UNIT - III

Probability and Distributions:

* Random Variables:-

→ Random Experiment:-

Probabilitic situation is called Random Experiment

→ Trail:-

Each performance in random experiment is called Trail

→ outcome:-

The result of a trail in random experiment is called outcome

→ Sample space:-

→ The set of all possible outcomes of a random experiment is called sample space. It is denoted by 'S'

→ A Sample space whose elements are finite (or) countable, then the sample space is called 'Discrete Sample space'.

→ A sample space whose elements are infinite (or) uncountable, then the sample space is called 'Continuous sample space'.

→ Event:-

Any non empty sub set of sample space is known as 'event'. Event is denoted by 'E' probability of an event is defined as

$$P(E) = \frac{\text{No. of elementary events in } E}{\text{No. of elementary events in } S}$$

Ex:- Tossing a coin, we get either head or tail

Sample space $S = \{H, T\}$

Tossing 2 coins, then the sample space is denoted by

$S = \{HH, HT, TH, TT\}$

Throwing a die, then $S = \{1, 2, 3, 4, 5, 6\}$.

Throwing 2 dies, then

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Note:-

- For tossing a coin n times. then the sample space contain 2^n elements
- For throwing a coin n times then the sample space contain 6^n elements

→ Random Variable:-

→ A real variable 'x' whose value is determined by all possible outcomes is called a Random variable.

→ If 'x' takes finite no. of values then 'x' is called 'Discrete Random Variable'.

→ If 'x' takes infinite no. of values then 'x' is called 'Continuous Random Variable'.

→ Probability Distribution

If 'x' takes the values $x_1, x_2, x_3, \ldots x_n$ with probabilities $P(x_1), P(x_2), P(x_3) \ldots P(x_n)$ such that $p(x_i) \geq 0$

(or)

$P(x = x_i) = p(i)\ \forall\ i = 1, 2, 3, \ldots n$ and $\sum_{i=1}^{n} P(x_i) = 1$

then $p(x_i)$ is called probability distribution (or) Discrete probability distribution function.

$X = x$	x_1	x_2	x_3	......	x_n
$P(X = x_i)$	P_1	P_2	P_3	......	P_n

- **Mean:-**

The mean value of a random variable X is defined as

$$E(x) = \sum_{i=1}^{n} x_i\, p(x_i)$$

(or)

$$\mu = \sum_{i=1}^{n} x_i P_i$$

- **Variance:-**

Variance of a random variable x is defined as

$$Var(x) = \sigma^2 = \sum_{i=1}^{n} x_i^2 P_i - \left[\sum_{i=1}^{n} x_i P_i\right]^2$$

(or)

$$\sigma^2 = E(x^2) - [E(x)]^2$$

- **standard deviation:-**

standard deviation is defined as $\sigma = \sqrt{\sigma^2} = \sqrt{E(x^2) - (E(x))^2}$

→ mean of a function $g(x)$ is defined by

$$E[g(x)] = \sum_{i=1}^{n} g(x_i) \cdot P(x_i)$$

- **Cumulative distribution (or) Probability distribution function:-**

It is defined by

$$F(u) = p(X \leq x) = \sum_{x_i \leq x} P(u_i)$$

- **Probability density function:-**

It is defined by

$$f_X(x) = \frac{d}{dx}\left[F_X(x)\right]$$

→ If 'x' is a discrete random variable and 'k', is a constant then S.T.

i, $E(x+k) = E(x) + k$

ii, $E(k) = k$

iii, $E(kx) = k\,E(x)$

iv, $E(ax+b) = a\,E(x) + b$

→ If 'x, y' are discrete random variables then

$$E(x+y) = E(x) + E(y)$$

$$E(xy) = E(x) \cdot E(y)$$

→ S.T $V(ax+b) = a^2 V(x)$

$$V(ax) = a^2 V(x)$$

$$V(x+b) = V(x)$$

If 'x' is a discrete random variable $V(x)$ is variance of x.

Problems:-

1. Let 'X' denotes the no. of heads in a single toss of 4 coins. Determine i, Probability of $x < 2$ $P(x<2)$

ii, $P(1 < x \leq 3)$

iii, $P(2 \leq x \leq 4)$

Sol:- Given that x = The no. of heads in tossing 4 coins.

$$S = \begin{Bmatrix} HHHH, & HHHT, & HHTH, & HHTT \\ HTHH, & HTHT, & HTTH, & HTTT \\ THHH, & THHT, & THTH, & THTT \\ TTHH, & TTHT, & TTTH, & TTTT \end{Bmatrix}$$

The no. of heads may be 0, 1, 2, 3 & 4.

$$X = 0, 1, 2, 3, 4.$$

$$P(X=0) = \frac{\text{No. of ways of having heads}}{\text{Total}} = \frac{1}{16}$$

$$P(X=1) = \frac{4}{16}$$

$$P(X=2) = \frac{6}{16}$$

$$P(X=3) = \frac{4}{16}$$

$$P(X=4) = 1/16$$

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· Probability distribution table

$X = x$	0	1	2	3	4
$P(X=x)$	$1/16$	$4/16$	$6/16$	$4/16$	$1/16$

i, $P(x<2) = P(x=0) + P(x=1)$

$= \frac{1}{16} + \frac{4}{16}$

$= 5/16.$

ii, $P(2 < x \le 3) = P(x=2) + P(x=3)$

$= \frac{6}{16} + \frac{4}{16}$

$= \frac{10}{16}$

$= \frac{5}{8}.$

iii, $P(2 \le x \le 4) = P(x=2) + P(x=3) + P(x=4)$

$= \frac{6}{16} + \frac{4}{16} + 1/16$

$= 11/16$

2. Find mean and variance of the uniform probability distributi
$f(x) = 1/x$ for $x = 1, 2, 3, \ldots n$

Sol:- $x = 1, 2, 3, \ldots n$

$P(x) = f(x) = 1/x.$

$X = x$	1	2	3	.....	n
$P(X=x)$	$1/1$	$1/2$	$1/3$	.....	$1/n$

mean $= \Sigma x p(x)$

$= 1\cdot 1 + 2\cdot\frac{1}{2} + 3\cdot\frac{1}{3} + \ldots + n\cdot 1/n$

$= 1+1+ \ldots\ldots$ (n times)

$\boxed{E(x) = n}$

Variance $= E(x^2) - (E(x))^2$

$= \Sigma x^2 f(x) - n^2$

$= [1\cdot 1/1 + 2^2\cdot 1/2 + 3^2\cdot 1/3 + \ldots + n^2\cdot 1/n] - n^2$

$$= [1+2+3+\dots+n] - n^2$$

$$= \frac{n(n+1)}{2} - n^2$$

$$= \frac{n^2+n-2n^2}{2}$$

$$\sigma^2 = \frac{n-n^2}{2}.$$

3. Calculate mean, variance and standard deviation for the following distribution.

$X=x$	0.3	0.2	0.1	0	1	2	3
$P(X=x)$	0.05	0.10	0.30	0	0.3	0.15	0.1

Sol:-

Mean :-

$$E(X) = \sum_{i=1}^{n} x_i P_i$$

$$= \sum_{i=0.3}^{3} x_i P_i$$

$$= (0.3)(0.05) + (0.2)(0.10) + (0.1)(0.30) + 0(0) + (1)(0.3) + 2(0.15) + 3(0.1)$$

$$= 0.015 + 0.02 + 0.03 + 0 + 0.3 + 0.3 + 0.3$$

$$\boxed{E(X) = 0.965}$$

Variance :-

$$\sigma^2 = \Sigma x_i^2 P_i - [E(X)]^2$$

$$= (0.3)^2(0.05) + (0.2)^2(0.10) + (0.1)^2(0.30) + 0^2.(0) + 1^2(0.3) + 2^2(0.15) + 3^2(0.1) - [0.965]^2$$

$$= 1.8115 - 0.931225$$

$$\boxed{\sigma^2 = 0.880275}$$

standard deviation :-

$$\sigma = \sqrt{E(x^2) - [E(x)]^2}$$

$$= \sqrt{0.880275}$$

$$= 0.938229$$

$$\boxed{\sigma = 0.938229}$$

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4. A random variable 'x' has the following distribution

$X = x$	0	1	2	3	4	5	6	7
$P(X=x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find i, k ii, $P(X \geq 6)$ iii, $P(X<6)$ iv, $P(0<X<5)$

v, If $P(X \leq k) > 1/2$ find the minimum value of k.

vi, mean vii, variance

Sol:- i, k.

we know that $\sum_{i=1}^{n} x\,p(i) = 1$

$$0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$$

$$10k^2 + 9k = 1$$

$$10k^2 + 9k - 1 = 0$$

$$(10k - 1)(k+1) = 0$$

$$k = 1/10 \text{ or } k = -1$$

$\therefore$ Here $k \neq -1$

$\therefore$ So $\boxed{k = 1/10}$

ii, $P(X \geq 6) = P(X=6) + P(X=7)$

$$= 2k^2 + 7k^2 + k$$

$$= 9k^2 + k$$

$$= \frac{9}{100} + \frac{1}{10}$$

$$= 0.09 + 0.1$$

$$= 0.19$$

iii, $P(X<6) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$

$$= 0 + k + 2k + 2k + 3k + k^2$$

$$= k^2 + 8k$$

$$= \frac{1}{100} + \frac{8}{10}$$

$$= 0.01 + 0.8$$

$$= 0.81$$

iv, $P(0<X<5) = P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$$= 0 + k + 2k + 2k + 3k$$

$$= 8k = 0.8$$

v) If $P(X \le k) > 1/2 \Rightarrow k = ?$

if $k = 0$ $\quad P(X \le 0) = P(X = 0) = 0$

$k = 1$ $\quad P(X \le 1) = P(X=0) + P(X=1)$

$= 0 + k$

$= 0.1$

$k = 2$ $\quad P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$= 0 + k + 2k$

$= 0.1 + 0.2$

$= 0.3$

$k = 3.$ $\quad P(X \le 3) = P(X \le 2) + P(X = 3)$

$= 0.3 + 2k$

$= 0.3 + 0.2$

$= 0.5$

$k = 4$ $\quad P(X \le 4) = P(X \le 3) + P(X = 4)$

$= 0.5 + 3k$

$= 0.8 > 0.5$

$\therefore$ The value of k is 4 such that $P(X \le k) > 1/2$

vi) mean

$$E(X) = \sum_{i=1}^{n} x_i P(x_i)$$

$= 0(0) + 1(k) + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2)$
$+ 7(7k^2 + k)$

$= 66k^2 + 30k$

$= 0.66 + 3$

$\boxed{E(X) = 3.66}$

vii) Variance

$$\sigma^2 = E(X^2) - [E(X)]^2$$

$= 0(0) + 1^2(k) + 2^2(2k) + 3^2(2k) + 4^2(3k) + 5^2(k^2) + 6^2(2k^2)$
$+ 7^2(7k^2 + k) - (3.66)^2$

$= 440k^2 + 124k - (3.66)^2$

$= 4.40 + 12.4 - 13.3956$

$= 16.8 - 13.3956$

$= 3.4044$

5. 2 dice are thrown let 'x' assign to each point (a,b) in sample space S. The maximum of its no. s

i.e. $X(a,b) = \max(a,b)$

Find the probability distribution 'x' is a random variable with $X(s) = \{1, 2, 3, 4, 5, 6\}$ and also find mean, variance and standard deviation of the distribution.

i. $P(X \leq 3)$ ii. $P(1 \leq X \leq 3)$ iii. $P(X > 4)$

Sol:- 2 dices are thrown

No. of elements in sample space are $n(s) = 36$

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

Given that $X(a,b) = \max(a,b)$

here 'X' is a random variable.

for max no: 1, favourable cases are $\{(1,1)\}$

$$P(X=1) = \frac{1}{36} = \frac{\text{no. of elements in favourable cases}}{\text{total no. of elements in sample space}}$$

for max no. 2, favourable cases are $\{(1,2)(2,2)(2,1)\}$

$$P(X=2) = \frac{3}{36}$$

$$P(X=3) = \frac{5}{36}$$

$$P(X=4) = 7/36$$

$$P(X=5) = 9/36$$

$$P(X=6) = 11/36$$

Probability distribution table is

$X=x$	1	2	3	4	5	6
$P(X=x)$	1/36	3/36	5/36	7/36	9/36	11/36

mean:-

$$E(X) = \sum_{i=1}^{6} x_i\, P(x_i)$$

$$= 1\cdot\frac{1}{36} + 2\cdot\frac{3}{36} + 3\cdot\frac{5}{36} + 4\cdot\frac{7}{36} + 5\cdot\frac{9}{36} + 6\cdot\frac{11}{36}$$

$= \frac{161}{36}$

$= 4.472$

variance:-

$\sigma^2 = E(x^2) - [E(x)]^2$

$= \Sigma x^2 p(x) - (4.472)^2$

$= 1 \cdot \frac{1}{36} + 4 \cdot \frac{3}{36} + 9 \frac{5}{36} + 16 \cdot \frac{7}{36} + 25 \frac{9}{36} + 36 \cdot \frac{11}{36} - (4.472)^2$

$= \frac{791}{36} - 19.999$

$= 21.972 - 19.999$

$= 1.973.$

Standard deviation:-

$\sigma = \sqrt{\sigma^2}$

$= \sqrt{1.973}$

$= 1.405$

6. Let 'x' is the minimum of 2 no. s. that appear a pair of dice is thrown is once. Determine i, discrete probability distribution ii, mean iii, variance iv, $P(x \le 2)$ v, $P(1 \le x \le 4)$ vi, $P(x < 6)$

Sol: 2 dices are thrown [Pair = 2]

No. of elements in sample space are $n(s) = 36$

$$S = \begin{bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{bmatrix}$$

for min no. 1, favourable cases are $\{(1,1)(1,2)(1,3)(1,4)(1,5)(1,6)(2,1)(3,1)(4,1)(5,1)(6,1)\}$

$P(x=1) = 11/36$

$P(x=2) = 9/36$

$P(x=3) = 7/36$

$P(x=4) = 5/36$

$P(x=5) = 3/36$

$P(x=6) = 1/36$

Probability distribution table

$X=x$	1	2	3	4	5	6
$P(X=x)$	$11/36$	$9/36$	$7/36$	$5/36$	$3/36$	$1/36$

Mean:-

$$E(x) = \sum_{i=1}^{6} x_i P(x_i)$$

$$= 1\cdot\frac{11}{36} + 2\cdot\frac{9}{36} + 3\cdot\frac{7}{36} + 4\cdot\frac{5}{36} + 5\cdot\frac{3}{36} + 6\cdot\frac{1}{36}$$

$$= \frac{91}{36}$$

$$= 2.528$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= 1\cdot\frac{11}{36} + 2^2\cdot\frac{9}{36} + 3^2\cdot\frac{7}{36} + 4^2\cdot\frac{5}{36} + 5^2\cdot\frac{3}{36} + 6^2\cdot\frac{1}{36} - (2.528)^2$$

$$= \frac{301}{36} - (2.528)^2$$

$$= 8.361 - 6.391$$

$$= 1.970.$$

standard deviation:-

$$\sigma = \sqrt{\sigma^2}$$

$$= \sqrt{1.970}$$

$$= 1.404.$$

$$P(X \le 2) = P(X=1) + P(X=2)$$

$$= \frac{11}{36} + \frac{9}{36}$$

$$= \frac{\cancel{20}\ 5}{\cancel{36}\ 9}$$

$$= 5/9$$

$$P(1 \le X \le 4) = P(X=1) + P(X=2) + P(X=3) + P(X=4)$$

$$= \frac{11}{36} + \frac{9}{36} + \frac{7}{36} + \frac{5}{36}$$

$$= \frac{32}{36}$$

$$= 8/9$$

$$P(X<6) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= \frac{11}{36} + 9/36 + 7/36 + 5/36 + 3/36$$

$$= 35/36$$

Continuous Probability Distribution:-

→ **Probability density function:-**

Let $f(x)$ be the probability density function such that

i. $f(x) \geq 0 \quad \forall x \in R$

ii. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ and

$$\frac{d}{dx}[f(x)] = f(x)$$

$$P(E) = \int_{E} f(x)\,dx.$$

→ **Probability Distribution function:-**

Probability Distribution function $F(x)$ is defined as

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\,dx \text{ and}$$

$$P(X > x) = \int_{x}^{\infty} f(x)\,dx$$

→ **Mean:-**

The mean value of a continuous random variable 'x' is defined as $\mu = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

→ **Variance:-**

$$\sigma^2 = V(X) = E(x^2) - [E(x)]^2$$
$$= E(x-\mu)^2$$
$$= \int_{-\infty}^{\infty} x^2 f(x)\,dx - \left[\int_{-\infty}^{\infty} x \cdot f(x)\,dx\right]^2$$

→ **standard deviation:-**

$$\sigma = \sqrt{\sigma^2} = \sqrt{var(X)}$$

→ **median:-**

Let 'm' denotes median of continuous probability distribution and it is denoted by

$$\int_{-\infty}^{m} f(x)\,dx = \int_{m}^{\infty} f(x)\,dx = 1/2$$

→ **mean deviation:-**

$$M.D = \int_{-\infty}^{\infty} |x-\mu| f(x)\,dx \qquad \mu = \text{mean.}$$

→ **mode:-**

Mode of the distribution is the value of x for which $f(x)$ is maximum.

$$P(a \leq x \leq b) = \int_{a}^{b} f(x)\,dx$$

Problems:-

1. If the probability density function of a random variable is given by $f(x) = \begin{cases} k(1-x^2) & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

Find i, k ii, Probability that a random variable having this probability density will take on a value between 0.1 & 0.2 and also greater than 0.5.

