UNIT II - LASERS & FIBER OPTICS

LASERs

LASER is an acronym for Light Application by Stimulated Emission of Radiation.

- Albert Einstein in 1917 theoretically proved the process of stimulated emission.
- In 1954, C.H. Townes operated a microwave device MASER – "Microwave Amplification by Stimulated Emission of Radiation".
- In 1960, T.H. Maiman first achieved laser action at optical frequency using Ruby laser.
- In 1961, A. Javan developed first gas laser that is He-Ne laser.
- In 1964, C.K.N. Patel developed CO_2 laser.
- In 1962, Nathan and Hall developed GaAs semiconductor laser called diode laser.

Characteristics of laser:

The important characteristics of laser are:

1. High monochromaticity
2. High directionality
3. High intensity (brightness)
4. High coherence

1. Monochromaticity: Monochromaticity is represented in terms of line width or spectral width of the source $\Delta\nu$ which is the frequency spread of a spectral line. Frequency spread $\Delta\nu$ is related to wavelength spread $\Delta\lambda$ by

$$\Delta\lambda = \left(\frac{-c}{\nu^2}\right)\Delta\nu$$

For white light source $\Delta\lambda \approx 300$ nm

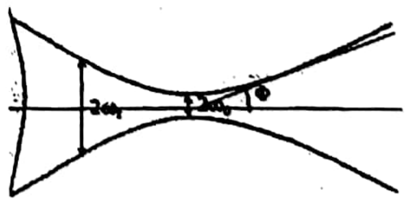
For gas discharge lamp $\Delta\lambda \approx 0.01$ nm

For Laser light source $\Delta\lambda \approx 0.001$ nm

So laser light is highly monochromatic light.

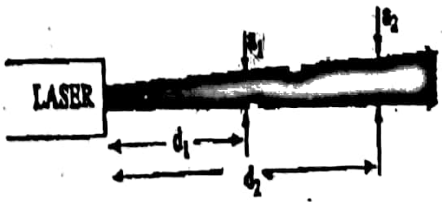
~~**2. Directionality**: The degree of directionality is represented in terms of beam divergence.~~

The beam divergence Φ is given in terms of the minimum spot size ω_o as $\Phi = (1.22\lambda/2\omega_o)$



For a typical small laser the beam divergence is about 1 milli radian. Laser beam increases in size about 1mm for every meter of beam travel for a beam divergence of 1 milli radian.

At d1 and d2 distances from the laser window, if the diameter of the spots are measured to be a1 and a2 respectively then the angle of divergence in degrees can be expressed as $\Phi = \frac{(a_2-a_1)}{2(d_2-d_1)}$.



For laser light divergence $\phi = 10^{-3}$ *radians*. Since the divergence of light is very low, we say that the laser light is highly directional.

3. Intensity: In laser many numbers of photons are in phase with each other, the amplitude of the resulting wave becomes 'nA' and hence the intensity of laser is proportional to $(nA)^2$. Thus due to coherent addition of amplitude and negligible divergence the intensity increases enormously. 1mw He-Ne laser is highly intense than the sun.

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4. Coherence: A predictable correlation of the amplitude and phase at any one point with other point is called coherence. There are two types of coherences.

i) Temporal coherence

ii) Spatial coherence

Temporal coherence (or longitudinal coherence):

The predictable correlation of amplitude and phase between any two points on a wave train is called temporal coherence.

Spatial coherence (or transverse coherence):

The predictable correlation of amplitude and phase at one point on the wave train w. r .t another point on a second wave, then the waves are said to be spatially coherence.

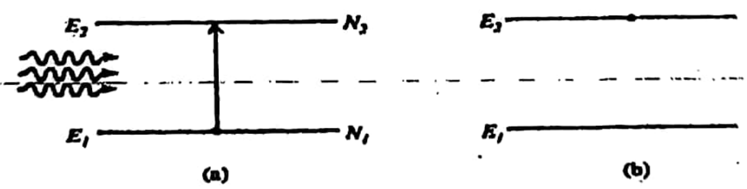
Laser light is highly coherent because two waves in laser beam maintain high degree of spatial and temporal coherences.

Interaction of radiation with matter:

In lasers the interaction between matter and light is of three different types.

1. Stimulated absorption (or) induced absorption.
2. Spontaneous emission.
3. Stimulated emission

1. Stimulated absorption (or) induced absorption:

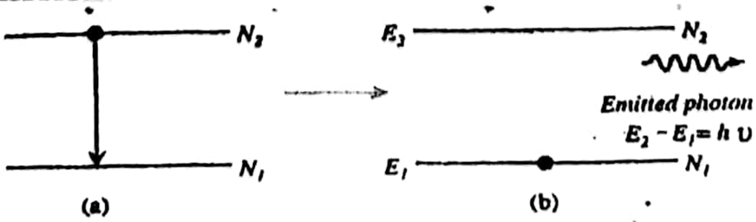


(a) Before absorption *(b) After absorption*

$$A + h\upsilon \rightarrow A^*$$

Initially let the atom is in ground state E_1. When a photon of energy $h\upsilon = E_2 - E_1$ incident on this atom then the atom is excited to higher energy level (excited state) E_2 by absorbing incident energy. This is called stimulated absorption.

2. Spontaneous emission:



(a) Before emission (b) After emission

$$A^* \rightarrow A + h\upsilon$$

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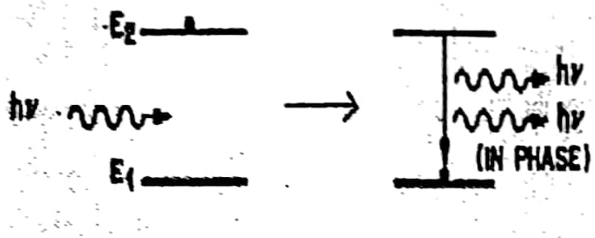
Initially the atom is in the excited state. This excited atom comes to ground state by emitting photon of Energy $h\upsilon = E_2 - E_1$ on its own after the lifetime of excited state (i.e. 10^{-8}sec). This type of emission without any external agency is called spontaneous emission. Different atoms of the medium emit photons at different times and in different directions. Hence there is no phase relationship among the emitted photons. So they are incoherent and intensity of light is very low.

Ex: Light from ordinary source (tube light, electric bulb, candle flame).

3. Stimulated emission:



$$A^* + h\upsilon \rightarrow A + 2h\upsilon$$

As shown in fig. a photon having energy $h\upsilon = E_2 - E_1$ incident on an atom present in the excited state. The atom is stimulated to make transition to ground state and gives off a photon of energy $h\upsilon = E_2 - E_1$. The emitted photon is in phase with the incident photon. The two photons travel in the same direction and they possess same energy and frequency. They are coherent and the intensity of light is very high. This type of forceful emission is called stimulated emission.

Ex: Light from LASER.

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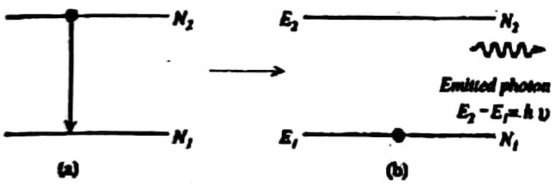
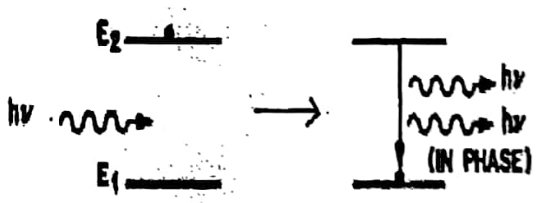
Differences between spontaneous and stimulated emission:

Spontaneous emission	Stimulated emission
1. Emission of light photon takes place immediately (10^{-8} sec) without any inducement during the transition of atoms from higher energy level to the lower energy level.	1. Emission of light photon takes place by the inducement of a photon having energy equal to the energy difference between transition energy levels $h\upsilon = E_2 - E_1$.
2. Single photon is emitted.	2. Two photons with same energy are emitted.
3. The energy of emitted photon is equal to the energy difference of two energy levels.	3. The energy of the emitted photons is double the energy of stimulated photons.
4. This was postulated by Bohr.	4. This was postulated by Einstein.
5. Incoherent radiation	5. Coherent radiation
6. Less intensity	6. High intensity
7. Polychromatic radiation	7. Monochromatic radiation
8. Less directionality	8. High directionality
9. More angular spread during propagation	9. Less angular spread during propagation
10. Ex. Light from sodium or mercury vapor lamp.	10. Ex. Light from LASER



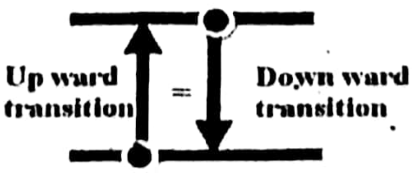
(a) Before emission (b) After emission



Einstein coefficients:

It establishes the relation between the three coefficients i.e. stimulated absorption, spontaneous emission and stimulated emission coefficients.

Let N_1 be the number of atoms per unit volume with energy E_1 and N_2 be the number of atoms per unit volume with energy E_2 and ρ(ν) be the density of photons. Let 'n' be the number of photons per unit volume at frequency 'ν' such that $E2 - E1 = h\vartheta$. The energy density of interacting photons $\rho(\nu) = nh\vartheta$. When the photons interact with ground level atoms, both upward (absorption) and downward (emission) transition occurs.
At the equilibrium the upward transitions must be equal to downward transitions.



