Write 25 lines on each page

MATHEMATICAL EXPECTATION (PROBLEMS)

(1) The Probability distribution of a random variable X is given below. Find (i) $E(X)$ (ii) $var(X)$ (iii) $E(2X-3)$ (iv) $Var(2X-3)$

x :	-2	-1	0	1	2
$P[X=x]$:	0.2	0.1	0.3	0.3	0.1

Sol:-

$$E(X) = \sum x\, P(x)$$
$$= (-2)(0.2) + (-1)(0.1) + (0)(0.3) + (1)(0.3) + (2)(0.1)$$
$$= -0.4 - 0.1 + 0 + 0.3 + 0.2$$
$$E(x) = 0$$

(ii) $Var(x) = E(x^2) - [E(x)]^2$

$$E(x^2) = \sum x^2 P(x)$$
$$= (-2)^2(0.2) + (-1)^2(0.1) + (0)^2(0.3) + (1)^2(0.3) + (2)^2(0.1)$$
$$= 4(0.2) + 0.1 + 0 + 0.3 + 0.4$$
$$= 0.8 + 0.1 + 0 + 0.3 + 0.4$$
$$E(x^2) = 1.6$$

$\boxed{E(x)=0}$

$$Var(X) = E(x^2) - [E(x)]^2$$
$$= 1.6 - 0$$
$$Var(x) = 1.6$$

$\boxed{E(c)=c}$

$$E(2x-3) = 2E(x) - E(3)$$
$$= 2E(x) - 3$$
$$= 2(0) - 3$$

$\boxed{\because E(x)=0}$

$$E(2x-3) = -3$$

$$V(2x-3) = V(2x) - V(3)$$

$\boxed{\begin{array}{l} V(c)=0 \\ V(ax)=a^2V(x) \end{array}}$

$V(5x) = 5^2V(x)$

$$= 2^2 V(x) - 0$$
$$= 4V(x)$$
$$= 4(1.6)$$

$\boxed{V(x)=1.6}$

Problem (2)

Mean and Standard Deviation of a random Variable X are 5 and 4 respectively. Find $E(x^2)$ and Standard Deviation of $(5-3x)$

Sol:- Given $E(X) = \mu = 5$ (Mean)

$S.D = \sigma = 4$

Variance $\sigma^2 = (4)^2$

$Var = 16$

$\boxed{\begin{array}{l} Variance = \sigma^2 \\ S.D = \sqrt{Var} \\ = \sqrt{\sigma^2} \\ S.D = \sigma \end{array}}$

$Var(X) = E(x^2) - [E(X)]^2$

$16 = E(x^2) - (5)^2$

Given $\boxed{\because E(X) = 5}$

$E(x^2) = 16 + 25$

$\boxed{E(x^2) = 41}$

$\boxed{\begin{array}{l} Var(ax) = a^2 V(X) \\ Var(-3x) = (-3)^2 (X) \\ = 9x \end{array}}$

$Var(c) = 0$

$Var(5-3x) = Var(5) + Var(-3x)$

$= 0 + (-3)^2 \cdot Var(X)$

$= 9 Var(X)$

$= 9(16)$

$= 144$

Given $\boxed{Var(X) = 16}$

$S.D(5-3x) = \sqrt{Var(5-3x)}$

$= \sqrt{144}$

$S.D(5-3x) = 12$

$\boxed{S.D\ \sigma = \sqrt{Var}}$

Problem :- A Machine produces an average of 500 items during the first week of the month. an average of 400 items during the last week of the month. The Pros for there being 0.68 and 0.32 respectively. Determine the Expected value of the production.

Sol:- Let x be the random variable which denotes the items produced by the machines.

The prob distribution becomes:

x:	500	400
$P(X=x)$:	0.68	0.32

Sol:- Expected value of the production $E(x) = \sum x \cdot P(x)$

$$E(x) = (500)(0.68) + (400)(0.32)$$
$$= 468$$

Problem (9) The monthly demand for TITAN watches is known to have the following prob distribution.

