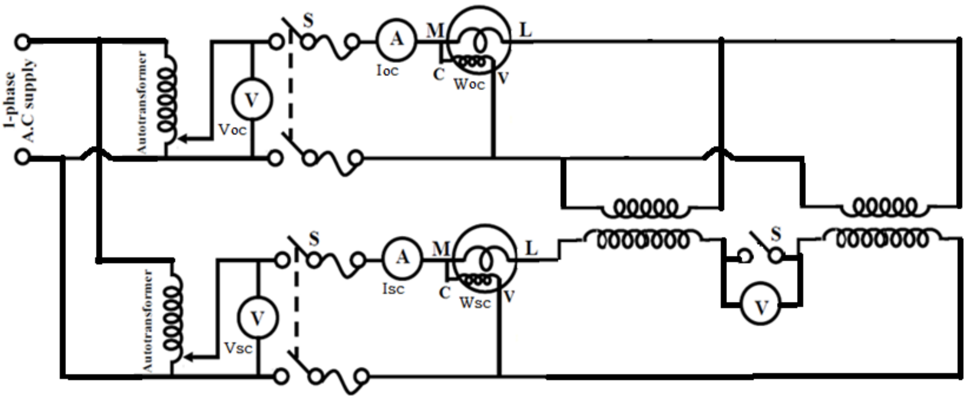
The regulation of the transformer is calculated from the $r_{e(HV)}$ and $x_{e(HV)}$ as

$$\%reg = \frac{I_{HV} r_{eHV} \cos\phi \pm I_{HV} x_{eHV} \sin\phi}{\mathrm{V_{HV}}} \times 100 \text{ where } + \text{is for lagging pf and} - \text{is for leading pf}$$

12. Explain Sumpner's test or back to back test

Ans: Purpose of Sumpner’s test or back to back test on transformer is to determine efficiency, voltage regulation considering the **heating under loaded** conditions.

1. Two identical transformers are required to conduct the Sumpner's test
2. Both transformers are connected to supply such that one transformer is loaded on another.
3. Both Primaries are connected in parallel and both secondaries are connected in series opposition which is checked by the voltmeter showing zero volts when the switch S is closed.

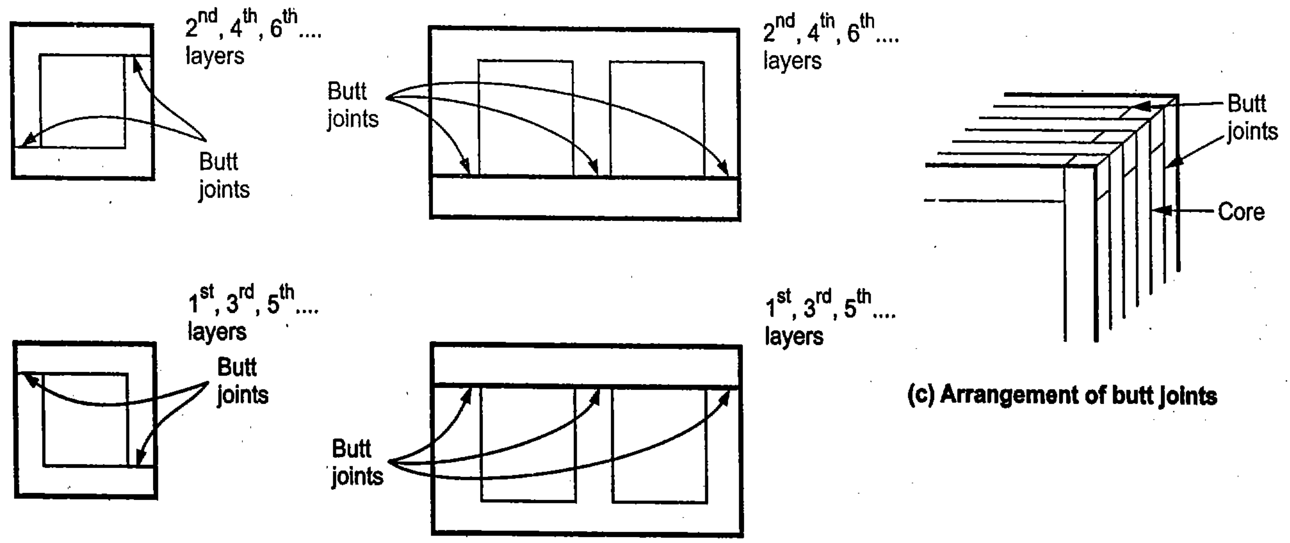


Procedure for sumpner’s test:

1. Both the emf's cancel each other, as transformers are identical. In this case, as per superposition theorem, no current flows through secondary. And thus the no load test is simulated.
2. The current drawn from V_{oc} is $2I_0=I_{oc}$ and the input power measured by wattmeter W_{oc} is equal to iron losses of both transformers. i.e. iron loss per transformer $P_i = W_{oc}/2$.
3. Now, a small voltage V_{sc} is injected into secondary with the help of a low voltage transformer.
4. The voltage V_{sc} is adjusted so that, the rated current I_{sc} flows through the secondary. In this case, both primaries and secondary’s carry rated current.
5. Thus short circuit test is simulated and wattmeter W_{sc} shows total full load copper losses of both transformers. i.e. copper loss per transformer $P_{Cu} = W_{sc}/2$.
6. From above test results, the full load efficiency of each transformer is calculated and is given as

$$\%\eta = \frac{xVA\cos\phi}{xVA\cos\phi + \frac{W_{oc}}{2} + x^2\frac{W_{sc}}{2}} \times 100$$

2-1 Civil BEEE UNIT-3



(c) Arrangement of butt joints

(a) Core type

(b) Shell type

Fig. 4.3.3 Staggering in transformer

The **advantages of staggering** in transformer are,

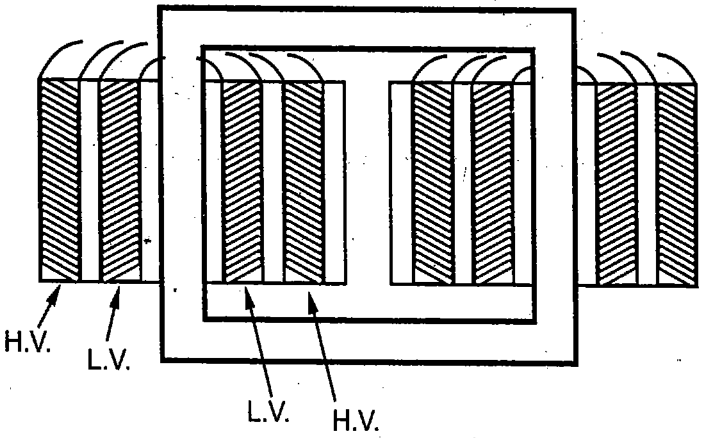
1. It avoids continuous air gap.
2. The reluctance of magnetic circuit gets reduced.
3. The continuous air gap reduces the mechanical strength of the core. The staggering helps to increase the mechanical strength of the core.

In large transformers, the core is corrugated. The tank walls are corrugated means having folds and ridges. It has following advantages.

1. It allows changes in the effective volume of the oil with temperature. It helps in accomodating expansion and contraction of oil.
2. It helps to dissipate the losses effectively, by increasing the surface area of the tank. It provides large heat radiation area.

4.3.2 Types of Windings

The coils used are wound on the limbs and are insulated from each other. In the basic transformer shown in the Fig. 4.2.1, the two windings wound are shown on two different limbs i.e. primary on one limb while secondary on other limb. But due to this leakage flux increases which affects the transformer



H.V. = High Voltage
L.V. = Low Voltage
Both windings on same limb to avoid leakage flux

Fig. 4.3.4 (a) Cylindrical concentric coils

performance badly. Similarly it is necessary that the windings should be very close to each other to have high mutual inductance. To achieve this, the two windings are split into number of coils and are wound adjacent to each other on the same limb. A very common arrangement is cylindrical concentric coils as shown in the Fig. 4.3.4 (a).

Such cylindrical coils are used in the core type transformer. These coils are mechanically strong. These are wound in the helical layers. The different layers are insulated from each other by paper, cloth or mica. The low voltage winding is placed near the core from ease of insulating it from the core. The high voltage is placed after it.

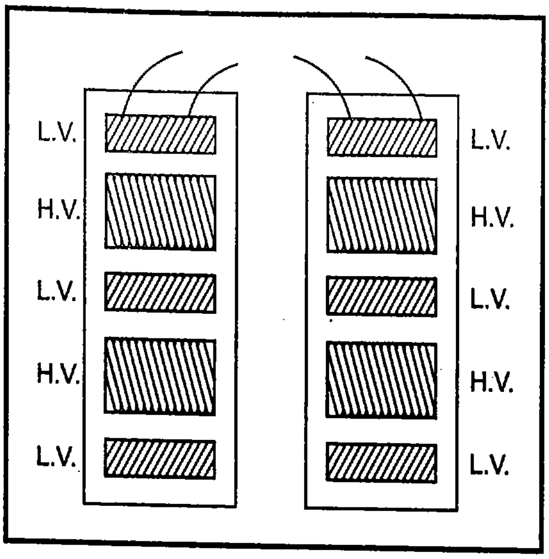


Fig. 4.3.4 (b) Sandwich coils

The other type of coils which is very commonly used for the shell type of transformer is **sandwich coils.** Each high voltage portion lies between the two low voltage portion sandwiching the high voltage portion. Such subdivision of windings into small portions reduces the leakage flux. Higher the degree of subdivision, smaller is the reactance. The **sandwich coil** is shown in the Fig. 4.3.4 (b). The top and bottom coils are low voltage coils. All the portions are insulated from each other by paper.

