③ # Higher Order Differential Equations

Solutions of Higher order Homogeneous Differential Equations:

H.O.D.E

$f(D)y = 0$ → Homogeneous H.D.E → solution → y = complementary function (C.F)

$f(D)y = X$ → Non-Homogeneous H.DE → solution → y = Complementary function + Particular Integral (C.F + P.I)

Solve the following Higher order Differential Equations:

① $\dfrac{d^3y}{dx^3} - 7\dfrac{dy}{dx} - 6y = 0.$

② $\dfrac{d^4y}{dx^4} + 13\dfrac{d^2y}{dx^2} + 36y = 0$

③ $\dfrac{d^4x}{dt^4} + 4x = 0.$

④ $(D^4 + 4)y = 0$

⑤ $y'' - 2y' + 10y = 0$ given $y(0) = 4$, $y'(0) = 1$

⑥ $\dfrac{d^3y}{dx^3} + 6\dfrac{d^2y}{dx^2} + 12\dfrac{dy}{dx} + 8y = 0$ under the conditions $y(0) = 0$ and $y'(0) = 0$, $y''(0) = 2$.

⑦ $\dfrac{d^4x}{dt^4} = m^4x$. show that $x = c_1\cos mt + c_2\sin mt + c_3\cosh mt + c_4\sinh mt$.

⑧ $(D^3 + 1)y = 0.$

⑨ $(D^4 + 6D^3 + 11D^2 + 6D)y = 0.$

④ Given D.E is $(D^4+4)y=0$. $\rightarrow$ ①

An A.E is $m^4+4=0$

$$(m^2)^2+(2)^2=0$$

$$(m^2+2)^2-2m^2(2)=0$$

$$(m^2+2)^2-4m^2=0$$

$$(m^2+2)^2-(2m)^2=0$$

$$(m^2+2+2m)(m^2+2-2m)=0$$

$m^2+2m+2=0$ and $m^2-2m+2=0$

$m=\frac{-2\pm\sqrt{4-8}}{2}$ and $m=\frac{2\pm\sqrt{4-8}}{2}$

$=\frac{-2\pm 2i}{2}$ $\qquad$ $=\frac{2\pm 2i}{2}$

$=\frac{\cancel{2}(-1\pm i)}{\cancel{2}}$ $\qquad$ $=\frac{\cancel{2}(1\pm i)}{\cancel{2}}$

$m=-1\pm i$ $\qquad$ $m=1\pm i$.

$\therefore$ The roots $-1\pm i$, $1\pm i$ are complex and distinct roots.

$\therefore$ The complementary function (C.F) is

$$e^{-1(x)}[c_1\cos x+c_2\sin x]+e^{1(x)}[c_3\cos x+c_4\sin x]$$

$\therefore$ The solution of Equ^{n}① is $y=$ C.F

$$y=e^{-x}[c_1\cos x+c_2\sin x]+e^{x}[c_3\cos x+c_4\sin x].$$

⑤ Given D.E is $y''-2y'+10y=0$

$$D^2y-2Dy+10y=0$$

$$(D^2-2D+10)y=0$$

An A.E is $m^2-2m+10=0$

$$m=\frac{2\pm\sqrt{4-40}}{2}$$

$$=\frac{2\pm\sqrt{-36}}{2}$$

$$= \frac{2 \pm 6i}{2}$$

$$= \frac{\cancel{2}(1 \pm 3i)}{\cancel{2}}$$

$$m = 1 \pm 3i$$

$\therefore$ The roots $1 \pm 3i$ are complex and distinct roots.

$\therefore$ The complementary function $= e^{(1)x}[c_1 \cos 3x + c_2 \sin 3x]$

$\therefore$ The solution is $y = C.F$

$$y = e^{x}[c_1 \cos 3x + c_2 \sin 3x] \rightarrow ①$$

Given that $y(0) = 4$ and $y'(0) = 1$

$x = 0;\ y = 4$ $\qquad$ $x = 0,\ y' = 1$

at $x = 0$, $y = 4$

from ①,
$$y = e^0[c_1 \cos 3(0) + c_2 \sin 3(0)]$$

$$4 = 1[c_1 \cos 0 + c_2 \sin 0]$$

$$4 = c_1(1) + c_2(0)$$

$$\Rightarrow \boxed{c_1 = 4}$$

from ①

at $x = 0$, $y' = e^x[c_1 \cos 3x + c_2 \sin 3x] + e^x[c_1(-\sin 3x)3 + c_2(\cos 3x)(3)]$

$$y' = e^x[c_1 \cos 3x + c_2 \sin 3x] + e^x[-3c_1 \sin 3x + 3c_2 \cos 3x]$$

at $x = 0$, $y' = 1$ and $c_1 = 4$

$$\therefore 1 = e^0[4.\cos 3(0) + c_2 \sin 3(0)] + e^0[-3(4) \sin 3(0) + 3c_2 \cos 3(0)]$$

$$1 = (1)[4(1) + c_2(0)] + (1)[-12(0) + 3c_2(1)]$$

$$1 = (4 + 0) + (0 + 3c_2)$$

$$1 = 4 + 3c_2$$

$$3c_2 = 1 - 4$$

$$3c_2 = -3$$

$$\boxed{c_2 = -1}$$

$$\therefore y = e^x[4 \cos 3x - \sin 3x]$$

⑥ $\frac{d^3y}{dx^3} + 6\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 8y = 0$

$D^3y + 6D^2y + 12Dy + 8y = 0$

$(D^3+6D^2+12D+8)\,y = 0$

A.E is $m^3+6m^2+12m+8=0$

$(m+2)(m^2+4m+4)=0$

$$\begin{array}{r|rrrr} -2 & 1 & 6 & 12 & 8 \\ & 0 & -2 & -8 & -8 \\ \hline & 1 & 4 & 4 & \boxed{0} \end{array}$$

$m+2=0,\quad m^2+4m+4=0$

$m=-2,\quad (m+2)(m+2)=0$

$m=-2,\ m=-2$

∴ The roots $-2,-2,-2$ are real and repeated roots.

Now, $C.F = c_1e^{-2x} + c_2e^{-2x}(x) + c_3e^{-2x}(x^2)$

∴ The solution is $y = C.F$

$y = c_1e^{-2x} + c_2e^{-2x}(x) + c_3e^{-2x}x^2$

$y = e^{-2x}[c_1 + c_2x + c_3x^2] \rightarrow ①$

Given that $y(0)=0$, $y'(0)=0$ and $y''(0)=2$.

at $x=0$, $y=0$.

$0 = e^{-2(0)}[c_1 + c_2(0) + c_3(0)^2]$

$0 = (1)[c_1+0+0]$

$\Rightarrow \boxed{c_1=0}$

from ①,

$y' = e^{-2x}(-2)[c_1 + c_2x + c_3x^2] + e^{-2x}[0 + c_2 + 2c_3x] \rightarrow ②$

at $x=0$, $y'=0$.

$0 = e^{-2(0)}(-2)[c_1 + c_2(0) + c_3(0)] + e^{-2(0)}[c_2 + 2c_3(0)]$

$0 = (1)(-2)[0+0] + (1)[c_2+0]$

$0 = -2c_1 + c_2$

$\Rightarrow \boxed{c_2=0}$

from ②,

$$y'' = -2\cdot e^{-2x}(-2)[c_1 + c_2x + c_3x^2] + (-2)e^{-2x}[0 + c_2 + 2c_3x]$$
$$+ -2e^{-2x}[c_2 + 2c_3x] + e^{-2x}[0 + 2c_3]$$
$$= 4e^{-2x}[c_1 + c_2x + c_3x^2] - 2e^{-2x}(c_2 + 2c_3x)$$
$$-2e^{-2x}(c_2 + 2c_3x) + e^{-2x}2c_3.$$

$$= 4e^{-2x}(c_1 + c_2x + c_3x^2) - 4e^{-2x}(c_2 + 2c_3x) + 2.e^{-2x}c_3$$

at $x = 0$, $y'' = 2$

$$2 = 4e^{-2(0)}(c_2(0) + c_3(0)^2) - 4e^{-2(0)}[c_2 + 2c_3(0)] + 2e^{-2(0)}c_3$$

$$2 = 4(1)[0+0] - 4(1)[c_2 + 0] + 2(1)c_3$$

$$2 = 4(0) - 4c_2 + 2c_3$$ Wrong

$$2 = 0 - 4(2) + 2c_3$$

$$2 = -8 + 2c_3$$

$$\cancel{2}c_3 = \cancel{10}\,5$$

$$\boxed{c_3 = 5}$$

$$y' = -2e^{-2x}\cdot[c_1 + c_2x + c_3x^2] + e^{-2x}[c_2 + 2c_3x]$$

$$y' = e^{-2x}[-2c_1 - 2c_2x - 2c_3x^2 + c_2 + 2c_3x]$$

$$y'' = e^{-2x}(-2)[-2c_1 - 2c_2x - 2c_3x^2 + c_2 + 2c_3x] + e^{-2x}[0 - 2c_2 - 2c_3(2x) + 0 + 2c_3]$$

$$= -2e^{-2x}[-2c_1 - 2c_2x - 2c_3x^2 + c_2 + 2c_3x] + e^{-2x}[-2c_2 - 4xc_3 + 2c_3]$$

at $x=0$, $y'' = 2$

$$2 = -2e^{-2(0)}[-2(0) - 2(0)x - 2c_3(0)^2 + 0 + 2c_3(0)] + e^{-2(0)}[-2(0) - 4(0)c_3 + 2c_3]$$

$$2 = -2(1)\cdot[0] + (1)[2c_3]$$

$$2 = 0 + 2c_3$$

$$\cancel{2} = \cancel{2}c_3 \Rightarrow \boxed{c_3 = 1}$$

$\therefore c_1 = 0, \quad c_2 = 0, \quad c_3 = 1$

from ①,

$$\therefore y = e^{-2x}[c_1 + c_2x + c_3x^2]$$
$$= e^{-2x}[0 + 0 + 1] \Rightarrow y = e^{-2x}$$

① $\frac{d^3y}{dx^3} - 7\frac{dy}{dx} - 6y = 0 \longrightarrow ①$

$D^3y - 7Dy - 6y = 0$

$(D^3 - 7D - 6)y = 0$

An Auxiliary equⁿ is $m^3 - 7m - 6 = 0$

$(m+1)(m^2 - m - 6) = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -7 & -6 \\ & 0 & -1 & +1 & 6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$m+1 = 0$ and $m^2 - m - 6 = 0$

$\boxed{m=-1}$

$m^2 - 3m + 2m - 6 = 0$

$m(m-3) + 2(m-3) = 0$

$(m-3)(m+2) = 0$

$\boxed{m = -2, 3.}$

$\therefore m = -1, -2, 3.$

$\therefore$ The roots are real and distrinct.

Now, $C.F = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}$

Now, the solution of equⁿ ① is $y = C.F$

$y = c_1 e^{-x} + c_2 e^{-2x} + c_3 e^{3x}$

② Given D.E is $\frac{d^4y}{dx^4} + 13\frac{d^2y}{dx^2} + 36y = 0$

$D^4y + 13D^2y + 36y = 0$

$(D^4 + 13D^2 + 36)y = 0$

An Auxiliary equⁿ is $m^4 + 13m^2 + 36 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 13 & 36 \\ & 0 & -1 & & \\ \hline & 1 & -1 & & \end{array}$$

(3). Given D.E is $\frac{d^4x}{dt^4} + 4x = 0. \rightarrow (1)$

$(\sqrt{2})^4$ $\sqrt{2}\sqrt{2}\sqrt{2}\sqrt{2}$ $(\sqrt{2})^2(\sqrt{2})^4$

$$D^4x + 4x = 0$$

$$(D^4+4)x = 0.$$

An A.E is $m^4 + 4 = 0$

$$(m^2)^2 + (2)^2 = 0$$

$$(m^2+2)^2 - 2(2)m^2 = 0$$

$$(m^2+2)^2 - (2m)^2 = 0$$

$$(m^2+2+2m)(m^2+2-2m) = 0.$$

$$(m^2+2m+2)(m^2-2m+2) = 0$$

$m = \frac{-2 \pm \sqrt{4-8}}{2}$ $\qquad$ $m = \frac{2 \pm \sqrt{4-8}}{2}$

$= \frac{-2 \pm 2i}{2}$ $\qquad$ $= \frac{2 \pm 2i}{2}$

$= \frac{\cancel{2}(-1 \pm i)}{\cancel{2}}$ $\qquad$ $= \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$

$= -1 \pm i$ $\qquad$ $= 1 \pm i$

$m = -1 \pm i,\ 1 \pm i.$

$\therefore$ The roots are complex and distrinct.

Now the C.F $= e^{-t}[c_1 \cos t + c_2 \sin t] + e^{t}[c_3 \cos t + c_4 \sin t]$

$\therefore$ The solution of equ^n (1) is $x = C.F$

$$x = e^{-t}[c_1\cos t + c_2 \sin t] + e^{t}[c_3 \cos t + c_4 \sin t]$$

4 (7) Given D.E is $\frac{d^4x}{dt^4} = m^4 x. \rightarrow (1)$

$$D^4x = m^4x.$$

$$D^4x - m^4x = 0$$

$$x(D^4 - m^4) = 0$$

An A.E is $m^4 -$

(8) Given D.E is $(D^3+1)y=0 \rightarrow (1)$

An A.E is $m^3+1=0$

$$m^3+(1)^3=0$$

$$(m+1)^3-3m\cdot(m+1)=0$$

$$(m+1)[(m+1)^2-3m]=0$$

$$(m+1)[m^2+1+2m-3m]=0$$

$$(m+1)(m^2-m+1)=0$$

$$m=-1, \quad m=\frac{1\pm\sqrt{1-4}}{2}$$

$$=\frac{1\pm\sqrt{3}i}{2}$$

$$m=-1, \frac{1\pm\sqrt{3}i}{2}$$

∴ The roots are real, complex and distinct.

Now, $C.F = c_1e^{-x} + e^{\frac{1}{2}x}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\right) + c_2\sin\frac{\sqrt{3}}{2}\right]$

Now the solution of equ (1) is $y=C.F$

$$y=c_1e^{-x}+e^{\frac{1}{2}x}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\right)+c_2\sin\frac{\sqrt{3}}{2}\right]$$

(9) Given D.E is $(D^4+6D^3+11D^2+6D)y=0$

An A.E is $m^4+6m^3+11m^2+6m=0$

$$(m+1)(m+2)(m^2+3m)=0.$$

$$m+1=0, \quad m+2=0, \quad m^2+3m=0$$

$$m=-1, \quad m=-2, \quad m(m+3)=0$$

$$m=0, \quad m=-3.$$

$\therefore m=0,-1,-2,-3.$

∴ The roots are real and distinct

Now, the $C.F = c_1e^{(0)x}+c_2e^{-x}+c_3e^{-2x}+c_4e^{-3x}$

∴ the solution of equ (1) is $y=C.F$

$$y=c_1e^{(0)x}+c_2e^{-x}+c_3e^{-2x}+c_4e^{-3x}$$

(1) $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$

sol:- $D^3y - 6.D^2y + 11.Dy - 6y = 0$

$(D^3 - 6D^2 + 11D - 6)\, y = 0.$

$m^3 - 6m^2 + 11m - 6 = 0.$ (auxiliary equation)

$(m^2 - 5m + 6)(m-1) = 0$

$$\begin{array}{c|cccc} 1 & 1 & -6 & 11 & -6 \\ & 0 & 1 & -5 & 6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

$m - 1 = 0$ and $m^2 - 5m + 6 = 0$

$m = 1$ $\quad m^2 - 3m - 2m + 6 = 0$

$m(m-3) - 2(m-3) = 0$

$m = 2, \; m = 3.$

The roots are real and distinct.

$c_1e^x + c_2e^{2x} + c_3e^{3x}$

$\therefore$ The solution is $y = $ C.F (complementary function)

$y = c_1e^x + c_2e^{2x} + c_3e^{3x}$

(2) $\frac{d^3y}{dx^3} - 8y = 0.$

sol:- $D^3y - 8y = 0$

$(D^3 - 8)\, y = 0$

An auxiliary equn is $m^3 - 8 = 0.$

$m^3 = 8$

$m^3 = 2^3$

$\boxed{m = 2}$

$m^3 - 2^3 = 0$

$(m-2)(m^2 + 2m + 4) = 0.$

$m - 2 = 0$ and $m^2 + 2m + 4 = 0$

$\boxed{m = 2}$ $\quad m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$m = \frac{-2 \pm \sqrt{4-16}}{2}$$

$$= \frac{-2 \pm \sqrt{-12}}{2}$$

$$= \frac{-2 \pm 2\sqrt{3}i}{2}$$

$$= \frac{\cancel{2}(-1 \pm \sqrt{3}i)}{\cancel{2}}$$

$$\boxed{m = -1 \pm \sqrt{3}i.}$$

$m = 2, \ -1+\sqrt{3}i, \ -1-\sqrt{3}i.$

Now, Complementary function is

$$c_1 e^{2x} + c_2 e^{(-1+\sqrt{3}i)x} + c_3 e^{(-1-\sqrt{3}i)x}$$

$$= e^{-x}\left[c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right] + c_3 e^{2x}.$$

Now the solution is $y = C.F$

$$y = e^{-x}\left[c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x\right] + c_3 e^{2x}$$

<u>Non-Homogeneous Higher Order D.E:</u>

① ~~$\frac{d^3y}{dx^3} + y = 3+5e^{x}$~~

~~$\frac{d^3y}{dx^3} + y = 3+5e^{x}$~~

<u>Type-I</u>

② $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x$

<u>sol:-</u> $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = -2\cosh x \longrightarrow ①$

Equn ① is a non-homogeneous H.O.D Equn.

$D^2y + 4Dy + 5y = -2\cosh x.$

$(D^2+4D+5)y = -2\cosh x.$

An Auxiliary Equn is $m^2+4m+5=0$

$$m = \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm 2i}{2} = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

$$m = -2 \pm i$$

$\therefore$ The roots are complex and distinct.

Now, $C.F = e^{-2x}[c_1 \cos x + c_2 \sin x]$

And Particular

The particular Integral of the Equ^n ①

$$P.I = \frac{1}{f(D)} X$$

$$\sin hx = \frac{e^x - e^{-x}}{2}$$

$$\cos hx = \frac{e^x + e^{-x}}{2}$$

$$= \frac{1}{D^2+4D+5} -2\cos hx$$

$$= \frac{1}{D^2+4D+5} -\cancel{2}\left(\frac{e^x+e^{-x}}{\cancel{2}}\right)$$

$$= -\left[\frac{+1}{D^2+4D+5}(e^x+e^{-x})\right]$$

$$= -\left[\frac{1}{D^2+4D+5}e^x + \frac{1}{D^2+4D+5}e^{-x}\right]$$

$$= -\left[\frac{1}{(1)^2+4(1)+5}e^x + \frac{1}{(-1)^2+4(-1)+5}e^{-x}\right]$$

$$= -\left[\frac{1}{1+4+5}e^x + \frac{e^{-x}}{1-4+5}\right]$$

$$= -\left[\frac{e^x}{10} + \frac{e^{-x}}{2}\right]$$

$$\cancel{= + \left[\frac{1+5}{10}\right]}$$

$$\cancel{= -\left[\frac{6}{10}\right]}$$

$$\cancel{P.I = -\frac{3}{5} e^x}$$

$\therefore$ The solution of equ^n ① is $y = C.F + P.I$

$$y = e^{-2x}[c_1 \cos x + c_2 \sin x] - \frac{1}{10}e^x - \frac{1}{2}e^{-x}.$$

③

$$\frac{d^2y}{dx^2} - 4y = (1+e^x)^2 \rightarrow ①$$

$$D^2y - 4y = (1+e^x)^2$$

$$(D^2-4)y = 1+(e^x)^2 + 2e^x$$

An A.E is $m^2 - 4 = 0$

$$m^2 - (2)^2 = 0$$

$$(m+2)(m-2) = 0$$

$m = -2, 2.$

$\therefore$ The roots are real and distinct.

Now, The C.F $= c_1 e^{-2x} + c_2 e^{2x}$

Now the $P.I = \frac{1}{f(D)} (x)$

$$= \frac{1}{D^2-4} (1+e^x)^2$$

$$= \frac{1}{D^2-4} (1+e^{2x}+2e^x)$$

$$P.I = \frac{1}{D^2-4}(1) + \frac{1}{D^2-4} e^{2x} + \frac{1}{D^2-4} 2e^x.$$

$$= \frac{1}{D^2-4} e^{(0)x} + \frac{1}{D^2-4} e^{2x} + \frac{1}{D^2-4} 2e^x \longrightarrow ②$$

(P.I - 1) (P.I - 2) (P.I - 3)

$$PI_1 = \frac{1}{D^2-4} e^{(0)x} = \frac{1}{0-4} e^{(0)x} = \frac{-1}{4}.$$

$$PI_2 = \frac{1}{D^2-4} e^{2x}$$

$$= \frac{x}{2D-0} e^{2x} = \frac{x}{2(2)} e^{2x} = \frac{x}{4} e^{2x}$$

$$PI_3 = \frac{1}{D^2-4} 2e^x = \frac{2}{(1)^2-4} e^x = 2 \cdot \frac{1}{1-4} e^x = \frac{-2}{3} e^x.$$

Equ^n ②,

$$P.I = \frac{-1}{4} + \frac{x}{4} e^{2x} - \frac{2}{3} e^x.$$

Now the solution is $y = C.F + P.I$

$$y = c_1 e^{-2x} + c_2 e^{2x} - \frac{1}{4} + \frac{x}{4} e^{2x} - \frac{2}{3} e^x.$$

⑧

$$(D+2) \cdot (D-1)^2 y = e^{-2x} + 2\sin hx \longrightarrow ①$$

$$(D+2)(D^2-1-2D) y = e^{-2x} + 2\sin hx$$

An A.E is $(m+2)(m-1)^2 = 0.$

$m+2=0$, $(m-1)^2 = 0.$

$m=-2$, $(m-1)(m-1)=0$

$m=1$

$\therefore m = 1, 1, -2$

$\therefore$ The roots are real and district, repeat.

Now, the C.F $= C_1 e^{x} + C_2 x.e^{x} + C_3 e^{-2x}$.

Now the Particular Integral $= \frac{1}{F(D)} (x)$

$$P.I = \frac{1}{(D+2)(D-1)^2} (e^{-2x} + 2\sinh x)$$

$$= \frac{1}{(D+2)(D-1)^2} \left[e^{-2x} + \cancel{2}\left(\frac{e^{x}-e^{-x}}{\cancel{2}}\right)\right]$$

$$= \frac{1}{(D+2)(D-1)^2} \left[e^{-2x} + e^{x} - e^{-x}\right]$$

$$= \underbrace{\frac{1}{(D+2)(D-1)^2} e^{-2x}}_{(PI_1)} + \underbrace{\frac{1}{(D+2)(D-1)^2} e^{x}}_{(PI_2)}$$

$$- \underbrace{\frac{1}{(D+2)(D-1)^2} e^{-x}}_{(PI_3)} \longrightarrow (2)$$

$$PI_1 = \frac{1}{(D+2)\cancel{(D-1)^2}} e^{-2x}$$

$$= \frac{x}{(1+0)\ 2\cancel{(D-1)}(1-0)} e^{-2x}$$

$$= \frac{x}{2(\cancel{-2}-1)} \cancel{e^{-2x}} = \cancel{\frac{x}{-6}} \cancel{e^{-2x}} = \cancel{\frac{-x}{6}} \cancel{e^{-2x}}$$

$$\cancel{PI_2 = \frac{1}{(D+2)(D-1)^2} e^{x}}$$

$$\cancel{= \frac{x}{\ }}$$

$$PI_1 = \frac{1}{(D+2)(D-1)^2} e^{-2x}$$

$$= \frac{x}{(D+2)\ 2(D-1) + (D-1)^2(1+0)} e^{-2x}$$

$$= \frac{x}{(-2+2)\ 2(-2-1) + (-2-1)^2} e^{-2x}$$

$$= \frac{x}{0 + (-3)^2} e^{-2x} = \frac{x}{9} e^{-2x}$$

$$PI_2 = \frac{1}{(D+2)(D-1)^2} e^{x}$$

$$= \frac{x}{(D+2).2(D-1) + (D-1)^2(1+0)} e^{x}$$

$$\cancel{= \frac{x}{(1+2)\ 2(1-1) + (1-1)}}$$

$$= \frac{x}{(D-1)\left[2(D+2) + (D-1)\right]} e^{x}$$

$$= \frac{x^2}{(D-1)[2(1+0)+(1-0)]+[2(D+2)+(D-1)](1-0)} e^x$$

$$= \frac{x^2}{(1-1)[2+1]+[2(1+3)+(1-1)](1)} e^x$$

$$= \frac{x^2}{0+2(3)+0} e^x$$

$$= \frac{x^2}{6} e^x.$$

$$PI_3 = \frac{1}{(D+2)(D-1)^2} e^{-x}$$

$$= \frac{1}{(-1+2)(-1-1)^2} e^{-x}$$

$$= \frac{1}{(1)(-2)^2} e^{-x} = \frac{1}{4} e^{-x}.$$

from ②,

$$P.I = \frac{x}{9} e^{-2x} + \frac{x^2}{6} e^x + \frac{1}{4} e^{-x}.$$

Now the solution of eqn ① is $y = C.F + P.I$

$$y = c_1 e^x + c_2 x e^x + c_3 e^{-2x} + \frac{x}{9} e^{-2x} + \frac{x^2}{6} e^x + \frac{1}{4} e^{-x}$$

⑨ Given D.E is $\frac{d^2y}{dx^2} - 4y = \cosh(2x-1) + 3^x$ → ①

$$D^2y - 4y = \cosh(2x-1) + 3^x$$

$$(D^2-4)y = \cosh(2x-1) + 3^x$$

An AE is $m^2 - 4 = 0$

$$m^2 - (2)^2 = 0$$

$$(m+2)(m-2) = 0$$

$$m = -2, 2.$$

∴ The the roots are real and district.

Now, the $C.F = c_1 e^{-2x} + c_2 e^{2x}$.

