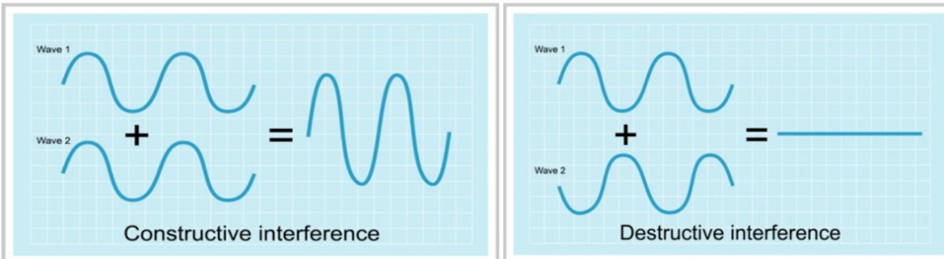
INTERFERENCE

Interference:

When two or more waves are superimposed then there is a modification of intensity or amplitude in the region of superposition. This modification of intensity or amplitude in the region of super position is called **Interference.**

When the resultant amplitude is the sum of the amplitudes due to two waves, the interference is known as **Constructive interference** and when the resultant amplitude is equal to the difference of two amplitudes, the interference is known as **destructive interference**.



PRINCIPLE OF SUPERPOSITION:

This principle states that the resultant displacement of particle in a medium acted upon by two or more waves simultaneously is the algebraic sum of displacements of the same particle due to individual waves in the absence of the others.

Consider two waves traveling simultaneously in a medium. At any point let y_1 be the displacement due to one wave and y_2 be the displacement of the other wave at the same instant.

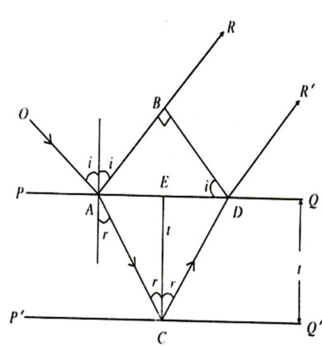
Then the resultant displacement due to the presence of both the waves is given by

$$y = y_1 \pm y_2$$

$+ve$ Sign has to be taken when both the displacements y_1 & y_2 are in the same direction

$-ve$ Sign' has to be taken when both the displacements y_1 & y_2 are in the opposite direction.

INTERFERENCE IN THIN FILMS



Consider a thin film of thickness t and refractive index μ. A ray of light OA incident on the surface at an angle i is partly reflected along AB and partly refracted into medium along AC, making an angle of refraction r .at C it is again partly reflected along CD. Similar refractions occur at E.

To find the path difference between the rays, draw DB perpendicular to AB

Then the path difference $= \mu(AC + CD) - AB$ …………….(1)

From triangle ACE

$$\cos r = \frac{CE}{AC}$$

$$AC = \frac{CE}{\cos r} = \frac{t}{\cos r} \quad \text{………………..(2)}$$

From triangle CDE

$$\cos r = \frac{CE}{CD}$$

$$CD = \frac{CE}{\cos r} = \frac{t}{\cos r} \quad \text{……………….(3)}$$

From triangle ABD

$$\cos(90 - i) = \frac{AB}{AD}$$

$$AB = AD\cos(90 - i) = 2AE\sin i \quad \text{………..(4)} \qquad (\because AD = 2AE)$$

FROM triangle ACE

$$\sin r = \frac{AE}{AC} \Rightarrow AE = AC\sin r$$

$$AE = \frac{t\sin r}{\cos r} \qquad (\because AC = \frac{t}{\cos r})$$

From Eq (4)

$$AB = \frac{2t\sin r}{\cos r} \times \sin i$$

$$AB = \frac{2t\sin r\sin i}{\cos r} \times \frac{\sin r}{\sin r}$$

$$AB = \frac{2\mu t\sin^2 r}{\cos r} \quad \text{………………………(5)} \qquad (\because \mu = \frac{\sin i}{\sin r})$$

On substituting the values of AC, CD & AB from Eq(2),(3)&(5) in Eq(1) ,we get

The path difference $= \mu(\frac{t}{\cos r} + \frac{t}{\cos r}) - \frac{2\mu t\sin^2 r}{\cos r}$

$$= \frac{2\mu t}{\cos r}(1 - \sin^2 r) = \frac{2\mu t\cos^2 r}{\cos r} = 2\mu t\cos r$$

$\therefore$ The path difference $= 2\mu t\cos r$

According to the theory of reversibility, when the light ray reflected at rarer-denser interface, it introduces an extra phase difference π (or) path difference of $\frac{\lambda}{2}$

$\therefore$ The actual path difference $= 2\mu t\cos r - \frac{\lambda}{2}$

Case.1: **condition for maximum intensity**

We know that the intensity is maximum when path difference= $n\lambda$

$\therefore$ From Eq.(6) $2\mu t\cos r - \frac{\lambda}{2} = n\lambda$

$$2\mu t\cos r = (2n+1)\frac{\lambda}{2}$$

Case.2: **condition for minimum intensity**

We know that the intensity is minimum when path difference $= (2n+1)\frac{\lambda}{2}$

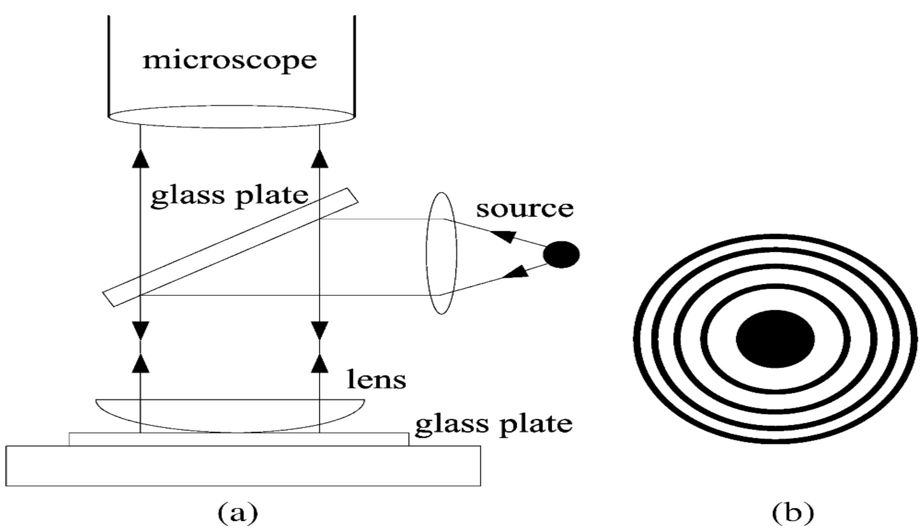
$\therefore$ from Eq.(6) $2\mu t\cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$$2\mu t\cos r = (n+1)\lambda$$

NEWTON'S RING EXPERIMENT

A Plano convex lens(L) having large focal length is placed with its convex surface on the glass plate(G_2).a gradually increasing air film will be formed between the plane glass plate and convex surface of Plano convex lens. The thickness of the air film will be zero at the point of contact and symmetrically increases as we go radially from the point of contact.

A monochromatic light of wavelength 'λ' is allowed to fall normally on the lens with the help of glass plate (G_1) kept at 45^0 to the incident monochromatic beam. A part of the incident light rays are reflected up at the convex surface of the lens and the remaining light is transmitted through the air film. Again a part of this transmitted light is reflected at on the top surface of the glass plate (G_1).both the reflected rays combine to produce an interference pattern in the form of alternate bright and dark concentric circular rings, known as Newton rings. The rings are circular because the air film has circular symmetry. These rings can be seen through the travelling microscope.



THEORY

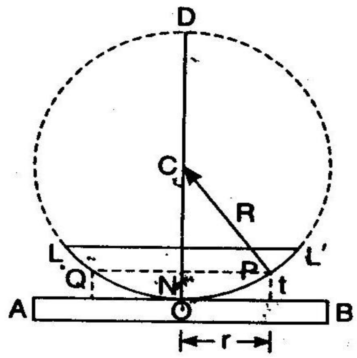
Consider a Plano convex lens is placed on a glass plate. Let R be the radius of curvature and r be the radius of NEWTON ring, corresponding to constant film thickness.