Demand (X):	1	2	3	4	5	6	7	8
Prob:	0.08	0.12	0.19	0.24	0.16	0.10	0.07	0.04

Find the expected demand for watches. also compute variance and mean.

Sol- Mean $E(X) = \sum x\, P(x)$

$$= (1)(0.08) + (2)(0.12) + (3)(0.19) + (4)(0.24) + (5)(0.16) + (6)(0.10) + (7)(0.07) + (8)(0.04)$$

$$= 4.06$$

$$\text{Variance}(X) = E(X^2) - [E(X)]^2$$

$$E(X^2) = \sum x^2 P(X)$$
$$= (1)^2(0.08) + (2)^2(0.12) + (3)^2(0.19) + (4)^2(0.24) + (5)^2(0.16) + (6)^2(0.10) + (7)^2(0.07) + (8)^2(0.04)$$

$$= 19.7$$

$$\text{Var}(X) = E(x^2) - [E(X)]^2$$
$$= 19.7 - (4.06)^2 \Rightarrow 19.7 - 16.48$$
$$\text{Var}(X) = 3.21$$

Problem

→ A discreate r.v X has the following prob mass function

x	-2	-1	0	1	2	3
$P(x=x)$	0.2	k	0.1	$2k$	0.1	$2k$

Find K, mean and variance.

Sol:- We know that $\sum P(x) = 1$

$$= 0.2 + k + 0.1 + 2k + 0.1 + 2k = 1$$

$$5k + 0.4 = 1$$

$$5k = 0.6$$

$$k = \frac{0.6}{5}$$

$$k = \frac{6}{(10)(5)} = \frac{\cancel{6}\,3}{\cancel{50}\,25} = 3/25$$

Hence the prob distribution is

x:	-2	-1	0	1	2	3
$P(x=x)$:	$\frac{2}{10}$	$\frac{3}{25}$	$\frac{1}{10}$	$\frac{6}{25}$	$\frac{1}{10}$	$\frac{6}{25}$

Rough: $\boxed{0.1 = \frac{1}{10}}$

mean $E(X) = \sum x\,P(x)$

$$= (-2)(2/10) + (-1)\left(\frac{3}{25}\right) + (0)\left(\frac{1}{10}\right) + (1)\left(\frac{6}{25}\right) + (2)(1/10) + (3)(6/25)$$

$$= \frac{6}{25}$$

Variance $= E(x^2) - [E(x)]^2$

$$E(x^2) = (-2)^2\left(\frac{2}{10}\right) + (-1)^2\left(\frac{3}{25}\right) + (0)^2(1/10) + (1)^2\left(\frac{6}{25}\right) + (2)^2(1/10) + (3)^2(6/25)$$

$$= \frac{73}{250}$$

$$Var(X) = E(x^2) - E(x)^2$$

$$= \frac{73}{250} - \left(\frac{6}{25}\right)^2 \Rightarrow \frac{293}{625}$$

Problem

(1) ~~Then~~ A fair dice is tossed. let the r.v X denote the twice the number appearing on the dice. Write the prob distribution of X Calculate mean and variance.

Sol:- let X be the random variable which denotes twice the number appearing on the dice.