Now, the particular Integral $= \frac{1}{F(D)} X.$

$$= \frac{1}{D^2-4} [\cosh(2x-1) + 3^x]$$

$$= \frac{1}{D^2-4} \cosh(2x-1) + \frac{1}{D^2-4} 3^x$$

$$= \frac{1}{D^2-4}\left[\cosh(2x)\cdot\cosh(1) - \sinh(2x)\cdot\sinh(1)\right] + \frac{1}{D^2-4}3^x$$

$$= \frac{1}{D^2-4}\cosh(2x)\cosh(1) - \frac{1}{D^2-4}\sinh(2x)\cdot\sinh(1) + \frac{1}{D^2-4}3^x$$

$$P.I = \cosh(1)\underbrace{\frac{1}{D^2-4}\cosh(2x)}_{PI_1} - \sinh(1)\underbrace{\frac{1}{D^2-4}\sinh(2x)}_{PI_2} + \underbrace{\frac{1}{D^2-4}3^x}_{PI_3} \longrightarrow (2)$$

$$P.I_1 = \frac{1}{D^2-4}\cosh 2x$$

$$= \frac{1}{D^2-4}\frac{e^{2x}+e^{-2x}}{2}$$

$$= \frac{1}{2}\left[\frac{1}{D^2-4}e^{2x} + \frac{1}{D^2-4}e^{-2x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{2D}e^{2x} + \frac{x}{2D}e^{-2x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{4}e^{2x} + \frac{x}{-4}e^{-2x}\right]$$

$$= \frac{1}{2}\cdot\frac{x}{4}\left[e^{2x} - e^{-2x}\right]$$

$$= \frac{x}{4}\left(\frac{e^{2x}-e^{-2x}}{2}\right) = \underline{\frac{x}{4}\sinh(2x)}$$

$\cosh(a \pm b) =$
$\cosh a \cdot \cosh b \pm \sinh a \sinh b$

$\sinh(a \pm b) =$
$\sinh a \cosh b \pm \cosh a \sinh b$

$$PI_2 = \frac{1}{D^2-4}\sinh(2x)$$

$$= \frac{1}{D^2-4}\left(\frac{e^{2x}-e^{-2x}}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^2-4}e^{2x} - \frac{1}{D^2-4}e^{-2x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{2D}e^{2x} - \frac{x}{2D}e^{-2x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{4}e^{2x} - \left(\frac{x}{-4}\right)e^{-2x}\right]$$

$$= \frac{1}{2}\left[\frac{x}{4}e^{2x} + \frac{x}{4}e^{-2x}\right]$$

$$= \frac{x}{4}\left[\frac{e^{2x}+e^{-2x}}{2}\right] = \frac{x}{4}\cosh(2x)$$

$$PI_3 = \frac{1}{D^2-4}3^x$$

$$= \frac{1}{D^2-4}e^{\log 3^x}$$

$$= \frac{1}{D^2-4}e^{x\log 3}$$

$$= \frac{1}{D^2-4}e^{(\log 3)x}$$

$$= \frac{1}{(\log 3)^2-4}e^{(\log 3)x}$$

$$= \frac{1}{(\log 3)^2-4}3^x.$$

$$P.I = \frac{x}{4}\sinh(2x)\cosh(1) - \frac{x}{4}\cosh(2x)\sinh(1) + \frac{1}{(\log 3)^2 - 4} 3^x$$

$$= \frac{x}{4}\left[\sinh(2x)\cosh(1) - \cosh(2x)\sinh(1)\right] + \frac{1}{(\log 3)^2 - 4} 3^x$$

$$= \frac{x}{4}\sinh(2x-1) + \frac{1}{(\log 3)^2 - 4} 3^x.$$

$\therefore$ the solution of equⁿ ① is $y = C.F + P.I$

$$y = c_1 e^{-2x} + c_2 e^{2x} + \frac{x}{4}\sinh(2x-1) + \frac{1}{(\log 3)^2 - 4} \cdot 3^x.$$

Wednesday 30/10/19

Type - II

④ $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$

sol:

$$D^2y + 2Dy + y = e^{2x} - \cos^2 x.$$

$$(D^2 + 2D + 1)y = e^{2x} - \cos^2 x.$$

An A.E is $m^2 + 2m + 1 = 0$

$$m^2 + m + m + 1 = 0$$

$$m(m+1) + 1(m+1) = 0$$

$$(m+1)(m+1) = 0$$

$$\therefore m = -1, -1$$

$\therefore$ The roots are real and repeat.

Now, the $C.F = c_1 e^{-x} + c_2 x.e^{-x}$.

$$PI = \frac{1}{f(D)} X$$

$$= \frac{1}{D^2 + 2D + 1}(e^{2x} - \cos^2 x)$$

$$= \underbrace{\frac{1}{D^2 + 2D + 1} e^{2x}}_{PI_1} - \underbrace{\frac{1}{D^2 + 2D + 1}\cos^2 x}_{PI_2}$$

$$PI_1 = \frac{1}{D^2 + 2D + 1} e^{2x}$$

$$= \frac{1}{4 + 4 + 1} e^{2x} = \frac{1}{9} e^{2x}$$

$$PI_2 = \frac{1}{D^2+2D+1}\cos^2 x$$

$$\boxed{\cos^2 x = \frac{1+\cos 2x}{2} \qquad \sin^2 x = \frac{1-\cos 2x}{2}}$$

$$= \frac{1}{D^2+2D+1}\left(\frac{1+\cos 2x}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^2+2D+1}(1+\cos 2x)\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2+2D+1}(1) + \frac{1}{D^2+2D+1}(\cos 2x)\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2+2D+1}e^{(0)x} + \frac{1}{D^2+2D+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[\frac{1}{0+0+1}e^{(0)x} + \frac{1}{-4+2D+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2D-3}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{2D-3}\times\frac{2D+3}{2D+3}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{2D+3}{4D^2-9}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{2D+3}{4(-4)-9}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{2D+3}{-16-9}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{2D+3}{-25}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{2D+3}{25}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{25}(2D\cos 2x + 3\cos 2x)\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{25}(2 - \sin 2x(2) + 3\cos 2x)\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{25}(-4\sin 2x + 3\cos 2x)\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{25}(3\cos 2x - 4\sin 2x)\right]$$

$$= \frac{1}{2} - \frac{1}{50}(3\cos 2x - 4\sin 2x)$$

$$= \frac{1}{2} - \frac{3}{50}\cos 2x + \frac{2}{25}\sin 2x.$$

$$PI = \frac{1}{9}e^{2x} + \frac{1}{2} - \frac{3}{50}\cos 2x + \frac{2}{25}\sin 2x$$

Now, the solution is $y = CF + P.I$

$$y = \frac{1}{9}e^{2x} + \frac{1}{2} - \frac{3}{50}\cos 2x + \frac{2}{25}\sin 2x + C_1 e^{-x} + C_2 x e^{-x}$$

(5). $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{-x} + \sin 2x$

Sol:- $D^3y + 2D^2y + Dy = e^{-x} + \sin 2x$

$(D^3 + 2D^2 + D)y = e^{-x} + \sin 2x$

A.E is $m^3 + 2m^2 + m = 0$

$(m+1)(m^2+m) = 0$

$$\begin{array}{r|cccc} -1 & 1 & 2 & 1 & 0 \\ & 0 & -1 & -1 & 0 \\ \hline & 1 & 1 & 0 & \boxed{0} \end{array}$$

$m+1=0 \qquad m(m+1) = 0$

$m = -1, \qquad m+1 = 0$

$m = -1, \; m = 0$

$\therefore m = -1, -1, 0$

$\therefore$ The roots are real and repeat.

Now, $C.F = C_1 e^{-x} + C_2 x e^{-x} + C_3 e^{(0)x}$.

$$PI = \frac{1}{D^3+2D^2+D}(e^{-x} + \sin 2x)$$

$$= \underbrace{\frac{1}{D^3+2D^2+D}e^{-x}}_{PI_1} + \underbrace{\frac{1}{D^3+2D^2+D}\sin 2x}_{PI_2}$$

$$PI_1 = \frac{1}{D^3+2D^2+D}e^{-x}$$

$$= \frac{x}{3D^2+4D+1}e^{-x}$$

$$= \frac{x^2}{6D+4}e^{-x}$$

$$= \frac{x^2}{-6+4}e^{-x} = \frac{-x^2}{2}e^{-x}$$

$$PI_2 = \frac{1}{D^3+2D^2+D}\sin 2x$$

$$= \frac{1}{D^2 \cdot D + 2D^2 + D}\sin 2x$$

$$= \frac{1}{(-4)D + 2(-4) + D}\sin 2x$$

$$= \frac{1}{-4D - 8 + D}\sin 2x$$

$$= \frac{1}{-3D-8} \sin 2x$$

$$= \frac{1}{-3D-8} \times \frac{-3D+8}{-3D+8} \sin 2x$$

$$= \frac{-3D+8}{9D^2-64} \sin 2x$$

$$= \frac{-3D+8}{9(-4)-64} \sin 2x$$

$$= \frac{-3D+8}{-36-64} \cdot \sin 2x$$

$$= \frac{\cancel{-}(3D-8)}{\cancel{-}100} \sin 2x$$

$$= \frac{1}{100}\left[3D \sin 2x - 8 \sin 2x\right]$$

$$= \frac{1}{100}\left[3 \cos 2x (2) - 8 \sin 2x\right]$$

$$= \frac{\cancel{6}^{3}}{\cancel{100}_{50}} \cdot \cos 2x - \frac{\cancel{8}^{2}}{\cancel{100}_{25}} \sin 2x$$

$$= \frac{3}{50} \cos 2x - \frac{2}{25} \sin 2x.$$

$$PI = \frac{-x^2}{2} e^{-x} + \frac{3}{50} \cos 2x - \frac{2}{25} \sin 2x$$

Now, the solution is $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 x e^{-x} + c_3 e^{(0)x} - \frac{x^2}{2} e^{-x} + \frac{3}{50} \cos 2x - \frac{2}{25} \sin 2x$$

⑥ $(D^2+D+1)y = (1+\sin x)^2$

sol: $(D^2+D+1)y = 1 + \sin^2 x + 2\sin x$

An A.E is $m^2+m+1 = 0$

$$m = \frac{-1 \pm \sqrt{1-4}}{2}$$

$$\therefore m = \frac{-1 \pm \sqrt{3} i}{2}$$

$$\therefore m = \frac{-1}{2} \pm \frac{\sqrt{3}}{2} i$$

∴ The roots are complex and distinct.

Now, $C.F = e^{-\frac{1}{2}x}\left[c_1 \cos \frac{\sqrt{3}}{2} x + c_2 \sin \frac{\sqrt{3}}{2} x\right]$

$$PI = \frac{1}{D^2+D+1}(1+\sin^2 x + 2\sin x)$$

$$= \frac{1}{D^2+D+1}(1) + \frac{1}{D^2+D+1}\sin^2 x + \frac{1}{D^2+D+1} 2\sin x$$

$$\quad PI_1 \qquad PI_2 \qquad PI_3 \longrightarrow (2)$$

$$PI_1 = \frac{1}{D^2+D+1} e^{(0)x}$$

$$= \frac{1}{0+0+1} e^{(0)x}$$

$$= (1) e^{(0)x}.$$

$$PI_2 = \frac{1}{D^2+D+1} \sin^2 x$$

$$= \frac{1}{D^2+D+1}\left(\frac{1-\cos 2x}{2}\right)$$

$$= \frac{1}{2}\left[\frac{1}{D^2+D+1} - \frac{1}{D^2+D+1}.\cos 2x\right]$$

$$= \frac{1}{2}\left[\frac{1}{D^2+D+1} e^{(0)x} - \frac{1}{D^2+D+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[\frac{1}{0+0+1} e^{(0)x} - \frac{1}{-4+D+1}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{D-3}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{1}{D-3}\times\frac{D+3}{D+3}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{D+3}{D^2-9}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{D+3}{-4-9}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 - \frac{D+3}{-13}\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{D+3}{13}.\cos 2x\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{13}(D\cos 2x + 3.\cos 2x)\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{13}(-\sin 2x(2) + 3\cos 2x)\right]$$

$$= \frac{1}{2}\left[1 + \frac{1}{13}(-2\sin 2x + 3\cos 2x)\right]$$

$$= \frac{1}{2} - \frac{\sin 2x}{13} + \frac{3}{2}\cos 2x.$$

$$PI_3 = \frac{1}{D^2+D+1} 2\sin x.$$

$$= 2 \frac{1}{D^2+D+1} \sin x$$

$$= 2 \frac{1}{-\cancel{1}+D+\cancel{1}} \sin x.$$

$$= 2 \frac{1}{D} \sin x.$$

$$= 2(-\cos x).$$

$$PI = 1 + \frac{1}{2} - \frac{1}{13}\sin 2x + \frac{3}{2}\cos 2x - 2\cos x.$$

Now, the solution of ~~eqn~~ is $y = C.F + P.I$

$$y = e^{-1/2 x}\left[c_1\cos\left(\frac{\sqrt{3}}{2}\right)x + c_2\sin\frac{\sqrt{3}}{2}x\right] + 1 + \frac{1}{2} - \frac{1}{13}\sin 2x + \frac{3}{2}\cos 2x - 2\cos x.$$

① $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x.$

Given D.E is $D^3y + D^2y + Dy + y = \sin 2x.$

$$(D^3 + D^2 + D + 1)y = \sin 2x.$$

An A.E is $m^3 + m^2 + m + 1 = 0.$

$$(m+1)(m^2+1) = 0.$$

$m+1 = 0,$ $\quad \cancel{m^2 + (1)^2 = 0}$

$\cancel{(m+1)^2 - 2m = 0}$

$m = -1,$ $\quad m = \pm i$

$\therefore$ The roots are real, complex and distinct.

$$C.F = c_1 e^{-x} + e^{(0)x}[c_1\cos x + c_2\sin x]$$

$$PI = \frac{1}{D^3+D^2+D+1} \sin 2x$$

$$= \frac{1}{-4D-4+D+1} \sin 2x$$

$$= \frac{1}{-3D-3} \sin 2x$$

$$= \frac{1}{-3D-3} \times \frac{-3D+3}{-3D+3} \sin 2x$$

$$= \frac{-3D+3}{9D^2-9} \sin 2x$$

$$= \frac{-3D+3}{9(-4)-9} \sin 2x$$

$$= \frac{-3D+3}{-36-9} \sin 2x$$

$$= \frac{-3D+3}{-45} \sin 2x$$

$$= \frac{\cancel{-}(3D-3)}{\cancel{-}45} \sin 2x$$

$$= \frac{\cancel{3}(D-1)}{\cancel{45}\,15} \sin 2x$$

$$= \frac{D-1}{15} \sin 2x$$

$$= \frac{1}{15}\left[D\cdot \sin 2x - \sin 2x\right]$$

$$= \frac{1}{15}\left[\cos 2x.(2) - \sin 2x\right]$$

$$= \frac{1}{15}\left[2\cos 2x - \sin 2x\right]$$

$$P.I. = \frac{1}{15}\left[2\cos 2x - \sin 2x\right]$$

∴ The solution of equ^n (1) is $y = C.F + P.I$

$$y = c_1 e^{-x} + e^{(0)x}\left[c_1 \cos x + c_2 \sin x\right] + \frac{1}{15}\left[2\cos 2x - \sin 2x\right]$$

② $\frac{d^2y}{dx^2} + \frac{dy}{dx} = \cos 2x$

sol: $D^2y + Dy = \cos 2x$.

$(D^2 + D)\, y = \cos 2x$.

An A.E is $m^2 + m = 0$

$m(m+1) = 0$

$m = 0, \quad m = -1$

∴ The roots are real and ~~and~~ distrinct.

Now $C.F = c_1 e^{(0)x} + c_2 e^{-x}$.

$$PI = \frac{1}{D^2 + D} \cos 2x$$

$$= \frac{1}{-4 + D} \cos 2x$$

$$= \frac{1}{-4+D} \times \frac{-4-D}{-4-D} \cos 2x$$

$$= \frac{-4-D}{16 - D^2} \cos 2x$$

$$= \frac{-4-D}{16 - (-4)}$$

$$= \frac{-4-D}{20} \cos 2x$$

$$= \frac{-1}{20} \left[4\cos 2x + D\cos 2x\right]$$

$$= \frac{-1}{20} \left[4\cos 2x + (-\sin 2x)\, 2\right]$$

$$= \frac{-1}{20} \left[4\cos 2x - 2\sin 2x\right]$$

$$= \frac{-1}{5} \cos 2x + \frac{1}{10} \sin 2x.$$

Now the solution is $y = C.F + PI$

$$y = c_1 e^{(0)x} + c_2 e^{-x} - \frac{1}{5}\cos 2x + \frac{1}{10}\sin 2x$$

③ $(D^3+1)\,y = 2\cos^2 x$.

Given D.E. is $(D^3+1)\,y = 2\cos^2 x$

An A.E is $m^3 + 1 = 0$

$(m+1)(m^2 - m + 1) = 0$

$m = -1, \quad m = \frac{-1 \pm \sqrt{1-4}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$

∴ The roots are real, ~~and~~ complex and distrinct.

Now, $C.F = c_1 e^{-x} + e^{-1/2 x}\left[c_2 \cos \frac{\sqrt{3}}{2}x + c_3 \sin \frac{\sqrt{3}}{2}x\right]$

$PI = \frac{1}{D^3+1} 2\cos^2 x$

$= 2\left[\frac{1}{D^3+1} \cos^2 x\right]$

$= 2\left[\frac{1}{D^3+1}\left(\frac{1+\cos 2x}{2}\right)\right]$

$= \frac{1}{D^3+1}(1) + \frac{1}{D^3+1}\cos 2x$

$= \frac{1}{D^3+1} e^{(0)x} + \frac{1}{D^3+1}\cos 2x$

$= \frac{1}{(0)0+1} e^{(0)x} + \frac{1}{-4D+1}\cos 2x$

$= 1 + \frac{1}{-4D+1} \times \frac{-4D-1}{-4D-1}\cos 2x$

$= 1 + \frac{-4D-1}{16D^2-1}\cos 2x.$

$= 1 - \frac{4D+1}{16D^2+1}\cos 2x.$

$= 1 - \frac{4D+1}{-64+1}\cos 2x$

$= 1 + \frac{4D+1}{63}\cos 2x$

$= 1 + \frac{4D}{63}\cos 2x + \frac{1}{63}\sin 2x$

$= 1 + \frac{4}{63}(-\sin 2x)(2) + \frac{1}{63}\sin 2x$

$= 1 - \frac{8}{63}\sin 2x + \frac{1}{63}\sin x.$

~~the solution~~

$\therefore$ The solution of is $y = C.F + PI$

$y = c_1 e^{-x} + e^{-1/2x}\left[\cos \frac{\sqrt{3}}{2}x + \sin \frac{\sqrt{3}}{2}x\right] + 1 - \frac{8}{63}\sin 2x + \frac{1}{63}\sin x.$

(8) $(D^2-3D+2)y = 6e^{-3x} + \sin 2x$

Soln Given D.E is $(D^2-3D+2)y = 6e^{-3x} + \sin 2x \rightarrow (1)$

A.E is $m^2-3m+2=0$

$m^2-m-2m+2=0$

$m(m-1)-2(m-1)=0$

$(m-1)(m-2)=0$

$m = 1, 2$

$\therefore$ The roots are real and distinct.

Now, $C.F = c_1 e^{x} + c_2 e^{2x}$

Now, the $PI = \frac{1}{D^2-3D+2}\left(6 e^{-3x} + \sin 2x\right)$

$$= \underbrace{6 \cdot \frac{1}{D^2-3D+2} e^{-3x}}_{PI_1} + \underbrace{\frac{1}{D^2-3D+2} \sin 2x}_{PI_2} \longrightarrow (2)$$

$PI_1 = 6 \frac{1}{D^2-3D+2} e^{-3x}$

$= 6 \frac{1}{9+9+2} e^{-3x}$

$= \cancel{6}^{3} \frac{1}{\cancel{20}_{10}} e^{-3x} \quad = \frac{3}{10} e^{-3x}$

$PI_2 = \frac{1}{D^2-3D+2} \sin 2x.$

$= \frac{1}{-4-3D+2} \sin 2x.$

$= \frac{1}{-3D-2} \sin 2x$

$= \frac{1}{-3D-2} \times \frac{-3D+2}{-3D+2} \sin 2x$

$= \frac{-3D+2}{9D^2-4} \sin 2x$

$= \frac{-3D+2}{-36-4} \sin 2x = \frac{-3D+2}{-40} \sin 2x.$

$= \frac{-3}{-40}[D \sin 2x] + \frac{\cancel{2}}{\cancel{-40}_{20}}[\sin 2x]$

$= \frac{3}{\cancel{40}_{20}} \cos 2x (\cancel{2}) - \frac{1}{20} \sin 2x.$

$= \frac{3}{20} \cos 2x - \frac{1}{20} \sin 2x.$

from (2),

$PI = \frac{3}{10} e^{-3x} + \frac{3}{20} \cos 2x - \frac{1}{20} \sin 2x.$

$= \frac{1}{10}\left[3 \cdot e^{-3x} + \frac{3}{2} \cos 2x - \frac{1}{2} \sin 2x\right]$

$\therefore$ The solution of equ^n (1) is $y = C.F + P.I$

$$y = c_1 e^{x} + c_2 e^{2x} + \frac{1}{10}\left[3 \cdot e^{-3x} + \frac{3}{2} \cos 2x - \frac{1}{2} \sin 2x\right]$$

⑨ $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$

Sol: Given D.E is $\frac{d^2y}{dx^2} + 4y = e^x + \sin 2x$.

$$D^2y + 4y = e^x + \sin 2x.$$

$$(D^2+4)\, y = e^x + \sin 2x \quad \rightarrow ①$$

An AE is $m^2+4=0$

$$m^2 = -4$$

$$m = \frac{0 \pm \sqrt{0-16}}{2}$$

$$= \frac{\pm\sqrt{16}\, i}{2}$$

$$= \frac{\pm \cancel{4}^{2} i}{\cancel{2}}$$

$$m = \pm 2i$$

∴ The roots are complex and distrinct.

Now, the C.F $= e^{(0)x}[c_1 \cos 2x + c_2 \sin 2x]$

Now, the P.I $= \frac{1}{D^2+4}(e^x + \sin 2x)$

$$= \underbrace{\frac{1}{D^2+4} e^x}_{P.I_1} + \underbrace{\frac{1}{D^2+4} \sin 2x}_{PI_2} \quad \longrightarrow ②$$

$$\cancel{= \frac{1}{1+4} e^x + \frac{1}{-4+4}}$$

$$PI_1 = \frac{1}{D^2+4} e^x = \frac{1}{1+4} e^x = \frac{1}{5} e^x$$

$$PI_2 = \frac{1}{D^2+4} \sin 2x$$

$$= \frac{x}{2D} \sin 2x$$

$$= \frac{x}{2} \frac{1}{D} \sin 2x$$

$$= \frac{x}{2} \cdot \frac{-\cos 2x}{2}$$

$$= -\frac{x}{4} \cdot \cos 2x.$$

$$PI = \frac{1}{5} e^x - \frac{x}{4} \cos 2x.$$

Now the solution of Equ ① is $y = C.F + P.I$

$$y = e^{(0)x}\{c_1 \cos 2x + c_2 \sin 2x\} + \frac{1}{5} e^x - \frac{x}{4} \cos 2x.$$

(7) $(D^2-4D+3)y = \sin 3x \cos 2x$.

Given DE is $(D^2-4D+3)y = \sin 3x \cdot \cos 2x$ → (1).

An A.E is $m^2-4m+3=0$

$m^2-m-3m+3=0$

$m(m-1)-3(m-1)=0$

$(m-1)(m-3)=0$

$m=1, 3.$

∴ The roots are real and distrinct.

Now, the $C.F = c_1e^{x}+c_2e^{3x}$

$$P.I = \frac{1}{D^2-4D+3}\sin 3x \cdot \cos 2x$$

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A+B)+\sin(A-B)]$

$\cos A \cdot \cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$

$\sin A \cdot \sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$

$$= \frac{1}{D^2-4D+3}\frac{1}{2}[\sin 5x+\sin x]$$

$$= \frac{1}{2}\left[\frac{1}{D^2-4D+3}\sin 5x + \frac{1}{D^2-4D+3}\sin x\right]$$

$$= \underbrace{\frac{1}{2}\frac{1}{D^2-4D+3}\sin 5x}_{PI_1} + \underbrace{\frac{1}{2}\frac{1}{D^2-4D+3}\sin x}_{PI_2} \longrightarrow (2).$$

$$PI_1 = \frac{1}{D^2-4D+3}\sin 5x$$

$$= \frac{1}{-25-4D+3}\sin 5x$$

$$= \frac{1}{-4D-22}\sin 5x$$

$$= \frac{1}{-4D-22}\times\frac{-4D+22}{-4D+22}\sin 5x$$

$$= \frac{-4D+22}{16D^2-484}\sin 5x.$$

$$= \frac{-4D+22}{16(-25)-484}\sin 5x$$

$$= \frac{-4D+22}{-400-484}\sin 5x$$

$$= \frac{\cancel{-}(4D-22)}{\cancel{-}884}\sin 5x$$

$$=\frac{1}{884}\left[4\,(D\sin 5x)-22\sin 5x\right]$$

$$=\frac{1}{884}\left[4.\cos 5x\,(5)-22\sin 5x\right]$$

$$=\frac{\cancel{20}^{5}}{\cancel{884}_{221}}\cos 5x-\frac{\cancel{22}^{11}}{\cancel{884}_{442}}\sin 5x$$

$$=\frac{5}{221}\cos 5x-\frac{11}{442}\sin 5x.$$

$$PI_2=\frac{1}{D^2-4D+3}\sin x$$

$$=\frac{1}{-1-4D+3}\sin x$$

$$=\frac{1}{-4D+2}\sin x=\frac{1}{-4D+2}\times\frac{-4D-2}{-4D-2}\sin x$$

$$=\frac{-4D+2}{16D^2-4}\sin x$$

$$=\frac{-4D-2}{16(-1)-4}\sin x$$

$$=\frac{-(4D+2)}{-16-4}\sin x$$

$$=\frac{+\cancel{2}(2D+1)}{+\cancel{20}_{10}}\sin x=\frac{1}{10}\left[2(D\sin x)+\sin x\right]$$

$$=\frac{1}{10}\left[2.\cos x+\sin x\right]$$

$$=\frac{2}{10}\cos x+\frac{1}{10}\sin x.$$

$$=\frac{1}{5}\cos x+\frac{1}{10}\sin x.$$

$$PI=\frac{1}{2}\left[\frac{5}{221}\cos 5x-\frac{11}{442}\sin 5x+\frac{1}{5}\cos x+\frac{1}{10}\sin x\right]$$

Now the solution of Equ^n ① is $y=C.F+P.I$

$$y=c_1e^{x}+c_2e^{3x}+\frac{1}{2}\left[\frac{5}{221}\cos 5x-\frac{11}{442}\sin 5x+\frac{1}{5}\cos x+\frac{1}{10}\sin x\right]$$

(10) $\frac{d^3y}{dx^3}+y=\cos(2x-1)$

Given D.E is $\frac{d^3y}{dx^3}+y=\cos(2x-1)$

$$D^3y+y=\cos(2x-1)$$

$$(D^3+1)\,y=\cos(2x-1)\quad\rightarrow ①$$

An A.E is $m^3+1=0$.

$$(m+1)(m^2-m+1)=0$$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & 0 & 1 \\ & 0 & -1 & 1 & -1 \\ \hline & 1 & -1 & 1 & \boxed{0} \end{array}$$

$$(m+1)(m^2-m+1)=0$$

$m+1=0, \quad m^2-m+1=0$

$m=-1, \quad m=\frac{1\pm\sqrt{1-4}}{2}$

$$=\frac{1\pm\sqrt{3}i}{2}$$

$\therefore m=-1,\ \frac{1}{2}\pm\frac{\sqrt{3}}{2}i$

$\therefore$ The roots are real, complex and distinct.