Review Questions

1. *Discuss the constructional features of transformers. Draw neat diagrams.*

   JNTU : May-03, 04, 05, Nov.-03, 05, Dec.-04, March-06, Marks 8

2. *What are the different parts of a transformer and explain their functions clearly.*

   JNTU : Nov.-06, Marks 16

4.4 Construction of Single Phase Transformers

JNTU : Nov.-05, 08

The various constructions used for the single phase transformers are,

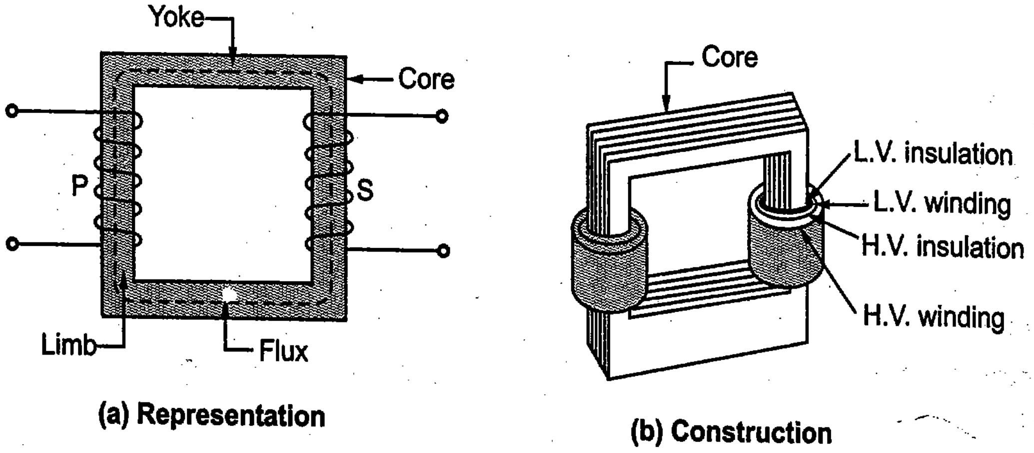
1. Core type 2. Shell type and 3. Berry type

4.4.1 Core Type Transformer

It has a single magnetic circuit. The core is rectangular having two limbs. The winding encircles the core. The coils used are of cylindrical type. As mentioned earlier, the coils are wound in helical layers with different layers insulated from each other by paper or mica. Both the coils are placed on both the limbs. The low voltage coil is placed inside near the core while high voltage coil surrounds the low voltage coil. Core is made up of large number of thin laminations.

As the windings are uniformly distributed over the two limbs, the natural cooling is more effective. The coils can be easily removed by removing the laminations of the top yoke, for maintenance.

The Fig. 4.4.1 (a) shows the schematic representation of the core type transformer while the Fig. 4.4.1 (b) shows the view of actual construction of the core type transformer.



(a) Representation (b) Construction

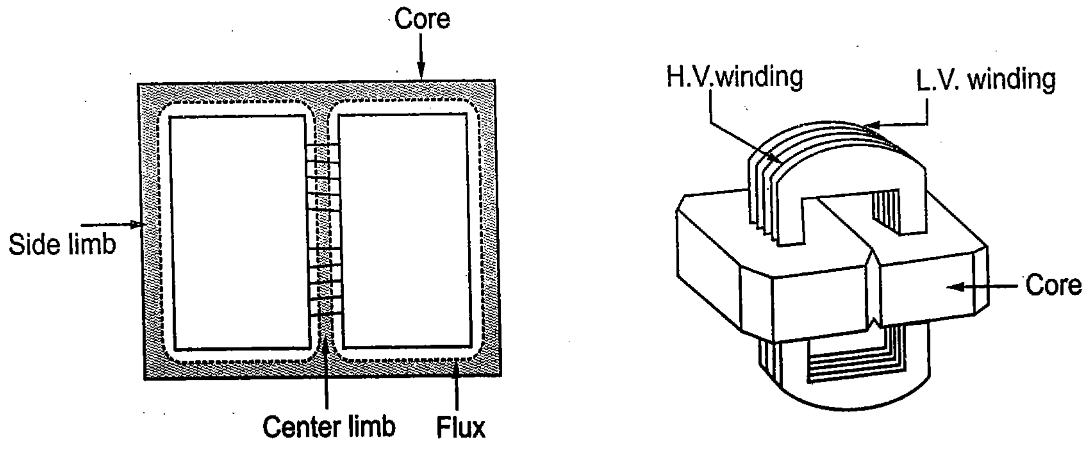
Fig. 4.4.1 Core type transformer

4.4.2 Shell Type Transformer

It has a double magnetic circuit. The core has three limbs. Both the windings are placed on the central limb. The core encircles most part of the windings. The coils used are generally multilayer disc type or sandwich coils. As mentioned earlier, each high voltage coil is in between two low voltage coils and low voltage coils are nearest to top and bottom of the yokes.

The core is laminated. While arranging the laminations of the core, the care is taken that all the joints at alternate layers are staggered. This is done to avoid narrow air gap at the joint, right through the cross-section of the core. Such joints are called over lapped or imbricated joints. Generally for very high voltage transformers, the shell type construction is preferred. As the windings are surrounded by the core, the natural cooling does not exist. For removing any winding for maintenance, large number of laminations are required to be removed.

The Fig. 4.4.2 (a) shows the schematic representation while the Fig. 4.4.2 (b) shows the outaway view of the construction of the shell type transformer.



(a) Representation **(b) Construction**

Fig. 4.4.2 Shell type transformer

4.4.3 Berry Type Transformer

This has distributed magnetic circuit. The number of independent magnetic circuits are more than 2. Its core construction is like spokes of a wheel. Otherwise it is symmetrical to that of shell type.

Diagramatically it can be shown as in the Fig. 4.4.3.

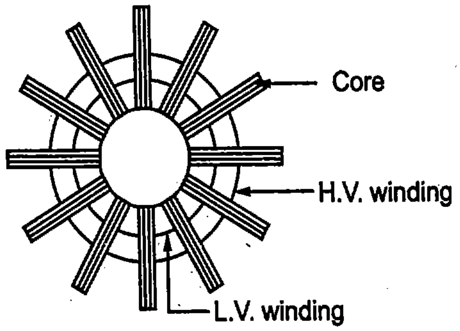


Fig. 4.4.3 Berry type transformer

Review Question

1. *Explain the constructional details of core and shell type transformers.*

JNTU : Nov.-05, 08, Marks 8

4.5 Comparison of Core and Shell Type Transformers

JNTU : Nov.-04, 07, May-05

The comparison of core type and shell type transformers is given in the Table 4.5.1.

Sr. No.	Core type	Shell type
1.	The winding encircles the core.	The core encircles most part of the windings.
2.	The cylindrical type of coils are used.	Generally, multilayer disc type or sandwich coils are used.
3.	As windings are distributed, the natural cooling is more effective.	As windings are surrounded by the core, the natural cooling does not exist.
4.	The coils can be easily removed from maintenance point of view.	For removing any winding for the maintenance, large number of laminations are required to be removed. This is difficult.
5.	The construction is preferred for low voltage transformers.	The construction is used for very high voltage transformers.

6.	It has a single magnetic circuit.	It has a double magnetic circuit.
7.	In a single phase type, the core has two limbs.	In a single phase type, the core has three limbs.

Table 4.5.1

Review Question

1. *Distinguish between core type and shell type transformers.*

JNTU : Nov.-04, 07, May-05, Marks 6

4.6 E.M.F. Equation of a Transformer

JNTU : May-05, 08, 13, Dec.-03, Nov.-04, 05, 08, 12, March-05, 06

When the primary winding is excited by an alternating voltage V_1, it circulates alternating current, producing an alternating flux ϕ. The primary winding has N_1 number of turns. The alternating flux ϕ linking with the primary winding itself induces an e.m.f. in it denoted as E_1. The flux links with secondary winding through the common magnetic core. It produces induced e.m.f. E_2 in the secondary winding. This is mutually induced e.m.f. Let us derive the equations for E_1 and E_2.

