Wave Optics

Principle of Super position:-

When two or more waves travel simultaneously in a medium then the resultant displacement of the individual waves then they are super imposed with each other is equal to the algebraic sum of the displacements of individual waves. This principle is called 'Super Position.' (or) 'Principle of Super Position'.

Explanation:-

Let y_1, y_2 be the displacements of any two waves travels simultaneously in a medium.

When the two waves super imposed with each other then according to the principle of super position. The resultant displacement (y) can we written as.

$$\boxed{y = y_1 \pm y_2}$$

Case (i):-

If the resultant displacement

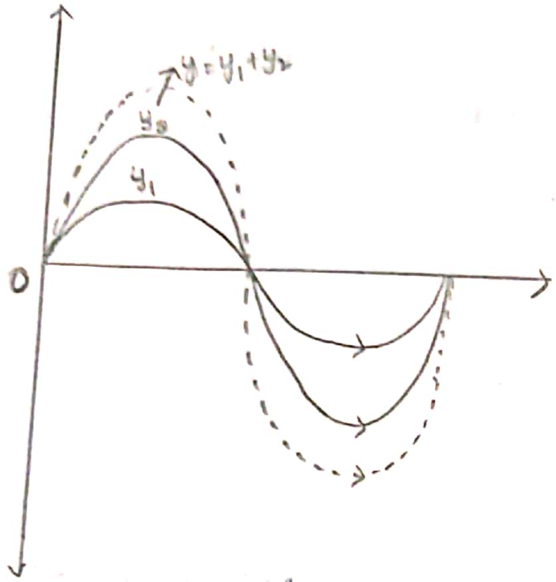
$$\boxed{y = y_1 + y_2}$$

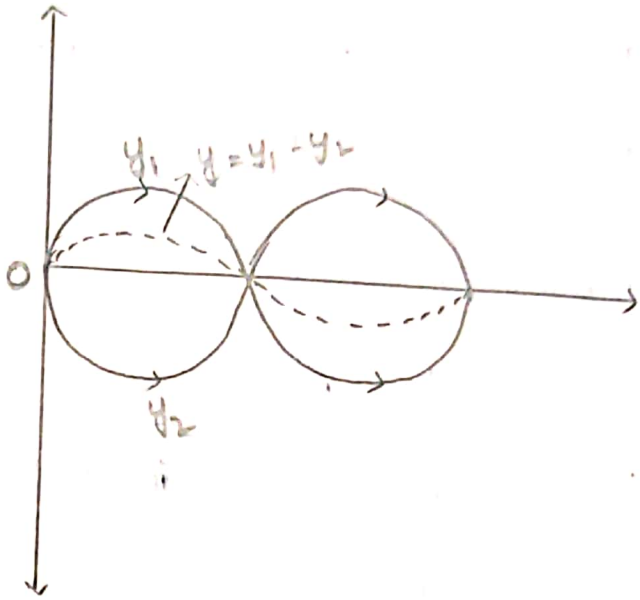
i.e., the two waves are travel in same direction.

Case-(ii) :-

If suppose the two waves travel in opposite direction then the resultant displacement.

$$\boxed{y = y_1 - y_2}$$





Coherent Waves :-

The waves which are having same amplitude same wavelength and constant phase difference are called 'coherent Waves'.

Coherent Sources :-

The light sources which can exhibits coherent waves are called 'Coherent Sources'.

Interference :-

When the two coherent waves super imposed with each other than the formation of maximum intensity and minimum intensity (or) bright fringe and drak fringe in the region of super position. This phenomenon is called 'Interference'

(or)

When the two coherent waves super imposed with each other then the intensity or amplitude can be modify in the region of super position. This phenomenon is called 'Interference'.

Types of Interference :-

The interference pattern is mainly two types.

They are : ① Constructive interference.
② Destructive interference.

① Constructive Interference :-

In which the resultant amplitude is sum of the individual amplitudes of the waves then the interference is called constructive interference.

Let $a_1, a_2,$ be the amplitudes of the any two waves. If 'A' be the resultant amplitude of the waves then according to the constructive interference.

Resultant Amplitude $A = a_1 + a_2$

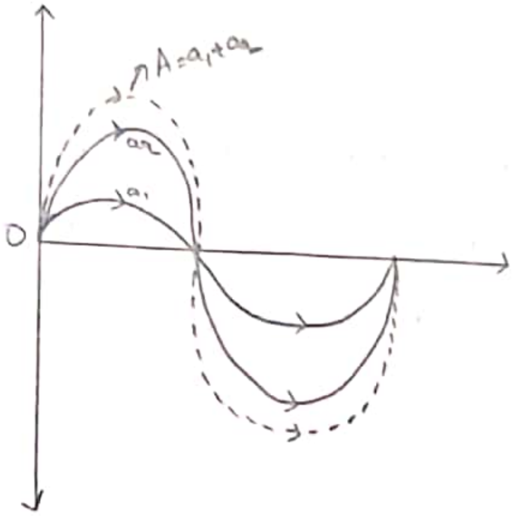
$a_1 = a_2 \approx a$

$I = (a+a)^2$

$$\boxed{I_{max} = 4a^2}$$

$A^2 = (a_1+a_2)^2$

$$\boxed{I = (a_1+a_2)^2}$$



② **Destructive Interference :-**

The Interference in which the resultant amplitude is difference of the individual amplitudes of the waves then then the interference is called Destructive Interference.

Let a_1, a_2 be the amplitude of any two waves. If 'A' be the resultant amplitude of the waves then according to the destructive interference

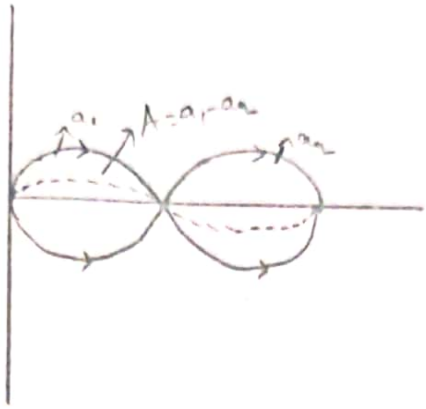
$$A = a_1 - a_2$$

$$A^2 = (a_1 - a_2)^2$$

$$I = (a_1 - a_2)^2$$

$$I = (a - a)^2$$

$$\boxed{I_{min} = 0}$$



Conditions when the two waves super impose with each other:-

let y_1, y_2 be the displacements of any two waves which can be taken as $y_1 = a_1 \sin \omega t \longrightarrow (1)$

$$y_2 = a_2 \sin(\omega t + \phi) \longrightarrow (2)$$

If suppose 'y' be the resultant displacement of above two waves then we can be written as

$$y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \phi)$$

$$= a_1 \sin \omega t + a_2 (\sin \omega t . \cos \phi + \cos \omega t \sin \phi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t . \cos \phi + a_2 \cos \omega t \sin \phi$$

$$= \sin \omega t (a_1 + a_2 \cos \phi) + a_2 \cos \omega t \sin \phi$$

let $a_1 + a_2 \cos \phi = R \cos \theta \longrightarrow (3)$

$$a_2 \sin \phi = R \sin \theta \longrightarrow (4)$$

eq^n $(3)^2 + (4)^2$

$$[a_1 + a_2 \cos \phi]^2 + [a_2 \sin \phi]^2 = R^2 \cos^2 \theta + R^2 \sin^2 \theta$$

$$a_1^2 + a_2^2 \cos^2\phi + 2a_1 a_2 \cos\phi + a_2^2 \sin^2\phi = R^2 \cos^2\theta + R^2 \sin^2\theta$$

$$a_1^2 + a_2^2 [\cos^2\phi + \sin^2\phi] + 2a_1 a_2 \cos\phi = R^2 [\cos^2\theta + \sin^2\theta]$$

$$\boxed{a_1^2 + a_2^2 + 2a_1 a_2 \cos\phi = R^2} \quad \text{——} \ (5)$$

if $a_1 = a_2 \approx a$

$$a^2 + a^2 + 2a^2 \cos\phi = R^2$$

$$2a^2 + 2a^2 \cos\phi = R^2$$

$$R^2 = 2a^2 [1 + \cos\phi]$$

$$= 2a^2 \left[2\cos^2 \frac{\phi}{2}\right]$$

$$\boxed{R^2\ I = 4a^2 \cos^2\left(\frac{\phi}{2}\right)}$$

The above equation can represents the resultant intensity of two waves when they are super imposed with each other.

<u>Case (i)</u>:- For the constructive interference (or) maximum intensity (or) Bright Fringe.

The phase difference values should be $0, 2\pi, 4\pi, 6\pi, \ldots 2n\pi$

i.e., $\boxed{\phi = 2n\pi}$ Where $n = 1, 2, 3 \ldots\ldots$

$\therefore$ For the constructive interference the values of path difference $= 0, \lambda, 2\lambda, 3\lambda, 4\lambda, \ldots\ldots n\lambda$.

i.e., for the constructive interference

$$\boxed{\text{path different} = n\lambda}$$

Case (ii):- For the constructive interference (or) minimum intensity (or) dark fringe.

The phase destructive interference the values of path difference

$$\frac{\lambda}{2}, \frac{3}{2}\lambda, \frac{5}{2}\lambda, \frac{7}{2}\lambda, \ldots\ldots (2n+1)\frac{\lambda}{2}.$$

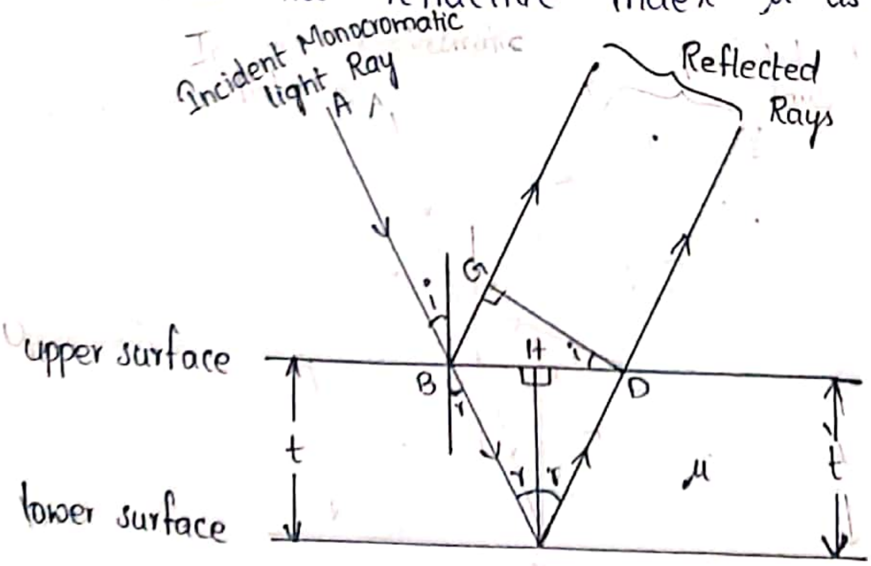
$$\text{path difference} = (2n+1)\frac{\lambda}{2}.$$

Relation between Path difference and phase difference:-

$$\boxed{\text{Phase difference} = \frac{2\pi}{\lambda}(\text{path difference})}$$

Interference in Thin films:-

Let us consider a thin film whose thickness is 't' and the refractive index 'μ' as shown in the figure.



When a monocromatic light ray $(\overrightarrow{AB})$ incident on the thin film the some portion of incident light ray reflects from the upper surface of the thin film as BGF.

The remaining portion of incident light ray refracts into the film as $\overrightarrow{BC}$ and again it will reflects from the lower surface of the thin film emerges through the film as $\overrightarrow{CDE}$ as shown in the figure.

Here the two reflected rays reflecting from the upper and lower surface of the thin film super imposed with each other than from the interference pattern.

From the figure 'i' be the incident angle.

'r' be the refracted angle.

$\overline{CH}$ be the perpendicular line to the upper surface drawn from the lower surface at the pt 'c'.

'$\overline{DG}$' be the perpendicular line drawn to the reflected ray $\overrightarrow{BF}$ from the point 'D'.

From the figure the path difference b/w to the two affected rays can be written as

$$\boxed{\Delta = (BC + CD)\mu - BG \quad (1)}$$

Where,

(BC + CD) be the path travelled by the second ray in the film.

From the ΔBCH,

$$\cos r = \frac{CH}{BC}$$

$$\cos r = \frac{t}{BC} \qquad [\because CH = t]$$

$$\boxed{BC = \frac{t}{\cos r}} \quad \text{——— (2)}$$

Similarly, from the ΔCHD,

$$\cos r = \frac{CH}{CD}$$

$$\cos r = \frac{t}{CD}$$

$$\boxed{CD = \frac{t}{\cos r}} \quad \text{——— (3)}$$

(2) + (3) $\Longrightarrow$ $BC + CD = \dfrac{t}{\cos r} + \dfrac{t}{\cos r}$

$$BC + CD = \frac{2t}{\cos r}$$

$$\boxed{(BC + CD)\mu = \frac{2\mu t}{\cos r}} \quad \text{——— (4)}$$

For the calculation for BG first we should calculate

$$\boxed{BD = BH + HD} \quad \text{——— (5)}$$

From the ΔBCH

$$\tan r = \frac{BH}{CH}$$

$$\tan r = \frac{BH}{t}$$

$$\boxed{BH = t \tan r} \text{ --- } (6)$$

From the ΔCHD,

$$\tan r = \frac{HD}{CH}$$

$$\tan r = \frac{HD}{t}$$

$$\boxed{HD = t \tan r} \text{ --- } (7)$$

(6) + (7) $\Rightarrow$ $BH + HD = BD$

$$BD = t \tan r + t \tan r$$

$$\boxed{BD = 2t \tan r} \text{ --- } (8)$$

From the ΔBDG,

$$\sin i = \frac{BG}{BD}$$

$$\sin i = \frac{BG}{2t \tan r}$$

$$\boxed{BG = \sin i \,(2t \tan r)} \text{ --- } (9)$$

We know that,

Snell's law $\mu = \frac{\sin i}{\sin r}$

$$\boxed{\sin i = \mu \sin r} \text{ --- } (10)$$

Substitute the eq (4) & (11) in (1) eq^n

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r} \sin^2 r$$

$$= \frac{2\mu t}{\cos r}\left[1 - \sin^2 r\right]$$

$$= \frac{2\mu t}{\cos r} [\cos^2 r]$$

$$\boxed{\Delta = 2\mu t \cos r} \quad \text{———} \quad (12)$$

When the light travel from rarer to denser medium then 'π' phase difference (or) $\lambda/2$ path difference will occur in b/w the reflected and refracted rays.