(i) Prob distribution of X.

x	2	4	6	8	10	12
$P(x)$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$	$1/6$

(ii) mean $\mu = \sum x\,P(x)$

$$= (2)(1/6) + (4)(1/6) + (6)(1/6) + (8)(1/6) + (10)(1/6) + (12)(1/6)$$

$$= 7.$$

Variance $\sigma^2 = \sum x^2 P(x) - \left[\sum x\,P(x)\right]^2$

$$\boxed{\begin{array}{c} Var(X) \\ E(X^2) - [E(X)]^2 \end{array}}$$

$$\sum x^2 P(x) = (2)^2(1/6) + (4)^2(1/6) + (6)^2(1/6) + (8)^2(1/6) + (10)^2(1/6) + (12)^2(1/6)$$

$$= \frac{4}{6} + \frac{16}{6} + \frac{36}{6} + \frac{64}{6} + \frac{100}{6} + \frac{144}{6}$$

$$= 0.6666 + 2.6666 + 6 + 10.6666 + 16.6666 + 24$$

$$= \cancel{36.6664}\ 60.6664$$

$\therefore$ Variance $(X) = \sum x^2 P(x) - \left[\sum x\,P(x)\right]^2$

$$= \cancel{36.6664}\ 60.6664 - (7)^2$$

$$= 60.6664 - 49$$

$$= 11.67$$

Problem

A sample of 3 items is selected at random from a box containing 10 items of which 4 are defective. Find the expected number of defectives items.

Sol. Let x be the r.v which denotes the defective items.

Total number of items = 10.
Number of good items = 6
Number of defective items = 4

$P(x=0) = P(\text{no. defective items}) = \frac{6_{C_3}}{10_{C_3}} = 1/6$. $\boxed{n_{C_r} = \frac{n!}{(n-r)!r!}}$

$P(x=1) = P(\text{one defective item}) = \frac{6_{C_2} \cdot 4_{C_1}}{10_{C_3}} = 1/2$

$P(x=2) = P(\text{two defective items}) = \frac{6_{C_1} \cdot 4_{C_2}}{10_{C_3}} = \frac{3}{10}$.

$P(x=3) = P(\text{Three defective items}) = \frac{4_{C_3}}{10_{C_3}} = \frac{1}{30}$

Hence the prob distribution

X	0	1	2	3
$P(x=x)$	$1/6$	$1/2$	$3/10$	$1/30$

Expected number of defectives $E(X) = \sum X \cdot P(X)$

$= (0)(1/6) + (1)(1/2) + (2)(\frac{3}{10}) + (3)(1/30)$.

$E(X) = 1.2$.

Variance $= E(x^2) - [E(x)]^2$

$E(X^2) = \sum x^2 P(X)$

Problem

A player tosses two fair coins. He wins ₹100 if a Head appears and ₹200 if two heads appear. On the other hand He losses ₹500 if no head appears. Determine the expected value of the game. Is the game favourable to the Players?.

Sol:- Let X be the random variable which denotes the number of heads appearing in tosses of two fair coins

$$S = \{HH, HT, TH, TT\}.$$

$P(x_1) = P(X=0) = P(\text{no. heads}) = 1/4$

$P(x_2) = P(X=1) = P(\text{one head}) = 2/4 = 1/2$

$P(x_3) = P(X=2) = P(\text{two heads}) = 1/4$

Amount to be lost if no head appears $= x_1 = -500$ Rs.

Amount to be win if one head is appear $x_2 = 100$ ₹

Amount to be won if two heads appear $x_3 = ₹\,200$

x:	-500	100	200
$P(x)$:	$1/4$	$1/2$	$1/4$

Expected value of the game $E(X) = \sum x\,P(x)$

$$(-500)(1/4) + (100)(1/2) + (200)(1/4)$$

$$= ₹\ -25/-$$

Hence The game is not favourable to the Player.

Problem

Amit plays a game of tossing a dice. If a number less than 3 appears. He gets ₹ a. Otherwise he has to pay ₹ 10. If the game is fair. Find a

Sol: Let x be the random Variable which denotes tossing a dice.

Prob of getting a number less than 3 is 1 (or) 2

$$P(x_1) = \frac{2}{6} = 1/3$$

Prob of getting a number more than (or) equal 3 is 3, 4, 5 and 6

$$P(x_2) = \frac{4}{6} = 2/3$$

Amount to be received for number less than 3 = x_1 = ₹ a.