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Relation between Einstein coefficients (B_{12} , A_{12} & B_{21}):

Upward transition (Stimulated absorption):

Stimulated absorption rate depends upon the number of atoms available in the lowest energy state as well as the energy density of photons.

$$\text{Stimulated absorption rate } \alpha.\rho(\upsilon)$$

$$\text{Stimulated absorption rate } \alpha.N_1$$

$$\text{Stimulated absorption rate } = B_{12}\rho(\upsilon)N_1 \text{..............(1)}$$

Where B_{12} is the Einstein coefficient of stimulated absorption.

Downward transition (Emission):

Spontaneous emission: The atom in the excited state returns to ground state emitting a photon of energy (E) = E2 – E1 =hv, spontaneously known as spontaneous emission. The spontaneous emission rate depends up on the number of atoms present in the excited state.

$$\text{Spontaneous emission rate } \alpha.N_2$$

$$\text{Spontaneous emission rate } = A_{21}N_2 \text{.................(2)}$$

Where A_{21} is the Einstein coefficient of spontaneous emission.

Stimulated emission: The atom in the excited state can also returns to the ground state by applying external energy, thereby emitting two photons which are having same energy as that of incident photon. This process is called as stimulated emission. Stimulated emission rate depends upon the number of atoms available in the excited state as well as the energy density of incident photons.

$$\text{Stimulated emission rate } \alpha N_2$$

$$\text{Stimulated emission rate } \alpha.\rho(\upsilon)$$

$$\text{Stimulated emission rate } = B_{21}\rho(\upsilon)N_2 \text{..............(3)}$$

Where B_{21} is the Einstein coefficient of stimulated emission

At thermal equilibrium

Up ward transition = Down ward transition

∴ Stimulated absorption = spontaneous emission + stimulated emission

$$B_{12}\rho(\upsilon)N_1 = A_{21}N_2 + B_{21}\rho(\upsilon)N_2$$

$$(B_{12}N_1 - B_{21}N_2)\rho(\upsilon) = A_{21}N_2$$

$$\rho(\upsilon) = \frac{A_{21}N_2}{B_{12}N_1 - B_{21}N_2}$$

$$\rho(\upsilon) = \frac{A_{21}N_2}{B_{21}N_2(\frac{B_{12}N_1}{B_{21}N_2} - 1)}$$

$$\rho(\upsilon) = \frac{A_{21}}{B_{21}(\frac{B_{12}N_1}{B_{21}N_2} - 1)}$$

According to Boltzman distribution law

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$$N_1 = N_0 e^{-E_1/kT}$$

$$N_2 = N_0 e^{-E_2/kT}$$

$$\frac{N_1}{N_2} = e^{E_2 - E_1/KT} = e^{\frac{h\upsilon}{KT}}$$

$$\therefore \rho(\upsilon) = \frac{A_{21}}{B_{21}(\frac{B_{12}}{B_{21}} e^{\frac{h\upsilon}{KT}} - 1)} \text{.................}(4)$$

According to the plank energy distribution law

$$\rho(\upsilon) = \frac{8\pi h\upsilon^3}{c^3(e^{\frac{h\upsilon}{KT}} - 1)} \text{.................}(5)$$

On comparing Eq (4) *and* (5)

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3} \text{.................}(6)$$

$$\frac{B_{12}}{B_{21}} = 1 \Rightarrow B_{12} = B_{21} \text{...........}(7)$$

The Eq (6) & (7) *are called Einstein Relations*

Conditions for light Amplifications:

At thermal equilibrium,

$$\frac{\text{Stimulated emission}}{\text{Spontaneous emission}} = \frac{B_{21}N_2\rho(v)}{A_{21}N_2} = \frac{B_{21}}{A_{21}}\rho(v) \text{........}(8) \quad \text{And}$$

$$\frac{\text{Stimulated emision}}{\text{Stimulated absorption}} = \frac{B_{21}N_2\rho(v)}{B_{12}N_1\rho(v)} = \frac{N_2}{N_1} \text{........}(9)$$

From Eq(8) & (9) *we concluded that for* getting *light amplification(gettingLaser)*

1. *The radiation density* $\rho(v)$ *is to be made larger.*
2. $N_2 > N_1$ (*Population inversion*)

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Population inversion:

Let us consider two level energy system of energies E_1 and E_2 as shown in figure. Let N_1 and N_2 be the populations of energy levels E_1 and E_2. The number of atoms present in an energy level is known as population of that energy level. At ordinary conditions, the population in the ground or lower state is always greater than the population in the excited or higher states.

The stage of making, population of higher energy level greater than the population of lower energy level is called population inversion.

According to Boltzmann's distribution the population of an energy level E_i at temperature T is given by

$$N_i = N_0 e^{\left(\frac{-E_i}{KT}\right)}$$

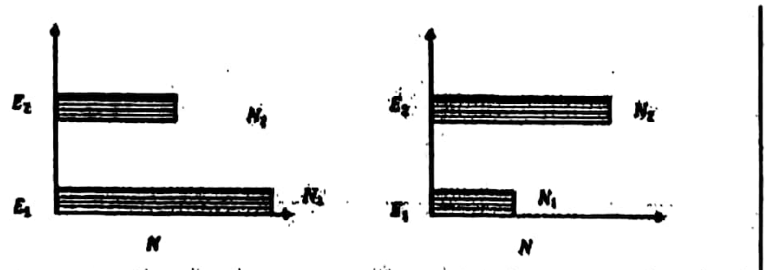
Where N_0 *is* the population of the lower level or ground state and K is the Boltzmann's constant.

From the above equation the population of energy levels E1 & E2 are given by

$$N_1 = N_0 e^{-E_1/kT}$$

$$N_2 = N_0 e^{-E_2/kT}$$

At ordinary conditions $N_1 > N_2$ i.e., the population in the lower state is always greater than the population in the higher states. The stage of making, population of higher energy level greater than the population of lower energy level ($N_2 > N_1$) is called population inversion or inverted population.



At normal conditions $N_1 > N_2$ After population inversion is achieved $N_2 > N_1$

Conditions for population inversion are:

a) The system should posses at least a pair of energy levels ($E_2 > E_1$), separated by an energy equal to the energy of a photon (hν).

b) There should be a continuous supply of energy to the system such that the atoms must be raised continuously to the excited state.

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Pumping mechanisms or Methods to produce population inversion:

The process of raising the particles from ground state to excited state to achieve population inversion is called pumping.

Methods of achieving population inversion:

Excitation of atom can be done by number of ways. The most commonly used excitation methods are

1. Optical pumping
2. Electrical discharge pumping
3. Chemical pumping
4. Injection current pumping

Optical pumping:

- Optical pumping is a process in which light is used to raise the atoms from a lower energy level to higher level to create population inversion.
- Optical pumping is used in solid state laser.
- The solid materials have very broad absorption band, so sufficient amount of energy is absorbed from the emission band of flash lamp to create population inversion.
- Xenon flash tubes are used for optical pumping.

Ex: Ruby laser, Nd:YAG Laser, Nd: Glass Laser

Electrical discharge pumping:

- In electric discharge pumping, atoms are excited into excited sate by collisions with fast moving electrons in electric discharge tube.
- Electrical discharge pumping is used in gas lasers.
- Since gas lasers have very narrow absorption band, so optical pumping is not suitable for gas lasers.
- Ex: He-Ne laser, CO_2 laser, Argon-ion laser, etc

Chemical pumping:

- In this method the chemical energy released during the chemical process, that energy will excite the atoms to higher level and create population inversion.
- Whenever hydrogen reacts with fluorine, it liberates lot of heat energy. By utilizing this heat energy the atoms excites into higher states and create population inversion.
- Ex: HF and DF lasers.

Injection current pumping:

- This pumping mechanism is used in semiconductor lasers.
- In semiconductor lasers, by passing high currents across the junction, the population inversion will create.
- In semiconductors lasers the population inversion always creates among majority and minority charge carriers.
- Ex: InP and GaAs lasers

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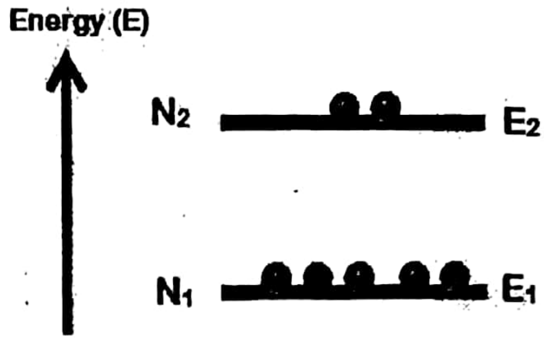
Pumping schemes or Levels of laser:

Two level pumping scheme:

Consider a system with two energy levels E_1 and E_2 in which the total number of atoms N_0 are distributed. Let N_1 and N_2 be the number of atoms in these states.