$\therefore$ The perfect path difference b/w to effective rays can be written as.

$$\boxed{\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

<u>Condition</u> <u>for</u> <u>Constructive</u> <u>Interference</u> <u>(or)</u> <u>Maximum</u> <u>Interference</u> <u>(or)</u> <u>Bright</u> <u>fringe</u>

For the constructive interference the path difference b/w the two effective rays should be equal to '$n\lambda$'.

i.e., $2\mu t \cos r + \frac{\lambda}{2} = n\lambda$

$$2\mu t \cos r = n\lambda - \frac{\lambda}{2}$$

$$= \frac{2n\lambda - \lambda}{2}$$

$$\boxed{2\mu t \cos r = (2n-1)\lambda/2} \quad \text{———} \quad (13)$$

Condition for Destructive Interference (or) Minimum Intensity (or) Dark fringe

For the destructive interference the path difference b/w the two effective rays should be equal to $(2n+1)\frac{\lambda}{2}$.

i.e., $2\mu t \cos r + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2} - \frac{\lambda}{2}$$

$$= \left(2n\left(\frac{\lambda}{2}\right)+\left(\frac{\lambda}{2}\right)+\frac{\lambda}{2} - \frac{\lambda}{2}\right)$$

$$\boxed{2\mu t \cos r = n\lambda} \quad \text{—— (14)}$$

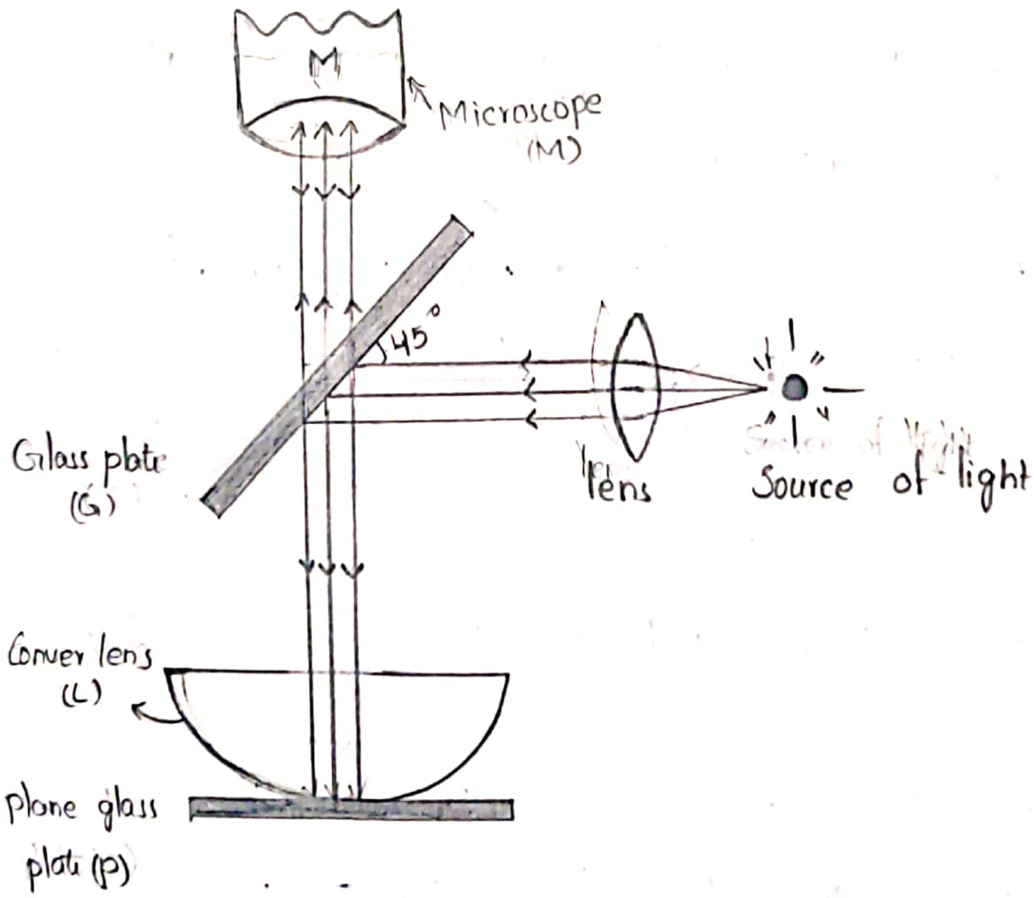
Conditions for sustainable Interference :-

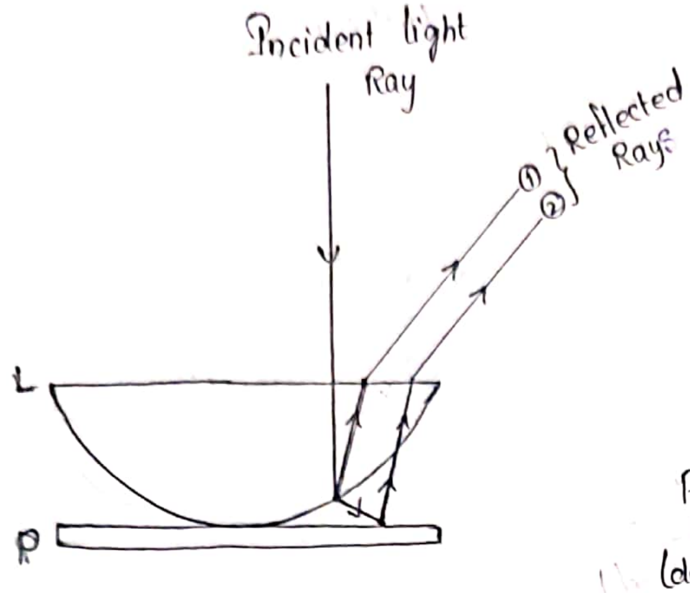
1. The light wave should be coherent (i.e., the light waves have same wavelength, same frequency, same amplitude, same Intensity and constant phase difference).
2. The width of the slit should be narrow. The distance b/w the two slits (d) should be minimum.
3. The distance b/w the slits and the screen should be maximum.
4. The background of the screen should be dark.

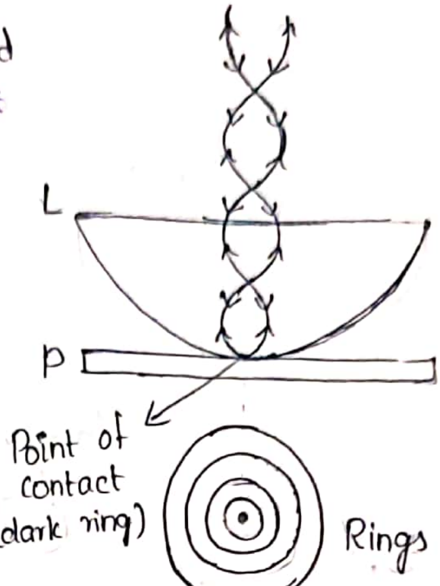
Newton Rings (or) Interference pattern in uniform thin films:-

Newton Rings are the alternating bright and dark rings which are formed due to the interference of two light rays. First the formation of these interfered alternating bright and dark rings was observe by newton. Hence these are called Newton Rings.

"Newton Rings are formed due to two reflected light rays reflecting from the upper and lower surface of the air film which is formed in between plano convex lens and plane glass plate."







Experimental Arrangement:-

The experimental arrangement of formation of Newton Rings as shown in the figure.

Where 's' is the light source which can emit light rays. These light rays incident on the 45° arrangement of glass plate then the light rays falls normally on the plano convex lens from the glass plate.

Due to two reflected rays reflecting from the upper and lower surface of air film ($\mu=1$) which is formed in between plano convex lens (L) and plane Glass plate (P), then form interference pattern. Due to circular symmetry we can observe alternating bright and dark rings as shown in the figure.

The entire formation of Newton Rings can be observe by the microscope (M) which is place top of the experimental arrangement

Theory :-

We know that the path difference between the two reflected rays in the case of thin film interference

It can be written as

$$\Delta = 2\mu t \cos r \pm \lambda/2 \quad \text{——} (1)$$

In the case of Newton Rings experiment the thin film is air film. For the air film $\mu = 1$

For the normal reflection $r = 0$

Substitute these two conditions in eq^n (1)

$$\Delta = 2(1)t \cos 0 \pm \lambda/2$$

$$\Delta = 2t \pm \lambda/2 \quad \text{——} (2)$$

Condition for Bright Ring :-

$$2t + \lambda/2 = n\lambda$$

$$2t = n\lambda - \frac{\lambda}{2}$$

$$2t = \frac{2n\lambda - \lambda}{2}$$

$$\boxed{2t = (2n-1)\frac{\lambda}{2}} \quad \text{——} (3)$$

Condition for Dark Ring:-

$$2t \pm \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$2t = (2n)\frac{\lambda}{2} + \lambda/2 - \lambda/2$$

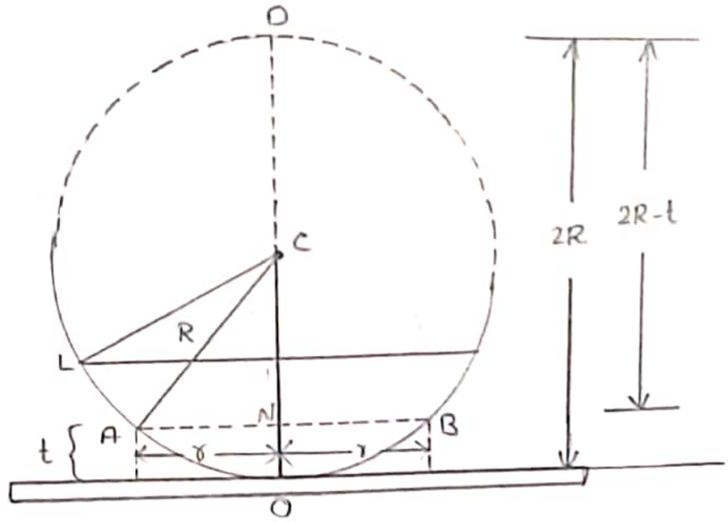
$$\boxed{2t = n\lambda} \quad \text{—— (4)}$$

Where 't' variable thickness of air film

Calculation of thickness of air film:-

let us consider 't' be the variable thickness of air film, 'r' be the radius of any one of the consider ring.

let us draw a circle by using the radius of curvature of plano convex lens.



According to the property of the circle from figure.

$$ON \times ND = NA \times NB$$

$$t \times (2R-t) = r \times r$$

$$2Rt - t^2 = r^2$$

for $R >>>> t$

Neglect t^2 from above term

$$2Rt = r^2$$

$$t = \frac{r^2}{2R} \quad \text{—— (5)}$$

(or)

$$t = \frac{(D/2)^2}{2R}$$

$$t = \frac{D^2}{8R} \quad \text{—— (6)}$$

Substitute eq^n (6) in eq^n (3)

$$\cancel{2}\left(\frac{D^2}{\cancel{8}_4 R}\right) = (2n-1)\frac{\lambda}{2}$$

$$D^2 = (2n-1)\frac{\cancel{4}^2 \lambda R}{\cancel{2}}$$

$$\boxed{D^2 = (2n-1)\, 2\lambda R} \quad \text{—— (7)}$$

$$\boxed{D_{n_{bright}} = \sqrt{2n-1}\,\sqrt{2\lambda R}} \quad \text{—— (8)}$$

The above equation can be represents the diameter of the n^{th} bright ring.

$$\boxed{D_{n_{bright}} \propto \sqrt{2n-1}}$$

$$\boxed{D_{n_{bright}} \propto \sqrt{\text{odd natural numbers}}}$$

Now substitute eq^n (6) in (4)

$$x\left(\frac{D^2}{4R}\right) = n\lambda$$

$$\boxed{D^2_{n_{dark}} = 4n\lambda R} \quad \text{--- (9)}$$

$$\boxed{D_{n_{dark}} = \sqrt{4n\lambda R}} \quad \text{--- (10)}$$

$$D_{n_{dark}} \propto \sqrt{n}$$

$$\boxed{D_{n_{dark}} \propto \sqrt{\text{natural numbers}}}$$

Applications of Newton Rings Experiment:-

1. Determination of wavelength of light (λ) :-

let us consider the condition for the n^{th} dark ring.

$$\boxed{D_n^2 = 4n\lambda R} \quad \text{--- (1)}$$

Similarly, let us consider the diameter of m^{th} dark ring

$$\boxed{D_m^2 = 4m\lambda R} \quad \text{--- (2)}$$

$$(1) - (2) \Longrightarrow D_n^2 - D_m^2 = 4n\lambda R - 4m\lambda R$$

$$= 4\lambda R(n-m)$$

$$\boxed{\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}} \quad \text{--- (3)}$$

2. Determination of radius of curvature of plano convex lens:-

We know that the wavelength of light

$$\lambda = \frac{D_n^2 - D_m^2}{4(n-m)R}$$

$$\boxed{R = \frac{D_n^2 - D_m^2}{4(n-m)}}$$

3. Determination of Refractive index of a liquid (μ):-

If suppose the entire experiment formation of a Newton rings will be represent in a liquid whose refractive index is 'μ', then the diameter of n^{th} & m^{th} dark rings will be change $D_n'^2$ and $D_m'^2$.