Now, the $C.F=c_1e^{-x}+e^{\frac{1}{2}x}\left[c_2\cos\frac{\sqrt{3}}{2}x+c_3\sin\frac{\sqrt{3}}{2}x\right]$

$$P.I=\cancel{\frac{1}{\cos(2x-1)}}\ \frac{1}{D^3+1}\cos(2x-1)$$

$$=\frac{1}{D^3+1}\left[\cos 2x.\cos(1)+\sin 2x\sin(1)\right]$$

$$=\underbrace{\cos(1)\frac{1}{D^3+1}\cos 2x}_{PI_1}+\underbrace{\sin(1)\frac{1}{D^3+1}\sin 2x}_{PI_2}\quad\longrightarrow (2)$$

$$PI_1=\cos(1)\frac{1}{D^3+1}\cos 2x$$

$$=\cos(1)\frac{1}{D^2\cdot D+1}\cos 2x$$

$$=\cos(1)\frac{1}{-4D+1}\cos 2x$$

$$=\cos(1)\frac{1}{-4D+1}\times\frac{-4D-1}{-4D-1}\cos 2x$$

$$=\cos(1)\frac{-4D-1}{16D^2-1}\cos 2x$$

$$=\cos(1)\frac{-(4D+1)}{16(-4)-1}\cos 2x$$

$$=\cos(1)\frac{+(4D+1)}{\cancel{-}65}\cos 2x$$

$$=\frac{\cos(1)}{65}\left[4(D\cos 2x)+\cos 2x\right]$$

$$=\frac{\cos(1)}{65}\left[4.(-\sin 2x)(2)+\cos 2x\right]$$

$$=\frac{\cos(1)}{65}\left[-8\sin 2x+\cos 2x\right]$$

$$PI_2 = \sin(1)\frac{1}{D^3+1}\sin 2x$$

$$= \sin(1)\frac{1}{-4D+1}\sin 2x$$

$$= \sin(1)\frac{1}{-4D+1}\times\frac{-4D-1}{-4D-1}\sin 2x$$

$$= \sin(1)\frac{-4D-1}{16D^2-1}\sin 2x$$

$$= \sin(1)\frac{-(4D+1)}{+16(4)-1}\sin 2x$$

$$= \sin(1)\frac{+(4D+1)}{+65}\sin 2x$$

$$= \frac{\sin(1)}{65}\left[4(D\sin 2x)+\sin 2x\right]$$

$$= \frac{\sin(1)}{65}\left[4.\cos 2x(2)+\sin 2x\right]$$

$$= \frac{\sin(1)}{65}\left[8\cos 2x+\sin 2x\right]$$

$$PI = \frac{\cos(1)}{65}\left(-8\sin 2x+\cos 2x\right)+\frac{\sin(1)}{65}\left(8\cos 2x+\sin 2x\right)$$

∴ The solution of eqn is, $y = C.F + P.I$.

$$y = c_1e^{-x} + e^{-\frac{1}{2}x}\left[c_1\cos\frac{\sqrt{3}}{2}x + c_2\sin\frac{\sqrt{3}}{2}x\right]$$

$$+\frac{\cos(1)}{65}\left(-8\sin 2x+\cos 2x\right)+\frac{\sin(1)}{65}\left(8\cos 2x+\sin 2x\right)$$

Monday
4/4/2019

Type - III

① $\frac{d^2y}{dx^2}+\frac{dy}{dx} = x^2+2x+4$

Sol: Given D.E is $D^2y + Dy = x^2+2x+4$.

$$(D^2+D)y = x^2+2x+4 \rightarrow ①$$

An A.E is $m^2+m=0$

$m(m+1)=0$

$m=0,-1$

∴ The roots are real and distinct.

Now, $C.F = c_1e^{(0)x} + c_2e^{-x}$

$$P.I = \frac{1}{D^2+D}(x^2+2x+4)$$

$$= \frac{1}{D(D+1)}(x^2+2x+4)$$

$$= \frac{1}{D(1+D)}(x^2+2x+4)$$

$$= \frac{1}{D}(1+D)^{-1}(x^2+2x+4)$$

$$= \frac{1}{D}[1-D+D^2-D^3+\cdots](x^2+2x+4)$$

$$= \frac{1}{D}[x^2+2x+4-(2x+2)+2]$$

$$= \frac{1}{D}[x^2+\cancel{2x}+4-\cancel{2x}-\cancel{2}+\cancel{2}]$$

$$= \frac{1}{D}(x^2+4)$$

$$P.I. = \frac{x^3}{3}+4x.$$

Now the solution of equ① is $y = C.F + P.I$

$$y = c_1 e^{(0)x} + c_2 e^{-x} + \frac{x^3}{3} + 4x.$$

② $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1+x^2$

sol: Given D.E is $D^3y - D^2y - 6Dy = 1+x^2$

$$(D^3-D^2-6D)y = 1+x^2 \longrightarrow ①$$

An A.E is $m^3-m^2-6m = 0.$

$$\begin{array}{c|cccc} 3 & 1 & -1 & -6 & 0 \\ & 0 & 3 & 6 & 0 \\ \hline & 1 & 2 & 0 & 0 \end{array}$$

$$(m-3)(m^2+2m) = 0$$

$$(m-3)\,m(m+2) = 0$$

$$m=0,\quad m=-2,\quad m=3.$$

∴ The roots are real and distinct.

$$C.F = c_1 e^{(0)x} + c_2 e^{-2x} + c_3 e^{3x}$$

$$P.I = \frac{1}{D^3-D^2-6D}(1+x^2)$$

$$= \frac{1}{+6D\left(\frac{D^2-D}{6}-1\right)}(1+x^2)$$

$$= \frac{1}{-6D\left[1-\left(\frac{D^2-D}{6}\right)\right]}(1+x^2)$$

$$= \frac{-1}{6D}\left[1-\left(\frac{D^2-D}{6}\right)\right]^{-1}(1+x^2)$$

$$= \frac{-1}{6D}\left[1+\left(\frac{D^2-D}{6}\right)+\left(\frac{D^2-D}{6}\right)^2+\cdots\right](1+x^2)$$

$$= \frac{-1}{6D}\left[(1+x^2)+\frac{D^2-D}{6}(1+x^2)+\left(\frac{D^2-D}{6}\right)^2(1+x^2)\right]$$

$$= \frac{-1}{6D}\left[1+x^2+\frac{1}{6}\left[2-(0+2x)\right]+\left(\frac{D^4+D^2-2D^3}{36}\right)(1+x^2)\right]$$

$$= \frac{-1}{6D}\left[1+x^2+\frac{1}{6}(2-2x)+\frac{1}{36}(0+2-0)\right]$$

$$= \frac{-1}{6D}\left[1+x^2+\frac{\cancel{2}}{\cancel{6}_3}(1-x)+\frac{1}{\cancel{36}_{18}}(\cancel{2})\right]$$

$$= \frac{-1}{6D}\left[1+x^2+\frac{1-x}{3}+\frac{1}{18}\right]$$

$$\cancel{= \frac{1}{6D}\left(x^2-x+2\right)+\frac{1}{18}}$$

$$\cancel{= \frac{-1}{6}\left[\frac{1}{D}(x^2)-\frac{1}{D}(x)+\frac{1}{D}(2)\right]+\frac{1}{D}\left(\frac{1}{18}\right)}$$

$$\cancel{= \frac{-1}{6}\left[\frac{x^3}{3}-\frac{x^2}{2}+2x\right]+\frac{1}{18}x}$$

$$= \frac{-1}{6D}\left[1+x^2+\frac{1}{3}-\frac{x}{3}+\frac{1}{18}\right]$$

$$= \frac{-1}{6}\left[\frac{1}{D}(1)+\frac{1}{D}(x^2)-\frac{1}{D}\frac{1}{3}-\frac{1}{D}\frac{x}{3}+\frac{1}{D}\frac{1}{18}\right]$$

$$= \frac{-1}{6}\left[x+\frac{x^3}{3}-\frac{1}{3}x-\frac{1}{3}\frac{x^2}{2}+\frac{1}{18}x\right]$$

$$= \frac{-1}{6}\left[x+\frac{x^3}{3}-\frac{x}{3}-\frac{x^2}{6}+\frac{x}{18}\right]$$

$$= \frac{-1}{6}\left[\frac{18x+6x^3-6x-3x^2+x}{18}\right]$$

$$P.I. = \frac{-1}{108}\left(6x^3-3x^2+13x\right)$$

Now, the solution of equ^n (1) is $y = C.F + P.I$

$$y = c_1 e^{(0)x}+c_2 e^{-2x}+c_3 e^{3x}-\frac{1}{108}\left(6x^3-3x^2+13x\right)$$

⑤ $\frac{d^2y}{dx^2} - 4y = x^2 + 2x$.

sol: Given D.E is $D^2y - 4y = x^2 + 2x$

$$(D^2-4)\,y = x^2 + 2x \rightarrow ①$$

The A.E is $m^2 - 4 = 0$.

$$(m+2)(m-2) = 0$$

$$m = 2, -2$$

∴ The roots are real and distinct.

$C.F = c_1 e^{2x} + c_2 e^{-2x}$

$$PI = \frac{1}{D^2-4}(x^2+2x)$$

$$= \frac{1}{4\left(\frac{D^2}{4}-1\right)}(x^2+2x)$$

$$= \frac{1}{-4\left(1-\frac{D^2}{4}\right)}(x^2+2x)$$

$$= \frac{-1}{4}\left(1-\frac{D^2}{4}\right)^{-1}(x^2+2x)$$

$$= \frac{-1}{4}\left[1+\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2+\cdots\right](x^2+2x)$$

$$= \frac{-1}{4}\left[(x^2+2x)+\frac{1}{4}D^2(x^2+2x)+\frac{1}{16}D^4(x^2+2x)\right]$$

$$= \frac{-1}{4}\left[x^2+2x+\frac{1}{4}(2)+0\right]$$

$$= \frac{-1}{4}\left[x^2+2x+\frac{1}{2}\right]$$

$$= \frac{-1}{4}\left[\frac{2x^2+4x+1}{2}\right]$$

$$P.I = \frac{-1}{8}\left[2x^2+4x+1\right]$$

Now, the solution of Eq^n ① is $y = C.F + P.I$

$$y = c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{8}\left[2x^2+4x+1\right].$$

(8) $(D^3-D)z = 2y+1+4\cos y+2e^y$

Given D.E is $(D^3-D)z = 2y+1+4\cos y+2e^y \rightarrow (1)$

An A.E is $m^3-m=0$

$m(m^2-1)=0$

$m(m+1)(m-1)=0$

$m=0,-1,1$

$\therefore$ The roots are real and distinct.

$C.F = c_1e^{(0)x}+c_2e^{-x}+c_3e^{x}$

$P.I = \frac{1}{D^3-D}(2y+1+4\cos y+2e^y)$

$= \frac{1}{D^3-D}2y + \frac{1}{D^3-D}(1) + \frac{1}{D^3-D}4\cos y + \frac{1}{D^3-D}2e^y$

$= 2\frac{1}{D^3-D}y + \frac{1}{D^3-D}e^{(0)y} + (4)\frac{1}{D^3-D}\cos y + 2\cdot\frac{1}{D^3-D}e^y \rightarrow (2)$

$PI_1 \qquad PI_2 \qquad PI_3 \qquad PI_4$

$PI_1 = 2\cdot\frac{1}{D^3-D}y$

$= 2\frac{1}{D(D^2-1)}y$ (3)

$= \frac{2}{-D}\frac{1}{(1-D^2)}y = \frac{-2}{D}(1-D^2)^{-1}y$

$= \frac{-2}{D}[1+D^2+(D^2)^2+(D^2)^3+\cdots]y$

$= \frac{-2}{D}[y+D^2(y)+0+0]$

$= \frac{-2}{D}[y+0]$

$= \frac{-2}{D}(y)$

$= -2\frac{1}{D}(y)$

$= -\cancel{2}\cdot\frac{y^2}{\cancel{2}}$

$= -y^2$

$$PI_2 = \frac{1}{D^3-D} e^{(0)y}$$

$$= \frac{y}{3D^2-1} e^{(0)y}$$

$$= \frac{y}{0-1} e^{(0)y} = \underline{-y}$$

$$PI_3 = 4\frac{1}{D^3-D} \cos y$$

$$= 4\frac{y}{3D^2-1} \cos y$$

$$= 4\frac{y}{3(-1)-1} \cos y$$

$$= \cancel{4}\frac{y}{-\cancel{4}} \cos y$$

$$= \underline{-y\cdot\cos y}$$

$$PI_4 = 2\frac{1}{D^3-D} e^y$$

$$= 2\frac{y}{3D^2-1} e^y$$

$$= 2\frac{y}{3(1)-1} e^y$$

$$= \cancel{2}\frac{y}{\cancel{2}} e^y$$

$$= \underline{y\cdot e^y}$$

$$PI = -y^2 - y - y\cos y + ye^y.$$

Now the solution of equ^n (1) is $z = C.F + P.I$

$$z = c_1 e^{(0)x} + c_2 e^{-x} + c_3 e^{x} - y^2 - y - y\cos y + ye^y.$$

(3) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$ $\rightarrow$ (1)

An A.E is $(m-2)^2 = 0$

$(m-2)(m-2) = 0$

$m = 2, 2$

$\therefore$ The roots are real and repeat.

$$C.F = c_1 e^{2x} + c_1 x\cdot e^{2x}.$$

$$P.I = \frac{1}{(D-2)^2} 8(e^{2x} + \sin 2x + x^2)$$

$$\cancel{=} 8\cdot \frac{1}{2\left[\frac{D^2}{2}-1\right]^2}(e^{2x} + \sin 2x + x^2)$$

$$\cancel{=} -\frac{8}{2} \frac{1}{\left[1-\frac{D^2}{2}\right]^2}(e^{2x} + \sin 2x + x^2)$$

$$\cancel{=} -4\left[1-\frac{D^2}{2}\right]^{-2}(e^{2x} + \sin 2x + x^2)$$

$$\cancel{=} -4\{$$

$$= 8\left[\frac{1}{(D-2)^2}e^{2x} + \frac{1}{(D-2)^2}\cdot\sin 2x + \frac{1}{(D-2)^2}\cdot x^2\right]$$

$PI_1 \quad PI_2 \quad PI_3 \rightarrow (2)$

$$PI_1 = \frac{1}{(D-2)^2}e^{2x}$$

$$= \frac{x}{2(D-2)}e^{2x}$$

$$= \frac{x^2}{2(1)}e^{2x}$$

$$= \frac{x^2}{2}\cdot e^{2x}$$

$$PI_2 = \frac{1}{D^2-4D+4}\sin 2x$$

$$= \frac{1}{\cancel{-4}-4D+\cancel{4}}\sin 2x$$

$$= \frac{-1}{4}\frac{1}{D}\sin 2x$$

$$= \frac{+1}{4}\left(\frac{-\cos 2x}{2}\right)$$

$$= \frac{1}{8}\cos 2x$$

$$PI_3 = \frac{1}{(D-2)^2}\cdot x^2 = \frac{1}{-2\left(1-\frac{D^2}{2}\right)^2}x^2$$

$$= \frac{-1}{2}\left(1-\frac{D^2}{2}\right)^{-2}\cdot x^2$$

$$= \frac{-1}{2}\left[1+\frac{\cancel{2}D^2}{\cancel{2}}+3\left(\frac{D^2}{2}\right)^2+\cdots\right]x^2$$

$$= \frac{-1}{2}\left[x^2+D^2(x^2)+0\right]$$

$$= \frac{-1}{2}\left[x^2+2\right]$$

from (2),

$$PI = 8\left[\frac{x^2}{2}e^{2x}+\frac{1}{8}\cos 2x-\frac{1}{2}(x^2+2)\right]$$

$$= \cancel{8}\left[\frac{4x^2e^{2x}+\cos 2x-4x^2-8}{\cancel{8}}\right]$$

$$= 4x^2e^{2x}+\cos 2x-4x^2-8$$

Now the solution of equ (1) is $y = C.F + P.I$

$$y = c_1e^{2x}+c_2\cdot xe^{2x}+4x^2e^{2x}+\cos 2x-4x^2-8$$

(6) $\frac{d^2y}{dx^2}+y = e^{2x}+\cosh 2x+x^3.$

Given DE is $D^2y+y = e^{2x}+\cosh 2x+x^3$

$$(D^2+1)\,y = e^{2x}+\cosh 2x+x^3 \rightarrow (1)$$

An A.E is : $m^2+1=0$

$$m = \frac{+0\pm\sqrt{0-4}}{2}$$

$$= \frac{\pm\cancel{2}i}{\cancel{2}}$$

$$= \pm i$$

$\therefore$ The roots are ~~real~~ complex and distinct.

$$C.F = e^{(0)x}\left[c_1 \cos x + c_2 \sin x\right]$$

$$P.I = \frac{1}{D^2+1}\left[e^{2x} + \cosh 2x + x^3\right]$$

$$= \underbrace{\frac{1}{D^2+1} e^{2x}}_{PI_1} + \underbrace{\frac{1}{D^2+1} \cosh 2x}_{PI_2} + \underbrace{\frac{1}{D^2+1} x^3}_{PI_3}$$

$$PI_1 = \frac{1}{D^2+1} e^{2x}$$

$$= \frac{1}{4+1} e^{2x} = \frac{1}{5} e^{2x}$$

$$PI_2 = \frac{1}{D^2+1} \cdot \cosh 2x$$

$$= \frac{1}{D^2+1} \cdot \frac{e^{2x}+e^{-2x}}{2}$$

$$= \frac{1}{2}\left(\frac{1}{D^2+1} e^{2x} + \frac{1}{D^2+1} e^{-2x}\right]$$

$$= \frac{1}{2}\left(\frac{1}{4+1} e^{2x} + \frac{1}{4+1} e^{-2x}\right)$$

$$= \frac{1}{2}\left[\frac{1}{5} e^{2x} + \frac{1}{5} e^{-2x}\right)$$

$$= \frac{1}{10}\left(e^{2x} + e^{-2x}\right)$$

$$PI_3 = \frac{1}{D^2+1} \cdot x^3$$

$$= \frac{1}{1+D^2} x^3.$$

$$= (1+D^2)^{-1} x^3.$$

$$= \left[1 - D^2 + D^4 - D^6 + \cdots\right] x^3$$

$$= x^3 - D^2(x^3) + D^4(x^3) - D^6(x^3)$$

$$= x^3 - 3x^2 + 6x - 6$$

from ③

$$PI = \frac{1}{5} e^{2x} + \frac{1}{10}\left(e^{2x} + e^{-2x}\right) + x^3 - 3x^2 + 6x - 6.$$

Now the solution of eqⁿ ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1 \cos x + c_2 \sin x\right] + \frac{1}{5}\left[e^{2x} + \frac{1}{2}\left(e^{2x} + e^{-2x}\right)\right] + x^3 - 3x^2 + 6x - 6$$

⑦ $(D-1)^2(D+1)^2 y = \sin^2\frac{x}{2} + e^x + x.$

Sol:- Given D.E is $(D-1)^2(D+1)^2 = \sin^2 x/_2 + e^x + x \rightarrow ①$

An A.E is $(m-1)^2(m+1)^2 = 0$

$m = 1, 1, \quad m = -1, -1$

∴ The roots are real and repeat.

$C.F = c_1 e^x + c_2 x.e^x + c_3 e^{-x} + c_4 x.e^{-x}.$

$$PI = \frac{1}{(D-1)^2(D+1)^2}\left(\sin^2 x/_2 + e^x + x\right)$$

$$= \frac{1}{(D-1)^2(D+1)^2}\left[\frac{1-\cos x}{2} + e^x + x\right]$$

$$= \frac{1}{(D-1)^2(D+1)^2}\left(\frac{1-\cos x}{2}\right) + \frac{1}{(D-1)^2(D+1)^2}e^x + \frac{1}{(D-1)^2(D+1)^2}x.$$

$$= \underbrace{\frac{1}{2}\frac{1}{(D-1)^2(D+1)^2}(1)}_{PI_1} - \underbrace{\frac{1}{2}\frac{1}{(D-1)^2(D+1)^2}\cos x}_{PI_2} + \underbrace{\frac{1}{(D-1)^2(D+1)^2}e^x}_{PI_3} + \underbrace{\frac{1}{(D-1)^2(D+1)^2}x}_{PI_4} \rightarrow ②$$

$$PI_1 = \frac{1}{2}\frac{1}{(D-1)^2(D+1)^2}e^{(0)x}$$

$$= \frac{1}{2}\frac{1}{(1)(1)}e^{(0)x} = \frac{1}{2}.$$

$$PI_2 = \frac{1}{2}\frac{1}{(D-1)^2(D+1)^2}\cos x$$

$$\neq \frac{1}{2}\frac{x}{(D-1)^2\, 2(D+1) + (D+1)^2\, 2(D-1)}\cos x$$

$$\neq \frac{1}{2}\frac{x^2}{}$$

$$= \frac{1}{2}\frac{1}{[(D-1)(D+1)]^2}\cos x$$

$$= \frac{1}{2}\frac{1}{(D^2-1)^2}\cos x$$

$$= \frac{1}{2}\frac{x}{2(D^2-1)\,2D}\cos x$$

$$= \frac{1}{8}\frac{x^2}{1(2D) + (D^2-1)(1)}\cos x$$

$$= \frac{1}{8}\frac{x^2}{(1)[2(1)] + (1-1)(1)}\cos x$$

$$= \frac{1}{2}\frac{1}{(-1-1)^2}\cos x$$

$$= \frac{1}{2}\frac{1}{(-2)^2}\cos x$$

$$= \frac{1}{2}\cdot\frac{1}{4}\cos x$$

$$= \frac{1}{8}\cos x$$

$$= \frac{x^2}{8} \frac{1}{(2+0)}\cos 3x$$

$$\Rightarrow \frac{x^2}{16}\cos 3x$$

$$PI_3 = \frac{1}{(D-1)^2(D+1)^2} e^x$$

$$\Rightarrow \frac{1}{(D^2-1)^2} e^x$$

$$= \frac{x}{2(D^2-1)}$$

$$= \frac{x}{(D-1)^2}$$

$$= \frac{1}{(D^2-1)^2} e^x$$

$$= \frac{x}{2(D^2-1)(2D)} e^x = \frac{1}{4}\frac{x}{D(D^2-1)} e^x$$

$$= \frac{1}{4}\frac{x^2}{D(2D-0)+(D^2-1)(1)} e^x$$

$$= \frac{1}{4}\frac{x^2}{(1)\ 2(1)+(0)(1)} e^x$$

$$= \frac{1}{4}\frac{x^2}{2} e^x \quad = \frac{x^2}{8} e^x$$

$$PI_4 = \frac{1}{(D-1)^2(D+1)^2} x$$

$$= \frac{1}{(D^2-1)^2} x = \frac{1}{(-1)(1-D^2)^2} x$$

$$= (1-D^2)^{-2} x$$

$$= [1+2(D^2)+3(D^2)^2+4(D^2)^3+5(D^2)^4+\ldots] x$$

$$= (x+2D^2x+3D^4x+4D^6x+\ldots)$$

$$= x+2(0)+3(0)+4(0)+\ldots$$

$$= x+0+0$$

$$= x$$

$$PI = \frac{1}{2} + \frac{1}{8}\cos 3x + \frac{x^2}{8}e^x + x$$

Now the solution of Eq^n ① is $y = C.F + P.I$

$$y = c_1e^x + c_2x.e^x + c_3e^{-x} + c_4x.e^{-x} + \frac{1}{2} + \frac{1}{8}\cos 3x + \frac{x^2}{8}e^x + x$$

wednesday
6/11/19 Type-4.

⑦ $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x^2e^{3x} + \sin 2x.$

Sol:- Given D.E is $D^2y - 3Dy + 2y = x^2 \cdot e^{3x} + \sin 2x$

$$(D^2 - 3D + 2)\,y = x^2 \cdot e^{3x} + \sin 2x \longrightarrow ①$$

An A.E is $m^2 - 3m + 2 = 0$

$m^2 - m - 2m + 2 = 0$

$m(m-1) - 2(m-1) = 0$

$(m-1)(m-2) = 0$

$m = 1, 2.$

∴ The roots are real and distinct.

$C.F = c_1e^{x} + c_2e^{2x}$

$$PI = \frac{1}{(D^2-3D+2)}\left(x \cdot e^{3x} + \sin 2x\right)$$

$$= e^{3x}\underbrace{\frac{1}{(D+3)^2 - 3(D+3) + 2}x^2}_{PI_1} + \underbrace{\frac{1}{(D\cancel{+0})^2 - 3(D\cancel{+0}) + 2}\sin 2x}_{PI_2}$$

$$PI_1 = e^{3x}\frac{1}{D^2 + \cancel{9} + 6D - 3D - \cancel{9} + 2}x^2$$

$$= e^{3x}\frac{1}{D^2 + 3D + 2}x^2$$

$$= e^{3x}\frac{1}{2\left(\frac{D^2+3D}{2} + 1\right)}x^2$$

$$= \frac{e^{3x}}{2}\frac{1}{1 + \left(\frac{D^2+3D}{2}\right)}x^2$$

$$= \frac{e^{3x}}{2}\left(1 + \frac{D^2+3D}{2}\right)^{-1}x^2$$

$$= \frac{e^{3x}}{2}\left[1 - \left(\frac{D^2+3D}{2}\right) + \left(\frac{D^2+3D}{2}\right)^2 - \left(\frac{D^2+3D}{2}\right)^3 + \cdots\right]x^2$$

$$= \frac{e^{3x}}{2}\left[x^2 - \left(\frac{D^2+3D}{2}\right)x^2 + \left(\frac{D^2+3D}{2}\right)^2x^2 - \cdots\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - \frac{1}{2}\left[D^2(x^2) + 3D(x^2)\right] + \left(\frac{D^4 + 9D^2 + 6D^3}{4}\right)x^2\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - \frac{1}{2}[2 + 3(2x)] + \frac{1}{4}[D^4(x^2) + 9D^2(x^2) + 6D(x^2)]\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - \frac{1}{2}(2+6x) + \frac{1}{4}(0 + 9(2) + 6(2x))\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - 1 - 3x + \frac{1}{4}(18 + 12x)\right]$$

$$= \frac{e^{3x}}{2}\left(x^2 - 1 - 3x + \frac{1}{2}(9 + 6x)\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - 1 - 3x + \frac{9}{2} + \cancel{\frac{6x}{2}}^{3x}\right)$$

$$= \frac{e^{3x}}{2}\left[x^2 - 1 - \cancel{3x} + \frac{9}{2} + \cancel{3x}\right]$$

$$= \frac{e^{3x}}{2}\left[x^2 - 1 + \frac{9}{2}\right] - 3x)$$

$$-1 + \frac{9}{2} = \frac{-2+9}{2} = \frac{7}{2}$$

$$\cancel{= \frac{e^{3x}}{2} \frac{1}{2}[2x^2 - 2 + 9]}$$

$$\cancel{= \frac{e^{3x}}{4}(2x^2 + 7)}$$

$$= \frac{e^{3x}}{2}\left[x^2 - 3x + \frac{7}{2}\right)$$

$$PI_2 = \frac{1}{D^2 - 3D + 2}\sin 2x$$

$$= \frac{1}{-4 - 3D + 2}\sin 2x$$

$$= \frac{1}{-3D - 2}\sin 2x$$

$$= \frac{1}{-3D-2} \times \frac{-3D+2}{-3D+2}\sin 2x$$

$$= \frac{-3D+2}{9D^2 - 4}\sin 2x$$

$$= \frac{-3D+2}{9(-4) - 4}\sin 2x$$

$$= \frac{-3D+2}{-36-4}\sin 2x$$

$$= \frac{+(3D-2)}{+40}\sin 2x$$

$$= \left(\frac{3D-2}{40}\right)\sin 2x$$

$= \frac{1}{40}\left[3D\sin 2x - 2\sin 2x\right]$

$= \frac{1}{40}\left[3\cdot\cos 2x(2) - 2\sin 2x\right]$

$= \frac{1}{40}\left[6\cos 2x - 2\sin 2x\right]$

$= \frac{\cancel{2}}{\cancel{40}_{20}}\left[3\cos 2x - \sin 2x\right]$

$= \frac{1}{20}\left[3\cos 2x - \sin 2x\right]$

$PI = \frac{e^{3x}}{2}\left[x^2 - 3x + \frac{7}{2}\right] + \frac{1}{20}\left[3\cos 2x - \sin 2x\right]$

Now the solution of equⁿ① is $y = C.F + P.I$

$y = c_1 e^x + c_2 e^{2x} + \frac{e^{3x}}{2}\left[x^2 - 3x + \frac{7}{2}\right] + \frac{1}{20}\left[3\cos 2x - \sin 2x\right]$

① $(D^2 - 4D + 3)y = e^x \cos 2x$

sol: Given D.E is $(D^2 - 4D + 3)y = e^x \cos 2x \rightarrow ①$

~~An~~ A.E is $m^2 - 4m + 3 = 0$

$m^2 - m - 3m + 3 = 0$

$m(m-1) - 3(m-1) = 0$

$(m-1)(m-3) = 0$

$m = 1, 3.$

∴ The roots are real and distinct.