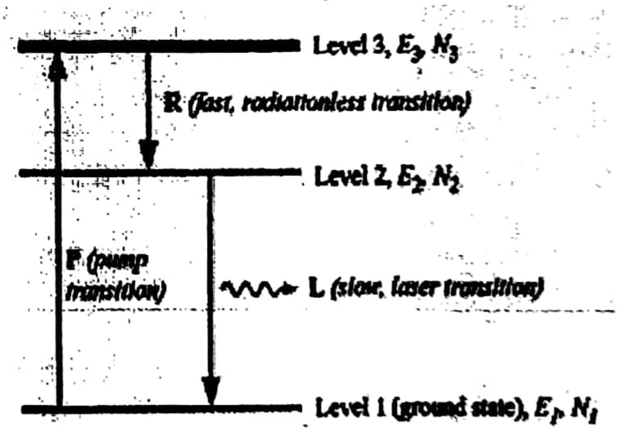
$$N_1 = N_0 e^{-E_1/kT}$$ and
$$N_2 = N_0 e^{-E_2/kT}$$
$$\frac{N_1}{N_2} = e^{E_2-E_1/KT} = e^{\frac{h\nu}{KT}}$$

If $h\nu > kT$ then $N_1 > N_2$
If $h\nu < kT$ then $N_1 = N_2$
If $h\nu = kT$ then $N_1 = e.N_2$

That is the number of atoms in lower energy state is always greater than or equal to number of atoms in the higher energy state. Therefore in a two level system population inversion cannot be achieved and hence it is not suitable for the production of laser.

Three level pumping scheme:

In three level pumping scheme, three energy levels are involved in lasing action. They are the ground state energy (E_1), excited state energy (E_3) and metastable state energy (E_2) as shown in fig.



In the laser, initially all the lasing atoms are present in the ground state energy level,(E_1). Light photons of energy (E_3-E_1) is made to incident on the ground state lasing atoms. The atoms in the ground state energy level will absorb the energy of photons and make transition to excited state energy (E_3). The atoms in the excited energy level (E_3), will remain for very short duration of the order of 10^{-8} sec and make non radiative transaction to metastable state E_2. The atoms will remain for longer duration (10^{-3} sec) in metastable state. If photons are continuously supplied to ground to ground state atoms then more number of atoms makes transitions to excited state and then to metastable state fastly, so that more number of atoms will accumulate in metastable than in ground state. This is known as population inversion. Now a chance proton of transition from (E_2-E_1) = $h\nu$ can trigger stimulated emission as shown in fig. The atoms present in E_2 energy level will make stimulated transition to E_1 energy level producing laser.

This scheme requires very high pumping power. Once the stimulated emission starts then quickly the metastable state becomes empty, hence population inversion ends: The system has to wait till the population inversion is to be re-established. Because of this, three level laser produces pulses of laser light.

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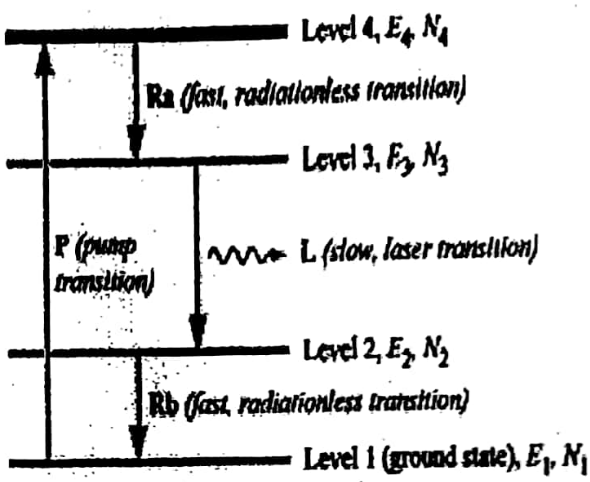
Four level pumping scheme:

In four level pumping scheme, four energy levels are involved in lasing action. They are the ground state energy (E_1), excited state energy (E_4) and metastable state energy (E_3) and lower lasing level (E_2).as shown in fig.



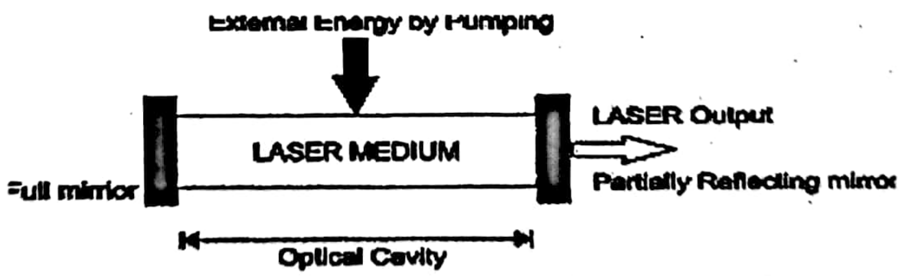
In the laser, initially all the lasing atoms are present in the ground state energy level,(E_1).light photons of energy E_4-E_1 is made to incident on the ground state lasing atoms. The atoms in the ground state energy level will absorb the energy of photons and make transition to excited state energy (E_4).the atoms in the excited energy level (E_4), will remain for very short duration of the order of 10^{-9} sec and quickly drop down to the metastable state (E_3).after some time population inversion between the states E_3 and E_2.A spontaneous emitted photon of energy E_3-E2= hv can initiates stimulated emissions. This makes transition of atoms from E_3 to E_2 level. The atoms in the E_2 level will undergo non radiative transition to ground state E_1 level.

In this scheme the lower lasing level is nearly vacant, hence less pumping power is sufficient to get population inversion. The four level laser produces continuous wave (CW) output laser.

Construction and components of laser:

Any laser system consists of three important components.

1. Source of energy.
2. Active medium.
3. Optical resonant cavity.



Components of LASER

Source of energy: To get laser emission, first we must have population inversion in the system. The source of energy supplies sufficient amount of energy to the active medium by which the atoms (or) molecules in it can be excited to the higher energy level. As a result we get population inversion in an active medium.

Active Medium (or) Laser Medium: This is the medium where stimulated emission of radiation takes place. After receiving energy from the source, the atoms or molecules get

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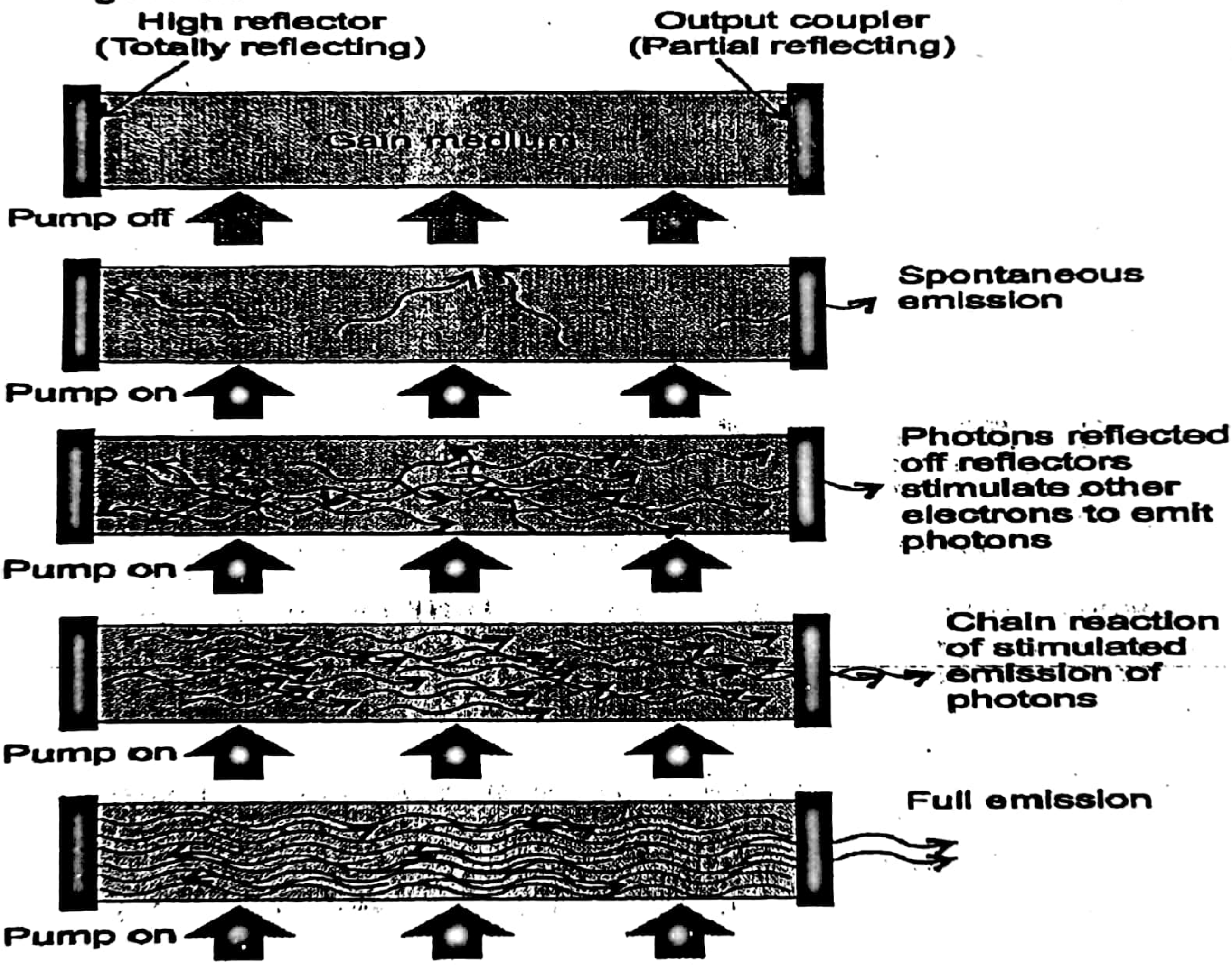
excited to higher energy levels. While transition to a lower energy level, the emitted photons start the stimulated emission process which result in laser emission.