We know that $\boxed{D_n^2 - D_m^2 = 4(n-m)\lambda R}$ —— (1)

When the liquid is introduced then

$$\boxed{\mu \{D_n'^2 - D_m'^2\} = 4(n-m)\lambda R} \quad \text{—— (2)}$$

$$\frac{(2)}{(1)} \Rightarrow \frac{\mu (D_n'^2 - D_m'^2)}{D_n^2 - D_m^2} = \frac{4(n-m)\lambda R}{4(n-m)\lambda R}$$

$$\boxed{\mu = \frac{D_n^2 - D_m^2}{D_n'^2 - D_m'^2}}$$

Difraction :-

The bending property of light is called Diffraction.

→ The Scientist grimaldi was first observed the diffraction pattern in ~~1665~~. 1665.

When the light is incident on an obstacle or small aperture whose sizes are comparable with the wave-length of light then the light can bends around the edges or corners of the obstacle then enter into the geometrical shadow. This phenomenon of bending property of light is called "diffraction of light".

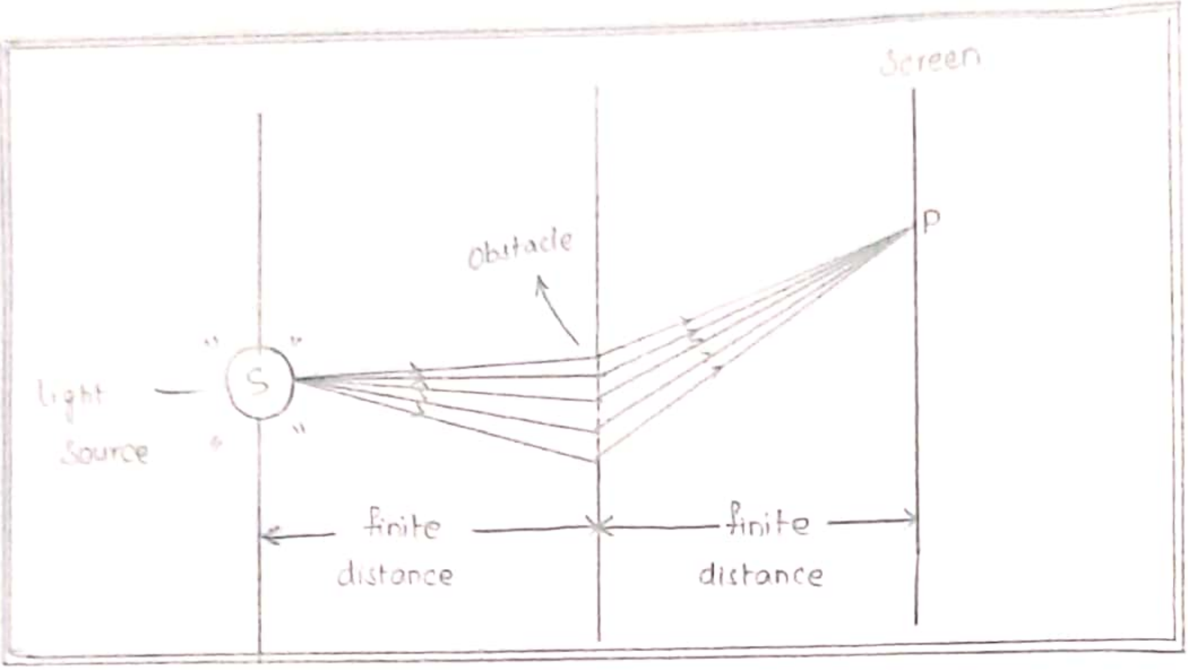
Types of diffraction:-

There are two types of diffraction pattern. They are :- ① Fresnel's diffraction.

② Fraunhofor diffraction.

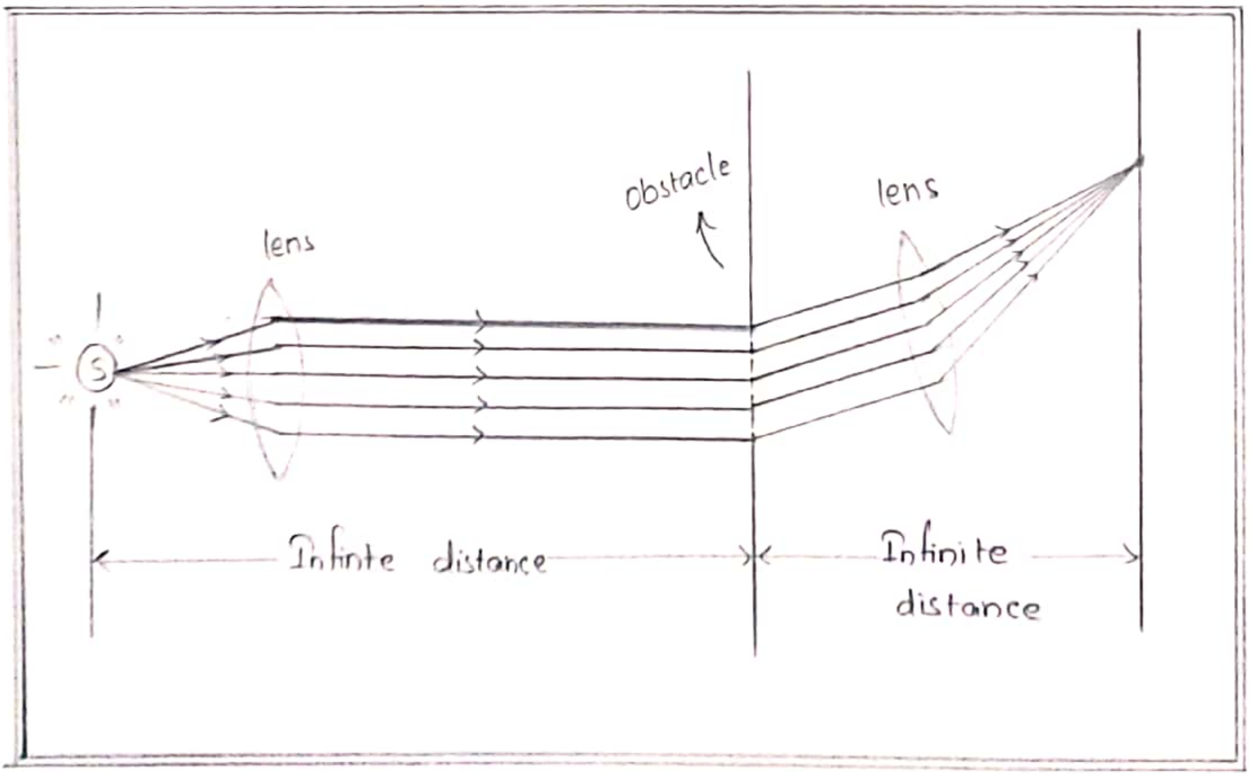
① Fresnel's diffraction :-

The diffraction in which the obstacle is finite distance from the source and the Screen then the diffraction is called Fresnel's diffraction.

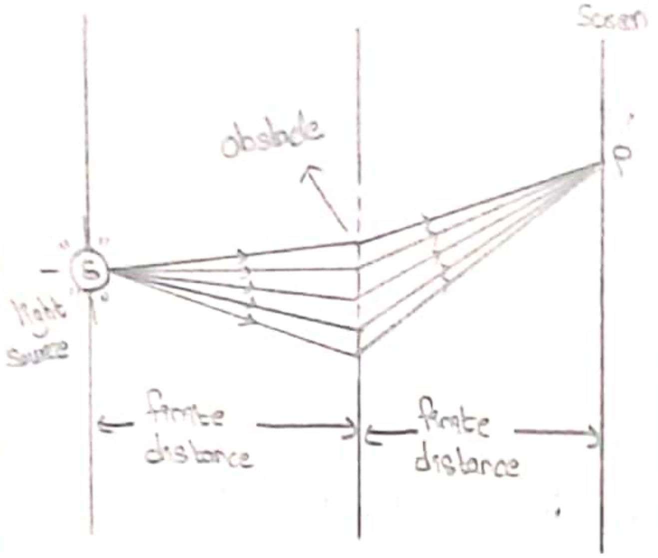


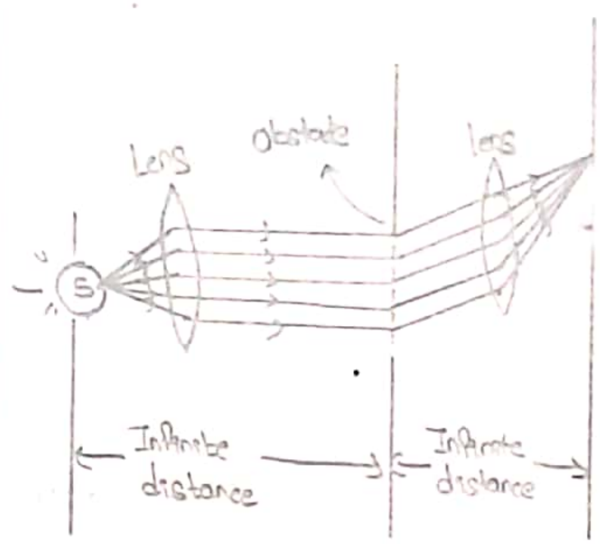
② **fraunhofor diffraction** :-

The diffraction in which the obstacle is infinite distance from the source and the screen then the diffraction is called fraunhofor diffraction.



Difference between Fresnel's and Fraunhofor diffraction:-



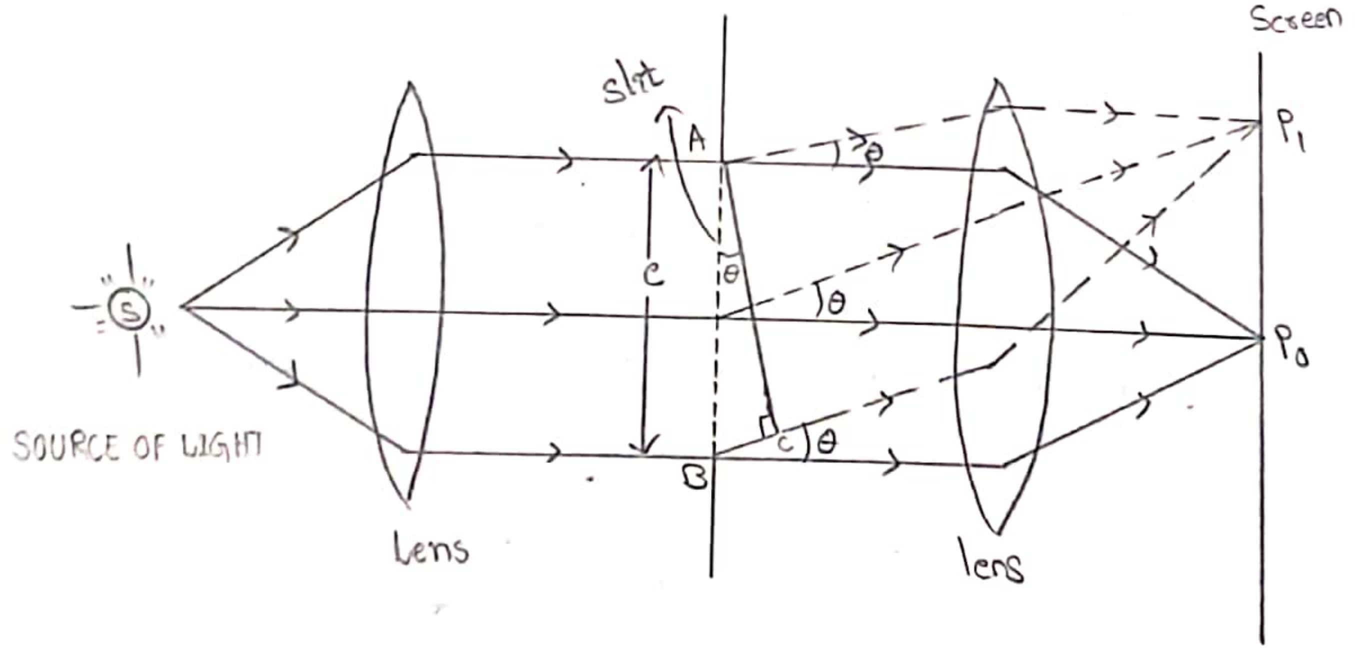


Fresnel's diffraction	Fraunhofor diffraction
1. In this diffraction the obstacle is present finite distance from the source and the screen.	1. In this diffraction the obstacle is present infinite distance from the source and the screen
2. In this diffraction point sources or light from narrow slits can be used as light source.	2. Extended sources (or) highly illuminated sources are used as source of light.
3. Spherical (or) Cylindrical wave fronts are participate in this diffraction pattern.	3. Plane wave fronts are partici-pate in this diffraction pattern
4. No lenses is used in this diffraction pattern	4. Lenses are used in this diffraction pattern.
5. We can observe diffraction pattern in only one direction.	5. We can observe diffraction pattern in all directions.