As one of the rays suffers reflection at denser medium, so a further phase changes of π or path difference of $\frac{\lambda}{2}$ takes place.

The path difference between the rays $=2\mu t \cos r + \frac{\lambda}{2}$ ---------------------------- **(i)**

For air $\mu = 1$, and normal incidence $r = 0$

$\therefore$ **Path difference** $= 2t + \frac{\lambda}{2}$



AT THE POINT OF CONTACT

The thickness of the air film t=0, μ=1 & for normal incidence r = 0.

Then the path difference = $\frac{\lambda}{2}$.

If the Then the path difference = $\frac{\lambda}{2}$ **then the corresponding phase difference is π.so that gives a dark spot is formed at the centre.**

For bright ring

$$2t + \frac{\lambda}{2} = n\lambda$$

$$2t = (2n-1)\frac{\lambda}{2} \quad \text{------------------------------------ (ii)}$$

For Dark ring

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{--- (iii)}$$

In the above fig, from the property of the circle

$$NP \times NQ = NO \times ND$$

$$r \times r = 2t \times (2R - t)$$

$$r^2 = 2Rt - t^2$$

As t is small, t^2 is very small. So t^2 is neglected.

$$\therefore \; r^2 = 2Rt$$

$$t = \frac{r^2}{2R} \quad \Rightarrow \quad t = \frac{D^2}{8R} \text{ ------------------------ (iv)}$$

Thus for bright ring

From Eq (ii) & (iv)

$$\frac{2D^2}{8R} = (2n-1)\frac{\lambda}{2}$$

$$\boxed{D_n^2 = 2(2n-1)\lambda R} \text{ -- (v)}$$

Thus for dark ring

From Eq. (iii) & (iv)

$$\frac{2D^2}{8R} = n\lambda$$

$$D_n^2 = 4Rn\lambda \text{ .... (vi)}$$

$$\boxed{D_n^2 = 4Rn\lambda}$$

Determination of wave length of monochromatic light

From Eq(vi) $D_n^2 = 4Rn\lambda$

For $n = m$, $D_m^2 = 4Rm\lambda$

$$\therefore \; D_m^2 - D_n^2 = 4Rm\lambda - 4Rn\lambda \quad = 4R\lambda(m-n)$$

$$\therefore \quad \lambda = \frac{D_m^2 - D_n^2}{4R(m-n)} \text{ ------------------------------------(vii)}$$

This is the expression for wave length of monochromatic light.

Determination of refractive index of a liquid

The experimental set up as shown in fig. is used to find the refractive index of a liquid.

To find the refractive index of a liquid, the plane glass plate and Plano convex lens set up is placed in a small metal container. The diameter of n^{th} and m^{th} dark rings are determined, when there is air between Plano convex lens and plane glass plate.

Then we have,

$$D_m^2 - D_n^2 = 4Rm\lambda - 4Rn\lambda$$
$$= 4R\lambda(m-n).$$

Now the given liquid whose refractive index (μ) is to be introduced in to the space between Plano convex lens and plane glass plate without disturbing the experimental set up.

Then the diameters of Newton's rings are changed. Now the diameter of n^{th} and m^{th} dark rings are measured.

Then $D_m^2 - D_n^2$ **= 4Rλ (m-n)/μ-------------------------- (viii)**

Therefore from (vii) & (viii)μ= $\underline{D_m^2 - D_n^2}$

CONDITIONS TO GET STATIONARY INTERFERENCE FRINGES

1. The two sources should be coherent.
2. The two sources must emit continuous waves of the same wavelength and same frequency.
3. The distance between the two sources (d) should be small.
4. The distance between the sources and the screen (D) should be large.
5. To view interference fringes, the back ground should be dark.
6. The amplitude of interfering waves should be equal.
7. The sources must be narrow, i.e., they must be extremely small.
8. The source must be monochromatic source.

Production of Colors in thin films:

With monochromatic light alternate dark and bright interference fringes are obtained. With white light, the fringes obtained are colored. it is because the path difference $2\mu t\cos r - \frac{\lambda}{2}$ depends upon $\mu, t \,\&\, r$

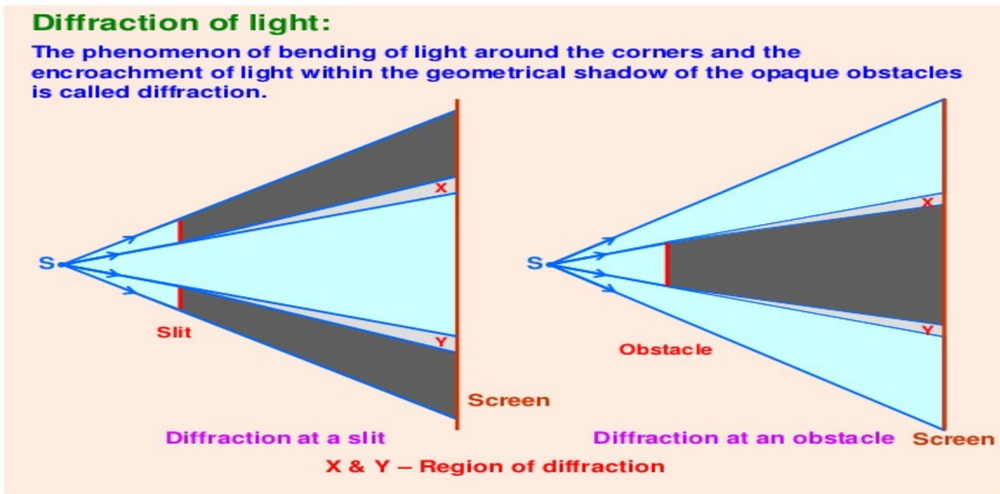
(i) Even if t and r kept constant, the path difference will change with $\mu \,\&\, \lambda$ of light used. White light composed of various colors from violet to red. The path difference also changes due to reflection at denser medium by $\frac{\lambda}{2}$ as $\lambda_V \langle \lambda_R$.

(ii) If the thickness of the film varies with uniformly, if at beginning it is thin, which will appear black. as path difference varies with thickness of the film, it appears different colors with white light.

(iii) If the angle of incidence changes, the angle of refraction is also changes, so that with white light, the film appears various colors when viewed from different directions.

DIFFRACTION

"When light is incident on the obstacles or small apertures whose size is comparable to wavelength of light, then there is a departure from straight line propagation, the light bends round the corners of the obstacles and enters into geometrical shadow. This bending of light is called diffraction."



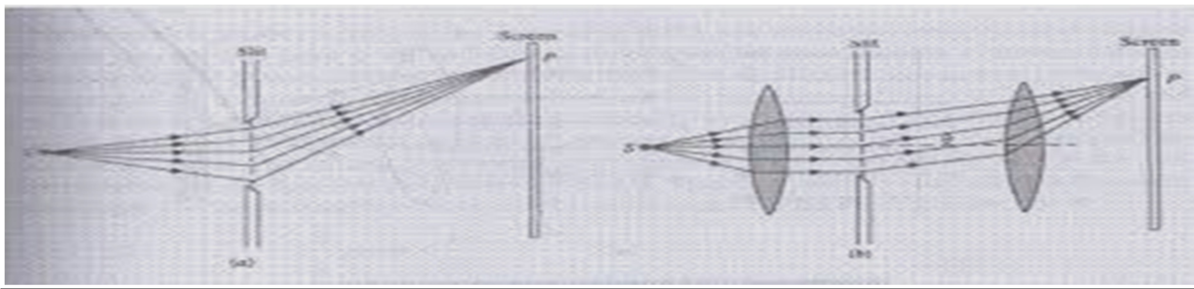
Differences between Interference and diffraction

INTERFERENCE	DIFFRACTION
1. Superposition is due to two separate wave fronts originating from two coherent sources.	1. Superposition is due to secondary wavelets originating from different parts of same wave front.
2. Interference fringes may or may not be of same width.	2. Diffraction fringes are not of the same width
3. Points of minimum intensity are perfectly dark	3. Points of minimum intensity are not perfectly dark.
4. All bright bands are of uniform intensity	4. All bright bands are not of same intensity.