Optical cavity (or) Resonator: The active medium is enclosed between a fully reflecting mirror and a partially reflecting mirror. These mirrors constitute an optical cavity (or) resonator. The reflective portion of the mirrors reflects the incident radiation back into the active medium. These reflected radiations enhance the stimulated emission process with in the active medium. As a result we get high- intensity monochromatic and coherent laser light through the non-reflecting portion of the mirror.

Lasing action:



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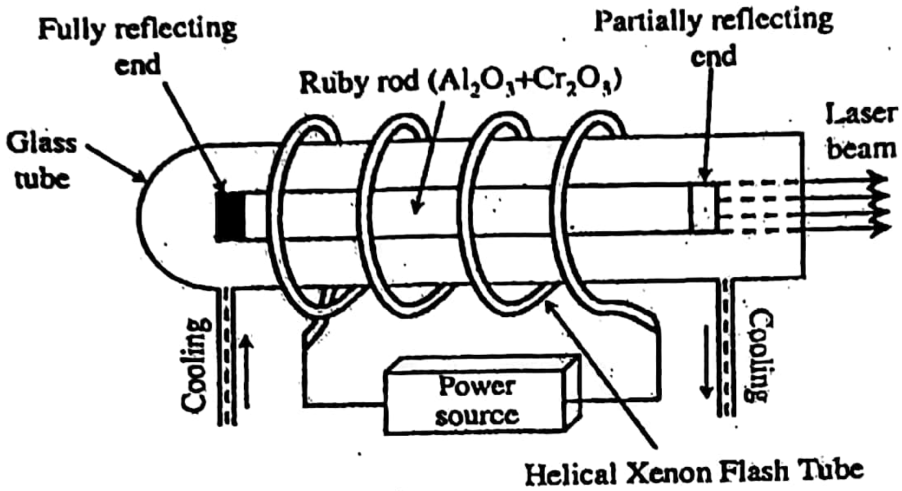
Ruby Laser:

Ruby laser is a three level solid state laser constructed by T.H. Maiman in the year 1960.



A schematic diagram of Ruby laser

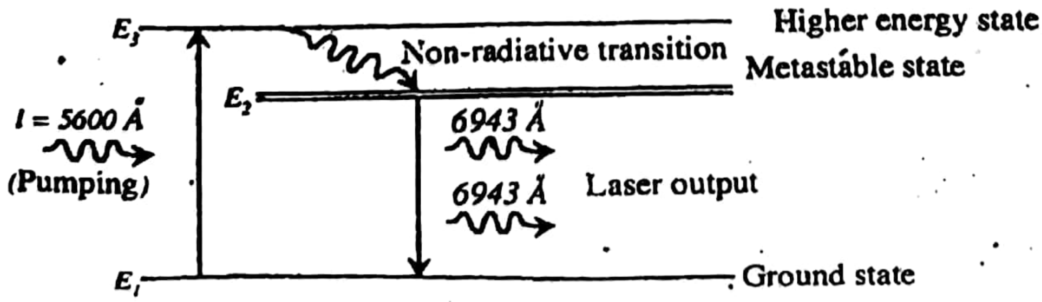
Basically Ruby ($Al_2O_3Cr_2O$) is a Al_2O_3 crystal containing 0.05% of chromium atoms.

1. Active medium: Ruby rod.
2. Active centers: Chromium atoms
3. Pumping method : Optical pumping
4. Exciting source: Helical Xenon flash lamp
5. Cavity Resonator: The partially silvered face and fully silvered face of ruby rod
6. Power output: 10^4-10^5 Watts
7. Nature of output: Pulsed
8. Wavelength: 6943 Å

Construction:

Ruby rod of 4 cm length and 5 mm diameter is kept inside the glass tube. The end faces are grounded and polished such that the end faces are exactly parallel to each other. One of the faces is silvered fully to get full reflection and other partially silvered to get partial reflection. The two silver faces acts as resonating cavity. The Ruby rod is surrounded by helical Xenon flash tube which acts as exciting source. It provides optical pumping for Cr^{3+} ions to move to the higher energy levels. Glass tube is provided with cold water circulation to prevent the damage due to heating from Xenon flash lamp.

Working (Energy level diagram):



Energy level diagram of Chromium ion

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The energy levels of Cr^{3+} ions in the crystal lattice are shown in the fig. When xenon flash lamp is switched on, it produces very high intensity radiation. The Cr^{3+} ions in the ground state E_1 absorb sufficient energy of wavelength 5600 Å and excited to higher energy state E_3. The Cr^{3+} ions make spontaneous transitions $E_3 \rightarrow E_1$ and $E_3 \rightarrow E_2$ through non-radiative transitions. Since E_2 is metastable state and close to E_3 the transition $E_3 \rightarrow E_2$ is more dominant than the transition $E_3 \rightarrow E_1$. In the course of time the population in E_2 increases and therefore population inversion is achieved between E_2 and E_1.

After completion of life time in E_2 one of the Cr^{3+} ions decay spontaneously to E_1 giving out a photon of wavelength 6943 Å. The emitted photons moving parallel to the active system are reflected back by the silvered surfaces. These photons trigger stimulated emission for Cr^{3+} ions in E_2, hence the rate of stimulated emission increases. When the intense beam is produced, it escapes from semi-silvered surface in the form of laser of wavelength 6943 Å.

Applications of Ruby laser: Ruby laser is used in

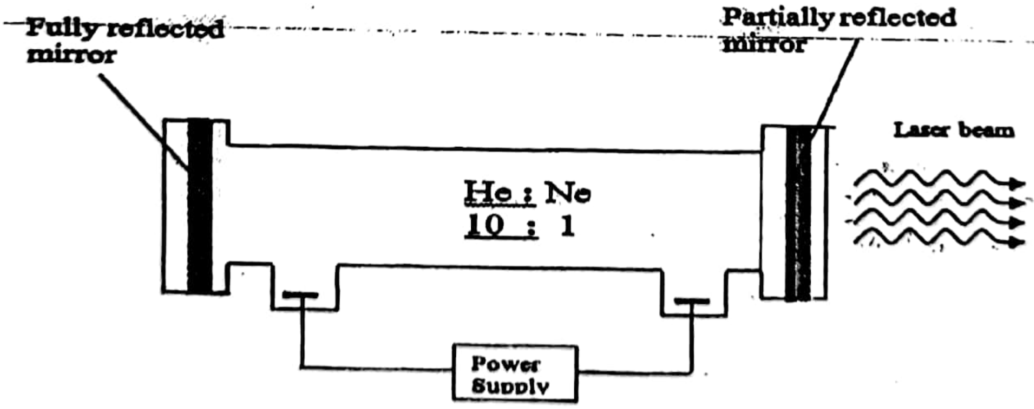
1. Pulsed holography
2. LIDAR
3. Remote sensing
4. Opthalmology
5. Drilling small areas.

Drawbacks of Ruby laser:

1. The laser beam is not continuous and contains pluses.
2. The efficiency of laser beam is very poor.
3. The laser requires high pumping power to achieve population inversion.

He - Ne laser:

He-Ne laser was the first gas laser fabricated by Ali Javan and others in 1961.



1. Active medium: mixture of He and Ne in the ratio 10:1
2. Active centers: Ne atoms
3. Pumping method: Electrical pumping
4. Exciting Source: Electric discharge
5. Resonating Cavity: Partially & fully reflecting Mirrors
6. Power output: 0.5-50 mW
7. Nature of output: Continuous waveform
8. Wavelength of laser: 6328 Å.

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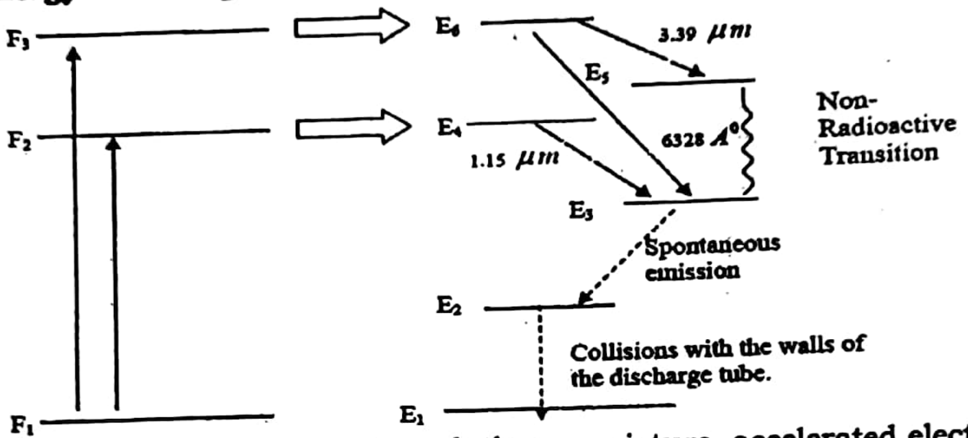
Construction:
In He-Ne gas laser, He and Ne gases are taken in the ratio 10:1 in the gas discharge tube of length 80 cm and diameter 1 cm made up of quartz. Two reflecting mirrors are fixed on either ends of the discharge tube, in that, one is partially reflecting and the other is fully reflecting which serve as optical cavity or resonator. The out power of these lasers depends on the length of the discharge tube and pressure of the gas mixture.
When the two windows are set at Brewster's angle, the output laser is linearly polarized.