Differences between Interference and Diffraction:

Interference	Diffractions
1. The interaction takes place b/w two sperate wave fronts ~~organi~~ Originating from the coherent sources.	1. The interaction takes place between the secondary wave lets originating different points of the exposed paths of the same wave front.
2. The fringe width may (or) may not be equal.	2. The fringe width of various fringes are never equal.
3. All the bright fringes have the same intensity.	3. The bright fringes or have various of intensity.
4. The dark fringes are perfectly dark.	4. There ~~are~~ is no perfect dark fringe.

Fraunhofar diffraction due to Single slit :-



let us consider a slit (i.e., single slit) AB whose width is e

$$\boxed{AB = e}$$

When a plane wave length front w.r.t incident on the single slit then we can observe the resultant intensity of the non deviated rays at P. On the screen that can represents principle maxima (or) central maxima.

At the same time we can observe the resultant diffraction pattern of diffracted rays with the diffraction angle 'θ'. at P_1 on the screen as shown in figure.

From the figure the path difference b/w the two effective rays is BC.

From ΔABC, $\sin\theta = \frac{BC}{AB}$

$$\sin\theta = \frac{BC}{e}$$

$$\boxed{BC = e\sin\theta} \longrightarrow (1)$$

$\therefore$ The phase difference $\delta = \frac{2\pi}{\lambda}$ (path difference)

$$= \frac{2\pi}{\lambda}(BC)$$

$$\boxed{\delta = \frac{2\pi}{\lambda}(e\sin\theta)} \longrightarrow (2)$$

If suppose 'n' number of waves are passing through the single slit then the phase difference b/w the any two successive light rays can be written as

$$d = \frac{\text{Total phase } \overset{\text{difference}}{\cancel{\text{diffraction}}}}{n}$$

$$= \frac{1}{n} \times \frac{2\pi}{\lambda} (e \sin\theta)$$

$$\boxed{d = \frac{2\pi}{n\lambda} (e \sin\theta)} \quad \text{—— (3)}$$

If suppose 'a' be the amplitude of a wave, 'n' be the no. of waves, 'd' be the phase difference b/w two light rays then according to vector addition method the resultant amplitude (R) can be written as

$$R = \frac{a \sin\left(\frac{nd}{2}\right)}{\sin(d/2)} \quad \text{—— (4)}$$

Now substitute eq^n (3) in (4)

$$R = \frac{a \sin\left[\frac{\cancel{n}}{2} \times \frac{2\pi}{\cancel{n}\lambda} (e \sin\theta)\right]}{\sin\left[\frac{1}{\cancel{2}} \times \frac{\cancel{2}\pi}{n\lambda} (e \sin\theta)\right]}$$

$$R = \frac{a \sin\left[\pi/\lambda \, (e \sin\theta)\right]}{\sin\left[\frac{\pi}{n\lambda} (e \sin\theta)\right]} \quad \text{—— (5)}$$

let $$\boxed{\alpha = \frac{\pi}{\lambda} (e \sin\theta)} \quad \text{—— (6)}$$

Substitute eqⁿ ⑥ in ③

$$R = \frac{a \sin\alpha}{\sin(\alpha/n)}$$

let us assume $\sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n}$

$$R = \frac{a \sin\alpha}{\alpha/n}$$

$$R = \frac{(na)\sin\alpha}{\alpha}$$

We know that $\boxed{na = A}$

$$\boxed{R = A\left(\frac{\sin\alpha}{\alpha}\right)} \quad \text{—— (7)}$$

The above equation can be represents the resultant amplitude of the waves.

The resultant intensity of the waves.

$$I = R^2$$

$$I = \left[A\left(\frac{\sin\alpha}{\alpha}\right)\right]^2$$

$$\boxed{I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2} \quad \text{—— (8)}$$

Condition for principle maxima (or) Central maxima:-

let us consider the resultant amplitude

$$R = A\left[\frac{\sin\alpha}{\alpha}\right]$$

$$R = \frac{A}{\alpha}[\sin\alpha]$$

$$R = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + \dots\dots\right].$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + \dots\dots\right]$$

$\therefore$ For the maximum value of 'R' the negative terms in the above expansion should be equal to '0'.

i.e., $\boxed{\alpha = 0}$ —— (9)

$$\frac{\pi}{\lambda}(e \sin\theta) = 0$$

$$\sin\theta = 0$$

$$\boxed{\theta = 0}$$

i.e., The central maxima (or) principle maxima occurs at $\boxed{\theta = 0}$.

$\therefore$ The resultant amplitude $R = A$

$$R^2 = A^2$$

$$\boxed{I_{max} = A^2}$$

Condition for minima (or) Secondary minima :-

let us consider the resultant amplitude

$$R = A\left[\frac{\sin\alpha}{\alpha}\right]$$

For the minimum value of R, $\sin\alpha = 0$

i.e., $\boxed{\alpha = \pm m\pi}$

$$\boxed{\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \dots\dots \pm m\pi} \text{ —— (10)}$$

$\therefore$ We know that $\alpha = \frac{\pi}{\lambda}(e \sin\theta)$

$$\frac{\pi}{\lambda}(e\sin\theta) = \pm m\pi$$

$$\boxed{e\sin\theta = \pm m\lambda}$$

Condition for Secondary maxima :-

For the secondary maxima $\frac{dI}{d\alpha} \approx 0$

$$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

$$A^2 \frac{d}{d\alpha}\left(\frac{\sin\alpha}{\alpha}\right)^2 = 0$$

$$A^2 \, 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\sin\alpha}{\alpha}\right] = 0$$

$$2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha(1)}{\alpha^2}\right] = 0$$

$$2A^2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0$$

$$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0$$

$$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \frac{\sin\alpha}{\cos\alpha}$$

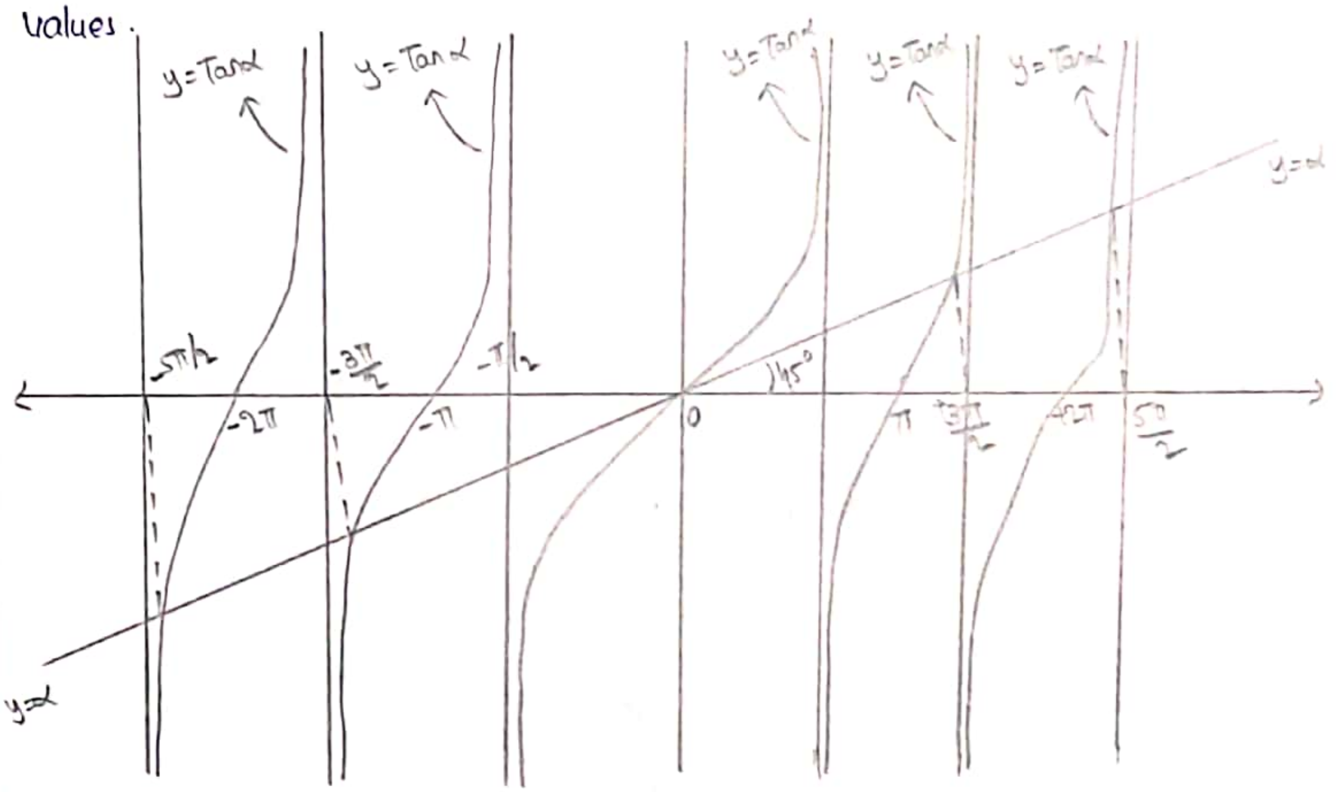
$$\boxed{\alpha = \tan\alpha}$$

The above equation can be written as

$$\boxed{y = \alpha}$$

$$\boxed{y = \tan\alpha}$$

Now plot a graph between $\boxed{y=\alpha}$ and $y = \tan\alpha$ then the intersecting points in the graph gives the secondary maxima values.



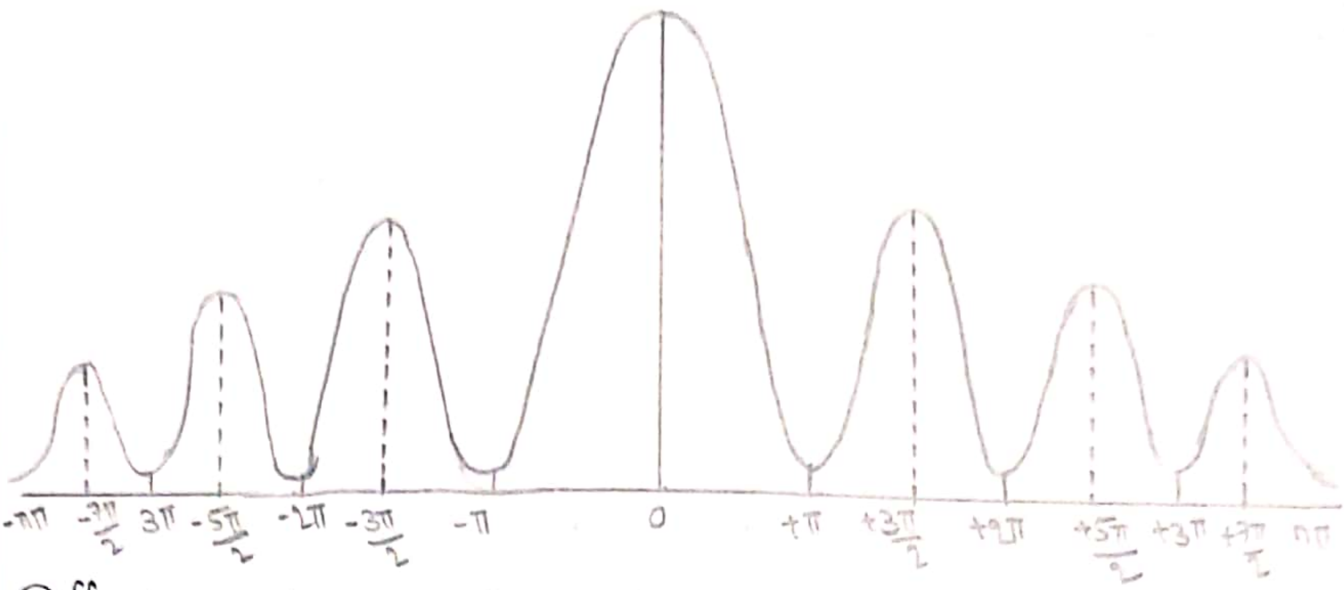
From the graph the intersecting points are $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots \pm(2n+1)\frac{\pi}{2}$ gives secondary maxima.

Resultant graph:-

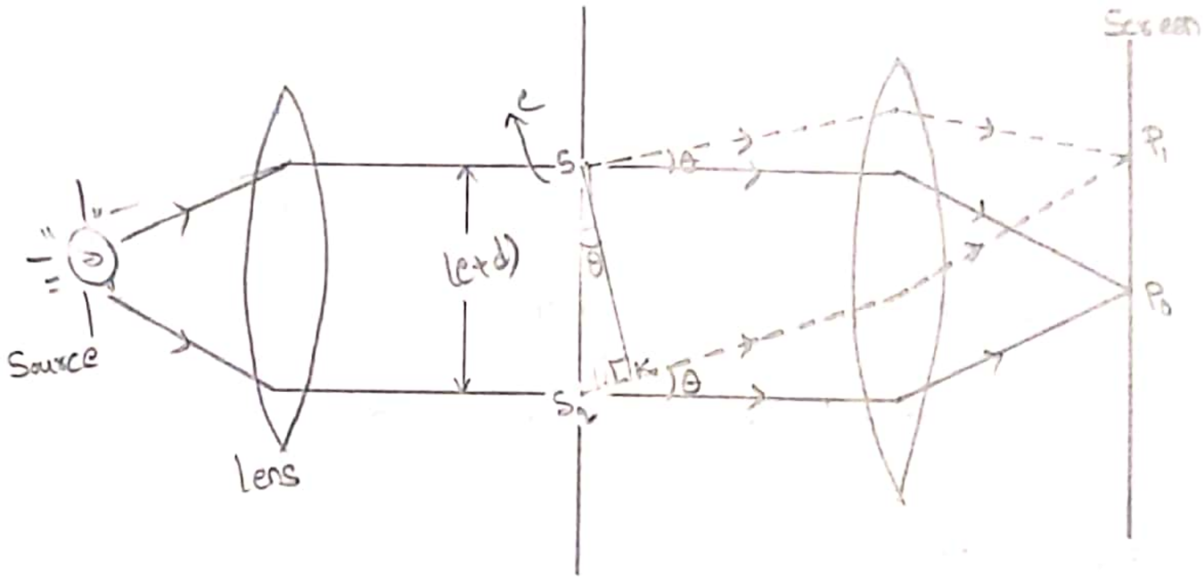
We know that the condition for principle maxima $\alpha = 0$ (or) $\theta = 0$.

