UNIT-III

Formal Grammar:

* Introduction
* classification of formal Grammar.
    1. chomsky Hirearchy.
    2. Types.

* **Introduction:—**

Mathematically A formal Grammar is a tuple like

$G = (V, T, P, S)$ where,

V = finite and non Empty set of non-Terminal symbols (or) Variables.

variables are represented by upper case letters

T = Finite and non Empty set of Terminal symbols Represented by lower case letters and some special symbols are there.

P = It is a set of production rules are of the form

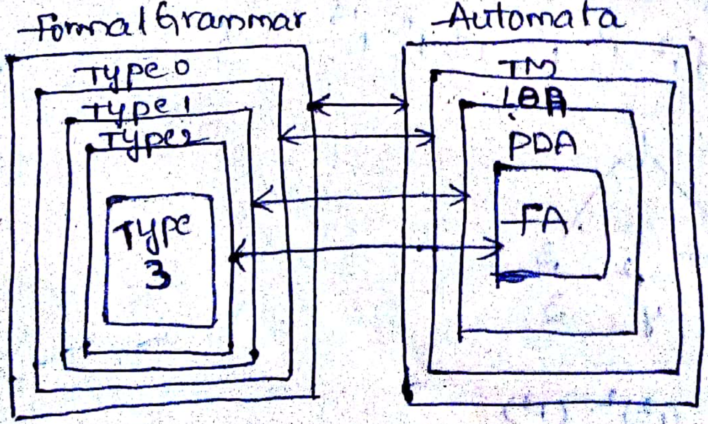
$$P \rightarrow \alpha \rightarrow \beta$$
$$\alpha \in V$$
$$\beta \in (V \cup T)^*$$

S → It is the starting symbol of the Grammar is always, a variable which is $S \in V$.

NOTE:— Grammar's are used to describe a language

* **classification of Grammar:—**

- using chomsky hierarchy.



Type 3 Grammar:—

* It is also called as Regular grammar.
* Type 3 Grammar is defined as $G = (V, T, P, S)$ where,

$V \to$ set of variables.
$T \to$ set of Terminals
$P \to$ set of production rules are of the form

$\boxed{\begin{array}{l} A \to Ba \\ A \to a \end{array}}$ — According to left linear grammar

(or)

$\boxed{\begin{array}{l} A \to aB \\ A \to a \end{array}}$ — According to Right linear grammar.

Ex:— $A \to aB$
$A \to Ba$
$A \to a$
$A \to \epsilon$

where.
$(A, B) \in V$
$a \in T^*$

* Type 3 Grammar is used to generating Regular language
* Regularlanguages are recognised (or) accepted by finite automata. i.e; NFA (or) DFA.

Type 2 Grammar:—

* It is also called as Context-free grammar.
* ~~ty~~ context-free grammar is defined as $G = (V, T, P, S)$

where $V \to$ finite set of variables
$T \to$ finite set of Terminals
$P \to$ finite set of production rules are of the form

$$\alpha \to \beta$$

where $\alpha \in V$
$\beta \in (V \cup T)^*$

Ex:— $S \to aSa$
$S \to bSb$
$S \to ab$
$S \to \epsilon$

* Context-free grammars are used to generate context-free language.

* context-free language recognised (or) Accepted by pushdown Automata.

Type 1 Grammar :—

* It is also called as context - Sensitive Grammar.
* A CSG is defined as $G=(V,T,P,S)$ where

$V=$ finite set of variables

$T=$ finite set of Terminals.

$P=$ set of production rules are of the form $\alpha \to \beta$

where, $\alpha \in (V \cup T)^+$

$\beta \in (V \cup T)^*$

length of $|\alpha| \leq$ length of $|\beta|$

Ex:- $S \to aBb$

$bB \to aa$

$B \to b$

* CSG is used to generating Context - Sensitive language
* CSL recognised (or) Accepted by Linear Bounded Automata

Type 0 Grammar:—

* It is also called also Recursive - Grammar (or) Recursive Enumerable grammar (or) phrase structured Grammar.
* mathematically Recursive grammar is defined as

$G=(V,T,P,S)$ where

$V \to$ finite set of variables

$T \to$ finite set of Terminals

$P \to$ set of production rules are of the form.

