PROBABILITY

UNIT - III

Introduction:- Galelio (1564-1624) of an Italian Mathematician was first to attempt a quantative measure of Probability while dealing with some Problems related to the Theory of dice in Gambling. For the first foundation of mathematical theory probability was laid in middle of 17^{th} century by the French mathematician Pascal and Fermates while solving no. of Problems of French Gamblers.

Russian mathematicians are also have made a very valuable contribution on modern theory of Probability. Favous mathematical scientist chebychev's who founded russian school of Mathematics and Statistics. They are proposed on definations on Probability.

Random experiment:- Consider an experiment can throw repeated ascentically identical conditions, does not gives unique results. But result is any one of the several possible out comes. This type of an experiment is called random experiment.

Ex:- Tossing a coin, rolling a dice... etc.

Sample Space:- Set of all Possible out comes of an experiment is known as sample space. It is denoted with S (or) Ω

Ex:- ① n Coins are ~~thrown~~ Tossed / Thrown sample space is 2^n

② n dice are rolled sample space is 6^n.

③ In a Pack of Cards if we draw x cards then sample space is $^{52}C_x$.

④ In an urn contains m-white n-blue. k-yellow balls if we draw x balls then sample space $^{m+n+k}C_x$.

Event:- the out come of an experiment is known as an event.

Ex:- ① Throwing a coin and getting a head is an event.

② Rolling a dice and getting 5^{th} face is an event.

Event

→ Simple event

→ Compound event.

Simple event:- An event is said to be simple event if it has single Possible out comes.

Ex:- Rolling a dice getting sixth face.

Compound event:- An event is said to be compound event it can be split into two.

Ex:- Rolling a dice getting sum is 9.

$S = \{(6,3)\ (3,6)\ (5,4)\ (4,5)\}$.

Exhaustive Cases:- The Total number of possible out comes in an experiment is known as exhaustive cases.

Ex:- (1) In tossing a coin total exhaustive cases $\underline{2}$

(2) In rolling a dice total exhaustive cases are $\underline{6}$

Favourable Cases:- The number of cases favourable to an event in a trial is known as favourable cases.

Mutually exclusive ~~cases~~ events:- The events are said to be mutually exclusive if happening of one event is prevents to the happening of another event is called mutually exclusive events.

Independent events:- The events are said to be Independent events happening of one event has no influence of happening of other event are said to be Independent events.

Mathematical (or) classical defination of Probability :- An experiment may results n exhaustive mutual exclusive. In which of them "m" are favourable to an event "E" then Probability of getting an event E is

$$P(E) = \frac{\text{Favourable Cases}}{\text{Total exhaustive Cases}} = \frac{m}{n}.$$

$$\boxed{P(E) = \frac{m}{n}}$$

Total Probability = 1

$$P(E) + P(\bar{E}) = 1$$

$$P(\bar{E}) = 1 - P(E)$$

$$P(\bar{E}) = 1 - \frac{m}{n}.$$

Limits of Probability $\boxed{0 \leq P \leq 1}$

(i) $P(E) = 1$ then that event is called certain event.

(ii) $P(E) = 0$ then that event is called impossible event.

Problems.

(1) Two dice are rolled. What is the Probability of getting sum is 9.

Sol:- Favourable cases for getting sum 9 is $\{(5,4)(4,5)(3,6)(6,3)\} = 4$

Total Exhaustive case for two dice are rolled $= 6^2 = 36$.

Prob of getting sum is 9 $= \frac{\text{Favourable cases}}{\text{Total exhaustive cases}} = \frac{4}{36}$.

(2) A card is drawn from well shuffled pack what is the prob of getting a king.

Sol:- Favourable cases for getting a king $= 4_{c_1}$.

Total exhaustive cases for drawing a card $= 52_{c_1}$.

Prob of getting a king $= \frac{\text{Favourable cases}}{\text{Total exhaustive case}} = \frac{4_{c_1}}{52_{c_1}}$

$= \frac{4}{52}$

(3) Two coins are rolled what is the prob of getting (i) Both heads (ii) both tails (iii) one head.

Sol:- Two coins are rolled Total exhaustive cases $n = 2^2 = 4$.

$\{(HH)\ (TT)\ (HT)\ (TH)\}$.

(i) Favourable cases for both heads $= 1$

Prob of getting both heads $= \frac{F.C.}{T.E.C} = \frac{1}{4}$

(ii) Favourable cases for both tails = 1

Prob of getting both tails = $\frac{F.C}{T.E.C} = \frac{1}{4}$.

(iii) Favourable cases for one head = 2

Prob of getting one head = $\frac{F.C}{T.E.C} = \frac{1}{2}$.

Problem (4): A stockist has 20 items in a lot of which 12 are non defectives, if customer selects 3 items from a lot.

(i) What is the prob of 3 are non defectives.

(ii) What is the prob of 2 are non defective and one is defective.

Sol:- (i) Total exhaustive cases for drawing three items = $^{20}C_3$.

Favourable cases for drawing 3 non defectives = $^{12}C_3$.