There are two types of Diffractions are there, they are

1. Fresnel Diffraction
2. Fraunhofer Diffraction

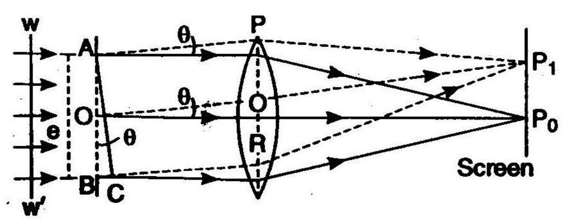
Fresnel diffraction **Fraunhofer diffraction**



Differences between Fresnel Diffraction and Fraunhofer Diffraction

Fresnel Diffraction	Fraunhofer Diffraction
1. Eighter a point source or an illuminated narrow slit is used.	1. Extended source at infinite distance is used.
2. The wave front undergoing diffraction is either spherical or cylindrical.	2. The wave front undergoing diffraction is plane wave front.
3. The source and screen are at finite distances from the obstacle.	3. The source and screen are at infinite distances from the obstacle.
4. No lens is used to focus the rays.	4. Converging lens is used to focus the rays.

FRAUNHOFER DIFFRACTION AT SINGLE SLIT:



Consider a slit AB of width "e" and a plane wave front WW^1 of monochromatic light of wavelength "λ" is incident normally on the slit. The diffracted light through the slit is focused with the help of a convex lens on a screen. The screen is placed at the focal plane of the lens. Here the secondary wave lets spared out to the right in all directions.

The waves travelling along OP_o are brought out to focus at P_o by the lens. Hens P_O is the bright central image.

The secondary wavelets at angle "θ" with normal are focused at P_1 on the screen. Depending upon path difference, P_1onmay be of maximum (or) minimum intensity point.

To find intensity at P_1 we drawn a normal AC from A to the light ray at B the path difference between the wave lets from A and B in the direction "θ" is given by

From $TraingleABC,\ \sin\theta = \frac{BC}{AB} \Rightarrow BC = AB\sin\theta = e\sin\theta$

$\therefore$ Phase difference = $\frac{2\pi}{\lambda} e \sin\theta$

Let us consider the width of the slit is divided into 'n' equal parts. Then the phase difference between any two consecutive waves from these parts would be.

$$\frac{1}{n}(total.phase) = \frac{1}{n}\left(\frac{2\pi}{\lambda}.e\sin\theta\right) = d \quad \text{(say)}$$

$\because$ **Resultant amplitude** $R = \dfrac{a\sin\frac{nd}{2}}{\sin\frac{nd}{2}}$

$$\therefore R = \frac{a\sin\left(\frac{n}{2}\times\frac{2\pi}{n\lambda}.e\sin\theta\right)}{\sin\left(\frac{2\pi}{2n\lambda}.e\sin\theta\right)}$$

$$= \frac{a\sin\left(\frac{\pi e\sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e\sin\theta}{n\lambda}\right)}$$

Let $\alpha = \frac{\pi e\sin\theta}{\lambda}$. Then

$$R = \frac{a\sin\alpha}{\sin\frac{\alpha}{n}}$$

As $\frac{\alpha}{n}$ is small, $\sin\frac{\alpha}{n} \approx \frac{\alpha}{n}$

$$\therefore R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = na\frac{\sin\alpha}{\alpha}$$

Now the intensity

$$\mathrm{I} = \mathrm{R}^2 = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2 \;.......(1)$$

Principal Maximum:

$$R = A\frac{\sin\alpha}{\alpha} = \frac{A}{\alpha}\left[\alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} - \frac{\alpha^7}{7!} + ...\right]$$

$$= A\left[1 - \frac{\alpha^2}{3!} + \frac{\alpha^4}{5!} - \frac{\alpha^6}{7!} + ...\right]$$

The value of R will be maximum, when α=0, i.e $\frac{\pi e\sin\theta}{\lambda} = 0$ or $\sin\theta = 0$

Or θ = 0

$\therefore$ Maximum intensity I=R^2 =A^2, this is occurred at θ = 0, this maximum is known as principal maximum.

Minimum intensity Positions:

The intensity will be minimum, when sin α=0.

$\therefore \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, ... \pm m\pi$

$\alpha = \pm m\pi$

$\frac{\pi e \sin\theta}{\lambda} = \pm m\pi$

$$e\sin\theta = \pm m\lambda$$

In this way, we obtain the points of min. inf. on either side of the principle maxima.

Secondary maxima:

In addition to principle maxima at α=0. There are weak secondary maxima between equally spaced minima. The points of secondary maxima obtained as follows.

$I = A^2\left(\frac{\sin\alpha}{\alpha}\right)^2$

$\frac{dI}{d\alpha} = A^2.2\frac{\sin\alpha}{\alpha}.\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$

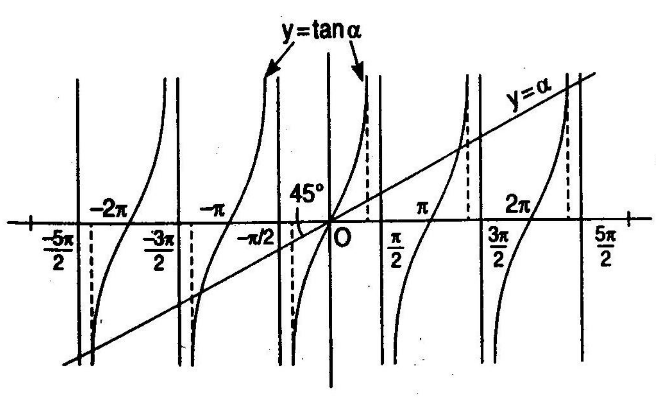
From above either sin α=0, or α cosα -sin α =0 if sin α=0, it is min. intensity position. Hence positions of maximum are obtained by

$\alpha\cos\alpha - \sin\alpha = 0$

$\alpha\cos\alpha = \sin\alpha$

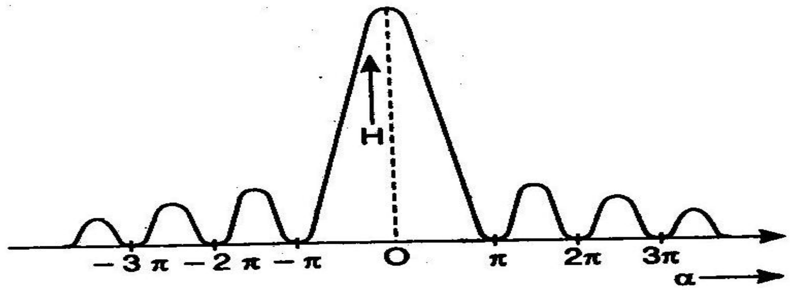
$$\alpha = \tan\alpha \qquad \text{---------------- (2)}$$

The values of α satisfying the above equation are obtained graphically by plotting curves $y = \alpha$, $y = \tan\alpha$ on the same graph. The points of intersection of two curves give the values of α which satisfy the equation (2)

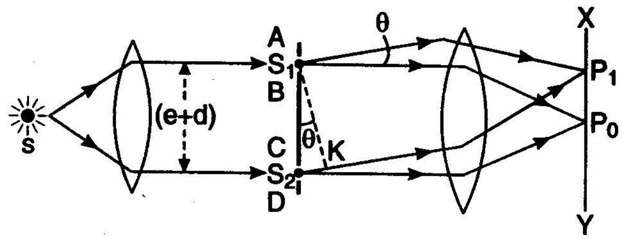


From fig The points of intersections are $\alpha = 0, \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}, ...,$ at these points we get secondary maxima

Intensity distribution graph



FRAUN HOFFER DIFFRACTION AT DOUBLE SLIT:



Let A B and CD be two parallel slits of equal width 'e' separated by an opaque distance d. The distance between the corresponding middle points of the two slits is $(e+d)$. Let a parallel beam of monochromatic beam of wave length λ be incident normally upon to the two slits. .

When a wave front is incident normally on both slits all the points with in the slits become the sources of secondary wavelets. The secondary waves traveling in the direction of incident light come to focus at P_o while the secondary waves traveling in the direction making an angle with θ the incident light come to focus at P_1.