$CF = c_1 e^x + c_2 e^{3x}$

$P.I = \frac{1}{D^2 - 4D + 3} e^x \cdot \cos 2x$

$= e^x \frac{1}{(D+1)^2 - 4(D+1) + 3} \cos 2x$

$= e^x \frac{1}{D^2 + \cancel{1} + 2D - 4D - \cancel{4} + \cancel{3}} \cos 2x$

$= e^x \frac{1}{D^2 - 2D} \cos 2x$

$= e^x \frac{1}{-4 - 2D} \cos 2x$

$= e^x \frac{1}{-4 - 2D} \times \frac{-4 + 2D}{-4 + 2D} \cos 2x$

$$= e^{x} \frac{-4+2D}{16-4D^2} \cos 2x$$

$$= e^{x} \frac{-4+2D}{16-4(-4)} \cos 2x$$

$$= e^{x} \frac{-4+2D}{16+16} \cos 2x$$

$$= e^{x} \frac{2D-4}{32} \cos 2x$$

$$= \frac{e^{x}}{32} (2 \cdot D\cos 2x - 4\cos 2x)$$

$$= \frac{e^{x}}{32} (2.(-\sin 2x)(2) - 4\cos 2x)$$

$$= \frac{e^{x}}{32} (-4\sin 2x - 4\cos 2x)$$

$$= \frac{-e^{x}}{8} (\sin 2x + \cos 2x)$$

Now the solution of equ^n ① is $y = C.F + P.I.$

$$y = c_1 e^{x} + c_2 e^{3x} + -\frac{e^{x}}{8} \sin 2x - \frac{e^{x}}{8} \cos 2x$$

② $(D^4 - 1) y = \cos x . \cosh x$

Sol:- Given D.E is $(D^4-1) y = \cos x . \cosh x \rightarrow ①$

An A.E is $m^4 - 1 = 0$

$$(m^2)^2 - (1)^2 = 0$$

$$(m^2+1)(m^2-1) = 0.$$

$$m^2+1 = 0, \quad m^2-1 = 0$$

$$m = \pm i, \quad m = \pm 1$$

∴ The roots are real, imaginary and distinct.

$$C.F = c_1 e^{x} + c_2 e^{-x} + e^{(0)x}[c_1 \cos x + c_2 \sin x]$$

$$P.I = \frac{1}{D^4-1} \cos x . \cosh x$$

$$= \frac{1}{D^4-1} \cos x \left(\frac{e^{x}+e^{-x}}{2}\right)$$

$$= \frac{1}{2}\frac{1}{D^4-1}\left(e^{x}.\cos x + e^{-x}.\cos x\right)$$

$$= \frac{1}{2}\left[\underbrace{\frac{1}{D^4-1}e^{x}.\cos x}_{PI_1} + \underbrace{\frac{1}{D^4-1}.e^{-x}\cos x}_{PI_2}\right] \rightarrow (2)$$

$$PI_1 = \frac{1}{D^4-1} e^{x}.\cos x$$

$$= e^{x}\frac{1}{(D+1)^4-1}.\cos x.$$

$$= e^{x}\frac{1}{[(D+1)^2]^2-1}\cos x$$

$$= e^{x}\frac{1}{(D^2+2D+1)^2-1}\cos x.$$

$$= e^{x}\frac{1}{(D^2)^2+4D^2+\cancel{1}+4D^3+4D+2D^2\cancel{-1}}\cos x$$

$$= e^{x}\frac{1}{(D^2)^2+6D^2+4D^3+4D}\cos x$$

$$= e^{x}\frac{1}{(-1)^2+6(-1)+4(-1)D+4D}\cos x$$

$$= e^{x}\frac{1}{1-6-\cancel{4D}+\cancel{4D}}\cos x$$

$$= e^{x}\frac{1}{-5}\cos x = \underline{\underline{\frac{-e^{x}}{5}\cos x}}$$

$$PI_2 = \frac{1}{(D^2+1)(D^2-1)} e^{-x}\cos x$$

$$= e^{-x}\frac{1}{[(D-1)^2+1][(D-1)^2-1]}\cos x$$

$$= e^{-x}\frac{1}{[D^2+1-2D+1]\,(D^2+\cancel{1}-2D\cancel{-1})}\cos x$$

$$= e^{-x}\frac{1}{(D^2-2D+2)(D^2-2D)}\cos x$$

$$= e^{-x}\frac{1}{D^4-2D^3-2D^3+\cancel{4D^2}+2D^2-4D}\cos x$$

$$= e^{-x}\frac{1}{D^4-4D^3+6D^2-4D}\cos x.$$

$$= e^{-x} \frac{1}{(-1)^2 - 4(-1)D + 6(-1) - 4D} \cos x$$

$$= e^{-x} \frac{1}{1 + \cancel{4D} - 6 - \cancel{4D}} \cos x$$

$$= e^{-x} \cdot \frac{1}{-5} \cdot \cos x = \frac{-e^{-x}}{5} \cos x$$

from ②,

$$P.I = -\frac{e^{x}}{5} \cos x - \frac{e^{-x}}{5}$$

Now the solution of equⁿ① is $y = C.F + P.I$

$$y = c_1 e^{x} + c_2 e^{-x} + e^{(0)x}[c_1 \cos x + c_2 \sin x] - \frac{e^{x}}{5} \cos x - \frac{e^{-x}}{5}.$$

③ $\frac{d^2y}{dx^2} - 4y = x.\sinh x$

sol: Given D-E is $D^2y - 4y = x.\sinh x$

$$(D^2 - 4)y = x.\sinh x \rightarrow ①$$

An Auxiliary Equation is $m^2 - 4 = 0$

$$m^2 - (2)^2 = 0$$

$$(m+2)(m-2) = 0$$

$$m = -2, 2.$$

∴ The roots are real and distinct.

$$C.F = C_1 e^{-2x} + C_2 e^{2x}$$

$$P.I = \frac{1}{D^2 - 4} x.\sinh x$$

$$= \frac{1}{D^2 - 4} x\left(\frac{e^{x} - e^{-x}}{2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{D^2 - 4}\right)\left[x.e^{x} - x.e^{-x}\right]$$

$$= \frac{1}{2}\left[\underbrace{\frac{1}{D^2 - 4} x e^{x}}_{PI_1} - \underbrace{\frac{1}{D^2 - 4} e^{-x} x}_{PI_2}\right] \rightarrow ②$$

$$P.I_1 = \frac{1}{D^2 - 4} x.e^{x}$$

$$= e^{x} \frac{1}{(D+1)^2 - 4} x.$$

$$= e^{x} \frac{1}{\cancel{D}4\left(1 - \frac{(D+1)^2}{4}\right)} x.$$

$$= \frac{-e^x}{4}\left[1-\frac{(D+1)^2}{4}\right]^{-1} x$$

$$= \frac{-e^x}{4}\left[1+\frac{(D+1)^2}{4}+\left(\frac{(D+1)^2}{4}\right)^2+\dots\right] x$$

$$= \frac{-e^x}{4}\left[x+\frac{(D+1)^2}{4}x+\frac{((D+1)^2)^2}{16}x\right]$$

$$= \frac{-e^x}{4}\left[x+\frac{D^2+1+2D}{4}x+\frac{(D^2+2D+1)^2}{16}\cdot x\right]$$

$$= \frac{-e^x}{4}\left[x+\frac{1}{4}(D^2(x)+x+2Dx)+\left(\frac{D^4+4D^2+1+4D^3+4D+2D^2}{16}\right)x\right]$$

$$= \frac{-e^x}{4}\left[x+\frac{1}{4}[0+x+2]+\frac{1}{16}[0+0+x+0+4+0]\right]$$

$$= \frac{-e^x}{4}\left[x+\frac{x}{4}+\frac{\cancel{2}}{\cancel{4}_2}+\frac{1}{16}(x+4)\right]$$

$$= \frac{-e^x}{4}\left[x+\frac{x}{4}+\frac{1}{2}+\frac{x}{16}+\frac{1}{4}\right]$$

$$= \frac{-e^x}{4}\left[\frac{16x+4x+8+x+4}{16}\right]$$

$$= \frac{-e^x}{4}\left(\frac{21x+12}{16}\right)$$

$$= \frac{-e^x}{4}\left(\frac{21x}{16}+\frac{\cancel{12}^{3}}{\cancel{16}_{4}}\right)$$

$$= \frac{-e^x}{4}\left(\frac{21x}{16}+\frac{3}{4}\right)$$

$$P.I_2 = \frac{1}{D^2-4}\,e^{-x}.x$$

$$= e^{-x}\frac{1}{(D-1)^2-4}\,x$$

$$= e^{-x}\frac{1}{D^2+1-2D-4}\,x$$

$$= e^{-x}\frac{1}{D^2-2D-3}\,x$$

$$= e^{-x}\frac{1}{-3\left(1-\left(\frac{D^2-2D}{3}\right)\right)}\,x$$

$$= -\frac{e^{-x}}{3}\left[1-\left(\frac{D^2-2D}{3}\right)\right]^{-1}x$$

$$= \frac{-e^{-x}}{3}\left[1+\left(\frac{D^2-2D}{3}\right)+\left(\frac{D^2-2D}{3}\right)^2+\cdots\right]x$$

$$= -\frac{e^{-x}}{3}\left[x+\left(\frac{D^2-2D}{3}\right)x+\frac{D^4+4D^2-4D^3}{9}\cdot x\right]$$

$$= -\frac{e^{-x}}{3}\left[x+\frac{1}{3}(D^2x-2Dx)+0\right]$$

$$= -\frac{e^{-x}}{3}\left[x+\frac{1}{3}(0-2)\right]$$

$$= -\frac{e^{-x}}{3}\left(x-\frac{2}{3}\right)$$

$$= -\frac{e^{-x}}{9}(3x-2)$$

$$P.I = \frac{-e^{x}}{4}\left(\frac{21x}{16}+\frac{3}{4}\right)-\frac{e^{-x}}{9}(3x-2)$$

Now the solution of Equ^n ① is $y = C.F + P.I$

$$y = c_1e^{-2x}+c_2e^{2x}-\frac{e^{x}}{4}\left(\frac{21x}{16}+\frac{3}{4}\right)-\frac{e^{-x}}{9}(3x-2)$$

④ $\frac{d^2y}{dx^2}+y = x^2\sin 2x$

Sol: Given D.E is $D^2y+y = x^2\sin 2x$

$$(D^2+1)y = x^2\sin 2x \rightarrow ①$$

A.n A.E is $m^2+1=0$

$$m = \pm i$$

∴ The roots are complex and distinct.

$$C.F = e^{(0)x}[c_1\cos x+c_2\sin x]$$

$$P.I = \frac{1}{D^2+1}x^2\sin 2x.$$

$$= I.P\left\{\frac{1}{D^2+1}x^2(\cos 2x+i\sin 2x)\right]$$

$$= I.P\left[\frac{1}{D^2+1}\cdot x^2 e^{2ix}\right]$$

$$= I.P.e^{2ix}\left[\frac{1}{(D+2i)^2+1}\cdot x^2\right]$$

$$= I.P\, e^{2ix}\frac{1}{1+(D+2i)^2}x^2$$

$= I.p\ e^{2ix} \left[1+(D+2i)^2\right]^{-1} \cdot x^2$

$= I.p\ e^{2ix} \left[1-(D+2i)^2+\left((D+2i)^2\right)^2+\cdots\right] x^2$

$= I.p\ e^{2ix} \left[x^2-(D^2+4i^2+4Di)x^2+(D^2+4i^2+4Di)^2 x^2\right]$

$(D^2-4+4Di)$

$= I.p\ e^{2ix} \left[x^2-(D^2x^2-4x^2+4iDx^2)+\left[D^4+16+16D^2i^2-8D^2-32Di+8Di\right] x^2\right]$

$= I.p\ e^{2ix} \left[x^2-(2x-4x^2+4i(2x))+(0+16x^2-16(2)-8(2)-32i(2x)+8)\right]$

$= I.p\ e^{2ix} \left[x^2-2x+4x^2-8xi+16x^2-32-16-64xi\right]$

$P.I. = I.p\ e^{2ix}\left[21x^2-72xi-2x-48\right]$

Now the solution of equ^n ① is $y = C.F + P.I.$

$y = e^{(0)x}\left[c_1\cos x+c_2\sin x\right]+I.p\ e^{2ix}(21x^2-72xi-2x-48)$.

⑤ $(D^4+2D^2+1)y = x^2\cos x$.

Sol: Given D.E is $(D^4+2D^2+1)y = x^2\cos x \rightarrow ①$

A.E is $m^4+2m^2+1=0$

$(m^2+1)^2=0$

$(m^2+1)(m^2+1)=0$

$m=\pm i,\ m=\pm i$

∴ The roots are complex and repeat.

$C.F = e^{(0)x}\left[(c_1+c_2x)\cos x+(c_3+c_4x)\sin x\right]$

$P.I = \frac{1}{(D^2+1)^2} \cdot x^2\cos x$

$= R.p\left[\frac{1}{(D^2+1)^2}\cdot x^2 (\cos x+i\sin x)\right]$

$= R.p\left[\frac{1}{(D^2+1)^2}\ x^2\ e^{ix}\right]$

$= R.p\ e^{ix}\left[\frac{1}{((D+i)^2+1)^2}x^2\right]$

$= R.p\ e^{ix}\ \frac{1}{(1+D^2)^2}\ x^2$

$= \cancel{R.p}\ e^{ix}\ (1+D^2)^{-2}.x^2$

$= R.p\ e^{ix}\ [1 - 2D^2 + 3(D^2)^2 + \cdots]x^2$

$= R.p\ e^{ix}\ [x^2 - 2D^2x^2 + 3D^4.x^2 + \cdots]$

$= R.p\ e^{ix}\ \frac{1}{((D+i)^2+1)^2}\ x^2$

$= R.p\ e^{ix}\ \frac{1}{[1+(D+i)^2]^2}\ x^2$

$= R.p\ e^{ix}\ [1+(D+i)^2]^{-2}.x^2$

$= R.p\ e^{ix}\ [1 - 2(D+i)^2 + 3((D+i)^2)^2 - 4((D+i)^2)^3 + \cdots]x^2$

$= R.p\ e^{ix}\ [x^2 - 2(D^2+i^2+2Di)x^2 + 3\underbrace{(D^2+i^2+2Di)^2}_{(D^2-1+2Di)}x^2]$

$= R.p\ e^{ix}\ [x^2 - 2(D^2x^2 - x^2 + 2Dix^2) + 3(D^4+1+4D^2i^2 - 2D^2 - 4Di + 4D^3i)\ x^2]$

$= R.p\ e^{ix}[x^2 - 2(2 - x^2 + 2i(2x)) + 3(0 + x^2 - 4(2) - 2(2) - 4i(2x) + 0)]$

$= R.p\ e^{ix}[x^2 - 4 + 2x^2 - 8xi + 3x^2 - 24 - 12 - 24xi]$

$P.I. = R.p\ e^{ix}[6x^2 - 32xi - 40]$

Now the solution of eqn ① is $y = C.F + P.I$

$y = e^{(0)x}[(C_1 + C_2x)\cos x + (C_3 + C_4x)\sin x] + R.p\ e^{ix}[6x^2 - 32xi - 40]$

⑧ $\frac{d^4y}{dx^4} - y = e^x \cos x$

Sol: Given D.E is $D^4y - y = e^x\cos x$

$(D^4-1)\,y = e^x\cos x \rightarrow ①$

An A.E is $m^4 - 1 = 0$

$(m^2)^2 - (1)^2 = 0$

$(m^2+1)(m^2-1) = 0$

$m = \pm i,\ m = \pm 1$

∴ The roots are real, complex and distinct.

$C.F = C_1e^{-x} + C_2e^{x} + e^{(0)x}[C_3\cos x + C_4\sin x]$

$$P.I = \frac{1}{D^4-1} e^x \cos x$$

$$= e^x \frac{1}{(D+1)^4-1} \cos x$$

$$= \cancel{e^x \frac{1}{(D^2+1+2D)^2-1}} \cos x$$

$$= \cancel{e^x \frac{1}{(-1+1)^2-1}} \cos x$$

$$\neq \cancel{e^x \frac{1}{0-1}} \cos x$$

$$\cancel{P.I \neq -e^x \cos x}$$

$$= e^x \frac{1}{(D^2+1+2D)^2-1} \cos x$$

$$= e^x \frac{1}{D^4+\cancel{1}+4D^2+2D^2+4D+4D^3-\cancel{1}} \cos x$$

$$= e^x \frac{1}{(-1)(-1)+4(-1)+2(-1)+4D+4(-1)D} \cos x$$

$$= e^x \cdot \frac{1}{1-4-2+\cancel{4D}-\cancel{4D}} \cos x$$

$$= e^x \frac{1}{-5} \cos x$$

$$P.I = -\frac{e^x}{5} \cos x$$

Now the solution of Eqn ① is $y = C.F + P.I$

$$y = c_1 e^{-x} + c_2 e^x + e^{(0)x}\left[c_3 \cos x + c_4 \sin x\right] - \frac{e^x}{5}\cos x.$$

⑨ $(D^2-2D)y = e^x \sin x$

Sol: Given D.E is $(D^2-2D)y = e^x \sin x \rightarrow$ ①

An A.E is $m^2-2m=0$

$m(m-2)=0$

$m=0, 2$

$\therefore$ The roots are real and distinct.

$$C.F = c_1 e^{(0)x} + c_2 e^{2x}$$

$$P.I = \frac{1}{D^2-2D} \cancel{\sin x}\, e^x \sin x$$

$$= e^x \frac{1}{(D+1)^2-2(D+1)} \sin x$$

$$= e^x \frac{1}{D^2+1+\cancel{2D}-\cancel{2D}-2} \sin x$$

$$= e^x \frac{1}{D^2-1} \sin x$$

$$= e^x \frac{1}{-1-1} \sin x$$

$$= e^x \frac{1}{-2} \sin x$$

$$P.I = -\frac{e^x}{2} \sin x$$

Now the solution of Eqn ① is $y = C.F + P.I$

$$y = c_1 e^{(0)x} + c_2 e^{2x} - \frac{e^x}{2}\sin x.$$

(10) $y'' - 2y' + 2y = x + e^x \cos x$

sol:- Given D.E is $y'' - 2y' + 2y = x + e^x \cos x$

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x + e^x \cos x$$

$$D^2y - 2Dy + 2y = x + e^x \cos x$$

$$(D^2 - 2D + 2)y = x + e^x \cos x \rightarrow (1)$$

A.E is $m^2 - 2m + 2 = 0$

$$m = \frac{2 \pm \sqrt{4-8}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2}$$

$$= \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$m = 1 \pm i$$

$\therefore$ The roots are complex and distinct.

$$C.F = e^x [c_1 \cos x + c_2 \sin x]$$

$$P.I = \frac{1}{D^2 - 2D + 2}(x + e^x \cos x) = \underbrace{\frac{1}{D^2-2D+2}x}_{PI_1} + \underbrace{\frac{1}{D^2-2D+2}e^x \cos x}_{PI_2} \rightarrow (2)$$

$$PI_1 = \frac{1}{2\left(\frac{D^2-2D}{2} + 1\right)}(x)$$

$$= \frac{1}{2\left(1 + \frac{D^2 - 2D}{2}\right)}(x)$$

$$= \frac{1}{2}\left[1 + \left(\frac{D^2-2D}{2}\right)\right]^{-1}(x)$$

$$= \frac{1}{2}\left[1 - \left(\frac{D^2-2D}{2}\right) + \left(\frac{D^2-2D}{2}\right)^2 - \dots\right]x$$

$$= \frac{1}{2}\left[x - \left(\frac{D^2-2D}{2}\right)x + \left(\frac{D^4+4D^2-4D^3}{2}\right)x\right]$$

$$= \frac{1}{2}\left[x - \frac{1}{2}(D^2x - 2Dx) + 0\right]$$

$$= \frac{1}{2}\left[x - \frac{1}{2}(0-2)\right]$$

$$= \frac{1}{2}[x+1]$$

$$PI_2 = \frac{1}{D^2-2D+2}e^x \cdot \cos x$$

$$= e^x \frac{1}{(D+1)^2 - 2(D+1) + 2}\cos x$$

$$= e^x \frac{1}{D^2+1+\cancel{2D}-\cancel{2D}-\cancel{1}+\cancel{1}} \cos x$$

$$= e^x \frac{1}{D^2+1} \cos x$$

$$= e^x \frac{x}{2D} \cos x$$

$$= e^x \frac{x}{2} \cdot \frac{1}{D}(\cos x)$$

$$PI_2 = \frac{x \cdot e^x}{2} \sin x.$$

$$P.I = \frac{1}{2}(x+1) + \frac{x \cdot e^x}{2} \sin x.$$

Now the solution of Equⁿ ① is $y = C.F + P.I$

$$y = e^x [c_1 \cos x + c_2 \sin x] + \frac{1}{2}(x+1) + \frac{x \cdot e^x}{2} \sin x.$$

(12) $\frac{d^2y}{dx^2} + 2y = x^2 e^{3x} + e^x (\cos 2x)$

Sol: Given D.E is $D^2y + 2y = x^2 e^{3x} + e^x (\cos 2x)$

$$(D^2+2)\, y = x^2 e^{3x} + e^x \cos 2x \longrightarrow ①$$

An A.E is $m^2 + 2 = 0$

$$m^2 = -2$$

$$m = \sqrt{-2}$$

$$m = \pm\sqrt{2}\, i.$$

∴ The roots are complex and distinct.

$$C.F = e^{(0)x} [c_1 \cos\sqrt{2}x + c_2 \sin\sqrt{2}x]$$

$$P.I = \frac{1}{D^2+2} (x^2 e^{3x} + e^x \cos 2x)$$

$$= \underbrace{\frac{1}{D^2+2} x^2 e^{3x}}_{PI_1} + \underbrace{\frac{1}{D^2+2} e^x \cos 2x}_{PI_2} \longrightarrow ②$$

$$P.I_1 = \frac{1}{D^2+2} e^{3x} \cdot x^2$$

$$= e^{3x} \frac{1}{(D+3)^2+2} x^2$$

$$= e^{3x} \frac{1}{D^2+9+6D+2} x^2$$

$$= e^{3x} \frac{1}{D^2+6D+11} x^2$$

$$= e^{3x} \frac{1}{11\left(\frac{D^2+6D}{11}+1\right)} x^2$$

$$= e^{3x} \frac{1}{11\left(1+\frac{D^2+6D}{11}\right)} x^2$$

$$= \frac{e^{3x}}{11} \left(1+\frac{D^2+6D}{11}\right)^{-1} x^2$$

$$= \frac{e^{3x}}{11} \left[1-\left(\frac{D^2+6D}{11}\right)+\left(\frac{D^2+6D}{11}\right)^2 - \cdots\right] x^2$$

$$= \frac{e^{3x}}{11} \left[x^2-\left(\frac{D^2+6D}{11}\right)x^2+\left(\frac{D^2+6D}{11}\right)^2 x^2 - \cdots\right]$$

$$= \frac{e^{3x}}{11} \left[x^2-\frac{1}{11}\left(D^2x^2+6Dx^2\right)+\frac{1}{121}\left(D^4+36D^2+12D^3\right)x^2\right]$$

$$= \frac{e^{3x}}{11}\left[x^2-\frac{1}{11}\left(2+6(2x)\right)+\frac{1}{121}\left(0+36(2)+0\right)\right]$$

$$= \frac{e^{3x}}{11}\left(x^2-\frac{2}{11}-\frac{12x}{11}+\frac{72}{121}\right)$$

$$= \frac{e^{3x}}{11}\left(x^2-\frac{12x}{11}+\frac{50}{121}\right)$$

$$P.I_2 = \frac{1}{D^2+2} e^x \cos 2x$$

$$= e^x \frac{1}{(D+1)^2+2} \cos 2x$$

$$= e^x \frac{1}{D^2+2D+3} \cos 2x$$

$$= e^x \frac{1}{-4+2D+3} \cos 2x$$

$$= e^x \frac{1}{2D-1} \cos 2x$$

$$= e^x \frac{1}{2D-1} \times \frac{2D+1}{2D+1} \cos 2x$$

$$= e^{x}\frac{2D+1}{4D^2-1}\cos 2x$$

$$= e^{x}\frac{2D+1}{4(-4)-1}\cos 2x$$

$$= e^{x}\frac{2D+1}{-17}\cos 2x$$

$$= \frac{-e^{x}}{17}\left(2D\cos 2x + (1)\cos 2x\right)$$

$$= -\frac{e^{x}}{17}\left(2(-\sin 2x)2 + \cos 2x\right)$$

$$= -\frac{e^{x}}{17}\left(-4\sin 2x + \cos 2x\right)$$

$$= \frac{e^{x}}{17}\left(4\sin 2x - \cos 2x\right)$$

$$P.I = \frac{e^{3x}}{11}\left(x^2 - \frac{12x}{11} + \frac{50}{121}\right) + \frac{e^{x}}{17}\left(4\sin 2x - \cos 2x\right)$$

Now the solution of equ^n ①, is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos\sqrt{2}x + c_2\sin\sqrt{2}x\right] + \frac{e^{3x}}{11}\left(x^2 - \frac{12x}{11} + \frac{50}{121}\right) + \frac{e^{x}}{17}\left(4\sin 2x - \cos 2x\right)$$

(11) $(D^3 + 2D^2 + D)y = x^2e^{2x} + \sin^2 x$.