Prob of getting 3 are non defectives = $\frac{F.C}{T.E.C}$

$$= \frac{^{12}C_3}{^{20}C_3}$$

(ii) Favourable cases for two are non defective and one is defective =

$^{12}C_2 \cdot {^8C_1}$

Total Exhaustive cases = $^{20}C_3$.

Prob of getting two are non defective and one is defective $= \frac{F.C}{T.E.C} = \frac{12_{C_2} \cdot 8_{C_1}}{20_{C_3}}$.

→ Conditional Probability:-

If A and B are two dependent events then,

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

i.e Prob of occurrance of event A, when event B is already happend.

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

i.e Prob of occurrance of event B, when event A is already happend.

Note:- If $A_1, A_2, A_3, \ldots A_n$ are Independent events the prob of getting atleast one of the event is

$$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = 1 - P(\overline{A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n})$$

$$= 1 - P(\bar{A_1} \cap \bar{A_2} \cap \bar{A_3} \cap \ldots \cap \bar{A_n})$$

$$= 1 - [P(\bar{A_1}) \cdot P(\bar{A_2}) \cdot P(\bar{A_3}) \ldots P(\bar{A_n})].$$

Problem :- (1) A problem in statistics given to three students A, B and C whose chances are solving are $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. What is the prob of the problem will be solve atleast one of them, are all three students try independently.

Sol:- Prob of that problem solved by student A $P(A) = \frac{1}{2}$

Prob of that problem solved by student B $P(B) = \frac{1}{4}$.

Prob of that problem solved by student C $P(C) = \frac{1}{4}$.

$$P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(\bar{B}) = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(\bar{C}) = 1 - P(C) = 1 - \frac{1}{4} = \frac{3}{4}$$

Prob of that problem solved atleast by one of them is

$$P(A \cup B \cup C) = 1 - [P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})]$$

$$= 1 - \left(\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{3}{4}\right)$$

$$= 1 - \frac{9}{32} = \frac{23}{32}$$

Problem :- (2) The prob of A will live up to 60 years is $\frac{3}{4}$. Prob of B will live up to 60 years is $\frac{2}{3}$. What is the prob that

(i) Both A and B are live up to 60 years.

(ii) Both die before reaching 60 years.

Sol:- Prob of A will live up-to 60 yrs $P(A)=\frac{3}{4}$.

Prob of B will live up to 60 yrs $P(B)=2/3$.

(i) Both A and B are live up to 60 years

$$P(A\cap B)= P(A)\cdot P(B)$$

$$=\frac{3}{4}\cdot\frac{2}{3} = \frac{6}{12}\Rightarrow 1/2$$

(ii) Both die before reaching 60 years $P(A\cup B)= 1-P(\overline{A\cup B})$

$$=1-\{P(\bar{A})\cdot P(\bar{B})\}.$$

$P(\bar{A})=1-P(A)$
$=1-3/4 = 1/4$,

$P(\bar{B}) = 1-P(B)$
$= 1-2/3 = 1/3$.

$$P(A\cup B) = 1-\{P(\bar{A})\cdot P(\bar{B})\}.$$

$$=1-\{1/4\cdot 1/3\}.$$

$$= 1-\frac{1}{12}$$

$$=\frac{11}{12}$$

Problem ③ A Bag contains 25 tickets numbered 1 to 25. one ticket is drawn what is the prob of getting multiples of 2 or 5.

Sol:- Let A_1 be the multiple of 2 and A_2 be the multiple of 5.

Total exhaustive cases for drawing a ticket is 25_{C_1}

Favourable Cases for multiple of 2 is

$= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24\} = {}^{12}C_1$

Prob of getting a ticket and Multiple of 2 is

$$P(A_1) = \frac{{}^{12}C_1}{{}^{25}C_1} = \frac{12}{25}.$$

Favourable Cases for Multiple of 5 is

$\{5, 10, 15, 20, 25\} = {}^{5}C_1$

Prob of getting a ticket and Multiple of 5 is

$$P(A_2) = \frac{{}^{5}C_1}{{}^{25}C_1} = \frac{5}{25}.$$

Favourable Cases for Multiple of 2 and 5 is

$\{10, 20\} = {}^{2}C_1$.

Prob of getting Multiple of 2 and 5 are

$$P(A_1 \cap A_2) = \frac{{}^{2}C_1}{{}^{25}C_1} = \frac{2}{25}.$$

Prob of getting Multiple of 2 (or) 5 is

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= \frac{12}{25} + \frac{5}{25} - \frac{2}{25}$$

$$P(A_1 \cup A_2) = \frac{15}{25} = \frac{3}{5}$$

$$= 0.6$$

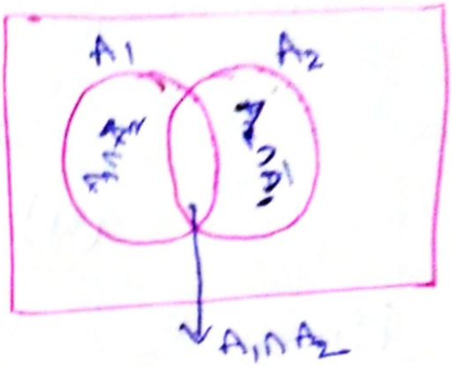
Theorem

Addition theorem on Probability for n=2 events

Statement:- If A_1 and A_2 are two disjoint events. then

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2).$$

Proof:- From venn diagram.