$\alpha \to \beta$

$\alpha \in (V \cup T)^{+}$

$\beta \in (V \cup T)^*$

$|\alpha| \geq |\beta|$

Ex:- $S \to aAbB$

$aAbB \to aB$

$abB \to aA$

$A \to \epsilon$

* Recursive Grammars are used to generating recursive language (or) Recursive-Enumerable language (or) phrase structured language.

* Recursive languages are recognised are accepted by Turing machine.

Relationship b/w formal grammar and automata :-

1. Type 3 $\subseteq$ Type 2 $\subseteq$ Type 1 $\subseteq$ Type 0

2. FA $\subseteq$ PDA $\subseteq$ LBA $\subseteq$ TM

Context - Free Grammar :

* Introduction
* Design of CFL
* closure properties of CFL.

*Introduction :-

Context-free Grammar is a Grammar which is defined by four tuples like $G=(V,T,P,S)$ where,

V – It is finite and non-Empty set of non-terminal symbols (or) variables.

T – finite and non-Empty set of Terminal symbols.

P – finite and non-Empty set of production rules are of the form $\alpha \rightarrow \beta$

$\alpha \in V$

$\beta \in (V \cup T)^*$

Ex:- $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow aa|bb$

$S \rightarrow \epsilon$

$S \rightarrow$ It is starting symbol.

Context free-language :-

Let $G=(V,T,P,S)$ be a Context-free grammar. The CFG generating a language 'L' is called Context-free language

* It is denoted by L(G)..
* Content - free languages are organized by PDA.

Design of CFL :—

1) Construct a CFL for the following set. $\{\epsilon, a, aa, aaa, \cdots a^n\}$

sol:- Given set $\{\epsilon, a, aa, aaa, aaaa, \cdots a^n\}$

minimum string $= \epsilon$
Next minimum string $= a$
Maximum string $= a^n$

$S \to a^n$
$\Downarrow$
$a \cdot a^{n-1} \Rightarrow S \to aS$
$\Downarrow$
$a \cdot a \cdot a^{n-2}$ $\quad S \to \epsilon$
$\Downarrow$ $\quad S \to a$
$a \cdot a \cdot a \cdot a^{n-3}$

CFG:-
$S \to aS$
$S \to \epsilon$
$S \to a$

$L = \{a^n \mid n \geq 0\}$ //.

2) Construct a CFL for the following set $\{\epsilon, ab, aabb, \cdots\}$

sol:- Minimum String $= \epsilon$
Next minimum string $= ab$
Maximum string $= a^n b^n$

$S \to a^n b^n$
$S \to a\,\underline{a^{n-1} \cdot b^{n-1}}\,b$
$S \to aa\,\underline{a^{n-2}\, b^{n-2}}\,bb$