Given D.E is $(D^3 + 2D^2 + D)y = x^2e^{2x} + \sin^2 x \longrightarrow ①$

An Auxiliary Equation is $m^3 + 2m^2 + m = 0$

$$m(m^2 + 2m + 1) = 0$$

$$m(m^2 + m + m + 1) = 0$$

$$m\left[m(m+1) + 1(m+1)\right] = 0$$

$$m(m+1)(m+1) = 0$$

$$m = 0, -1, -1$$

∴ The roots are real and repeat.

$$C.F = c_1e^{(0)x} + c_2e^{-x} + c_3x.e^{-x}.$$

$$P.I = \frac{1}{D^3+2D^2+D}x^2e^{2x} + \sin^2 x$$

$$= \frac{1}{D^3+2D^2+D}x^2.e^{2x} + \frac{1}{D^3+2D^2+D}\cdot\frac{1-\cos 2x}{2}$$

$$= \frac{1}{D^3+2D^2+D}x^2e^{2x} + \frac{1}{D^3+2D^2+D}\frac{1}{2} - \frac{1}{D^3+2D^2+D}\frac{1}{2}.\cos 2x \longrightarrow ②$$

$$PI_1 = e^{2x}\cdot\frac{1}{(D+2)^3+2(D+2)^2+(D+2)}x^2$$

$$= e^{2x}\frac{1}{D^3+8+6D^2+12D+2D^2+8+8D+D+2}x^2$$

$$= e^{2x}\frac{1}{D^3+8D^2+21D+18}x^2$$

$$= e^{2x}\frac{1}{18\left(1+\frac{D^3+8D^2+21D}{18}\right)}x^2$$

$$= \frac{e^{2x}}{18}\left(1+\frac{D^3+8D^2+21D}{18}\right)^{-1}x^2$$

$$= \frac{e^{2x}}{18}\left[1-\frac{D^3+8D^2+21D}{18}+\left(\frac{D^3+8D^2+21D}{18}\right)^2 - \cdots\right]x^2$$

$$= \frac{e^{2x}}{18}\left[1-\frac{1}{18}\left[D^3+8D^2+21D\right]x^2+\left[D^6+64D^4+441D^2+16D^5+336D^3+42D^4\right]x^2\right]$$

$$= \frac{e^{2x}}{18}\left(1-\frac{1}{18}\left[0+8(2)+21(2x)\right]+\left[0+0+441(2)+0+0+0\right]\right)$$

$$= \frac{e^{2x}}{18}\left[1-\frac{1}{18}\left[16+42x\right]+882\right]$$

$$= \frac{e^{2x}}{18}\left[1-\frac{\cancel{2}}{\cancel{18}_9}(8+21x)+882\right]$$

$$= \frac{e^{2x}}{18}\left[1-\frac{8}{9}-\frac{\cancel{21}^{7}x}{\cancel{9}_3}+882\right]$$

$$1-\frac{8}{9}+882 = \frac{9-8+7938}{9} = \frac{7930+9}{9} = \frac{7939}{9}$$

$$= \frac{e^{2x}}{18}\left[\frac{7939}{9}-\frac{7x}{3}\right]$$

$$PI_2 = \frac{1}{2}\frac{1}{D^3+2D^2+D}e^{(0)x}$$

$$= \frac{1}{2}\frac{x}{3D^2+4D+1}e^{(0)x}$$

$$= \frac{1}{2}\frac{x}{1}e^{(0)x} = \frac{x}{2}$$

$$PI_3 = \frac{1}{2}\frac{1}{D^3+2D^2+D}\cos 2x$$

$$= \frac{1}{2}\frac{1}{-4D-8+D}\cos 2x$$

$$= \frac{1}{2}\frac{1}{-8-3D}\cos 2x$$

$$= \frac{1}{2}\frac{1}{-8-3D}\times\frac{-8+3D}{-8+3D}\cos 2x$$

$$= \frac{1}{2}\frac{-8+3D}{64-9D^2}\cos 2x.$$

$$=\frac{1}{2}\frac{-8+3D}{64+36}\cos 2x$$

$$=\frac{1}{2}\frac{-8+3D}{100}\cos 2x$$

$$=\frac{3D-8}{200}\cdot\cos 2x$$

$$=\left(\frac{3D}{200}-\frac{1}{25}\right)\cos 2x$$

from ①,

$$P.I=\frac{e^{2x}}{18}\left(\frac{7939}{9}-\frac{7x}{3}\right)+\frac{x}{2}-\left(\frac{3D}{200}-\frac{1}{25}\right)\cos 2x$$

Now the solution of equⁿ ① is $y = C.F + P.I$

$$y=c_1e^{(0)x}+c_2e^{-x}+c_3x.e^{-x}+\frac{e^{2x}}{18}\left(\frac{7939}{9}-\frac{7x}{3}\right)+\frac{x}{2}-\left(\frac{3D}{100}-\frac{1}{25}\right)\cos 2x$$

Formulas:

$$\boxed{\begin{aligned} e^{i\theta} &= \cos\theta+i\sin\theta \\ e^{i(2\theta)} &= \cos 2\theta+i\sin 2\theta \\ e^{i(n\theta)} &= \cos n\theta+i\sin n\theta \end{aligned}}$$

$$\boxed{\begin{aligned} \sin^2 x &= \frac{1-\cos 2x}{2} \\ \cos^2 x &= \frac{1+\cos 2x}{2} \end{aligned}}$$

$$\boxed{\begin{aligned} \sin A\cdot\cos B &= \frac{1}{2}\left[\sin(A+B)+\sin(A-B)\right] \\ \cos A\cdot\cos B &= \frac{1}{2}\left[\cos(A-B)+\cos(A+B)\right] \\ \sin A\cdot\sin B &= \frac{1}{2}\left[\cos(A-B)-\cos(A+B)\right] \end{aligned}}$$

Type-IV

(4) $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$.

Given D.E. is $\frac{d^2y}{dx^2} + 4y = x^2 \sin 2x$

$$D^2y + 4y = x^2 \sin 2x$$

$$(D^2+4)y = x^2 \sin 2x \rightarrow (1)$$

An Auxiliary equation is $m^2+4=0$

$$m^2 = -4$$

$$m = \pm 2i$$

$\therefore$ The roots are complex and distinct.

$$C.F = \cancel{c_1} e^{(0)x}\left(c_1 \cos 2x + c_2 \sin 2x\right)$$

$$P.I = \frac{1}{D^2+4} x^2 \sin 2x$$

$$= \frac{1}{D^2+4} x^2 \; I.p \; e^{i(2x)}$$

$$= I.p \left[\frac{1}{D^2+4} x^2 e^{i(2x)}\right]$$

$$= I.p \left[e^{2ix} \frac{1}{(D+2i)^2+4} x^2\right]$$

$$= I.p \left[e^{2ix} \frac{1}{D^2 \cancel{-4} + 4Di \cancel{+4}} x^2\right]$$

$$= I.p \left[e^{2ix} \frac{1}{4Di\left(\frac{D^{\cancel{2}}}{4\cancel{D}i}+1\right)} x^2\right]$$

$$= I.p \left[e^{2ix} \frac{1}{4Di\left(\frac{D}{4i}+1\right)} x^2\right]$$

$$= I.p \left[\frac{e^{2ix}}{4Di}\left(1+\frac{D}{4i}\right)^{-1} x^2\right]$$

$$= I.p \left[\frac{e^{2ix}}{4Di}\left(1-\frac{D}{4i}+\left(\frac{D}{4i}\right)^2-\left(\frac{D}{4i}\right)^3+\ldots\right)x^2\right]$$

$$= I.p \left[\frac{e^{2ix}}{4Di}\left(x^2-\frac{D}{4i}x^2+\frac{D^2}{(-16)}x^2\right)\right]$$

$$= I.p \left[\frac{e^{2ix}}{4Di}\left(x^2-\frac{1}{\cancel{4}_2 i}(\cancel{2}x)-\frac{1}{\cancel{16}_8}(\cancel{2})\right)\right]$$

$$= I.P\left[\frac{e^{2ix}}{4Di}\left(x^2 - \frac{x}{2i} - \frac{1}{8}\right)\right]$$

$$= I.P\left[\frac{e^{2ix}}{4Di}\times\frac{i}{i}\left(x^2 - \frac{xi}{2i^2} - \frac{1}{8}\right)\right]$$

$$= I.P\left[\frac{e^{2ix}}{-4D}\, i\left(x^2 + \frac{x}{2}i - \frac{1}{8}\right)\right]$$

$$= I.P\left[\frac{ie^{2ix}}{-4}\left(\frac{x^3}{3} + \frac{x^2}{4}i - \frac{1}{8}x\right)\right]$$

$$= I.P\left[\frac{ie^{2ix}}{-4}\left(\frac{x^3}{3} + \frac{x^2}{4}i - \frac{x}{8}\right)\right]$$

$$= I.P\left[\frac{i}{-4}(\cos 2x + i\sin 2x)\left(\frac{x^3}{3} + \frac{x^2}{4}i - \frac{x}{8}\right)\right]$$

$$= I.P\left[\frac{-i}{4}(\cos 2x + i\sin 2x)\left(\left(\frac{x^3}{3} - \frac{x}{8}\right) + i\left(\frac{x^2}{4}\right)\right)\right]$$

$$= I.P\left[\frac{-i}{4}\cos 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) + i\frac{x^2}{4}\cos 2x + i\sin 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{x^2}{4}\sin 2x\right]$$

$$= I.P\left[\frac{-i}{4}\left(\cos 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{x^2}{4}\sin 2x + i\left(\frac{x^2}{4}\cos 2x + \sin 2x\left(\frac{x^3}{3} - \frac{x}{8}\right)\right)\right)\right]$$

$$= I.P\left[\frac{-i}{4}\left(\cos 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{x^2}{4}\sin 2x\right) + \frac{1}{4}\left(\frac{x^2}{2}\cos 2x + \sin 2x\left(\frac{x^3}{8} - \frac{x}{8}\right)\right)\right]$$

$$= I.P\frac{1}{4}\left[\cancel{-}\frac{x^2}{2}\cos 2x + \sin 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{i}{4}\left[\cos 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{x^2}{4}\sin 2x\right]\right]$$

$$= -\frac{1}{4}\cos 2x\left(\frac{x^3}{3} - \frac{x}{8}\right) - \frac{x^2}{4}\sin 2x$$

$$P.I = \frac{1}{4}\left[\frac{x^2}{4}\sin 2x - \left(\frac{x^3}{3} - \frac{x}{8}\right)\cos 2x\right]$$

Now the solution of eqn (1) is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos 2x + c_2\sin 2x\right] + \frac{1}{4}\left[\frac{x^2}{4}\sin 2x - \left(\frac{x^3}{3} - \frac{x}{8}\right)\cos 2x\right]$$

(5) $(D^4+2D^2+1)y = x^2\cos x$.

Given D.E is $(D^4+2D^2+1)y = x^2\cos x \rightarrow (1)$

An Auxiliary equn is $m^4+2m^2+1=0$

$$(m^2+1)^2 = 0.$$

$$(m^2+1)(m^2+1) = 0$$

$$m=\pm i, \quad m=\pm i$$

$\because$ The roots are complex and repeated roots.

$$C.F = e^{(0)x}[c_1\cos x + c_2\sin x] + xe^{(0)x}[c_3\cos x + c_4\sin x]$$

$$= c_1\cos x + c_2\sin x + c_3 x\cos x + c_4 x\sin x$$

$$= (c_1+c_3x)\cos x + (c_2+c_4x)\sin x$$

$$P.I = \frac{1}{D^4+2D^2+1} x^2\cos x$$

$\boxed{(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4}$

$$= \frac{1}{(D^2+1)^2} x^2\cos x.$$

$$= \frac{1}{D^4+2D^2+1} x^2\cdot(R.P\ e^{ix})$$

$$= R.P\left[\frac{1}{D^4+2D^2+1} x^2\cdot e^{ix}\right]$$

$$= R.P\left[e^{ix}\frac{1}{(D+i)^4+2(D+i)^2+1} x^2\right]$$

$$= R.P\left[e^{ix}\frac{1}{D^4+4D^3i+6D^2i^2+4Di^3+i^4+2(D^2+i^2+2Di)+1} x^2\right]$$

$$= R.P\left[e^{ix}\frac{1}{D^4+4D^3i-6D^2-\cancel{4Di}+\cancel{1}+2D^2-\cancel{2}+\cancel{4Di}+\cancel{1}} x^2\right]$$

$$= R.P\left[e^{ix}\frac{1}{D^4-4D^2+4D^3i} x^2\right]$$

$$= R.P\left[e^{ix}\frac{1}{4D^2\left(\frac{D^4+4D^3i}{4D^2}-1\right)} x^2\right]$$

$$= R.P\left[\frac{e^{ix}}{-4D^2}\frac{1}{\left(1-\frac{D^2+4Di}{4D^2}\right)} x^2\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(1-\frac{D^2+4Di}{4}\right)^{-1}x^2\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left[1+\left(\frac{D^2+4Di}{4}\right)+\left(\frac{D^2+4Di}{4}\right)^2+\cdots\right]x^2\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(x^2+\frac{1}{4}(D^2x^2+4Dix^2)+\frac{1}{16}(D^4x^2+16D^2i^2x^2+8D^3i\,x^2)\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left\{x^2+\frac{1}{4}[2+4i(2x)]\right\}+\frac{1}{16}(0-16(2)+0)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(x^2+\frac{2}{4}+\frac{8xi}{4}-\frac{1}{16}(32)\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(x^2-\frac{1}{2}+2xi-2\right)\right]$$

$$-\frac{1}{2}-2 = \frac{-1-4}{2} = \frac{-5}{2}$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(x^2+2xi-\frac{5}{2}\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4D^2}\left(x^2+2xi-\frac{5}{2}\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4}\frac{1}{D}\left(\frac{x^3}{3}+2i\frac{x^2}{2}-\frac{5}{2}x\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4}\left(\frac{1}{3}\frac{x^4}{4}+i\frac{x^3}{3}-\frac{5}{2}\frac{x^2}{2}\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4}\left(\frac{x^4}{12}+i\frac{x^3}{3}-\frac{5x^2}{4}\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{4}\left(\frac{x^4+4ix^3-15x^2}{12}\right)\right]$$

$$= R.P\left[\frac{-e^{ix}}{48}(x^4+4ix^3-15x^2)\right]$$

$$= R.P\left[\frac{-1}{48}(\cos x+i\sin x)(x^4+4ix^3-15x^2)\right]$$

$$= R.P\left[\frac{-1}{48}\left(\cos x(x^4-15x^2)+\cos x\cdot 4ix^3+i\sin x(x^4-15x^2)+i^2 4x^3\sin x\right)\right]$$

$$= R.P\left[\frac{-1}{48}\left(\cos x(x^4-15x^2)-4x^3\sin x\right)+i\left(\cos x\,4x^3+\sin x(x^4-15x^2)\right)\right]$$

$$= R.P\left[\frac{-1}{48}\left(\cos x(x^4-15x^2)-4x^3\sin x\right)+i\left(4x^3\cos x+(x^4-15x^2)\sin x\right)\right]$$

$$P.I = \frac{-1}{48}\left[(x^4-15x^2)\cos x - 4x^3\sin x\right]$$

$$P.I. = \frac{1}{48}\left[4x^3\sin x - (x^4-15x^2)\cos x\right]$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = (c_1+c_3x)\cos x + (c_2+c_4x)\sin x + \frac{1}{48}\left[4x^3\sin x - (x^4-15x^2)\cos x\right]$$

8/11/19
Saturday **Type - V**

② $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\cdot e^x \sin x$

Sol: Given D.E is $D^2y - 2Dy + y = x\cdot e^x \sin x$

$$(D^2-2D+1)y = xe^x\sin x \rightarrow ①$$

An Auxiliary Equ^n is $m^2-2m+1=0$

$$m^2-m-m+1=0$$

$$m(m-1)-1(m-1)=0$$

$$(m-1)(m-1)=0$$

$$m=1,1$$

∴ The roots are real and repeat.

$$C.F = c_1e^x + c_2x\cdot e^x.$$

$$P.I = \frac{1}{D^2-2D+1}\, x.e^x\sin x$$

$$= \underbrace{x\frac{1}{(D-1)^2}e^x\sin x}_{PI_1} - \underbrace{\frac{2(D-1)}{[(D-1)^2]^2}e^x\sin x}_{PI_2} \rightarrow ②$$

$$PI_1 = x\cancel{e^x}\frac{1}{D^2-2D+1}e^x\sin x$$

$$= xe^x\frac{1}{(D+1)^2-2(D+1)+1}\sin x$$

$$= x\cdot e^x\frac{1}{D^2+\cancel{1}+\cancel{2D}-\cancel{2D}-\cancel{2}+\cancel{1}}\sin x$$

$$= x\cdot e^x\frac{1}{D^2}\sin x$$

$$= xe^x(-\sin x)$$

$$= -xe^x\sin x$$

$$PI_2 = \frac{2(D-1)}{(D^2-2D+1)^2} e^x \sin x$$

$$= 2(D-1) \frac{1}{D^4-4D^2+1-4D^3+2D^2-4D} e^x \sin x$$

$$= 2(D-1) \frac{1}{D^4-4D^3-2D^2-4D+1}$$

$$= 2(D-1) \frac{1}{(D^2-2D+1)^2} e^x \sin x$$

$$= 2(D-1) e^x \frac{1}{[(D+1)^2-2(D+1)+1]^2} \sin x$$

$$= 2(D-1) e^x \frac{1}{(D^2+2D+1-2D-2+1)^2} \sin x$$

$$= 2(D-1) e^x \frac{1}{(D^2)^2} \sin x$$

$$= 2(D-1) e^x \frac{1}{D^4} \sin x$$

$$= 2(D-1) e^x \sin x.$$

$$= 2e^x (D\sin x - \sin x)$$

$$PI_2 = 2e^x (\cos x - \sin x)$$

$$PI = -x e^x \sin x - 2e^x(\cos x - \sin x)$$

$$= -xe^x \sin x - 2e^x \cos x + 2e^x \sin x$$

$$= e^x(2\sin x - x\sin x - 2\cos x)$$

$$= 2e^x \sin x - xe^x \sin x - 2e^x \cos x$$

$$= 2e^x \sin x + e^x(-x\sin x - 2\cos x)$$

Now the solution of equⁿ ① is $y = C.F + P.I$

$$y = c_1 e^x + c_2 x e^x + 2e^x \sin x + e^x(-x\sin x - 2\cos x)$$

(5) $(D^2-1)y = x\sin x + (1+x^2)e^x$.

Sol:- Given D.E is $(D^2-1)y = x\sin x + (1+x^2)e^x \rightarrow (1)$

An Auxiliary eqn is $m^2-1=0$

$m^2=1$

$m=\pm 1$

$\therefore$ The roots are real and distinct.

$C.F = c_1e^x + c_2e^{-x}$

$$P.I = \frac{1}{D^2-1}\left[x\sin x + (1+x^2)e^x\right]$$

$$= \underbrace{\frac{1}{D^2-1}x\sin x}_{PI_1} + \underbrace{\frac{1}{D^2-1}(1+\cancel{x^2})e^x}_{PI_2} + \underbrace{\frac{1}{D^2-1}x^2 e^x}_{PI_3} \rightarrow (2)$$

$$PI_1 = \frac{1}{D^2-1}x\sin x$$

$$= x\frac{1}{D^2-1}\sin x - \frac{2D}{(D^2-1)^2}\sin x$$

$$= x\cdot\frac{1}{-1-1}\sin x - 2D\frac{1}{(-1-1)^2}\sin x$$

$$= \frac{x}{-2}\sin x - \cancel{2}D\frac{1}{\cancel{4}\,2}\sin x$$

$$= -\frac{x}{2}\sin x - \frac{1}{2}\cos x$$

$$PI_2 = \frac{1}{D^2-1}\cdot e^x$$

$$= \frac{x}{2D}e^x$$

$$= \frac{x}{2(1)}e^x = \frac{x}{2}e^x$$

$$PI_3 = \frac{1}{D^2-1}x^2e^x$$

$$= e^x\frac{1}{(D+1)^2-1}x^2$$

$$= e^x\frac{1}{D^2+\cancel{1}+2D-\cancel{1}}x^2$$

$$= e^x\frac{1}{2D\left(1+\frac{D}{2}\right)}x^2$$

$$= \frac{e^x}{2D}\left(1+\frac{D}{2}\right)^{-1}x^2$$

$$= \frac{e^x}{2D}\left(1-\frac{D}{2}+\left(\frac{D}{2}\right)^2-\cdots\right)x^2$$

$$= \frac{e^x}{2D}\left(x^2 - \frac{D}{2}x^2 + \frac{D^2}{4}x^2\right)$$

$$= \frac{e^x}{2D}\left[x^2 - \frac{1}{\cancel{2}}(\cancel{2}x) + \frac{1}{\cancel{4}_2}(\cancel{2})\right]$$

$$= \frac{e^x}{2D}\left(x^2 - x + \frac{1}{2}\right)$$

$$= \frac{e^x}{2}\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{2}x\right)$$

from ②,

$$PI = -\frac{x}{2}\sin x - \frac{1}{2}\cos x + \frac{x}{2}e^x + \frac{e^x}{2}\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{2}x\right)$$

Now the solution of equⁿ ① is $y = C.F + P.I$

$$y = c_1 e^x + c_2 e^{-x} - \frac{x}{2}\sin x - \frac{1}{2}\cos x + \frac{x}{2}e^x + \frac{e^x}{2}\left(\frac{x^3}{3} - \frac{x^2}{2} + \frac{1}{2}x\right)$$

⑦ $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x \sin x$

sol: Given D.E is $D^2y + 3Dy + 2y = xe^x \sin x$

$$(D^2 + 3D + 2)y = xe^x \sin x \quad \rightarrow ①$$

An Auxiliary equⁿ is $m^2 + 3m + 2 = 0$

$$m^2 + m + 2m + 2 = 0$$

$$m(m+1) + 2(m+1) = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2$$

∴ The roots are real and distinct.

$$C.F = c_1 e^{-x} + c_2 e^{-2x}.$$

$$P.I = \frac{1}{D^2 + 3D + 2} xe^x \sin x$$

$$= \frac{1}{D^2 + 3D + 2} e^x (x \sin x)$$

$$= e^x \frac{1}{(D+1)^2 + 3(D+1) + 2} x \sin x$$

$$= e^x \frac{1}{D^2 + 2D + 1 + 3D + 3 + 2} x \sin x$$

$$= e^x \frac{1}{D^2 + 5D + 6} x \sin x$$

$$= e^x \left[x \frac{1}{D^2 + 5D + 6} \sin x - \frac{2D + 5}{(D^2 + 5D + 6)^2} \sin x\right]$$

$$= e^x\left[x\frac{1}{-1+5D+6}\sin x - \frac{2D+5}{(-1+5D+6)^2}\sin x\right]$$

$$= e^x\left[x\frac{1}{5D+5}\sin x - \frac{2D+5}{(5D+5)^2}\sin x\right]$$

$$= e^x\left[x\frac{1}{5(D+1)}\sin x - \frac{2D+5}{25(D+1)^2}\sin x\right]$$

$$= e^x\left[\frac{x}{5}\frac{1}{D+1}\times\frac{D-1}{D-1}\sin x - \frac{2D+5}{25}\frac{1}{D^2+2D+1}\sin x\right]$$

$$= e^x\left[\frac{x}{5}\frac{D-1}{D^2-1}\sin x - \frac{2D+5}{25}\frac{1}{-\cancel{1}+2D+\cancel{1}}\sin x\right]$$

$$= e^x\left[\frac{x}{5}\frac{D-1}{-1-1}\sin x - \frac{2D+5}{25}\frac{1}{2D}\sin x\right]$$

$$= e^x\left[\frac{-x}{10}(D\cdot\sin x - \sin x) - \frac{2D+5}{50}(-\cos x)\right]$$

$$= e^x\left[\frac{-x}{10}(\cos x - \sin x) + \frac{2}{50}(-\sin x) + \frac{5}{50}\cos x\right]$$

$$= e^x\left[\frac{x}{10}(\sin x - \cos x) - \frac{1}{25}\sin x + \frac{1}{10}\cos x\right]$$

$$= e^x\left[\frac{x}{10}\sin x - \frac{x}{10}\cos x - \frac{1}{25}\sin x + \frac{1}{10}\cos x\right]$$

$$= e^x\left[\frac{1}{10}\cos x(1-x) + \frac{1}{5}\sin x\left(\frac{x}{2}-\frac{1}{5}\right)\right]$$

$$= e^x\left[\frac{1}{10}\cos x(1-x) + \frac{1}{5}\sin x\left(\frac{5x-2}{10}\right)\right]$$

$$= e^x\left[\frac{1}{10}\cos x(1-x) + \frac{1}{50}\sin x(5x-2)\right]$$

$$P.I = \frac{e^x}{10}\left[\cos x(1-x) + \frac{1}{5}\sin x(5x-2)\right]$$

Now the solution of equⁿ ① is $y = C.F + P.I$

$$y = c_1e^{-x} + c_2e^{-2x} + \frac{e^x}{10}\left[\cos x(1-x) + \frac{1}{5}\sin x(5x-2)\right].$$

① $(D^2-4)\,y = x\cos 2x$

Sol:- Given D.E is $(D^2-4)y = x\cos 2x \rightarrow (1)$

An Auxiliary equn is $m^2-4=0$

$$m^2-2^2=0$$

$$(m+2)\,(m-2)=0$$

$$m=2,-2.$$

$\therefore$ The roots are real and distinct.

$$C.F = c_1e^{2x} + c_2e^{-2x}$$

$$P.I = \frac{1}{D^2-4}\,x\,(\cos 2x)$$

$$= x.\frac{1}{D^2-4}\cos 2x - \frac{2D}{(D^2-4)^2}.\cos 2x$$

$$= x.\frac{1}{-4-4}\cos 2x - \frac{2D}{(-4-4)^2}\cdot\cos 2x$$

$$= x\,\frac{1}{-8}\cos 2x - \frac{\cancel{2}D}{\cancel{64}\,32}\cos 2x$$

$$= \frac{-x}{8}\cdot\cos 2x - \frac{1}{32}D(\cos 2x)$$

$$= \frac{-x}{8}\cos 2x - \frac{1}{\cancel{32}\,16}(-\sin 2x)\cancel{2}$$

$$P.I = -\frac{x}{8}\cos 2x + \frac{1}{16}\sin 2x$$

Now the solution of equn① is $y = C.F + P.I$

$$y = c_1e^{2x} + c_2e^{-2x} - \frac{x}{8}\cos 2x + \frac{1}{16}\sin 2x.$$

③ $\dfrac{d^2y}{dx^2} + 4y = x\sin x$

Sol:- Given D.E is $\dfrac{d^2y}{dx^2} + 4y = x\sin x$

$$D^2y + 4y = x\sin x$$

$$(D^2+4)\,y = x\sin x \rightarrow (1)$$

An Auxiliary equn is $m^2+4=0$

$$m^2=-4$$

$$m=\pm 2i$$

$\therefore$ The roots are complex and distinct.

$$C.F = e^{(0)x}[c_1\cos 2x + c_2\sin 2x]$$

$$P.I = \frac{1}{D^2+4}\cdot x\sin x$$

$$= x\frac{1}{D^2+4}\sin x - \frac{2D}{(D^2+4)^2}\sin x$$

$$= x\frac{1}{-1+4}\sin x - \frac{2D}{(-1+4)^2}\sin x$$

$$= \frac{x}{3}\sin x - \frac{2D}{9}\cdot\sin x$$

$$P.I = \frac{x}{3}\cdot\sin x - \frac{2}{9}\cos x.$$

Now the solution of equⁿ ① is $y = C.F + P.I$

$$y = e^{(0)x}[c_1\cos 2x + c_2\sin 2x] + \frac{x}{3}\sin x - \frac{2}{9}\cos x.$$

④ $\frac{d^2y}{dx^2} - 9y = x\cos 2x$

<u>sol:</u> Given D.E is $D^2y - 9y = x\cos 2x$

$$(D^2-9)\,y = x\cos 2x \rightarrow ①$$

An Auxiliary equⁿ is $m^2-9=0$

$$m^2-3^2=0$$

$$(m-3)(m+3)=0$$

$$m = 3, -3.$$

∴ The roots are real and distinct.

$$C.F = c_1e^{3x} + c_2e^{-3x}.$$

$$P.I = \frac{1}{D^2-9}\cdot x\cdot\cos 2x$$

$$= x\cdot\frac{1}{D^2-9}\cos 2x - \frac{2D}{(D^2-9)^2}\cos 2x$$

$$= x\frac{1}{-4-9}\cos 2x - \frac{2D}{(-4-9)^2}\cos 2x$$

$$= x\frac{1}{-13}\cos 2x - \frac{2D}{+169}\cos 2x$$

$$= \frac{-x}{13}\cos 2x - \frac{2}{169}(-\sin 2x)\,2$$

$$= \frac{-x}{13}\cos 2x + \frac{4}{169}\sin 2x$$

13×13, 39, 13, 169

Now the solution of equⁿ ① is $y = CF + PI$

$$y = c_1e^{3x} + c_2e^{-3x} - \frac{x}{13}\cos 2x + \frac{4}{169}\sin 2x.$$

⑥ $(D^2-1)y = x\sin 3x + \cos x$

Sol:-

Given D.E is $(D^2-1)y = x\sin 3x + \cos x$. $\rightarrow$ ①

An Auxiliary equn is $m^2-1=0$

$(m+1)(m-1)=0$

$m=1,-1$

$\therefore$ The roots are real and distinct.

