UNIT-I

THREE-PHASE INDUCTION MOTORS

Objectives:

- To familiarize the students with the advantages of three phase induction motors.
- To familiarize the students with the constructional details, working principle of three phase induction motors.
- To make students understand the concepts of rotating magnetic fields and power flow in three phase induction motors.
- To familiarize the students with torque slip characteristic of induction motor.

Micro syllabus: Construction details – types of rotors-squirrel cage rotor- slip ring rotor-comparison between cage and wound rotors- production of rotating magnetic field- principle of operation – slip of an Induction motor–rotor emf, rotor frequency, rotor current and pf at standstill and during running conditions- phasor diagram-power flow diagram of an induction motor- rotor power input-rotor copper loss-mechanical power developed-interrelationship between rotor power input, rotor copper loss and mechanical power developed-Torque developed-torque equation-expressions for maximum torque and starting torque for squirrel cage and slip ring Induction Motor- condition for maximum starting torque-condition for maximum torque-torque slip characteristic- relation between starting torque and maximum torque-equivalent circuit-crawling and cogging.

Outcomes:

Students will be able to

- Understand the concepts of rotating magnetic fields.
- Demonstrate the knowledge of the working principle of three phase induction motor.
- Solve various numerical problems related to the efficiency of the induction motor considering all the types of losses.
- draw the torque slip characteristic
- understand the maximum torque and starting torque expressions

1.1. Introduction:

- A 3 phase induction motor can be used for different applications with various speed and load requirements.
- Electric motors can be found in almost every production process today. Getting the most out of your application is becoming more and more important in order to ensure cost-effective operations.
- The three-phase induction motors are the most widely used electric motors in industry. They run at essentially constant speed from no-load to full-load.
- However, the speed is frequency dependent and consequently these motors are not easily adapted to speed control. We usually prefer d.c. motors when large speed variations are required.
- Nevertheless, the 3-phase induction motors are simple, rugged, low-priced, easy to maintain and can be manufactured with characteristics to suit most industrial requirements. Like any electric motor, a 3-phase induction motor has a stator and a rotor. The stator carries a 3-phase winding (called stator winding) while the rotor carries a short-circuited winding (called rotor winding).
- Only the stator winding is fed from 3-phase supply. The rotor winding derives its voltage and power from the externally energized stator winding through electromagnetic induction and hence the name.
- The induction motor may be considered to be a transformer with a rotating secondary and it can, therefore, be described as a "transformer type" a.c. machine in which electrical energy is converted into mechanical energy.
- Induction motors are constructed both for single phase and three phase operations.
- Three phase induction motors are widely used for industrial applications such as in lifts, cranes, pumps, exhaust fans, lathes, etc., where as single phase induction motors are used mainly for domestic electric appliances such as fans, refrigerators, washing machines, exhaust pumps, etc.

1.1.1 Advantages:

(i) It has simple and rugged construction.
(ii) It is relatively cheap.
(iii) It requires little maintenance.
(iv) It has high efficiency and reasonably good power factor.
(v) It has self-starting torque.

1.1.2 Disadvantages:

(i) It is essentially a constant speed motor and its speed cannot be changed easily.
(ii) Its starting torque is inferior to d.c. shunt motor.

1.2 Construction

- The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by three phase induction motors because of its simple and rugged construction, low cost, good operating characteristics, absence of commutator and good speed regulation.
- In three phase induction motor the power is transferred from stator to rotor winding through induction. The Induction motor is also called asynchronous motor as it runs at a speed other than the synchronous speed.
- Like any other electrical motor induction motor also have two main parts namely stator and rotor. A 3-phase induction motor has two main parts (i) stator and (ii) rotor.
- The rotor is separated from the stator by a small air-gap which ranges from 0.4 mm to 4 mm, depending on the power of the motor.
- The main body of the Induction Motor comprises of two major parts as shows in Figure 1.1

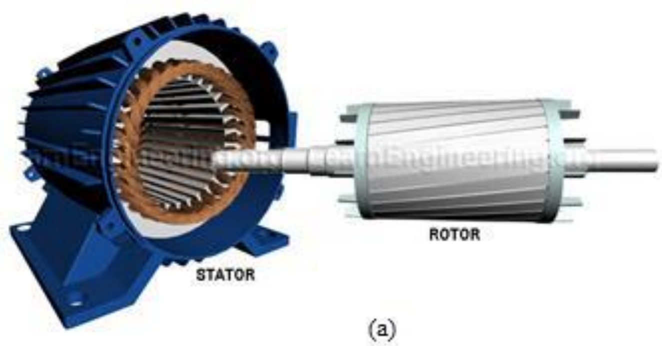
i) Shaft for transmitting the torque to the load. This shaft is made up of steel.

ii) Bearings for supporting the rotating shaft.

iii) One of the problems with electrical motor is the production of heat during its rotation. In order to overcome this problem we need fan for cooling.

iv) For receiving external electrical connection Terminal box is needed.

v) There is a small distance between rotor and stator which usually varies from 0.4 mm to 4 mm. Such a distance is called air gap.