$\therefore S \to aSb$
$S \to \epsilon$
$S \to ab$

$\therefore$ CFG :- $S \to aSb$
$S \to \epsilon$
$S \to ab$

$\therefore L = \{a^n b^n \mid n \geq 0\}$

3) construct a CFL for the following set $\{a, b, ab, aabb, aaabbb, \ldots\}$

sol:- Minimum string = a/b

Maximum string = $a^n b^n$

$S \rightarrow a^n b^n$

$\rightarrow a a^{n-1} b^{n-1} b$ $\Rightarrow$ $S \rightarrow aSb$

$S \rightarrow a$

$\rightarrow a a a^{n-2} b^{n-2} bb$ $S \rightarrow b.$

$\therefore$ CFG $S \rightarrow asb$

$S \rightarrow a$

$S \rightarrow b$

$\therefore L = \{a^n b^n \mid n \geq 1\}$

4) construct a CFG to generate the language $L = \{a^n b^{2n} / n \geq 1\}$

sol:- Minimum string = abb

maximum string = $a^n b^{2n}$

$S \rightarrow a^n b^{2n}$

$S \rightarrow a a^{n-1} b^{2n-2} bb \Rightarrow S \rightarrow aSbb$

$S \rightarrow abb$

$\therefore$ CFG = $S \rightarrow aSbb$

$S \rightarrow abb.$

5) construct CFG for the following CFL

$L = \{0^i 1^{i+1} \mid i \geq 0\}$

sol: $L = \{0^i 1^{i+1} \mid i \geq 0\}$

$= 0^i 1^i . 1$

$A \rightarrow 0^i 1^i$

$\rightarrow 0 0^{i-1} 1^{i-1} 1$

$S \rightarrow A1$

$\rightarrow 0A1$

$A \rightarrow 0A1$

$A \rightarrow \epsilon$

$A \rightarrow 01$

CFG: $S \rightarrow A1$

$A \rightarrow 0A1$

$A \rightarrow \epsilon$

$A \rightarrow 01$

6) construct a CFL from the following Language

$L = \{a^m b^m c^n \mid m, n \geq 0\}$

sol:-

$\underbrace{a^m b^m}_{A} \underbrace{c^n}_{B}$

$A \to a^n b^n$

$\to a\, a^{n-1} b^{n-1} b$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to c^n$

$B \to c c^{n-1}$

$B \to cB$

$B \to \epsilon$

$B \to c$

CFG:- $S \to AB$

$A \to aAb$

$A \to \epsilon$

$A \to ab$

$B \to cB$

$B \to \epsilon$

$B \to c$

Closure properties of CFL:-

- content free languages are closed under union
- " " " " " concatenation.
- " " " " " Kleene closure
- " " " " " Reversal

- content free languages are not closed under Complement
- " " " " " " Intersection
- " " " " " " difference.

Derivation:-

* Introduction * Types of Derivation * Derivation tree

Derivation is a process of generating a string from a given grammar.

Derivation process can be represented graphically is called Derivation tree (or)

* left most derivation * Rightmost derivation.

Left most derivation:- with example

In this, we can replace a left most Variable to obtain the given input string.

Right most derivation:-

In this, in each step we can replace a Right most variable to obtain the given input string.

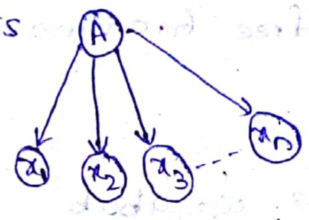
Derivation tree :-

Let $G = (V, T, P, S)$ be a CFG. Then there is a derivation tree for G. If and only if.

* The root node of the Tree is labelled with start symbol of G
* All leaf nodes of Tree are labelled by Terminals (or) special symbols of G.
* The interior nodes are labelled by variables of G.
* If any production rule in G. is are the form.

$A \rightarrow x_1 x_2 x_3 --- x_n$ then the

derivation tree is



find the i) left most derivation

ii) Right most derivation

iii) parse tree for the i/p string id+id*id

from the following grammar. $E \rightarrow E+E$

$E \rightarrow E*E$

$E \rightarrow id$

Sol:- The given grammar is $E \rightarrow E+E$

$E \rightarrow E*E$

$E \rightarrow id$

Input string. id+id*id.

LMD:- $E \rightarrow E+E$

$\rightarrow$ ~~E+E*E~~

$\rightarrow id+E$

$\rightarrow id+E*E$

$\rightarrow id+id*E$

$\rightarrow id+id*id$.