$C.F = c_1 e^{x} + c_2 e^{-x}$

$$P.I = \frac{1}{D^2-1}(x\sin 3x + \cos x)$$

$$= \underbrace{\frac{1}{D^2-1} x\sin 3x}_{PI_1} + \underbrace{\frac{1}{D^2-1}\cos x}_{PI_2} \rightarrow ②$$

$$PI_1 = \frac{1}{D^2-1} x\sin 3x$$

$$= x\frac{1}{D^2-1}\sin 3x - \frac{2D}{(D^2-1)^2}\sin 3x$$

$$= x\frac{1}{-9-1}\sin 3x - \frac{2D}{(-9-1)^2}\sin 3x$$

$$= x\frac{1}{-10}\sin 3x - \frac{\cancel{2}D}{\cancel{100}\,50}\sin 3x$$

$$= \frac{-x}{10}\sin 3x - \frac{1}{50}\cos 3x\,(3)$$

$$= \frac{-x}{10}\sin 3x - \frac{3}{50}\cos 3x.$$

$$PI_2 = \frac{1}{D^2-1}\cos x.$$

$$= \frac{1}{-1-1}\cos x$$

$$= \frac{1}{-2}\cos x$$

$$= \frac{-1}{2}\cos x$$

from ②,

$$P.I = \frac{-x}{10}\sin 3x - \frac{3}{50}\cos 3x - \frac{1}{2}\cos x.$$

Now the solution of equn ① is $y = C.F + P.I$

$$y = c_1 e^{x} + c_2 e^{-x} - \frac{x}{10}\sin 3x - \frac{3}{50}\cos 3x - \frac{1}{2}\cos x.$$

11/11/2019
Monday

General Method:

③ $\frac{d^2y}{dx^2} + a^2y = \sec ax.$

Sol:- Given D.E is $D^2y + a^2y = \sec ax$

$(D^2+a^2)y = \sec ax \rightarrow ①$

An Auxiliary Equn is $m^2+a^2=0$

$m^2 = -a^2$

$m = \pm ai$

∴ The roots are complex and distinct.

$C.F = e^{(0)x}[c_1 \cos ax + c_2 \sin ax]$

$P.I = \frac{1}{D^2+a^2} \sec ax$

$= \frac{1}{(D+ai)(D-ai)} \sec ax$

$= \frac{1}{2ai}\left(\frac{1}{D-ai} - \frac{1}{D+ai}\right) \sec ax$

$= \frac{1}{2ai}\left[\underbrace{\frac{1}{D-ai} \sec ax}_{PI_1} - \underbrace{\frac{1}{D+ai} \sec ax}_{PI_2}\right] \rightarrow ②$

$PI_1 = \frac{1}{D-ai} \sec ax$

$= e^{iax}\int \sec ax \cdot e^{-iax}\, dx$

$= e^{iax}\left[\int \sec ax\,(\cos ax - i\sin ax)\, dx\right]$

$= e^{iax}\left[\int \sec ax \cos ax\, dx - i\int \sin ax \cdot \sec ax\, dx\right]$

$= e^{iax}\left[\int (1)\, dx - i\int \tan ax\, dx\right]$

$= e^{iax}\left(x - i\,\frac{\log(\sec ax)}{a}\right)$

$= e^{iax}\left[x - \frac{i}{a}\log(\sec ax)\right]$

$PI_2 = \frac{1}{D+ai} \sec ax$

$= \frac{1}{[D-(-ai)]} \sec ax$

$$= e^{-iax}\int \sec ax\, e^{iax}\,dx$$

$$= e^{-iax}\int \sec ax.(\cos ax + i\sin ax)\,dx$$

$$= e^{-iax}\int \sec ax.\cos ax\,dx + i\int \sec ax.\sin ax.\,dx$$

$$= e^{-iax}\int (1)\,dx + i\int \tan ax\,dx$$

$$= e^{-iax}\left[x + i\frac{\log(\sec ax)}{a}\right]$$

$$= e^{-iax}\left[x + \frac{i}{a}\log(\sec ax)\right]$$

$$P.I = \frac{1}{2ai}\left[e^{iax}\left[x - \frac{i}{a}\log(\sec ax)\right] - e^{-iax}\left[x + \frac{i}{a}\log(\sec ax)\right]\right]$$

$$= \frac{1}{2ai}\left[e^{iax}.x - e^{iax}\frac{i}{a}\log(\sec ax) - e^{-iax}x - e^{-iax}\frac{i}{a}\log(\sec ax)\right]$$

$$= \frac{1}{2ai}\left[x\left(e^{iax} - e^{-iax}\right) - \frac{i}{a}\log(\sec ax)\left(e^{iax} + e^{-iax}\right)\right]$$

$$= \frac{1}{2ai}\left[x.2i\sin ax - \frac{i}{a}\log(\sec ax)\,2\cos ax\right]$$

$$P.I = \frac{x}{a}\sin ax - \frac{1}{a^2}\log(\sec ax)\cos ax.$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}(c_1\cos ax + c_2\sin ax) + \frac{x}{a}\sin ax - \frac{1}{a^2}\cos ax.\log(\sec ax).$$

⑤ $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{e^x}$.

Sol:- Given D.E is $D^2y + 3Dy + 2y = e^{e^x}$

$$(D^2 + 3D + 2)y = e^{e^x} \rightarrow ①$$

An Auxiliary equ^n is $m^2 + 3m + 2 = 0$

$$m^2 + m + 2m + 2 = 0$$

$$m(m+1) + 2(m+1) = 0$$

$$(m+1)(m+2) = 0$$

$$m = -1, -2.$$

∴ The roots are real and distinct.

$$C.F = c_1 e^{-x} + c_2 e^{-2x}$$

$$P.I = \frac{1}{(D+1)(D+2)} e^{e^x}$$

$$= \frac{1}{2}\left(\frac{1}{D+1} - \frac{1}{D+2}\right) e^{e^x}$$

$$= \frac{1}{2}\left[\underset{PI_1}{\frac{1}{D+1} e^{e^x}} - \underset{PI_2}{\frac{1}{D+2} e^{e^x}}\right] \rightarrow (2)$$

$$PI_1 = \frac{1}{D+1} e^{e^x}$$

$$= \frac{1}{D-(-1)} e^{e^x}$$

$$= e^{-x} \int e^{e^x} e^{x} dx$$

$$= e^{-x} \int e^{e^x} e^x dx$$

$$\boxed{\begin{array}{l} e^x = t \\ e^x dx = dt \end{array}}$$

$$= e^{-x} \int e^t dt$$

$$= e^{-x} e^t$$

$$= e^{-x} . e^{e^x}$$

$$PI_2 = \frac{1}{D+2} e^{e^x}$$

$$= \frac{1}{D-(-2)} e^{e^x}$$

$$= e^{-2x} \int e^{e^x} e^{2x} . dx$$

$$= e^{-2x} \int e^{e^x} . e^x e^x dx$$

$$= e^{-2x} \int e^t . t . dt$$

$$= e^{-2x} e^t (t-1)$$

$$= e^{-2x} e^{e^x} (e^x - 1)$$

$$PI = \frac{1}{2}\left[e^{-x} . e^{e^x} - e^{-2x} e^{e^x} (e^x - 1)\right]$$

$$= \frac{1}{2}\left[e^{-x} e^{e^x} - e^{-2x} e^{e^x} e^x + e^{-2x} e^{e^x}\right]$$

$$= \frac{1}{2}\left[e^{-x} e^{e^x} - e^{-2x} e^{e^x} e^x + e^{-x} e^{-x} . e^{e^x}\right]$$

$$= \frac{1}{2}\left\{e^{-x} e^{e^x} - e^{-x} \cancel{e^{-x}} e^{e^x} \cancel{e^x} + e^{-2x} e^{e^x}\right\}$$

$$= \frac{1}{2}\left[e^{-x} \cancel{e^{e^x}} - \cancel{e^{-x} e^{e^x}} + e^{-2x} e^{e^x}\right]$$

$$= \frac{1}{2} e^{-2x} e^{e^x}$$

(4) $\frac{d^2y}{dx^2} + 4y = 4 \tan 2x$.

Sol: Given D.E is $D^2y + 4y = 4 \tan 2x$

$$(D^2+4) y = 4 \tan 2x \rightarrow (1)$$

An Auxiliary equ^n is $m^2 + 4 = 0$

$$m^2 = -4$$

$$m = \pm 2i$$

$\therefore$ The roots are complex and distinct.

$$C.F = e^{(0)x}\left[c_1 \cos 2x + c_2 \sin 2x\right]$$

$$P.I = \frac{1}{D^2+4}\, 4\tan 2x$$

$$= \frac{1}{(D-2i)(D+2i)}\, 4\tan 2x$$

$$= \frac{1}{4i}\left(\frac{1}{D-2i} - \frac{1}{D+2i}\right) 4\tan 2x$$

$$= \frac{1}{i}\left(\frac{1}{D-2i} - \frac{1}{D+2i}\right)\tan 2x$$

$$= \frac{1}{i}\Big[\underbrace{\frac{1}{D-2i}\tan 2x}_{PI_1} - \underbrace{\frac{1}{D+2i}\tan 2x}_{PI_2}\Big] \rightarrow ②$$

$$PI_1 = \frac{1}{D-2i}\tan 2x$$

$$= e^{2ix}\int \tan 2x\, e^{-2ix}\, dx$$

$$= e^{2ix}\int \tan 2x \cdot e^{-i(2x)}\, dx$$

$$= e^{2ix}\int \tan 2x \cdot (\cos 2x - i\sin 2x)\, dx$$

$$= e^{2ix}\int \sin 2x \cdot dx - i\int \tan 2x \cdot \sin 2x\, dx$$

$-2i$

$$= e^{2ix}\left[\left(-\frac{\cos 2x}{2}\right) - i\int \frac{\sin^2(2x)}{\cos(2x)}\, dx\right]$$

$$= e^{+2ix}\left[-\frac{\cos 2x}{2} - i\int \frac{1-\cos^2 2x}{\cos 2x}\, dx\right]$$

$$= e^{2ix}\left[-\frac{\cos 2x}{2} - i\int \frac{1}{\cos 2x}\, dx + i\int \cos 2x\, dx\right]$$

$$= e^{2ix}\left[-\frac{\cos 2x}{2} - i\,\frac{\log(\sec 2x + \tan 2x)}{2} + i\,\frac{\sin(2x)}{2}\right]$$

$$= e^{2ix}\left(-\frac{\cos 2x}{2} - i\,\frac{\log(\sec 2x + \tan 2x)}{2} + i\,\frac{\sin 2x}{2}\right)$$

$$P.I_2 = \frac{1}{D+2i}\tan 2x$$

$$= \frac{1}{D-(-2i)}\tan 2x$$

$\cos ax + i\sin ax$
$\cos ax - i\sin ax$

$= e^{-2ix}\int \tan 2x \cdot e^{2ix}\,dx$

$= e^{-2ix}\int \tan 2x\,(\cos 2x + i\sin 2x)\,dx$

$= e^{-2ix}\int (\tan 2x\cdot\cos 2x + i\tan 2x\cdot\sin 2x)\,dx$

$= e^{-2ix}\int \sin 2x\,dx + i\int \frac{\sin^2 2x}{\cos 2x}\,dx$

$= e^{-2ix}\int -\frac{\cos 2x}{2} + i\int \frac{1}{\cos 2x}\,dx - i\int \frac{\cos^2 2x}{\cancel{\cos 2x}}\,dx$

$= e^{-2ix}\left[-\frac{\cos 2x}{2} + i\frac{\log(\sec 2x + \tan 2x)}{2} - i\frac{\sin 2x}{2}\right]$

$P.I = \frac{1}{i}\left[e^{2ix}\,\frac{-\cos 2x}{2} + \frac{i\log(\sec 2x + \tan 2x)}{2}e^{2ix} - i\,\frac{\sin 2x}{2}\,e^{2ix}\right]$

$-\left[e^{-2ix}\,\frac{-\cos 2x}{2} + i\,\frac{\log(\sec 2x + \tan 2x)}{2}e^{-2ix} - \frac{i}{2}\sin 2x\, e^{-2ix}\right]$

log

$= \frac{1}{i}\left[-\frac{\cos 2x}{2}\,2i\sin 2x + \frac{i}{2}\log(\sec 2x + \tan 2x)\,2\cos 2x - i\,\frac{\sin 2x}{2}\,2\cos 2x\right.$

$$\begin{vmatrix} e^{-x} & e^{-2x} \\ -e^{-x} & -2e^{-2x} \end{vmatrix}$$

$\cancel{\cos x} - i\sin x - \cancel{\cos x} - i\sin x$

$\cos x + i\cancel{\sin x} + \cos x - \cancel{i\sin x}$

$2\cos x$

$-e^{-x}\,2e^{-2x} + e^{-x}\cdot e^{-2x}$

$-2e^{-3x} + e^{-3x}$

$-e^{-3x}$

$$C.F = e^{(0)x}\left(c_1\cos 2x + c_2\sin 2x\right).$$

$$P.I = \frac{1}{D^2+4}\, 4\tan 2x$$

$$= 4\,\frac{1}{D^2+4}\tan 2x.$$

$$= 4\,\frac{1}{(D+2i)(D-2i)}\tan 2x$$

$$= 4\,\frac{-1}{4i}\left(\frac{1}{D+2i} - \frac{1}{D-2i}\right)\tan 2x$$

$$= \frac{-1}{i}\left(\frac{1}{D+2i} - \frac{1}{D-2i}\right)\tan 2x$$

$$= \frac{-1}{i}\left(\underbrace{\frac{1}{D+2i}\tan 2x}_{P.I_1} - \underbrace{\frac{1}{D-2i}\tan 2x}_{PI_2}\right) \rightarrow (2)$$

$$PI_1 = \frac{1}{D+2i}\tan 2x$$

$$= \frac{1}{D-(-2i)}\tan 2x$$

$$= e^{-2ix}\int \tan 2x \cdot e^{2ix}\,dx$$

$$= e^{-2ix}\int \tan 2x\,(\cos 2x + i\sin 2x)\,dx$$

$$= e^{-2ix}\int \frac{\sin 2x}{\cancel{\cos 2x}}\cancel{\cos 2x}\,dx + i\int \frac{\sin 2x}{\cos 2x}\sin 2x\,dx$$

$$= e^{-2ix}\left[\frac{-\cos 2x}{2} + i\int \frac{\sin^2 2x}{\cos 2x}\,dx\right]$$

$$= e^{-2ix}\left[\frac{-\cos 2x}{2} + i\int \frac{1-\cos^2 2x}{\cos 2x}\,dx\right]$$

$$= e^{-2ix}\left[\frac{-\cos 2x}{2} + i\int \sec 2x\,dx - i\int \cos 2x\,dx\right]$$

$$= e^{-2ix}\left[\frac{-\cos 2x}{2} + i\,\frac{\log(\sec 2x + \tan 2x)}{2} - i\,\frac{\sin 2x}{2}\right]$$

$$= e^{-2ix}\left[\frac{-1}{2}\cos 2x + \frac{i}{2}\log(\sec 2x + \tan 2x) - \frac{i}{2}\sin 2x\right]$$

$$PI_2 = \frac{1}{D-2i}\tan 2x$$

$$= e^{2ix}\int \tan 2x \cdot e^{-2ix}\,dx$$

$$= e^{2ix}\int \tan 2x\,(\cos 2x - i\sin 2x)\,dx$$

$$= e^{2ix}\int \sin 2x\,dx - i\int \frac{\sin^2 2x}{\cos 2x}\,dx$$

$$= e^{2ix}\left[\frac{-\cos 2x}{2} - i\int \frac{1-\cos^2 2x}{\cos 2x}\,dx\right]$$

$$= e^{2ix}\left[-\frac{\cos 2x}{2} - i\int \sec 2x\,dx + i\int \cos 2x\,dx.\right.$$

$$= e^{2ix}\left[\frac{-1}{2}\cos 2x - i\,\frac{\log(\sec 2x+\tan 2x)}{2} + i\,\frac{\sin 2x}{2}\right]$$

$$= e^{2ix}\left[\frac{-1}{2}\cos 2x - \frac{i}{2}\log(\sec 2x+\tan 2x) + \frac{i}{2}\sin 2x\right]$$

$$P.I = \frac{-1}{i}\left[e^{-2ix}\left[\frac{-1}{2}\cos 2x + \frac{i}{2}\log(\sec 2x+\tan 2x) - \frac{i}{2}\sin 2x\right] - e^{2ix}\left[\frac{-1}{2}\cos 2x - \frac{i}{2}\log(\sec 2x+\tan 2x) + \frac{i}{2}\sin 2x\right]\right]$$

$$= \frac{-1}{i}\left[e^{-2ix}\frac{-1}{2}\cos 2x + \frac{i}{2}e^{-2ix}\log(\sec 2x+\tan 2x) - \frac{i}{2}e^{-2ix}\sin 2x + e^{2ix}\frac{1}{2}\cos 2x + \frac{i}{2}e^{2ix}\log(\sec 2x+\tan 2x) - \frac{i}{2}e^{+2ix}\sin 2x\right]$$

$$= \frac{-1}{i}\left[\frac{1}{2}\cos 2x\left[e^{2ix} - e^{-2ix}\right] + \frac{i}{2}\log(\sec 2x+\tan 2x)\left[e^{2ix}+e^{-2ix}\right] - \frac{i}{2}\sin 2x\left[e^{2ix}+e^{-2ix}\right]\right]$$

$$= \frac{-1}{i}\left[\frac{1}{\cancel{2}}\cos 2x\,(\cancel{2}i\sin 2x) + \frac{i}{\cancel{2}}\log(\sec 2x+\tan 2x)\,\cancel{2}\cos 2x - \frac{i}{\cancel{2}}\sin 2x\,\cancel{2}\cos 2x\right]$$

$$= -\cancel{\cos 2x\sin 2x} - \log(\sec 2x+\tan 2x)\cos 2x + \cancel{\sin 2x\cos 2x}.$$

$$P.I = -\log(\sec 2x + \tan 2x)$$

Now the solution of Equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos 2x + c_2\sin 2x\right] - \log(\sec 2x+\tan 2x)$$

① $\dfrac{d^2y}{dx^2} + a^2y = \tan ax$

Sol: Given D.E is $D^2y + a^2y = \tan ax$

$$(D^2+a^2)y = \tan ax \rightarrow ①$$

An Auxiliary Equ^n is $m^2+a^2=0$

$$m^2 = -a^2$$

$$m = \pm ai$$

∴ The roots are complex and distinct.

$$C.F = e^{(0)x}\left[c_1\cos ax + c_2\sin ax\right]$$

Tuesday
12/11/2019

Method of variation of Parameter:

② $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$.

Given D.E is $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

$$D^2y - 6Dy + 9y = \frac{e^{3x}}{x^2}$$

$$(D^2 - 6D + 9)y = \frac{e^{3x}}{x^2} \rightarrow ①$$

An Auxiliary equn is $m^2 - 6m + 9 = 0$

$$(m-3)^2 = 0$$
$$(m-3)(m-3) = 0$$
$$m = 3, 3.$$

∴ The roots are real and repeat.

$$C.F = c_1 e^{3x} + c_2 x e^{3x}$$

Let us take $y_1 = e^{3x}$ and $y_2 = xe^{3x}$

The P.I is of the form $P.I = u_1y_1 + u_2y_2$

$u_1 = -\int$

$$P.I = \frac{1}{D^2+a^2} \tan ax$$

$$= \frac{1}{(D+ai)(D-ai)} \tan ax$$

$$= \frac{-1}{2ai}\left(\frac{1}{D+ai} - \frac{1}{D-ai}\right) \tan ax$$

$$= \frac{-1}{2ai}\left[\underbrace{\frac{1}{D+ai} \tan ax}_{PI_1} - \underbrace{\frac{1}{D-ai} \tan ax}_{PI_2}\right] \rightarrow ②$$

$$PI_1 = \frac{1}{D+ai} \tan ax$$

$$= \frac{1}{D-(-ai)} \tan ax$$

$$= e^{-aix} \int \tan ax \, e^{aix} \, dx$$

$$= e^{-aix} \int \tan ax (\cos ax + i \sin ax) \, dx$$

$$= e^{-aix} \int \frac{\sin ax}{\cos ax} \cos ax \, dx + i \int \frac{\sin^2 ax}{\cos ax} \, dx$$

$$= e^{-aix}\left[-\frac{\cos ax}{a} + i\int \sec ax\,dx - i\int \cos ax\,dx\right]$$

$$= e^{-aix}\left[\frac{-1}{a}\cos ax + i\,\frac{\log(\sec ax + \tan ax)}{a} - i\cdot\frac{\sin ax}{a}\right]$$

$$= e^{-aix}\left[\frac{-1}{a}\cos ax + \frac{i}{a}\log(\sec ax + \tan ax) - \frac{i}{a}\sin ax\right]$$

$$PI_2 = \frac{1}{D-ai}\tan ax$$

$$= e^{aix}\int \tan ax\, e^{-aix}\,dx.$$

$$= e^{aix}\int \tan ax\,(\cos ax - i\sin ax)\cdot dx$$

$$= e^{aix}\int \frac{\sin ax}{\cancel{\cos ax}}\cancel{\cos ax} - i\int \frac{\sin^2 ax}{\cos ax}\,dx$$

$$= e^{aix}\left[\frac{-\cos ax}{a} - i\int \sec ax\,dx + i\int \cos ax\,dx\right]$$

$$= e^{aix}\left[\frac{-1}{a}\cos ax - i\,\frac{\log(\sec ax + \tan ax)}{a} + i\,\frac{\sin ax}{a}\right]$$

$$= e^{aix}\left[\frac{-1}{a}\cos ax - \frac{i}{a}\log(\sec ax + \tan ax) + \frac{i}{a}(\sin ax)\right]$$

$$P.I = \frac{-1}{2ai}\left[e^{-aix}\left[\frac{-1}{a}\cos ax + \frac{i}{a}\log(\sec ax + \tan ax) - \frac{i}{a}\sin ax\right] - e^{aix}\left[\frac{-1}{a}\cos ax - \frac{i}{a}\log(\sec ax + \tan ax) + \frac{i}{a}\sin ax\right]\right]$$

$$= \frac{-1}{2ai}\left[\frac{-1}{a}e^{-aix}\cos ax + \frac{i}{a}e^{-aix}\log(\sec ax + \tan ax) - \frac{i}{a}e^{-aix}\sin ax + \frac{1}{a}e^{aix}\cos ax + \frac{i}{a}e^{aix}\log(\sec ax + \tan ax) - \frac{i}{a}e^{aix}\sin ax\right]$$

$$= \frac{-1}{2ai}\left[\frac{1}{a}\cos ax\left(e^{aix} - e^{-aix}\right) + \frac{i}{a}\log(\sec ax + \tan ax)\left(e^{aix} + e^{-aix}\right) - \frac{i}{a}\sin ax\left[e^{aix} + e^{-aix}\right]\right]$$

$$= \frac{-1}{2ai}\left[\frac{1}{a}\cos ax\, 2i\sin ax + \frac{i}{a}\log(\sec ax + \tan ax)\, 2\cos ax\cdot - \frac{i}{a}\sin ax\, 2\cos ax\right]$$

$$= \frac{-1}{a^2}\cancel{\sin ax\cos ax} - \frac{1}{a^2}\cdot\log(\sec ax + \tan ax) + \frac{1}{a^2}\cdot\cancel{\sin ax\cos ax}$$

$$P.I = \frac{-1}{a^2}\log(\sec ax + \tan ax)$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos ax + c_2\sin ax\right] - \frac{1}{a^2}\log(\sec ax + \tan ax)$$

② $\frac{d^2y}{dx^2} + y = \cancel{\cos x} \; \operatorname{cosec} x$

Sol:- Given D.E is $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

$$D^2y + y = \operatorname{cosec} x$$

$$(D^2+1)y = \operatorname{cosec} x \rightarrow ①$$

An Auxiliary equn is $m^2+1=0$

$$m^2 = -1$$

$$m = \pm i$$

∴ The roots are complex and distinct.

$$C.F = e^{(0)x}\left[c_1 \cos x + c_2 \sin x\right]$$

$$P.I = \frac{1}{D^2+1} \cancel{\cos ax} \operatorname{cosec} x$$

$$= \frac{1}{(D+ai)(D-i)} \cos ax \operatorname{cosec} x$$

$$= \left(\frac{1}{D+ai} - \frac{1}{D-i}\right) \cos ax \operatorname{cosec} x$$

$$= \frac{-1}{2i}\left[\frac{1}{D+i} - \frac{1}{D-i}\right] \cancel{\cos ax} \operatorname{cosec} x$$

$$= \frac{-1}{2i}\left[\underbrace{\frac{1}{D+i} \operatorname{cosec} x}_{PI_1} - \underbrace{\frac{1}{D-i} \operatorname{cosec} x}_{PI_2}\right] \rightarrow ②$$

$$PI_1 = \frac{1}{D-(-i)} \operatorname{cosec} x$$

$$= e^{-ix}\int \operatorname{cosec} x \cdot e^{ix}\, dx$$

$$= e^{-ix}\int \operatorname{cosec} x\,(\cos x + i \sin x)\, dx$$

$$= e^{-ix}\int \frac{1}{\sin x} \cos x + i \int \cancel{\frac{1}{\sin x}} \cdot \cancel{\sin x}\, dx$$

$$= e^{-ix}\int \cot x\, dx + i\int (1)\, dx$$

$$= e^{-ix}\left[\log(\sin x) + i x\right]$$

$$PI_2 = \frac{1}{D-i} \operatorname{cosec} x$$

$$= e^{ix}\int \operatorname{cosec} x \cdot e^{-ix}\, dx$$

$$= e^{ix}\int \operatorname{cosec} x\,(\cos x - i \sin x)\, dx$$

$$= e^{ix}\int \cot x\, dx - i\int (1)\, dx$$

$$= e^{ix}\left[\log(\sin x) - ix\right]$$

$$P.I = \frac{-1}{2i}\left[e^{-ix}\left[\log(\sin x) + ix\right] - e^{ix}\left[\log(\sin x) - ix\right]\right]$$

$$= \frac{-1}{2i}\left[e^{-ix}\log(\sin x) + ixe^{-ix} - e^{ix}\log(\sin x) + e^{ix}ix\right]$$

$$= \frac{-1}{2i}\left[\log(\sin x)\left(e^{-ix} - e^{ix}\right) + ix.\left(e^{ix} + e^{-ix}\right)\right]$$

$$= -\frac{1}{2i}\left[\log(\sin x)(-2i\sin x) + ix\, 2\cos x\right]$$

$$= +\log(\sin x).\sin x - x.\cos x$$

$$P.I = \sin x.\log(\sin x) - x.\cos x$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos x + c_2\sin x\right] + \sin x.\log(\sin x) - x\cos x.$$

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* M.O.V.O.P :- continuous:

where $U_1 = -\int \frac{y_2 x}{W}\,dx$ and $U_2 = -\int \frac{y_1 x}{W}\,dx$

Wronskin $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

$$= \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & 3xe^{3x} \end{vmatrix}$$

$xe^{3x}(3) + e^{3x}$

$$= \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}(1+3x) \end{vmatrix}$$

$$= e^{3x}.e^{3x}(1+3x) - 3xe^{3x}.e^{3x}$$

$$= e^{6x} + \cancel{3xe^{6x}} - \cancel{3xe^{6x}}$$

$$\boxed{W = e^{6x}}$$

$$U_1 = -\int \frac{\cancel{x}e^{3x} \cdot \frac{e^{3x}}{x\cancel{y}}}{e^{6x}} dx \qquad U_2 = -\int \frac{e^{3x} \cdot \frac{e^{3x}}{x^2}}{e^{6x}} dx$$

$$= -\int \frac{\frac{1}{x} \cancel{e^{6x}}}{\cancel{e^{6x}}} dx \qquad = -\int \frac{1}{x^2} dx$$

$$= -\int x^{-2} dx$$

$$= -\int \frac{1}{x} dx \qquad = -\left(\frac{x^{-1}}{-1}\right)$$

$$= -\log x \qquad = \frac{1}{x}.$$

Now the $P.I = -\log x\, e^{3x} + \frac{1}{\cancel{x}} \cdot \cancel{x} e^{3x} = e^{3x}(1-\log x)$

~~Now the solution of equ n① is y = C.F + P.I~~

Now the solution of equn① is $y = C.F + P.I$

$y = c_1 e^{3x} + c_2 x e^{3x} + e^{3x}(1-\log x)$.