(a)

(b)

Fig 1.1 Three phase induction motor (a) squirrel cage rotor (b) slip ring rotor.

1.2.1. Stator:

- As its name indicates, stator is a stationary part of induction motor.
- A stator winding is placed in the stator of induction motor and the three phase supply is given to it. Stator is made up of number of stampings in which different slots are cut to receive 3 phase winding circuit which is connected to 3 phase AC supply.
- The three phase windings are arranged in such a manner in the slots that they produce a rotating magnetic field after AC supply is given to them.
- The windings are wound for a definite number of poles depending upon the speed requirement, as speed is inversely proportional to the number of poles, given by the formula:

$$N_s = 120f/p$$

Where Ns= synchronous speed
f = Frequency
p = no. of poles

Figure 1.2 Stator of three phase induction motor.

- It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses.
- A number of evenly spaced slots are provided on the inner periphery of the laminations [See Fig.(1.2)]. The insulated winding connected to form a balanced 3-phase star or delta connected circuit.
- The 3-phase stator winding is wound for a definite number of poles as per requirement of speed. Greater the number of poles, lesser is the speed of the motor and vice-versa. When 3-phase supply is given to the stator winding, a rotating magnetic field of constant magnitude is produced. This rotating field induces currents in the rotor by electromagnetic induction.

The stator of the three phase induction motor consists of three main parts:.

i. Stator Frame:

- ❖ It is the outer most part of the three phase induction motor.
- ❖ Its main function is to support the stator core and the field winding. It acts as a covering and it provide protection and mechanical strength to all the inner parts of the induction motor. The frame is either made up of die cast or fabricated steel.
- ❖ The frame of three phase induction motor should be very strong and rigid as the air gap length of motor is very small, otherwise rotor will not remain concentric with stator, which will give rise to unbalanced magnetic pull.

ii. Stator Core:

- ❖ The main function of the stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick.
- ❖ All the stampings are stamped together to form stator core, which is then housed in stator frame. The stamping is generally made up of silicon steel, which helps to reduce the hysteresis loss occurring in motor.

iii. Stator Winding or Field Winding:

- ❖ The slots on the periphery of stator core of the motor carries three phase windings. This three phase winding is supplied by three phase ac supply.
- ❖ The three phases of the winding are connected either in star or delta depending upon which type of starting method is used.
- ❖ The squirrel cage motor is mostly started by star – delta starter and hence the stator of squirrel cage motor is delta connected.
- ❖ The slip ring three phase induction motor are started by inserting resistances so, the stator winding of slip ring induction motor can be connected either in star or delta.
- ❖ The winding wound on the stator of three phase induction motor is also called field winding and when this winding is excited by three phase ac supply it produces a rotating magnetic field.

1.2.2. Rotor:

- ❖ The rotor is a rotating part of induction motor. The rotor is connected to the mechanical load through the shaft.
- ❖ Rotor consists of cylindrical laminated core with parallel slots that carry conductor bars. Conductors are heavy copper or aluminium bars which

fits in each slots. These conductors are brazed to the short circuited end rings.

- ❖ The slots are not exactly made parallel to the axis of the shaft but are slotted a little skewed for the following reason, they reduce magnetic hum or noise and they avoid stalling of motor.
- ❖ The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types: Squirrel cage type and Wound type.

1.2.2.1 Squirrel cage rotor:

- ❖ **Squirrel cage three phase induction motor:** The rotor of the squirrel cage three phase induction motor is cylindrical in shape and have slots on its periphery.
- ❖ The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cadge rotor beside) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of motor more smooth and quieter. The squirrel cage rotor consists of aluminum, brass or copper bars.
- ❖ These aluminum, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor.
- ❖ The rotor conductors are permanently shorted by the copper or aluminum rings called the end rings. In order to provide mechanical strength these rotor conductors are braced to the end rings and hence form a complete closed circuit resembling like a cage and hence got its name as “squirrel cage induction motor”.
- ❖ The squirrel cage rotor winding is made symmetrical. As the bars are permanently shorted by end rings, the rotor resistance is very small and it is not possible to add external resistance as the bars are permanently shorted.
- ❖ The absence of slip ring and brushes make the construction of Squirrel cage three phase induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adapting any number of pole pairs.
- ❖ The below diagram shows squirrel cage induction rotor having aluminum bars short circuited by aluminum end rings.
- ❖ It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot.
- ❖ All these bars are joined at each end by metal rings called end rings. [See Fig. (1.3)]. This forms a permanently short-circuited winding which is indestructible.
- ❖ The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator.
- ❖ Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of 3-phase induction motors use

squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances.

- ❖ However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque.