Sol:- i, we know that

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{-\infty}^{0} f(x)\, dx + \int_{0}^{1} f(x) dx + \int_{1}^{\infty} f(x) dx = 1$$

$$\int_{-\infty}^{0} 0\, dx + \int_{0}^{1} (k - kx^2) dx + \int_{1}^{\infty} 0\, dx = 1$$

$$k(x)_0^1 - k\left(\frac{x^3}{3}\right)_0^1 = 1$$

$$k - \frac{k}{3} = 1$$

$$\frac{2k}{3} = 1$$

$$\boxed{k = 3/2}$$

ii, $P(0.1 \leq x \leq 0.2)$.

we know that $p(a \leq x \leq b) = \int_a^b f(x) dx$

$$P(0.1 \leq x \leq 0.2) = \int_a^b f(x)\, dx$$

$$= \int_{0.1}^{0.2} k - kx^2\, dx$$

$$= k(x)_{0.1}^{0.2} - k\left(\frac{x^3}{3}\right)_{0.1}^{0.2}$$

$$= k\left[0.2 - 0.1\right] - k\left[\frac{(0.2)^3}{3} - \frac{(0.1)^3}{3}\right]$$

$$= k[0.1] - k[0.0027 - 0.0003]$$

$$= k[0.1 + 0.0003 - 0.0027]$$

$$= \frac{3}{2}[0.0976]$$

$$= 0.1464$$

$\therefore P(0.1 \leq x \leq 0.2) = 0.1464$.

iii) $P(x>0.5)$

$$P(x>x) = \int_x^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{\infty} f(x)\,dx$$

$$= \int_{0.5}^{1} f(x)dx + \int_{1}^{\infty} f(x)\,dx$$

$$= k\int_{0.5}^{1} (1-x^2)dx + 0$$

$$= k\left[x - \frac{x^3}{3}\right]_{0.5}^{1}$$

$$= k\left[1 - \frac{1}{3} - 0.5 + \frac{(0.5)^3}{3}\right]$$

$$\boxed{\cancel{= k[0.1973 - 0.0977]}}$$

$$= \frac{3}{2}[0.6667 - 0.4583]$$

$$P(x>0.5) = 0.3126$$

2. If a random variable has the probability density $f(x)$ is defined as $f(x) = \begin{cases} 2.e^{-2x} & x>0 \\ 0 & x\le 0 \end{cases}$

Find probability between 1 & 3. probability greater than 0.5.

Sol: Given $f(x) = \begin{cases} 2.e^{-2x} & x>0 \\ 0 & x\le 0 \end{cases}$

i) $P(1\le x\le 3) = \int_1^3 f(x)\,dx$

$$= \int_1^3 2.e^{-2x}\,dx$$

$$= 2.\left[\frac{e^{-2x}}{-2}\right]_1^3$$

$$= -\left[e^{-6} - e^{-2}\right]$$

$$= e^{-2} - e^{-6}$$

$$= \frac{1}{e^2} - \frac{1}{e^6}$$

$$= 0.1329$$

ii) $P(x>0.5) = \int_{0.5}^{\infty} f(x)\,dx$

$$= \int_{0.5}^{\infty} 2.e^{-2x}\,dx$$

$$= 2\left[\frac{e^{-2x}}{-2}\right]_{0.5}^{\infty}$$

$$= -\left[e^{-\infty} - e^{-1}\right]$$

$$= \frac{1}{e} - \frac{1}{e^{\infty}}$$

$$= 0.3679$$

3. The probability density function $f(x)$ of a continuous random variable is given by $f(x) = c\,e^{-|x|}$ $[-\infty < x < \infty]$ show that $c = 1/2$ and find mean, variance of the distribution. And also find the probability that the variable lies b/w 0 & 4.

Sol:- We know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} c\,e^{-|x|}\,dx = 1$$

$$\boxed{\begin{aligned} & c\left[\frac{e^{-|x|}}{-1}\right]_{-\infty}^{\infty} = 1 \\ & -c\left[\frac{1}{e^{+\infty}} - \frac{1}{e^{\infty}}\right] = 1. \end{aligned}}$$

$$\int_{-\infty}^{0} c\,e^{x}\,dx + \int_{0}^{\infty} c\,e^{-x}\,dx = 1$$

$$c\left[\frac{e^{x}}{1}\right]_{-\infty}^{0} + c\left[\frac{e^{-x}}{-1}\right]_{0}^{\infty} = 1$$

$$c\left[e^{0} - e^{-\infty} - e^{-\infty} + e^{-0}\right] = 1$$

$$c\left[2e^{0} - \frac{2}{e^{\infty}}\right] = 1$$

$$2ce^{0} = 1$$

$$\boxed{c = 1/2}$$

$$\int_{-\infty}^{\infty} f(x)\,dx = \begin{cases} 2\int_{0}^{\infty} f(x)\,dx & f(x) \text{ is even} \\ 0 & f(x) \text{ is odd.} \end{cases}$$

Now. $f(x) = c\,e^{-|x|}$

$f(-x) = c\,e^{-|-x|}$

$= c\,e^{-|x|} = f(x)$

even function.

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 2\int_{0}^{\infty} f(x)\,dx$$

$$1 = 2c\int_{0}^{\infty} e^{-|x|}\,dx$$

$$1 = 2c\left[\frac{e^{-|x|}}{-1}\right]_{0}^{\infty}$$

$$1 = 2c\,[-0 + 1]$$

$$\boxed{c = 1/2}$$

Mean:-

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

Now $f(x) = c\, e^{-|x|}$

now $x f(x) = x c e^{-|x|} = g(x)$

now $g(-x) = -x c e^{-|x|}$

$= -x c e^{-|x|}$

$= -g(x)$

$\therefore x f(x)$ is odd function.

$$\therefore \int_{-\infty}^{\infty} x f(x)\, dx = 0$$

$$\boxed{E(x) = 0}$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$

Now $g(x) = x^2 f(x) = x^2 c e^{-|x|} = E(x^2)$

$g(-x) = (-x)^2 c\, e^{-|-x|} = x^2 c e^{-|x|} = g(x)$

$\therefore g(x)$ is even function.

Now $\sigma^2 = 2 \int_{+0}^{\infty} x^2 c e^{-|x|} dx - (0)^2$

$= 2c \int_{+0}^{\infty} x^2 e^{-|x|} dx$

$= 2c \left[x^2 \frac{e^{-x}}{-1} - 2(x) \frac{e^{-x}}{1} + 2 \frac{e^{-x}}{-1} \right]_0^{\infty}$

$= 2c \left[-\frac{x^2}{e^x} - \frac{2x}{e^x} + \frac{(-2)}{e^x} \right]_0^{\infty}$

$= \cancel{2}(1/\cancel{2}) [0 - 0 + 0 - 0 + 0 - (-2)]$

$$\boxed{\sigma^2 = 2}$$

$P(0 \le x \le 4) = \int_0^4 f(x)\, dx.$

$= c \int_0^4 e^{-|x|} dx$

$= c \int_0^4 e^{-x} dx$

$= 1/2 \left[\frac{e^{-x}}{-1} \right]_0^4$

$$= \frac{-1}{2}\left[\frac{1}{e^4} - \frac{1}{e^0}\right]$$

$$= \frac{-1}{2}\left[0.0183 - 1\right]$$

$$= 0.4908.$$

4. Probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{1}{2}\cdot \sin x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

Find mean, mode & median of the distribution & also find probability b/w 0 to $\pi/2$.

Sol:- Given

$$f(x) = \begin{cases} 1/2 \ \sin x & 0 \le x \le \pi \\ 0 & \text{otherwise} \end{cases}$$

mean :-

$$E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{0} x\cdot f(x)\, dx + \int_{0}^{\pi} x\cdot f(x) dx + \int_{\pi}^{\infty} x f(x)\, dx.$$

$$= 0 + \int_{0}^{\pi} x\cdot\frac{1}{2}\sin x\, dx + 0$$

$$= \frac{1}{2}\int_{0}^{\pi} x \sin x\, dx$$

$$= \frac{1}{2}\left[x(-\cos x) - (1)(-\sin x)\right]_0^{\pi}$$

$$= \frac{1}{2}\left[\pi(-\cos\pi) - 0 + 0 + 0\right]$$

$$= \frac{\pi}{2}.$$

mode :-

Value of 'x' for which $f(x)$ is maximum

Given $f(x) = \frac{1}{2}\sin x$

$$f'(x) = \frac{-\cos x}{2} = 0$$

$$\cos x = 0$$

$$\boxed{x = \pi/2}$$

$$f''(x) = \frac{1}{2}(-\sin x)$$

$$f''(\pi/2) = \frac{1}{2}(-\sin \pi/2)$$

$$= \frac{-1}{2} < 0$$

$\therefore f(x)$ has maximum value at $x = \pi/2$

$$\boxed{\text{mode} = \pi/2}$$

median:

we know that

$$\int_{M}^{\infty} f(x)\,dx = \int_{-\infty}^{M} f(x)\,dx = 1/2$$

$$\int_{0}^{M} f(x)\,dx = \int_{M}^{\pi} f(x)\,dx = 1/2$$

now $\displaystyle\int_{0}^{M} f(x)\,dx = \pi/2$

$$\frac{1}{2}\int_{0}^{M} \sin x\,dx = \pi/2$$

$$\left[-\cos x\right]_0^M = \pi$$

$$-\cos M + \cos 0 = \pi$$

$$-\cos M = 0$$

$$\boxed{M = \pi/2}$$

$$P(0 < x < \pi/2) = \int_{0}^{\pi/2} f(x)\,dx$$

$$= \frac{1}{2}\int_{0}^{\pi/2} \sin x\,dx$$

$$= \frac{1}{2}\left[-\cos x\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left[-\cos \pi/2 + \cos 0\right]$$

$$= 1/2$$

5. A continuous random variable has the following probability density function $f(x) = \begin{cases} kxe^{-dx} & x \geq 0,\ d>0 \\ 0 & \text{otherwise} \end{cases}$

Find k, mean, variance.

Sol:- we know that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx = 1$$

$$0 + k\int_{0}^{\infty} xe^{-dx}\,dx = 1$$

$$k\left[x\frac{e^{-dx}}{-d} - \frac{e^{-dx}}{d^2}\right]_0^{\infty} = 1.$$

$$k\left[\frac{\infty}{-de^{\infty}} - \frac{1}{d^2e^{\infty}} - 0 + \frac{1}{d^2}\right] = 1$$

$$k\left[0 - 0 - 0 + \frac{1}{d^2}\right] = 1$$

$$\boxed{k = d^2}$$

Mean:-

$$\int_{-\infty}^{\infty} x\,f(x)\,dx = \int_{-\infty}^{0} x\cdot f(x)\,dx + \int_{0}^{\infty} x\cdot f(x)\,dx$$

$$= 0 + \int_{0}^{\infty} kx^2 e^{-dx}\,dx$$

$$= k\left[x^2\frac{e^{-dx}}{-d} - \frac{2x}{d^2}e^{-dx} + 2\frac{e^{-dx}}{-d^3}\right]_0^{\infty}$$

$$= d^2\left[\frac{\infty^2}{-de^{+\infty}} - \frac{2\infty}{d^2e^{-\infty}} + \frac{2}{-d^3e^{\infty}} - 0 + 0 + \frac{2}{d^3}\right]$$

$$= \frac{2}{d^3}\times d^2$$

$$= \frac{2}{d}$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \int_{0}^{\infty} x^3 k e^{-dx}\,dx - \left(\frac{2}{d}\right)^2$$

$$= k\left[x^3\frac{e^{-dx}}{-d} - 3x^2\frac{e^{-dx}}{d^2} + 6x\frac{e^{-dx}}{-d^3} - 6\frac{e^{-dx}}{d^4}\right]_0^{\infty}$$

$$- 4/d^2$$

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$$= d^2\left[0 - 0 + 0 - 0 - 0 + 0 - 0 + \frac{6}{d^4}\right] - \frac{4}{d^2}$$

$$= \frac{6}{d^2} - \frac{4}{d^2}$$

$$\sigma^2 = \frac{2}{d^2}$$

6. A continuous random variable has the distribution function is

$$F(x) = \begin{cases} 0 & \text{if } x \le 1 \\ k(x-1)^4 & \text{if } 1 \le x \le 3 \\ 1 & \text{if } x > 3 \end{cases}$$

find k, mean, f(x).

Sol: w.k.T. $f(x) = \frac{d}{dx} F(x)$

$$= \begin{cases} 0 & \text{if } x \le 1 \\ 4k(x-1)^3 & \text{if } 1 \le x \le 3 \\ 0 & \text{if } x > 3. \end{cases}$$

(i) k

w.k.T $\int_{-\infty}^{\infty} f(x)\,dx = 1.$

$$\int_{-\infty}^{1} f(x)\,dx + \int_{1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx = 1$$

$$0 + \int_{1}^{3} 4k(x-1)^3\,dx + 0 = 1.$$

$$4k\left[\frac{(x-1)^4}{4}(1)\right]_1^3 = 1$$

$$4k\left[\frac{2^4}{4} - 0\right] = 1$$

$$k = 1/24 = 1/16$$

(ii) mean.

w.k.T $\int_{-\infty}^{\infty} x f(x)\,dx = E(x)$

$$E(x) = \int_1^3 4k\, x(x-1)^3\,dx$$

$$= 4(1/16)\int_1^3 x(x-1)^3\,dx$$

$$= \frac{1}{4}\int_0^2 (t+1)t^3\,dt$$

$$= \frac{1}{4}\int_0^2 (t^4 + t^3)\,dt$$

Let
$x - 1 = t$
$dx = dt$
$x = t+1$
$x = 1$
$1 - 1 = t = 0$
$x = 3$
$3 - 1 = t = 2$

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$$= \frac{1}{4}\left[\frac{t^5}{5} + \frac{t^4}{4}\right]_0^2$$

$$= \frac{1}{4}\left[\frac{2^5}{5} + \frac{2^4}{4} + 0\right]$$

$$= \frac{\cancel{32}8}{5\times \cancel{4}} + \frac{\cancel{16}}{\cancel{4}\times \cancel{4}}$$

$$= \frac{8}{5}$$

7. Is the function $f(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & x < 0 \end{cases}$ density? Also find cumulative probability distribution $F(2)$

Sol:- we know that

the density of function $f(x) = 1$.

Now. $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{\infty} f(x)\,dx$

$$= 0 + \int_0^{\infty} e^{-x}\,dx$$

$$= -[e^{-\infty} - e^{-0}]$$

$$= 1.$$

$\therefore$ $f(x)$ is density function.

$$F(2) = P(x \leq 2) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \int_{-\infty}^{2} f(x)\,dx$$

$$= \int_{-\infty}^{0} f(x)\,dx + \int_{0}^{2} f(x)\,dx$$

$$= 0 + \int_0^2 e^{-x}\,dx$$

$$= -[e^{-2} - e^{-0}]$$

$$= \frac{1}{e^0} - \frac{1}{e^2}$$

$$= 0.8647\ldots$$

8. If 'x' is a continuous random variable and $Y = ax + b$ then prove that $E(Y) = aE(x) + b$ and variance $V(Y) = a^2 V(x)$ where V is variance and a, b are constants.

Sol:- Given $Y = ax + b \rightarrow (1)$

taking Expectations on both sides

$$E(Y) = E(ax + b) \qquad \therefore E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$$E(Y) = \int_{-\infty}^{\infty} (ax+b) f(x) dx \qquad \therefore \int_{-\infty}^{\infty} f(x) dx = 1$$

$$= a \int_{-\infty}^{\infty} x f(x) dx + b \int_{-\infty}^{\infty} f(x) dx$$

$$= a E(x) + b(1)$$

$$\therefore E(Y) = aE(x) + b \rightarrow (2)$$

from (1) & (2)

$$E(Y) = aE(x) + b$$

$$Y = ax + b$$

w.k.T $\quad V(x) = E(x - \mu)^2$

$$V(Y) = E(Y - \mu)^2$$

$$= E(ax + \cancel{b} - aE(x) - \cancel{b}]^2$$

$$= E[a(x - E(x))]^2$$

$$= E[a^2 (x - E(x))^2]$$

$$= a^2 [E(x - E(x))^2]$$

$$\therefore = a^2 [E(x - \mu]^2]$$

$$= a^2 [V(x)]$$

$$\therefore V(Y) = a^2 V(x)$$

9. If 'x' is a continuous random variable and k is constant then prove that i, $V(x+k) = V(x)$

ii, $V(kx) = k^2 V(x)$

Sol:- $V(x+k) = E[(x+k)^2] - [E(x+k)]^2$

$$= \int_{-\infty}^{\infty} (x+k)^2 f(x) dx - \left[\int_{-\infty}^{\infty} (x+k) f(x) dx\right]^2$$

$$= \int_{-\infty}^{\infty} x^2 f(x) dx + \int_{-\infty}^{\infty} 2kx f(x) dx + \int_{-\infty}^{\infty} k^2 f(x) dx - \left[\int_{-\infty}^{\infty} x f(x) dx + \int_{-\infty}^{\infty} k f(x) dx\right]^2$$

$= E(X^2) + 2k E(X) + k^2 - [E(X) + k]^2$

$= E(X^2) + 2k \cancel{E(X)} + \cancel{k^2} - [E(X)]^2 - 2k \cancel{E(X)} - \cancel{k^2}$

$= E(X^2) - [E(X)]^2$

$= V(X)$

ii, $V(kx) = E[(kx)^2] - [E(kx)]^2$

$= \int_{-\infty}^{\infty} k^2 x^2 f(x) dx - \left[\int_{-\infty}^{\infty} kx f(x) dx\right]^2$

$= k^2 \int_{-\infty}^{\infty} x^2 f(x) dx - k^2 \left[\int_{-\infty}^{\infty} x f(x) dx\right]^2$

$= k^2 \left[\int_{-\infty}^{\infty} x^2 f(x) dx - \left[\int_{-\infty}^{\infty} x f(x) dx\right]^2\right]$

$= k^2 \left[E(X^2) - [E(X)]^2\right]$

$= k^2 V(X)$

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* Poisson Distribution:-

A random variable 'x' is said to be poisson distribution if its probability density is defined as

$$P(X = r) = \begin{cases} e^{-\lambda} \dfrac{\lambda^r}{r!} & r = 0, 1, 2, \dots \infty \\ 0 & \text{otherwise} \end{cases}$$

(or)

$$P(X = x) = \begin{cases} e^{-\lambda} \dfrac{\lambda^x}{x!} & x = 0, 1, 2, 3 \dots \infty \\ 0 & \text{otherwise} \end{cases}$$

where $\lambda > 0$ is a parameter

mean:-

$$E(x) = \mu = \sum_{r=0}^{\infty} r\, p(r)$$

$$= \sum_{r=0}^{\infty} \cancel{r} \cdot e^{-\lambda} \frac{\lambda^r}{\cancel{r}(r-1)!}$$

$$= \sum_{r=0}^{\infty} e^{-\lambda} \frac{\lambda^r}{(r-1)!}$$

$$= e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{(r-1)!}$$

if $r = 0$

$(r-1)! = (-1)!$ $\downarrow$ not defined

$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^r}{(r-1)!}$$

$$= e^{-\lambda} \left[\frac{\lambda}{1} + \frac{\lambda^2}{1!} + \frac{\lambda^3}{2!} + \frac{\lambda^4}{3!} + \dots \lambda \right]$$

$$= e^{-\lambda} \lambda \left[1 + \lambda + \frac{\lambda^2}{2} + \frac{\lambda^3}{6} + \dots \right]$$

$$= e^{-\lambda} \lambda e^{\lambda}$$

$$\boxed{\mu = \lambda}$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \Sigma r^2 p(r) - (\lambda)^2$$

$$= \Sigma [r(r+1) + r] p(r) - (\lambda)^2$$

$$= \sum_{r=0}^{\infty} r(r-1) p(r) + \sum_{r=0}^{\infty} r\, p(r) - (\lambda)^2$$

$$= \sum_{r=0}^{\infty} \cancel{r}(\cancel{r-1}) e^{-\lambda} \frac{\lambda^r}{(\cancel{r})(\cancel{r-1})(r-2)!} + \lambda - \lambda^2$$

$$= e^{-\lambda} \sum_{r=2}^{\infty} \frac{\lambda^r}{(r-2)!} + \lambda - \lambda^2$$

$$= e^{-\lambda} \left[\frac{\lambda^2}{0!} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \dots \right] + \lambda - \lambda^2$$