Amount to be paid for numbers more than (or) equal to 3 = x_2 = ₹ −10.

$$x: \quad a \qquad -10$$

$$P(x) = \quad 1/3 \qquad 2/3$$

$$E(x) = \Sigma x\, P(x)$$

$$= (a)(1/3) + (-10)(2/3)$$

$$= \frac{a}{3} - \frac{20}{3}$$

for a fair game $E(x) = 0$

$$\frac{a}{3} - \frac{20}{3} = 0$$

$$\frac{a}{\cancel{3}} = \frac{20}{\cancel{3}}$$

$$\boxed{a = 20}$$

ANU

Problem :- The Prob that there is an atleast one error an account Statement prepared by A is 0.2 and for B and C they are 0.25 and 0.4 respectively. A, B and C prepared 10, 16 and 20 Statements. Find the expected number of correct statements in all.

Sol:- Let $P(x_1)$, $P(x_2)$ and $P(x_3)$ be the probabilities of the events that there is no error in the account statements prepared by A, B and C respectively.

$P(x_1) = 1 -$ Prob of atleast one error in the Statement prepared by A.

$= 1 - 0.2$

$= 0.8$

Similarly $P(x_2) = 1 - 0.25$

$= 0.75$

$P(x_3) = 1 - 0.4$

$= 0.6$

also $x_1 = 10$, $x_2 = 16$, $x_3 = 20$

x :	10	16	20
$P(x)$:	0.8	0.75	0.6

Expected number of Correct Statements

$E(X) = \sum x P(x)$

$= (10)(0.8) + (16)(0.75) + (20)(0.6)$

$= 32$

Q ⇒ Define moment Generating function? write its Properties?. (M.G.F)

Def of MGF :- If x is a random Variable $E[e^{tx}]$ is called moment Generating function of random variable x. It is denoted with $M_x(t)$ defined as

$$\boxed{M_x(t) = E[e^{tx}]}$$

Properties :- ① Prove that ~~$M_{at}(x)$~~ $M_{ax}(t) = M_x(at)$ where 'a' is a Constant.

Proof :- By The Defination $M_x(t) = E[e^{tx}]$.

L.H.S $M_{ax}(t) = E[e^{tax}]$.

$= M_x(at)$

$= R.H.S$.

② Prove that $M_{x-a}(t) = e^{-at} M_x(t)$, where 'a' is a Constant.

Proof :- L.H.S $M_{x-a}(t) = E[e^{t(x-a)}]$.

$= E[e^{tx - ta}]$.

$= E[e^{tx} \cdot e^{-ta}]$.

$= e^{-ta} \cdot E[e^{tx}]$.

$= R.H.S$.

$\boxed{E(cx) = cE(x)}$

③ Prove that $M_{\frac{x-a}{h}}(t) = e^{-at/h} \cdot M_x(t/h)$, where 'a' is a Constant

Proof!- By The Defination $M_x(t) = E[e^{tx}]$.

L.H.S $M_{\frac{x-a}{h}}(t) = E\left[e^{t\left(\frac{x-a}{h}\right)}\right]$.

$= E\left[e^{\frac{tx}{h} - \frac{ta}{h}}\right]$.

Prop. $\boxed{E[cx] = cE[x]}$

$= E\left[e^{\frac{tx}{h}} \cdot e^{-\frac{ta}{h}}\right]$

$= e^{-\frac{ta}{h}} E\left(e^{\frac{tx}{h}}\right]$.

$= e^{-\frac{ta}{h}} \cdot M_x(t/h)$

$M_{\left(\frac{x-a}{h}\right)}(t) = R.H.S$

④ Additive property:- Sum of the n independent random variables M.G.F is equals to Product of their respective M.G.F.

Proof!- let x_1, x_2, x_3 be two independent r.v's.

$M_{x_1+x_2}(t) = M_{x_1}(t) \cdot M_{x_2}(t)$

By The Defination $M_x(t) = E[e^{tx}]$.