According to the theory of diffraction at a single slit. The amplitude R due to all the wavelets diffracted from each slit in a dissection θ is given by.

$$R = A\frac{\sin\alpha}{\alpha} \text{ where } \alpha = \frac{\pi e \sin\theta}{\lambda}$$

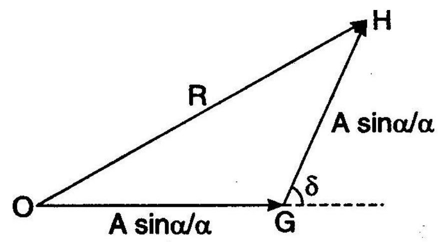
Thus for simplicity we can take two slits as equivalent to two sources S_1 and S_2 placed at mid points of the slits and each slit sending a wavelet of amplitude $\frac{A\sin\alpha}{\alpha}$ in the direction θ.

∴ Resultant amplitude at a point P_1 on the screen will be a result of interference between two waves of amplitude $\frac{A\sin\alpha}{\alpha}$ and having a phase difference.

The path difference between the wavelets from S_1 and S_2 in the dissection θ = S_2k.

$$path.difference = (e+d)\sin\theta$$

$$\therefore \; phase.difference(\delta) = \frac{2\pi}{\lambda}(e+d)\sin\theta$$



From figure $R\cos\theta = \frac{A\sin\alpha}{\alpha} + \frac{A\sin\alpha}{\alpha}\cos\delta$ ....(1)

$R\sin\theta = \frac{A\sin\alpha}{\alpha}\sin\delta$ ....(2)

Squaring & adding eq(1)&(2)

$$I = R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 + \left(\frac{A\sin\alpha}{\alpha}\right)^2\cos\delta + 2\left(\frac{A\sin\alpha}{\alpha}\right)^2\cos\delta + \left(\frac{A\sin\alpha}{\alpha}\right)^2\sin\delta$$

$$= \left(\frac{A\sin\alpha}{\alpha}\right)^2\left[2 + 2\cos\delta\right]$$

$$= 2\left(\frac{A\sin\alpha}{\alpha}\right)^2\left[1 + \cos\delta\right]$$

$$= 2\left(\frac{A\sin\alpha}{\alpha}\right)^2 2\cos^2\frac{\delta}{2}$$

$$I = 4\left(\frac{A\sin\alpha}{\alpha}\right)^2\cos^2\beta \text{........(3)}, \text{ Where } \beta = \frac{\delta}{2} = \frac{\pi}{\lambda}(e+d)\sin\theta$$

Discussion of Intensity:

From equation (3) the resultant intensity depending upon the following two factors.

1. $A^2\frac{\sin^2\alpha}{\alpha^2}$ Which is same as the intensity in the case of single slit diffraction thus it gives intensity distribution in the diffraction pattern.
2. $\cos^2\beta$ Which gives the intensity pattern due to two waves interfere.

The resultant intensity at any point on the screen is given by the product of these two factors.

$\therefore$ Diffraction term $\frac{\sin^2\alpha}{\alpha^2}$ gives the

(i)Central maximum at $\theta = 0$

(ii)Minimum intensity positions $\alpha = \pm m\pi$

$$\frac{\pi e\sin\theta}{\lambda} = \pm m\pi$$

$$e\sin\theta = \pm m\lambda$$

.

(iii) .Secondary maxima obtained at $\alpha = \pm 3\frac{\pi}{2}, \pm 5\frac{\pi}{2}$,...............

On taking these three points plotted as graph as shown in the fig(a).

The interference term $\cos^2\beta$ gives the maximum

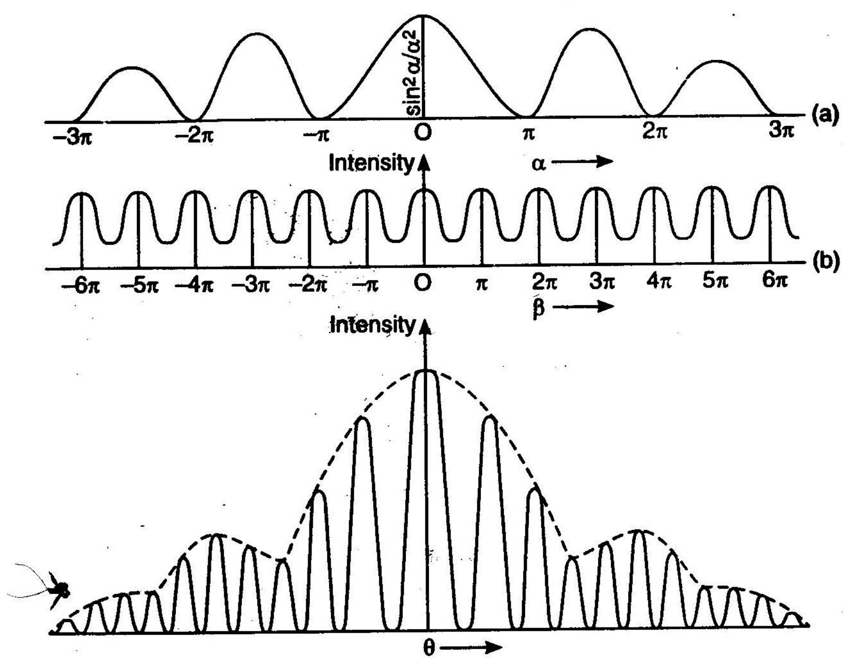
$$\cos^2\beta = 1 \Rightarrow \beta = \pm m\pi$$

$$\frac{\pi}{\lambda}(e+d)\sin\theta = \pm m\pi$$

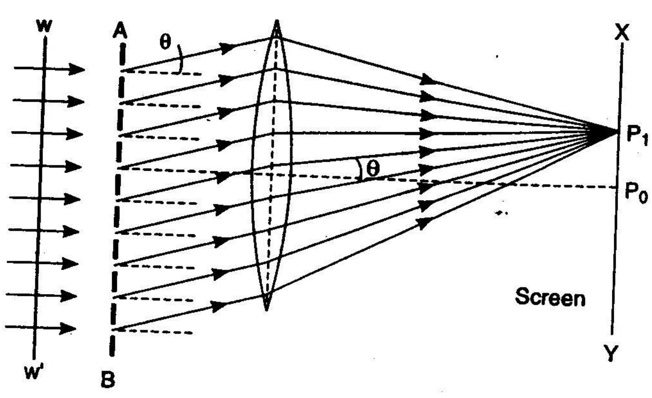
$$(e+d)\sin\theta = \pm m\lambda$$

This is plot as shown in fig.(b)

The resultant intensity graph is as shown in fig. (c)



Diffraction at N-Parallel slits [Diffraction grating]



An arrangement consists of large no. of parallel slits of same width and separated by equal opaque spaces is known as diffraction grating.

If there are N slits.

The path difference between any two consecutive slits is $=(e+d)\sin\theta$

$\therefore$ Phase difference = $\frac{2\pi}{\lambda}(e+d)\sin\theta = 2\beta$

By the method of vector addition of amplitudes

$$R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}$$

In this case $a = \frac{A\sin\alpha}{\alpha}$, $n = N$ and $d = 2\beta$

$$\therefore\ R = \frac{A\sin\alpha}{\alpha}.\frac{\sin N\beta}{\sin\beta}$$

$$I = R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \frac{\sin\beta}{\beta}\left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

The factor $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ gives the distribution of intensity due to single slit. While the factor $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ gives the distribution of intensity as a combined effect of all the slits.