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$= e^{-d} d^2 [1 + \frac{d}{1!} + \frac{d^2}{2!} + \frac{d^3}{3!} + \cdots] + d - d^2$

$= e^{-d} d^2 e^{+d} + d - d^2$

$= d^2 + d - d^2$

$\boxed{\sigma^2 = d}$

standard deviation:-

$\sigma = \sqrt{\sigma^2}$

$\boxed{\sigma = \sqrt{d}}$

Recurrence formula:-

we have $P(x) = e^{-d} \frac{d^x}{x!} \rightarrow ①$

$\parallel^{ly}$ $P(x+1) = e^{-d} \frac{d^{x+1}}{(x+1)!} \rightarrow ②$

② ÷ ①

$\Rightarrow \frac{P(x+1)}{P(x)} = \frac{\cancel{e^{-d}} d^{x+1} (x)!}{\cancel{e^{-d}} d^{x} (x+1)!}$

$\frac{P(x+1)}{P(x)} = \frac{d}{x+1}$

$P(x+1) = \frac{d}{x+1} \cdot P(x) \qquad x = 0, 1, 2, 3, \cdots$

Problems:-

1. If a random variable has poission distribution such that $P(1) = P(2)$ Find mean, $P(4)$, $P(x \geq 1)$, $P(1 < x < 4)$

Sol:- Given that $P(1) = P(2)$

$\cancel{e^{-d}} \frac{d}{1!} = \cancel{e^{-d}} \frac{d^2}{2!}$

$\boxed{d = 2}$

mean :- $\mu = d$

$\boxed{\mu = 2}$

$P(4) = \frac{d}{4} P(3)$ (or) $P(4) = e^{-d} \frac{d^4}{4!}$

$= e^{-2} \frac{2^4}{24}$

$= 0.135335 \times 0.6667$

$= 0.090223.$

ii. $P(x \geq 1) = 1 - P(0)$

$= 1 - e^{-\lambda} \frac{\lambda^0}{0!}$

$= 1 - e^{-2} \cdot 1$

$= 1 - 0.13533$

$= 0.86466$

iii, $P(1 < x < 4) = P(2) + P(3)$

$= e^{-\lambda} \frac{\lambda^2}{2!} + e^{-\lambda} \frac{\lambda^3}{3!}$

$= \frac{1}{e^2} \left[\frac{4^2}{2_1} + \frac{8}{6_3} \right]$

$= e^{-2} \cdot 3.3333$

$= 0.45112.$

2. In a poisson distribution $P(x=1) \cdot \frac{3}{2} = P(x=3)$. Find $P(x \geq 1), P(x \leq 3)$

Sol: Given that

$P(x=1) \cdot \frac{3}{2} = P(x=3)$

$\cancel{e^{-\lambda}} \frac{\lambda}{1!} \frac{3}{2} = \cancel{e^{-\lambda}} \frac{\lambda^{3}}{3!}$

$\lambda^2 = \frac{3 \times 3}{2}$

$\boxed{\lambda = \pm 3}$

but $\lambda > 0 \therefore \boxed{\lambda = 3}$

$P(x \geq 1) = 1 - P(0)$

$= 1 - e^{-\lambda} \frac{\lambda^0}{0!}$

$= 1 - e^{-3}$

$= 1 - 0.049727$

$= 0.95021$

$P(x \leq 3) = P(0) + P(1) + P(2) + P(3)$

$= e^{-\lambda} \frac{\lambda^0}{0!} + e^{-\lambda} \frac{\lambda}{1!} + e^{-\lambda} \frac{\lambda^2}{2!} + e^{-\lambda} \frac{\lambda^3}{3!}$

$= e^{-3} \left[1 + 3 + \frac{9}{2} + \frac{27}{6} \right]$

$= e^{-3} (13)$

$= 0.647232.$

3. If variance of a poisson variable is 3. Then find $P(x=0)$, $P(0 < x \leq 3)$, $P(1 \leq x < 4)$.

Sol: Given $\sigma^2 = 3$

$\boxed{\lambda = 3}$

$P(x=0) = e^{-\lambda} \frac{\lambda^0}{0!}$

$= e^{-3}$

$= 0.04979$

$P(0 < x \leq 3) = P(1 \leq x < 4)$

$= P(1) + P(2) + P(3)$

$= e^{-\lambda} (\lambda/1!) + e^{-\lambda} \left(\frac{\lambda^2}{2!} \right) + e^{-\lambda} (\lambda^3/3!)$

$= e^{-3} \left[\frac{3}{1} + \frac{9}{2} + \frac{27}{6} \right]$

$= 0.597444$

4. Using recurrence formula find the probability where $x = 0,1,2,3,4,$ it mean is 3.

Sol:- Given $\mu = \lambda = 3.$

$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

$P(0) = e^{-\lambda} \frac{\lambda^0}{0!}$

$= e^{-3}$

$= 0.04979$

$P(1) = e^{-3} \frac{(3)}{1!}$

$= 3e^{-3}$

$= 0.14936$

$P(2) = e^{-\lambda} \frac{\lambda^2}{2!}$

$= e^{-3} \frac{3^2}{2}$

$= 0.224042$

$P(3) = e^{-\lambda} \frac{\lambda^3}{3!}$

$= e^{-3} \frac{3^3}{6}$

$= 0.224042$

$P(4) = e^{-\lambda} \frac{\lambda^4}{4!}$

$= e^{-3} \frac{3^4}{24}$

$= 0.168031$

$P(5) = e^{-\lambda} \frac{\lambda^5}{5!}$

$= e^{-3} \frac{3^5}{120}$

$= 0.10082.$

5. If 'x' is a poisson variable such that $3 * P(4) = \frac{1}{2} P(X=2) + P(X=0)$. Find mean of random variable x and $P(X <= 2)$.

Sol:- Given that

$3P(4) = \frac{1}{2} P(2) + P(0)$

$3 \cancel{e^{-\lambda}} \frac{\lambda^4}{4!} = \frac{1}{2} \cancel{e^{-\lambda}} \frac{\lambda^2}{2!} + \cancel{e^{-\lambda}} \frac{\lambda^0}{1!}$

$\frac{\lambda^4}{8} = \frac{\lambda^2}{4} + 1.$

$2 | 8, 4$
$2 | 4, 2$
$2, 1$

$\lambda^4 - 2\lambda^2 - 8 = 0$

$(\lambda^2 - 4)(\lambda^2 + 2) = 0$

$\lambda^2 = 4 \quad \lambda^2 = -2$

$\lambda = \pm 2$

$\lambda > 0 \therefore$ mean $= \lambda = 2.$

Now $P(X \leq 2) = P(0) + P(1) + P(2)$

$= e^{-\lambda}\left[\frac{\lambda^0}{1} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!}\right]$

$= e^{-2}[1 + 2 + 2]$

$= 5/e^2$

$= 0.67668.$

6. If x is poisson variable such that $P(2) = 9P(4) + 90P(6)$. Find $P(x<2)$, $P(x>4)$ $P(x>=1)$

Sol: Given that $P(2) = 9P(4) + 90P(6)$

$$\cancel{e^{-\lambda}}\frac{\lambda^2}{2!} = 9\cancel{e^{-\lambda}}\frac{\lambda^{4^2}}{4!} + 90\cancel{e^{-\lambda}}\frac{\lambda^{6^4}}{6!}$$

$$\frac{1}{2} = \frac{\overset{3}{\cancel{9}}\lambda^2}{\underset{8}{\cancel{24}}} + \frac{\overset{1}{\cancel{90}}\lambda^4}{\underset{8}{\cancel{720}}}$$

$$\begin{array}{r|l} 2 & 8, 8, 2 \\ 2 & 4, 4, 1 \\ 2 & 2, 2, 1 \\ & 1, 1, 1 \end{array}$$

$$\lambda^4 + 3\lambda^2 - 4 = 0$$

$$(\lambda^2 - 1)(\lambda^2 + 4) = 0$$

$$\begin{array}{r|l} 2 & 2, 6, 24 \\ 3 & 1, 3, 12 \\ & 1, 1, 4 \end{array}$$

$$\lambda^2 = \pm 1 \qquad \lambda^2 = -4$$

$$\therefore \lambda > 0 \quad \boxed{\therefore \lambda = 1}$$

Now $P(x<2) = P(0) + P(1)$

$$= e^{-\lambda}\left(\frac{\lambda^0}{0!}\right) + e^{-\lambda}\frac{\lambda^1}{1!}$$

$$= e^{-1}[1 + 1]$$

$$= \frac{2}{e}$$

$$= 0.7357$$

Now $P(x>4) = 1 - [P(0) + P(1) + P(2) + P(3) + P(4)]$

$$= 1 - \left[e^{-1}\left[1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24}\right]\right]$$

$$= 1 - \left[\frac{24 + 24 + 12 + 4 + 1}{24\,e}\right]$$

$$= 1 - 0.9963$$

$$= 0.00365$$

Now $P(x \geq 1) = 1 - P(0)$

$$= 1 - e^{-1}[1]$$

$$= 1 - 1/e$$

$$= 0.63212$$

7. The average number of accidents on any day on a national highway is (1.8). Determine the probability that the no. of accidents are i, Atleast 1 ii, Atmost 1.

Sol: Given that

The average no. of accidents on any day on a national highway = mean = $\mu = 1.8$.

i, Atleast 1 $= P(x \geq 1)$

$$= 1 - P(0)$$

$$= 1 - e^{-\lambda}\frac{\lambda^0}{0!}$$

$$= 1 - e^{-1.8}$$

$$= 0.8347$$

ii, Atmost 1 $= P(x \leq 1)$

$$= P(0) + P(1)$$

$$= e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!}\right]$$

$$= e^{-1.8}[1 + (1.8)]$$

$$= 2.8\,e^{-1.8}$$

$$= 0.46284.$$

8. The average no. of phone calls per minute coming into a switch board between 2 pm to 4pm is 2.5 determine the probability that during one particular minute, there will be i, 4 or fewer & ii, more than 6.

Sol: Given the average is $= 2.5$

i, 4 or fewer $= P(X \le 4)$

$$= P(0) + P(1) + P(2) + P(3) + P(4)$$

$$= e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!}\right]$$

$$= e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24}\right]$$

$$= e^{-2.5}\left[1 + 2.5 + 3.125 + 2.6041 + \overset{1.6272}{\cancel{0.6422}}\right]$$

$$= \frac{1}{12.1824}\left[\overset{10.85630}{\cancel{9.2723}}\right]$$

$$= \cancel{0.76112} \quad 0.89118.$$

ii, more than 6 $= P(X \ge 6)$

$$= 1 - \left[\left[P(0) + P(1) + P(2) + P(3) + P(4) + P(5)\right] + P(6)\right]$$

$$= 1 - \left[\left[\overset{0.89118}{\cancel{0.76112}} + e^{-2.5}\frac{\lambda^5}{120}\right] + e^{-2.5}\frac{\lambda^6}{720}\right]$$

$$= \cancel{0.17907}$$

$$= 1 - \left[0.89118 + \frac{1}{12.1824}\frac{(2.5)^5}{120} + \frac{1}{12.1824}\frac{(2.5)^6}{720}\right]$$

$$= 1 - \left[0.89118 + 0.0668 + 0.0778\right]$$

$$= 0.01418$$

9. Fit a poisson distribution to the following data

x	0	1	2	3	4
$f(x)$	109	65	22	3	1

Sol:

$$\text{mean} = \frac{\Sigma x f(x)}{\Sigma f(x)}$$

$$= \frac{0(109) + 1(65) + 2(22) + 3(3) + 4(1)}{109 + 65 + 22 + 3 + 1}$$

$$= \frac{0 + 65 + 44 + 9 + 4}{200}$$

$$= \frac{122}{200}$$

$$\mu = 0.61$$

$\boxed{\lambda = 0.61}$, $N = 200$, $n = 4$

వివరణ:-

By poisson distribution:-

$$P(x) = e^{-d}\frac{d^x}{x!}$$

Expected frequency :- ~~f(x) =~~

Now.

$$f(x) = N \cdot P(x)$$

Now

$f(0) = N \cdot P(0)$

$= 200 \cdot e^{-d}\frac{d^0}{0!}$

$= 200 \cdot \frac{1}{e^{0.61}}$

$= 108.67$

$= 109$

$f(1) = N \cdot P(1)$

$= 200 \cdot e^{-d}\frac{d^1}{1!}$

$= 200\, e^{-(0.61)}(0.61)$

$= 66.289$

$= 66$

$f(2) = N \cdot P(2)$

$= 200 e^{-d}\frac{d^2}{2!}$

$= 200 \cdot e^{-(0.61)}(0.61)^2/2$

$= 20.2180$

$= 20$

$f(3) = N \cdot P(3)$

$= 200 \cdot e^{-d}\frac{d^3}{3!}$

$= 200 \cdot e^{-0.61}\frac{(0.61)^3}{6}$

$= 4.111011$

$= 4$

$f(4) = N \cdot P(4)$

$= 200 \cdot e^{-d}\frac{d^4}{4!}$

$= 200 \cdot e^{-0.61}\frac{(0.61)^4}{24}$

$= 0.6269.$

$= 1$

x	0	1	2	3	4
observed	109	65	22	3	1
Expected.	108.67	66.289	20.2180	4.111	0.6269
	109	66	20	4	1.

Expected values are not agree with observed values.

10. Fit a poisson distribution to the following data

x	0	1	2	3	4	5	6	7
f(x)	305	365	210	80	28	9	2	1

Sol:-

$$\text{mean} = \frac{\Sigma x f(x)}{\Sigma f(x)}$$

$$= \frac{0(305)+1(365)+2(210)+3(80)+4(28)+5(9)+6(2)+7}{305+365+210+80+28+9+2+1}$$

$$= \frac{0+365+420+240+112+45+12+7}{}$$

$$= \frac{1201}{1000} = 1.201$$

$\lambda = 1.201$; $N = 1000$;

By poisson distribution.

$$P(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

Expected values or frequency : $f(x) = N \cdot P(x)$

Now

$f(0) = N \cdot P(0)$

$= 1000 \times e^{-1.201} \frac{\lambda^0}{0!}$

$= 300.8966$

$= 301$

$f(1) = N \cdot P(1)$

$= 1000 \times e^{-1.201} \frac{(\lambda)^1}{1!}$

$= 300.8966 \times (1.201) = 361.376$

$= 361$

$f(2) = N \cdot P(2)$

$= 1000 \times e^{-1.201} \frac{\lambda^2}{2!}$

$= 300.8966 \times \frac{(1.201)^2}{2}$

$= 217.0067$

$= 217$

$f(3) = N \cdot P(3)$

$= 1000 \times e^{-1.201} \frac{\lambda^3}{3!}$

$= 300.8966 \times \frac{(1.201)^3}{6}$

$= 86.8750$

$= 87$

$f(4) = N \cdot P(4)$

$= 1000 \times e^{-1.201} \frac{\lambda^4}{4!}$

$= 86.8750 \times \frac{1.201}{4}$

$= 26.084$

$= 26$

$f(5) = N \cdot P(5)$

$= 1000 \times e^{-1.201} \frac{\lambda^5}{5!}$

$= 26.084 \times \frac{1.201}{5}$

$= 6.2654$

$= 6$

$f(6) = N \cdot P(6)$

$= 1000 \times e^{-1.201} \frac{\lambda^6}{6!}$

$= 6.2654 \times \frac{1.201}{6}$

$= 1.2541$

$= 1$

$f(7) = N \cdot P(7)$

$= 1000 \times e^{-1.201} \frac{\lambda^7}{7!}$

$= 1.2541 \times \frac{1.201}{7}$

$= 0.2151$

$= 0.$

x	0	1	2	3	4	5	6	7
observed	305	365	210	80	28	9	2	1
Expected	300.8966	361.376	217.0067	86.87	26.064	6.265	1.254	0.2151
	301	361	217	87	26	6	1	0

The expected value are not agree with observed values

* Derivation of Poisson distribution :-

• Derive $P(X=r) = e^{-\lambda} \dfrac{\lambda^r}{r!}$ for $r = 0, 1, 2, 3, \ldots\ldots$

By binomial distribution

$$P(X=r) = {}^nC_r \, p^r \, q^{n-r}$$

$$= \frac{n!}{r!\,(n-r)!} \, p^r (1-p)^{n-r}$$

$$= \frac{(n)(n-1)(n-2)\ldots\ldots(n-r+1)(n-r)(n-r-1)\ldots\ldots(4)(3)(2)(1)}{(n-r)!\,r!} \, p^r \, \frac{(1-p)^n}{(1-p)^r}$$

$$= \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)\cancel{(n-r)!}}{\cancel{(n-r)!}\,r!} \, p^r \, \frac{(1-p)^n}{(1-p)^r}$$

$$= \frac{\lambda/p \left(\frac{\lambda}{p}-1\right)\left(\lambda/p-2\right)\ldots\ldots\left[\frac{\lambda}{p}-(r-1)\right]}{r!} \, p^r \, \frac{(1-p)^n}{(1-p)^r}$$

$$= \frac{\lambda(\lambda-p)(\lambda-2p)\ldots\ldots[\lambda-p(r-1)]}{\cancel{p^r}\,r!} \, \cancel{p^r} \, \frac{(1-p)^n}{(1-p)^r}$$

taking limit as $p \to 0$ & $n \to \infty$

$$= \frac{\lambda(\lambda)(\lambda)\ldots\ldots(\lambda)(r\text{ times})}{r!} \, \frac{\lim_{n\to\infty}\left[1-\lambda/n\right]^n}{\lim_{p\to 0}(1-p)^r}$$

$$= \frac{\lambda^r}{r!} \, \frac{\lim_{n\to\infty}\left[\left[1-\lambda/n\right]^{-n/\lambda}\right]^{-\lambda}}{(1)^r}$$

$$= \frac{\lambda^r}{r!} \cdot \frac{e^{-\lambda}}{1}$$

$$\lim_{x\to\infty}\left(1-\frac{1}{x}\right)^{-x} = e^{-x}$$

$$\therefore P(X=r) = e^{-\lambda}\,\frac{\lambda^r}{r!}$$

* Probability Distribution:-

→ Bernouli's distibution :-

A random variable x, x taken the values 0 and 1 and its probabilities q and p.