$$A_1 \cup A_2 = A_1 \cup (A_2 \cap \bar{A}_1)$$

Taking probabilities on both sides

$$P(A_1 \cup A_2) = P[A_1 \cup (A_2 \cap \bar{A}_1)].$$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2 \cap \bar{A}_1) \quad \text{--- (1)} \quad [\because A_1, A_2 \text{ are disjoint sets}].$$

also $A_2 = (A_1 \cap A_2) \cup (A_2 \cap \bar{A}_1)$

Taking probabilities on both sides.

$$P(A_2) = P[(A_1 \cap A_2) \cup (A_2 \cap \bar{A}_1)].$$

$$P(A_2) = P(A_1 \cap A_2) + P(A_2 \cap \bar{A}_1)$$

$$P(A_2) - P(A_1 \cap A_2) = P(A_2 \cap \bar{A}_1) \quad \text{--- (2)}$$

Substitute equation (2) in to equ (1).

$$P(A_1 \cup A_2) = P(A_1) + P(A_2 \cap \bar{A}_1)$$

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

Hence the proof.

Theorem Addition theorem on Probability for n-events :-

Statement :- $A_1, A_2, A_3, \ldots A_n$ are n events has been drawn from sample space. then.

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k)$$

$$+ (-1)^{n-1} \cdot P[A_1 \cap A_2 \cap A_3 \cap \ldots \cap A_n].$$

(or)

$$P(A_1 \cup A_2 \cup A_3 \ldots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k)$$

$$\ldots\ldots + (-1)^{n-1} \cdot P(A_1 \cap A_2 \cap A_3 \cap \ldots\ldots \cap A_n)$$

Proof :- This can be proved it by mathematical Induction.

For $\underline{n=2}$ events.

$$P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

It is true for $\underline{n=2}$ events.

For $\underline{n=3}$ events.

$$P(\underbrace{A_1}_{A_1} \cup \underbrace{A_2 \cup A_3}_{A_2}) = P(A_1) + P(A_2 \cup A_3) - P[A_1 \cap (A_2 \cup A_3)]$$

$$= P(A_1) + P(A_2 \cup A_3) - P[\underbrace{(A_1 \cap A_2)}_{A_1} \cup \underbrace{(A_1 \cap A_3)}_{A_2}]$$

$$= P(A_1) + P(A_2 \cup A_3) - [P(A_1 \cap A_2) + P(A_1 \cap A_3) - P(A_1 \cap A_2 \cap A_3)]$$

$$= P(A_1) + P(\underline{A_2 \cup A_3}) - P(A_1 \cap A_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_2 \cap A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3).$$

$$= P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_2 \cap A_3) - P(A_1 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

It is true for $\underline{n=3}$ events.

Let us assume that the given statement is true for $\underline{n=r}$ events.

$$P\left[\bigcup_{i=1}^{r} A_i\right] = \sum_{i=1}^{r} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k) \cdots (-1)^r \cdot P(A_1 \cap A_2 \cap A_3 \cdots A_r)$$

For $\underline{n = r+1}$ events

$$P\left(\bigcup_{i=1}^{r+1} A_i\right) = P\left[\underbrace{\bigcup_{i=1}^{r} A_i}_{A_1} \cup \underbrace{A_{r+1}}_{A_2}\right].$$

$$= P\left[\bigcup_{i=1}^{r} A_i\right] + P(A_{r+1}) - P\left[\bigcup_{i=1}^{r} A_i \cap A_{r+1}\right].$$

$$= \sum_{i=1}^{r} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k) \cdots (-1)^r \cdot P(A_1 \cap A_2 \cap \cdots A_r) + P(A_{r+1}) - P\left(\bigcup_{i=1}^{r} A_i \cap A_{r+1}\right)$$

$$= \sum_{i=1}^{r+1} P(A_i) - \sum\sum_{i<j} P(A_i \cap A_j) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k) \cdots$$

$$(-1)^{r-1} \cdot P(A_1 \cap A_2 \cap A_3 \cdots A_r) - \left[\sum_{i=1}^{r} P(A_i \cap A_{r+1}) - \right.$$

$$\sum\sum_{i<j} P(A_i \cap A_j \cap A_{r+1}) + \sum\sum\sum_{i<j<k} P(A_i \cap A_j \cap A_k \cap A_{r+1}) \cdots$$

$$\left. (-1)^{r-1} \cdot P(A_1 \cap A_2 \cap A_3 \cap \cdots A_r \cap A_{r+1}) \right]$$

It is true for $n = r+1$ events

It is also true for $n = r$ events. According to the Principle of Mathematical Induction The given statement is true for all Positive integers.