Principle maxima:

The intensity will be maximum when $\sin\beta = 0$

$$\beta = \pm n\pi, n = 0,1,2,3,.....$$

But at the same time Sin N β = 0. So that the factor $\left(\frac{\sin N\beta}{\sin\beta}\right)$ becomes indeterminate.

$$\therefore \lim_{\beta\to n\pi}\frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to n\pi}\frac{N\cos N\beta}{\cos\beta} = \pm N$$

$$\lim_{\beta\to n\Pi}\left(\frac{\sin N\beta}{\sin\beta}\right)^2 = N^2$$

$\therefore$ The Resultant intensity $I = \left(\frac{A\sin\alpha}{\alpha}\right)^2 N^2$

i.e. The principle maxima obtained for $\beta = \pm n\pi$

$$\frac{\pi(e+d)\sin\theta}{\lambda} = \pm n\pi$$

$$\boxed{(e+d)\sin\theta = \pm n\lambda}$$

Minimum Intensity Positions:

Intensity I is the minimum when $\sin N\beta = 0$, but $\sin\beta \neq 0$

$$\therefore N\beta = \pm\pi, \pm 2\pi, \pm 3\pi, ...$$

$$\frac{N\pi(e+d)\sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{N(e+d)\sin\theta = \pm m\lambda}$$

Where m having all values except

$0, N, 2N, ...nN.$

$i.e, m = 1, 2, ...(N-1), (N+1), ...(2N-1), (2N+1), ...$

Secondary maximum:

I maximum when

$$\frac{dI}{d\beta} = 0$$

$$\frac{d}{d\beta}[(A\frac{\sin\alpha}{\alpha})^2(\frac{\sin N\beta}{\sin\beta})^2] = 0$$

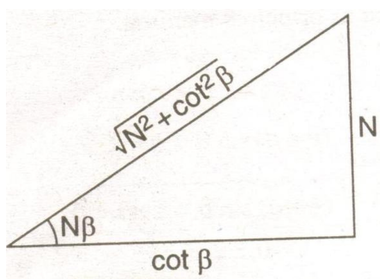
$$(\frac{A\sin\alpha}{\alpha})^2 2[\frac{\sin N\beta}{\beta}][\frac{N\sin\beta\cos N\beta - \sin N\beta\cos\beta}{\sin^2\beta}] = 0$$

$$N\sin\beta\cos N\beta - \sin N\beta\cos\beta = 0$$

$$N\sin\beta\cos N\beta = \sin N\beta\cos\beta$$

$$N\sin\beta = \cos\beta(\frac{\sin N\beta}{\cos N\beta})$$

$$\tan N\beta = \frac{N}{\cot\beta}$$



$$\therefore \sin N\beta = \frac{N}{\sqrt{(N^2 + \cot^2\beta)}}$$

$$\frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{(N^2 + \cot^2\beta)\sin^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta + \cos^2\beta}$$

$$= \frac{N^2}{N^2\sin^2\beta + 1 - \sin^2\beta}$$

$$\therefore \frac{\sin^2 N\beta}{\sin^2\beta} = \frac{N^2}{1 + (N^2 - 1)\sin^2\beta}$$

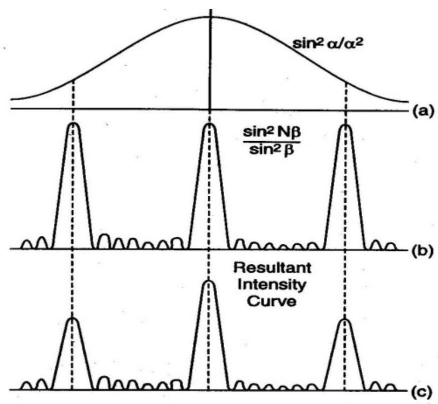
$$I_{\sec} = (A\frac{\sin\alpha}{\alpha})^2[\frac{N^2}{(N^2 - 1)\sin^2\beta + 1}$$

$$\therefore \frac{\text{intensity.of.secondary maxima}}{\text{intensity.of.principle maxima}}$$

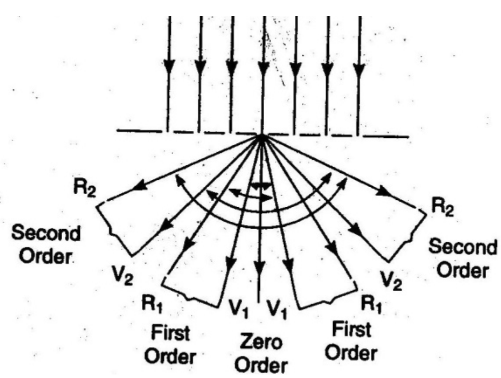
$$= \frac{N^2}{(1+(N^2-1)\sin^2\beta)\times N^2}$$

$$\therefore \frac{\text{intensity.of.secondary maxima}}{\text{intensity.of.principle maxima}} = \frac{1}{1+(N^2-1)\sin^2\beta}$$

From this we conclude that as the value of N increases the intensity of secondary maxima will decreases



GRATING SPECTRA



We know that the principle maxima in a grating are formed in a direction θ is given by

$$(e+d)\sin\theta = \pm n\lambda$$

Where $(e+d)$ grating element is θ is the angle diffraction and λ is Wave length

From the above equation, we conclude that

1. For a particular wave length λ, the angle of diffraction θ is different for different orders.
2. For white light and for an order n the light of different wave lengths will be diffracted in different directions. The longer the wavelength, greater is the angle of diffraction. So violet color being in the innermost position and red color in the outermost position.
3. Most of the intensity goes to zero order and rest is distributed among other orders thus the spectra become fainter as we go to higher orders.

Characteristics of grating spectra

1. Spectrum of different orders are situated symmetrically on both sides of zero order
2. Spectral lines are almost straight and quite sharp.
3. Spectral colors are in the order from violet to red.
4. Most of the intensity goes to zero order and rest is distributed among the other orders.

Maximum no. orders available with a grating

The principle maxima in grating satisfying the condition

$$(e+d)\sin\theta = n\lambda$$

$$n = \frac{(e+d)\sin\theta}{\lambda}$$

$$n_{\max} = \frac{(e+d)\sin 90^0}{\lambda}$$

$$n_{\max} = \frac{(e+d)}{\lambda}$$

DISPERSIVE POWER OF GRATING:

The dispersive power of grating is defined as the rate of variation of angle of diffraction with wavelength i.e., $\frac{d\theta}{d\lambda}$ is known as dispersive power of grating.

The condition for maxima is (e +d) sin θ = nλ

On differentiation we get (e+d) cosθ dθ = n dλ

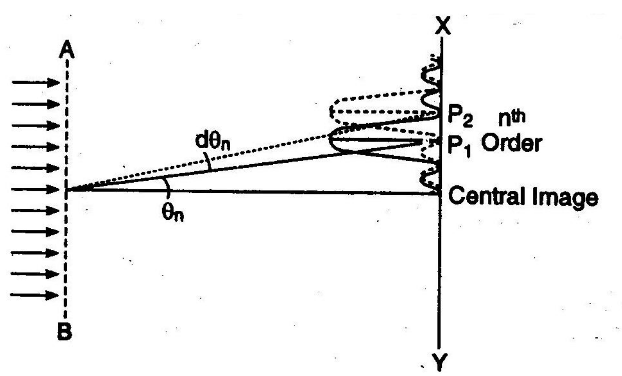
This is the expression for dispersive power of grating.

Conclusions :

- The dispersive power is directly proportional to diffraction order n.
- The dispersive power is inversely proportional to grating element (e+d).
- The dispersive power is inversely proportional to cosθ.

RESOLVING POWER OF GRATING:

The resolving power of a grating is defined as the capacity to form separate diffraction maxima of two wave lengths which are very close to each other



Let A B be a plane grating having grating element $(e+d)$ and N be the total no. of slits. let a beam of wavelengths λ and $\lambda + d\lambda$ is normally incident on the grating in the fig P_1 is the n_{th} primary maximum of wavelength λ at an angle of diffraction θ_n and P_2 is the n_{th} primary maximum of wavelength $\lambda + d\lambda$ at an angle of diffraction $(\theta_n + d\theta_n)$.

According to Rayleigh's criterion, the two wave lengths will be resolved if the principle maximum of one falls on the first minimum of the other.

The principle maximum of λ in the direction θ_n is given by

$$(e+d)\sin\theta_n = \pm n\lambda \text{..........(1)}$$

The wave length $(\lambda + d\lambda)$ form its n_{th} primary maxima in the direction $(\theta_n + d\theta_n)$

$$(e+d)\sin(\theta_n + d\theta_n) = \pm n(\lambda + d\lambda) \text{.............(2)}$$

The first minimum of wave length λ from in the direction $(\theta_n + d\theta_n)$

$$N(e+d)\sin(\theta_n + d\theta_n) = (nN+1)\lambda \text{.............(3)}$$

Multiplying eq(2) with N

$$N(e+d)\sin(\theta_n + d\theta_n) = \pm nN(\lambda + d\lambda) \text{............(4)}$$

From (3) & (4)

$$Nn(\lambda + d\lambda) = (nN+1)\lambda$$

$$nN\lambda + nNd\lambda = nN\lambda + \lambda$$

$$nNd\lambda = \lambda$$

$$\frac{\lambda}{d\lambda} = nN$$

But from eq (1) $n = \dfrac{(e+d)\sin\theta_n}{\lambda}$

$\therefore$ Resolving power of grating

$$\boxed{\frac{\lambda}{d\lambda} = \frac{N(e+d)\sin\theta_n}{\lambda}}$$

<u>PREVIOUS QUESTIONS</u>

1. What is meant by diffraction of light? Explain on the basis of Huygens wave theory.
2. Explain with necessary theory, the Fraunhofer diffraction due to 'n' slits.