RMD:- $E \rightarrow E+E$

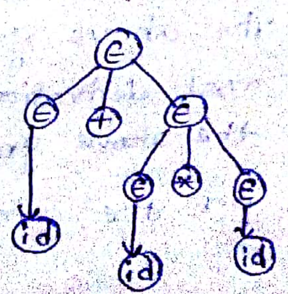
$\rightarrow$ ~~id.~~ $E+E*E$

$\rightarrow E+E*id$

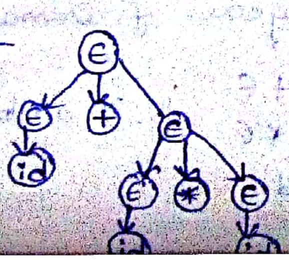
$\rightarrow E+id*id$

$\rightarrow id+id*id$

Parse tree:-



Parse tree:-



Ambiguious Grammar:-

* A CFG $G=(V,T,P,S)$ which generates two (or) more parse trees for given i/p string is called Ambiguious grammar.

* That means an Ambegeous grammar has two or more left most derivations (or) right most derivation (or) parse tree.

Ex:- Prove that $S \to aSbS$ is ambegeous for the i/p
$S \to bSaS$
$S \to \epsilon$

string abab

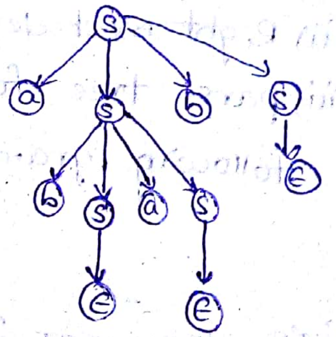
sol:- The given Context free Grammar is $S \to aSbS$
$S \to bSaS$
$S \to \epsilon$.

the input string is $w = abab$

① LMD:

$S \to aSbS$
$\to abSaSbS$
$\to ab\epsilon aSbS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

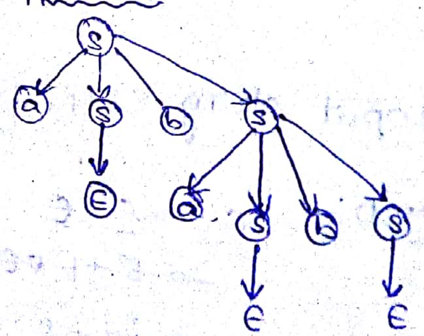
parsetree



② LMD:

$S \to aSbS$
$\to a\epsilon bS$
$\to abS$
$\to abaSbS$
$\to aba\epsilon bS$
$\to ababS$
$\to abab\epsilon$
$\to abab$

Parsetree



∴ The above grammar generates two parsetrees (or) two left most derivation for the same i/p string $w=abab$. Hence the above grammar is ambegeous grammar.

2) P.T the grammar $E \to E+E$
$E \to E*E$ is ambegeous for i/p
$E \to id$

string $id+id*id$.

sol:- The given context free grammar is

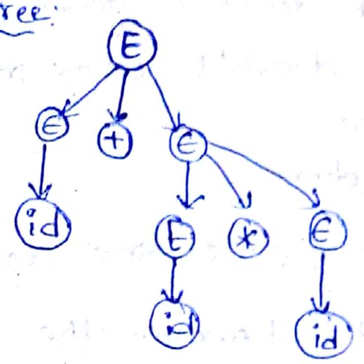
$E \to E+E$
$E \to E*E$
$E \to id$

The input string is $w = id+id*id$.

① LMD:

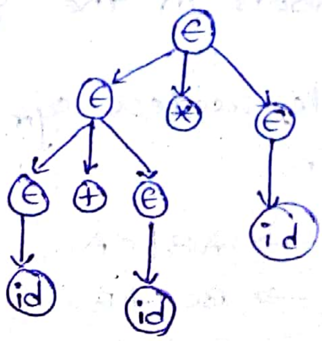
$E \to E+E$
$\to id+E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$

Parsetree:



② LMD:-

$E \to E*E$
$\to E+E*E$
$\to id+E*E$
$\to id+id*E$
$\to id+id*id.$



** (7m)

Simplification of CFG:

* Introduction
* Methods
  1. Elimination of useless symbols.
  2. Elimination of ε-productions
  3. Elimination of unit productions.

Introduction:-

It's means minimizing the no. of productions in the given CFG. that is reducing size of CFG. size of CFG is equal to no. of productions.