④ $y'' - 2y' + y = e^x \log x$.

Given D.E is ~~$m^2 - 2m + m$~~

$$D^2y - 2Dy + y = e^x \log x.$$

$$(D^2 - 2D + 1)y = e^x \log x \rightarrow ①$$

An Auxiliary equn is $m^2 - 2m + 1 = 0$

$$(m-1)^2 = 0$$

$$(m-1)(m-1) = 0$$

$$m = 1, 1.$$

∴ The roots are real and complex.

$C.F = c_1 e^x + c_2 x e^x$.

Let us take $y_1 = e^x$, $y_2 = xe^x$

The P.I is of the form $P.I = U_1 y_1 + U_2 y_2$

where $U_1 = -\int \frac{y_2 X}{W} dx$ and $U_2 = -\int \frac{y_1 X}{W} dx$

Wronskin $W = \begin{vmatrix} e^x & xe^x \\ e^x & x \cdot e^x + e^x \end{vmatrix}$

$$= \begin{vmatrix} e^x & xe^x \\ e^x & e^x(1+x) \end{vmatrix}$$

$$= e^x e^x (1+x) - e^x . x e^x$$

$$= e^{2x} + \cancel{xe^{2x}} - \cancel{xe^{2x}}$$

$$\boxed{W = e^{2x}}$$

$$U_1 = -\int \frac{\cancel{xe^x}.\ e^x \log x}{\cancel{e^{2x}}}\,dx \qquad U_2 = \int \frac{e^x.\ e^x \log x}{\cancel{e^{2x}}}\,dx$$

$$= -\int x \log x\ dx \qquad = -\int \log x\ dx$$

$$= -\left[\log x . \frac{x^2}{2} - \int \frac{1}{\cancel{x}} \frac{x^{\cancel{2}}}{2}\,dx\right] \qquad = -(x \log x - x)$$

$$= -\left[\frac{x^2}{2} \log x - \frac{1}{2}\int x\,dx\right] \qquad = -x \log x + x$$

$$= -\left[\frac{x^2}{2} \log x - \frac{1}{2}\frac{x^2}{2}\right]$$

$$= -\frac{x^2}{2} \log x + \frac{x^2}{4}$$

$$P.I = \left(-\frac{x^2}{2} \log x + \frac{x^2}{4}\right) e^x + (-x \log x + x)\, x . e^x$$

$$= -\frac{x^2}{2} \log x . e^x + \frac{x^2}{4} e^x + - x \log x\, x e^x + x . x e^x$$

$$= -\frac{x^2}{2} \log x . e^x + \frac{x^2}{4} e^x - x^2 \log x\, e^x + x^2 e^x$$

$$= \log x . e^x \left(-\frac{x^2}{2} - x^2\right) + x^2 e^x \left(\frac{1}{4} + 1\right)$$

$$= \log x . e^x \left(\frac{-x^2 - 2x^2}{2}\right) + x^2 e^x \left(\frac{1+4}{4}\right)$$

$$P.I = e^x . \log x \left(-\frac{3x^2}{2}\right) + x^2 e^x \left(\frac{5}{4}\right)$$

Now the solution of Equn ① is $y = C.F + P.I$

$$y = c_1 e^x + c_2 x e^x - e^x \log x . \left(\frac{3x^2}{2}\right) + \frac{5}{4} x^2 e^x .$$

⑥ $\dfrac{d^2y}{dx^2} + y = \dfrac{1}{1+\sin x}$.

Given D-E is $D^2y + y = \dfrac{1}{1+\sin x}$

$$(D^2+1)\,y = \frac{1}{1+\sin x} \rightarrow ①$$

An Auxiliary Equn is $m^2 + 1 = 0$

$$m^2 = -1$$

$$m = \pm i$$

∴ The roots are complex and distinct.

$C.F = e^{(0)x}[c_1 \cos x + c_2 \sin x]$

Let us take $y_1 = \cos x$ and $y_2 = \sin x$.

The P.I is of the form $P.I = u_1 y_1 + u_2 y_2$

where $u_1 = -\int \frac{y_2 X}{W} dx$ and $u_2 = \int \frac{y_1 X}{W} dx$

$$\text{Wronskin } (W) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$$

$$= \cos x \cdot \cos x + \sin x \cdot \sin x$$

$$= \cos^2 x + \sin^2 x$$

$$\boxed{W = 1}$$

$$u_1 = -\int \frac{\sin x \cdot \frac{1}{1+\sin x}}{1} dx$$

$$= -\int \sin x \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx$$

$$= -\int \sin x \left(\frac{1-\sin x}{1-\sin^2 x}\right) dx$$

$$= -\int \sin x \left(\frac{1-\sin x}{\cos^2 x}\right) dx$$

$$= -\int \left(\frac{\sin x}{\cos^2 x} - \frac{\sin^2 x}{\cos^2 x}\right) dx$$

$$= -\int \sec x \cdot \tan x \, dx + \int \tan^2 x \, dx$$

$$= -\sec x + \int (\sec^2 x - 1) dx$$

$$= -\sec x + \int \sec^2 x \, dx - \int (1) dx$$

$$= -\sec x + \tan x - x.$$

$$u_2 = -\int \frac{\cos x \frac{1}{1+\sin x}}{1} dx$$

$$= -\int \cos x \frac{1}{1+\sin x} \times \frac{1-\sin x}{1-\sin x}$$

$$= -\int \cos x \cdot \left(\frac{1-\sin x}{\cos^2 x}\right) dx$$

$$= -\int \sec x \, dx + \int \tan x \, dx$$

$$= -\log(\sec x + \tan x) + \log(\sec$$

$$P.I = (-\sec x + \tan x - x)\cos x + [-\log(\sec x + \tan x) + \log(\sec x)] \sin x$$

$$= -\sec x \cdot \cos x + \tan x \cdot \cos x - x\cos x - \log(\sec x + \tan x)\sin x + \log(\sec x) \sin x$$

$$= -1 + \cos x \cdot \tan x - x\cos x - \sin x[\log(\sec x + \tan x) - \log(\sec x)]$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos x + c_2\sin x\right] + \cos x\tan x - (x\cos x + 1)$$
$$- \sin x\left[\log(\sec x + \tan x) - \log(\sec x)\right]$$

$$= e^{(0)x}\left[c_1\cos x + c_2\sin x\right] + \sin x - (x\cos x + 1) - \sin x \cdot \log\left(\frac{\sec x + \tan x}{\tan x}\right)$$

(10) $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = \frac{1}{x^3}e^{-3x}$.

sol: Given D.E is $D^2y + 6Dy + 9y = \frac{1}{x^3}e^{-3x}$

$$(D^2 + 6D + 9)y = \frac{1}{x^3}e^{-3x} \rightarrow ①$$

An Auxiliary equ^n is $m^2 + 6m + 9 = 0$

$$(m+3)^2 = 0$$
$$(m+3)(m+3) = 0$$
$$m = -3, -3.$$

∴ The roots are real and repeat.

$C.F = c_1 e^{-3x} + c_2 x e^{-3x}$.

Let us take $y_1 = e^{-3x}$, $y_2 = xe^{-3x}$.

The P.I is of the form $P.I = u_1 y_1 + u_2 y_2$

where $u_1 = -\int \frac{y_2 X}{W}dx$, $u_2 = \int \frac{y_1 X}{W}dx$

$$\text{Wronskin } (W) = \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & x.e^{-3x}(-3) + e^{-3x} \end{vmatrix}$$

$$= \begin{vmatrix} e^{-3x} & xe^{-3x} \\ -3e^{-3x} & e^{-3x}(-3x+1) \end{vmatrix}$$

$$= e^{-3x}.e^{-3x}(-3x+1) + 3x.e^{-3x}.e^{-3x}$$

$$= \cancel{-3x.e^{-6x}} + e^{-6x} + \cancel{3xe^{-6x}}$$

$$\boxed{W = e^{-6x}}$$

$$U_1 = -\int \frac{\cancel{x e^{-3x}} \cdot \frac{1}{x^2} e^{-3x}}{\cancel{e^{-6x}}} dx \qquad U_2 = -\int \frac{e^{-3x} \cdot \frac{1}{x^3} e^{-3x}}{\cancel{e^{-6x}}} dx$$

$$= -\int x^{-2} dx \qquad\qquad = -\int x^{-3} dx$$

$$= -\left(\frac{x^{-1}}{-1}\right) \qquad\qquad = -\left(\frac{x^{-2}}{-2}\right)$$

$$= \frac{1}{x} \qquad\qquad = \frac{1}{2x^2}$$

$$P.I = \frac{1}{x} e^{-3x} + \frac{1}{2x^2} \cdot x \cdot e^{-3x} = \frac{1}{x} e^{-3x} + \frac{1}{2x} e^{-3x}$$

Now the solution of equⁿ① is $y = C.F + P.I$

$$y = c_1 e^{-3x} + c_2 x e^{-3x} + e^{-3x} \frac{1}{x}\left(1 + \frac{1}{2}\right)$$

$$y = c_1 e^{-3x} + c_2 x e^{-3x} + \frac{e^{-3x}}{x}\left(3/2\right).$$

① $\frac{d^2y}{dx^2} + 4y = 4\sec^2 2x.$

Sol:- Given DE is $D^2y + 4y = 4\sec^2 2x$

$$(D^2+4)y = 4\sec^2 2x \rightarrow ①$$

An Auxiliary equⁿ is $m^2+4=0$

$$m^2 = -4$$

$$m = \pm 2i$$

∴ The roots are complex and distinct

$$C.F = e^{(0)x}\left[c_1 \cos 2x + c_2 \sin 2x\right]$$

Let us take $y_1 = \cos 2x$, $y_2 = \sin 2x$

The P.I is of the form $P.I = U_1 y_1 + U_2 y_2$

where $U_1 = -\int \frac{y_2 X}{W} dx$ and $U_2 = -\int \frac{y_1 X}{W} dx$

$$\text{Wronskin value } (W) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2[\cos^2 2x + \sin^2 2x]$

$= 2(1)$

$\boxed{W = 2}$

$$U_1 = -\int \frac{\sin 2x \cdot \cancel{4}^{2} \sec^2 2x}{\cancel{2}} dx \qquad U_2 = -\int \frac{\cos 2x \cdot \cancel{4}^{2} \sec^2 2x}{\cancel{2}} dx$$

$\cancel{= -2\int \sin 2x\,(1+\tan^2 2x)\,dx}$

$\cancel{= -2\left[\int \sin 2x\,dx + \int \sin 2x \tan^2 2x\,dx\right]}$

$= -2\int \cos 2x \cdot \frac{1}{\cos^2 2x}\,dx$

$= -2\int \sec 2x\,dx$

$= -\cancel{2}\,\frac{\log(\sec 2x + \tan 2x)}{\cancel{2}}$

$= -\log(\sec 2x + \tan 2x)$

$= -2\int \sin 2x \frac{1}{\cos^2 2x}\,dx$

$= -2\int \tan 2x \sec 2x\,dx$

$= -\cancel{2}\,\frac{\sec 2x}{\cancel{2}} = -\sec 2x$

$P.I = -\sec 2x \cos 2x + (-\log(\sec 2x + \tan 2x)\sin 2x)$

$= -1 - \log(\sec 2x + \tan 2x)\sin 2x$

$= -[\sin 2x \cdot \log(\sec 2x + \tan 2x) + 1]$

Now the solution of Equn ① is $y = C.F + P.I$

$$y = e^{(0)x}[c_1 \cos 2x + c_2 \sin 2x] - [\sin 2x \cdot \log(\sec 2x + \tan 2x) + 1]$$

③ $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x.$

Sol:- Given D.E is $D^2y + y = \operatorname{cosec} x$

$(D^2+1)y = \operatorname{cosec} x \rightarrow ①$

An Auxiliary Equn is $m^2+1=0$

$m^2 = -1$

$m = \pm i$

∴ The roots are complex and distinct.

$C.F = e^{(0)x}[c_1 \cos x + c_2 \sin x]$

Let us take $y_1 = \cos x$, $y_2 = \sin x$.

The P.I is of the form $P.I = u_1 y_1 + u_2 y_2$.

where $u_1 = -\int \frac{y_2 X}{W} dx$ and $u_2 = -\int \frac{y_1 X}{W} dx$

Wronskin value $(W) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x$

$\boxed{W = 1}$

$u_1 = -\int \frac{\sin x \operatorname{cosec} x}{1} dx$

$= -\int (1) dx$

$= -x.$

$u_2 = -\int \frac{\cos x \cdot \operatorname{cosec} x}{1} dx$

$= -\int \cot x \, dx$

$= -\log(\sin x)$

$P.I = -x \cdot \cos x - \log(\sin x) \cdot \sin x$

$= -[x \cos x - \sin x \cdot \log(\sin x)]$

Now the solution of Eqn ① is $y = C.F + P.I$

$y = e^{(0)x}[c_1 \cos x + c_2 \sin x] - [x \cos x - \sin x \cdot \log(\sin x)]$

⑤ $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$

Sol: Given D.E is $D^2y - y = \frac{2}{1+e^x}$

$(D^2-1) y = \frac{2}{1+e^x} \rightarrow ①$

An Auxiliary Eqn is $m^2 - 1 = 0$

$m^2 = 1$

$m = \pm 1$

∴ The roots are real and distinct.

$C.F = c_1 e^x + c_2 e^{-x}$

Let us take $y_1 = e^x$, $y_2 = e^{-x}$

The P.I is of the form $P.I = u_1 y_1 + u_2 y_2$

where $u_1 = -\int \frac{y_2 X}{W} dx$ and $u_2 = -\int \frac{y_1 X}{W} dx$.

Wronskin value $= \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix}$

$$= -\cancel{e^x}\cdot\cancel{e^{-x}} - \cancel{e^x}\cdot\cancel{e^{-x}}$$

$$= -1-1$$

$$\boxed{W = -2}$$

$$u_1 = -\int \frac{e^{-x}\cdot\frac{\cancel{2}}{1+e^x}}{-\cancel{2}} dx \qquad u_2 = -\int \frac{e^x\cdot\frac{\cancel{2}}{1+e^x}}{-\cancel{2}} dx$$

$$= \int \frac{e^{-x}}{1+e^x}\cdot dx \qquad = \int \frac{e^x}{1+e^x} dx$$

$$= \int \cancel{e^{-x}}\cdot\frac{1}{\cancel{1+e^x}} dx \qquad = \int \frac{\cancel{e^{-x}}\cdot e^x}{e^{-x}+1} dx$$

$$\int \frac{1}{\cancel{e^x}+\cancel{e^{2x}}} dx \qquad = \frac{\log(e^{-x}+1)}{-e^{-x}}$$

$$= \int \frac{e^{-x}\cdot e^{-x}}{1+e^x} dx \qquad \boxed{\begin{matrix} 1+e^{-x} = t \\ -e^{-x}dx = dt \\ e^{-x}dx = -dt \end{matrix}} \qquad = -e^x\cdot\log(e^{-x}+1)$$

$$= \int \frac{t-1}{t}\cdot(-dt)$$

$$= -\int\left(1-\frac{1}{t}\right)dt$$

$$= -\int (1)dt + \int \frac{1}{t}dt$$

$$= -t + \log t$$

$$= -(1+e^{-x}) + \log(1+e^{-x})$$

$$P.I = \left[-(1+e^{-x}) + \log(1+e^{-x})\right]e^x + \left[-e^x\log(e^{-x}+1)\right]e^{-x}$$

$$= -e^x - \cancel{e^{-x}}\cancel{e^x} + e^x\log(1+e^{-x}) - \cancel{e^x}\log(e^{-x}+1)\cancel{e^{-x}}$$

$$= -e^x - 1 + e^x\log(1+e^{-x}) - \log(1+e^{-x})$$

$$= -e^x\left(1-\log(1+e^{-x})\right) - 1\left(1+\log(1+e^{-x})\right)$$

Now the solution of equn ① is $y = C.F + P.I$

$$y = c_1e^x + c_2e^{-x} - e^x\left(1-\log(1+e^{-x})\right) - \left(1+\log(1+e^{-x})\right)$$

(7) $\frac{d^2y}{dx^2} + y = \tan x.$

Sol:- Given D.E is $D^2y + y = \tan x$

$$(D^2+1)y = \tan x \rightarrow (1)$$

An Auxiliary equ^n is $m^2+1=0$

$$m^2 = -1$$

$$m = \pm i$$

∴ The roots are complex and distinct.

$$C.F = e^{(0)x}[c_1 \cos x + c_2 \sin x]$$

Let us take $y_1 = \cos x$ and $y_2 = \sin x$

The P.I of is of the form $P.I = u_1y_1 + u_2y_2$

where $u_1 = -\int \frac{y_2 X}{W} dx$ and $u_2 = -\int \frac{y_1 X}{W} dx$

Wronskin value $(W) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$$= \cos^2 x + \sin^2 x$$

$$\boxed{W=1}$$

$u_1 = -\int \frac{\sin x \cdot \tan x}{1} dx$

$= -\int \frac{\sin^2 x}{\cos x} dx$

$= -\int \frac{1-\cos^2 x}{\cos x} dx$

$= -\int \sec x \, dx + \int \cos x \, dx$

$= -\log(\sec x + \tan x) + \sin x$

$u_2 = -\int \frac{\cos x \cdot \tan x}{1} dx$

$= -\int \cos x \cdot \frac{\sin x}{\cos x} dx$

$= -\int \sin x \, dx$

$= -(-\cos x)$

$= \cos x$

$P.I = \{-\log(\sec x + \tan x) + \sin x\}\cos x + \cos x \sin x$

$= -\log(\sec x + \tan x) + \sin x \cdot \cos x + \sin x \cdot \cos x$

$= 2\sin x \cos x - \log(\sec x + \tan x)$

$P.I = \sin 2x - \log(\sec x + \tan x)$

Now the solution of equ^n (1) is $y = C.F + P.I$

$$y = e^{(0)x}[c_1 \cos x + c_2 \sin x] + \sin 2x - \log(\sec x + \tan x)$$

⑧ $y'' + y = \sec^2 x$.

sol:- Given D.E is $D^2y + y = \sec^2 x$.

$(D^2+1)\,y = \sec^2 x \rightarrow ①$

An Auxiliary equ^n is $m^2+1=0$

$m^2 = -1$

$m = \pm i$

∴ The roots are complex and distinct.

$C.F = e^{(0)x}\left[c_1\cos x + c_2 \sin x\right]$

Let us take $y_1 = \cos x$, $y_2 = \sin x$

The P.I is of the form $P.I = U_1 y_1 + U_2 y_2$

where $U_1 = -\int \frac{y_2 X}{W}\,dx$ and $U_2 = -\int \frac{y_1 X}{W}\,dx$

Wronskin value $(W) = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix}$

$= \cos^2 x + \sin^2 x$

$\boxed{W = 1}$

$U_1 = -\int \frac{\sin x \sec^2 x}{1}\,dx$

$= -\int \sin x\,(1+\tan^2 x)\,dx$

$= -\int \sin x\,dx - \int \sin x \cdot \frac{\sin^2 x}{\cos^2 x}\,dx$

$= -(-\cos x) - \int \sin x \left(\frac{1-\cos^2 x}{\cos^2 x}\right) dx$

$= \cos x - \int \sin x \cdot \frac{1}{\cos^2 x}\,dx + \int \sin x\,dx$

$= \cos x - \int \sec x \cdot \tan x\,dx + (-\cos x)$

$= \cancel{\cos x} - \sec x - \cancel{\cos x}$

$= \underline{-\sec x}$. (or)

$\boxed{\begin{aligned} U_1 &= -\int \sin x \cdot \sec^2 x\,dx \\ &= -\int \sec x \cdot \tan x\,dx \\ &= \underline{-\sec x} \end{aligned}}$

$U_2 = -\int \frac{\cos x \cdot \sec^2 x}{1}\,dx$

$= -\int \cancel{\cos x} \cdot \frac{1}{\cos^{\cancel{2}} x}\,dx$

$= -\int \sec x\,dx$

$= -\log(\sec x + \tan x)$

$P.I = -\sec x \cdot \cos x + \left[-\log(\sec x + \tan x)\right]\sin x$

$= -\left[1 + \sin x \cdot \log(\sec x + \tan x)\right]$

Now the solution of equ^n ① is $y = C.F + P.I$

$y = e^{(0)x}\left[c_1\cos x + c_2\sin x\right] - \left[1 + \sin x \cdot \log(\sec x + \tan x)\right]$

⑨ $\frac{d^2y}{dx^2}+y=x\sin x$.

Sol:- Given D.E is $D^2y+y=x\sin x$

$$(D^2+1)y=x\sin x \rightarrow ①$$

An Auxiliary equn is $m^2+1=0$

$$m^2=-1$$

$$m=\pm i$$

∴ The roots are ~~real~~ complex and distinct.

$$C.F=e^{0x}\left[c_1\cos x+c_2\sin x\right]$$

Let us take $y_1=\cos x$, $y_2=\sin x$

The P.I is of the form $P.I = u_1y_1+u_2y_2$

where $u_1=-\int\frac{y_2X}{W}dx$ and $u_2=-\int\frac{y_1X}{W}dx$

Wronskin value $(W)=\begin{vmatrix}\cos x & \sin x\\ -\sin x & \cos x\end{vmatrix}$

$$=\cos^2x+\sin^2x$$

$$\boxed{W=1}$$

$$u_1=-\int\frac{\sin x\cdot(x\sin x)}{1}dx$$

$$=-\int x\cdot\sin^2x\cdot dx$$

$$=-\left[x\cdot\frac{(\sin x)^3}{3}-\int 1\cdot\frac{(\sin x)^3}{3}dx\right]$$

$$=-\left[x\cdot\frac{(\sin x)^3}{3}-\frac{1}{3}\int\sin^3x\cdot dx\right]$$

$$=-\left[x\cdot\frac{(\sin x)^3}{3}-\frac{1}{3}\int\left(\frac{3\sin x-\sin 3x}{4}\right)dx\right]$$

$$=-\left[x\cdot\frac{(\sin x)^3}{3}-\frac{1}{3}\times\frac{3}{4}\int\sin x\,dx+\frac{1}{12}\int\sin 3x\cdot dx\right]$$

$$=-\left[\frac{x}{3}(\sin x)^3-\frac{1}{4}(-\cos x)+\frac{1}{12}\left(\frac{-\cos 3x}{3}\right)\right]$$

$$=-\frac{x}{3}(\sin x)^3+\frac{1}{4}\cos x-\frac{1}{36}\cos 3x$$

$$u_2=-\int\frac{\cos x\cdot x\sin x}{1}dx$$

$$=-\frac{1}{2}\int x\cdot\sin 2x\,dx$$

$$= \frac{-1}{2}\left[x\cdot\left(-\frac{\cos 2x}{2}\right) - \int (1)\left(-\frac{\cos 2x}{2}\right) dx\right]$$

$$= \frac{-1}{2}\left[-\frac{x}{2}\cos 2x + \frac{1}{2}\int \cos 2x\, dx\right]$$

$$= \frac{-1}{2}\left[-\frac{x}{2}\cos 2x + \frac{1}{2}\frac{\sin 2x}{2}\right]$$

$$= -\frac{1}{2}\left[-\frac{x}{2}\cos 2x + \frac{1}{4}\sin 2x\right]$$

$$= \frac{1}{4}\left[x\cdot\cos 2x - \frac{1}{2}\sin 2x\right]$$

$$P.I = \left(-\frac{x}{3}(\sin x)^3 + \frac{1}{4}\cos x - \frac{1}{36}\cos 3x\right)\cancel{\cos x}$$
$$+\frac{1}{4}\left(x\cos 2x - \frac{1}{2}\sin 2x\right)\sin x$$

$$\cancel{= -\frac{x}{3}(\sin x)^3}$$
$$= -\frac{x}{3}(\sin x)^3 + \frac{1}{4}\cos x + \frac{1}{4}\cos^2 x - \frac{1}{36}(\cos 3x),$$
$$+\frac{1}{4}x\cos 2x\cdot\sin x - \frac{1}{8}\sin 2x\cdot\sin x$$

$$\cancel{= -\frac{x}{3}(\sin x)^3\cdot\cos x}$$

Now

$$u_1 = -\int \sin x\cdot x\cdot\sin x\, dx$$

$$= -\int x\cdot\sin^2 x = -\int x\left(\frac{1-\cos 2x}{2}\right) = -\int\frac{x}{2}dx + \int x\frac{\cos 2x}{2}\, dx$$

$$= -\frac{1}{2}\int x\, dx + \frac{1}{2}\int \cos 2x\cdot x\, dx \quad -\frac{1}{2}\cdot\frac{x^2}{2} + \frac{1}{2}\frac{\sin 2x}{2}$$

$$= -\frac{1}{2}\frac{x^2}{2} + \frac{1}{2}\left(x\cdot\frac{\sin 2x}{2} + \int\frac{\cos 2x}{4}\right) \quad -\frac{x^2}{4} + \frac{1}{4}\sin 2x$$

$$= -\frac{x^2}{4} + \frac{1}{2}\left[\frac{x}{2}\sin 2x + \frac{\cos 2x}{4}\right]$$

$$= -\frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x.$$

$$P.I = \left(-\frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x\right)\cos x + \frac{1}{4}\left(x\cos 2x - \frac{1}{2}\sin 2x\right)\sin x$$