The secondary maxima occurs at $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots \pm(2n+1)\frac{\pi}{2}$.

The minima occurs at $\pm\pi, \pm 2\pi, \pm 3\pi, \ldots, \pm n\pi$.



Diffraction due to double slit :-



Let us consider two single slit S_1 & S_2 each single slit width is 'e'. Which are seperated by a distance 'd'.

∴ The mean distance b/w the two slits.

$$S_1 S_2 = e + d \quad \text{——} (1)$$

When the light passing through the two single slits then we can observe the both interference and diffraction pattern.

On the screen the point P_0 can represents the resultant intensity of non deviated light rays through the single slits S_1 & S_2 repectively.

At the same time the point P_1 on the screen can represents resultant intensity of the diffracted rays with the angle of diffraction 'θ'.

From the figure the path difference b/w the two effective rays equal to S_2k.

From $\Delta S_1 S_2 k$.

$$\sin\theta = \frac{S_2 k}{S_1 S_2}$$

$$\sin\theta = \frac{S_2 k}{e+d}$$

$$\boxed{S_2 k = e+d \ \sin\theta} \quad \text{--- } ②$$

∴ Phase difference can be written as

$$\delta = \left(\frac{2\pi}{\lambda}\right) \text{path difference}$$

$$= \frac{2\pi}{\lambda}(S_2 k)$$

$$\delta = \frac{2\pi}{\lambda}(e+d \ \sin\theta) \quad \text{--- } ③$$

Let us assume $\overline{OG}$ be the resultant amplitude of through the single slit S_1.

According to Fraunhofor single slit diffraction.

The resultant amplitude $\overrightarrow{OG} = \left[A \ \frac{\sin\alpha}{\alpha}\right]$

Similarly let us consider $\vec{GH}$ be the resultant vector of waves through the single slit S_2.

$$\therefore \vec{GH} = \left[A \frac{\sin\alpha}{\alpha}\right]$$

Let $\vec{OH}$ be the resultant vector (R) of both $\vec{OG}$ and $\vec{GH}$.

According to the triangle law of vectors from triangle.

The resultant vector $(\vec{OH})^2 = (\vec{OG})^2 + (\vec{GH})^2 + 2(\vec{OG})(\vec{GH}) \cos\delta$

$$R^2 = \left[A \frac{\sin\alpha}{\alpha}\right]^2 + \left[A \frac{\sin\alpha}{\alpha}\right]^2 + 2\left[\frac{A \sin\alpha}{\alpha}\right]\left[\frac{A \sin\alpha}{\alpha}\right] \cos\delta$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 + 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos\delta$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \left[1 + \cos\delta\right]$$

$$= 2\left(\frac{A \sin\alpha}{\alpha}\right)^2 \left(2\cos^2 \delta/2\right)$$

$$R^2 = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2(\delta/2) \quad \text{--- (4)}$$

$$R^2 = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2\left[\frac{2\pi/\lambda\,(e+d)\sin\theta}{2}\right]$$

$$= 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cos^2\left[\frac{\pi}{\lambda}(e+d)\sin\theta\right] \quad \text{--- (5)}$$

let, $\boxed{\frac{\pi}{\lambda}(e+d)\sin\theta = \beta}$ —— (6)

eq^n (6) in eq^n (5)

$$\boxed{I = 4\left(\frac{A \sin\alpha}{\alpha}\right)^2 \cdot \cos^2\beta} \quad \text{--- (7)}$$

The above equation can be represents the resultant intensity of both interference pattern and diffraction pattern.

The above equation the term $4\left(\frac{A\sin\alpha}{\alpha}\right)^2$ can represents diffraction pattern.

The term $\cos^2\beta$ can represents interference pattern.

Diffraction pattern:-

The above equation $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ can represents the diffraction pattern.

Condition for principle maxima:-

The principal maxima occurs at $\boxed{\alpha=0}$ or $\boxed{\theta=0}$

Condition for Secondary maxima:-

The secondary maxima occurs at $\pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \ldots \pm (2n+1)\pi/2$.

Condition for minima:-

The minima occurs at

$$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots \pm m\pi.$$

Interference Pattern:-

In the resultant intensity the factor $\cos^2\beta$ represents interference pattern.

Condition for bright:-

let us consider resultant intensity $I = \cos^2\beta$

For the maximum intensity $\cos^2\beta = 1$

$$\therefore \beta = \pm\pi, \pm 2\pi, \pm 3\pi, \dots \pm n\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm n\pi$$

$$(e+d)\sin\theta = n\lambda$$

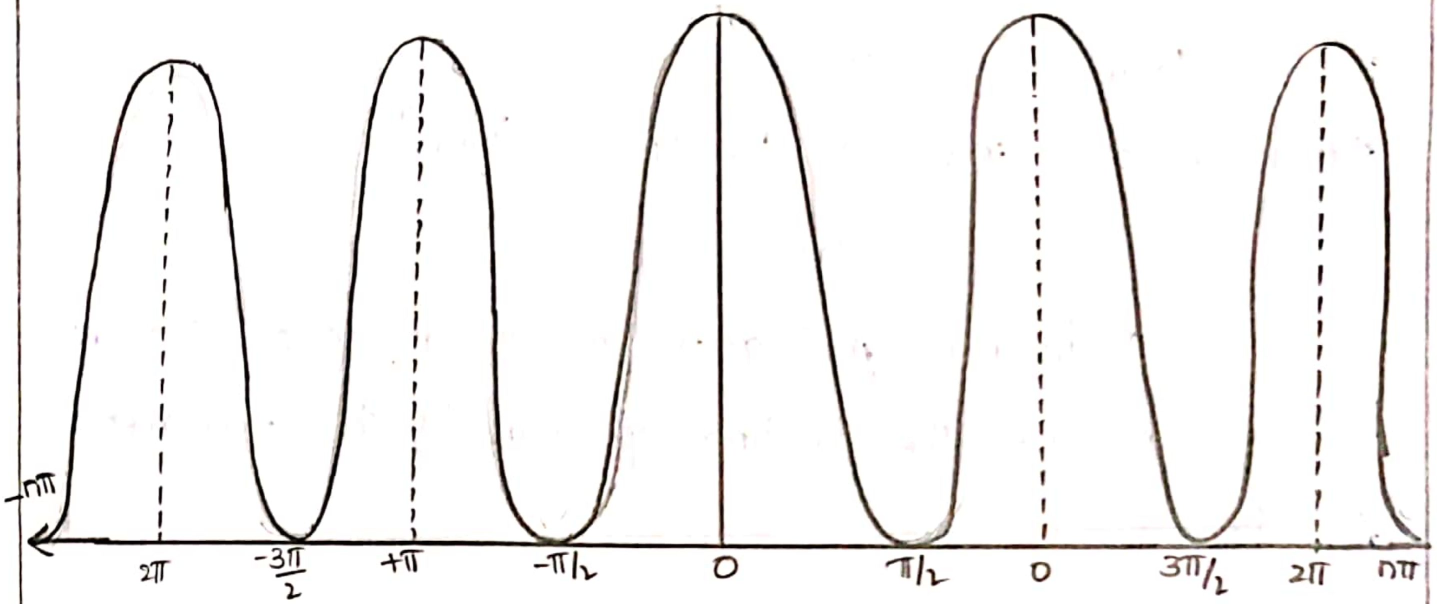
Condition for dark:-

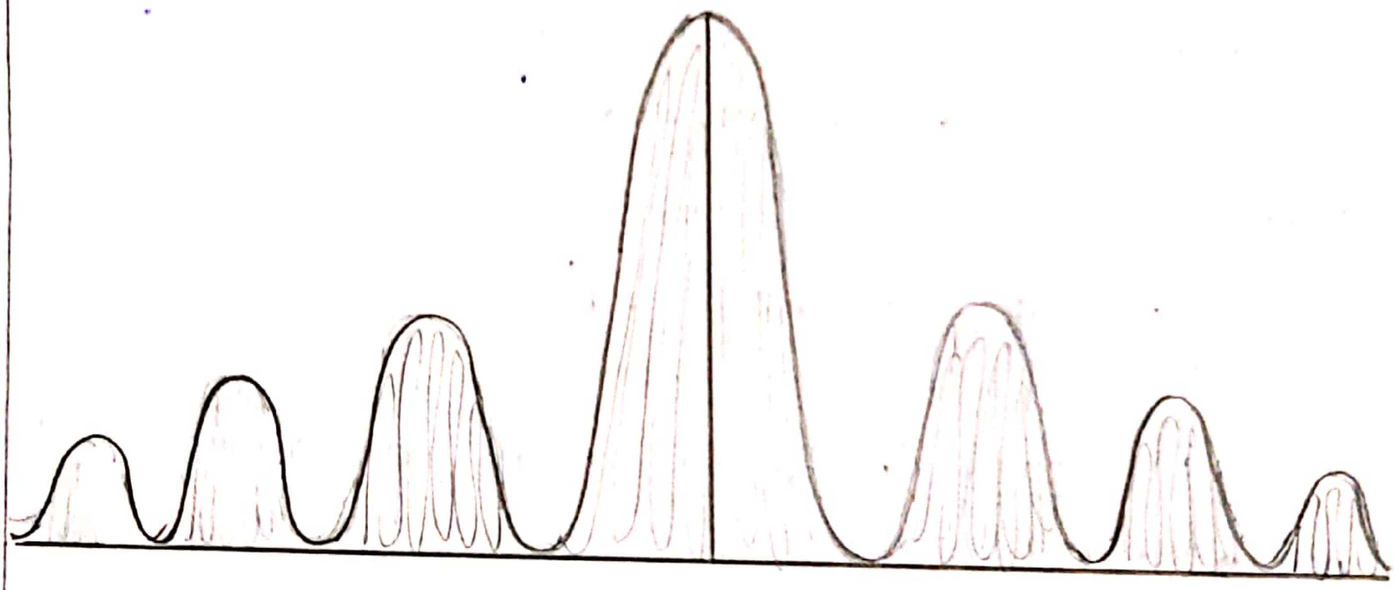
For the dark fringe $\cos^2\beta = 0$

$$\therefore \beta = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots \pm(2n+1)\pi/2 .$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = (2n+1)\frac{\pi}{2} .$$

$$(e+d)\sin\theta = (2n+1)\lambda/2$$


-nπ
2π
-3π/2
+π
-π/2
0
π/2
0
3π/2
2π
nπ



Diffraction Gratting :-

"Diffraction Gratting is a small glass plate which contains large number of equally spaced transparent parallel slits which are seperated by opaque lines."

(or)

Diffraction gratting is an optical device which is used to produce the gratting spectrum by the phenomenon of diffraction.

In the diffraction gratting 'e' be the width of the opaque line.

'd' be the width of the slit.

'e+d' be the distance between the two lines which is called gratting element.

$$\boxed{\therefore \text{Gratting element} = e+d}$$

If suppose 'N' be the no. of lines per inch on the gratting.

$$\therefore N(e+d) = 1''$$
$$= 2.54 \text{ cm.}$$

$$\boxed{e+d = \frac{2.54}{N} \text{ cm}}$$ (or)

$$\boxed{N = \frac{2.54}{(e+d)}} \text{ cm}$$

If suppose 'N' be the no.of lines per unit on the grating.

$$\therefore N(e+d) = 1$$

$$\boxed{(e+d) = \frac{1}{N}}$$

(or)

$$\boxed{N = \frac{1}{(e+d)}}$$

Generally a diffraction grating consists of 5000 to 30,000 lines per inch on the grating.

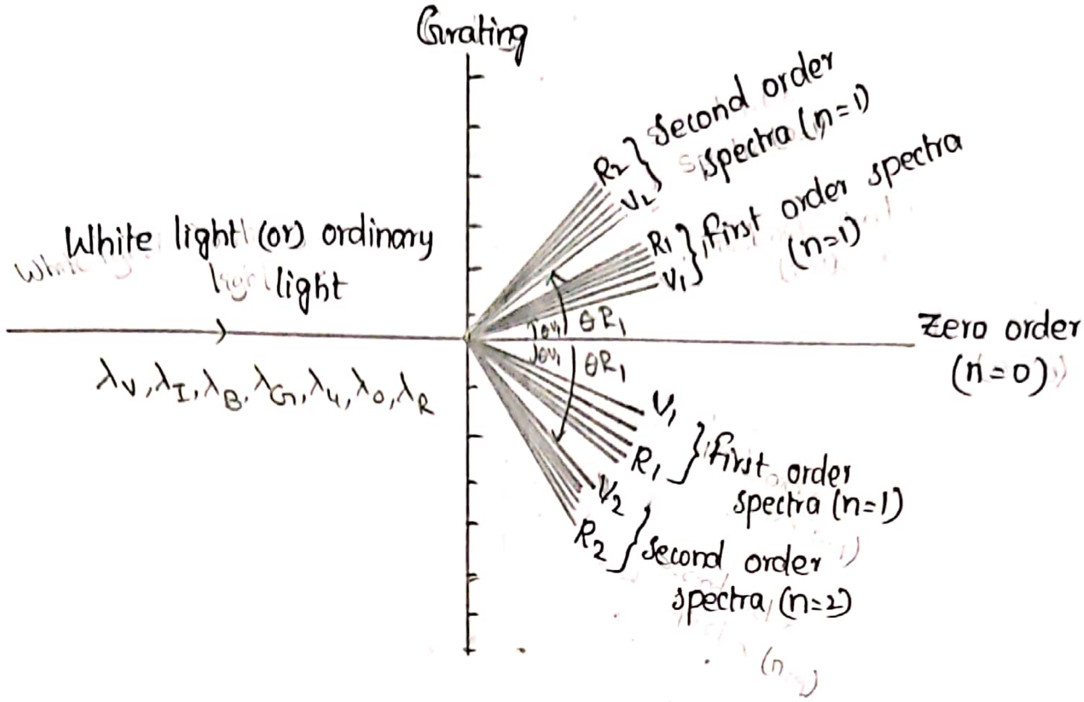
The lines on the grating can be rulled by a diamond point.