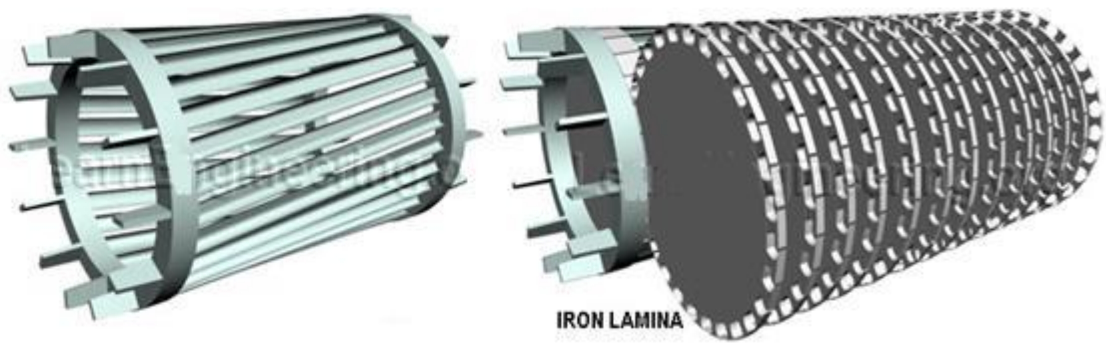


Fig. 1.3 squirrel cage rotor.

Advantages of squirrel cage induction rotor:
i) Its construction is very simple and rugged.
ii) As there are no brushes and slip rings, these motors requires less maintenance.
Applications:
Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc.

1.2.2.2 Wound rotor:

- ❖ **Slip ring or wound three phase induction motor:** In this type of three phase induction motor the rotor is wound for the same number of poles as that of stator but it has less number of slots and has less turns per phase of a heavier conductor.
- ❖ The rotor also carries star or delta winding similar to that of stator winding. The rotor consists of number of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form star connection.
- ❖ As its name indicates three phase slip ring induction motor consists of slip rings connected on same shaft as that of rotor. The three ends of three phase windings are permanently connected to these slip rings.
- ❖ The external resistance can be easily connected through the brushes and slip rings and hence used for speed control and improving the starting torque of three phase induction motor.

- ❖ The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances.
- ❖ At starting, the resistance is connected in rotor circuit and is gradually cut out as the rotor pick up its speed.
- ❖ When the motor is running the slip ring are shorted by connecting a metal collar, which connect all slip rings together and the brushes are also removed. This reduces wear and tear of the brushes.
- ❖ Due to presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor. It consists of a laminated cylindrical core and carries a 3- phase winding, similar to the one on the stator [See Fig. (1.4)].
- ❖ The rotor winding is uniformly distributed in the slots and is usually star-connected. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring.
- ❖ The three brushes are connected to a 3-phase star-connected rheostat as shown in Fig. (1.5). At starting, the external resistances are included in the rotor circuit to give a large starting torque.
- ❖ These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the three brushes are short circuited so that the wound rotor runs like a squirrel cage rotor.

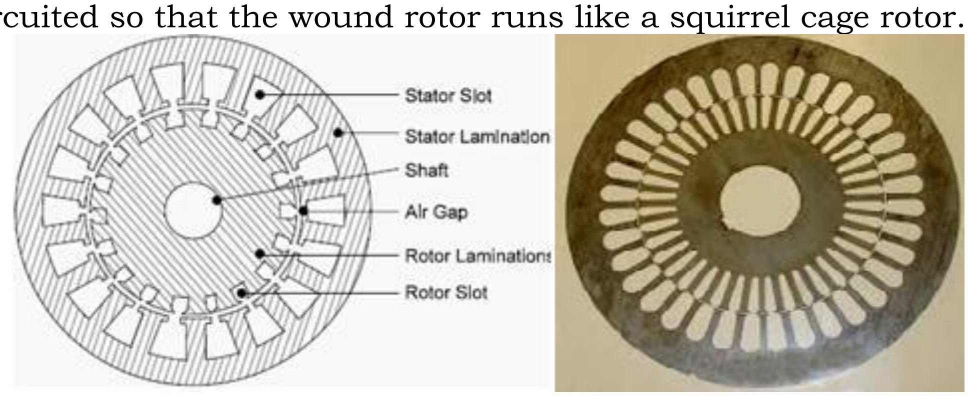


Fig 1.4 Lamination of stator and rotor.