Working (Energy level diagram):



When the electric discharge passed through the gas mixture, accelerated electrons collide with He atoms and excite them to higher levels F_2 and F_3. These states are metastable states, so the He atoms stay longer time in these states. During this time He atoms collide with Ne atoms in the ground level E_1 and exchange energy through collisions. This results in the excitation of Ne atoms to the levels E_4 and E_6 and de-excitation of He atoms to ground state F_1.

Due to the continuous excitation of Ne atoms, population inversion is achieved between the higher levels E_4 (E_6) and lower levels E_3 (E_5). The transitions $E_6 \rightarrow E_5$: λ=3.39 μm and $E4 \rightarrow E_3$: λ = 1.15 μm corresponds to IR region. The transition $E_6 \rightarrow E_3$ corresponds to visible red light λ= 6328 Å. The Ne atoms present in the E_3 level are de-excited into E_2 level, by spontaneously emitting a photon of wavelength 6000 Å. When a narrow discharge tube is used, the Ne atoms present in the level E_2 collide with the walls of the tube and get de-excited to ground level E_1. The excitation and de-excitation of He and Ne atoms is a continuous process and thus it gives continuous laser radiations.

Advantages:

1. He-Ne laser emits continuous laser radiation.
2. Due to the setting of end windows at Brewster's angle, the output laser is linearly polarized.
3. Gas lasers are more monochromatic and directional when compared with the solid state laser.

Applications: He-Ne laser is applied in the following fields,

1. Interferometry
2. Laser printing
3. Bar-code reading
4. In metrology in surveying, alignment etc.,
5. In three dimensional recording of objects called holography.

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Applications of laser:

Due to high intensity, high mono-chromaticity and high directionality of lasers, they are widely used in various fields like

1. Communication 2. Computers 3. Chemistry 4. Photography
5. Industry 6. Medicine 7. Military 8. Scientific research

1. Communication:

- In case of optical communication, semiconductors laser diodes are used as optical sources.
- More channels can be sent simultaneously, signal cannot be tapped as the band width is large, more data can be sent.
- A laser is highly directional and less divergence, hence it has greater potential use in space crafts and submarines.

2. Computers:

- In LAN (local area network), data can be transferred from memory storage of one computer to other computer using laser for short time.
- Lasers are used in CD-ROMS during recording and reading the data.

3. Chemistry:

- Lasers are used in molecular structure identification.
- Lasers are also used to accelerate some chemical reactions.
- Using lasers, new chemical compound can be created by breaking bonds between atoms or molecules.

4. Photography:

- Lasers can be used to get 3-D lens less photography.
- Lasers are also used in the construction of holograms.

5. Industry:

- Lasers can be used to blast holes in diamonds and hard steel.
- Lasers are also used as a source of intense heat.
- Carbon dioxide laser is used for cutting drilling of metals and nonmetals.
- High power lasers are used to weld or melt any material.
- Lasers are also used to cut teeth in saws and test the quality of fabric.

6. Medicine:

- Pulsed neodymium laser is employed in the treatment of liver cancer.
- Argon and carbon dioxide lasers are used in the treatment of liver and lungs.
- Lasers used in the treatment of Glaucoma.

7. Military:

- Lasers can be used as a war weapon.
- High energy lasers are used to destroy the enemy air-crafts and missiles.
- Lasers can be used in the detection and ranging likes RADAR.

8. Scientific research: Lasers are used

- in the field of 3D-photography.
- in Recording and reconstruction of hologram.
- to create plasma.
- to produce certain chemical reactions.
- in Raman spectroscopy to identify the structure of the molecule.

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Questions

1. What are the characteristics of laser beam? Explain.
2. (a) With a neat diagram explain (i) absorption (ii) spontaneous emission (iii) stimulated emission
   (b) Obtain the relation between Einstein's coefficients.
3. What is population inversion and how can it be achieved.
4. Describe the various methods to achieve population inversion in lasers.
5. Explain the three level and four level laser systems. What are the advantages of four level laser system over three level laser system?
6. With neat diagram, describe the construction and working of Ruby laser.
7. Distinguish between spontaneous and stimulated emissions.
8. Explain the construction and working of He-Ne laser with a neat diagram.

Problems

1) Evaluate the wavelength of radiation given out by a laser with $(E_2-E_1) = 3eV$.
   (Hint: $[E_2-E_1] = hC/\lambda$; Ans: 4410 Å)

2) The He-Ne laser emits laser beam of wavelength 632.8 nm. Calculate the energy difference in eV between the two energy levels of the Neon atom.
(Hint: $[E_2-E_1] = hC/\lambda$; Ans : 1.964 eV)

3) Calculate the relative population in the laser transition levels in a ruby laser in thermal equilibrium (T = 300 K). The wavelength is 6943 Å.
(Hint: $N_1/N_2 = \exp (E_2-E_1)/KT$; Ans: 1.064×10^{30})

Objective Questions

1. The spontaneous emission process depends upon
   a. Properties of energy states E1 and E2 b. incident energy supplied to the atoms
   c. both a and b d. none
2. The induced energy require for emission process
   a. absorption b. spontaneous emission
   c. Stimulated emission d. none
3. The number of atoms in the lower energy state is __________ that of the higher.
   a. more than b. less than c. equal to d. none
4. The life time of excited state is
   a. 10^{-3} sec **b. 10^{-8} sec** c. 10^{3} sec d. none
5. The life time of an atom in the metastable state is
   a. 10^{-3} sec b. 10^{-8} sec c. 10^{3} sec d. None
6. Laser stands for
   a. light amplification by spontaneous emission o radiation
   b. light modification by spontaneous emission of radiation
   c. light amplification by stimulated emission of radiation
   d. none
7. The laser beam has characteristics of
   a. perfect mono chromatic b. high intensity c. high coherence **d. all the above**
8. The mono chromatic light is
   a. laser b. sun light c. high coherence d. all the above
9. The amount of time in which an atom stays in the excited state is
   a. life time b. relaxation time c. met stable time d. none

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10. The expression for number of atoms in any energy state at temperature T is

a. $N=N_0\, e^{\frac{-E_l}{kT}}$ b. $N=N_0\, e^{\frac{E_l}{kT}}$ c. $N_0=N\, e^{\frac{-E_l}{kT}}$ d. none

11. The relation A_{21}/B_{21} = ____________

a. $\frac{8\pi h\vartheta^3}{c^3}$ b. $\frac{8\pi\vartheta^3}{c^3}$ c. $\frac{8\pi h\vartheta^3}{c^2}$ d. $\frac{8\vartheta^3}{c^3}$

12. The probable rate of transition for spontaneous emission of radiation is

a. $(P_{21})_{sp} = A_{21}$ b. $(P_{21})_{sp} = A_{12}$ c. $(P_{21})_{sp} = U(v)A_{21}$ d. $(P_{21})_{sp} = B_{21}$

13. If A21 is the Einstein coefficient for spontaneous emission then the spontaneous emission left time τ_{sp} is

a. A_{21} **b. $\frac{1}{A_{21}}$** c. $\frac{1}{\sqrt{A_{21}}}$ d. $\frac{1}{A_{21}^2}$

14. If is the Einstein coefficient for stimulate emission then the stimulated emission lift time τ_{sp} is

a. B_{21} b. $\frac{1}{B_{21}^2}$ c. $\frac{1}{\sqrt{B_{21}}}$ **d. $\frac{1}{B_{21}}$**

15. The stimulated emission of radiation was forwarded by

a. Bohr b. Sommerfeld **c. Einstein** d. Maimann

16. If N_1 and N_2 be the number of atoms in the lower and higher energy states E_1 and E_2 respectively then the condition for population inversion

a. $N_1 > N_2$ **b. $N_2 > N_1$** c. $N_1 = N_2$ d. $N_1 \geq N_2$

17. The Ruby laser was invented by

a. Einstein b. Bohr **c. Maiman** d. Ali Javan

18. Ruby laser is

a. Two level laser **b. Three level** c. Four level d. None

19. The active atoms in the ruby laser

a. Al^{3+} **b. Cr^{3+}** c. Fe^{3+} d. None

20. The emitted wave length of the laser beam in ruby laser is

a. 6328 Å b. 6493 Å **c. 6943 Å** d.5600 Å

21. The He-Ne laser is

a. Solid state laser **b. Continuous laser** c. Pulsed Laser d. None

22. The ratio of Ne and He atoms in He-Ne laser is

a. 10:1 b. 1:2 **c. 1:10** d. 1:1

23. The emitted wave length of the laser beam in He-Ne laser is

a. 6328 Å b. 6493 Å c. 6943 Å d.5600 Å

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FIBER OPTICS

Introduction:

Fiber optics is a branch of physics which deals with the transmission and reception of light waves using optical fibers which acts as a guiding media. The transmission of light waves by optical fiber was first demonstrated by John Tyndall in 1870.

Advantages of optical fibers:

- Higher information carrying capacity.
- Light in weight and small in size.
- No possibility of internal noise and cross talk generation.
- No hazards of short circuits as in case of metals.
- Can be used safely in explosive environment.
- Low cost of cable per unit length compared to copper or G.I cables.
- No need of additional equipment to protect against grounding and voltage problems.
- Nominal installation cost.
- Using a pair of copper wires only 48 independent speech signals can be sent simultaneously whereas using an optical fiber 15000 independent speech signals can be sent simultaneously.