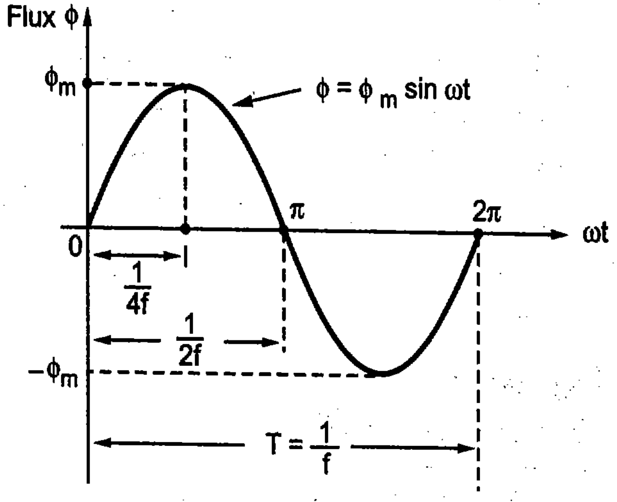


Fig. 4.6.1 Sinusoidal flux

The primary winding is excited by purely sinusoidal alternating voltage. Hence the flux produced is also sinusoidal in nature having maximum value of ϕ_m as shown in the Fig. 4.6.1.

The various quantities which affect the magnitude of the induced e.m.f. are :

ϕ = Flux, and ϕ_m = Maximum value of flux

N_1 = Number of primary winding turns, N_2 = Number of secondary winding turns

f = Frequency of the supply voltage

E_1 = R.M.S. value of the primary induced e.m.f.

E_2 = R.M.S. value of the secondary induced e.m.f.

From Faraday's law of electromagnetic induction the average e.m.f. induced in each turn is proportional to the average rate of change of flux.

$\therefore$ Average e.m.f. per turn = Average rate of change of flux

$$\therefore \quad \phi_m = B_m \times A = 6 \times 36 \times 10^{-4} = 0.0216 \text{ Wb}$$

$$E_1 = 4.44\,\phi_m f N_1 \quad \text{i.e.} \quad 2200 = 4.44 \times 0.0216 \times 50 \times N_1$$

$$\therefore \quad N_1 = 458.792 \approx \mathbf{459}$$

$$\frac{N_1}{N_2} = \frac{E_1}{E_2} \quad \text{i.e.} \quad N_2 = \frac{N_1 \times E_2}{E_1} = 52.13 \approx \mathbf{52}$$

Review Questions

1. *Derive the e.m.f. equation of a 1-phase transformer.*

   JNTU : May-05, 08, March-06, Nov.-04, 05, Marks 8

2. *What is an ideal transformer ?* **JNTU : Nov.-08, May-08, Marks 4**
3. *What is kVA rating of a transformer ?*
4. *A single phase transformer has 480 turns on primary and 90 turns on the secondary. The mean length of flux path in the core is 1.8 m and joints are equivalent to an air gap of 1 mm. The maximum value of the flux density is to be 1.1 T when a potential difference of 2200 volts at 50 Hz is applied to the primary. Assume value of magnetic field strength corresponding to the flux density of 1.1 T in the core to be 400 A/m.*

   Calculate

   i) The cross-section area of the core ii) Maximum value of the magnetizing current

   iii) Secondary voltage on no load. **[Ans.: i) 0.01876 m^2 ii) 1.500 A iii) 412.5 volts]**

5. *A single phase transformer has 350 primary and 1050 secondary turns. The primary is connected to 400 V, 50 Hz a.c. supply. If the net cross sectional area of the core is 50 cm^2, calculate i) The maximum value of the flux density in the core ii) The induced e.m.f. in the secondary winding.* **[Ans.: B_m = 1.0296 Wb/m^2, E_2 = 1200 V]**
6. *A single phase transformer has 500 turns on primary and 1000 turns on secondary. The voltage per turn in the primary winding is 0.2 volts. Calculate,*

   i) Voltage induced in the primary winding ii) Voltage induced in the secondary winding

   iii) The maximum value of the flux density if the cross section area of the core is 200 cm^2

   iv) kVA rating of the transformer if the current in primary at full load is 10 A, the frequency is 50 Hz. **[Ans.: i) E_1 = 100 volts, ii) E_2 = 200 volts, iii) ϕ_m = 9.009 × 10^{-4} Wb, iv) B_m = 0.045 web/m^2 or Tesla]**

4.7 Ideal Transformer on No Load

JNTU : Nov.-04, 05, 06, 08, 12, May-05, 08, 13, March-06

Consider an ideal transformer on no load as shown in the Fig. 4.7.1. The supply voltage is V_1 and as it is an no load the secondary current $I_2 = 0$.

The primary draws a current I_1 which is just necessary to produce flux in the core. As it is magnetising the core, it is called **magnetising current** denoted as I_m. As the transformer is ideal, the winding resistance is zero and it is purely inductive in nature. The magnetising current I_m is very small and lags V_1 by 30° as the winding is purely inductive. This I_m produces an alternating flux ϕ which is in phase with I_m.

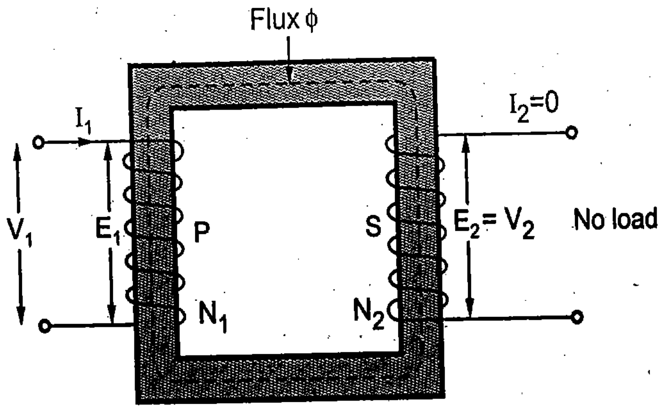


Fig. 4.7.1 Ideal transformer on no load

The flux links with both the winding producing the induced e.m.f.s E_1 and E_2, in the primary and secondary windings respectively. According to Lenz's law, the induced e.m.f. opposes the cause producing it which is supply voltage V_1. Hence E_1 is in antiphase with V_1 but equal in magnitude. The induced E_2 also opposes V_1 hence in antiphase with V_1 but its magnitude depends on N_2. Thus E_1 and E_2 are in phase.

The phasor diagram for the ideal transformer on no load is shown in the Fig. 4.7.2.

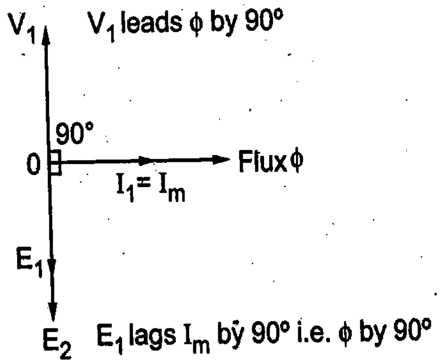


Fig. 4.7.2 Phasor diagram for ideal transformer on no load

It can be seen that flux ϕ is reference. I_m produces ϕ hence in phase with ϕ. V_1 leads I_m by 90° as winding is purely inductive so current has to lag voltage by 90°.

E_1 and E_2 are in phase and both opposing supply voltage V_1.

The power input to the transformer is $V_1 I_1 \cos (V_1 \wedge I_1)$ i.e. $V_1 I_m \cos (90°)$ i.e. zero. This is because on no load output power is zero and for ideal transformer there are no losses hence input power is also zero. Ideal no load p.f. of transformer is zero lagging.

4.7.1 Practical Transformer on No Load

Actually in practical transformer iron core causes hysteresis and eddy current losses as it is subjected to alternating flux. While designing the transformer the efforts are made to keep these losses minimum by,

1. Using high grade material as silicon steel to reduce hysteresis loss.
2. Manufacturing core in the form of laminations or stacks of thin laminations to reduce eddy current loss.

Apart from this there are iron losses in the practical transformer. Practically primary winding has certain resistance hence there are small primary copper loss present.

Thus the primary current under no load condition has to supply the iron losses i.e. hysteresis loss and eddy current loss and a small amount of primary copper loss. This current is denoted as I

8

Synchronous Machines

8.1 Introduction

It is known that the electric supply used, now a days for commercial as well as domestic purposes, is of alternating type.

Similar to d.c. machines, the a.c. machines associated with alternating voltages, are also classified as generators and motors.

The machines generating a.c. e.m.f. are called **alternators** or **synchronous generators**. While the machines accepting input from a.c. supply to produce mechanical output are called **synchronous motors**. Both these machines work at a specific constant speed called **synchronous speed** and hence in general called **synchronous machines**.

8.2 Difference between D.C. Generator and Alternator

It is seen that in case of a d.c. generator, basically the nature of the induced e.m.f. in the armature conductors is of alternating type. By using commutator and brush assembly it is converted to d.c. and made available to the external circuit. If commutator is dropped from a d.c. generator and induced e.m.f. is tapped outside from an armature directly, the nature of such e.m.f. will be alternating. Such a machine without commutator, providing an a.c. e.m.f. to the external circuit is called an **alternator**. The obvious question is how is it possible to collect an e.m.f. from the rotating armature without commutator ?