$M_{x_1+x_2}(t) = E\left[e^{t[x_1+x_2]}\right]$.

$= E\left[e^{tx_1 + tx_2}\right]$.

$= E\left[e^{tx_1} \cdot e^{tx_2}\right]$.

$= E[e^{tx_1}] \cdot E[e^{tx_2}]$.

$= M_{x_1}(t) \cdot M_{x_2}(t)$

(5) Prove That $M_0(t) = 1$.

Proof:- By The Defination $M_x(t) = E[e^{tx}]$.

L.H.S $M_0(t) = E[e^{t(0)}]$.

$= E[e^{0}]$.

$= E(1)$

$= 1$

R.H.S.

(6) $|M_x(t)| \leq 1$

Q:⟹ Define characterstic function of random variable X and write its Properties.

Proof:- If X is a random variable $E[e^{itx}]$ is called characterstic function of The r.v X.

it is denoted with $\phi_x(t)$ defined as.

$$\boxed{\phi_x(t) = E[e^{itx}]}$$

Properties:- (1) Prove That $\phi_{ax}(t) = \phi_x(at)$

where 'a' is a Const.

Proof:- By the Def $\phi_x(t) = E[e^{itx}]$.

L.H.S $\phi_{ax}(t) = E[e^{itax}]$.

$= \phi_x(at)$

$=$ R.H.S.

(2) Prove that $\phi_{x-a}(t) = e^{-iat} \cdot \phi_x(t)$ where 'a' is Const.

Proof:- By the Defination $\phi_x(t) = E[e^{itx}]$.

L.H.S $\phi_{x-a}(t) = E[e^{it(x-a)}]$.

$= E[e^{itx - ita}]$.

$= E[e^{itx} \cdot e^{-ita}]$.

$= e^{-ita} \cdot E[e^{itx}]$.

$= e^{-ita} \cdot \phi_x(t)$

$=$ R.H.S

(3) Prove that $\phi_{\left(\frac{x-a}{h}\right)}(t) = e^{-\frac{iat}{h}} \cdot \phi_x(t/h)$, where 'a' is a Const.

Proof:- By the Defination $\phi_x(t) = E[e^{itx}]$.

$\phi_{\left(\frac{x-a}{h}\right)}(t) = E\left[e^{it\left(\frac{x-a}{h}\right)}\right]$.

$= E\left[e^{it\left[\frac{x}{h} - \frac{a}{h}\right]}\right]$.

$= E\left[e^{\frac{itx}{h}} \cdot e^{-\frac{ita}{h}}\right]$

$= e^{-\frac{ita}{h}} \cdot E\left[e^{\frac{itx}{h}}\right]$.

$= e^{-\frac{ita}{h}} \cdot \phi_x(t/h)$

$=$ R.H.S

(4) Additive property:— sum of the independent random variables characteristic function is equals to product of their respective characteristic function.

let x_1, x_2 are two independent r.v's

$$\phi_{x_1+x_2}(t) = \phi_{x_1}(t) \cdot \phi_{x_2}(t)$$

$$\text{L.H.S} \quad \phi_{x_1+x_2}(t) = E\left[e^{it(x_1+x_2)}\right].$$

$$= E\left[e^{itx_1 + itx_2}\right].$$

$$= E\left[e^{itx_1} \cdot e^{itx_2}\right].$$

$$= E\left[e^{itx_1} \cdot e^{itx_2}\right].$$

$$= E\left[e^{itx_1}\right] . E\left[e^{itx_2}\right].$$

$$= \phi_{x_1}(t) \cdot \phi_{x_2}(t)$$

$$= \text{R.H.S.}$$

(5) Prove That $\phi_0(t) = 1$

By The Defination $\phi_x(t) = E[e^{itx}]$.