Bernouli's distribution defined as $p(X=x) = p^{x} \cdot q^{1-x}$

x	0	1
$p(x=x)$	q	p

where $p+q=1$

Mean:-

$$E(x) = \Sigma x p(x)$$
$$E(x) = 0 \cdot q + 1 \cdot p$$
$$E(x) = p$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$
$$= \Sigma x^2 p(x) - (p)^2$$
$$= 0^2 q + 1^2 p - p^2$$
$$= p - p^2$$
$$= p(1-p)$$
$$= pq$$

standard deviation:-

$$\sigma = \sqrt{pq}$$

→ Binomial distribution:-

A random variable x has the binomial distribution if its probability density function is defined as

$$P(X=r) = \begin{cases} {}^{n}C_{r}\, p^{r} . q^{n-r}, & r = 0,1,2,\dots n,\ p+q=1 \\ 0 & \text{otherwise} \end{cases}$$

(or)

$$P(X=x) = \begin{cases} {}^{n}C_{x}\, p^{x} . q^{n-x}, & x = 0,1,2,\dots n,\ p+q=1 \\ 0 & \text{otherwise.} \end{cases}$$

mean:-

$$E(x) = \sum_{r=0}^{n} r\, P(r)$$

$$= 0.P(0) + 1.P(1) + 2.P(2) + 3.P(3) + \dots + n.P(n)$$

$$= 1.P(1) + 2.P(2) + 3.P(3) + \dots + n.P(n)$$

$$= 1.\,{}^{n}C_{1}\, p^{1} q^{n-1} + 2\left[{}^{n}C_{2}\, p^{2} q^{n-2}\right] + 3\left[{}^{n}C_{3}\, p^{3} q^{n-3}\right] + \dots + n\left[{}^{n}C_{n}\, p^{n} q^{0}\right]$$

$$= n\,pq^{n-1} + n(n-1)p^{2}q^{n-2} + \frac{n(n-1)(n-2)}{2} p^{3} q^{n-3} + \dots + np^{n}$$

$$= np\left[q^{n-1} + (n-1)pq^{n-2} + \frac{(n-1)(n-2)}{2} p^{2} q^{n-3} + \dots + p^{n-1}\right]$$

w.k.T.

$$(x+a)^{n} = {}^{n}C_{0}\, a^{0} x^{n} + {}^{n}C_{1}\, a^{1} x^{n-1} + {}^{n}C_{2}\, a^{2} x^{n-2} + \dots + {}^{n}C_{n}\, a^{n}$$

$$= a^{0}x^{n} + na^{1}x^{n-1} + \frac{(n-1)n}{2!} a^{2} x^{n-2} + \dots + a^{n}.$$

Substitute a with q, x with p, n with $n-1$.

$$\boxed{\cancel{(p+q)^{n-1} \neq p^{n-1} + (n-1)qp^{n-2}}}$$

$$(p+q)^{n-1} = p^{n-1} + (n-1)q\,p^{n-2} + \frac{(n-2)(n-1)}{2!} q^{2} p^{n-3} + \dots - q^{n}$$

$$\therefore E(x) = np\left[(q+p)^{n-1}\right]$$

but $p+q=1$

$$= np\,(1)^{n-1}$$

$$\boxed{E(x) = np}$$

$$\boxed{\mu = np}$$

Variance:-

$$\sigma^2 = E(x^2) - [E(x)]^2$$

$$= \Sigma(x^2 p(x)) - (np)^2$$

$$= \sum_{x=0}^{n} [x(x-1)+x]p(x) - (n^2p^2)$$

$$= \sum_{x=0}^{n} x(x-1).p(x) + \sum_{x=0}^{n} x.p(x) - n^2p^2$$

$$= \sum_{x=0}^{n} x(x-1).p(x) + np - n^2p^2$$

$$= \sum_{x=0}^{n} x(x-1)\, {}^nC_r\, p^r q^{n-r} + np - n^2p^2$$

$$= 0+0+2.{}^nC_2\, p^2q^{n-2} + 3.2\, {}^nC_3\, p^3q^{n-3} + \dots + n(n-1)p^n\, {}^nC_n + np - n^2p^2$$

$$= \cancel{2}\frac{n(n-1)}{\cancel{2!}}p^2q^{n-2} + 3.\cancel{2}\frac{n(n-1)(n-2)}{\cancel{3!}}p^3q^{n-3} + \dots + n(n-1)p^n + np - n^2p^2$$

$$= n(n-1)p^2\left[q^{n-2} + \frac{n-2}{1!}pq^{n-3} + \frac{(n-2)(n-3)}{2!}p^2q^{n-4} + \dots p^{n-2}\right] + np - n^2p^2$$

Note: $(a+x)^n \longrightarrow$ substitute a, with q
x with p
n with $n-2$

$$\sigma^2 = n(n-1)p^2[(q+p)^{n-2}] + np - n^2p^2$$

$$= n(n-1)p^2 + np - n^2p^2$$

$$= n^2p^2 - np^2 + np - n^2p^2$$

$$= np[1-p]$$

$$\boxed{\sigma^2 = npq}$$

Standard deviation:-

$$\sigma = \sqrt{\sigma^2}$$

$$= \sqrt{npq}$$

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Problems:-

1. 10 coins are thrown simultaneously find the probability of getting atleast i. 7 heads ii. 6 heads

Sol:- Here $n = 10$ coins

Let p = Probability of getting head in tossing one coin

q = Probability of getting tail in tossing one coin

$p = q = 1/2$

i. Probability of getting ~~head~~ atleast 7 ~~times~~ heads

$$P(X \geq 7) = P(X=7) + P(X=8) + P(X=9) + P(X=10)$$

$$= {}^{10}C_7\, p^7 q^3 + {}^{10}C_8\, p^8 q^2 + {}^{10}C_9\, p^9 q^1 + {}^{10}C_{10}\, p^{10} q^0$$

$$= {}^{10}C_3\, \frac{1}{2^7} \cdot \frac{1}{2^3} + {}^{10}C_2\, \frac{1}{2^8} \cdot \frac{1}{2^2} + {}^{10}C_1\, \frac{1}{2^9} \cdot \frac{1}{2} + {}^{10}C_{10}\, \frac{1}{2^{10}}(1)$$

$$= \frac{10(9)(8)}{2 \times 3} \cdot \frac{1}{2^{10}} + \frac{10(9)}{2}\, \frac{1}{2^{10}} + \frac{10}{1!}\, \frac{1}{2^{10}} + 1 \cdot \frac{1}{2^{10}}$$

$$= \frac{1}{2^{10}}\left[120 + 45 + 10 + 1\right]$$

$$= \frac{1}{2^{10}}[176]$$

$$= \frac{176}{1024}$$

$$= 0.171875$$

$$\boxed{\therefore P(X \geq 7) = 0.171875}$$

ii. Probability of getting atleast 6 heads

$$P(X \geq 6) = P(X=6) + P(X \geq 7)$$

$$= {}^{10}C_6\, p^6 q^4 + 0.171875$$

$$= {}^{10}C_4\, \frac{1}{2^{10}} + 0.171875$$

$$= \frac{10 \times 9 \times 8 \times 7}{2 \times 3 \times 4} + 0.171875$$

$$= \frac{210}{1024} + 0.171875$$

$$= 0.205078 + 0.171875$$

$$= 0.376953$$

2. 2 dices are thrown 5 times. Find the probability of getting 7 as sum i, atleast once ii, ~~one~~ two times iii, $P(1 < X < 5)$

Sol: Here $n = 5$

p = probability of getting sum as 7 $= 6/36 = 1/6$

q = probability of not getting sum as 7 $= \frac{30}{36} = 5/6$

$$P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$$

$$P(X = r) = {}^{5}C_{r}\, p^{r} q^{5-r}.$$

now

i, Probability of getting 7 as sum atleast once

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + P(X=4) + P(X=5)$$

$$= {}^{5}C_{1}\, p^1 q^4 + {}^{5}C_{2}\, p^2 q^3 + {}^{5}C_{3}\, p^3 q^2 + {}^{5}C_{4}\, p^4 q^1 + {}^{5}C_{5}\, p^5 q^0$$

$$= 5 \cdot \frac{1}{6}\left(\frac{5}{6}\right)^4 + \frac{5 \times 4}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^3 + \frac{5 \times 4 \times 3}{2 \times 3}\left(\frac{1}{6}\right)^3 (5/6)^2 + \frac{5}{1} \times \left(\frac{1}{6}\right)^4\left(\frac{5}{6}\right)$$

$$+ 1 \cdot \left(\frac{1}{6}\right)^5 (5/6)^0$$

$$= \left(\frac{1}{6}\right)^5 \left[5^5 + 10(5^3) + 10(5^2) + 5^2 + 1\right]$$

$$= 0.0001\,[4651]$$

$$\boxed{P(X \geq 1) = 0.4651}$$

ii, Probability of getting 7 as sum ~~atleast~~ 2 times

$\cancel{P(X \leq 1) = 1 - P(X < 1)}$

$\cancel{= 1 -}$

$$P(X=2) = {}^{5}C_{2}\, (1/6)^2 (5/6)^3$$

$$= \frac{5 \times 4}{2} \cdot \frac{5^3}{6^5}$$

$$= 10\left(\frac{5^3}{6^5}\right)$$

$$= \frac{1250}{6^5}$$

$$= 0.16075$$

iii, $P(1 < X < 5) = P(X=2) + P(X=3) + P(X=4)$

$$= {}^{5}C_{2} \left(\frac{1}{6}\right)^2 (5/6)^3 + {}^{5}C_{3}\, (1/6)^3 (5/6)^2 + {}^{5}C_{4}\, (1/6)^4 (5/6)^1$$

$$= 10 \cdot \frac{5^3}{6^5} + 10\,\frac{5^2}{6^5} + 5 \times \frac{5}{6^5}$$

$$= \frac{1}{6^5}\left[1250 + 250 + 25\right] = 0.19612.$$

3. The mean & variance of a binomial distributions are 4 & 4/3 respectively find $P(X \geq 1)$

Sol: Given that $np = \text{mean} = 4$

$\text{Variance} = npq = 4/3$

$4q = 4/3$

$\boxed{q = 1/3}$

$\therefore$ wkT $p + q = 1$

$p + 1/3 = 1$

$p = 1 - 1/3$

$\boxed{p = 2/3}$

now $np = 4$

$n \frac{2}{3} = 4$

$\boxed{n = 6}$

now $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - {}^6C_0\, p^0 q^6$

$= 1 - 1 \cdot \left(\frac{2}{3}\right)^0 \left(1/3\right)^6$

$= 1 - \frac{1}{3^6}$

$= 1 - 0.00137$

$= 0.99863.$

4. The mean & variance of a binomial distrib variate 'X' with parameter μ & σ^2 are 16 & 8. find, $P(X \geq 1)$ & $P(X > 2)$

Sol: Given mean $= \mu = 16 = np$

$\text{variance} = \sigma^2 = 8 = npq$

$npq = 8$ w.k.T $p + q = 1$ $np = 16$

$16q = 8$ $p = 1 - 1/2$ $n \frac{1}{2} = 16$

$\boxed{q = 1/2}$ $\boxed{p = 1/2}$ $\boxed{n = 32}$

i. $P(X \geq 1) = 1 - P(X < 1)$

$= 1 - {}^{32}C_0\, p^0 q^{32}$

$= 1 - \frac{1}{2^{32}}$

$= 1 - 0.0000000002$

$= 0.9999999998$

ii. $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - P(X=0) + P(X=1) + P(X=2)$

$= 1 - \left[\frac{1}{2^{32}} + \frac{32}{2^{32}} + \frac{496}{2^{32}}\right]$

$= 1 - \frac{1}{2^{32}}\left[1 + 32 + 496\right]$

$= 1 - 0.0000001232$

$= 0.9999998768$

5. Assume that 50% of all Engineering students are good in mathematics. Determine the probability that among 18 Engineering students i, Exactly 10

ii, atmost 8

iii, Atleast 10

iv, atleast 2 and atmost 9.

Sol:- Here $n = 18$

p = Probability that students are good in mathematics

q = Probability that students are not good in mathematics

$$p = q = 1/2$$

i, $P(x=10) = {}^{18}C_{10}\, p^{10} q^{18-10}$

$$= {}^{18}C_8 \frac{1}{(2)^{18}}$$

$$= \frac{\cancel{18}^{3} \times 17 \times \cancel{16} \times \cancel{15}^{3} \times \cancel{14} \times 13 \times \cancel{12}^{2} \times 11}{1 \times \cancel{2} \times \cancel{3} \times \cancel{4} \times \cancel{5} \times \cancel{6} \times \cancel{7} \times \cancel{8}} \times \frac{1}{2^{18}}$$

$$= 0.16692$$

ii, $P(x \le 8) = {}^{18}C_0 p^0 q^{18} + {}^{18}C_1 p^1 q^{17} + {}^{18}C_2 p^2 q^{16} + {}^{18}C_3 p^3 q^{15} + {}^{18}C_4 p^4 q^{14} + {}^{18}C_5 p^5 q^{13} + {}^{18}C_6 p^6 q^{12} + {}^{18}C_7 p^7 q^{11} + {}^{18}C_8 p^8 q^{10}$

$$= \frac{1}{2^{18}}\left[{}^{18}C_0 + {}^{18}C_1 + {}^{18}C_2 + {}^{18}C_3 + {}^{18}C_4 + {}^{18}C_5 + {}^{18}C_6 + {}^{18}C_7 + {}^{18}C_8\right]$$

$$= \frac{1}{2^{18}}\left[1 + 18 + 153 + 816 + 3060 + 8568 + 18564 + 31824 + 43758\right]$$

$$= \frac{106762}{262144}$$

$$= 0.40726$$

iii, $P(x \ge 10) = \frac{1}{2^{18}}\left[{}^{18}C_{10} + {}^{18}C_{11} + {}^{18}C_{12} + {}^{18}C_{13} + {}^{18}C_{14} + {}^{18}C_{15} + {}^{18}C_{16} + {}^{18}C_{17} + {}^{18}C_{18}\right]$

$$= \frac{1}{2^{18}}\left[43758 + 31824 + 18564 + 8568 + 3060 + 816 + 153 + 18 + 1\right]$$

$$= 0.40726$$

iv, $P(2 \le x \le 9) = \left[{}^{18}C_2 + {}^{18}C_3 + {}^{18}C_4 + {}^{18}C_5 + {}^{18}C_6 + {}^{18}C_7 + {}^{18}C_8 + {}^{18}C_9\right]\frac{1}{2^{18}}$

$$= \frac{1}{2^{18}}\left[153+816+3060+8568+18564+31824 + 43758+48620\right]$$

$$= 0.59266$$

6. 20% of items produced from a factory ~~are~~ are defective. Find the probability that in a sample of 5 choosen at random
i, None is defective ii, one is defective iii, $P(1 < x < 4)$

Sol:- $n=5$

p = Probability that denotes defective items $= 1/5$

q = Probability that denotes non defective items $= 4/5$

$$P(x=r) = {}^{n}C_r\, p^r q^{n-r}$$

$$P(x=r) = {}^{5}C_r\, p^r q^{5-r}$$

i, None is defective. $= P(x=0)$

$$= {}^{5}C_0\, p^0 q^5$$

$$= 1 \cdot (4/5)^5$$

$$= 0.3276$$

ii, one is defective $= P(x=1)$

$$= {}^{5}C_1\, p^1 q^4$$

$$= \cancel{5} \cdot \frac{1}{\cancel{5}} \cdot \left(\frac{4}{5}\right)^4$$

$$= 0.4096$$

iii, $P(1 < x < 4) = P(x=2) + P(x=3)$

$$= {}^{5}C_2\, p^2 q^3 + {}^{5}C_3\, p^3 q^2$$

$$= 2 \cdot {}^{5}C_2\, p^2 q^3$$

$$= \cancel{2} \cdot \frac{5 \cdot 4}{1 \cdot \cancel{2}} \; \frac{1}{5^2} \; \frac{4^3}{5^3}$$

$$= 0.256$$

7. In 8 throws of a die 5 or 6 is considered a success, find the mean of no. of success and standard deviation.

Sol:- Here $n=8$

p = probability that we get 5 or 6 in throwing a die

$$= \frac{2}{6} = 1/3$$

q = Probability that we can't get 5 or 6 in throwing a die

$$= 4/6 = 2/3$$

mean $= np$

$= 8 \times \frac{1}{3}$

$= 2.6667$

Variance $= npq$

$= 2.6667 \times \frac{2}{3}$

$= 1.7778$

Standard deviation $= \sqrt{\sigma^2} = \sqrt{1.7778} = 1.3333.$

8. 2 dices are thrown 120 times, find the average no. of times, in which, the no. on the first die exceeds the no. on the second die.

Sol:

$$S = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

P = probability that no. on 1^{st} die exceed no. on 2^{nd} die

$= \frac{15}{36} = 5/12$

$q = 1 - 5/12 = 7/12$

$n = 120$

mean $= np$

$= \cancel{120}^{10} \times \frac{5}{\cancel{12}}$

$= 50$

The average no. of times in which, the no. on the first die exceeds the no. on the 2^{nd} die $= 50$.

9. If 3 of 20 tyres are defective & 4 of them are randomly choosen for inspection, what is the probability that only 1 of the defective tyre.

Sol: Here $n = 20$; $p = \frac{3}{20}$; $q = \frac{17}{20}$, 4 tires are randomly choosen

$P(x=1) = {}^4C_1 \, p^1 q^3$

$= \cancel{4} \cdot \frac{3}{\cancel{20}_5} \cdot \frac{(17)^3}{(20)^3}$

$= 0.368475$

Fitting of a Binomial distribution.