   (Or)

   Give the theory of plane diffraction grating. Obtain the condition for the formation of nth order maximum.
3. Distinguish between Interference and Diffraction.

   (Or)

   How is diffraction different from Interference?
4. Calculate the maximum number of orders possible for plane diffraction grating.
5. Write notes on Rayleigh's criterion.
6. Distinguish between Fresnel and Fraunhoffer diffractions.
7. Define Resolving power of grating. Derive the expression for Resolving power of a grating based on Rayleigh's criterion.
8. Describe the action of plane transmission grating in producing diffraction spectrum.
9. Show that grating with 500lines/cm cannot give a spectrum in 4^{th} order for the light of wave length 5890 A^0.

POLARIZATION

Interference and diffraction are the phenomenon which confirmed the wave nature of light. But the phenomenon could not establish whether light waves are longitudinal (or) transverse.
When the phenomenon of polarization was discovered it was established that light waves are transverse in nature.

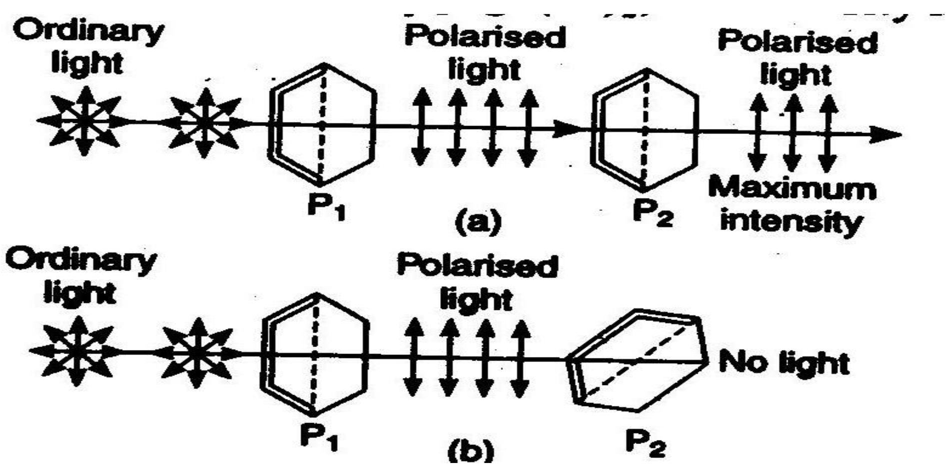
Polarization is a property of waves that describes the orientation of their oscillations. In a transverse wave if all the vibrations are confined in a single direction, it is said to be polarized.

Polarization: It is the process of converting ordinary light into polarized light.

Polarized wave: the wave which is unsymmetrical about the direction of propagation is called polarized wave.

Polarized light: The light which has acquired the property of **one sidedness** is called polarized light

POLARIZATION OF LIGHT WAVES



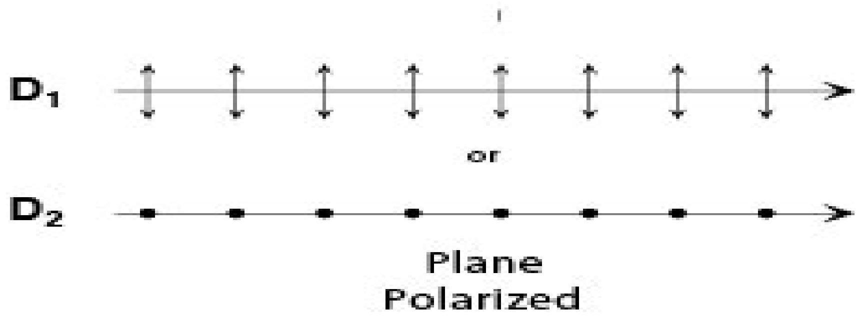
When a ordinary light is passed through a pair of tourmaline crystal plates with their planes parallel to each other, then the maximum intensity is obtained. When their planes perpendicular to each other, the intensity is zero. This shows that light is a transverse wave motion

TYPES OF POLARIZED LIGHT

There are three different types of polarized light.

1. Plane polarized light
2. Circularly polarized light.
3. Elliptically polarized light.

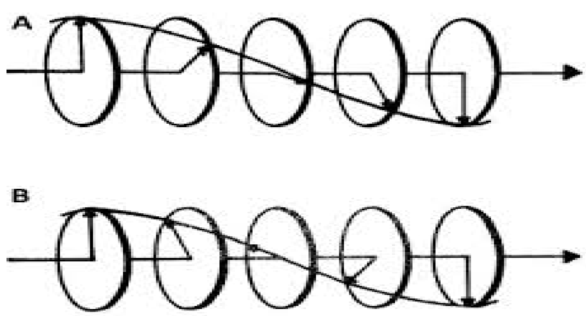
1. Plane polarized light: When the vibrations of light are confined along a single direction, the light is said to be plane polarized light. (Eighter in the direction along the plane of the paper (or) in the direction along the perpendicular to the plane of the paper)

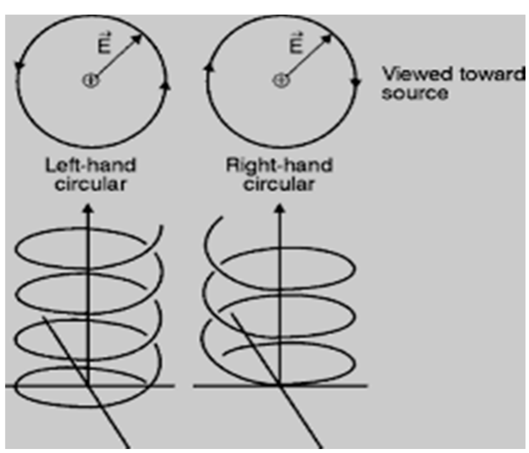


2. Circularly polarized light:

The projection of a wave on a plane intercepting the axis of propagating gives a circle with the amplitude vector remaining constant.

I.e. The vector rotates in the clock wise direction with respect to the direction of propagation; it results in right, circularly polarized light while the rotation anti-clock wise direction results in left circularly polarized light.

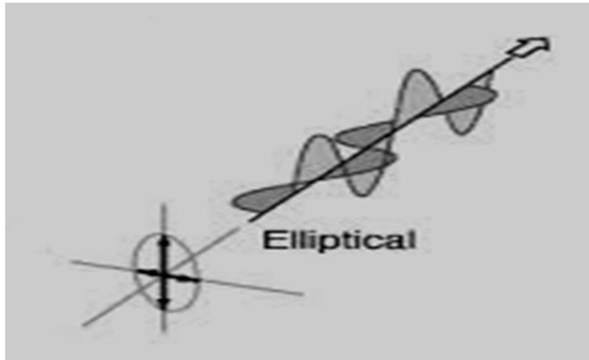




If the vibrations are along a circle, the light is said to be circularly polarized light.

3. Elliptically polarized light:

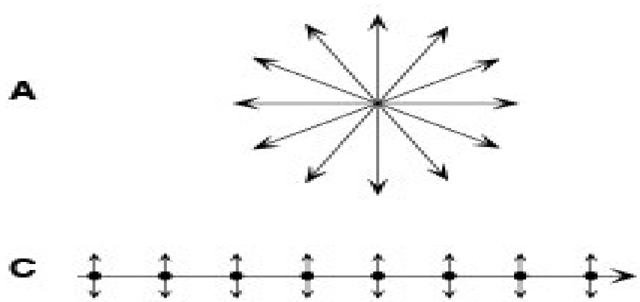
The projection of a wave on a plane intercepting the axis of propagating gives a ellipse and amplitude vector is not constant but various periodically.