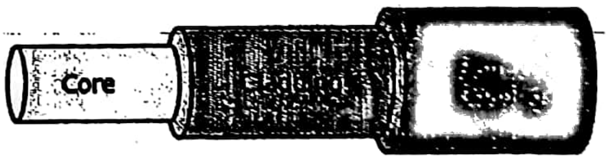
Optical Fiber:

Optical fiber is a long, thin transparent dielectric material made up of glass or plastic. It carries electromagnetic waves of optical frequencies (visible and infrared) from one end of the fiber to the other end by means multiple total internal reflections. Thus optical fiber works as guiding medium in optical communication systems.

Construction:

An optical fiber is a very thin, flexible transparent material made with plastic or glass. It has cylindrical shape consisting of three layers or sections

1) Core
2) Cladding
3) Sheath or outer jacket or buffer jacket



Parts of an Optical fiber

1) Core: Optical fiber consists of an inner cylindrical material made up of glass or plastic called core. Light is transmitted within the core which has refractive index n_1. It is a denser medium. Core is made up of silica (SiO_2).

2) Cladding: The core is surrounded by a cylindrical shell of glass or plastic called cladding. It has refractive index n_2 which is less than the refractive index of core i.e., ($n_1 > n_2$). It acts as a rarer medium. Typical refractive index values are $n_1 = 1.48$ and $n_2 = 1.46$. The core diameter is ≈ 50 μm and the thickness of cladding is ≈ 1 to 2 wavelengths of light propagate through the fiber. To lower the refractive index of cladding the silica is doped with phosphorous or bismuth material.

3) Sheath or outer or buffer Jacket: The cladding is surrounded by a polyurethane jacket called sheath. This layer protects the fiber from the surrounding atmosphere. It provides necessary toughness and tensile strength to the fiber.

Many fibers are grouped to form a cable. A cable may contain one to several hundred such fibers.

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Working Principle of Optical Fiber:

Optical fiber works on the principle of total internal reflection. The light launched inside the core through its one end propagates to the other end due to total internal reflection at the core and cladding interface.

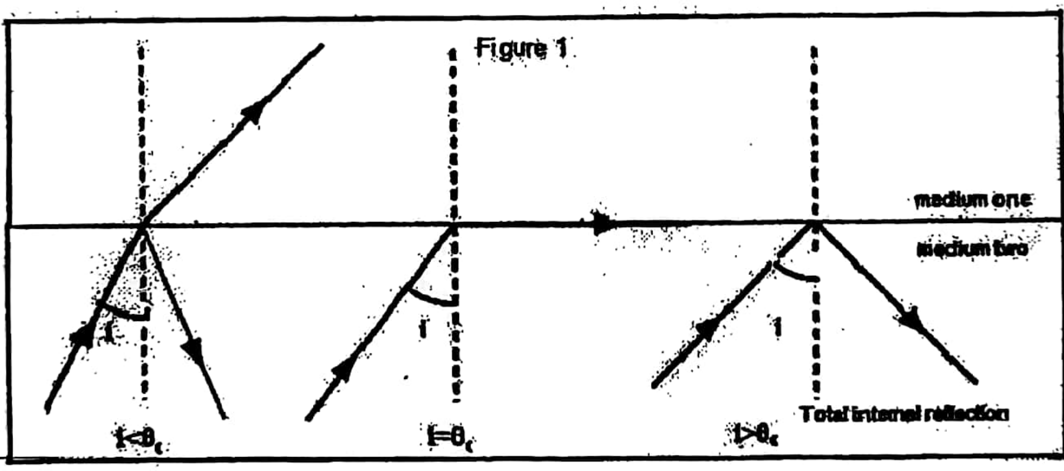
Total internal reflection:

Conditions for total internal reflection:

1) The light ray should move from denser to rarer medium.
2) The refractive index of core must be greater than cladding i.e. $n_1 > n_2$
3) The angle of incidence (i) must be greater than the critical angle (θ_C) i.e. $i > \theta_C$.
4) The critical angle $\theta_C = \sin^{-1}(\frac{n_2}{n_1})$

Explanation:

Let us consider a denser medium and a rarer medium of refractive indices n1 and n2 respectively and n1>n2. Let a light ray move from denser to rare medium with 'i' as the angle of incidence and 'r' as angle of refraction. The refracted ray bends away from the normal as it travels from denser to rarer medium with increase of angle of incidence 'i'.



In this we get three cases

Case-1: When $i < \theta_C$, Then the light ray refracts into rarer medium as shown in figure.

Case-2: When $i = \theta_C$, then the light ray traverses along the two media as shown in figure. For the two media, applying Snell's law

$$n_1 \text{ Sin } i = n_2 \text{ Sin } r$$

At critical angle $i = \theta_C$ and $r = 90^o$

$$n_1 \text{ Sin } \theta_C = n_2 \text{ Sin } 90^o$$

$$n_1 \text{ Sin } \theta_C = n_2$$

$$\text{Sin } \theta_C = \frac{n_2}{n_1}$$

For air $n_2 = 1$

$$\theta_C = \sin^{-1}(\frac{1}{n_1})$$

Case-3: When $i > \theta_C$, then the light ray reflected back into the same medium as shown in figure. This phenomenon is called total internal reflection.

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Acceptance angle, Acceptance and Numerical aperture:

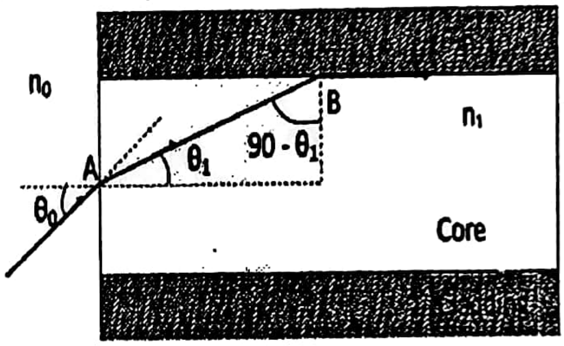
Acceptance angle:

Definition: The maximum angle of incidence at the core of an optical fiber so that the light can be guided though the fiber by total internal reflection is called as acceptance angle.

Let n_0, n_1 and n_2 be the refractive indices of air, core and cladding media. Let a light ray OA is incident on the interface of air medium and core medium with an angle of incidence θ_0 then the light ray refracts into the core medium with an angle of refraction θ_1 and the refracted ray AB is again incident on the interface of core and cladding with an angle of incident $(90^0 - \theta_1)$.

If $(90^0 - \theta_1)$ is equal to the critical angle of core and cladding media then the ray travels along the interface of core and cladding along the path BC. If the angle of incident at the interface of air and core $\theta_1 < \theta_0$, then $(90^0 - \theta_1)$ will be greater than the critical angle. Therefore, the total internal reflection takes place.

Applying Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$ at point A



$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\sin\theta_0 = \frac{n_1}{n_0}\sin\theta_1 \text{ ------- (1)}$$

Applying Snell's law at point B

$$n_1 \sin(90 - \theta_1) = n_2 \sin 90^0$$

$$n_1 \cos\theta_1 = n_2$$

$$\cos\theta_1 = \frac{n_2}{n_1}$$

$$\sin\theta_1 = \sqrt{1 - \cos^2\theta_1}$$

$$\sin\theta_1 = \sqrt{1 - \frac{n_2^2}{n_1^2}}$$

$$\sin\theta_1 = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \text{ --------(2)}$$

Substituting eq.(2) in eq.(1), we get

$$\sin\theta_0 = \frac{n_1}{n_0}\frac{\sqrt{n_1^2 - n_2^2}}{n_1}$$

$$\sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

$$\theta_0 = \sin^{-1}\left(\frac{\sqrt{n_1^2 - n_2^2}}{n_0}\right) \text{ ------ (3)}$$

For air medium $n_0 = 1$

Acceptance angle $\theta_0 = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$ ------ (4)

Acceptance cone: Rotating the acceptance angle about the fiber axis gives the acceptance cone of the fiber. Light launched at the fiber end within this acceptance cone alone will be accepted and propagated to the other end of the fiber by total internal reflection. Larger acceptance angles make launching easier.

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Numerical aperture:

Definition: Numerical aperture is defined as the light gathering capacity of an optical fiber and it is directly proportional to the acceptance angle.

Numerically it is equal to the sin of the acceptance angle.

NA = Sin (acceptance angle)

$$NA = Sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

Fractional change in refractive index $\Delta = (\frac{n_1 - n_2}{n_1})$ -----(5)

$$\Delta n_1 = (n_1 - n_2)$$

$$NA = \sqrt{(n_1 - n_2)(n_1 + n_2)}$$

$$NA = \sqrt{\Delta n_1 (n_1 + n_2)}$$

Since $n_1 \approx n_2$, So $(n_1 + n_2) = 2 n_1$

$$NA = \sqrt{\Delta n_1 (2n_1)}$$

$$NA = n_1\sqrt{2\Delta} \text{ ----------- (6)}$$

The above equation gives a relationship between numerical aperture and fractional change in relative refractive index.

Numerical aperture of the fiber depends only on refractive indices of the core and cladding materials and is not a function of fiber dimensions.