Hence the Proof

Theorem

Multiplication theorem on Probability for n-events:-

Statement:- $A_1, A_2, A_3 \ldots A_n$ are n events defined on a sample space then

$$P[A_1 \cap A_2 \cap A_3 \cap \cdots A_n] = P(A_1) \cdot P(A_2 / A_1) \cdot P(A_3 / A_1 \cap A_2) \cdots$$

$$\cdots P(A_n / A_1 \cap A_2 \cap A_3 \cdots A_n).$$

Proof:- This can be proved it by mathematical induction.

For $n=2$ events

according to Conditional Probability

$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2/A_1)$

It is true for $n=2$ events.

$$\boxed{\begin{array}{c} P(A_2/A_1) = \dfrac{P(A_1 \cap A_2)}{P(A_1)} \\ P(A_1) \cdot P(A_2/A_1) = P(A_1 \cap A_2) \end{array}}$$

For $n=3$ events

$$P(A_1 \cap A_2 \cap A_3) = P(A_1 \cap A_2) \cdot P(A_3/A_1 \cap A_2)$$

$$= P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2)$$

It is true for $n=3$ events.

let us take the given statement is true for $n=r$ events

$$P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_r) = P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2) \cdots$$

$$\cdots P(A_r/A_1 \cap A_2 \cap A_3 \cap \dots \cap A_{r-1})$$

For $n=r+1$ events

$$P(A_1 \cap A_2 \cap A_3 \dots \cap A_{r+1}) = P(\underbrace{A_1 \cap A_2 \cap A_3 \dots \cap A_r}_{A_1} \cap \underbrace{A_{r+1}}_{A_2})$$

$$= P(A_1 \cap A_2 \cap A_3 \cap \dots \cap A_r) \cdot P[A_{r+1}/A_1 \cap A_2 \cap A_3 \dots A_r]$$

$$\boxed{\begin{array}{c} \text{Rough} \\ P(A_1 \cap A_2) = P(A_1) \cdot P(A_2/A_1) \end{array}}$$

$$P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2) \dots P[A_r/A_1 \cap A_2 \cap \dots \cap A_{r-1}] \cdot P[A_{r+1}/A_1 \cap A_2 \cap \dots A_r]$$

$$= P(A_1) \cdot P(A_2/A_1) \cdot P(A_3/A_1 \cap A_2) \cdots P(A_r/A_1 \cap A_2 \cap \cdots A_{r-1}) \cdot$$

$$P(A_{r+1}/A_1 \cap A_2 \cap \cdots \cap A_r)$$

It is true for $\underline{n = r+1}$ events.

It is also true for $\underline{n = r}$ events.

According to Mathematical Induction the given statement is true for all positive Integers.

Hence the proof.

UNIT-III

RANDOM VARIABLES

$$\text{Random variable} \begin{cases} \text{Discreate Random Variable} \rightarrow \text{Prob mass function} \\ \text{Contineous Random Variable} \rightarrow \text{Prob Density function} \end{cases}$$

Random Variable :- A real valued function defined on a sample space is Called Random variable. It can takes The Values on real line ($-\infty$ to $+\infty$)

(or)

Random Variable means a real number x can associated with out comes of random experiment. It can takes any one of the Several Possible out comes with definate Probability.

Ex:- (1) In tossing a coin getting a head is denoted with random variable x, it can takes the values $x = \{0, 1\}$.

is sample space $S = \{H, T\}$.

random variable x is getting Head $x = \{0, 1\}$.

Ex:- In tossing two coins getting a Head is denoted with random variable x.

It Can takes the Values $X=\{0,1,2\}$

ie Sample Space $S=\{HH, TT, TH, HT\}$.

random variable X is getting Head $X=\{0,1,2\}$.

Discreate random variable :- A random Variable X Can assumes atmost Countable number of Values (or) finate number of Values. then it is Called discreate random variable.

(or)

A real Valued function defined on a discreate sample Space is Called discreate random Variable.

Contineous Random Variable :- A random Variable X Can assumes infinate (or) uncountable number of Values then it is Called Contineous random Variable.

(or)

A real Valued function defined on a Contineous Sample Space is Called Contineous random Variable.

Probability mass function (P.m.f) :-

Suppose X is discreate random variable taking atmost Countable number of values ie $x_1, x_2, x_3 \dots x_n$ of some probability $P(x_1), P(x_2), P(x_3) \dots P(x_n)$ are must satisfies the following conditions.

(i) $P(x_i) \geq 0$

(ii) $\sum_{i=1}^{n} P(x_i) = 1$

The function $P(x_i)$ is called probability mass function of the random variable X.

x_i	x_1	x_2	x_3	....	x_n
$P(x_i)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	....	$P(x_n)$

Mean $\mu_1' = \sum_{i=1}^{n} x_i P(x_i)$

$\mu_2' = \sum_{i=1}^{n} x_i^2 P(x_i)$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

Variance. $\mu_2 = \sum_{i=1}^{n} x_i^2 P(x_i) - \left[\sum_{i=1}^{n} x_i P(x_i)\right]^2$

Probability density function (P.d.f):-

consider the small interval $[x - \frac{1}{2}dx, x + \frac{1}{2}dx]$ of length of dx round the point x. f(x) is any continuous function of x such that f(x) dx represented the probability density function of random variable X. a must satisfies the following conditions.