If the vibrations are along an ellipse, the light is said to be elliptically polarized light.

Unpolarized light:

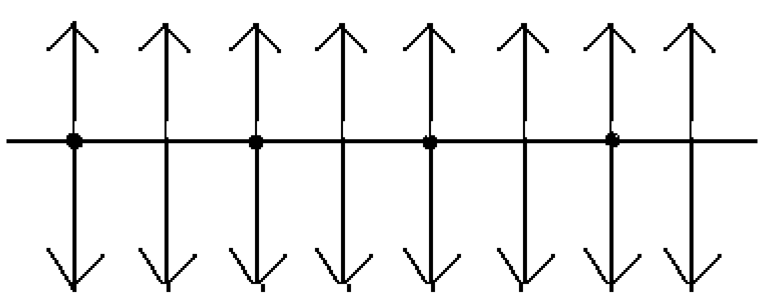
Unpolarized light (or) ordinary light has vibrations both parallel and perpendicular to the plane of the paper.



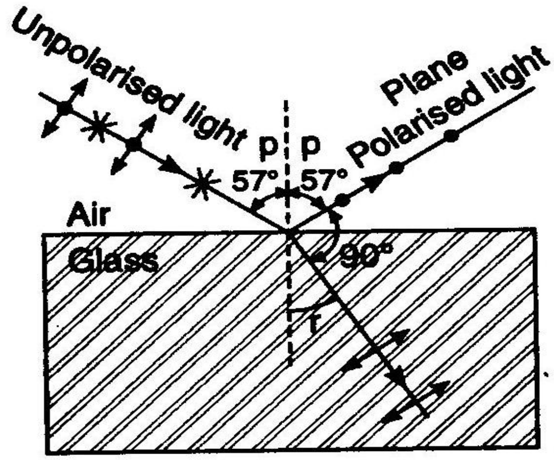
non-Polarized

Partially polarized light:

If the linearly polarized light contains small additional component of unpolarised light it becomes partially plane polarized light.



1. POLARIZATION BY REFLECTION (BREWSTER'S LAW)



Brewster observed that for a particular angle of incidence is known as angle of polarization .The refracted light is completely plane polarized in the plane if incidence.

Brewster proved that the tangent of the angle of polarization (P) is numerically equal to refractive index of material.

$$\boxed{\mu = \tan P}$$

This is known as Brewster's law.

He also proved that the reflected and refracted rays are perpendicular to each other.

The angle between reflected and refracted rays

From Brewster's law

$$\mu = \tan p ....(1)$$

From Snell's law

$$\mu = \frac{\sin p}{\sin r} ....(2)$$

$$from (1) and (2)$$

$$\frac{\sin p}{\cos p} = \frac{\sin p}{\sin r}$$

$$\cos p = \sin r$$

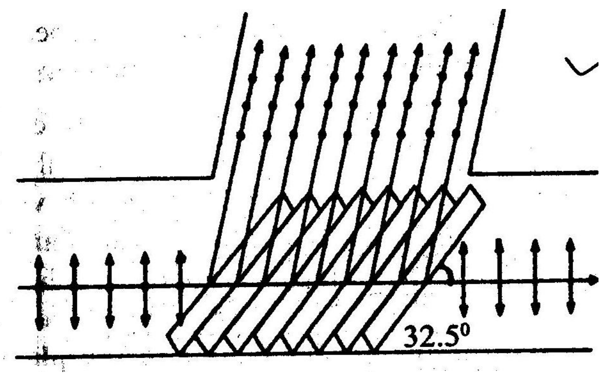
$$\cos p = \cos(90 - r)$$

$$p = 90 - r$$

$$p + r = 90$$

The angle between reflected and, refracted ray is 90^o

2. POLARIZATION BY REFRACTION (PILE OF PLATES)

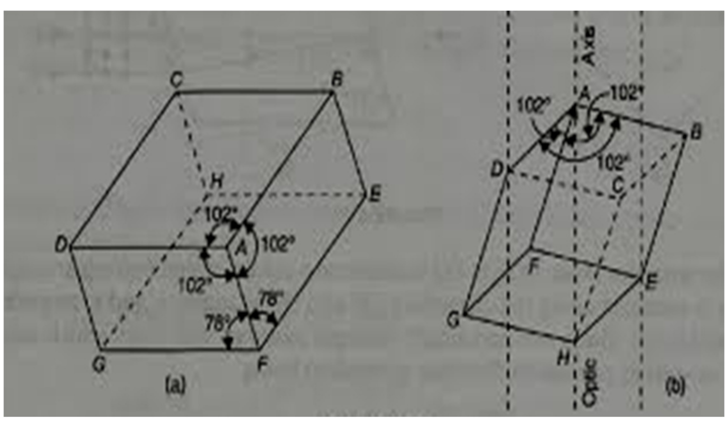


We know that when unpolarized light is incident at polarizing angle the reflected light is completely plane polarized and transmitted light contains a greater proportion of light vibrating parallel to the plane of incidence. if the process of reflection at polarizing angle is repeated using no. of plates all inclined at polarizing angle, finally the transmitted light become s purely plane polarized. Such an arrangement is known as pile of plates.

GEOMETRY OF CALCITE CRYSTAL

Calcite is a transparent color less crystal. Chemically it is hydrated calcium carbonate. It was at one time found in large quantities in ICELAND. Hence it is also known as ICELAND SPAR.

It consist six faces of parallegrams having angles of 102^0 and 78^0. The corners A and H are said to be blunt corners. The other corners of the crystal consist of two acute and one obtuse angle.



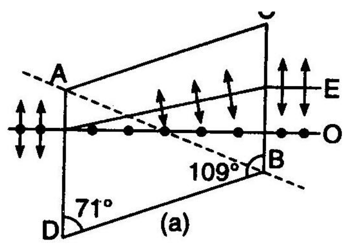
Blunt corner:

The corner where three obtuse angles meet .that corner is called Blunt corner

Principle axis:

It is the line passing through the any one of the blunt corner and making equal angles with the three faces which meet at this corner.

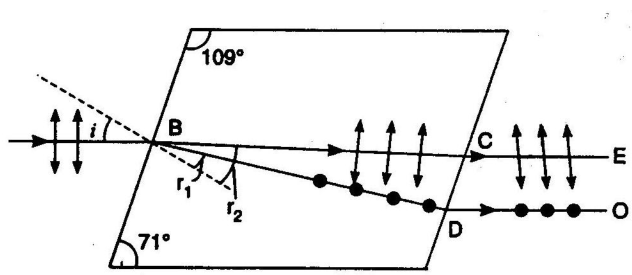
Principle section:



Any plane which contains principle axes and is perpendicular to two opposite faces is called a principle section.

3. BIREFRINGENCE (POLARIZATION BY DOUBLE REFRACTION):

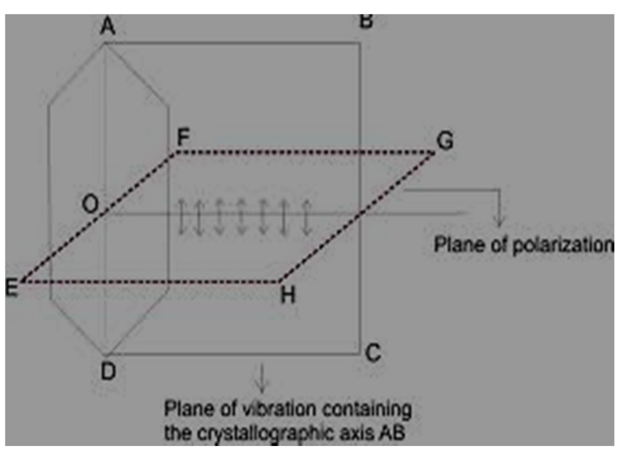
When a beam of ordinary un polarized light is passed through a calcite crystal, the refracted light split into two refracted rays .this phenomenon is called double refraction



Among the two rays one which always obey the ordinary laws of refraction and having vibrations perpendicular to the principle section is known as ordinary ray and The other which do not obey general laws of refraction and having vibrations in the principle section is called extraordinary ray .