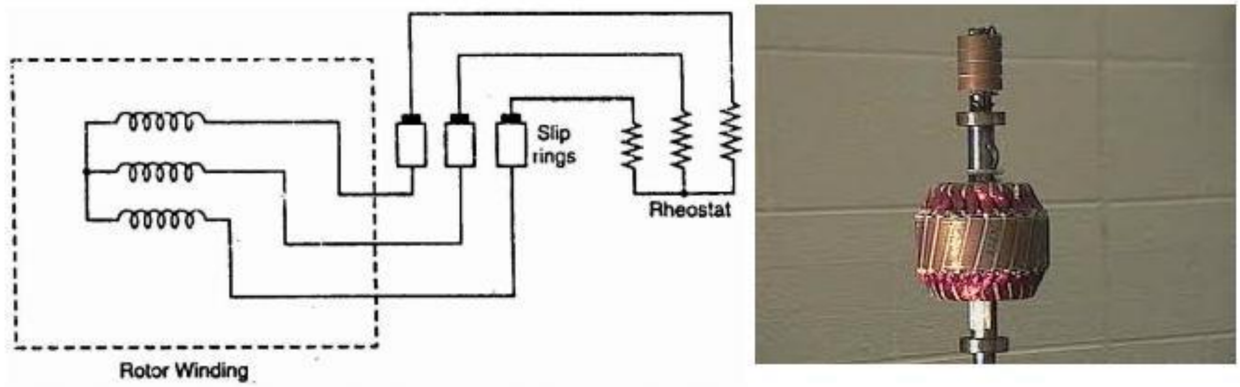


Figure 1.5 Slip ring rotor.

Advantages of slip ring induction motor are:
1. It has high starting torque and low starting current.
2. Possibility of adding additional resistance to control speed.

Application:
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc.

2.3. Working Principle of three phase induction motor:

- The two essential parts of a three phase induction motor are (a) a three phase stator winding, and (b) a closed rotor winding.
- To start a motor, a three phase supply is connected across the terminals. The rotor gets its excitation by electromagnetic induction.
- When three-phase supply is connected across the stator windings, a rotating magnetic field, constant in magnitude but rotating at synchronous speed is produced. The speed depends upon the supply frequency and the no. of poles for which the winding is made. The expression for synchronous speed is given by

$$N_s=120f/P$$

In fig 1.6 a simplified connection diagram of a three phase induction motor is shown. For simplicity, on the stator of a three-phase, 2 pole winding made only with six coils has been shown. Coil R-R' represents R phase winding. Similarly Y-Y' and B-B' are the phase windings of Y and B phases respectively.

- The phase windings are displaced in space by 120^o.
- On the rotor is shown six coils, 1-1', 2-2', 3-3' etc., are placed on slots.
- The direction of the rotating magnetic field produced by the stator ampere turns will depend upon the sequence in which the supply terminals Rs, Ys and Bs are connected across the stator winding terminals R, Y and B.

In the fig, the stator has been shown wound for two poles. The rotor will also have two poles induced in it. Let us assume that the stator field is rotating in anticlockwise direction. The position of stator poles at a particular instant of time has been shown in the figure. The rotating magnetic field produced by the stator will induce emf in the rotor conductors. The direction of induced emf in the rotor conductors can be determined thus:

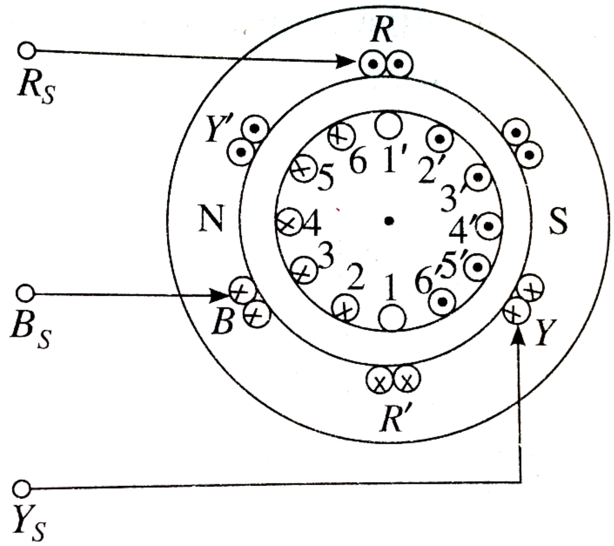


Fig 1.6 Three phase supply connected across the three phase stator windings of an Induction Motor

Assume that the stator field which is rotating in the anti-clockwise direction is made stationary. The rotor conductors then can be assumed to be rotating in clock wise direction with respect to the stator field. Applying Fleming's Right Hand Rule, the direction of induced emf can be determined and will be as shown by crosses and dots in fig. Emf induced in the coil 4-4' will be maximum at this position. Emf induced by the coil 1-1' will be zero. Emf induced in the coils 3-3' and 5-5' will be somewhat less than that induced in the coil 4-4'. Emf induced in the coil 2-2' and coil 6-6' will still be less than that induced in coil 4-4'. The magnitude and direction of induced emf in various coils at the particular instant under consideration are shown graphically in the fig. Since the rotor winding is a closed one, the rotor induced emf will cause the flow of current in the rotor conductors. Assume, for time being to be a purely resistive circuit. The distribution of current flowing through the rotor conductors will be same as that of the induced emf, since in a resistive circuit current is in time-phase with the voltage. Due to the current flow in the rotor conductors the rotor will be magnetized. The position of the rotor poles and the direction of the torque developed on the rotor are shown in the fig. 1.7(a)