Methods:- $S \to AB$
$A \to a$
$A \to aA$
$B \to SB$

Elimination of useless symbols:-

useful symbol:- A variable is said to be useful if and only if

* It generates a terminal string.
* It is used in derivation of a string at least one time

useless symbol :-

* A variable is said to be useless if and only if.
  * It doesn't generates a terminal string.
  * It doesn't used in derivation of a string at least one time.

Procedure :

step 1 :- Determine useless symbols in the grammar.

step 2 :- Remove the productions which contains useless symbols in the grammar.

Ex :- eliminate useless symbols from the following grammar.

$S \rightarrow AB \mid cA$
$B \rightarrow Bc \mid AB$
$A \rightarrow a$
$c \rightarrow aB \mid b$

sol :- The given CFG is

$S \rightarrow AB$
$S \rightarrow cA$
$B \rightarrow Bc$
$B \rightarrow AB$
$A \rightarrow a$
$c \rightarrow aB$
$c \rightarrow b$

In the given grammar 'B' doesn't generating a terminal string.

$S \rightarrow \underline{A}B$
$S \rightarrow a\underline{B}$
$\rightarrow a\underline{B}c$
$\rightarrow a\underline{A}Bc$
$\rightarrow aaB\underline{c}$
$\rightarrow aaBb$
$\rightarrow aa\underline{A}Bb$
$\rightarrow aaaBb$
⋮

so, ∴ 'B' is useless symbol.

so, we can eliminate the productions which contains 'B'.

∴ The reduced CFG is

$S \rightarrow cA$
$A \rightarrow a$
$c \rightarrow b$.

2) Elimination of ∈-production :-

∈-production :- A production is of the form $A \rightarrow \epsilon$ is called ∈-production (or) NULL production.

procedure :-

step 1 :- If the grammar contains $A \rightarrow \epsilon$ then replace 'A' with ∈ in the remaining productions.

step 2 :- Remove $A \rightarrow \epsilon$ from the grammar.

Ex :- Remove ∈-productions from the following grammar

$A \rightarrow 0B1 \mid 1B1$

$B \rightarrow 0B \mid 1B \mid \epsilon$

sol :- the given CFG is $A \rightarrow 0B1$

$A \rightarrow 1B1$

$B \rightarrow 0B$

$B \rightarrow 1B$

$B \rightarrow \epsilon$.

$A \rightarrow 0B1$
$\rightarrow 0\epsilon 1$
$\rightarrow 01$

$\therefore A \rightarrow 0B1$
$A \rightarrow 01$

$B \rightarrow 0B$
$\rightarrow 0\epsilon$
$\rightarrow 0$

$\therefore B \rightarrow 0B$
$B \rightarrow 0$

$A \rightarrow 1B1$
$\rightarrow 1\epsilon 1$
$\rightarrow 11$

$\therefore A \rightarrow 1B1$
$A \rightarrow 11$

$B \rightarrow 1B$
$\rightarrow 1\epsilon$
$\rightarrow 1$

$\therefore B \rightarrow 1B$
$B \rightarrow 1$

After eliminating $B \rightarrow \epsilon$ the resultant CFG is

$A \rightarrow 0B1$ $\quad$ $B \rightarrow 1B$

$A \rightarrow 01$ $\quad$ $B \rightarrow 1$

$A \rightarrow 1B1$

$A \rightarrow 11$

$B \rightarrow 0B$

$B \rightarrow 0$

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** Normal forms :-

* Introduction

* Types of Normal forms

1. chomsky Normal Form (CNF)

2. Greiback Normal Form (GNF).

Introduction :-

In CFG each production of the form $\alpha \rightarrow \beta$ where $\alpha \in V$ that means β contains any no.of non-terminal symbols $\beta \in (V \cup T)$ and any no. of terminal symbols. But, we need to have a grammar in specific form i.e; we can decide the no. of non terminals and terminals on R.H.S of the grammar. This can be implemented by using "normalization of CFG".