Diffraction Spectrum (or) Grating Spectrum:-

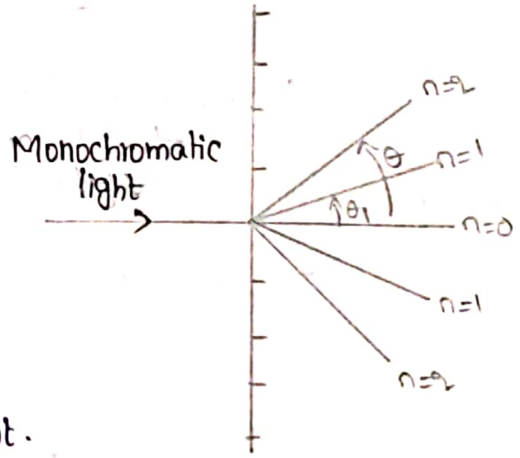
"When the light falls on the grating then the light can diffract and from the spectra (collection of 7 colours) due to diffraction which is called Diffraction spectrum or Gratting spectrum."

* The intensity of the spectral lines can be enplain by using the equation.

$$\boxed{(e+d) \sin\theta = n\lambda}$$



Which is called grating equation.



Where (e+d) = Grating element

θ = Angle of diffraction

n = order of diffraction

(or)

Order of spectra

λ = Wavelength of light.

* For a particular order of spectra (n is constant) the wavelength of the spectral line will be change with the angle of diffraction.

i.e., $\boxed{\theta \propto \lambda}$, Where n = constant

* For a particular wavelength of light (i.e., monochromatic light) the order of the spectrum will be change with respect to the angle of diffraction.

i.e., $\boxed{\theta \propto n}$ wher λ = constant.

* In any order of spectra the violet spectral line is in inner most position and the red colour spectral line is in outer most position to the zero order.

* The maximum intensity of light concentrated at the zero order and the remaining intensity of light.

* If the width of the slit in the grating is small then the spectral lines will be appear as sharp.

* **Maximum order of diffraction possible (n_{max}):-**

For the maximum order of diffraction (n_{max}).

$$\boxed{\theta = 90^\circ}$$

i.e., If $\boxed{\theta = 90^\circ}$ then $\boxed{n = n_{max}}$

$$(e+d)\sin\theta = n\lambda$$

$$\boxed{\theta = 90^\circ} \Rightarrow n = n_{max}$$

$$(e+d)\sin 90^\circ = n_{max}\lambda$$

$$(e+d) = n_{max}\lambda$$

$$n_{max} = \frac{(e+d)}{\lambda} \qquad \boxed{\because (e+d) = \frac{1}{N}}$$

$$\boxed{n_{max} = \frac{1}{N\lambda}}$$

* Dispersive power of grating:-

The change in the angle of diffraction with respect to the change in wavelength is called dispersive power of grating.

i.e., $D = \frac{d\theta}{d\lambda}$

let us consider $(e+d)\sin\theta = n\lambda$

Differentiating on both ~~di~~ sides

$(e+d)\cos\theta \cdot d\theta = n d\lambda$

$$\frac{d\theta}{d\lambda} = \frac{n}{(e+d)\cos\theta}$$

$$\boxed{D = \frac{d\theta}{d\lambda} = \frac{nN}{\cos\theta}} \qquad \left[\because \frac{1}{e+d} = N\right]$$

* Determination of Wave length:-

$(e+d)\sin\theta = n\lambda \Rightarrow \sin\theta = \frac{1}{(e+d)} n\lambda$

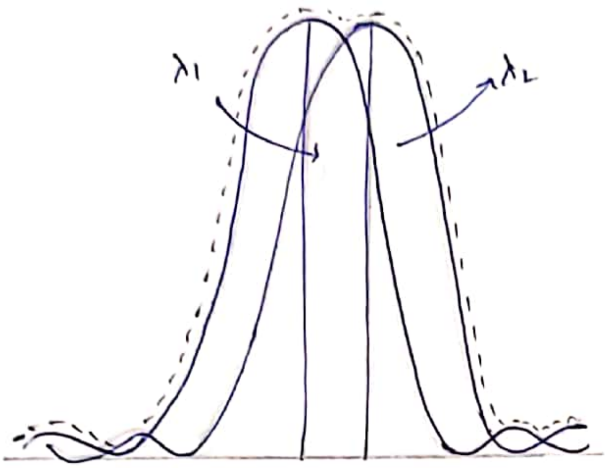
$\sin\theta = nN\lambda$

$$\boxed{\lambda = \frac{\sin\theta}{nN}}$$

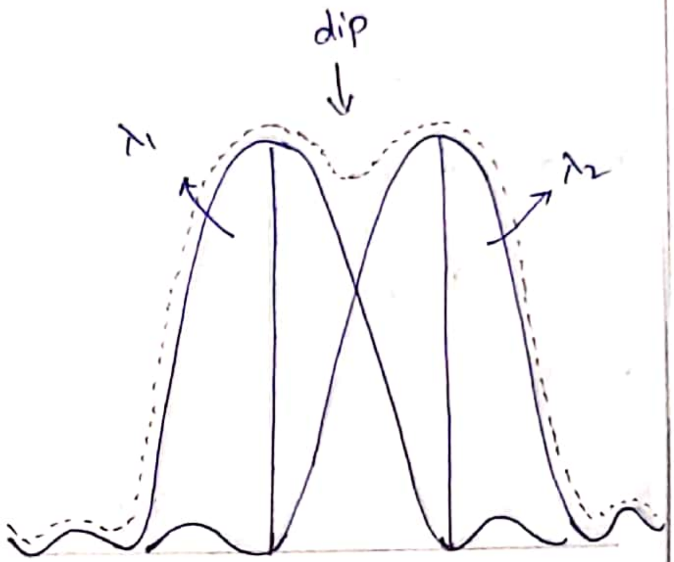
Resolving power of an optical instrument:-

"The capacity of an optical instrument to seperate two images or objects or wavelengths when they are near by each other is called Resolving Power of an optical instruments."

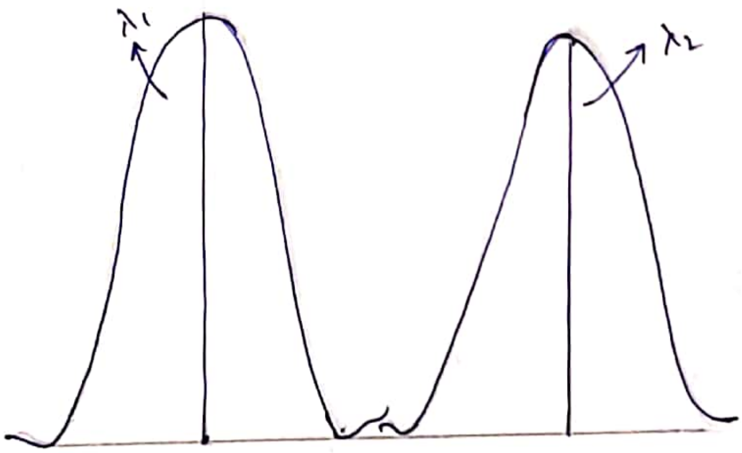
Rayleigh's creterian for resolving power of an optical instrument:-



ⓐ Not Resolved condition



ⓑ Just Resolved condition
(or)
Rayleigh's limit



ⓒ Well resolved condition

let us consider any two images whose wavelengths are λ_1 and λ_2 respectively. Figure 'a' shows the two images or in not resolved condition. i.e., The two images are merged with each other. Figure 'b' shows the Rayleigh's creterian (or) Rayleigh's limit of an optical instrument. ~~Accor~~

According to Rayleigh's creteria two images are said to be just resolved. " The principle maxima of one image coincides with the minima of the other image. Similarly the central maxima of other image and coincid with the minima of first image.

Fig (c) shows the two images are called well resolved condition.

Resolving Power of gratting:-

" The ability of an grating to seperate two images, when they are nearby each other is called Resolving Power of grating."

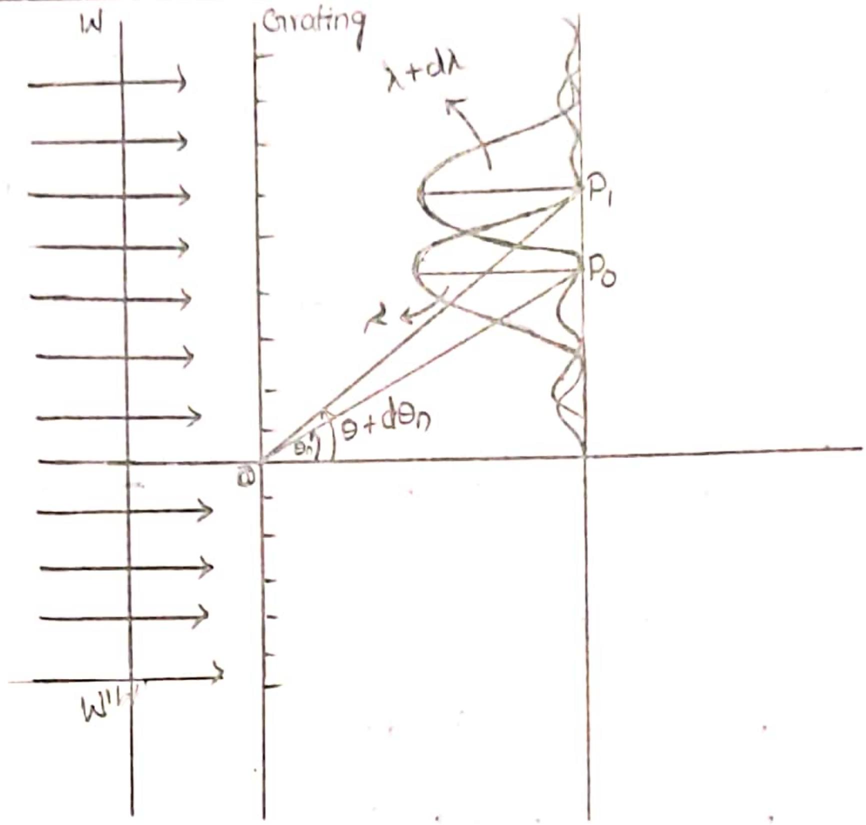
(Or)

" The ratio between the wavelength of one image(λ) and the wavelength of difference b/w the two images ($d\lambda$) is called Resolving Power of grating."

Resolving power of gratting

$$R = \frac{\lambda}{d\lambda}$$

let us consider any two images whose wavelengths are λ, $\lambda + d\lambda$ on the screen at P_0, P_1 respectively where θ_n, $\theta_n + d\theta_n$ are the diffraction angles to the central maxima's of both images from the point 'o'. Where ww' is the incident wave.



Let us consider any two images whose wavelengths are $\lambda, \lambda+d\lambda$.

Now let us consider the condition for principle maxima of the image whose wavelength is 'λ' at the angle of diffraction θ_n.

i.e., $\boxed{(e+d)\sin\theta_n = n\lambda}$ —— (1)

If 'N' be the number of slits on the grating then the above equation can be written as

$$N(e+d)\sin\theta_n = nN\lambda \quad \text{—— (2)}$$

i.e., $N, 2N, 3N, \ldots nN$ gives maxima.

Now let us consider the equation for minima of the image whose wavelength is 'λ' at the angle of diffraction $\theta_n+d\theta_n$.

$$\boxed{N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda} \quad —③$$

Now let us consider the condition for central maxima of the image whose wavelength is '$\lambda + d\lambda$' at the angle of diffraction '$\theta_n + d\theta_n$':

$$\boxed{N(e+d)\sin(\theta_n + d\theta_n) = Nn(\lambda + d\lambda)} \quad —④$$

From eq^n's ③ & ④

$$(nN+1)\lambda = nN(\lambda + d\lambda)$$

$$nN\lambda + \lambda = nN\lambda + nNd\lambda$$

$$\lambda = nNd\lambda$$

$$\boxed{\frac{\lambda}{d\lambda} = nN}$$

(or)

Resolving power of grating

$$\boxed{R = \frac{\lambda}{d\lambda} = nN}$$

i.e., The resolving power of grating is depends on the order of spectrum (n)

The Number of lines on grating (N).

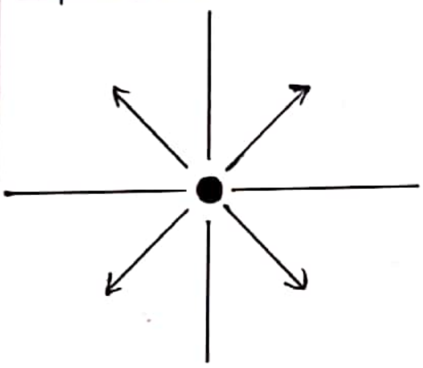
THE END

POLARIZATION

Polarization:-

The process of converting the unpolarised light into polarised light is called "polarisation".