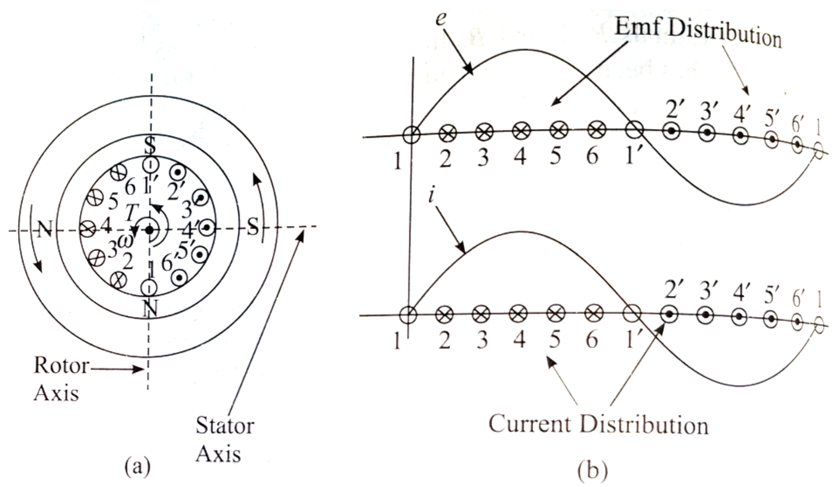


Fig.1.7 a) direction of currents in rotor conductors due to emf induced by stator rotating magnetic field assuming rotor circuit to be purely resistive b) direction of induced emf and current

It may be seen that the rotor will rotate in the same direction as the stator rotating magnetic field. If the direction of rotation of the rotating, magnetic field is reversed, the rotor will rotate in the opposite direction. The torque angle in this case (assuming the rotor circuit to be purely resistive one) is maximum i.e., 90^o. In actual practice, the rotor circuit is not purely resistive but will have inductive reactance in addition to resistance. The rotor current therefore will lag and rotor induced emf by some angle. The current distribution in various armature conductors has been shown in the fig.1.7 (c and d) From the fig,1.7(d) it can be seen that when the induced emf, say , in conductor 4 is maximum, current is maximum in conductor 6. In fig1.7, the position of rotor poles due to lagging current flow through the rotor is shown. The rotor field axis is now making an angle somewhat less than that in fig 1.7(a) with the stator field axis. The torque angle and hence the torque produced is therefore less when the rotor is an inductive circuit. To achieve higher toque at starting, therefore, in slip-ring type induction motors, extra resistance is connected in the rotor circuit so that the rotor becomes more resistive and torque angle is increased.

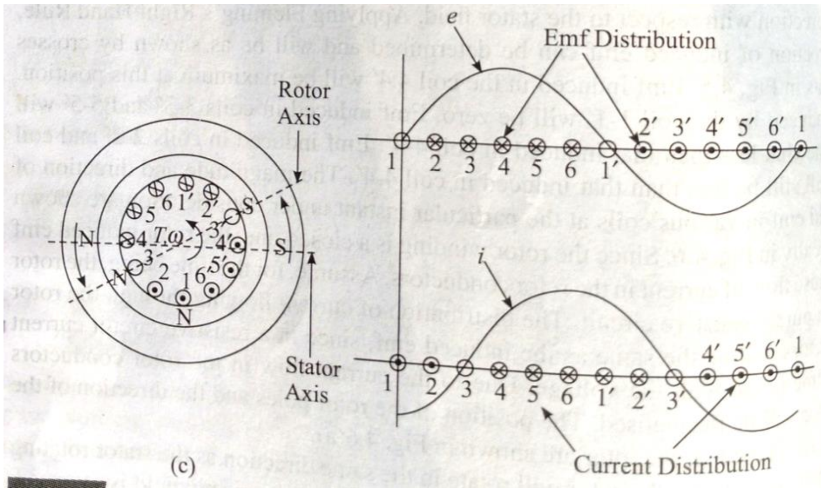


Fig. 1.7c) direction of current in rotor conductors considering rotor circuit inductance d) direction of induced emf and current

1.4 Production Of Rotating Magnetic Field With Three-Phase Winding And A Three Phase Supply

Figure 1.8(a) shows a stator with three-phase winding. One end of each of the phases is connected together. Other ends are kept free to be connected to the supply terminals. Each phase winding is shown to have been made of two coils. In practice, however, there will be more coils per phase and the windings will be distributed throughout the stator slots. When a three-phase supply, as shown in figure 1.8(b) is applied across the stator winding terminals, magnetic fields will be produced by the current flowing through the phase windings. The direction of field produced by each phase at any particular instant of time will be different as shown in figure 1.8(c).