→ Find the mean = $\frac{\Sigma x f(x)}{\Sigma f(x)} = np$

$\therefore N = \Sigma f(x)$

→ Find p, q values

→ By binomial distribution $P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

→ Find $P(0), P(1) \ldots \ldots P(n)$

→ Find the expected (or) theoritical frequencies

$f(x) = N \cdot P(x)$

→ Compare the given frequency and expected frequency.

* Fitting Binomial distribution

1. Fit a Binomial distribution to the following data

x	0	1	2	3	4	5
frequency $f(x)$	2	14	20	34	22	8

Sol: Here $n = 5$

$N =$ sum of frequencies

$= \Sigma f(x)$

$= 2 + 14 + 20 + 34 + 22 + 8$

$= 100$

$$\text{mean} = \frac{\Sigma x \cdot f(x)}{\Sigma f(x)}$$

$$= \frac{0(2) + 1\cdot(14) + 2(20) + 3(34) + 4(22) + 5(8)}{100}$$

$$= \frac{284}{100}$$

$\boxed{np = 2.84}$

$\therefore np = 2.84$

$p = \frac{2.84}{5}$

$\boxed{p = 0.568}$

w.k.T

$p + q = 1$

$q = 1 - p$

$\boxed{q = 0.432}$

By binomial distribution

$P(X = r) = {}^nC_r\, p^r\, q^{n-r}$

$= {}^5C_r\, (0.568)^r (0.432)^{n-r}$ $\quad r = 0, 1, 2, 3, 4, 5$

$P(X = 0) = {}^5C_0\, p^0\, q^5$

$= 1 \cdot (0.432)^5 = 0.01505$

$P(X = 1) = {}^5C_1\, p^1\, q^4$

$= 5 \cdot (0.568) \cdot (0.432)^4$

$= 0.09891$

$P(X = 2) = {}^5C_2\, p^2\, q^3$

$= \frac{5 \cdot 4}{2} (0.568)^2 (0.432)^3$

$= 0.2601$

$P(X = 3) = {}^5C_3\, p^3\, q^2$

$= 10\, (0.568)^3 (0.432)^2$

$= 0.34199$

$P(X = 4) = {}^5C_4\, p^4\, q^1$

$= 5 \cdot (0.568)^4 (0.432)$

$= 0.22483$

$P(X = 5) = {}^5C_5\, p^5\, q^0$

$= 1 \cdot (0.568)^5$

$= 0.05912$

మొత్తం:-

Expected (ర) theoritical frequency

$f(x) = N \cdot p(x)$

$f(0) = 100 \cdot P(0)$
$= 100 \cdot (0.01505)$
$= 1.505 = 2$

$f(1) = 100 \cdot P(1)$
$= 100\ (0.09891)$
$= 9.891 \approx 10$

$f(2) = 100 \cdot P(2)$
$= 100\ (0.2601)$
$= 26.01 = 26$

$f(3) = 100 \cdot P(3)$
$= 100 \cdot (0.34199)$
$= 34.199 = 34$

$f(4) = 100 \cdot P(4)$
$= 100\ (0.22483)$
$= 22.483 = 22$

$f(5) = 100 \cdot P(5)$
$= 100 \cdot (0.05912)$
$= 5.912 = 6.$

x	0	1	2	3	4	5
observed	2	14	20	34	22	8
Expected ~~observed~~	1.505	9.891	26.01	34.199	22.483	5.912
~~Expected~~.	2	10	26	34	22	6

Expected frequencies are not agree with observed frequencies.

2) 4 coins are tossed 160 times. The no. of times 'x' heads occur is given below.

X	0	1	2	3	4.
f(x)	8	34	69	43	6

Fit a binomial distribution to this data on the hypothesis that coins are unbiased.

Sol:- coins are unbiased. $p = q$, Here $n = 4$
$p = q = 1/2$

$N = \Sigma f(x)$
$= 160.$

$P(X = r) = {}^nC_r\ p^r\ q^{n-r}$
$= {}^4C_r \left(\frac{1}{2}\right)^r \left(\frac{1}{2}\right)^{n-r}$
$= {}^4C_r\ (1/2)^n \quad \therefore r = 0, 1, 2, 3, 4.$

Expected frequency:- $f(x) = N \cdot P(x)$

$f(0) = N \cdot P(0)$
$= 160 \cdot {}^4C_0\ (1/2)^4$
$= 10$

$f(1) = N \cdot P(1)$
$= 160\ {}^4C_1\ (1/2)^4$
$= 40$

$f(2) = N \cdot P(2)$
$= 160 \cdot {}^4C_2 (1/2)^4$
$= 60$

$f(3) = N \cdot P(3)$
$= 160 \cdot {}^4C_3 (1/2)^4$
$= 40$

$f(4) = N \cdot P(4)$
$= 160 \cdot {}^4C_4 (1/2)^4$
$= 10.$

X	0	1	2	3	4
observed	8	34	69	43	6
expected	10	40	60	40	10

Here expected frequencies not agree with observed frequencies.

Note:-
→ If the coin (or) die is unbiased then $p = q = 1/2$
→ If the coin (or) die is biased then $p \neq q$

3. 7 coins are tossed & the no. of heads are noted. The experiment is repeated 128 times and the following distribution is obtained

X	0	1	2	3	4	5	6	7
f(x)	7	6	19	35	30	23	7	1

Fit a binomial distribution i, the coin is unbiased.
ii, The nature of coin is unknown (or) coin is biased.

Sol: i, Given the coin is unbiased

$p = q = 1/2, \quad n = 7, \quad N = 128$

$P(X = x) = {}^nC_r \, p^r q^{n-r}$
$= {}^7C_r (1/2)^r (1/2)^{7-r}$
$= {}^7C_r \left(\frac{1}{2}\right)^7$

Expected ~~ddd~~ frequency :- $f(x) = N \cdot P(x)$

$f(0) = N \cdot P(0)$
$= 128 \cdot {}^7C_0 (1/2)^7$
$= \frac{128}{128} \cdot 1$
$= 1.$

$f(1) = N \cdot P(1)$
$= \cancel{128}\ 128 \cdot {}^7C_1 \frac{1}{2^7}$
$= \frac{128}{128} \cdot 7$
$= 7$

మొత్తం:-

$f(2) = N \cdot P(2)$
$= 128 \; {}^7C_2 \frac{1}{2^7}$
$= \frac{7 \times 6^3}{2}$
$= 21$

$f(3) = N \cdot P(3)$
$= 128 \cdot {}^7C_3 \frac{1}{2^7}$
$= \frac{7 \times 6 \times 5}{2 \times 3}$
$= 35$

$f(4) = N \cdot P(4)$
$= 128 \cdot {}^7C_4 \frac{1}{2^7}$
$= \frac{7 \times 6 \times 5 \times 4}{2 \times 3 \times 4}$
$= 35$

$f(5) = N \cdot P(5)$
$= 128 \cdot {}^7C_5 \frac{1}{2^7}$
$= \frac{7 \times 6^3}{2}$
$= 21$

$f(6) = N \cdot P(6)$
$= 128 \cdot {}^7C_6 (1/2^7)$
$= \frac{128}{128} \times 7$
$= 1.7 = 7$

$f(7) = N \cdot P(7)$
$= 128 \cdot {}^7C_7 (1/2^7)$
$= \frac{128}{128}$
$= 1.$

X	0	1	2	3	4	5	6	7
observed	7	6	19	35	30	23	7	1
Expected	1	7	21	35	35	21	7	1

ii, Given the coin is ~~un~~biased $N = 128$, $n = 7$

$$\text{mean} = \frac{\Sigma x f(x)}{\Sigma f(x)}$$

$$= \frac{0(7) + 1(6) + 2(19) + 3(35) + 4(30) + 5(23) + 6(7) + 7(1)}{128}$$

$$= \frac{6 + 38 + 105 + 120 + 115 + 42 + 7}{128}$$

$np = 3.3828$

$P = \frac{3.3828}{7}$

$\boxed{P = 0.4832}$

w.k.T $p + q = 1$

$\boxed{q = 0.5168}$

Expected frequency : $f(x) = N \cdot P(x)$

$f(0) = N \cdot P(0)$
$= 128 \cdot {}^7C_0 (P)^0 (q)^7$
$= 128 \cdot 1 \cdot 1 \cdot (0.5168)^7$
$= 1.2602$
$= 1.$

$f(1) = N \cdot P(1)$
$= 128 \cdot {}^7C_1 \; p^1 q^6$
$= 128 \cdot 7 \times 0.4832 \times (0.5168)^6$
$= 8.2484$
$= 8$

$f(2) = N \cdot P(2)$

$= 128 \cdot {}^7C_2 \, p^2 q^5$

$= 128 \cdot \frac{7 \cdot \cancel{6}^{3}}{\cancel{2}} (0.5168)^5 (0.4832)^2$

$= 23.1363$

$= 23$

$f(3) = N \cdot P(3)$

$= 128 \cdot {}^7C_3 \, p^3 q^4$

$= 128 \cdot 35 \cdot (0.5168)^4 (0.4832)^3$

$= 36.0536$

$= 36$

$f(4) = N \cdot P(4)$

$= 128 \cdot {}^7C_4 \, p^4 q^3$

$= 128 \cdot 35 \cdot (0.4832)^4 (0.5168)^3$

$= 33.7095$

$= 34$

$f(5) = N \cdot P(5)$

$= 128 \; {}^7C_5 \, p^5 q^2$

$= 128 \cdot 21 \cdot (0.4832)^5 (0.5168)^2$

$= 18.9107$

$= 19$

$f(6) = N \cdot P(6)$

$= 128 \cdot {}^7C_6 \, p^6 q^1$

$= 128 \cdot 7 \cdot (0.4832)^6 (0.5168)^1$

$= 5.8937$

$= 6.$

$f(7) = N \cdot P(7)$

$= 128 \cdot {}^7C_7 \, p^7 q^0$

$= 128 \cdot 1 \cdot 1 \; (0.4832)^7$

$= 0.7872$

$= 1.$

Here expected frequencies are not agree with the observed frequencies.

X	0	1	2	3	4	5	6	7
observed	7	6	19	35	30	23	7	1
Expected.	1.2602 1	8.2484 8	23.13 23	36.05 36	33.70 34	18.91 19	5.89 6	0.78 1.

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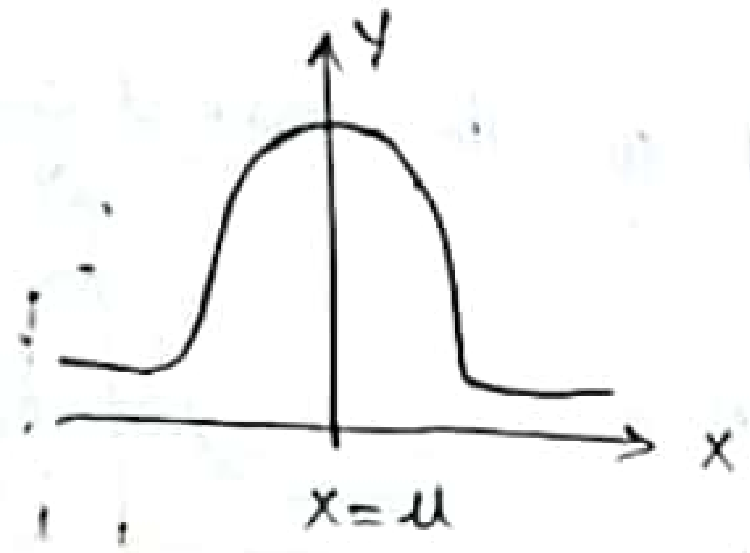
Normal Distribution :

A random variable 'x' is said to be normal distribution, if it's probability density function is defined as $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$

i.e. $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]}$.

$-\infty < \mu < \infty$
$-\infty < x < \infty$
$\sigma > 0$

characteristic of normal distribution :-



→ The graph of Normal distribution $y = f(x)$ in xy-plane is called Normal curve

→ The curve is Bell shaped & is symmetric about the line $x = \mu$. The 2 tails of Normal curve which extends to $-\infty$ and $+\infty$. In normal distribution

mean = mode = median.

→ The total area under the normal curve is called 'Total population'..

→ The probability between x_1 & x_2 is

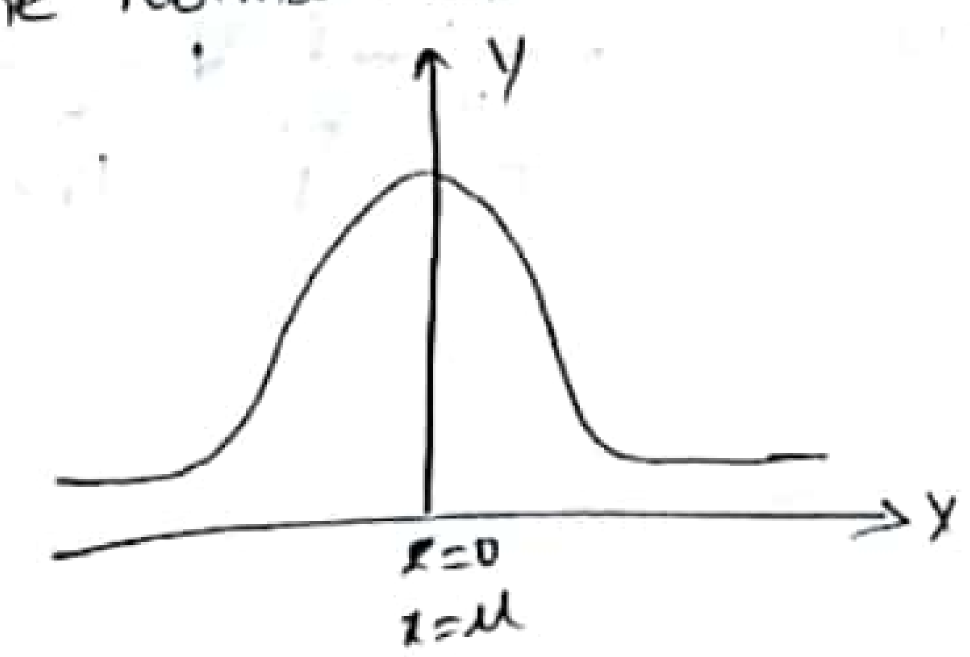
$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} f(x)\, dx$$

$$P(x_1 \le X \le x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{\left[\frac{-(x-\mu)^2}{2\sigma^2}\right]} dx.$$

→ The total area under the normal curve is unity.

→ The area from $x = \mu$ to left $= 0.5$
$x = \mu$ to right $= 0.5$

Area under the Normal curve :-



change the scale from x to z

we take $z = \frac{x - \mu}{\sigma}$

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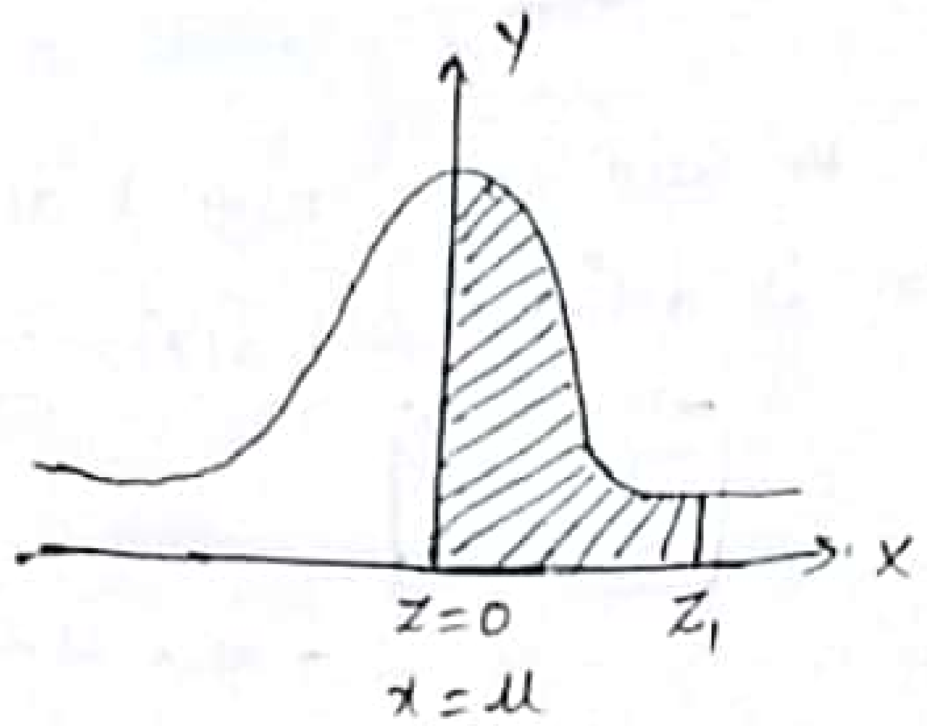


The area from $z=0$ to $z=z_1$ is denoted by $A(z)$

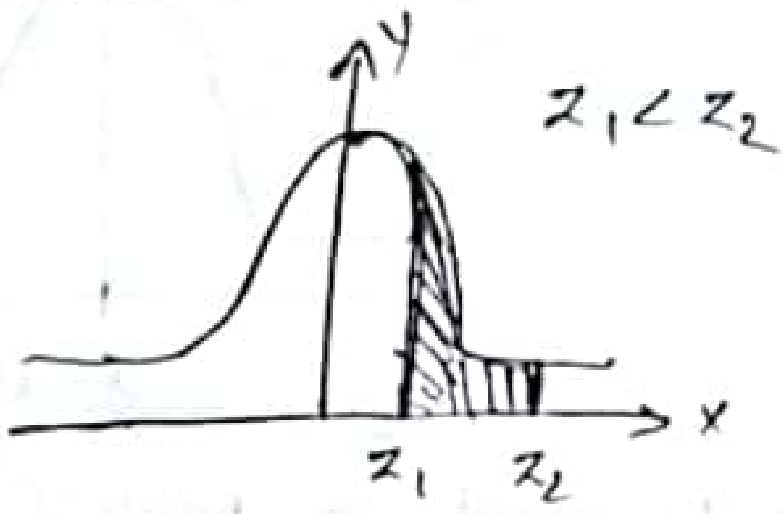
$\therefore A(-z) = A(z)$

* To find the probability density of the normal curve :-

1. $z_1 < z_2$.