Normalization :-

The process of Arranging the grammar with fixed no. of non-terminals and terminals on R.H.S of CFG is called normalization.

normal-forms are classified into two types

i) chomsky normal form.

ii) Greiback normal form

chomsky normal form :-

It is defined as $\alpha \to \beta$

non-terminal $\to$ Non-terminal. Non-terminal.
(or)
Non-terminal $\to$ Terminal.

conversion of CFG to CNF :-

Procedure :-

step 1 :- simplify the CFG.

step 2 :- convert the simplified CFG to CNF.

Ex:- convert the following CFG into chomsky normal form.

$S \to aaaaS$

$S \to aaaa$

Sol:- The given grammar is $S \to aaaaS$

$S \to aaaa$

Consider a non-terminal A that derives Terminal a.

$\therefore$ The production rule is $A \to a$. is in CNF

$S \to aaaaS$

$S \to A\boxed{AAAS}$ can be replaced by P_1

$S \to AP_1$ is in CNF.

$P_1 \to A\boxed{AAS}$ can be replaced by P_2

$P_1 \to AP_2$ is in CNF

$P_2 \to A\boxed{AS}$ can be replaced by P_3

$P_2 \to AP_3$ is in CNF

$P_3 \to AS$ is in CNF

$S \to aaaa$

$S \to A\boxed{AAA}$ can be replaced by P_4

$S \rightarrow AP_4$ is in CNF

$P_4 \rightarrow A\boxed{AA}$ can be replaced by P_5.

$P_4 \rightarrow AP_5$ is in CNF

$P_5 \rightarrow AA$ is in CNF.

The resultant Grammar CNF is

$S \rightarrow AP_1$ $\quad P_5 \rightarrow AA$

$S \rightarrow AP_4$ $\quad A \rightarrow a$

$P_1 \rightarrow AP_2$

$P_2 \rightarrow AP_3$

$P_3 \rightarrow AS$

$P_4 \rightarrow AP_5$

2) Convert the given CFG to CNF. $S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow a$

$S \rightarrow b$.

Sol:- The given grammar is $S \rightarrow aSa$

$S \rightarrow bSb$.

$S \rightarrow a$

$S \rightarrow b$

It is already in simplified form.

Consider a non-terminal A that derives a terminal a and the non-Terminal B. that derives the terminal b.

$\therefore$ The production rules is $A \rightarrow a$, $B \rightarrow b$. are in CNF.

(i) $S \rightarrow aSa$

$S \rightarrow A\boxed{SA}$ can be replaced by P_1

$S \rightarrow AP_1$ is in CNF.

$P_1 \rightarrow SA$ is in CNF.

(ii) $S \rightarrow bSb$

$S \rightarrow B\boxed{SB}$ can be Replaced by P_2

$S \rightarrow BP_2$ is in CNF

$P_2 \rightarrow SB$ is in CNF

(iii) $S \rightarrow a$ is in CNF

$S \rightarrow b$ is in CNF.

$\therefore$ The resultant grammar in CNF is

$S \rightarrow AP_1$

$S \rightarrow BP_2$

$S \to a$

$S \to b$

$P_1 \to SA$

$P_2 \to SB$

$A \to a$

$B \to b$.

Greibach Normal Form (GNF) :-

GNF is defined as

Non-terminal $\to$ Terminal. any no. of nonterminals

Non-terminal $\to$ Terminal.

Lemma 1:

Let CFG be $G=(V,T,P,S)$ and there is a production rule $A \to aB$ and $B \to \beta_1|\beta_2|\beta_3|\cdots|\beta_n$ then add the new production rule to GNF $A \to a\beta_1|a\beta_2|a\beta_3|\cdots|a\beta_n$ to GNF.

$\therefore$ B is replaced by $B \to \beta_1|\beta_2|\cdots|\beta_n$

Lemma 2:

Let CFG be $G=(V,T,P,S)$ and there is production rule

$A \to A\alpha_1|A\alpha_2|\cdots|A\alpha_n|\beta_1|\beta_2|\cdots|\beta_n$ then the production rules are added to GNF.