Key Point : *So the arrangement which is used to collect an induced e.m.f. from the* ***rotating armature*** *and make it available to the* ***stationary circuit*** *is called* ***slip ring and brush assembly****.*

8.2.1 Concept of Slip Rings and Brush Assembly

Whenever there is a need of developing a contact between rotating element and the stationary circuit without conversion of an e.m.f. from a.c. to d.c., the slip rings and brush assembly can be used.

In case of three phase alternators, the armature consist of three phase winding and an a.c. e.m.f. gets induced in these windings. After connecting windings in star or delta, the three ends of the windings are brought out. Across these terminals three phase supply is ranging from 125 r.p.m. to 500 r.p.m. The prime movers used to drive such rotor are generally water turbines and I.C. engines.

8.6.2 Smooth Cylindrical Type Rotor

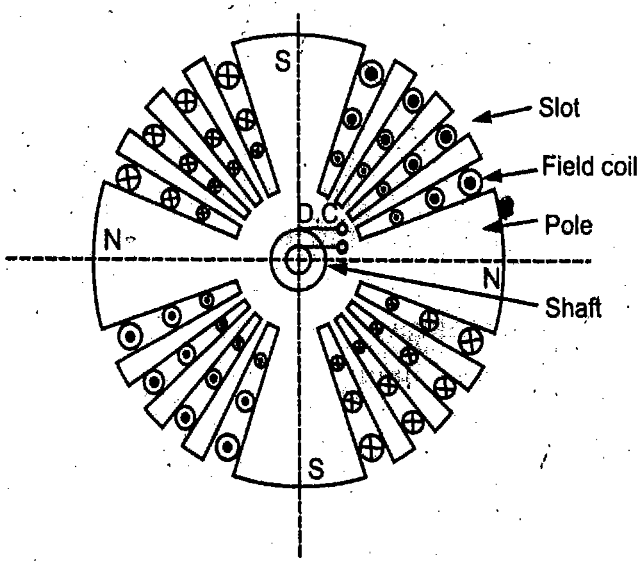


Fig. 8.4 Smooth cylindrical rotor

This is also called **non salient type** or **non-projected pole type of rotor.**

The rotor consists of smooth solid steel cylinder, having number of slots to accommodate the field coil. The slots are covered at the top with the help of steel or manganese wedges. The unslotted portions of the cylinder itself act as the poles. The poles are not projecting out and the surface of the rotor is smooth which maintains uniform air gap between stator and the rotor. These rotors have small diameters and large axial lengths. This is to keep peripheral speed within limits. The main advantage of this type is that these are mechanically very strong and thus preferred for high speed alternators ranging between 1500 to 3000 r.p.m. Such high speed alternators are called '**turboalternators**'. The prime movers used to drive such type of rotors are generally steam turbines, electric motors.

The Fig. 8.4 shows smooth cylindrical type of rotor.

Let us list down the differences between the two types in tabular form.

8.6.3 Difference between Salient and Cylindrical Type of Rotor

Sr. No.	Salient Pole Type	Smooth Cylindrical Type
1.	Poles are projecting out from the surface.	Unslotted portion of the cylinder acts as poles hence poles are non projecting.
2.	Air gap is non uniform.	Air gap is uniform due to smooth cylindrical periphery.
3.	Diameter is high and axial length is small.	Small diameter and large axial length is the feature.
4.	Mechanically weak.	Mechanically robust.
5.	Preferred for low speed alternators.	Preferred for high speed alternators i.e. for turboalternators.
6.	Prime mover used are water turbines, I.C. engines.	Prime movers used are steam turbines, electric motors.

7.	For same size, the rating is smaller than cylindrical type.	For same size, rating is higher than salient pole type.
8.	Separate damper winding is provided.	Separate damper winding is not necessary.

8.7 Working Principle

The alternators work on the principle of **electromagnetic induction**. When there is a relative motion between the conductors and the flux, e.m.f. gets induced in the conductors. The d.c. generators also work on the same principle. The only difference in practical alternator and a d.c. generator is that in an alternator the conductors are stationary and field is rotating. But for understanding purpose we can always consider relative motion of conductors with respect to the flux produced by the field winding.

Consider a relative motion of a single conductor under the magnetic field produced by two stationary poles. The magnetic axis of the two poles produced by field is vertical, shown dotted in the Fig. 8.5.

Let conductor starts rotating from position 1. At this instant, the entire velocity component is **parallel** to the flux lines. Hence there is no cutting of flux lines by the conductor. So $\frac{d\phi}{dt}$ at this instant is zero and hence induced e.m.f. in the conductor is also zero.

As the conductor moves from position 1 towards position 2, the part of the velocity component becomes perpendicular to the flux lines and proportional to that, e.m.f. gets induced in the conductor. The magnitude of such an induced e.m.f. increases as the conductor moves from position 1 towards 2.

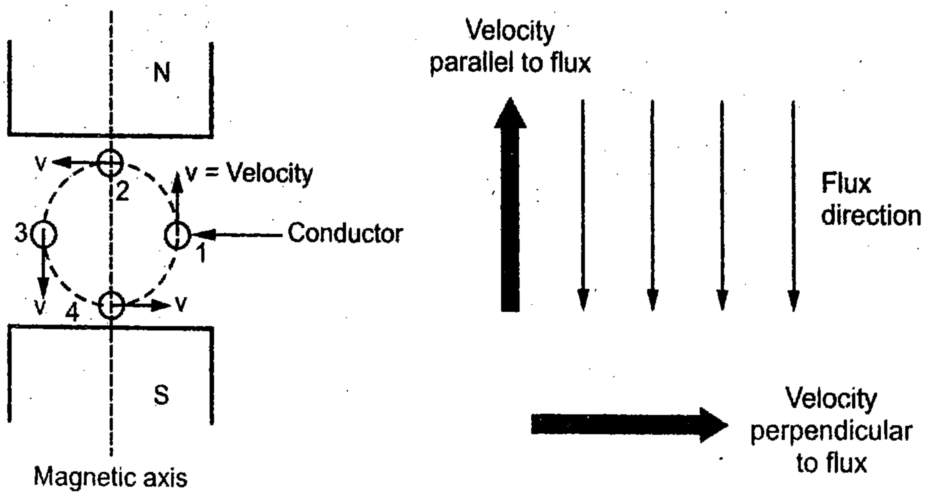


Fig. 8.5 Two pole alternator

At position 2, the entire velocity component is **perpendicular** to the flux lines. Hence there exists maximum cutting of the flux lines. And at this instant, the induced e.m.f. in the conductor is at its maximum.

As the position of conductor changes from 2 towards 3, the velocity component perpendicular to the flux starts decreasing and hence induced e.m.f. magnitude also starts decreasing. At position 3, again the entire velocity component is parallel to the flux lines and hence at this instant induced e.m.f. in the conductor is zero.

As the conductor moves from position 3 towards 4, the velocity component perpendicular to the flux lines again starts increasing. But the direction of velocity component now is opposite to the direction of velocity component exsisting during the movement of the conductor from position 1 to 2. Hence an induced e.m.f. in the conductor increases but in the opposite direction.

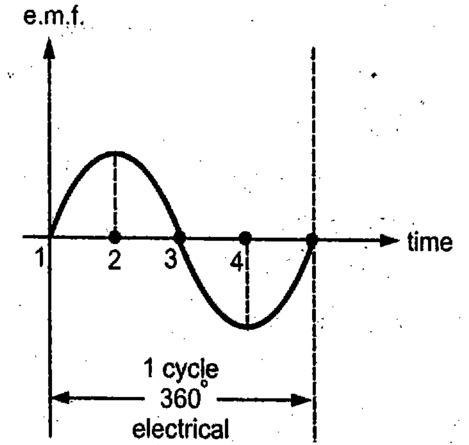


Fig. 8.6 Alternating nature of the induced e.m.f.

At position 4, it achieves maxima in the opposite direction, as the entire velocity component becomes **perpendicular** to the flux lines.

Again from position 4 to 1, induced e.m.f. decreases and finally at position 1, again becomes zero. This cycle continues as conductor rotates at a certain speed.

So if we plot the magnitudes of the induced e.m.f. against the time, we get an alternating nature of the induced e.m.f. as shown in the Fig. 8.6.

This is the working principle of an alternator.

8.7.1 Mechanical and Electrical Angle

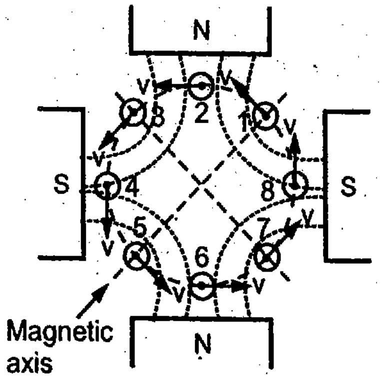


Fig. 8.7 (a) 4 Pole alternator

We have seen that for 2 pole alternator, one mechanical revolution corresponds to one electrical cycle of an induced e.m.f. Now consider 4 pole alternator i.e. the field winding is designed to produce 4 poles. Due to 4 poles, the magnetic axis exists diagonally shown dotted in the Fig. 8.7.