① $f(x) \geq 0$

② $\int_{-\infty}^{\infty} f(x) dx = 1$

Mean $\mu_1' = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.

$$\mu_2' = \int_{-\infty}^{\infty} x^2 f(x)\,dx.$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} x \cdot f(x)\,dx\right]^2$$

Problems.

1. A random variable x has the following Probability function.

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	K	$2K$	$2K$	$3K$	K^2	$2K^2$	$7K^2+K$

(i) Find the value of K. (ii) $P[x<6]$ (iii) $P[x \geq 6]$.
(iv) $P[0<x<5]$.

Sol:- We know that $\sum P(x_i) = 1$

$$0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$$
$$10K^2+9K=1$$
$$10K^2+9K-1=0$$
$$10K^2+10K-K-1=0$$
$$10K[K+1]-1[K+1]=0$$

$(K+1)=0$ | $10K-1=0$

$\cancel{\boxed{K=-1}}$ | $10K=1$

$K = 1/10$.

$\therefore K = 1/10$.

x:	0	1	2	3	4	5	6	7
$P(x)$:	0	0.1	0.2	0.2	0.3	0.01	0.02	0.17

(ii) $P[x<6] = P[x=0] + P[x=1] + P[x=2] + P[x=3] + P[x=4] + P[x=5]$

$= 0 + 0.1 + 0.2 + 0.2 + 0.3 + 0.01$

$= 0.81$

(iii) $P[x \geq 6] = P[x=6] + P[x=7]$.

$= 0.02 + 0.17$.

$= 0.19$.

$P[x \geq 6] = 1 - P[x<6]$

$= 1 - 0.81$

$= 0.19$.

$$\boxed{\begin{array}{l} P[x \geq a] + P[x<a] = 1 \\ P[x \geq a] = 1 - P[x<a] \end{array}}$$

(iv) $P[0<x<5] = P[x=1] + P[x=2] + P[x=3] + P[x=4]$.

$= 0.1 + 0.2 + 0.2 + 0.3$

$= \underline{0.8}$.

Problem (2) In Throwing a ~~f~~ dice find Probability distribution and also find
(i) $P[x \geq 5]$ (ii) $P[x<5]$ (iii) $P[x \geq 2]$ (iv) $P[0 \leq x \leq 4]$.

Sol:- A dice was thrown Prob distribution becomes.

x:	1	2	3	4	5	6
$P(x)$:	1/6	1/6	1/6	1/6	1/6	1/6

(i) $P[x \geq 5] = P[x=5] + P[x=6]$.

$= \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}$.

(ii) $P[x<5] = 1 - P[x \geq 5]$.

$= 1 - \frac{1}{3} = \frac{2}{3}$.

(iii) $P[x \geq 2] = P[x=2] + P[x=3] + P[x=4] + P[x=5] + P[x=6]$.

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$.

$= 5/6$

(iv) $P[0 \leq x \leq 4] = P[x=1] + P[x=2] + P[x=3] + P[x=4]$.

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$

$= \frac{4}{6} = 2/3$.

Problem

(3) Two dice are thrown find the Prob. distribution of Sum of the faces and also find (i) $P[x \geq 4]$ (ii) $P[x \leq 6]$ (iii) $P[1 \leq x < 6]$. and also find mean and Variance.

Sol:- Probability distribution of Sum of two dices is

Sum	Favourable Cases	Prob.
2	(1,1)	1/36
3	(1,2) (2,1)	2/36
4	(3,1) (1,3) (2,2)	3/36
5	(2,3) (3,2) (4,1) (1,4)	4/36
6	(3,3) (5,1) (1,5) (4,2) (2,4)	5/36
7	(1,6) (6,1) (5,2) (2,5) (4,3) (3,4)	6/36
8	(6,2) (2,6) (4,4) (5,3) (3,5)	5/36
9	(5,4) (4,5) (6,3) (3,6)	4/36
10	(5,5) (6,4) (4,6)	3/36
11	(6,5) (5,6)	2/36
12	(6,6)	1/36

Prob distribution :

x:	2	3	4	5	6	7	8	9	10	11	12
$P(x)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

(i) $P[x \geq 4] = P(x=4] + P(x=5] + P[x=6] + P[x=7] + P[x=8] + P[x=9] + P[x=10]$
$+ P[x=11] + P[x=12]$.

$= 3/36 + 4/36 + \frac{5}{36} + \frac{6}{36} + \frac{5}{36} + \frac{4}{36} + \frac{3}{36} + \frac{2}{36}$.

$= \frac{33}{36}$.

(or)

$P[x \geq 4] = 1 - P[x < 4]$.

$= 1 - \{P[x=2] + P(x=3]\}$

$= 1 - \{2/36 + \frac{3}{36}\}$.