Classification of optical fibers:

Based on the refractive index of core medium, optical fibers are classified into two categories.

i. Step index fiber
ii. Graded index fiber

Based on the number of modes of transmission, optical fibers are classified into two categories

i. Single mode fiber
ii. Multi mode fiber

Based on the material used, optical fibers are may broadly classified into four categories

i. All glass fibers
ii. All plastic fibers
iii. Glass core with plastic cladding fibers
iv. Polymer clad silica fibers

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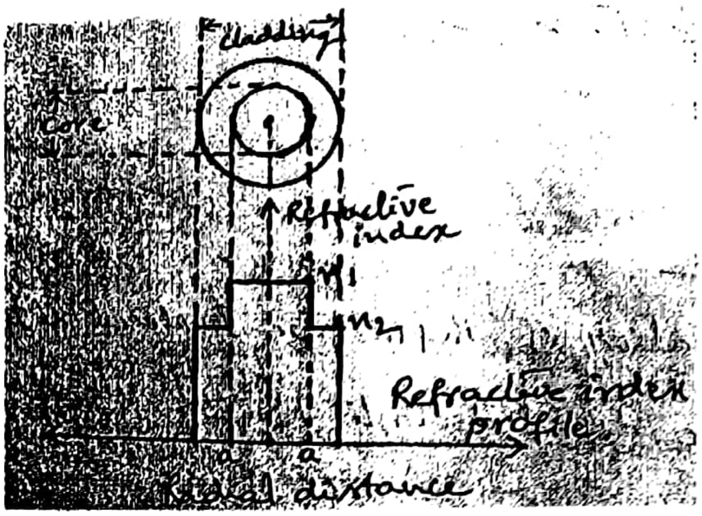
Step index optical fibers:

In step index fibers the refractive index is uniform throughout the core medium (n_1). As we go radially in the fiber the refractive index undergoes a step change ($n_1 \rightarrow n_2$) at the core-cladding interface. The variation refractive index with radial distance from the axis of the core is given by,

$n(r) = n_1$ for $r \leq a$

$n(r) = n_2$ for $r > a$

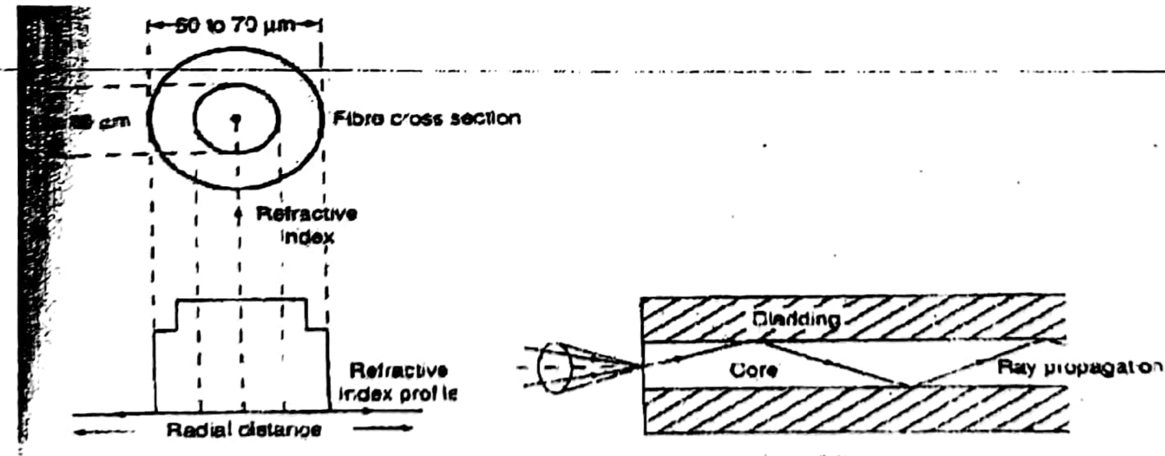


Based on the mode of propagation of light rays step index fibers are two types.

a) Single mode step index fibers
b) Multi mode step index fibers

Single mode step index fibers:

The core diameter of this fiber is about 8 to 10µm and outer diameter of cladding is 60 to 70 µm. There is only one path for ray propagation. So, it is called single mode fiber. The cross sectional view, refractive index profile and ray propagation are shown in below fig. In this fiber, the transmission of light is by successive total internal reflections i.e. it is a reflective type fiber. The shape of propagation of the optical signal is in zigzag manner. Nearly 80% of the fibers manufactured today in the world are single mode fibers. They are mainly used in submarine cable system. Lasers are used as light sources in these fibers.



(a) (b)

Multi mode step index fibers:

In this fiber the core and cladding diameters are much larger to have many paths for light propagation. The core diameter of varies from 50 to 200 µm and the outer diameter of cladding varies from 100 to 250 µm. The cross-sectional view, refractive index profile and ray propagations are shown in below fig. Light propagation in this fiber is by multiple total internal reflections i.e, it is a reflective type fiber.

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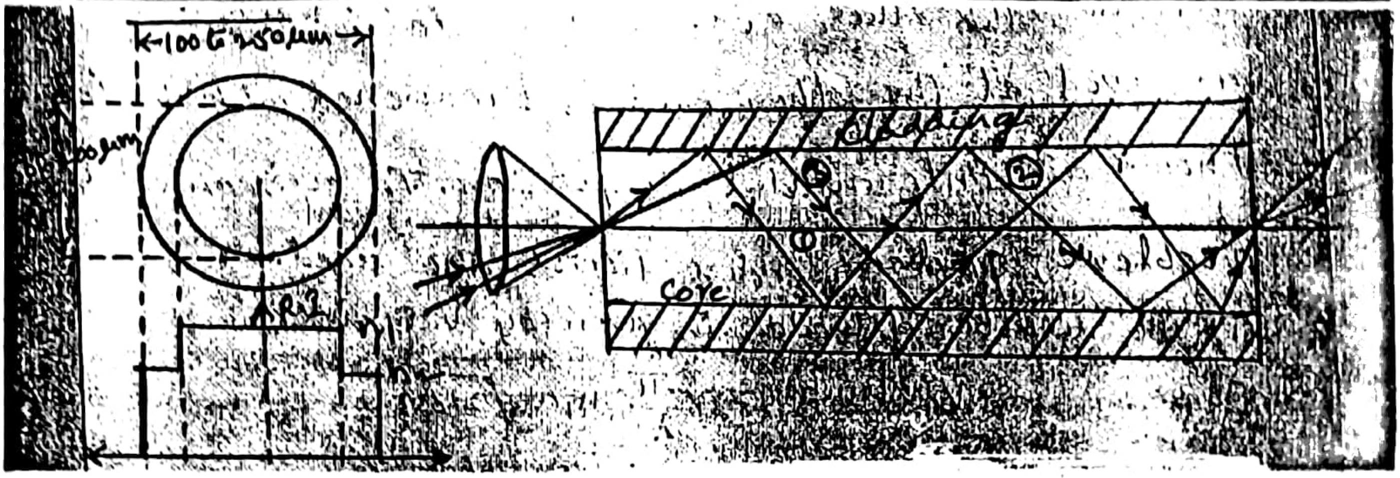


Generally the signal through the fiber is in digital form i.e. in the form of pulses representing 0s and 1s. From figure the ray 1 follows shortest path (i.e. travels along the axis of fiber) and the ray 2 follows longer path than ray 1. Hence the two rays reach the received end at different times. Therefore, the pulsed signal received at other end gets broadened. This is called intermodal dispersion. This difficulty is over come in graded index fibers.

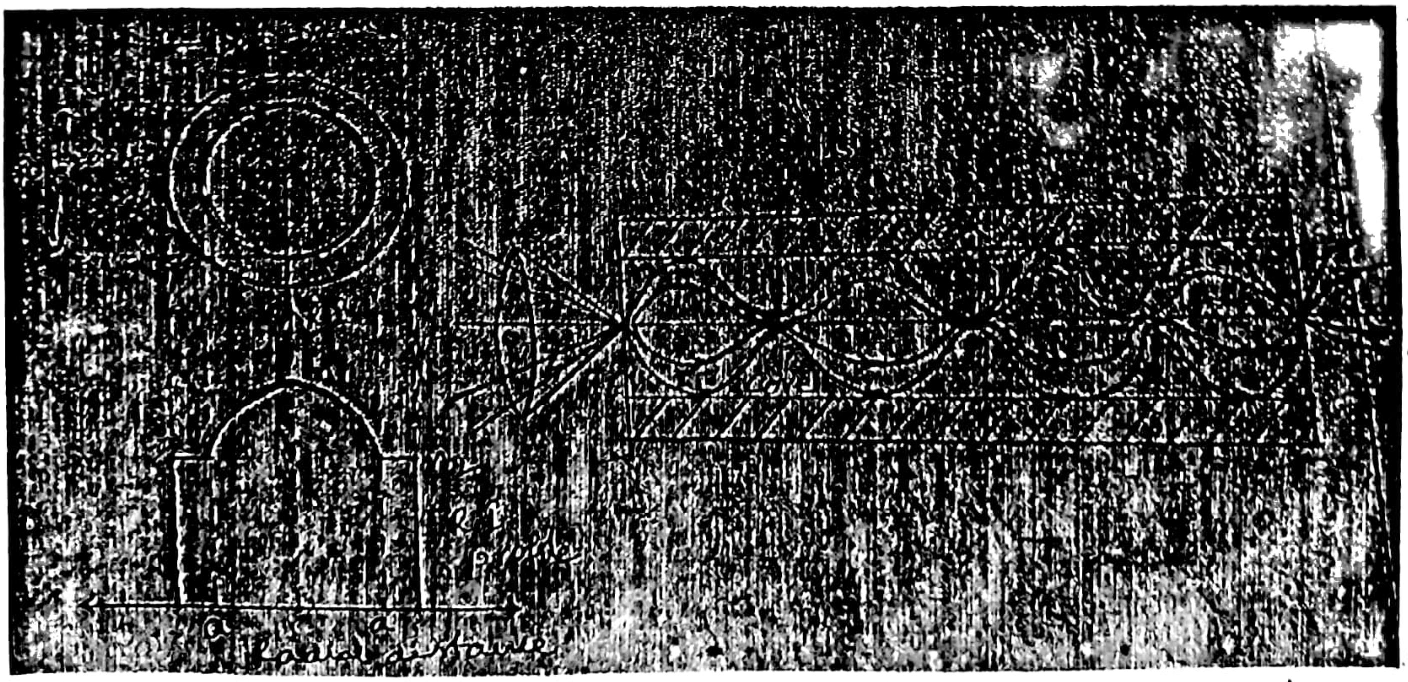
The number of modes is given by V-number = $\frac{2\pi}{\lambda} n_1 a \sqrt{2\Delta}$