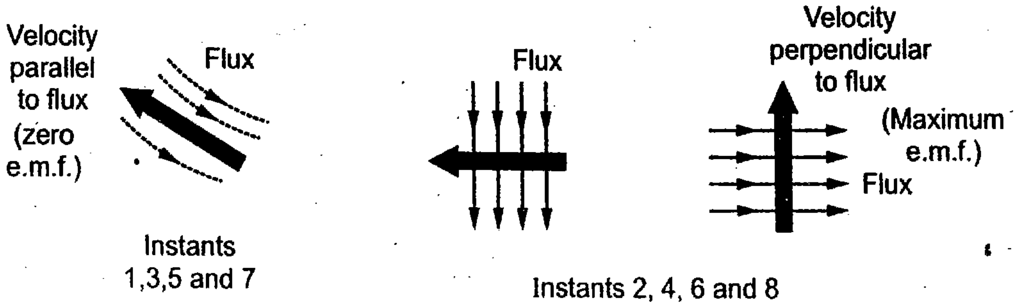


Fig. 8.7 (b) Veclocity components at different instants

5.1 Introduction

An electric motor is a device which converts an electrical energy into a mechanical energy. This mechanical energy then can be supplied to various types of loads. The motors can operate on d.c. as well as single and three phase a.c. supply. The motors operating on d.c. supply are called d.c. motors while motors operating on a.c. supply are called a.c. motors. As a.c. supply is commonly available, the a.c. motors are very popularly used in practice. The a.c. motors are classified as single and three phase induction motors, synchronous motors and some special purpose motors. Out of all these types, three phase induction motors are widely used for various industrial applications. Hence this chapter gives the emphasis on the working principle, types and features of three phase induction motors. The important advantages of three phase induction motors over other types are self starting property, no need of starting device, higher power factor, good speed regulation and robust construction. The working principle of three phase induction motors is based on the production of **rotating magnetic field**. Hence before beginning the actual discussion of three phase induction motors, let us discuss the production of rotating magnetic field from a three phase a.c. supply.

5.2 Rotating Magnetic Field (R.M.F.) JNTU : Nov.-04, 06, 12, March-06, May-08

The rotating magnetic field can be defined as the field or flux having constant amplitude but whose axis is continuously rotating in a plane with a certain speed. So if the arrangement is made to rotate a permanent magnet, then the resulting field is a rotating magnetic field. But in this method, it is necessary to rotate a magnet physically to produce rotating magnetic field.

But in three phase induction motors such a rotating magnetic field is produced by supplying currents to a set of **stationary** windings, with the help of three phase a.c. supply. The current carrying windings produce the magnetic field or flux. And due to interaction of three fluxes produced due to three phase supply, resultant flux has a constant magnitude and its axis rotating in space, without physically rotating the windings. This type of field is nothing but rotating magnetic field. Let us study how it happens ?

5.2.1 Production of R.M.F.

A three phase induction motor consists of three phase winding as its stationary part called **stator**. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by 120°. The windings are supplied by a balanced three phase a.c. supply. This is shown in the Fig. 5.2.1. The three phase windings are denoted as R-R′, Y-Y′ and B-B′.

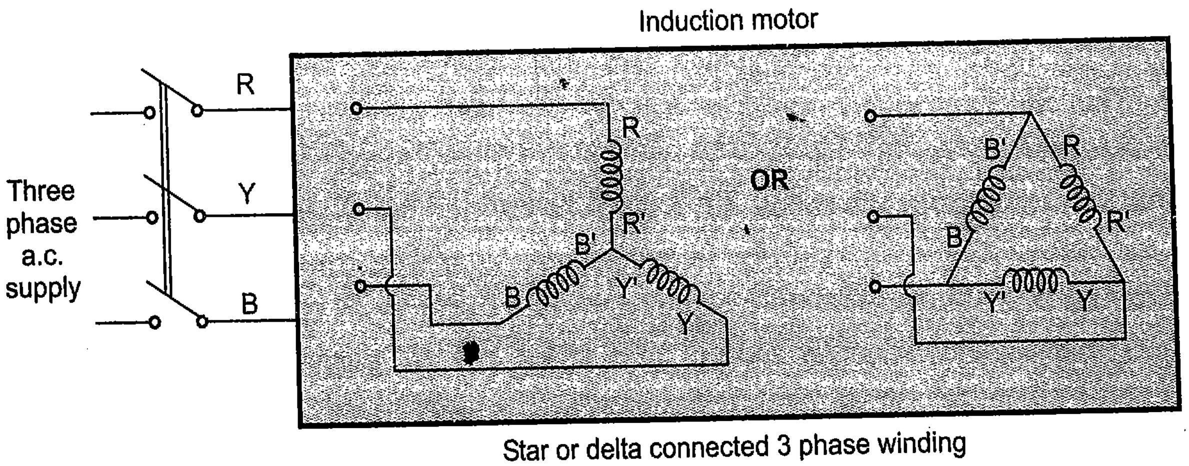


Fig. 5.2.1

The three phase currents flow simultaneously through the windings and are displaced from each other by 120° electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are separated from each other by 120°. If the phase sequence of the windings is R-Y-B, then mathematical equations for the instantaneous values of the three fluxes ϕ_R, ϕ_Y and ϕ_B can be written as,

$$\phi_R = \phi_m \sin(\omega t) = \phi_m \sin\theta \qquad \ldots (5.2.1)$$

$$\phi_Y = \phi_m \sin(\omega t - 120^\circ) = \phi_m \sin(\theta - 120^\circ) \qquad \ldots (5.2.2)$$

$$\phi_B = \phi_m \sin(\omega t - 240^\circ) = \phi_m \sin(\theta - 240^\circ) \qquad \ldots (5.2.3)$$

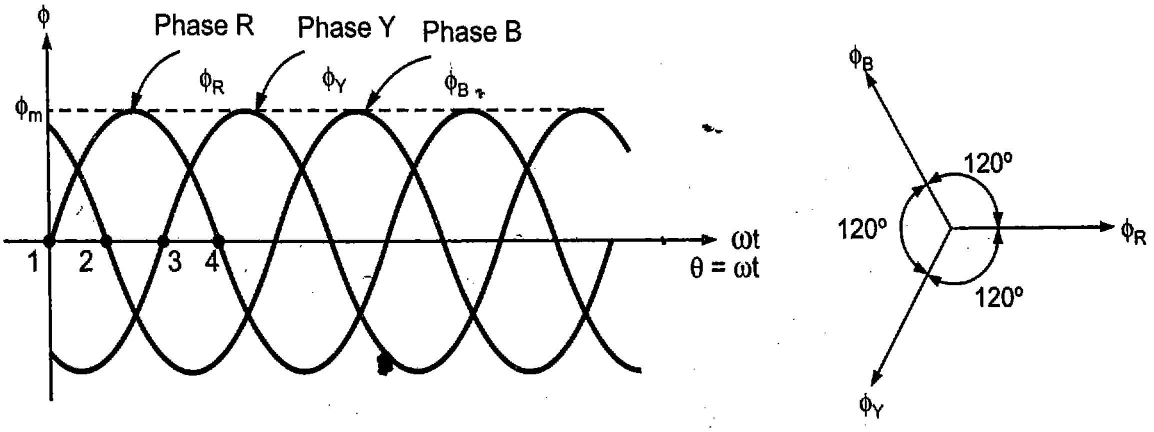
As windings are identical and supply is balanced, the magnitude of each flux is ϕ_m. Due to phase sequence R-Y-B, flux ϕ_Y lags behind ϕ_R by 120° and ϕ_B lags ϕ_Y by 120°. So ϕ_B ultimately lags ϕ_R by 240°. The flux ϕ_R is taken as reference while writing the equations.

The Fig. 5.2.2 (a) shows the waveforms of three fluxes in space. The Fig. 5.2.2 (b) shows the phasor diagram which clearly shows the **assumed positive directions** of each flux. Assumed positive direction means whenever the flux is positive it must be represented along the direction shown and whenever the flux is negative it must be represented along the opposite direction to the assumed positive direction.

Let ϕ_R, ϕ_Y and ϕ_B be the instantaneous values of three fluxes. The resultant flux ϕ_T is the phasor addition of ϕ_R, ϕ_Y and ϕ_B.

$$\therefore \qquad \bar{\phi}_T = \bar{\phi}_R + \bar{\phi}_Y + \bar{\phi}_B$$

Let us find ϕ_T at the instants 1, 2, 3 and 4 as shown in the Fig. 5.2.2 (a) which represents the values of θ as 0°, 60°, 120° and 180° respectively. The phasor addition can be performed by obtaining the values of ϕ_R, ϕ_Y and ϕ_B by substituting values of θ in the equations (5.2.1), (5.2.2) and (5.2.3).