$= 1 - \frac{5}{36}$

$= \frac{36-5}{36}$

$P[x \geq 4] = \frac{31}{36}$

(ii) $P[x \leq 6] = P[x=2] + P[x=3] + P[x=4] + P[x=5] + P[x=6]$.

$= \frac{1}{36} + \frac{2}{36} + \frac{3}{36} + \frac{4}{36} + \frac{5}{36}$.

$= \frac{15}{36}$

(iii) $P[1 \leq x < 6] = P[x=2] + P[x=3] + P[x=4] + P[x=5]$.

$= 1/36 + 2/36 + 3/36 + 4/36$

$= \frac{10}{36}$.

(iv) Mean $\mu_1' = \sum x \cdot P(x)$

Mean$(\mu_1') = (2)(1/36) + (3)(2/36) + (4)(3/36) + (5)(4/36) + (6)(5/36) + (7)(6/36) +$
$(8)(5/36) + (9)(4/36) + (10)(3/36) + (11)(2/36) + (12)(1/36)$

$= \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36}$

Mean $\mu_1' = \frac{252}{36} = 7$.

$\boxed{\therefore \text{mean } \mu_1' = 7}$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$\mu_2' = \sum x^2 P(x)$

$= (2)^2(1/36) + (3)^2(2/36) + (4)^2(3/36) + (5)^2(4/36) + (6)^2(5/36) + (7)^2(6/36) + (8)^2(5/36) + (9)^2(4/36) + (10)^2(3/36) + (11)^2(2/36) + (12)^2(1/36)$

$= \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36}$

$\mu_2' = \frac{1974}{36}$.

$\boxed{\mu_2' = 54.8333}$

Variance $= \mu_2' - (\mu_1')^2$

$= 54.8333 - (7)^2$

$= 54.8333 - 49$.

$\boxed{\text{Variance } \mu_2 = 5.8333}$

Problem (3) If the random variable x has the following Prob mass function.

$P[X=r] = P(r) = \frac{1}{2^r}$, $r = 1, 2, 3, 4, \ldots$

find (i) $P[r \text{ is even}]$ (ii) $P[r \geq 5]$.

Sol:- Given Prob mass function $P[X=r] = P(r) = \frac{1}{2^r}$

$r = 1, 2, 3, 4 \ldots$

$P(r) = \frac{1}{2^1}, \frac{1}{2^2}, \frac{1}{2^3}, \frac{1}{2^4}, \ldots\ldots$

(i) $P[r \text{ is even}] = P[r=2] + P[r=4] + P[r=6] + \ldots\ldots$

$= \frac{1}{2^2} + \frac{1}{2^4} + \frac{1}{2^4} + \frac{1}{2^6} + \ldots$

Sum of Infinate Series

"a" is first term

r is common ratio.

$S_\infty = \frac{a}{1-r}$

Here $\boxed{a = 1/2}$, $r = \frac{r_2}{r_1} = \frac{1/2^4}{1/2^2} = \frac{1}{2^4} \times \frac{2^2}{1}$

$\boxed{r = \frac{1}{2^2} = 1/4}$

$S_\infty = \frac{a}{1-r} = \frac{1/2}{1 - 1/4} = \frac{1/2}{3/4} = \frac{1}{\cancel{2}} \times \frac{\cancel{4}^2}{3}$

$= 2/3$

(ii) $P[r \geq 5] = 1 - P[r < 5]$

$= 1 - \{P(r=1) + P(r=2) + P(r=3) + P(r=4)\}$

$= 1 - \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + 1/16$

$= 1 - 0.9375$

$= 0.0625$

$\boxed{P[X \geq 5] = 0.0625}$

Problem ④ Probability mass function of random variable x is

$$P[X=x] = P(x) = \frac{x}{15}, \quad x = 1,2,3,4 \text{ and } 5.$$

find (i) $P[x=1 \text{ (or) } x=2]$ (ii) $P\left[\frac{1}{2} < x < \frac{5}{2} \,\middle|\, x > 1\right]$.

(iii) $P\left[\frac{1}{2} < x < \frac{5}{2}\right]$.

Sol:- Given Prob mass function of random variable x is

$$P[X=x] = P(x) = \frac{x}{15}, \quad x = 1,2,3,4 \text{ and } 5.$$

x :	1	2	3	4	5
P(x) :	1/15	2/15	3/15	4/15	5/15

(i) $P[x=1 \text{ (or) } x=2] = P[x=1] + P[x=2]$.

$$= \frac{1}{15} + \frac{2}{15}$$

$$= 3/15 = 1/5$$

(ii) $P\left[\frac{1}{2} < x < 5/2 \,\middle|\, x > 1\right] = P\left[0.5 < x < 2.5 \,\middle|\, x > 1\right]$.

x lies between 0.5 and 2.5 but it is greater than 1.

$\therefore P[x=2] = 2/15$

(iii) $P\left[1/2 < x < 5/2\right] = P[0.5 < x < 2.5]$.

$$= P[x=1] + P[x=2].$$

$$= 1/15 + 2/15 = 3/15$$

$$= 1/5$$

Problem :- In rolling a dice the number of faces is denoted with random variable x. then write the probability distribution. also find mean and variance.