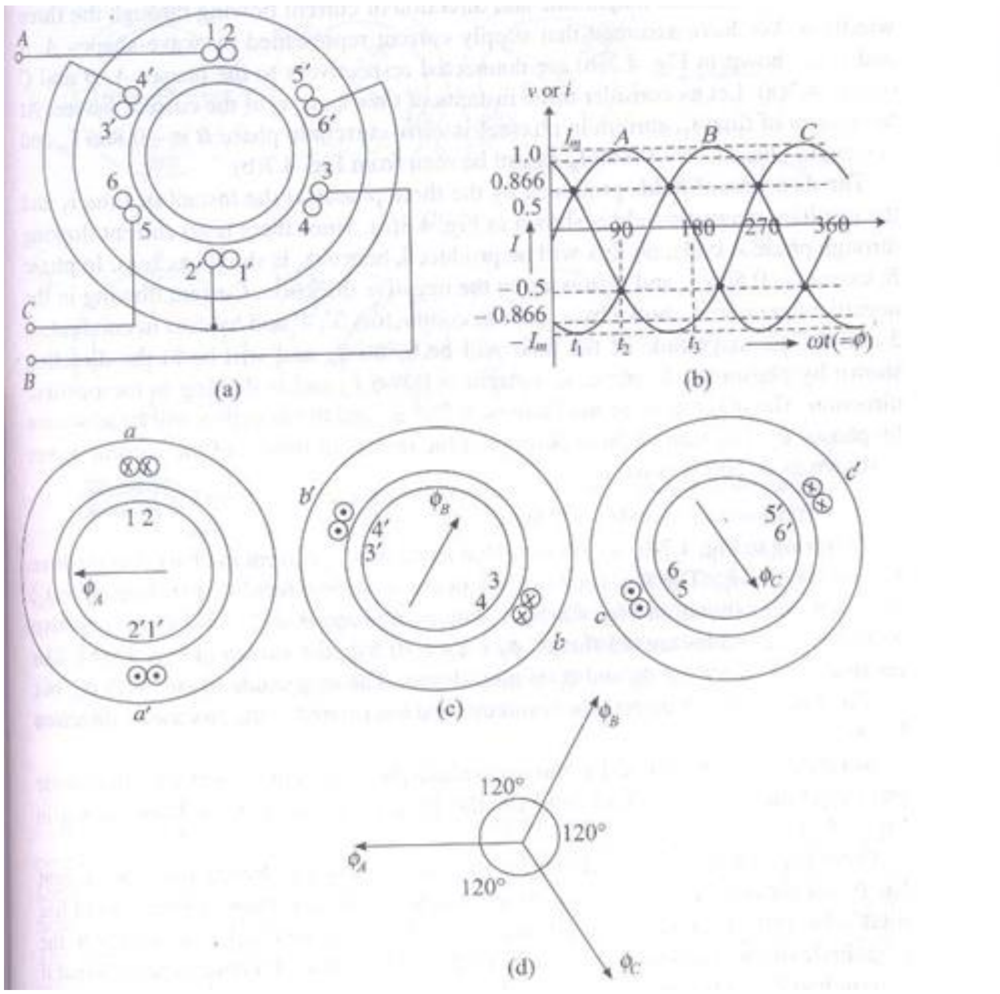


Figure 1.8 Magnetic fields produced due to current flowing in each phase of a three-phase stator winding (a) Three-phase winding with two coils per phase wound on stator; (b) three-phase supply; (c) flux produced by the individual phase currents; (d) phasor representation of the fields produced by the three phase winding mmfs.

These three magnetic fields will give rise to a resultant magnetic field. We will study the nature of the resultant magnetic field at different instants of time when the three phase currents will continue to flow through the three phase stator windings.

The directions of flux produced by the currents flowing through three phase windings are shown vectorially in Fig. 1.8(d). The magnitude and direction of the three phase currents flowing through the windings will change with time. In Fig. 1.8(d), while representing the flux produced by each phase through, we have assumed positive current of equal magnitude flowing through the windings. To determine the magnitude and direction of the resultant magnetic field we will take into account the actual magnitude and direction of current flowing through the three windings. We have assumed that supply current represented by wave shapes A, B and C as shown in Fig. 1.8(b) are connected respectively to the phases A, B and C of Fig. 1.8(a). Let us consider three instants of time t_1, t_2, t_3 of current waves. At the instant t_1, current in phase A is zero, current in phase B is $-0.866I_m$ and current in phase C is $+0.866I_m$ as can be seen from Fig. 1.8(b).