(a) Waveforms of three fluxes **(b) Assumed positive directions**

Fig. 5.2.2

Case 1 : $\theta = 0°$

Substituting in the equations (5.2.1), (5.2.2) and (5.2.3) we get,

$$\phi_R = \phi_m \sin 0° = 0$$

$$\phi_Y = \phi_m \sin(-120°) = -0.866\,\phi_m$$

$$\phi_B = \phi_m \sin(-240°) = +0.866\,\phi_m$$

The phasor addition is shown in the Fig. 5.2.3 (a). The positive values are shown in assumed positive directions while negative values are shown in opposite direction to the assumed positive directions of the respective fluxes. Refer to assumed positive directions shown in the Fig. 5.2.2 (b).

BD is drawn perpendicular from B on ϕ_T. It bisects ϕ_T.

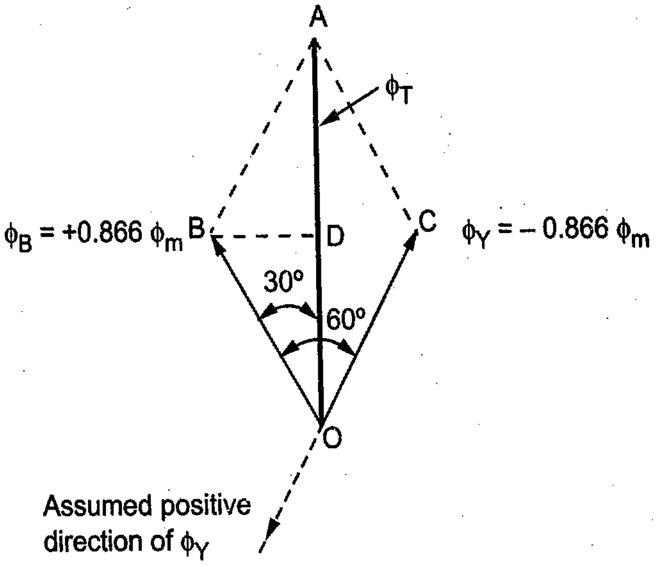


Fig. 5.2.3 (a) Vector diagram for θ = 0°

$$\therefore \quad OD = DA = \frac{\phi_T}{2}$$

In triangle OBD, $\angle BOD = 30°$

$$\therefore \quad \cos 30° = \frac{OD}{OB} = \frac{\phi_T/2}{0.866\phi_m}$$

$$\therefore \quad \phi_T = 2 \times 0.866\phi_m \times \cos 30°$$

$$= 1.5\,\phi_m$$

So magnitude of ϕ_T is 1.5 ϕ_m and its position is vertically upwards at $\theta = 0°$.

Case 2 : $\theta = 60^\circ$

Equations (5.2.1), (5.2.2) and (5.2.3) give us,

$$\phi_R = \phi_m \sin 60^\circ = +0.866\,\phi_m$$

$$\phi_Y = \phi_m \sin(-60^\circ) = -0.866\,\phi_m$$

$$\phi_B = \phi_m \sin(-180^\circ) = 0$$

So ϕ_R is positive and ϕ_Y is negative and hence drawing in appropriate directions we get phasor diagram as shown in the Fig. 5.2.3 (b).

Doing the same construction, drawing perpendicular from B on ϕ_T at D we get the same result as,

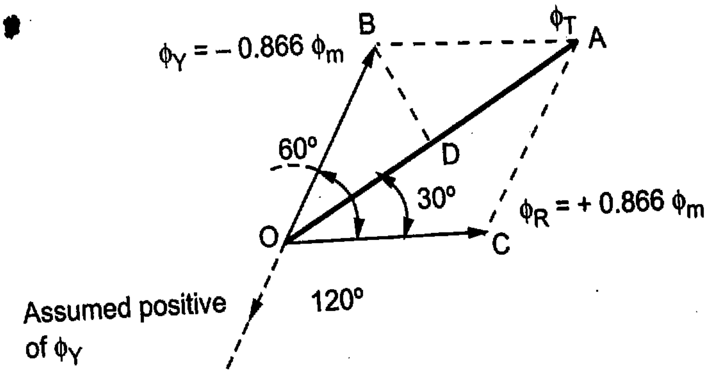


Fig. 5.2.3 (b) Vector diagram for θ = 60°

$$\phi_T = 1.5\,\phi_m$$

But it can be seen that **though its magnitude is 1.5 ϕ_m it has rotated through 60° in space, in clockwise direction, from its previous position.**

Case 3 : $\theta = 120^\circ$

Equations (5.2.1), (5.2.2) and (5.2.3) give us,

$$\phi_R = \phi_m \sin 120 = +0.866\,\phi_m$$

$$\phi_Y = \phi_m \sin 0 = 0$$

$$\phi_B = \phi_m \sin(-120) = -0.866\,\phi_m$$

So ϕ_R is positive and ϕ_B is negative. Showing ϕ_R and ϕ_B in the appropriate directions, we get the phasor diagram as shown in the Fig. 5.2.3 (c).

After doing the construction same as before i.e. drawing perpendicular from B on ϕ_T, it can be proved again that,

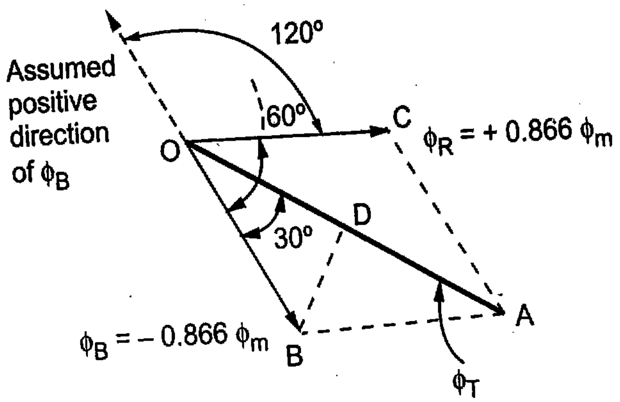


Fig. 5.2.3 (c) Vector diagram for θ = 120°

$$\phi_T = 1.5\,\phi_m$$

But the position of ϕ_T is such that it has rotated further through 60° from its previous position, in clockwise direction. **And from its position at θ = 0°, it has rotated through 120° in space, in clockwise direction.**

Case 4 : $\theta = 180°$

From the equations (5.2.1), (5.2.2) and (5.2.3),

$$\phi_R = \phi_m \sin(180°) = 0$$

$$\phi_Y = \phi_m \sin(60°) = +0.866\,\phi_m$$

$$\phi_B = \phi_m \sin(-60°) = -0.866\,\phi_m$$

So $\phi_R = 0$, ϕ_Y is positive and ϕ_B is negative. Drawing ϕ_Y and ϕ_B in the appropriate directions, we get the phasor diagram as shown in the Fig. 5.2.3 (d).

From phasor diagram, it can be easily proved that,

$$\phi_T = 1.5\,\phi_m$$

Thus the magnitude of ϕ_T once again remains same. But it can be seen that it has further rotated through 60° from its previous position in clockwise direction.

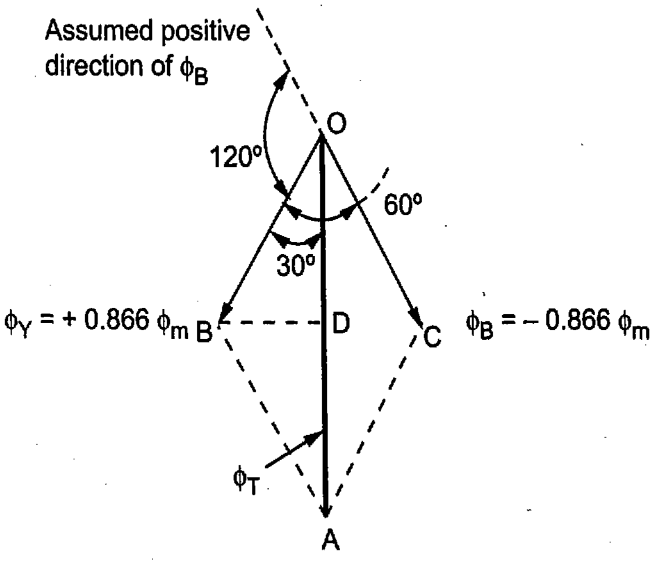


Fig. 5.2.3 (d) Vector diagram for θ = 180°

So for an electrical half cycle of 180°, the resultant ϕ_T has also rotated through 180°. This is applicable for the windings wound for 2 poles.

From the above discussion we have following conclusions :

a) The resultant of the three alternating fluxes, separated from each other by 120°, has a constant amplitude of 1.5 ϕ_m where ϕ_m is maximum amplitude of an individual flux due to any phase.

b) The resultant always keeps on rotating with a certain speed in space.

Key Point *This shows that when a three phase stationary windings are excited by balanced three phase a.c. supply then the resulting field produced is* ***rotating magnetic field.*** *Though nothing is physically rotating, the field produced is rotating in space having constant amplitude.*

5.2.2 Speed of R.M.F.

There exists a fixed relation between frequency f of a.c. supply to the windings, the number of poles P for which winding is wound and speed N r.p.m. of rotating magnetic

5.5 Working Principle

JNTU : Nov.-03, 04, 06, 08, May-04, 05, 13

Induction motor works on the principle of electromagnetic induction.