Representation of unpolarised light (or) ordinary light :-



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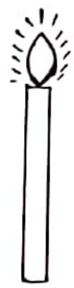

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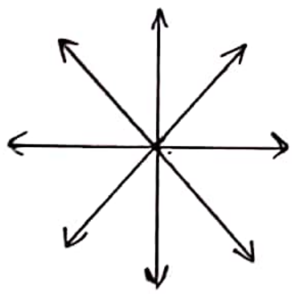

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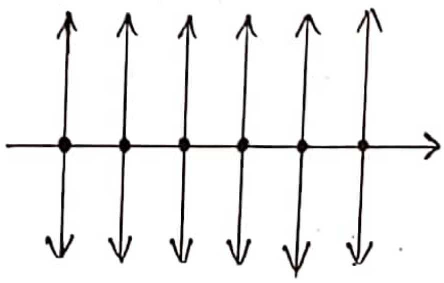


Types of polarised light :-

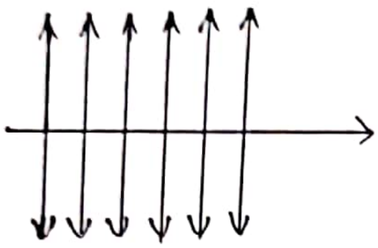
There are three types of polarised light.

They are :-

1. Plane polarized light
2. Circularly polarized light.
3. Eliptical polarized light.

Plane polarised light :-

If the electrical vibrations are belongs to one plane then it is called plane polarised light.

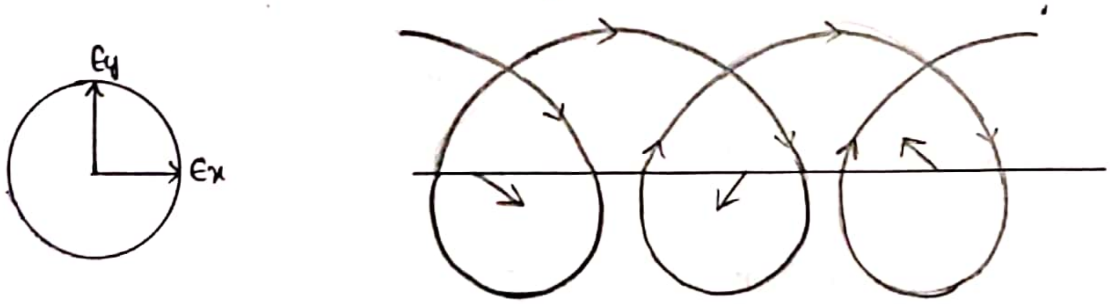


Vertical

Horizontal

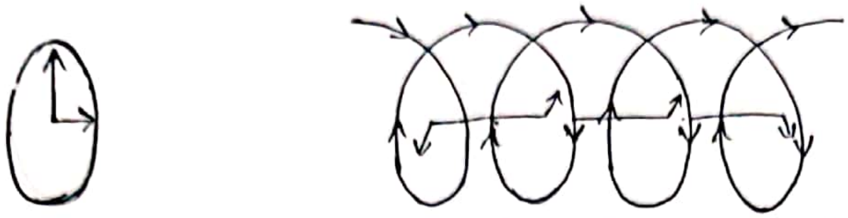
Circularly polarized light:-

If two plane polarised lights with equal magnitude and which are mutually perpendicular to each other (i.e., phase difference is $\pi/2$) super impose with each other, then from the circularly polarized light.



Representation of circularly polarised light.

Eliptical polarized light :- If two polarised lights with unequal magnitude and which are mutually perpendicular to each other (i.e., phase difference in $\pi/2$) Superimpose with each other then form elliptical polarised light.



Representation of elliptically polarization

Methods of production of polarised light:-

There are six types of methods to produce polarized light.

1. Polarization by reflection
2. Polarization by transition.
3. Polarization by refraction.
4. Polarization by selective absorption.
5. Polarization by scattering
6. Polarization by double refraction.

Polarization by reflection:-

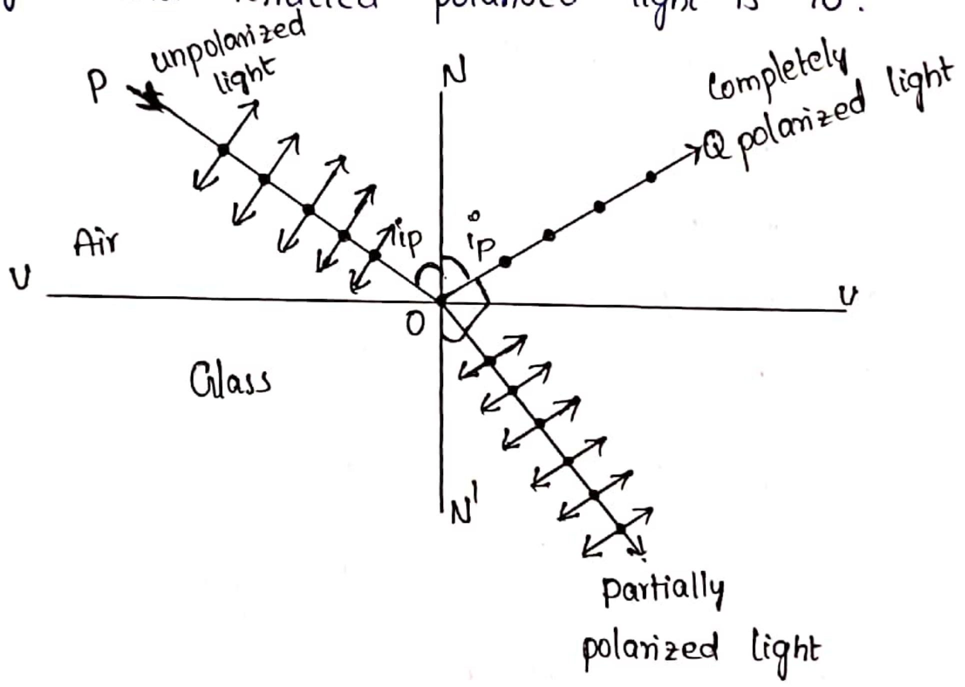
When an unpolarized light was reflected at the surface of some transparent medium such as glass, water etc.... The reflected medium such as light was found to be partially plane polarized. The degree of polarization changed with the angle of incidence. For the particular angle of incidence the reflected light was found to be completely plane polarized." The angle of incedence for which the reflected beam is completely plane polarized is known as polarizing angle or angle of polarization" This angle is also known as Brewester's angle.

Brewester's law:-

A tangential value of polarised angle is equal to the refractive index of the medium is known as Brewester's law.

$$\boxed{\mu = \tan i_p}$$

It can proves that the angle between the reflected polarised light and refracted polarised light is 90°.



Polarization by reflection

From the figure,

$\angle PON = i$ (angle of incidence)

$\angle N'OR = r$ (angle of refraction)

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad \text{——— ②}$$

From Brewester's law,

$$\mu = \tan i = \frac{\sin i}{\cos i} \quad \text{——— ③}$$

From equations ② and ③ we get

$$\cos i = \sin r$$

$$\sin(90 - i) = \sin r$$

$$90 - i = r$$

$$i + r = 90^\circ$$

From the figure,

$$\angle NOQ + \angle QOR + \angle NOR = 180^\circ$$

$$i + \angle QOR + r = 180$$

$$\angle QOR = 180^\circ - (i + r)$$

$$= 180^\circ - 90^\circ$$

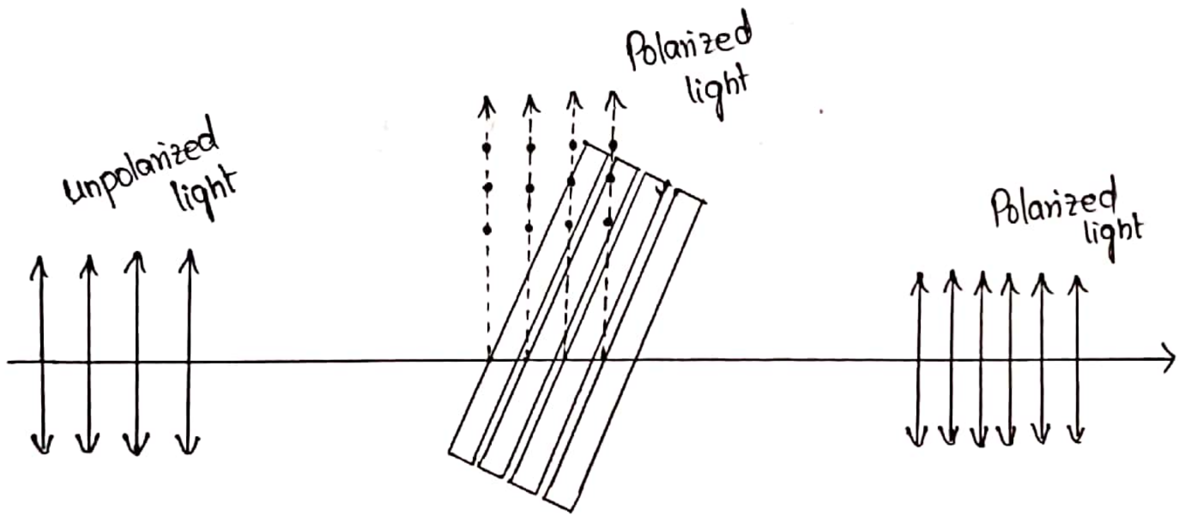
$$= 90^\circ$$

$$\boxed{\angle QOR = 90^\circ}$$

Hence it is proved that the reflected and refracted rays are at right angles.

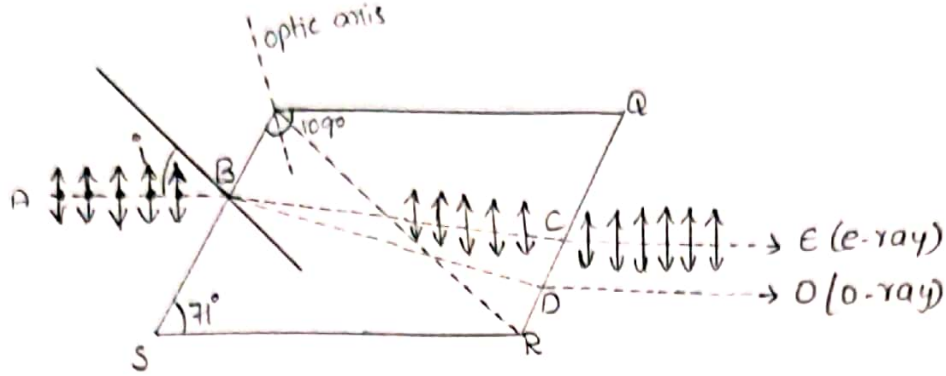
Polarization by Refraction:-

When an unpolarized light is incident at polarizing angle the reflected light is completely plane. Polarised and refracted light is partially plane polarized. The transmitted contains a greater proportion of light vibrating parallel to the plane of incidence. If the reflection at polarized angle is repeated using number of plates all inclined at polarizing angle, finally the transmitted light becomes purely plane polarized. Such an arrangement is known as pile of plates. The polarized light is perpendicular to the plane of incidence.



Double refraction:-

The phenomenon of splitting into two plane polarised refracted light rays when the unpolarised light is incident on the



Calcite crystal is called double refraction.

The resultant refracted rays are extraordinary ray or ordinary ray.

Extra ordinary ray:-

The ray which can not obeys the laws of refraction is called extra ordinary ray.

Ordinary ray:-

The ray which can be obey the laws of refraction is called ordinary ray.

Differences between ordinary ray and Extra ordinary ray:-

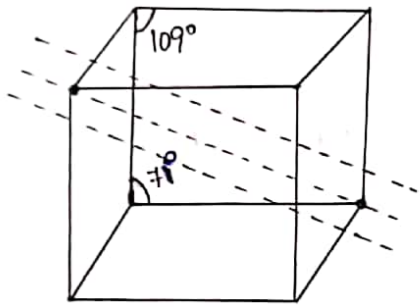
Ordinary ray	Extra Ordinary ray
① It can obey the class of fraction	① It cannot obey the class of refractions.
② It obeys snell's law $\mu = \frac{\sin i}{\sin r}$	② It cannot obey snell's law $\mu \neq \frac{\sin i}{\sin r}$
③ The velocity or speed of the ordinary ray is same in all direction	③ The velocity or speed of the extraordinary ray is different in different directions.
④ The vibrations are perpendicular to optic axis.	④ The vibrations are perpendicular to principal section.

(5) If $\mu_o > \mu_e$, then crystal is called Negative crystal.	(5) If $\mu_e > \mu_o$, then the crystal is called positive crystal.

Where, μ_o = refractive index of ordinary ray

μ_e = refractive index of extra ordinary ray.

Structure of calcite crystal:-



The shape of calcite crystal is rhombohedral. The common angle is 71°, 109°

Blunt corner:-

"A corner which has three obtuse angle is called Blunt corner."

Optic axis:-

"A line joining of two blunt corner is called optic axis."

Any line which is parallel to optic axis is also treated as optic axis.

Uniaxial Crystal:-

The crystal which has only one optic axis is called uniaxial crystals.

Ex: calcite, tourmaline.

Biaxial Crystal:-

The crystal which has two optic axis are called Biaxial crystal.

Ex: Borax

Nicol prism:-

Nicol prism is a optical device which is made from the calcite crystal which is used to produce the plane polarised light.