Sol:- A dice is rolled prob distribution becomes.

$$x : 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6.$$
$$P(x) : \tfrac{1}{6} \quad \tfrac{1}{6} \quad \tfrac{1}{6} \quad \tfrac{1}{6} \quad \tfrac{1}{6} \quad \tfrac{1}{6}.$$

Mean $\mu_1' = \sum x_i P(x_i)$

$$= (1)(\tfrac{1}{6}) + (2)(\tfrac{1}{6}) + (3)(\tfrac{1}{6}) + (4)(\tfrac{1}{6}) + (5)(\tfrac{1}{6}) + (6)(\tfrac{1}{6})$$

$$= \frac{21}{6}.$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$\mu_2' = \sum x_i^2 P(x_i)$$
$$= (1)^2(\tfrac{1}{6}) + (2)^2(\tfrac{1}{6}) + (3)^2(\tfrac{1}{6}) + (4)^2(\tfrac{1}{6}) + (5)^2(\tfrac{1}{6}) + (6)^2(\tfrac{1}{6})$$
$$= \frac{1}{6} + \frac{4}{6} + \frac{9}{6} + \frac{16}{6} + \frac{25}{6} + \frac{36}{6}$$

$$\mu_2' = \frac{91}{6}.$$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$= \frac{91}{6} - \left(\frac{21}{6}\right)^2$$

$$= \frac{91}{6} - \frac{441}{36}.$$

Variance $\mu_2 = \dfrac{546-441}{36} = \dfrac{105}{36}.$

Note:— If x is a Contineous random variable. f(x) is a Prob density function lies between a to b. then.

① r^{th} central moment $\mu_r = \int_a^b (x-\bar{x})^r \cdot f(x)\,dx$.

② r^{th} raw moment about the Point-A is

$$\mu_r^1 = \int_a^b (x-A)^r \cdot f(x)\,dx.$$

③ r^{th} raw moment about origin,

$$\mu_r^1 = \int_a^b (x-0)^r \cdot f(x)\,dx.$$

$$\mu_r^1 = \int_a^b x^r \cdot f(x)\,dx \quad \text{—— ①}$$

Put $r=1$ in eqn① $\mu_1^1 = \int_a^b x^1 \cdot f(x)\,dx = \text{Mean}.$

Put $r=2$ in eqn① $\mu_2^1 = \int_a^b x^2 \cdot f(x)\,dx.$

~~Put~~ Variance $\mu_2 = \mu_2^1 - (\mu_1^1)^2$

$$\mu_2 = \int_a^b x^2 f(x)\,dx - \left[\int_c^b x \cdot f(x)\,dx\right]^2.$$

Put $r=3$ in eqn① $\mu_3^1 = \int_a^b x^3 \cdot f(x)\,dx.$

Put $r=4$ in eqn① $\mu_4^1 = \int_a^b x^4 \cdot f(x)\,dx.$

Problems:

(1) If x is a Contineous random Variable With prob density function $f(x) = Ax^2, 0 \le x \le 1$ ~~obtain~~ Determine the Value of A and find mean and Variance.

Sol:- Given Prob density function $f(x) = Ax^2$, $0 \le x \le 1$

We Know that $\int_a^b f(x)\,dx = 1$

$$\int_0^1 Ax^2\,dx = 1$$

$$A\int_0^1 x^2\,dx = 1$$

$$A\left[\frac{x^3}{3}\right]_0^1 = 1$$

$$\frac{A}{3}[1-0] = 1$$

$$\boxed{A = 3}$$

$\therefore$ Prob density function $f(x) = 3x^2, \quad 0 \le x \le 1$

Mean $\mu_1' = \int_0^1 x \cdot f(x)\,dx$

$$= \int_0^1 x \cdot (3x^2)\,dx$$

$$= 3\int_0^1 x^3\,dx$$

$$= 3\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{3}{4}\left[x^4\right]_0^1$$

Mean $\mu_1' = \frac{3}{4}[1-0] = 3/4$

$\therefore$ Mean $\mu_1' = 3/4$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$\mu_2' = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx$$

$$= \int_0^1 x^2 \cdot (3x^2)\,dx$$

$$= 3\int_0^1 x^4 \cdot dx$$

$$= 3\left[\frac{x^5}{5}\right]_0^1$$

$$\mu_2' = \frac{3}{5}[1-0] = 3/5$$

$\therefore$ Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$= \frac{3}{5} - (3/4)^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$\mu_2 = \frac{48-45}{80} = 0.375$$

$$\boxed{\text{Variance } \mu_2 = 0.375}$$

Problem (2) The diameter of the electric cable say x is assumed to be a contineous random variable with prob density function $f(x) = 6x(1-x)$, $0 \le x \le 1$, . Check that cable is prob density function also calculate mean and variance.