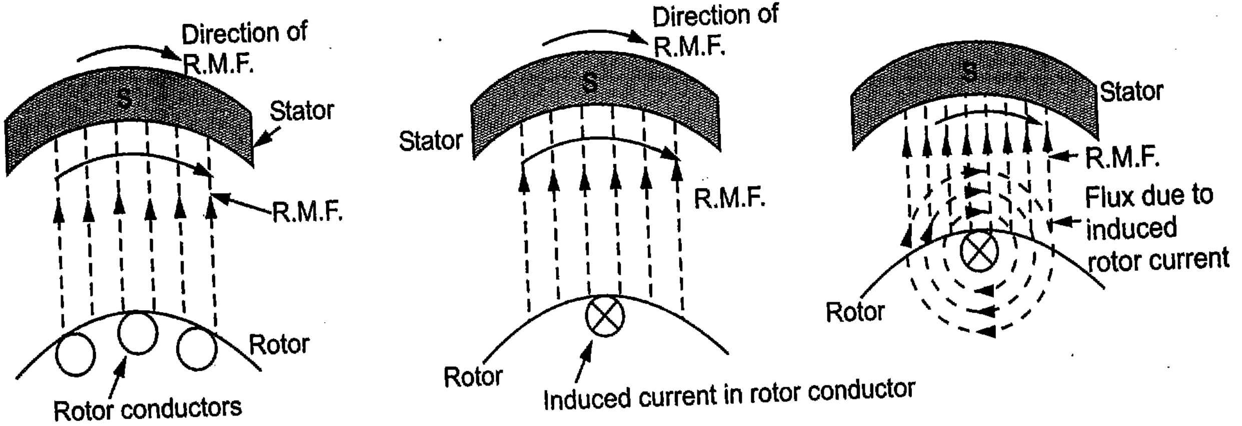
When a three phase supply is given to the three phase stator winding, a rotating magnetic field of constant magnitude is produced as discussed earlier. The speed of this rotating magnetic field is synchronous speed, N_s r.p.m.

$$N_s = \frac{120\,f}{P} = \text{Speed of rotating magnetic field}$$

where f = Supply frequency

P = Number of poles for which stator winding is wound.

This rotating field produces an effect of rotating poles around a rotor. Let direction of rotation of this rotating magnetic field is **clockwise** as shown in the Fig. 5.5.1 (a).



(a) (b) (c)

Fig. 5.5.1 Working principle

Now at this instant rotor is **stationary** and stator flux R.M.F. is **rotating**. So its obvious that there exists a relative motion between the R.M.F. and rotor conductors. Now the R.M.F. gets cut by rotor conductors as R.M.F. sweeps over rotor conductors. Whenever conductor cuts the flux, e.m.f. gets induced in it. So e.m.f. gets induced in the rotor conductors called **rotor induced e.m.f.** This is electro-magnetic induction. As rotor forms closed circuit, induced e.m.f. circulates current through rotor called **rotor current** as shown in the Fig. 5.5.1 (b). Let direction of this current is going into the paper denoted by a cross as shown in the Fig. 5.5.1 (b).

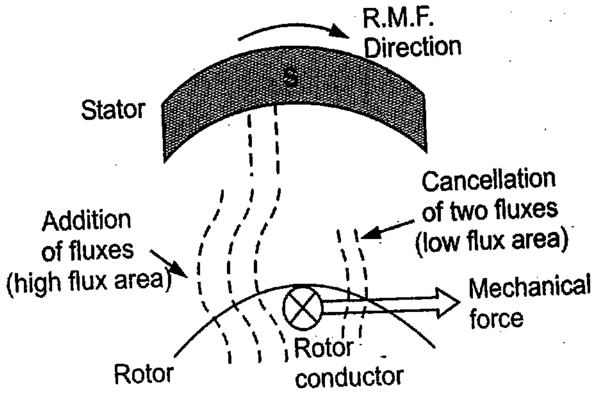


Fig. 5.5.1 (d) Interaction of fluxes

Any current carrying conductor produces its own flux. So rotor produces its flux called **rotor flux**. For assumed direction of rotor current, the direction of rotor flux is clockwise as shown in the Fig. 5.5.1 (c). This direction can be easily determined using right hand thumb rule. Now there are two fluxes, one R.M.F. and other rotor flux. Both the fluxes interact with each as shown in the Fig. 5.5.1 (d). On left of rotor conductor, two fluxes are in same direction hence add up to get high flux area. On right side, two fluxes cancel each other to produce low flux area. As flux lines act as stretched rubber band, high flux density area exerts a push on rotor conductor towards low flux density area. So rotor conductor experiences a force from left to right in this case, as shown in the Fig. 5.5.1 (d), due to **interaction of the two fluxes**.

As all the rotor conductors experience a force, the overall rotor experiences a torque and starts rotating. So **interaction of the two fluxes is very essential for a motoring action**. As seen from the Fig. 5.5.1 (d), the direction of force experienced is same as that of rotating magnetic field. Hence rotor starts rotating in the same direction as that of rotating magnetic field.

Alternatively this can be explained as : According to Lenz's law the direction of induced current in the rotor is so as oppose the cause producing it. The cause of rotor current is the induced e.m.f. which is induced because of relative motion present between the rotating magnetic field and the rotor conductors. Hence to oppose the relative motion i.e. to reduce the relative speed, the rotor experiences a torque in the same direction as that of R.M.F. and tries to catch up the speed of rotating magnetic field.

So, N_s = Speed of rotating magnetic field in r.p.m.

N = Speed of rotor i.e. motor in r.p.m.

$N_s - N$ = Relative speed between the two, rotating magnetic field and the rotor conductors.

Thus rotor always rotates in same direction as that of R.M.F.

5.5.1 Can $N = N_s$?

When rotor starts rotating, it tries to catch the speed of rotating magnetic field.

If it catches the speed of the rotating magnetic field, the relative motion between rotor and the rotating magnetic field will vanish ($N_s - N = 0$). In fact the relative motion is the main cause for the induced e.m.f. in the rotor. So induced e.m.f. will vanish and hence there cannot be rotor current and the rotor flux which is essential to produce the torque on the rotor. Eventually motor will stop. But immediately there will exist a relative motion between rotor and rotating magnetic field and it will start. But due to inertia of rotor, this does not happen in practice and rotor continues to rotate with a speed slightly less than the synchronous speed of the rotating magnetic field in the steady state. The **induction motor never rotates at synchronous speed**. The speed at which it rotates is hence called **subsynchronous speed** and motor sometimes called **asynchronous motor**.

$$\therefore \qquad N < N_s$$

So it can be said that rotor slips behind the rotating magnetic field produced by stator. The difference between the two is called **slip speed** of the motor.

$$N_s - N = \text{Slip speed of the motor in r.p.m.}$$

This speed decides the magnitude of the induced e.m.f. and the rotor current, which in turn decide the torque produced. The torque produced is as per the requirements of overcoming the friction and iron losses of the motor along with the torque demanded by the load on the motor.

Review Questions

1. *Can induction motor rotate at synchronous speed ? Why ?*
   JNTU : Nov.-03, 04, May-04, 05, Marks 6
2. *Explain the principle of operation of a 3-phase induction motor.*
   JNTU : Nov.-04, 06, 08, May-13, Marks 8

5.6 Slip of Induction Motor

JNTU : May-13

We have seen that rotor rotates in the same direction as that of R.M.F. but in steady state attains a speed less than the synchronous speed. The difference between the two speeds i.e. synchronous speed of R.M.F. (N_s) and rotor speed (N) is called slip speed. This slip speed is generally expressed as the percentage of the synchronous speed.

So slip of the induction motor is defined as the difference between the synchronous speed (N_s) and actual speed of rotor i.e. motor (N) expressed as a fraction of the synchronous speed (N_s). This is also called absolute slip or fractional slip and is denoted as 's'.

Thus $$\boxed{s = \frac{N_s - N}{N_s}} \qquad \text{... (Absolute slip)}$$

The percentage slip is expressed as,

$$\boxed{\% s = \frac{N_s - N}{N_s} \times 100} \qquad \text{... (Percentage slip)}$$

In terms of slip, the actual speed of motor (N) can be expressed as,

$$\boxed{N = N_s(1-s)} \quad \text{... (From the expression of slip)}$$

At start, motor is at rest and hence its speed N is zero.

$$\therefore \quad \boxed{s = 1 \text{ at start}}$$

This is maximum value of slip s possible for induction motor which occurs at start. While s = 0 gives us $N = N_s$ which is not possible for an induction motor. So **slip of induction motor cannot be zero** under any circumstances.