The crystals showing this phenomenon are known as doubly refracting crystals or **Birefringent crystals**.

PLANE OF POLARIZATION & PLANE OF VIBRATION:



When ordinary light is passed through a tourmaline crystal, the light is polarized and the vibrations are confined only in one direction which is perpendicular to the direction of propagation of light

The plane in which the vibrations of polarized light are confined .this plane is known as plane of vibration. This plane contains the direction of vibration as well as direction of propagation

The plane which has no vibrations the plane is known as plane of polarization. Thus a plane passing through the direction of propagation and perpendicular to the plane of vibration is known as plane of polarization

HUYGEN'S THEORY OF DOUBLE REFRACTION

1. When any wave front strikes a double refracting crystal, every point of the crystal becomes a source of two wave fronts

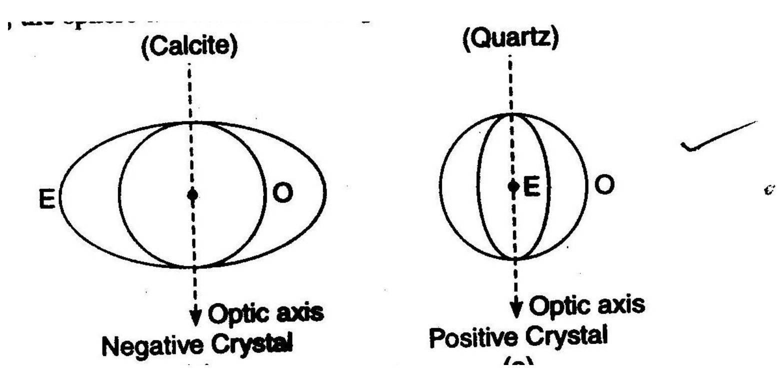
2. Ordinary wave front is spherical, because ordinary have same velocity in all directions

3. Extraordinary wave front is elliptical, because E-Ray has different velocities in different directions

4. The sphere and ellipsoid are touch each other along optic axis because the velocity of ordinary and extraordinary rays is same along optic axis

4. The crystal in which the velocity of ordinary is grater than extraordinary ray. That crystal are called Positive crystals

5. The crystals in which the velocity of extra ordinary is grater than ordinary ray .that crystals are called Negative crystal



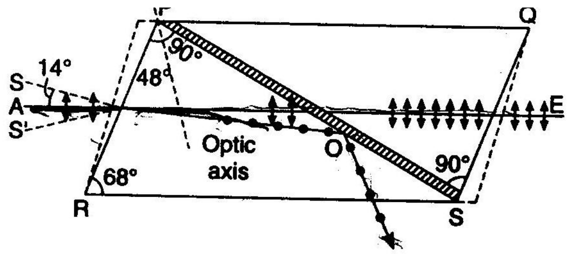
NICOL PRISM

When an ordinary light is transmitted through a calcite crystal, it splits into ordinary and extraordinary rays. Nicol eliminated the ordinary beam by utilizing the phenomenon of total internal reflection at Canada balsam separating the two pieces of calcite .this device is called NICOL PRISM

Construction:

A calcite crystal whose length is three times as that of its width is taken .The end faces of this crystal are grounded in such a way that the angle in the principle section becomes 68^0 and 112^0 .Then calcite crystal cut into two pieces . The cut surfaces are grounded and polished optically flat and then cemented together by Canada balsam. The refractive index of Canada balsam lies between refractive indices of O-ray and E-ray. i.e

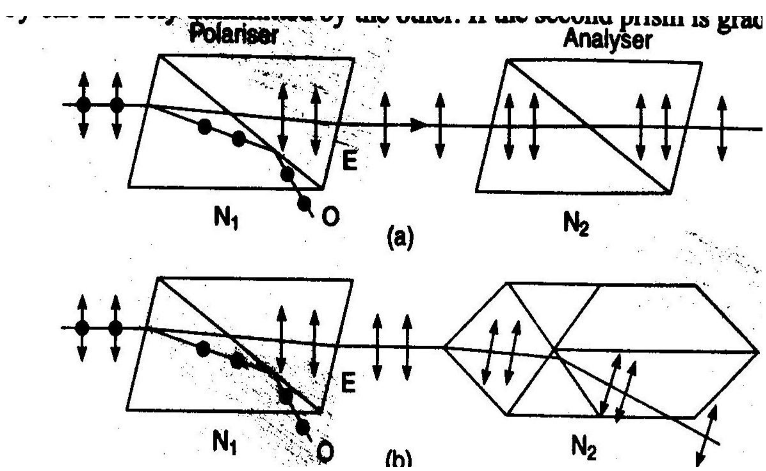
$$\mu_e < \mu_{cb} < \mu_o$$



Working:

When an ordinary beam of light incident on the Nicol prism, it split into ordinary plane polarized light and extraordinary plane polarized light. From the values of refractive indices the Canada balsam acts as a rarer medium for ordinary ray and denser medium for extraordinary ray. Moreover the dimensions of the crystal are so chosen that the angle of incidence of ordinary ray at the calcite-Canada balsam surface become grater than the corresponding critical angle .Under these conditions the ordinary ray under go total internal reflection and is eliminated. Only extraordinary transmitted

EXPLAIN HOW NICOL PRISM CAN BE USED BOTH AS POLARIZER AND ANALYZER



When two of Nicols placed co-axially then the first Nicol produces polarized light is known as polarizer while the second which analyzes the polarized light is known as analyzer.

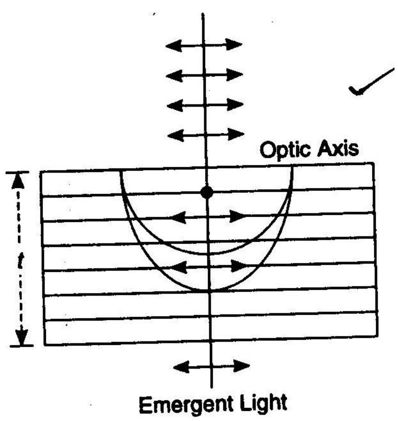
When two Nicols are placed with their planes parallel to each other. Then the extra ordinary Plane polarized transmitted by one is freely transmitted by the other. .if the second nicol is rotated gradually, then the intensity of E-ray gradually decreases and when the two nicols are at right angles to each other, no light comes from the second prism.

Thus first Nicol produces plane polarized light and second nicol detects it.

WAVE PLATES:

The wave plates are introduced specified path difference between o-ray and e-ray for particular wavelength.

HALF WAVE PLATE:



If the thickness of a crystal is taken such that it introduces a path difference of $\frac{\lambda}{2}$ or phase difference of π,then that crystal is called half wave plate.

Let μ_o, μ_e are the refractive indices of ordinary, extraordinary rays and t is the thickness of the calcite crystal

Then the path difference between ordinary and extraordinary ray $= \mu_e t - \mu_o t$

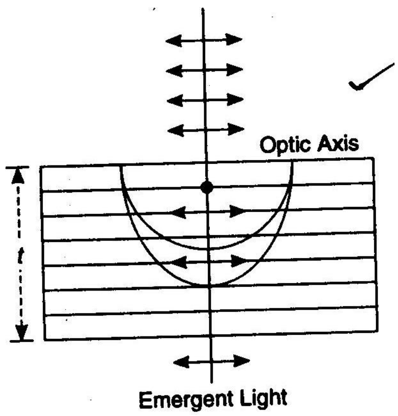
But for half wave plate $path.difference = \frac{\lambda}{2}$

$$\therefore \mu_e t - \mu_o t = \frac{\lambda}{2}$$

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

QUARTER WAVE PLATE



If the thickness of a crystal is taken such that it introduces a path difference of $\frac{\lambda}{4}$ or phase difference of $\frac{\pi}{2}$, then that crystal is called quarter wave plate.

Let μ_o, μ_e are the refractive indices of ordinary, extraordinary rays and t is the thickness of the calcite crystal

Then the path difference between ordinary and extraordinary ray $= \mu_e t - \mu_o t$

But for half wave plate $path.difference = \frac{\lambda}{4}$

$$\therefore \mu_e t - \mu_o t = \frac{\lambda}{4}$$

$$(\mu_e - \mu_o)t = \frac{\lambda}{4}$$

$$\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$$