   If $z_1\ z_2$ are positive then

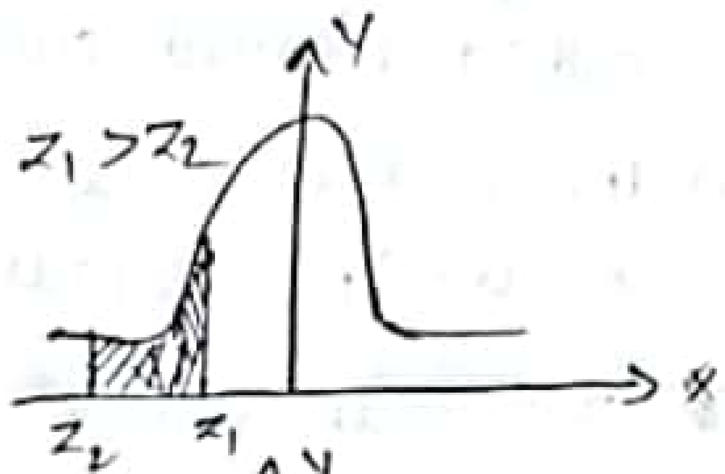
   $P(z_1 \le z \le z_2) = A(z_2) - A(z_1)$



2. $z_1 > z_2$

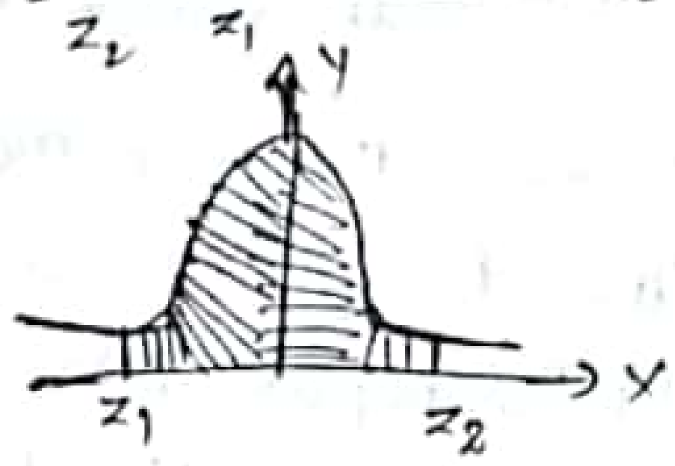
   If $z_1\ z_2$ are negative then

   $P(z_1 \le z \le z_2) = A(z_2) - A(z_1)$



3. If $z_1 < 0$ & $z_2 > 0$ then

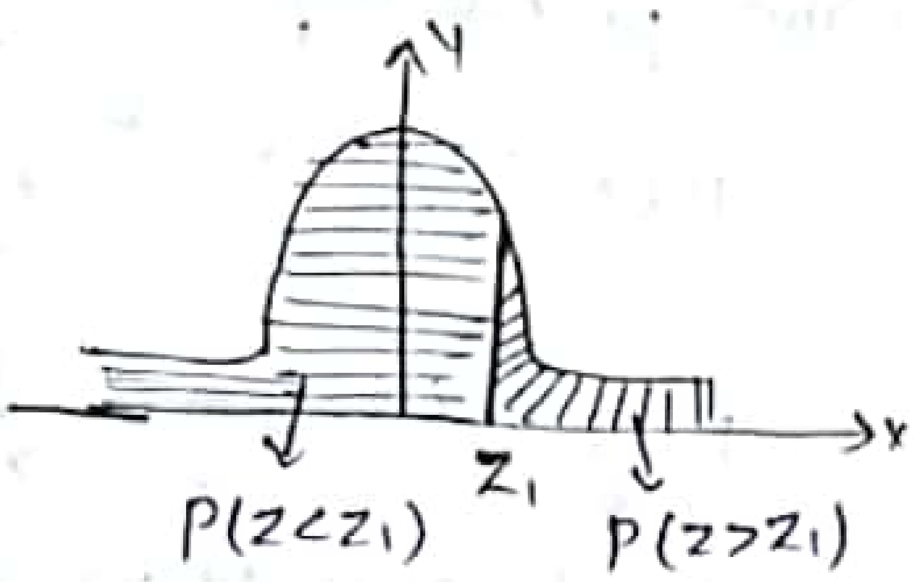
   $P(z_1 \le z \le z_2) = A(z_1) + A(z_2)$



4. If $z_1 > 0$ then

   $P(z > z_1) = \frac{1}{2} - A(z_1)$

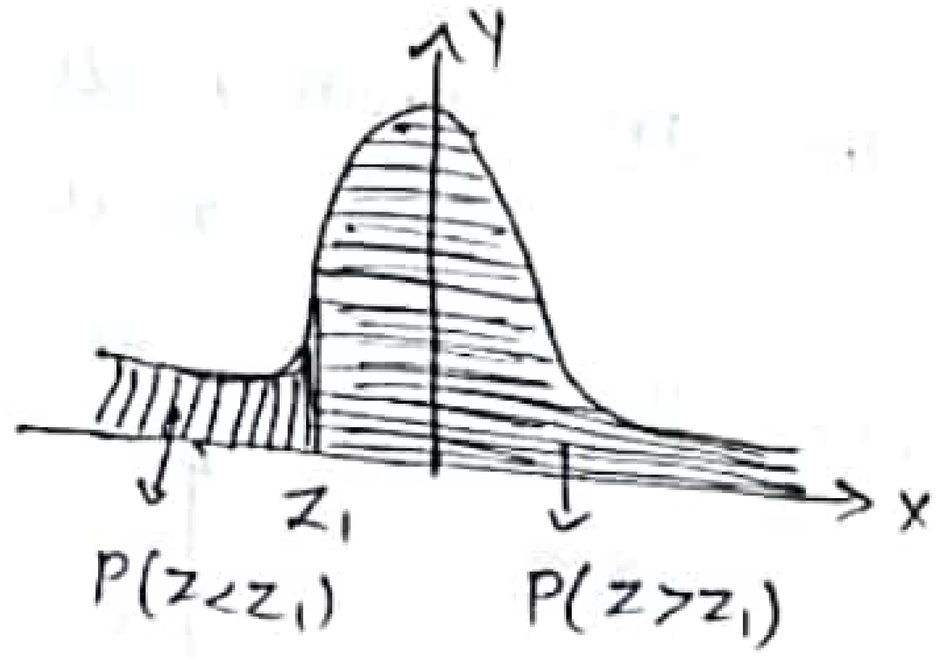
   $A(z < z_1) = \frac{1}{2} + A(z_1)$



5. If $z_1 < 0$ then

   $P(z < z_1) = \frac{1}{2} - A(z_1)$

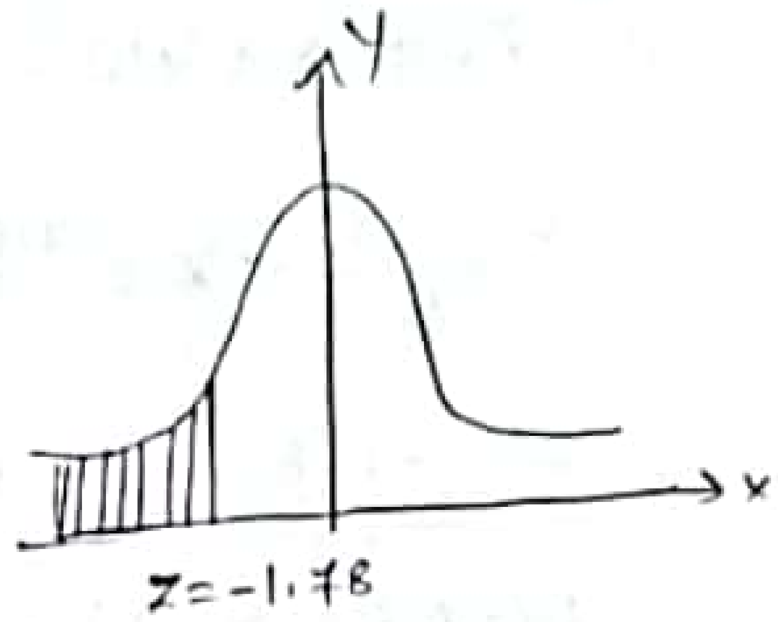
   $P(z > z_1) = \frac{1}{2} + A(z_1)$



Problems:-

1. If z is a normal variate find the probability that i, to the left of $z = -1.78$ ii, to the right $z = -1.45$ iii, corresponding to $-0.8 \le z \le 1.53$ iv, to the left of $z = -1.52$ & to the right of $z = 1.83$.

Sol: i, Probability to the left of z is -1.78



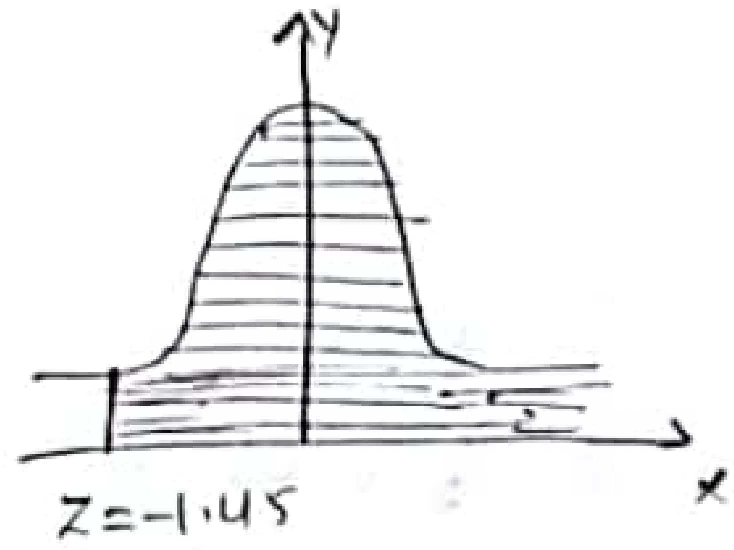
$$= P(z < 1.78) = \frac{1}{2} - A(-1.78)$$

$$= 0.5 - A(1.78)$$

$$= 0.5 - 0.4625$$

$$= 0.0375$$

ii, Probability to the right of $z = -1.45$



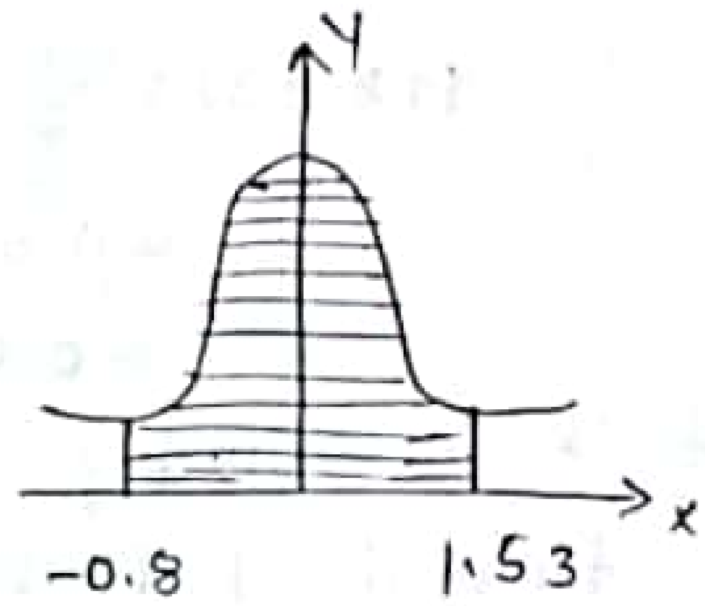
$$P(z > (-1.45)) = \frac{1}{2} + A(-1.45)$$

$$= \frac{1}{2} + A(1.45)$$

$$= \frac{1}{2} + 0.4265$$

$$= 0.9265$$

iii, Corresponding to $-0.8 \le z \le 1.53$.



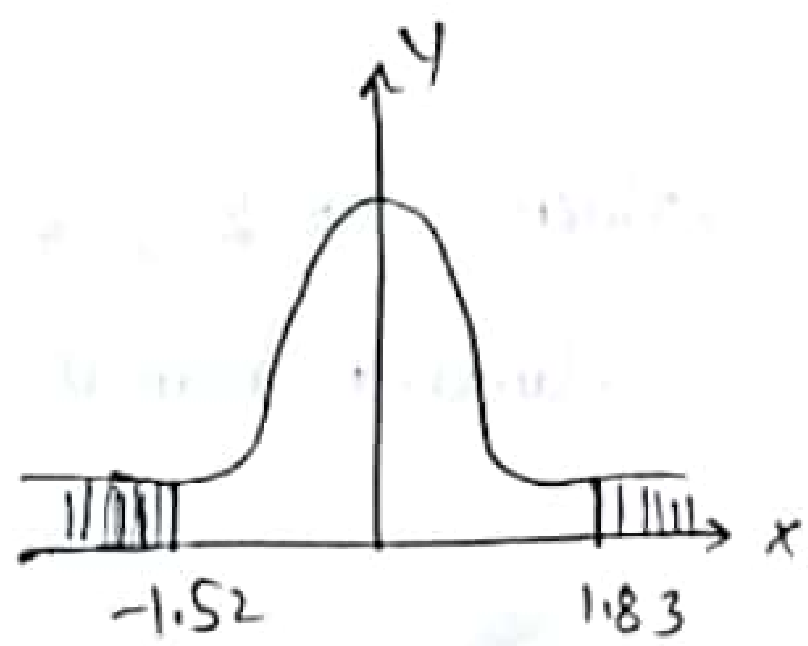
$$P(-0.8 \le z \le 1.53) = A(-0.8) + A(1.53)$$

$$= A(0.8) + A(1.53)$$

$$= 0.2881 + 0.4370$$

$$= 0.7251$$

iv, to the left of $z = -1.52$ & to the right of $z = 1.83$



$$= 1 - P(+1.83 \le z \le -1.52)$$

$$= 1 - [A(1.83) + A(1.52)]$$

$$= 1 - [0.4664 + 0.4357]$$

$$= 1 - [0.9021]$$

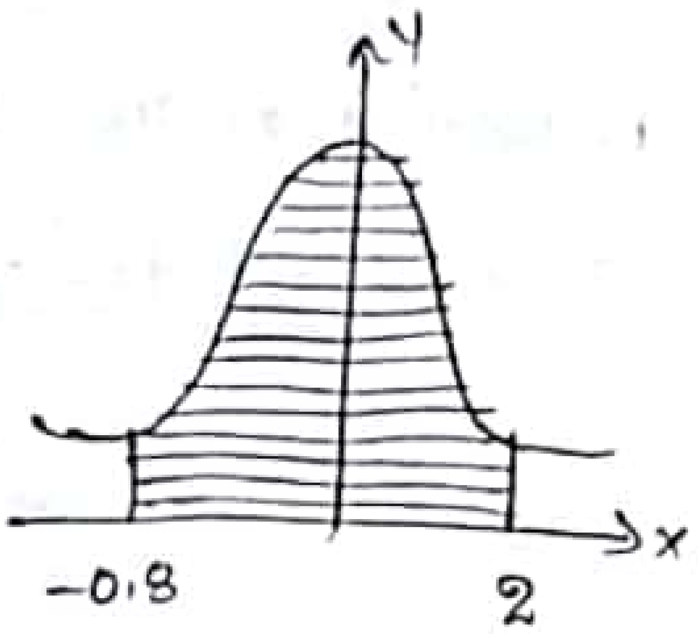
$$= 0.0979.$$

2. If 'x' is a normal variate with mean 30, and standard deviation 5 find probability that

i, $26 \le x \le 40$ ii, $x \ge 45$

Sol:- we know that $z = \frac{x-\mu}{\sigma}$, $\mu = 30$, $\sigma = 5$

i, $26 \le x \le 40$



$$\frac{26-30}{5} \le z \le \frac{40-30}{5}$$

$\Rightarrow -0.8 \le z \le 2.$

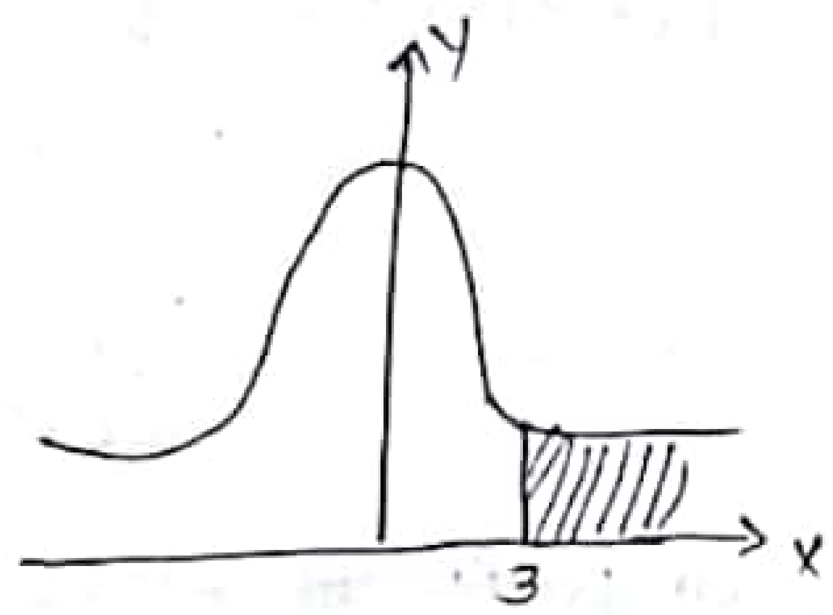
$P(-0.8 \le z \le 2) = A(-0.8) + A(2)$

$= A(0.8) + A(2)$

$= 0.2881 + 0.4772$

$= 0.7653$

ii, $x \ge 45$



$$z \ge \frac{45-30}{5}$$

$z \ge 3.$

$P(z \ge 3) = \frac{1}{2} - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$

3. 'x' is normally distributed with mean 2 & variance 0.1 then find i, $P(|x-2| \le 0.01)$ ii, $P(|x-2| > 0.01)$

Sol:- Let $|x| \le a$ $\quad |x-a| \le b$

$-a \le x \le a$ $\quad -b \le x-a \le b$

$a-b \le x \le a+b$

Given $\mu = 2$; $\sigma^2 = 0.1$; $\sigma = \sqrt{\sigma^2} = 0.316$

standard normal variable $z = \frac{x-\mu}{\sigma}$

$|x-a| \le b = a-b \le x \le a+b$

i. $P(|x-2| \le 0.01) = P(2-0.01 \le x \le 2+0.01)$

$= P(1.99 \le x \le 2.01)$

where $x = 1.99 \rightarrow z = \frac{1.99-2}{0.0316} = -0.03165$

$x = 2.01 \rightarrow z = \frac{2.01-2}{0.316} = 0.03165$

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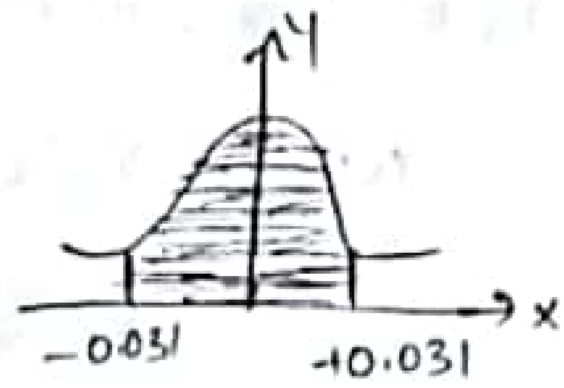
$\therefore P(-0.03165 \le z \le 0.03165) = A(-0.03165) + A(0.03165)$