The direction of fields produced by the three phases at the instant of time t_1 and the resultant magnetic field is shown in Fig. 1.8(a). Since there is no current flowing through phase A coils, no flux will be produced, hence Φ_A is shown as zero. In phase B, current is 0.866 I_m and is flowing in the negative direction. Current flowing in the negative direction is shown by a cross in conductors 3', 4' and by dots in conductors 3 and 4. The magnitude of the field will be $0.866\Phi_m$ and will be in the direction shown by phasor Φ_B. In phase C, current is 0.866 I_m and is flowing in the positive direction. The magnitude of field will be $0.866\Phi_m$ and the direction will be as shown by phasor Φ_C. The sum of these phasors at the instant of time t_1 of the current waves is shown as Φ_R and is equal to,

$$\Phi_R = 1.5\ \Phi_m$$

Referring to Fig. 1.8(b), we observe that at the time t_2 current in phase A is positive maximum, i.e., I_m. The flux produced by phase A ampere turns is therefore $\Phi_A = \Phi_m$, its direction is shown in Fig. 1.8(b). Currents in phase B and phase C are negative and their magnitudes are 0.5 fluxes, $\Phi_B = \Phi_C = -0.5\ \Phi_m$ are shown in Fig. 1.8(b). The resultant flux is also shown. The magnitude of resultant flux is 1.5 Φ_m, but its direction is now changed. The resultant field has rotated in the clockwise direction by 90^0.

Similarly flux produced by the individual phase windings and the magnitude and direction of the resultant field for the instant of time t_3 have been shown in figure 1.8(c).

From figure 1.8 it is observed that for the time t_1 to t_2 , i.e., by the time the current has flown for one-half cycle through the windings, the resultant magnetic field has rotated by 180^0 mechanical, i.e., by half a revolution in the clockwise direction, the magnitude of the resultant field has been $1.5Ø_m$ all the time. It could be shown that the current had flown for one cycle through the windings; the resultant field would have rotated by one complete rotation.

The supply frequency is generally 50cps. In one second therefore the resultant field will rotate by 50 revolutions. In one minute the resultant field will rotate by 50X60 = 3000 revolutions. This speed is called synchronous speed. Synchronous speed, in addition to the supply frequency, depends on the number of poles for which the winding is made. In this case the windings have been made for two poles. The relation between frequency, number of poles, and the synchronous speed, as mentioned earlier, is given by

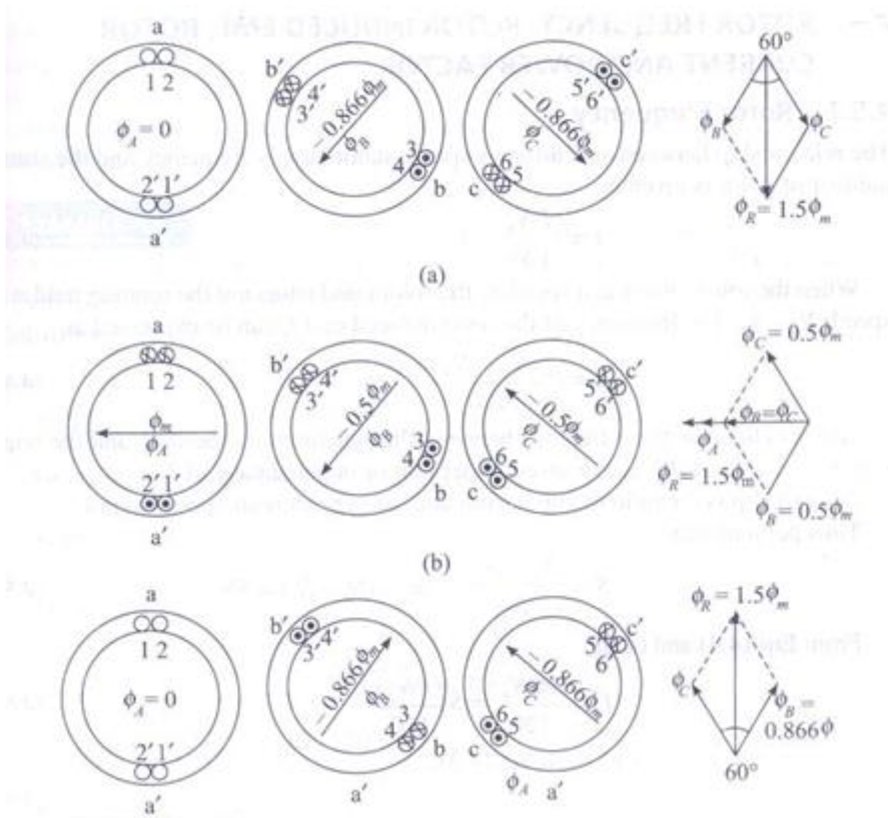


Figure 1.9 Directions of magnetic field produced by the ampere-turn of each phase and the resultant field produced by the three phases mmfs

$$N_s = \frac{120\,f}{P}$$

If the sequence of supply to the different phase windings is changed, i.e., if we connect A phase supply to B phase winding and B phase supply to A phase winding, keeping C phase winding supply connection unchanged, the resultant magnetic field will rotate in the opposite direction, in this case in the anticlockwise direction. The magnitude of the resultant magnetic field and its speed will however remain unchanged. To summarise

(a) When a three-phase supply is connected across a three-phase winding, a rotating field of constant magnitude rotating at synchronous speed is produced.