Now the solution of equ^n ① is $y = C.F + P.I$

$$y = e^{(0)x}\left[c_1\cos x + c_2\sin x\right] + \left(-\frac{x^2}{4} + \frac{x}{4}\sin 2x + \frac{1}{8}\cos 2x\right)\cos x$$
$$+\frac{1}{4}\left(x\cos 2x - \frac{1}{2}\sin 2x\right)\sin x.$$

14/11 thurs

Applications of Higher order d.e:

①

The equation of the L.C.R circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = 0.$$

Since $L=0.1$, $R=20$, $C=25\times10^{-6}$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$$

$$\frac{d^2q}{dt^2} + \frac{20}{0.1}\frac{dq}{dt} + \frac{q}{(0.1)(25\times10^{-6})} = 0$$

$$\frac{d^2q}{dt^2} + 200\frac{dq}{dt} + 400000\,q = 0. \rightarrow ①$$

Equⁿ ① is Higher order homogeneous d.e.:

∴ The solution is $q=$ complementary function.

$$D^2q + 200Dq + 400000q = 0$$

$$(D^2+200D+400000)q = 0.$$

An Auxiliary equⁿ is $m^2+200m+400000=0$.

$$m = \frac{-200 \pm \sqrt{(200)^2 - 4(1)400000}}{2(1)}$$

$$= \frac{-200 \pm \sqrt{40000 - 1600000}}{2}$$

$$= \frac{-200 \pm \sqrt{-1560000}}{2} \quad \because \boxed{1248.9996}$$

$$= \frac{-200 \pm 1249i}{2}$$

$$m = -100 \pm 624.5i$$

∴ The roots are complex and distinct.

$$C.F = e^{-100t}[c_1\cos(624.5)t + c_2\sin(624.5)t]$$

Now the solution of equⁿ ① is $q=C.F$

$$q = e^{-100t}[c_1\cos(624.5)t + c_2\sin(624.5)t]$$

Given that at $t=0$, $q=0.05$, $i=0$

at $t=0$, $q=0.05$

$$0.05 = e^{-100(0)}\left[c_1\cos(624.5)0 + c_2\sin(624.5)0\right]$$

$$0.05 = e^{(0)}\left[c_1(1)+c_2(0)\right]$$

$$\boxed{c_1 = 0.05}$$

$$i=\frac{dq}{dt} = e^{-100t}(-100)\left[c_1\cos(624.5)t + c_2\sin(624.5)t\right]$$
$$+e^{-100t}\left[c_1(-\sin(624.5)t)(624.5) + c_2\cos(624.5)t\cdot(624.5)\right]$$

at $t=0$, $i=0$

$$0 = e^{-100(0)}(-100)\left[c_1\cos(624.5)0 + c_2\sin(624.5)0\right]$$
$$+e^{-100t}\left[-c_1\sin(624.5)0\,(624.5) + c_2\cos((624.5)0)\cdot(624.5)\right]$$

$$0 = -100\left[c_1(1)+c_2(0)\right] + e^{-100(0)}(0 + c_2\,624.5)$$

$$0 = -c_1\,100 + c_2\cdot 624.5$$

$$0 = -0.05(100) + c_2\,624.5 \Rightarrow 0 = 5 + c_2\,624.5$$

$$\cancel{c_2\,624.5 = 0.05}$$

$$\cancel{c_2 = \frac{0.05}{624.5}}$$

$$c_2 = \frac{-5}{624.5}$$

$$c_2 = 0.008006405$$

$$\boxed{c_2 = 0.008}$$

(3) The eqn of the L.C.R. circuit is

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E\sin\omega t$$

$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = \frac{E}{L}\sin\omega t$$

$$\frac{d^2q}{dt^2} + 2s\frac{dq}{dt} + \omega^2 q = \frac{E}{L}\sin\omega t \rightarrow (1)$$

where $\omega^2 = \frac{1}{LC}$

$2s = \frac{R}{L}$

$$D^2q + 2SDq + \omega^2 q = \frac{E}{L}\sin\omega t$$

$$(D^2 + 2DS + \omega^2)q = \frac{E}{L}\sin\omega t$$

An auxiliary equ^n is $m^2 + 2Sm + \omega^2 = 0$

$$m = \frac{-2S \pm \sqrt{4S^2 - 4(1)\omega^2}}{2}$$

$$= \frac{-2S \pm \sqrt{4S^2 - 4\omega^2}}{2}$$

$$= \frac{\cancel{2}(-S \pm \sqrt{S^2 - \omega^2})}{\cancel{2}}$$

$$= -S \pm \sqrt{S^2 - \omega^2}$$

We have $R^2 < \frac{4L}{C}$

$$\frac{R^2}{4L} < \frac{1}{C}$$

$$\frac{R^2}{4L^2} < \frac{1}{LC}$$

$$\left(\frac{R}{2L}\right)^2 - \frac{1}{LC} < 0$$

$$\boxed{S^2 - \omega^2 < 0}$$

$$m = -S \pm \sqrt{S^2 - \omega^2}\, i$$

∴ The roots are complex and distinct.

Let $p = \sqrt{S^2 - \omega^2}$

$$m = -S \pm p i$$

$$C.F = e^{-St}[C_1 \cos pt + C_2 \sin pt].$$

The particular integral is of the form $= \frac{1}{f(D)} X$

$$= \frac{1}{D^2 + 2DS + \omega^2} \cdot \frac{E}{L}\sin\omega t.$$

$$= \frac{E}{L}\frac{1}{D^2 + 2SD + \omega^2}\sin\omega t$$

$$= \frac{E}{L}\frac{1}{\cancel{-\omega^2} + 2SD + \cancel{\omega^2}}\sin\omega t$$

$$= \frac{E}{L}.\frac{1}{2S}\left(\frac{1}{D}\sin\omega t\right)$$

$$= \frac{E}{2L\delta}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$P.I = \frac{-E}{2L\delta\omega}(\cos\omega t).$$

$$= \frac{-E}{R\omega}(\cos\omega t)$$

$$2\delta = \frac{R}{L} \Rightarrow \delta = \frac{R}{2L}$$

$$\boxed{R = 2L\delta}$$

Now the solution for equn ① is $q = C.F + P.I$

$$q = e^{-\delta t}[c_1\cos pt + c_2\sin pt) - \frac{E}{R\omega}(\cos\omega t) \rightarrow ②$$

we have $t=0,\ q=0$

$$0 = e^{-\delta(0)}[c_1\cos p(0) + c_2\sin p(0)) - \frac{E}{R\omega}\cos\omega(0)$$

$$0 = (1)[c_1(1) + c_2(0)] - \frac{E}{R\omega}(1)$$

$$\boxed{c_1 = \frac{E}{R\omega}}$$

$$i = \frac{dq}{dt} = e^{-\delta t}(-\delta)[c_1\cos pt + c_2\sin pt] + e^{-\delta t}[c_1(-\sin pt(p)) + c_2\cos pt(p)]$$

$$i = -\delta e^{-\delta t}[c_1\cos pt + c_2\sin pt] + e^{-\delta t}[-pc_1\sin pt + pc_2\cos pt] + \frac{E}{R\omega}\sin\omega t(\omega)$$

$$+ \frac{E}{R}\sin\omega t$$

we have $t=0,\ i=0$

$$0 = -\delta e^{-\delta(0)}[c_1\cos p(0) + c_2\sin p(0)) + e^{-\delta(0)}[-pc_1\sin p(0) + pc_2\cos p(0)] + \frac{E}{R}\sin\omega(0)$$

$$0 = -\delta(1)[c_1(1) + c_2(0)] + (1)[pc_1(0) + pc_2(1)] + \frac{E}{R}(0)$$

$$0 = -\delta c_1 + pc_2$$

$$0 = -\delta\frac{E}{R\omega} + pc_2$$

$$pc_2 = \frac{\delta E}{R\omega}$$

$$\boxed{c_2 = \frac{\delta E}{pR\omega}}$$

from (2),

$$q = e^{-St}\left[c_1 \cos pt + c_2 \sin pt\right] - \frac{E}{R\omega}\cos\omega t$$

$$= e^{-St}\left[\frac{E}{R\omega}\cos pt + \frac{ES}{PR\omega}\sin pt\right] - \frac{E}{R\omega}\cos\omega t$$

$$= \frac{E}{R\omega}\left[e^{-St}\left(\cos pt + \frac{S}{p}\sin pt\right) - \cos\omega t\right]$$

$$= \frac{E}{R\omega}\quad -\cos\omega t + e^{\frac{-Rt}{2L}}$$

$$q = \frac{E}{R\omega}\left[-\cos\omega t + e^{\frac{-Rt}{2L}}\left(\cos pt + \frac{R}{2Lp}\sin pt\right)\right]$$

$$i = \frac{dq}{dt} = e^{-St}(-S)\left[c_1\cos pt + c_2\sin pt\right] + e^{-St}\left[c_1(-\sin pt)p + c_2(\cos pt)\,p\right]$$

$$-\frac{E}{R\omega}(-\sin\omega t)\,\omega$$

$$i = \frac{dq}{dt} = \frac{E}{R\omega}\left[\sin\omega t\;\omega + e^{-R/2L\,t}\cdot\left(\frac{-R}{2L}\right)\left(\cos pt + \frac{R}{2Lp}\sin pt\right) + e^{\frac{-Rt}{2L}}\left[-\sin pt(p) + \frac{R}{2L\cancel{p}}\cos(pt)\,\cancel{p}\right]\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\left(\frac{R}{2L}\right)\left(\cos pt + \frac{R}{2Lp}\sin pt\right) - e^{\frac{-Rt}{2L}}p\cdot\sin pt + e^{\frac{-Rt}{2L}}\cdot\frac{R}{2L}\cdot\cos pt\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\cancel{\frac{R}{2L}\cos pt} - e^{\frac{-Rt}{2L}}\cdot\frac{R}{2L}\cdot\frac{R}{2Lp}\sin pt - e^{\frac{-Rt}{2L}}p\cdot\sin pt + e^{\frac{-Rt}{2L}}\cdot\cancel{\frac{R}{2L}\cos pt}\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\cdot\frac{R^2}{4L^2p}\sin pt - e^{\frac{-Rt}{2L}}\cdot p\cdot\sin pt\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\sin pt\left(\frac{S^2}{P} + P\right)\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\sin pt\left(\frac{S^2 + P^2}{P}\right)\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\sin pt\left(\frac{\cancel{S^2} + \omega^2 - \cancel{S^2}}{P}\right)\right]$$

$$= \frac{E}{R\omega}\left[\omega\cdot\sin\omega t - e^{\frac{-Rt}{2L}}\sin pt\left(\frac{\omega^2}{P}\right)\right]$$

$$= \frac{E}{R\omega} \omega \left[\sin\omega t - e^{\frac{-Rt}{2L}} \frac{\omega}{p} \cdot \sin pt \right]$$

$$i = \frac{E}{R} \left[\sin\omega t - e^{\frac{-Rt}{2L}} \frac{1}{p\sqrt{LC}} \sin pt \right]$$

④

The equ^n of the L.C.R circuit is

$$L \frac{d^2q}{dt^2} + R \cdot \frac{dq}{dt} + \frac{q}{C} = E \sin\omega t$$

$$\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = \frac{E}{L} \sin\omega t$$

$$\frac{d^2q}{dt^2}$$

② An uncharged condenser ..

Given that,

The equ^n of the L.C.R circuit is

$$L \frac{d^2q}{dt^2} + \frac{R}{\cancel{L}} \frac{dq}{dt} + \frac{q}{C} = E \sin \frac{t}{\sqrt{LC}}$$

Given that resistance is negligible.

then, $$L \frac{d^2q}{dt^2} + \frac{q}{C} = E \sin \frac{t}{\sqrt{LC}}$$

$$\frac{d^2q}{dt^2} + \frac{q}{LC} = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

$$D^2q + \omega^2 q = \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

$$(D^2+\omega^2)\, q = \frac{E}{L} \sin \frac{t}{\sqrt{LC}} \rightarrow ①$$

An Auxiliary equ^n is

$$m^2+\omega^2=0$$

$$m^2=-\omega^2$$

$$m = \pm \omega i$$

$\therefore$ The roots are complex and distinct.

$$C.F = e^{(0)t} \left[c_1 \cos\omega t + c_2 \sin\omega t \right]$$

$$P.I = \frac{1}{D^2+\omega^2} \frac{E}{L} \sin \frac{t}{\sqrt{LC}}$$

$$= \frac{E}{L} \cdot \frac{1}{D^2+\omega^2} \sin\omega t.$$

$$= \frac{E}{L} \frac{t}{2D+0} \sin\omega t$$

$$= \frac{Et}{2L} \frac{1}{D} \sin\omega t$$

$$= \frac{Et}{2L} \frac{-\cos\omega t}{\omega}$$

$$P.I = \frac{-Et}{2L\omega} \cos\omega t$$

Now the solution of Equn ① is $q = C.F + P.I$

$$q = c_1 \cos\omega t + c_2 \sin\omega t - \frac{Et}{2L\omega} \cos\omega t. \rightarrow ②$$

At $t=0$, $q=0$

$$0 = c_1 \cos\omega(0) + c_2 \sin\omega(0) - \frac{E(0)}{2L\omega} \cos\omega(0)$$

$$0 = c_1(1) + c_2(0) - 0$$

$$\Rightarrow \boxed{c_1 = 0}$$

from ①,

$$q = c_2 \sin\omega t - \frac{Et}{2L\omega} \cos\omega t. \rightarrow ③$$

$$i = \frac{dq}{dt} = c_2 \cos\omega t(\omega) - \frac{E}{2L\omega}\left[t.(-\sin\omega t)\omega + \cos\omega t(1)\right]$$

$$i = c_2\omega . \cos\omega t + \frac{Et}{2L} \sin\omega t - \frac{E}{2L\omega} \cos\omega t$$

At $t=0$, $i=0$

$$0 = c_2\omega . \cos\omega(0) + \frac{E(0)}{2L} \sin\omega(0) - \frac{E}{2L\omega} \cos\omega(0)$$

$$0 = c_2\omega + 0 - \frac{E}{2L\omega}$$

$$c_2\omega = \frac{E}{2L\omega} \Rightarrow \boxed{c_2 = \frac{E}{2L\omega^2}}$$

from ③,

$$q = \frac{E}{2L\omega^2} \sin\omega t - \frac{Et}{2L\omega} \cos\omega t.$$

$$= \frac{E\cancel{L}C}{2\cancel{L}} \sin\frac{t}{\sqrt{LC}} - \frac{Et\sqrt{LC}}{2L} \cos\omega t.$$

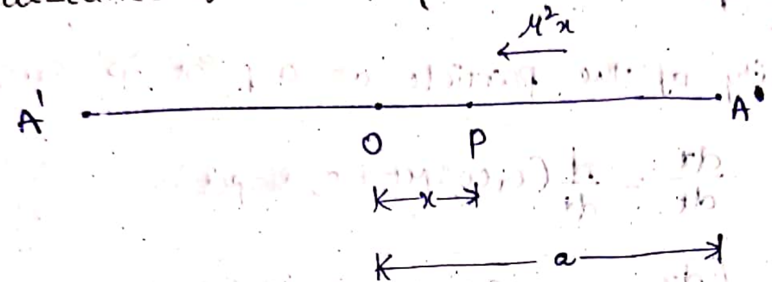
$$= \frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{t\sqrt{LC}}{LC} \cos\frac{t}{\sqrt{LC}}\right]$$

$$\boxed{q = \frac{EC}{2}\left[\sin\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}} \cos\frac{t}{\sqrt{LC}}\right]}$$

* Simple Harmonic Motion

Max distance = amplitude
a^2 = Intensity
velocity (v) = $\frac{dx}{dt}$
acceleration (a) = $\frac{dv}{dt}$

* ① A particle is said to execute S.H.M if it moves in a straight line such that its acceleration is always directed towards a fixed point in the line and is proportional to the distance of the particle from the fixed point.



→ Let 'O' be the fixed point in the line AA'.
→ Let 'P' be the position of the particle at any time 't'.
→ Where $\boxed{OP = x}$
→ Since the acceleration is always directed towards the point 'O', i.e, the acceleration is in the direction opposite to that in which 'x' increases.

→ Therefore, the equ^n of the motion of the particle is

$$\frac{d^2x}{dt^2} = -\mu^2 x.$$

(or)

$$\frac{d^2x}{dt^2} + \mu^2 x = 0$$

(or)

$$\boxed{D^2 x + \mu^2 x = 0}$$

$$\boxed{(D^2 + \mu^2)x = 0} \rightarrow ①$$

where $D = \frac{d}{dt}$

→ It is a linear differential equ^n with constant co-efficient.

i.e., $D^2 + \mu^2 = 0$ $\boxed{x \neq 0}$

$\rightarrow D^2 = -\mu^2$

$\rightarrow \boxed{D = \pm \mu i}$

∴ The solution of equ^n ① is

$$\boxed{x = c_1 \cos \mu t + c_2 \sin \mu t.} \longrightarrow ②$$

∴ The velocity of the particle at a point 'p' can be written as $\frac{dx}{dt} = \frac{d}{dt}(c_1 \cos \mu t + c_2 \sin \mu t)$

$$v = \boxed{\frac{dx}{dt} = -c_1 \mu \sin \mu t + c_2 \mu \cos \mu t} \longrightarrow ③$$

→ If the particle starts from the rest at 'A', where OA = a.

→ Therefore from ②

At $t = 0$, $x = a$

$a = c_1 \cos \mu(0) + c_2 \sin \mu t.$

$= c_1 (1) + c_2 (0)$

$\rightarrow \boxed{a = c_1}$

→ a or c_1 from ③ At $t = 0$, $v = 0$, $\frac{dx}{dt} = 0$

$v = \frac{dx}{dt} = -c_1 \mu \sin \mu(0) + c_2 \mu \cos \mu(0)$

$\frac{dx}{dt} = -c_1 \mu (0) + c_2 \mu (1)$

Substitution 'c_1' and 'c_2' value in ①

* $\boxed{x = a \cos \mu t} \longrightarrow ④$

$\therefore$ velocity $= \frac{dx}{dt} = -a\mu \sin \mu t \rightarrow (5)$

$$v = \frac{dx}{dt} = -a\mu\sqrt{1-\cos^2 \mu t})$$

Let $\cos \mu t = \frac{x}{a}$. then above equn can be written

as $= -a\mu\sqrt{1-\cos^2 \mu t}$

$$\frac{dx}{dt} = -a\mu\sqrt{1-\frac{x^2}{a^2}}$$

$$\frac{dx}{dt} = -a\mu\sqrt{\frac{a^2-x^2}{a^2}}$$

$$\frac{dx}{dt} = -\mu\sqrt{a^2-x^2} \rightarrow (6)$$

Time Period:

The time taken for one perfect oscillation is called time period. which is denoted T.

→ The time period can be written as $\boxed{T = \frac{2\pi}{\mu}}$

Frequency of the Oscillators:

The no. of oscillations fore second is called frequency of the oscillator.

→ which is denoted by n $n = \frac{1}{T}$

$$n = \frac{1}{2\pi/\mu}$$

$$\boxed{n = \frac{\mu}{2\pi}}$$

① A particle is executing S.H.M ...

Sol: Given amplitude $= 20$ cm
time $(T) = 4$ seconds

We know that, $T = \frac{2\pi}{\mu}$

$$4 = \frac{2\pi}{\mu}$$

$$\boxed{\mu = \pi/2}$$

We know that, $x = a\cos \mu t$

Case(i)

At $x_1 = 5$ cm, $\mu = \pi/2$, $a = 20$ cm

$$x_1 = a\cos \mu t$$

$$5 = 20 \cos \pi/2\, t$$

$$1/4 = \cos \pi/2\, t$$

$$\cos^{-1}(1/4) = \pi/2\, t$$

$$\boxed{t_1 = \frac{2}{\pi} \cos^{-1} 1/4}$$

Case(ii)

At $x_2 = 15$ cm, $\mu = \pi/2$, $a = 20$ c

$$x_2 = a \cos \mu t$$

$$15 = 20 \cos \pi/2\, t$$

$$3/4 = \cos \pi/2\, t$$

$$\cos^{-1}(3/4) = \pi/2\, t$$

$$\boxed{t_2 = \frac{2}{\pi} \cos^{-1} 3/4}$$

$$\therefore t_2 - t_1 = \frac{2}{\pi} \cos^{-1} 3/4 - \frac{2}{\pi} \cos^{-1} 1/4$$

$$= \frac{2}{\pi}\left[\cos^{-1}(3/4) - \cos^{-1}(1/4)\right]$$

$$= \frac{2}{180}\left[41.40962211 - 75.52248781\right]$$

$$= \frac{1}{90}(-34.1128657)$$

$$= -0.1379$$

$$\boxed{t_2 - t_1 \approx -0.38}\ \text{seconds}$$

② A particle moving in a straight line...

Sol: Given $x = a\cos\mu t$.

We know that the velocity, $V = -a\mu \sin \mu t$

(or)

$$V = -\mu \sqrt{a^2 - x^2}$$

case(i)

At displacement $= x_1$

velocity $= v_1$

$$v_1 = -\mu\sqrt{a^2 - x_1^2}$$

$$v_1^2 = \mu^2(a^2 - x_1^2)$$

case(ii) At displacement $= x_2$

velocity $= v_2$

$$v_2 = +\mu\sqrt{a^2 - x_2^2}$$

$$v_2^2 = \mu^2(a^2 - x_2^2)$$

$$\therefore v_2^2 - v_1^2 = \mu^2(a^2 - x_2^2) - \mu^2(a^2 - x_1^2)$$

$$= \cancel{a^2\mu^2} - \mu^2 x_2^2 - \cancel{\mu^2 a^2} + x_1^2\mu^2$$

$$v_2^2 - v_1^2 = \mu^2(x_1^2 - x_2^2)$$

$$\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2} = \mu^2$$

$$\mu = \left(\sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}\right)$$

We know that Time period $T = \frac{2\pi}{\mu}$.

$$T = \frac{2 \times \pi}{\sqrt{\frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}}}$$

$$T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$$

③ At the end of the three successive seconds. the distances of a point moving with S.H.M from its mean position are x_1, x_2, x_3 respectively. Show that the time of a complete oscillation is $\frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)}$

Sol: Given that x_1, x_2, x_3 are the distances.

Let at the positions the times can be taken as t, $t+1$, $t+2$ seconds respectively. seconds.

We know that, $x = a\cos\mu t$

$$x_1 = a\cos\mu t \rightarrow ①$$

$$x_2 = a\cos\mu(t+1) \rightarrow ②$$

$$x_3 = a\cos\mu(t+2) \rightarrow ③$$

By adding equ^n ① & ③

We get $x_1 + x_3 = a\cos\mu t + a\cos\mu(t+2)$

$$x_1+x_3 = a[\cos\mu t + \cos\mu(t+2)]$$

$$x_1+x_3 = a\ 2\cos\left(\frac{\mu t+\mu(t+2)}{2}\right)\cos\left(\frac{\mu t-\mu(t+2)}{2}\right)$$

$$= 2a\cos\left(\frac{\mu t+\mu t+2\mu}{2}\right)\cos\left(\frac{\cancel{\mu t}-\cancel{\mu t}-2\mu}{2}\right)$$

$$= 2a\cos\left(\frac{2\mu t+2\mu}{2}\right)\cos\left(\frac{-\cancel{2}\mu}{\cancel{2}}\right)$$

$$= 2a\cos\left(\frac{\cancel{2}\mu(t+1)}{\cancel{2}}\right)\cos(-\mu)$$

$$x_1+x_3 = 2a\cos\mu(t+1)\cdot\cos\mu$$

$$x_1+x_3 = 2\cdot\cos\mu\,[a\cos\mu(t+1)]$$

$$x_1+x_3 = 2\cos\mu\ x_2$$

$$\cos\mu = \frac{x_1+x_3}{2x_2}$$

$$\mu = \cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)$$

We know that,

$$\text{Time period } (T) = \frac{2\pi}{\mu}$$

$$T = \frac{2\pi}{\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right)}$$

④ A particle is executing S.H.M. ....

sol: Given amplitude = 5 meters.

time (T) = 4 seconds

W.K.T, $T = \frac{2\pi}{\mu}$

$$4 = \frac{2\pi}{\mu}$$

$$\boxed{\mu = \pi/2}$$

W.K.T, $x = a\cos\mu t_1$

case(i) At $x_1 = 4\,m$, $\mu = \pi/2$, $a = 5m$

$$x_1 = a\cos\mu t_1$$

$$4 = 5\cos\pi/2\ t_1$$

case(ii) At $x_2 = 2m$, $\mu = \pi/2$, $a = 5$

$$x_2 = a\cos\mu t_2$$

$$2 = 5\cos\pi/2\ t_2$$

$4/5 = \cos \pi/2 \, t$ $\qquad\qquad$ $2/5 = \cos \pi/2 \cdot t_2$

$\cos^{-1}(4/5) = \pi/2 \, t$ $\qquad\qquad$ $\cos^{-1}(2/5) = \pi/2 \, t_2$

$t_1 = \frac{2}{\pi} \cos^{-1}(4/5)$ $\qquad\qquad$ $t_2 = \frac{2}{\pi} \cos^{-1}(2/5)$

$\therefore t_2 - t_1 = \frac{2}{\pi} \cos^{-1}(2/5) - \frac{2}{\pi} \cos^{-1}(4/5)$

$= \frac{2}{180} \left[\cos^{-1}(2/5) - \cos^{-1}(4/5) \right]$

$= \frac{2}{180} \left[66.42 - 36.86 \right]$

$= \frac{2}{180} \times 29.56$

$= 0.3284$

$\boxed{t_2 - t_1 \cong 0.33}$ seconds.

⑤ At the end of the three successive seconds, - -

Sol:- Given that $x_1 = 1$, $x_2 = 5$, $x_3 = 5$

Time period $(T) = \frac{2\pi}{\theta}$

Let at the positions the times can be taken as,

t, $t+1$, $t+2$ seconds respectively.

W.K.T, $x = a \cos \mu t$

$x_1 = a \cos \mu t \Rightarrow 1 = a \cos \mu t \rightarrow ①$

$x_2 = a \cos \mu (t+1) \Rightarrow 5 = a \cos \mu (t+1) \rightarrow ②$

$x_3 = a \cos \mu (t+2) \Rightarrow 5 = a \cos \mu (t+2) \rightarrow ③$

By adding eqn ① & ③

$1+5 = a \cos \mu t + a \cos \mu (t+2)$

$6 = a \left[\cos \mu t + \cos \mu (t+2) \right]$

$6 = a \, 2 \cos \left(\frac{\mu t + \mu (t+2)}{2} \right) \cos \left(\frac{\mu t - \mu (t+2)}{2} \right)$

$6 = 2a \cos \left(\frac{\mu t + \mu t + 2\mu}{2} \right) \cos \left(\frac{\cancel{\mu t} - \cancel{\mu t} - 2\mu}{2} \right)$

$6 = 2a \cos \left(\frac{2\mu t + 2\mu}{2} \right) \cos \left(\frac{-\cancel{2}\mu}{\cancel{2}} \right)$

$3 = a \cos \frac{\cancel{2}(\mu t + \mu)}{\cancel{2}} \cos(-\mu)$

$3 = a \cos \mu (t+1) \cos \mu$

$3 = 5 \cos \mu$

$\cos \mu = \frac{3}{5}.$

$$\boxed{\therefore \cos \theta = \frac{3}{5}}$$