$= 2A(0.03165)$

$\doteq 2[0.0120]$

$= 0.0240.$



ii) $P(|x-2| > 0.01) = 1 - P(|x-2| \le 0.01)$

$= 1 - 0.0240$

$= 0.976.$

4. If the masses of 300 students are normally distributed with mean 68 kgs. and standard deviation 3 kgs. how many students have masses i) > 72 kgs ii) ≤ 64 kgs iii) between 65 and 71

Sol: Given that $\mu = 68$; $\sigma = 3$; $z = \dfrac{x-\mu}{\sigma}$

i) $P(x > 72)$

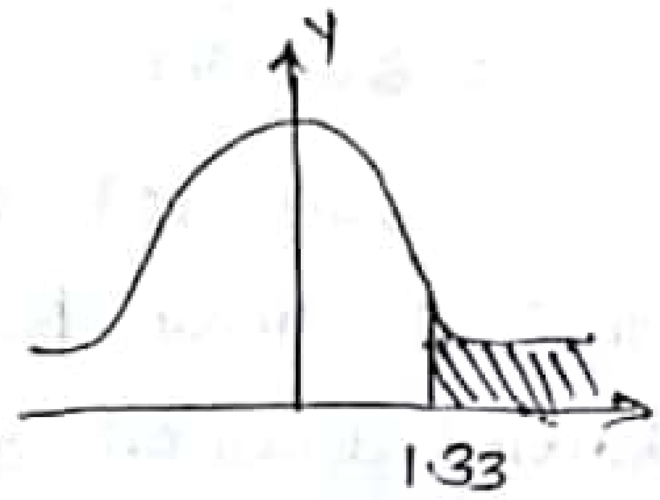
Here $x = 72 \Rightarrow z = \dfrac{72-68}{3} = 1.33$

$P(x > 72) = P(z > 1.33)$

$= 0.5 - A(1.33)$

$= 0.5 - 0.4082$

$= 0.0918$



The no. of students having mass > 72 kgs

$= P(x > 72) \times$ no. of students

$= 0.0918 \times 300$

$= 27.54$

$= 27$ (or) $28.$

ii) $P(x \le 64)$

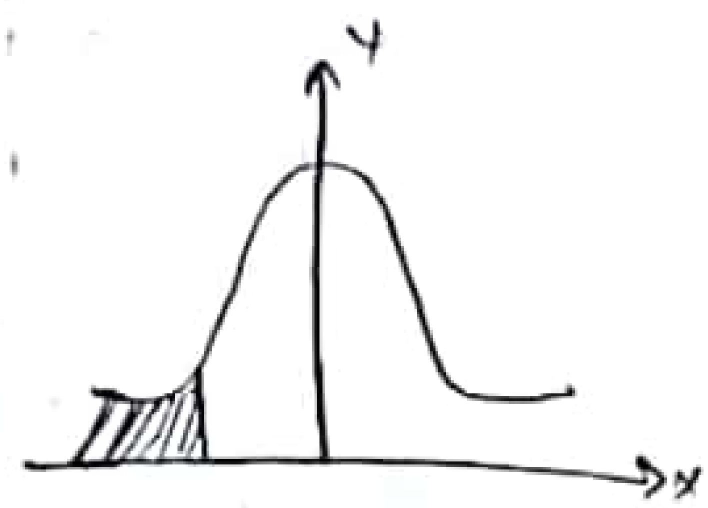
Here $x = 64 \Rightarrow z = \dfrac{64-68}{3} = -1.33$

$P(x > 72) = P(z > (-1.33))$

$= 0.5 - A(-1.33)$

$= 0.5 - 0.4082$

$= 0.0918$



No. of students having mass ≤ 64 is

$= P(z \le 1.33) \times 300$

$= 0.0918 \times 300$

$= 27.54$

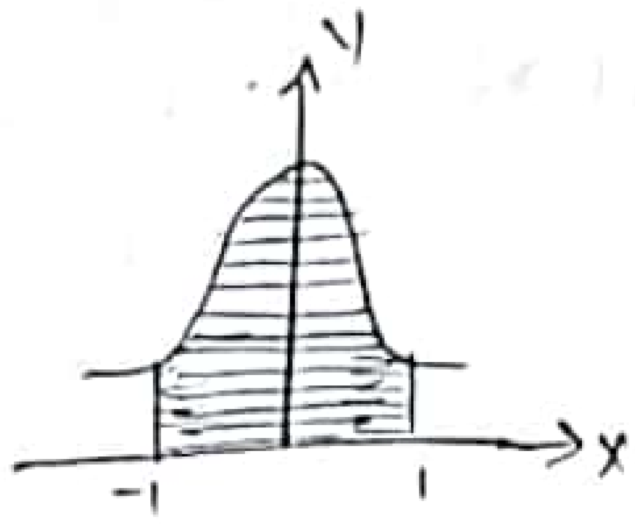
$= 27$ (or) 28

iii) $P(65 \le x \le 71)$

where $x = 65 \to z = \frac{65-68}{3} = -1$

$x = 71 \to z = \frac{71-68}{3} = 1.$

$P(65 \le x \le 71) = P(-1 \le z \le 1)$



$= A(-1) + A(1)$

$= A(1) + A(1)$

$= 2 * A(1)$

$= 2 * 0.3413$

$= 0.6826$

No. of students having marks between 65 & 71

is $= P(-1 \le z \le 1) * 300$

$= 0.6826 * 300$

$= 204.78$

$= 204$ (or) 205

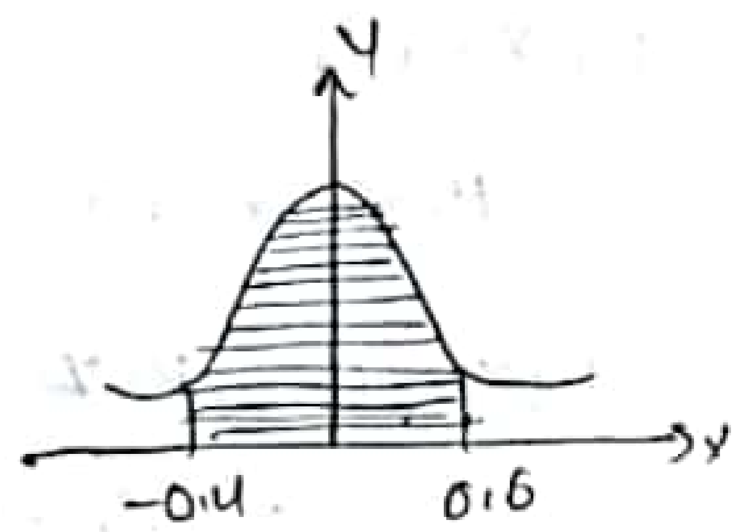
5. Given that the mean height of students in a class is 158 cm with standard deviation of 20 cm. find how many students heights lies between 150 cm & 170 cm. If there are 100 students in the class.

Sol: $P(150 \le x \le 170)$ $\sigma = 20cm$; $\mu = 158cm$

where $x = 150 \to z = \frac{150-158}{20} = -0.4$

$x = 170 \to z = \frac{170-158}{20} = 0.6$

$P(150 \le x \le 170) = P(-0.4 \le z \le 0.6)$



$= A(-0.4) + A(0.6)$

$= A(0.4) + A(0.6)$

$= 0.1554 + 0.2258$

$= 0.3812$

No. of students having weight b/w 150 cm & 170 cm is

$= 0.3812 \times 100$

$= 38.12$

$\simeq 38$

If 'x' is normal variable with mean 30 & standard deviation 5 then find probabilies that i, $|x-30| \le 5$ ii, $|x-30| > 5$.

Given $\mu = 30$, $\sigma = 5$ $\therefore |x-a| \le b$

$a-b \le x \le a+b$

i, $P(|x-30| \le 5) = P(30-5 \le x \le 30+5)$

$= P(25 \le x \le 35)$

= ~~[illegible]~~

where $x = 25 \rightarrow z = \frac{25-30}{5} = -1$

$x = 35 \rightarrow z = \frac{35-30}{5} = 1$.

$\therefore P(|x-30| \le 5) = P(-1 \le z \le 1)$

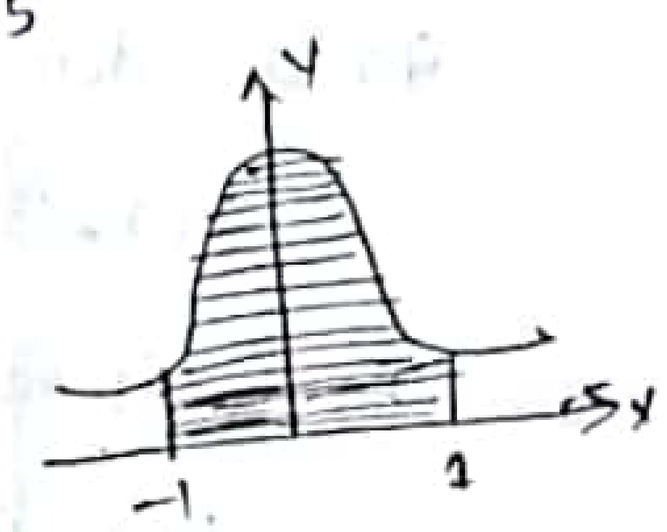
$= A(-1) + A(1)$

$= A(1) + A(1)$,

$= 2 \times A(1)$

$= 2 \times 0.3413$

$= 0.6826$



ii, $P(|x-30| > 5) = 1 - P(|x-30| \le 5)$

$= 1 - 0.6826$

$= 0.3176$

7. The mean & standard deviation of a normal variable are 8 & 4 find i, $P(5 \le x \le 10)$ ii, $P(x \ge 5)$

Sol:- $\mu = 8$, $\sigma = 4$

i, $P(5 \le x \le 10)$

where $x = 5 \rightarrow z = \frac{5-8}{4} = -0.75$

$x = 10 \rightarrow z = \frac{10-8}{4} = 0.5$

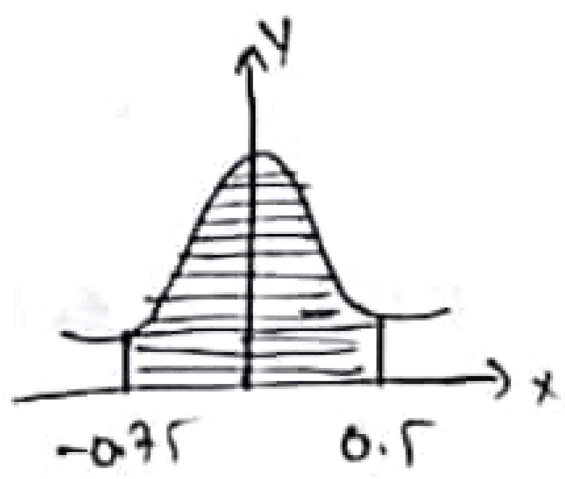
$\therefore P(5 \le x \le 10) = P(-0.75 \le z \le 0.5)$

$= A(-0.75) + A(0.5)$

$= A(0.75) + A(0.5)$

$= 0.2734 + 0.1916$

$= 0.4650$



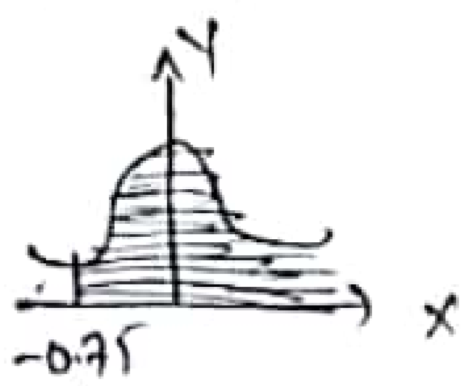
ii, $P(x \ge 5)$

where $x = 5$, $z = \frac{5-8}{4} = -0.75$

$P(z \ge -0.75) = 0.5 + A(0.75)$

$= 0.5 + 0.2734$

$= 0.7734$.



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8. In a normal distribution, 7% of items are under 35 & 89% of items are under 63. find mean & standard deviation of distribution.

(or)

If 7% of probability for a normal distribution is below 35 & 89% of probability is below 63. Then find mean & standard deviation of distribution.

Sol:- Let 'μ' be the mean and 'σ' be the standard deviation of Normal distribution.

Given that

7% of items are under 35

$$P(X<35) = 7\% = 0.07$$

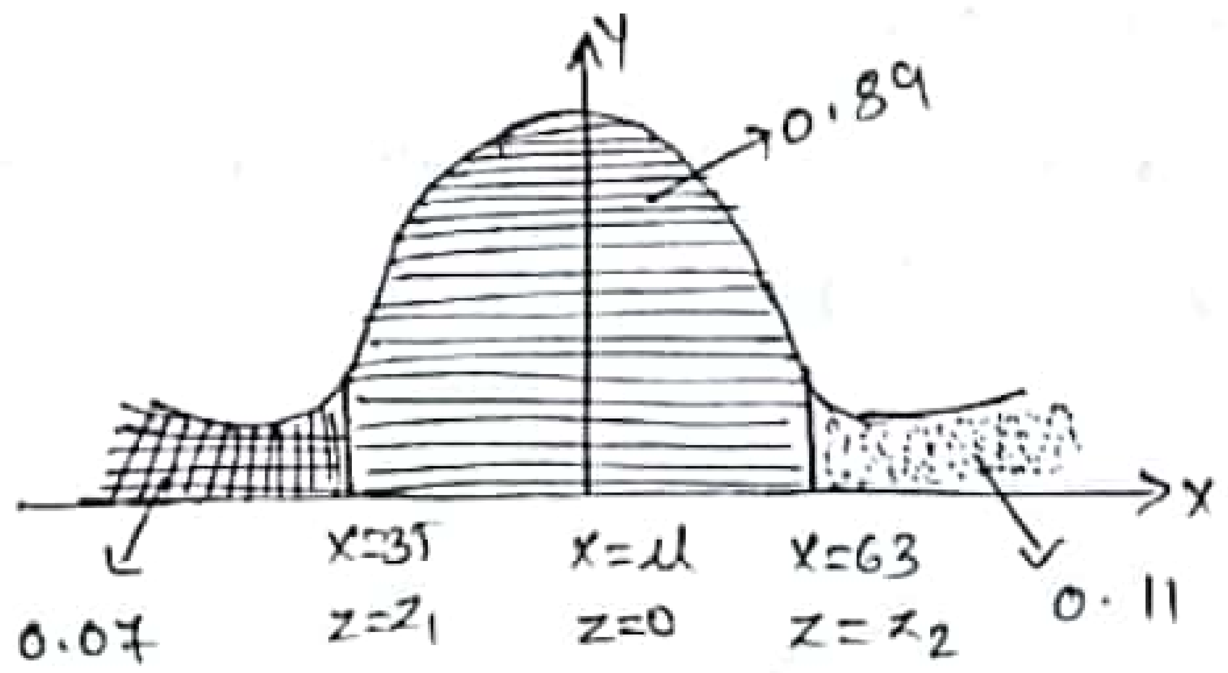
89% of items are under 63

$$P(X<63) = 89\% = 0.89$$

$$P(X\geq 63) = 1 - P(X<63)$$

$$= 1 - 0.89$$

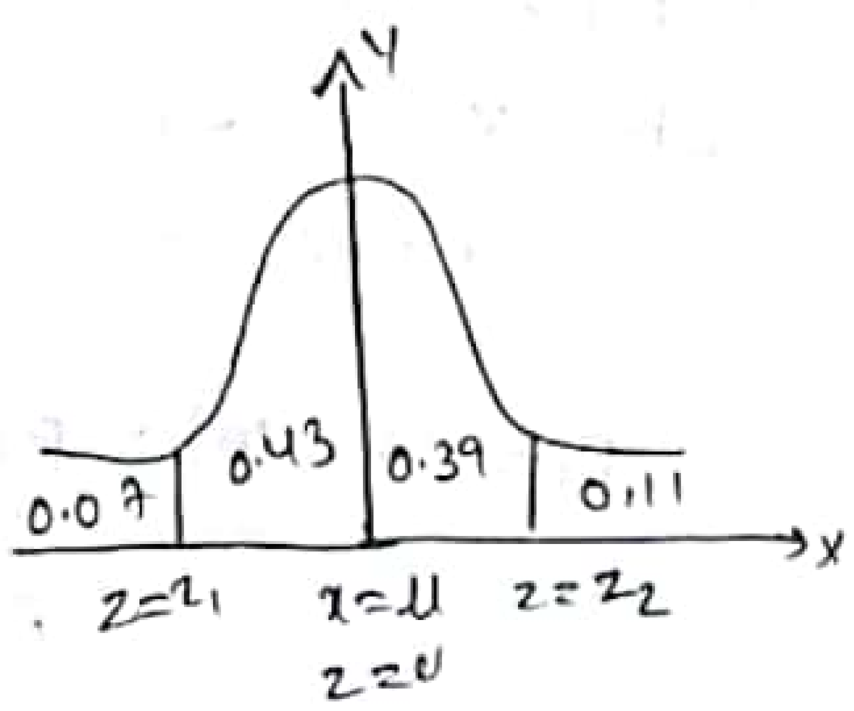
$$= 0.11$$



$$P(0<z<z_1) = \frac{1}{2} - P(X<35)$$

$$= 0.5 - 0.07$$

$$= 0.43$$



$$P(0<z<z_2) = \frac{1}{2} - P(X\geq 63)$$

$$= 0.5 - 0.11$$

$$= 0.39$$

we know that $P(0<z<z_1) = A(z_1)$

$$0.43 = A(z_1)$$

$$\boxed{z_1 = 1.48}$$

$$P(0<z<z_2) = A(z_2)$$

$$0.39 = A(z_2)$$

$$\boxed{z_2 = 1.23}$$

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we have $z = \frac{x-\mu}{\sigma}$

where $x = 35 \rightarrow z = \frac{35-\mu}{\sigma} = -z_1$ (say) $\rightarrow$ (1)

$x = 63 \rightarrow z = \frac{63-\mu}{\sigma} = z_2$ (say) $\rightarrow$ (2)

from (1) & (2)

$\frac{35-\mu}{\sigma} = -1.48$

$35 = \mu - 1.48\sigma \rightarrow$ (3)

$\frac{63-\mu}{\sigma} = 1.23$

$63 = \mu + 1.23\sigma \rightarrow$ (4)

from (3) & (4)

$35 = \mu - 1.48\sigma$

$63 = \mu + 1.23\sigma$

$(-) \quad (-) \quad (-)$

$+28 = +2.7\sigma$

$\sigma = \frac{28}{2.7}$

$\boxed{\sigma = 10.3321}$

substitute 'σ' in eq (3)

$35 = \mu - (1.48)(10.3321)$

$\mu = 35 + 15.2915$

$\boxed{\mu = 50.2915}$

$\therefore$ mean $= 50.2915$

$\therefore$ standard deviation $= 10.3321$

9. In a normal distribution 31% of item are under 45 & 8% of item are over 64 find mean & variance.

Sol: let 'μ' be the mean & 'σ^2' be the variance, 'σ' be the standard deviation of Normal distribution.