$$\phi_0(t) = E[e^{it(0)}].$$

$$= E(e^0)$$

$$= E(1)$$

$$= 1$$

$$= \text{R.H.S}$$

(6) $|\phi_x(t)| \leq 1$

Cumulant Generating function (C.G.F):-

If x is a random variable and its moment Generating function is $M_x(t)$ and cumulant Generating function is denoted with $K_x(t)$ is defined as

$$\boxed{K_x(t) = \log(M_x(t))}$$

Theorem

Cauchy Schwartz Inequality:-

If two random variables x and y taking the real values then

$$[E(xy)]^2 \leq E(x^2) \cdot E(y^2)$$

Proof:- let us consider the real valued function of the real variable is

$$Z(t) = E[x + ty]^2 \text{ which is non negative.}$$

ie $(x+ty)^2 \geq 0$.

$$\because Z(t) = E[x + ty]^2$$

$$= E[x^2 + 2xty + t^2y^2]$$

$$Z(t) = E(x^2) + 2t\,E(xy) + t^2 E(y^2)$$

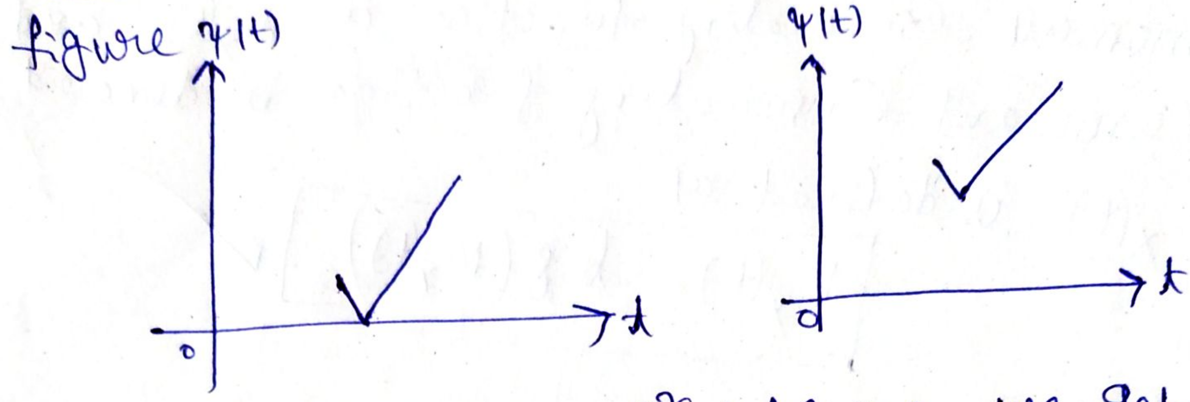
This is the form of quadratic equation.

$$Z(t) = At^2 + Bt + C$$

$$\boxed{\begin{array}{c} \text{Roots} \\ [y \to x] \\ Ax^2 + bx + c \end{array}}$$

where $A = E(y^2)$, $B = 2E(xy)$, $C = E(x^2)$

The function $\gamma(t)$ touches t-axis only one Point (or) not at all. as shown in the given figure



The Discriment $B^2-4AC > 0$ we get two distinct Points which is controduction to the above Statement.

$$\therefore B^2-4AC \leq 0$$

$$\left[2E(xy)\right]^2 - 4E(y^2)\cdot E(x^2) \leq 0$$

$$4\left[E(xy)\right]^2 - 4E(y^2)\cdot E(x^2) \leq 0$$

$$4\left[\left[E(xy)\right]^2 - E(x^2)\cdot E(y^2)\right] \leq 0$$

$$\left[E(xy)\right]^2 - E(x^2)\cdot E(y^2) \leq \frac{0}{4}$$

$$\left[E(xy)\right]^2 - E(x^2)\cdot E(y^2) \leq 0$$

$$\left[E(xy)\right]^2 \leq E(x^2)\cdot E(y^2)$$

Hence the Proof.