Number of modes of propagation N= $(V^2)/2$

Numerical aperture NA= $\sqrt{n_1^2 - n_2^2}$

Graded index optical fibers:

In graded index fibers the refractive index decreases continuously from center radially to the surface of the core. The refractive index is maximum at the center and minimum at the surface of core. This fiber can be single mode or multimode fiber. The refractive index profile is circularly symmetric. The propagation of light ray is not due to total internal reflection but by refraction.



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The variation of refractive index with radial distance from the axis of core is given by,

$$n(r) = n_1 \sqrt{1 - 2\Delta\left(\left(\frac{r}{a}\right)^{\alpha}\right)} \text{ for } r \le a$$

$$n(r) = n_1 (1 - 2\Delta) \approx n_2 \text{ for } r > a$$

The cross sectional view, refractive index profile and ray propagation of multimode graded index fiber are shown in above fig. The diameter of core varies from 50 to 200μm and outer diameter of cladding varies from 100 to 250 μm.

In graded index fiber, the light propagation is by refraction. Light rays travel at different speed in different paths of the fiber. Near the surface of the core, the refractive index is lower, so rays near the outer surface travel faster than the rays travel at the center. Because of this, all the rays arrive at the receiving end of the fiber approximately at the same time. So the intermodal dispersion is reduced. This fiber is costly.

Either laser or LED is used as light source. It is used in the telephone trunk between central offices.

The number of modes is given by V-number = $\frac{2\pi}{\lambda} a\sqrt{n_1^2 - n_2^2}$

Number of modes of propagation N= $(V^2)/4$

Numerical aperture NA= $n_1 \sqrt{2\Delta\left(1 - \left(\frac{r}{a}\right)^{\alpha}\right)}$

Differences between step index fibers and graded index fibers:

Step index fiber	Graded index fiber
1. In step index fibers the refractive index of the core medium is uniform through and undergoes an abrupt change at the interface of core and cladding.	1. In graded index fibers, the refractive index of the core medium is varying in the parabolic manner such that the maximum refractive index is present at the center of the core.
2. The diameter of core is about 10 μm in case of single mode fiber and 50 to 200 μm in multi mode fiber.	2. The diameter of the core is about 50 μm.
3. The transmitted optical signal will cross the fiber axis during every reflection at the core cladding boundary.	3. The transmitted optical signal will never cross the fiber axis at any time.
4. The shape of propagation of the optical signal is in zigzag manner.	4. The shape of propagation of the optical signal appears in the helical or spiral manner
5. Attenuation is more for multi mode step index fibers but Attenuation is less in single mode step index fibers	5. Attenuation is very less in graded index fibers
6. Numerical aperture is more for multi mode step index fibers but it is less in single mode step index fibers	6. Numerical aperture is less in graded index fibers
7. Light source is laser	7. Light source is either laser or LED

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Differences between single mode and multi mode fibers:

Single mode fiber	Multi mode fiber
1. In single mode optical fibers only one mode of propagation is possible	1. In multi mode optical fibers many number of modes of propagation are possible.
2. The diameter of core is about 10 μm.	2. The diameter of core is 50 to 200 μm.
3. The difference between the refractive indices of core and cladding is very small.	3. The difference between the refractive indices of core and cladding is also large compared to the single mode fibers.
4. There is no dispersion, so these are more suitable for communication.	4. Due to multi mode transmission, the dispersion is large, so these fibers are not used for communication purposes.
5. The process of launching of light into single mode fibers is very difficult.	5. The process of launching of light into single mode fibers is very easy.
6. Fabrication is very difficult and the fiber is costly.	6. Fabrication is very easy and the fiber is cheaper.

Applications of optical fibers:

1. Communication applications
   i) In communication systems in exchange of information between different computers
   ii) For exchange of information in cable televisions, space vehicles, submarines etc.
2. Sensing applications:
   i) Displacement sensor
   ii) Fluid level detector or Liquid level sensor
   iii) Temperature and pressure sensor
   iv) Chemical sensors
3. Medical applications
   i) Optical fibers are used in medicine, in the fabrication of endoscopy for the visualization of internal parts of the human body, laparoscopic surgery and key hole surgery
   ii) In ophthalmology, an optical fiber guided laser beam corrects the defects in vision.
   iii) In cardiology for laser angioplasty
   iv) The heart, respiratory system and pancreas can be investigated.
   v) By measuring absorption of light by the blood the portion of hemoglobin in the blood can be estimated.
4. Military applications
   i) An aircraft, a ship or a tank needs tons of copper wire for wiring of the communication equipment, control mechanisms, panel illumination etc. Use of optical fiber will reduce the weight and further maintain true communication.
   ii) Fiber guided missiles are used in recent wars.

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Questions

1. Explain briefly 'basic principle of an optical fiber' or explain the principle of total internal reflection in optical fibers.
2. Explain the terms numerical aperture and acceptance angle and derive expressions for the acceptance angle and numerical aperture in terms of fractional change in refractive index.
3. Write a note on the applications of an optical fiber.
4. Explain how the optical fibers are classified.
5. Describe different types of fibers based on refractive index profiles and number of modes.
6. Distinguish between step index fiber and graded index fiber.
7. Write the differences between single mode and multimode optical fiber.
8. With the help of suitable diagram explain the principle, construction and working of an optical fiber as a waveguide.

Problems

1. An optical fiber has a core material of refractive index of 1.55 and cladding material of refractive index of 1.50. The light is launched it in air. Calculate the its numerical aperture.
2. Calculate the angle of acceptance of a given optical fiber, if the refractive indices of the core and cladding are 1.563 and 1.498 respectively.
3. The numerical aperture of an optical fiber is 0.39. If the difference in the refractive indices of the material of its core and cladding is 0.05. Calculate refractive index of the core material.
4. Calculate fractional change in refractive for a given optical fiber if the refractive indices of the core and the cladding are 1.563 and 1.498 respectively.

Objective Questions

1. Light ray travels through the optical fiber by the principle of
   a) Reflection b) Transmission **c) Total Internal Reflection** d) None

2. The expression for Numerical Aperture is
   a) $\sqrt{(n_1^2-n_2^2)}$ b) $\sqrt{(n_1^2 * n_2^2)}$ c) $\sqrt{(n_1 - n_2)}$ d) $\sqrt{(n_1^2/n_2^2)}$

3. The refractive indices of step index fiber are
   a) Varies **b) Constant** c) Non-uniform d) None

4. Numerical Aperture depends on
   a) Acceptance Angle b) Critical Angle c) Refractive Angle d) None.

5. In step index fiber, the signal travels in ---------- form
   a) Helical b) Parabolic **c) Zig-zag** d) None

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6. In graded index fiber, the signal travels in ---------- form
   a) Helical b) Parabolic c) Zig-zag d) None

7. If Δ is a fractional difference of refractive indices of fiber, then its numerical aperture is
   a) $n_2\sqrt{(2\Delta)}$ **b) $n_1\sqrt{(2\Delta)}$** c) $n_1\sqrt{\Delta}$ d) $n_2\sqrt{\Delta}$

8. The attenuation in an optical fiber is a function of
   a) fiber material only b) wave length of light only
   c) length of the fiber only **d) all the above**

9. The refractive index of the core is
   a) greater than the cladding. b) less than the cladding.
   c) equal to cladding d) all the above.

10. To overcome the problem of inter model dispersion one has to use
   a) multimode fiber b) step index fiber **c) graded index fiber** d) plastic fiber.

11. When more than one mode is propagating, how is it dispersed?
   a) Dispersion **b) Inter-modal dispersion** c) Material dispersion d) Waveguide dispersion

12. Which of the following has more distortion?
   a) Single step-index fiber b) Graded index fiber
   c) Multimode step-index fiber d) Glass fiber

13. What is the principle of fiber optical communication?
   a) Frequency modulation b) Population inversion
   c) Total internal reflection d) Doppler Effect

14. The refractive index of a cladding of a fiber with refractive index 1.5 and numerical aperture 0.244 is
   a) 1.4 b) 1.325 **c) 1.48** d) 1.656

15. Source of light in fiber optic system
   a) LED and laser diode b) LED c) laser diode d) photo diode

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