⇒ The Nicol prism is invented by the scientist Nicol in 1828.

⇒ The Nicol prism can be used as a polarizer and analyser.

Principle:-

When an ordinary light is incident on a calcite crystal, it splits into O-ray and E-ray having orthogonal projections. The O-ray can undergo total internal reflection with in the Nicol prism and E-ray is transmitted through prism is a linearly polarized light.

Construction:-

A calcite crystal whose length is three times its breadth is taken for the construction of a Nicol prism. The unit cell of calcite crystal is a rhombohedron having principal section ABCD with angles 71° and 109° as shown in fig (a).

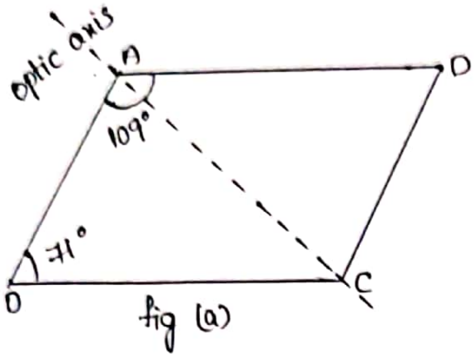


fig (a)

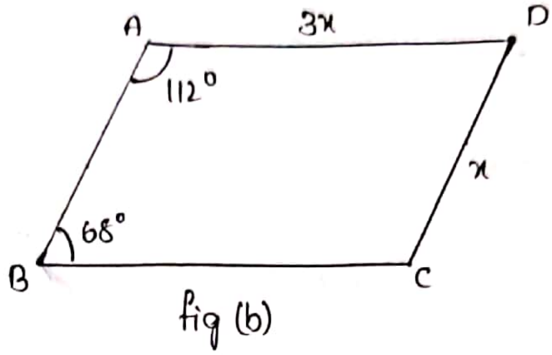
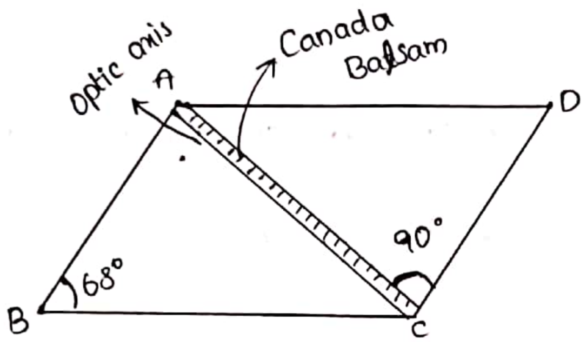


fig (b)



Construction of Nicol Prism.

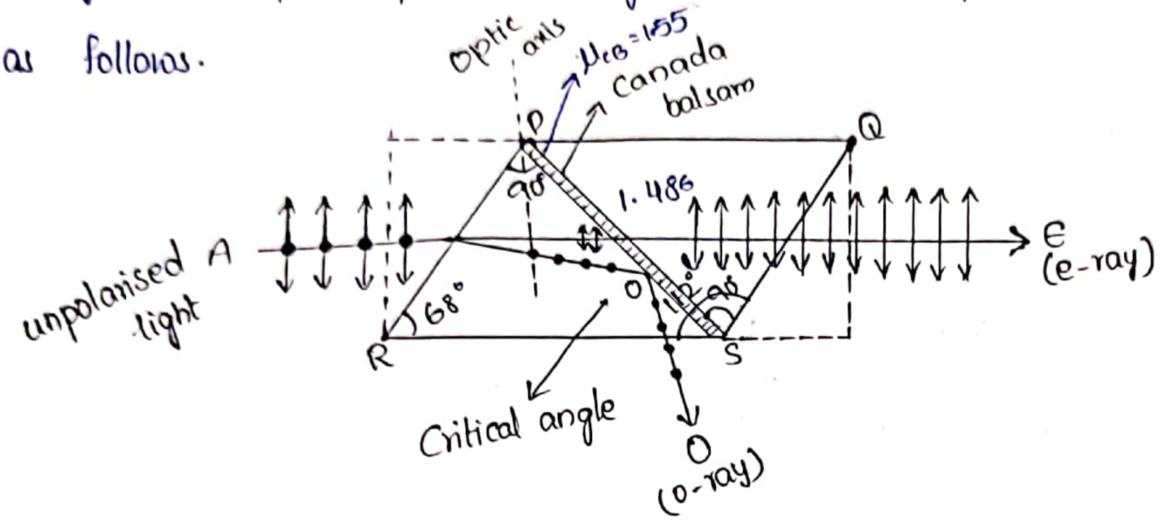
The end faces of the principal section AB and CD is cut. Such that the angles in the principal plane ABCD become 68° and 112° in place of 71° and 109° as shown in fig (b)

⇒ The resulting crystal is then cut diagonally i.e., along AC into two parts. The surface of each part is made optically flat and then these are polished.

⇒ The polished surfaces are joined together by 'canada balsam' as shown in fig. The resulting device is called as 'Nicol prism' which produces plane polarized light.

Working of a Nicol prism:-

A schematic diagram of Nicol prism is as shown in figure. The plane polarized light from Nicol prism can be produced as follows.



Working of a Nicol prism.

When a beam of plane polarized light is incident on the face PR with angle of 15°, it splits into two refracted rays: o-ray and e-ray. These two rays are plane polarized rays whose vibrations are at right angles to each other.

The refractive index of canada balsam is 1.55 whereas it is 1.658 for ordinary ray and 1.486 for extraordinary ray. Hence canda balsam layer can act as an optically rarer medium for o-ray and optically denser medium for e-ray.

The O-ray travels in the Nicol prism from denser to rarer medium and is incident at the canada balsam with an angle greater than the critical angle 69°, the O-ray is totally internally reflected from the crystal.

$$\boxed{\theta_c : \sin^{-1}\left(\frac{n_2}{n_1}\right) : \sin^{-1}\left(\frac{1.55}{1.656}\right) = 69^\circ}$$

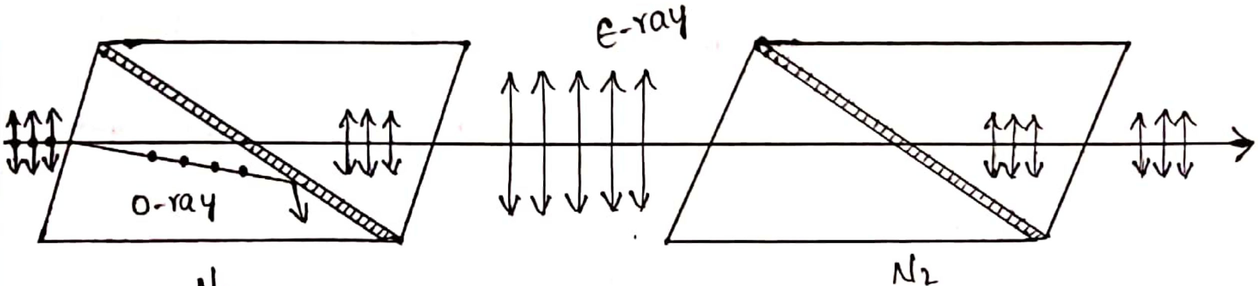
The E-ray is going from a rarer medium to a denser medium and is transmitted to the Nicol prism which is a plane polarized light.

Therefore, using the phenomenon of total internal reflection. The plane polarized light is generated from the Nicol prism.

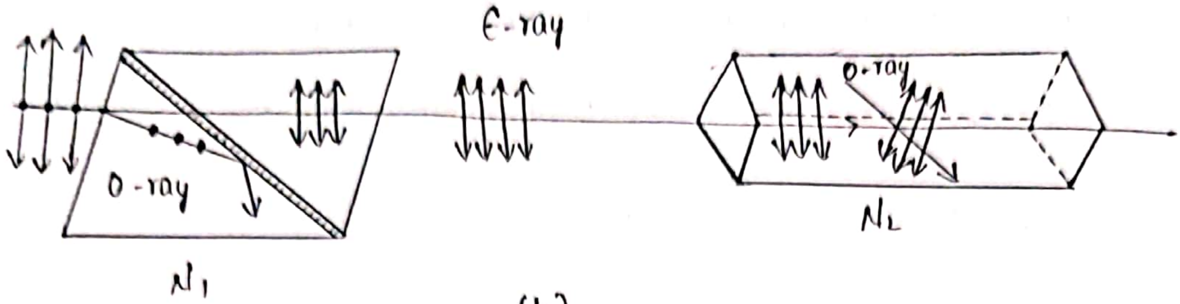
Thus, Nicol prism act as a polarizer and analyzer.

Nicol prism as a polarizer and Analyser:-

When a beam of use polarized light is incident on the Nicol prism N_1 the emergent beam from N_1 is plane polarized light. Therefore, the prism N_1 is called polarizer.



(a)



(b)

a) Principal planes parallel

b) Principal planes inclined at right angles to each other.

If the polarized beam from N_1 falls on another Nicol prism N_2, whose principal plane is parallel to that of N_1, E-ray is transmitted through N_2 with maximum intensity.

If the Nicol prism N_2 is slowly rotated with respect to N_1, the intensity of E ray gradually decreases.

When the principal plane of N_2 is perpendicular to the principal plane of N_1, there is no light from N_2. Because E-ray behaves as O-ray in the Nicol prism. N_2 and it is totally internally reflected by canada balsam layer.

If the Nicol prism N_2 is further rotated about its axis, the intensity of light coming out N_2, gradually increases and becomes maximum when the principal plane N_2 is again parallel to that of N_1.

This property can be used for detecting the plane polarized light, hence the Nicol prism N_2 acts as an analyzer

Thus, Nicol prisms N_1 and N_2 are known as polarizer and analyzer respectively.

Wave plates:-

Wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce path difference or phase difference between the ordinary ray and extraordinary ray.

Wave plates are mainly two types. They are

1. Half Wave plate
2. Quarter Wave plate

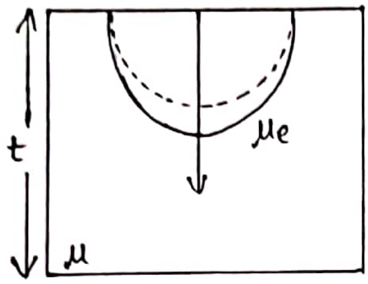
Half Wave Plate:-

Half wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce the path difference $\lambda/2$ or phase difference π between the ordinary ray and extraordinary ray.

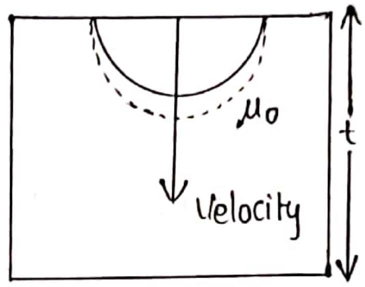
Calculation of thickness (t) of Half wave plate:-

let us consider a glass plate whose refractive index is μ and thickness is 't'.

let 'μ_o' be the refractive index of ordinary ray and 'μ_e' be the refractive index of extra ordinary ray.



Negative crystal



Positive crystal

The total path difference travelled by ordinary ray $= \mu_o t$

The total path difference travelled by extraordinary ray $= \mu_e t$

The path difference between two rays

$$\Delta = \mu_o t - \mu_e t$$

$$\Delta = (\mu_o - \mu_e)t$$

We know that the path difference introduced by Half wave plate, $\Delta = \frac{\lambda}{2}$

i.e., $\frac{\lambda}{2} = t(\mu_o - \mu_e)$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}} \quad \text{—} (1)$$

The above eq^n represents the thickness of Half-wave plate for negative crystal.

Similarly:

$$t = \frac{\lambda}{2(\mu_e - \mu_o)} \quad \text{——} \quad (2)$$

It represents the thickness of Half-wave plate for positive crystal.

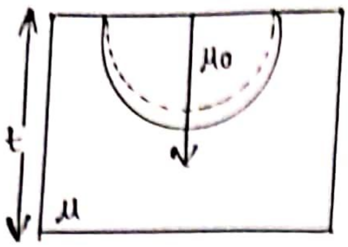
Quarter wave plate:-

Quarter wave plates are the uniaxial doubly refracting positive or negative crystals which can introduce the path difference $\frac{\lambda}{4}$ or phase difference $\frac{\lambda}{2}$ between the ordinary ray and extraordinary ray.

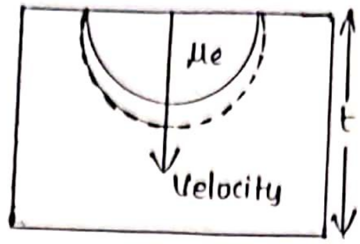
Calculation of thickness of Quarter wave plate:-

let us consider a glass plate whose refractive index is 'μ' and thickness is 't'.

let 'μ_o' be the refractive index of o-ray and 'μ_e' be the refractive index of E-ray.



Negative crystal



positive crystal

The total path travelled by O-ray : $\mu_0 t$

The total path travelled by E-ray - $\mu_e t$

The path difference between two rays

$$\Delta = \mu_0 t - \mu_e t$$

$$= (\mu_0 - \mu_e) t$$

We know that the path difference introduced by Quarter wave plate is $\Delta = \frac{\lambda}{4}$

$$\frac{\lambda}{4} = (\mu_0 - \mu_0) t$$

$$t = \frac{\lambda}{4(\mu_0 - \mu_e)} \quad \text{---} \quad (1)$$

It represents thickness of quarter wave plate for negative crystal.

Similarly, $t = \frac{\lambda}{4(\mu_0 - \mu_e)}$ ——— (2)

It represents thickness of Quarter wave plate for positive crystal.