$A \to \beta_1|\beta_2|\beta_3|\cdots|\beta_n$

$A \to \beta_1 Z|\beta_2 Z|\beta_3 Z|\cdots|\beta_n Z$

$Z \to \alpha_1|\alpha_2|\alpha_3|\cdots|\alpha_n$

$Z \to \alpha_1 Z|\alpha_2 Z|\alpha_3 Z|\cdots|\alpha_n Z$

Converting ~~CFG~~ CFG into GNF :-

Procedure:-

step 1 :- Simplify the CFG.

step 2 :- Converting simplified CFG into GNF.

Ex:- Convert the given CFG to GNF $S \to ABA$

$A \to aA|\epsilon$

$B \to bB|\epsilon$

Sol:- The Given CFG is $S \to ABA$

$A \to aA$

$A \rightarrow \epsilon$

$B \rightarrow bB$

$B \rightarrow \epsilon$

Simplified of given CFG :-

ⓐ elimination of ϵ-productions :-

$A \rightarrow \epsilon \quad B \rightarrow \epsilon$

① $S \rightarrow ABA$
$S \rightarrow \epsilon BA$
$S \rightarrow BA$

② $S \rightarrow ABA$
$S \rightarrow AB\epsilon$
$S \rightarrow AB$

③ $S \rightarrow ABA$
$S \rightarrow A\epsilon A$
$S \rightarrow AA$

④ $S \rightarrow ABA$
$S \rightarrow \epsilon\epsilon A$
$S \rightarrow A$

⑤ $S \rightarrow ABA$
$S \rightarrow \epsilon B\epsilon$
$S \rightarrow B$

$A \rightarrow aA$; $B \rightarrow bB$
$A \rightarrow a\epsilon$; $B \rightarrow b\epsilon$
$A \rightarrow a$; $B \rightarrow b$

∴ After eliminating $A \rightarrow \epsilon$, $B \rightarrow \epsilon$ from the grammar the resultant grammar is

$S \rightarrow ABA \qquad A \rightarrow aA$
$S \rightarrow BA \qquad A \rightarrow a$
$S \rightarrow AB \qquad B \rightarrow bB$
$S \rightarrow AA \qquad B \rightarrow b$
$S \rightarrow A$ ✓
$S \rightarrow B$ ✓

Elimination of unit productions :-

The above grammar has two unit productions like

$S \rightarrow A$ × $\qquad S \rightarrow B$ ×

$S \rightarrow aA \qquad S \rightarrow bB$
$S \rightarrow a \qquad S \rightarrow b$

[∴ since $A \rightarrow aA$, $A \rightarrow a$, $B \rightarrow bB$, $B \rightarrow b$]

∴ After elimination unit productions $S \rightarrow A$, $S \rightarrow B$ from the grammar. The resultant grammar is

$S \rightarrow ABA \qquad A \rightarrow aA$
$S \rightarrow BA \qquad A \rightarrow a$
$S \rightarrow AB \qquad B \rightarrow bB$
$S \rightarrow AA \qquad B \rightarrow b$
$S \rightarrow aA$
$S \rightarrow a$
$S \rightarrow bB$
$S \rightarrow b$

there is no useless production.

∴ The simplified CFG is

S → ABA
S → BA
S → AB
S → AA
S → aA
S → a
S → bB
S → b

A → aA
A → a
B → bB
B → b

Converting simplified CFG to GNF:

i) S → ABA
S → aABA ✓
S → aBA ✓

A → aA ✓
A → a ✓
B → bB ✓
B → b ✓

ii) S → BA
S → bBA ✓
S → bA ✓

iii) S → AB
S → aAB
S → aB ✓

iv) S → AA
S → aAA
S → aA

v) S → aA ✓
S → a ✓

vi) S → bB ✓
S → b ✓

∴ The resultant grammar is in GNF is

S → aABA | aBA | bBA | bA | aAB | aB | aAA | aA | bB | a | b
A → aA | a
B → bB | b.