Sol- Given prob density function $f(x) = 6x(1-x)$ $0 \le x \le 1$

To check given function is Prob density function or not we have to prove total prob = 1

$$\int_0^1 f(x)dx = \int_0^1 6x(1-x)\,dx$$

$$= 6\int_0^1 (x-x^2)\,dx$$

$$= 6\int_0^1 x\cdot dx - \int_0^1 x^2\cdot dx$$

$$= 6\left[\left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1\right]$$

$$= \frac{6}{2}[1-0] - \frac{6}{3}[1-0]$$

$$= \frac{\cancel{6}^3}{\cancel{2}_1} - \frac{\cancel{6}^2}{\cancel{3}_1}$$

$$= 3-2$$

$$\int_0^1 f(x)dx = 1$$

$\therefore$ Given function is Prob density function.

$\therefore$ mean $\mu_1' = \int_0^1 x\cdot f(x)dx$

$$\mu_1' = \int_0^1 x\cdot 6x(1-x)\,dx$$

$$= 6\int_0^1 x^2(1-x)\,dx$$

$$= 6\left[\int_0^1 x^2\cdot dx - \int_0^1 x^3\cdot dx\right]$$

$$= 6\left[\left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1\right]$$

$$= 6\left[\left(\frac{1}{3}-0\right) - \left(\frac{1}{4}-0\right)\right]$$

$$= 6\left[\frac{4-3}{12}\right] = \frac{1}{2}$$

$\therefore$ Mean $\mu_1' = \frac{1}{2}$

Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$\mu_2' = \int_0^1 x^2 \cdot f(x)\,dx$$

$$= \int_0^1 x^2\, 6x(1-x)\,dx$$

$$= 6\left[\int_0^1 x^3(1-x)\,dx\right]$$

$$= 6\left[\int_0^1 (x^3 - x^4)\,dx\right]$$

$$= 6\left[\int_0^1 x^3\,dx - \int_0^1 x^4\,dx\right]$$

$$= 6\left[\left[\frac{x^4}{4}\right]_0^1 - \left[\frac{x^5}{5}\right]_0^1\right]$$

$$= 6\left[\left(\frac{1}{4} - 0\right) - \left(\frac{1}{5} - 0\right)\right]$$

$$= 6\left[\frac{1}{4} - \frac{1}{5}\right]$$

$$= 6\left[\frac{5-4}{20}\right]$$

$$\mu_2' = \frac{6}{20}$$

$\therefore$ Variance $\mu_2 = \mu_2' - (\mu_1')^2$

$$= \frac{6}{20} - \left(\frac{1}{2}\right)^2$$

$$= \frac{6}{20} - \frac{1}{4}$$

$4 \overline{)4,20}$
$1,5$

Variance $= \dfrac{6-5}{20} = \dfrac{1}{20} = 0.05$

Problem (3) If X is continuous random variable with prob density function

$$f(x) = \begin{cases} ax & 0 \le x \le 1 \\ a & 1 \le x \le 2 \\ -ax+3a & 2 \le x \le 3 \\ 0 & \text{otherwise.} \end{cases}$$

(i) Find the constant value "a"

(ii) $P[X \le 1.5]$.

Sol:- (i) To find constant "a"

We know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{3} f(x)dx + \int_{3}^{\infty} f(x)dx = 1$$

$$0 + \int_0^1 ax\cdot dx + \int_1^2 a\cdot dx + \int_2^3 (-ax+3a)\,dx + 0 = 1$$

$$= a\left[\frac{x^2}{2}\right]_0^1 + a\left[x\right]_1^2 \cancel{a} - a\left[\frac{x^2}{2}\right]_2^3 + 3a\left[x\right]_2^3 = 1$$

$$= \frac{a}{2}[1-0] + a[2-1] - \frac{a}{2}[9-4] + 3a[3-2] = 1$$

$$= \frac{a}{2} + a - \frac{5a}{2} + 3a = 1$$

$$= \cancel{\frac{a+2a-5a+6a}{2} = 1} \qquad \frac{a+2a-5a+6a}{2} = 1$$

$$= \cancel{\frac{2a}{4a}} \qquad = \frac{4a}{2} = 1$$

$$2a = 1$$

$$\boxed{a = 1/2}$$

(ii) $P[X \le 1.5]$.

$$= \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{1.5} f(x)\,dx.$$

$$0 + \int_{0}^{1} ax\cdot dx + \int_{1}^{1.5} a\cdot dx.$$

$$= 0 + a\left[\frac{x^2}{2}\right]_0^1 + a\left[x\right]_1^{1.5}.$$

$$= \frac{a}{2}[1-0] + a[1.5-1].$$

$$= \frac{a}{2} + 0.5a$$

$$P[X \le 1]. = \frac{a}{2} + \frac{a}{2} = \frac{2a}{2}$$

$$= a.$$

$$P[x \le 1.5]. = 1/2$$

Problem.

→ If x is a Contineous r.v with Prob density function $f(x) = 6x^2 c(1-x)$, $0 \le x \le 1$: find Constant "c". also find Mean and Variance.

Ans

$c = 2$

Mean $\mu_1' = 3/5$

$\mu_2' = 2/5$.

Variance $\mu_2 = 1/25$