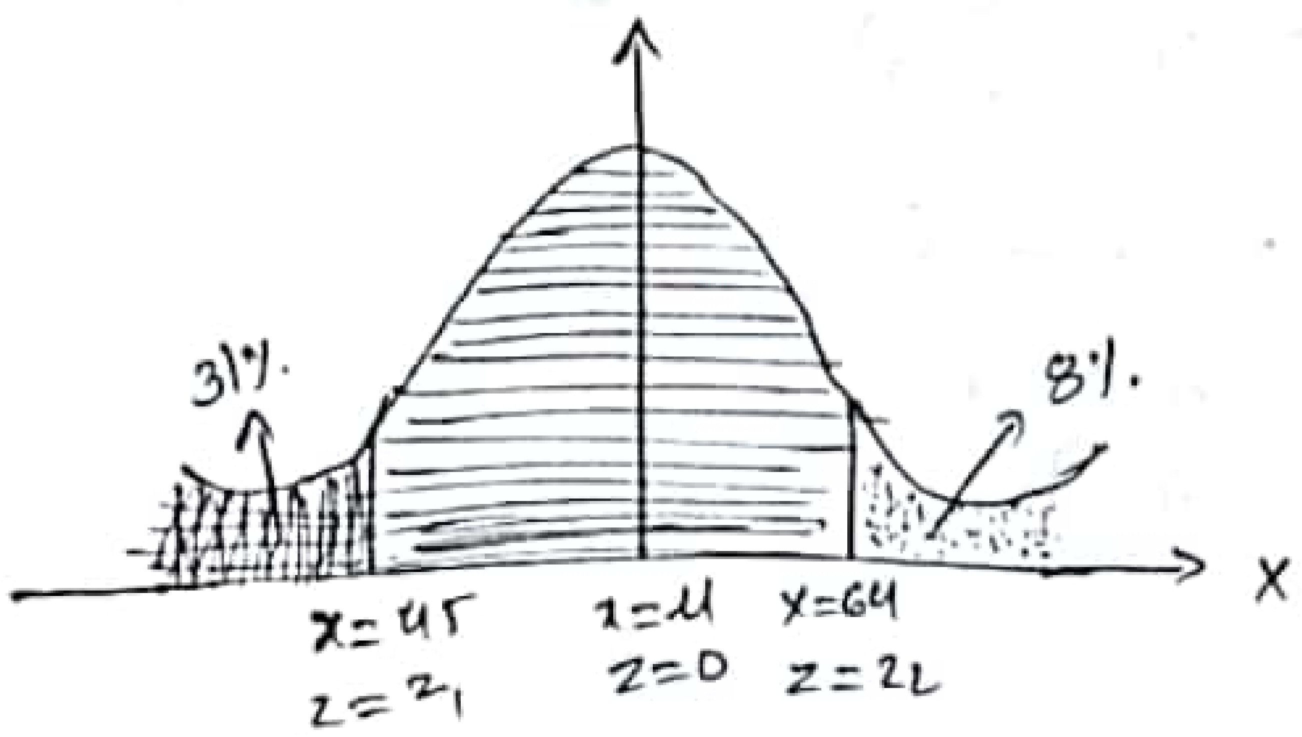
Given

31% of items are under 45

$P(x < 45) = 31\% = 0.31$

8% of items are over 64

$P(x > 64) = 8\% = 0.08$.



$$P(0 < z < z_1) = 0.5 - P(X \le 45)$$

$$A(z_1) = 0.5 - 0.31$$

$$A(z_1) = 0.19$$

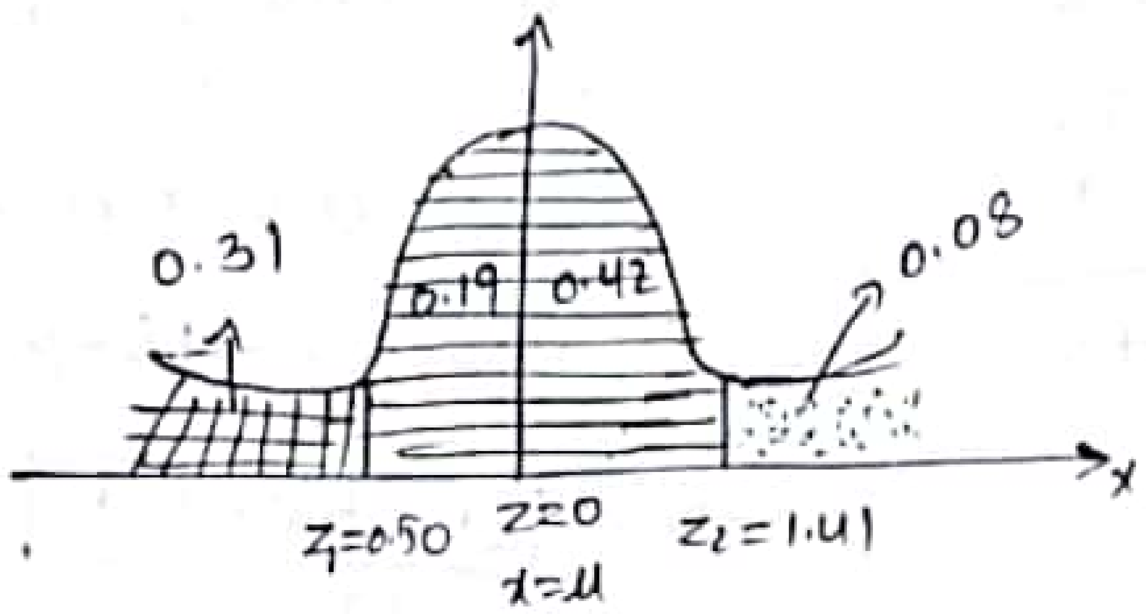
$$\boxed{z_1 = 0.50}$$

$$P(0 < z < z_2) = 0.5 - P(X \ge 64)$$

$$A(z_2) = 0.5 - 0.08$$

$$A(z_2) = 0.42$$

$$z_2 = 1.41.$$



we know that

$$z = \frac{x - \mu}{\sigma}$$

$$x = 45 \Rightarrow z = \frac{45 - \mu}{\sigma} = -z_1 \text{ (say)} \rightarrow (1)$$

$$x = 64 \Rightarrow z = \frac{64 - \mu}{\sigma} = z_2 \text{ (say)} \rightarrow (2)$$

from (1) & (2)

$$45 = \mu - (0.50)\sigma \rightarrow (3)$$

$$64 = \mu + (1.41)\sigma \rightarrow (4)$$

$$(-) \quad (-) \quad (-) \quad (-)$$

$$-19 = -(1.91)\sigma$$

$$\sigma = \frac{19}{1.91} \Rightarrow \boxed{\sigma = 9.9476}$$

$$\text{variance} = \sigma^2 = (9.9476)^2$$

$$\boxed{\sigma^2 = 98.9547}$$

substitute σ in eq (3)

$$45 = \mu - (0.50)(9.9476)$$

$$\mu = 45 + 4.9738$$

$$\boxed{\mu = 49.9738}$$

10. If 10% of probability for a normal distribution is below 35 and 5% above 90 then find mean & standard deviation?

Sol: Let $\mu \to$ be the mean & $\sigma \to$ standard deviation of the Normal distribution.

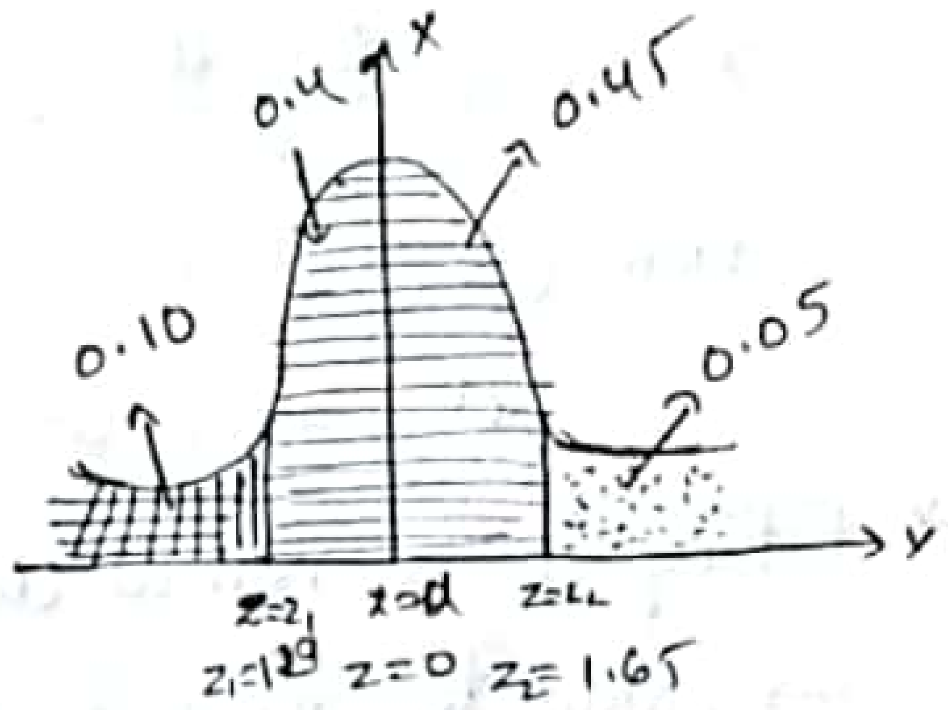
Given that

10% of probability is below 35

$P(X<35) = 10\% = 0.10$

5% of above 90

$P(X \geq 90) = 5\% = 0.05$



$P(0<z<z_1) = 0.5 - P(X<35)$

$A(z_1) = 0.5 - 0.10$

$\boxed{A(z_1) = 0.4} \Rightarrow \boxed{z_1 = 1.29}$

$P(0<z<z_2) = 0.5 - P(X \geq 90)$

$A(z_2) = 0.5 - 0.05$

$\boxed{A(z_2) = 0.45} \Rightarrow \boxed{z_2 = 1.65}$

we know that $z = \frac{x-\mu}{\sigma}$

$X = 35 \Rightarrow z = \frac{35-\mu}{\sigma} = -z_1 \Rightarrow 35 = \mu - z_1\sigma \to ①$

$X = 90 \Rightarrow z = \frac{90-\mu}{\sigma} = z_2 \Rightarrow 90 = \mu + z_2\sigma \to ②$

from ① & ②

$35 = \mu - 1.29\sigma$

$90 = \mu + 1.65\sigma$

$(-)$

$-55 = -2.94\sigma$

$\boxed{\sigma = 18.7075}$

$\boxed{\mu = 59.1326}$

11. The marks obtained in statistics in a certain examination found to be normally distributed. If 15% of the students ≥ 60 marks & 40% of student < 30 marks. Then find mean & 'σ'

Sol: $\mu \to$ mean ; $\sigma \to$ standard deviation.

Given that $P(X<30) = 0.40$

$P(X \geq 60) = 0.15$

Now $P(0<z<z_1) = 0.5 - 0.4 = 0.1$

$A(z_1) = 0.1$

$z_1 = 0.26$

$P(0<z<z_2) = 0.5 - 0.15$

$A(z_2) = 0.35$

$z_2 = 1.04$

w.r.t $z = \frac{x-\mu}{\sigma}$

$x = 30 \Rightarrow \frac{30-\mu}{\sigma} = -z_1 \Rightarrow 30 = \mu - (0.96)(\sigma) \rightarrow (1)$

$x = 60 \Rightarrow \frac{60-\mu}{\sigma} = z_2 \Rightarrow 60 = \mu + 1.04(\sigma) \rightarrow (2)$

from (1) & (2)

$\mu = 36, \quad \sigma = 23.0769.$

* Fitting of a normal distribution:-

→ calculate mean & standard deviation.

$$\mu = \frac{\Sigma x f(x)}{\Sigma f(x)} \; ; \; N = \Sigma f(x)$$

$$\sigma = \sqrt{\frac{\Sigma x^2 f(x)}{\Sigma f(x)} - \left[\frac{\Sigma x \cdot f(x)}{\Sigma f(x)}\right]^2}$$

→ calculate standard normal variable $z_i = \frac{x_i - \mu}{\sigma}$.

x_i = true lower limit of given x

→ Find area from $z=0$ to $z=z_i$

→ Find difference between areas of 2 successive limits

→ Expected frequency $= f(x) = N \cdot A$ where A is area.

Problems:-

*1. Fit a normal distribution to the following data.

class interval	60-62	63-65	66-68	69-71	72-74
frequency	5	18	42	27	8.

Sol:-

$$\mu = \frac{\Sigma x f(x)}{\Sigma f(x)}$$

$$= \frac{(61)5 + (64)18 + (67)(42) + 70(27) + 73(8)}{5+18+42+27+8}$$

$$= \frac{6745}{100}$$

$$\boxed{\mu = 67.45}$$

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$$\sigma^2 = \frac{\Sigma x^2 f(x)}{\Sigma f(x)} - \left[\frac{\Sigma x f(x)}{\Sigma f(x)}\right]^2$$

$$= \frac{(61)^2 5 + (64)^2 18 + (67)^2 (42) + (70)^2 (27) + (73)^2 8}{5+18+42+27+8} - (67.45)^2$$

$$= \frac{455803}{100} - (67.45)^2$$

$$\sigma^2 = 8.5275$$

$$\sigma = \sqrt{\sigma^2}$$

$$= \sqrt{8.5275}$$

$$\boxed{\therefore \sigma = 2.920}$$

we know that $z_i = \frac{x_i - \mu}{\sigma}$

class interval	observed $f(x)$	True lower limit	z_i	Area	difference b/w 2 limits areas	Expected $f(x)$
		59.5	-2.723	0.4967	-0.0422	4.22
60 - 62	5	62.5	-1.695 ~~0.4967~~	0.4545	-0.2091	20.91
63 - 65	18	65.5	-0.668	0.2454	-0.1086	10.86
66 - 68	42	68.5	0.359	0.1368	0.2794	27.94
69 - 71	27	71.5	1.387	0.4262	0.0758	7.58
72 - 74	8	74.5	2.414	0.4920		

2. Fit a normal distribution to the following data.

x	2	4	6	8	10
$f(x)$	1	4	6	4	1

Sol: $\mu = \frac{\Sigma x f(x)}{\Sigma f(x)}$ $\quad N = 1+4+6+4+1 = 16$

$$= \frac{2(1)+4(4)+6(6)+8(4)+10(1)}{1+4+6+4+1} = \frac{96}{16} = 6 \quad \boxed{\mu = 6}$$

$$\sigma^2 = \frac{\Sigma x^2 f(x)}{\Sigma f(x)} - \mu^2$$

$$= \frac{2^2(1)+4^2(4)+6^2(6)+8^2(4)+10^2(1)}{1+4+6+4+1} - 6^2 = 4$$

$$\therefore \sigma = \sqrt{\sigma^2} = \sqrt{4} = 2 \quad \boxed{\therefore \sigma = 2}$$

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x	observed $f(x)$	True lower limit	z_i	Area	difference areas b/w 2 limits	Expected $f(x)=N.A_i$
		1	-2.5	-0.4933		
2	1				~~0.0606~~ 0.0606	0.9696
		3	-1.5	-0.4332		
4	4				~~0.2416~~ 0.2416	3.8656
		5	-0.5	-0.1916		
6	6				~~0.3~~ 0.3832	6.1312
		7	0.5	0.1916		
8	4				0.2416	3.8656
		9	1.5	0.4332		
10	1				0.0606	0.9696
		11	2.5	0.4938		

3. Fit a normal distribution to the following data

class	5-9	10-14	15-19	20-24	25-29	30-34	35-39
frequency	1	10	37	36	13	2	1

Sol:- $\mu = \dfrac{\Sigma x f(x)}{\Sigma f(x)}$ $\quad N = \Sigma f(x)$

$$= \frac{7(1) + (12)10 + 17(37) + 22(36) + 27(13) + 32(2) + 37(1)}{1+10+37+36+13+2+1}$$

$$= \frac{2000}{100}$$

$\boxed{\mu = 20}$ $\quad \boxed{\therefore N = 100}$

$$\sigma^2 = \frac{\Sigma x^2 f(x)}{\Sigma f(x)} - \left[\frac{\Sigma x f(x)}{f(x)}\right]^2$$

$$= \frac{7^2(1) + (12)^2 10 + (17)^2(37) + (22)^2 36 + (27)^2(13) + (32)^2(2) + (37)^2(1)}{1+10+37+36+13+2+1} - (20)^2$$

$$= \frac{42500}{100} - 400$$

$\boxed{\sigma^2 = 25}$

$\sigma = \sqrt{\sigma^2} = \sqrt{25}$ $\quad \boxed{\sigma = 5}$

class	observed f(x)	True lower limit (x_1, x_2)	z_i (z_1, z_2)	b/w Area difference areas	Expected $f(x) = NA_i$
5-9	1	(4.5, 9.5)	(-3.1, -2.1)	0.0169	1.69
10-14	10	(9.5, 14.5)	(-2.1, -1.1)	0.1178	11.78
15-19	37	(14.5, 19.5)	((-1.1), -0.1)	0.3245	32.45
20-24	36	(19.5, 24.5)	(-0.1, 0.9)	0.3557	35.57
25-29	13	(24.5, 29.5)	[(0.9), 1.9]	0.3554	35.54
30-34	2	(29.5, 34.5)	[1.9, 2.9]	0.0268	02.68
35-39	1	(34.5 - 39.5)	[2.9, 3.9]	0.0019	0.19

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