Practically motor operates in the slip range of 0.01 to 0.05 i.e. 1 % to 5 %. The slip corresponding to full load speed of the motor is called **full load slip.**

Example 5.6.1 *A 4 pole, 3 phase induction motor is supplied from 50 Hz supply. Determine its synchronous speed. On full load, its speed is observed to be 1410 r.p.m. Calculate its full load slip.*

Solution : Given values are,

$$P = 4, \quad f = 50 \text{ Hz}, \quad N = 1410 \text{ r.p.m.}$$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

Full load absolute slip is given by,

$$s = \frac{N_s - N}{N_s} = \frac{1500-1410}{1500} = 0.06$$

$$\therefore \quad \% s = 0.06 \times 100 = \mathbf{6\ \%}$$

Example 5.6.2 *A 4 pole, 3 phase, 50 Hz, star connected induction motor has a full load slip of 4 %. Calculate full load speed of the motor.*

Solution : Given values are,

$$P = 4, \quad f = 50 \text{ Hz}, \quad \% s_{fl} = 4\ \%$$

$$s_{fl} = \text{Full load absolute slip} = 0.04$$

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

$$s_{fl} = \frac{N_s - N_{fl}}{N_s} \quad \text{where } N_{fl} = \text{Full load speed of motor}$$

$$\therefore \quad 0.04 = \frac{1500 - N_{fl}}{1500}$$

$$\therefore \quad N_{fl} = \mathbf{1440 \text{ r.p.m.}}$$

This is the full load speed of the motor.

2. *A 3-phase induction motor has a 4-pole, star connected stator winding. The motor runs on a 50 Hz supply with 200 V between lines. The rotor resistance and standstill rotor reactance per phase are 0.1 Ω and 0.9 Ω respectively. The ratio of rotor to stator turns in 0.67. Calculate :*
   i) Total torque at 4 % slip *ii) Maximum torque developed*
   iii) Speed at maximum torque *iv) Maximum mechanical power*
   Neglect stator impedance. **[Ans. : 40.4786 Nm, 63.5065 Nm, 1333.333 r.p.m., 8867.1801 W]**

3. *A 3-phase induction motor has a 4-pole, star connected stator winding. The motor runs on a 50 Hz supply with 200 V between lines. The rotor resistance and standstill rotor reactance per phase are 0.1 Ω and 0.9 Ω respectively. The ratio of rotor to stator turns in 0.67. Calculate :*
   i) Total torque at 4 % slip *ii) Maximum torque developed*
   iii) Speed at maximum torque *iv) Maximum mechanical power*
   Neglect stator impedance.
   [Ans. : i) 40.47 Nm, ii) 63.50 Nm, iii) 1333.33 r.p.m. iv) $P_m = T_m \times \omega_m$ = 8.867 kW]

5.11 Torque-Slip Characteristics

JNTU : Nov.-06, 12, May-07

As the induction motor is loaded from no load to full load, its speed decreases hence slip increases. Due to the increased load, motor has to produce more torque to satisfy load demand. The torque ultimately depends on slip as explained earlier. The behaviour of motor can be easily judged by sketching a curve obtained by plotting torque produced against slip of induction motor. The curve obtained by plotting torque against slip from s = 1 (at start) to s = 0 (at synchronous speed) is called **torque-slip characteristics** of the induction motor. It is very interesting to study the nature of torque-slip characteristics.

We have seen that for a constant supply voltage, E_2 is also constant. So we can write torque equation as,

$$\boxed{T \propto \frac{s R_2}{R_2^2 + (s X_2)^2}}$$

Now to judge the nature of torque-slip characteristics let us divide the slip range (s = 0 to s = 1) into two parts and analyze them independently.

i) Low slip region :

In low slip region, 's' is very very small. Due to this, the term $(s X_2)^2$ is so small as compared to R_2^2 that it can be neglected.

$$\therefore \quad \boxed{T \propto \frac{s R_2}{R_2^2} \propto s} \quad \text{... As } R_2 \text{ is constant.}$$

Hence in **low slip region torque is directly proportional to slip**. So as load increases, speed decreases, increasing the slip. This increases the torque which satisfies the load demand.

Hence the graph is straight line in nature.

At $N = N_s$, $s = 0$ hence $T = 0$. As no torque is generated at $N = N_s$, motor stops if it tries to achieve the synchronous speed. Torque increases linearly in this region, of low slip values.

ii) High slip region :

In this region, slip is high i.e. slip value is approaching to 1. Here it can be assumed that the term R_2^2 is very very small as compared to $(s X_2)^2$. Hence neglecting R_2^2 from the denominator, we get

$$\boxed{T \propto \frac{s R_2}{(sX_2)^2} \propto \frac{1}{s}}$$

where R_2 and X_2 are constants.

So in high slip region torque is inversely proportional to the slip. Hence its nature is like rectangular hyperbola.

Now when load increases, load demand increases but speed decreases. As speed decreases, slip increases. In high slip region as $T \propto 1/s$, torque decreases as slip increases. But torque must increase to satisfy the load demand. As torque decreases, due to extra loading effect, speed further decreases and slip further increases. Again torque decreases as $T \propto 1/s$ hence same load acts as an extra load due to reduction in torque produced. Hence speed further drops. Eventually motor comes to standstill condition. The **motor cannot continue to rotate at any point in this high slip region**. Hence this region is called **unstable region** of operation.

So torque - slip characteristics has two parts,

1. Straight line called **stable region of operation.**
2. Rectangular hyperbola called **unstable region of operation.**

Now the obvious question is upto which value of slip, torque-slip characteristic represents stable operation ?

In low slip region, as load increases, slip increases and torque also increases linearly. Every motor has its own limit to produce a torque. The maximum torque, the motor can produce as load increases is T_m which occurs at $s = s_m$. So linear behaviour continues till $s = s_m$.

If load is increased beyond this limit, motor slip acts dominantly pushing motor into high slip region. Due to unstable conditions, motor comes to standstill condition at such a load. Hence T_m i.e. maximum torque which motor can produce is also called

breakdown torque or **pull out torque**. So range s = 0 to s = s_m is called low slip region, known as stable region of operation. Motor always operates at a point in this region. And range s = s_m to s = 1 is called high slip region which is rectangular hyperbola, called unstable region of operation. Motor cannot continue to rotate at any point in this region.

At s = 1, N = 0 i.e. at start, motor produces a torque called **starting torque** denoted as T_{st}.

The entire torque-slip characteristics is shown in the Fig. 5.11.1.

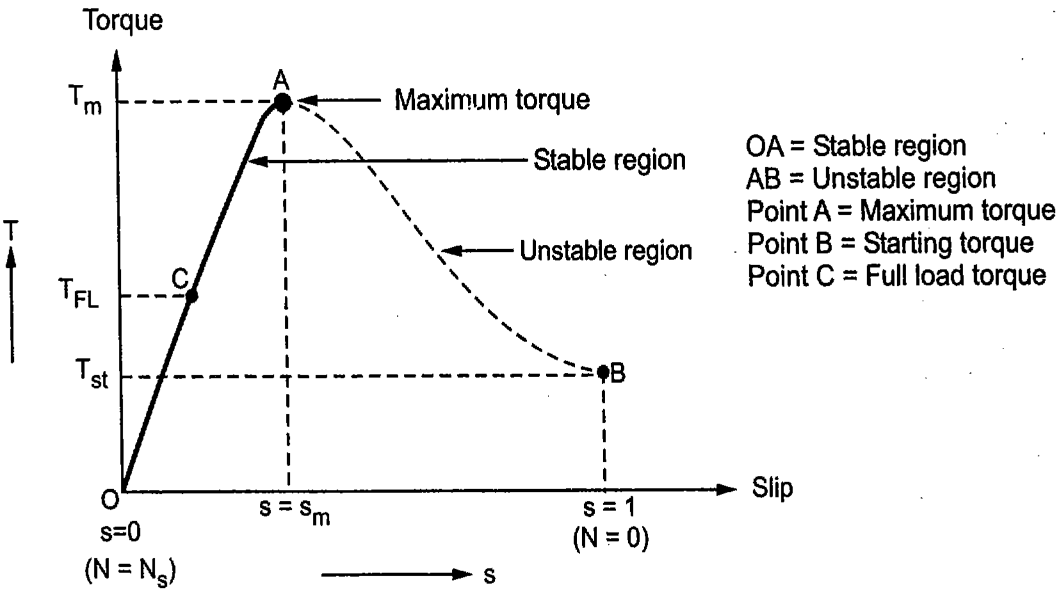


Fig. 5.11.1 Torque-slip characteristics

5.11.1 Full Load Torque

When the load on the motor increases, the torque produced increases as speed decreases and slip increases. The increased torque demand is satisfied by drawing more current from the supply.

The load which motor can drive safely while operating continuously and due to such load, the current drawn is also within safe limits is called **full load condition** of motor. When current increases, due to heat produced the temperature rises. The safe limit of current is that which when drawn for continuous operation of motor, produces a temperature rise well within the limits. Such a full load point is shown on the torque-slip characteristics as point C in the Fig. 5.11.1 and corresponding torque as $T_{F.L.}$.

The interesting thing is that the load on the motor can be increased beyond point C till maximum torque condition. But due to high current and hence high temperature rise there is possibility of damage of winding insulation, **if motor is operated for longer time** duration in this region i.e. from point C to B. But motor can be used to drive loads more than full load, producing torque upto maximum torque for short duration of time. Generally full load torque is less than the maximum torque.