② Convert the following CFG into GNF S → AA | 0
A → SS | 1

Sol: Given Grammar S → AA
S → 0
A → SS
A → 1

The simplified CFG is S → AA
S → 0
A → SS
A → 1

① $S \to AA|0$

$S \to SSA|0$

$S \to 0$

$S \to 0Z$

$Z \to SA$

$Z \to SAZ$

② $S \to AA|0$

$S \to 1A|0$

$S \to 1A$

$S \to 0$

③ $A \to SS$

$A \to 0S$

$A \to 0ZS$

$A \to 1AS$

$A \to 0S$

$Z \to SA$	$Z \to SAZ$
$Z \to 0A$	$Z \to 0AZ$
$Z \to 0ZA$	$Z \to 0ZAZ$
$Z \to 1AA$	$Z \to 1AAZ$
$Z \to 0A$	$Z \to 0AZ$

∴ The resultant grammar is

$S \to 0|0Z|1A$

$Z \to 0A|0ZA|1AA|0A|0AZ|0ZAZ|1AAZ|$

$A \to 0S|0ZS|1AS|1$

③ convert the given CFG to GNF $S \to CA$
$A \to a$
$C \to aB|b$

⇒ Given CFG is not a simplified Grammar

After eliminating the useless symbols the resultant CFG is. $S \to CA$
$A \to a$
$C \to b$

By Applying Lemma 1 $S \to CA$
$S \to bA$

∴ The resultant GNF is $S \to bA$
$A \to a$
$C \to b$

④ Convert the given CFG to GNF $S \to SS$
$S \to 0S1|01$

The given CFG is a simplified CFG

The resultant grammar is $S \to SS$
$S \to 0S1$
$S \to 01$

Replaced 0 by A, 1 by B

Then productions are $A \to 0$
$B \to 1$

$S \to SS$

$S \to ASB$

$S \to AB$

Applying Lemma ①

① $S \to SS$

$S \to ASBS$

$S \to 0SBS$

② $S \to SS$

$S \to ABS$

$S \to 0BS$

② $S \to ASB$

$S \to 0SB$

$S \to AB$

$S \to 0B$

The resultant grammar GNF is

$S \to 0SBS \mid 0BS \mid 0SB \mid 0B$

$A \to 0$

$B \to 1$

Pumping Lemma for CFL:—

pumping lemma is used for proving the given language is CFL (or) not.

Lemma :- let 'L' be any CFL, then there is a constant 'n' which depends only a part 'L' such that there exist a string $w = uvxyz$ such that
1. $|vy| \geq 1$
2. $|vxy| \leq n$
3. for $\forall\, i \geq 0$ uv^ixy^iz is in L

Then 'L' is said to be CFL. otherwise it is not a CFL.

① Prove that $L = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL.

The given language $L = \{a^n b^n c^n \mid n \geq 0\}$.

$L = \{\epsilon, abc, aabbcc, ----\}$

consider a constant n and the string $w = a^n b^n c^n$

consider a string $w \in L$

$w = abc$ for $n = 1$

$|w| = 3n$

for $i = 1$ $w = abc$

for $i = 2$

$w = uv^ixy^iz$

$w = uv^2xy^2z$

$w = a^{n-1} a^2 b^{n-1} b^2 c^n$

$w = a^{n+1} b^{n+1} c^n \notin L$

$a^n \quad b^n \quad c^n$

$\underset{u}{a^{n-1}} \quad \underset{v}{a} \quad \underset{x}{b^{n-1}} \quad \underset{y}{b} \quad \underset{z}{c^n}$

$\therefore$ The given language is not a CFL.

② show that the language $L = \{ss^T / s \in \{a,b\}^*\}$

Given language $L = \{ss^T / s \in \{a,b\}^*\}$

$L = \{\epsilon,$