(b) The direction of rotation of the rotating field so produced depends on the sequence of supply to the phase windings.

1.5 Speed of RMF:

The speed at which the rotating magnetic field revolves is called the synchronous speed (Ns). Referring to Fig. 1.9 the field has completed one revolution. Therefore, for a 2-pole stator winding, the field makes one revolution in one cycle of current. In a 4-pole stator winding, it can be shown that the rotating field makes one revolution in two cycles of current. In general,

for P poles, the rotating field makes one revolution in P/2 cycles of current (see Fig. 1.7).
∴ Cycles of current = 2/P × revolutions of field
or Cycles of current per second = 2/P × revolutions of field per second

Since revolutions per second are equal to the revolutions per minute (Ns) divided by 60 and the number of cycles per second is the frequency (f).

$$f = \frac{P}{2} \times \frac{N_s}{60} = \frac{P\,N_s}{120}$$

$$N_s = \frac{120\,f}{P}$$

The speed of the rotating magnetic field is the same as the speed of the alternator that is supplying power to the motor if the two have the same number of poles. Hence the magnetic flux is said to rotate at synchronous speed.

1.6 Slip:

We have seen above that rotor rapidly accelerates in the direction of rotating field. In practice, the rotor can never reach the speed of stator flux. If it did, there would be no relative speed between the stator field and rotor conductors, no induced rotor currents and, therefore, no torque to drive the rotor. The friction and windage would immediately cause the rotor to slow down. Hence, the rotor speed (N) is always less than the suited field speed (Ns). This difference in speed depends upon load on the motor. The difference between the synchronous speed Ns of the rotating stator field and the actual rotor speed N is called slip. It is usually expressed as a percentage of synchronous speed i.e.,

$$\%\text{ age slip S} = \frac{N_s - N}{N_s} \times 100$$

(i) The quantity Ns − N is sometimes called slip speed.
(ii) When the rotor is stationary (i.e., N = 0), slip, s = 1 or 100 %.
(iii) In an induction motor, the change in slip from no-load to full-load is hardly 0.1% to 3% so that it is essentially a constant-speed motor.

1.7 Rotor Frequency at operation condition:

The frequency of a voltage or current induced due to the relative speed between a winding and a magnetic field is given by the general formula;

$f = \frac{PN}{120}$

where N = Relative speed between magnetic field and the winding
P = Number of poles
For a rotor speed N, the relative speed between the rotating flux and the rotor is (Ns – N). Consequently, the rotor current frequency f_2 is given by;

$$f_2 = \frac{(N_s - N)P}{120}$$

$$f_2 = \frac{SN_sP}{120}$$
$$f_2 = s\, f_1$$

Where f_2 = rotor current frequency, S = slip and f1 = supply frequency (stator frequency). The relative speed between the rotating field and stator winding is (Ns − 0)= Ns. Therefore, the frequency of induced current or voltage in the stator winding is same as the supply frequency

$$f_1 = Ns\ P/120.$$

1.8 Rotor induced emf, current and power factor:

The rotating magnetic field produced by the stator ampere-turns will induce emf in both the stator and rotor windings. The induced emf will depend upon the magnitude of the rotating flux and the speed at which this flux cuts the stator and rotor conductors. When the rotor is stationary (i.e., at standstill) the stator flux cuts the rotor conductors at a speed Ns. Let E_{20} be the induced emf in the rotor winding when the rotor is not rotating. When the rotor starts rotating at a speed Nr, the rotating field cuts the rotor conductors at a speed (Ns-Nr) i.e., at SNs rpm. Since at Ns speed of flux cutting, induced emf in the rotor is E_{20}, at SNs speed of flux cutting induced emf in the rotor will be SE_{20}. Let E_2 be the induced emf in the rotor winding when the rotor is rotating.

Rotor induced emf, $E_2=SE_{20}$

At the instant of starting, slip is equal to one. Therefore at start, $E_2=E_{20}$(maximum emf is induced in the rotor). As the motor picks up speed, its slip decreases, and, therefore the rotor induced emf decreases. When the rotor approaches synchronous speed, its slip reduced to a very small value and hence rotor induced emf becomes very small. The rotor cannot attain synchronous speed because at synchronous speed no emf will be induced in the rotor and no torque will be produced. Rotor will, therefore, always rotate at a lower speed than synchronous speed.

Fig (1.10) shows the circuit of a 3-phase induction motor at any slip s. The rotor is assumed to be of wound type and star connected. Note that rotor e.m.f./phase and rotor reactance/phase are sE_2 and sX_2 respectively. The rotor resistance/phase is R_2 and is independent of frequency and, therefore, does not depend upon slip. Likewise, stator winding values R_1 and X_1 do not depend upon slip. Since the motor represents a balanced 3-phase load, we need to consider one phase only; the conditions in the other two phases being similar.

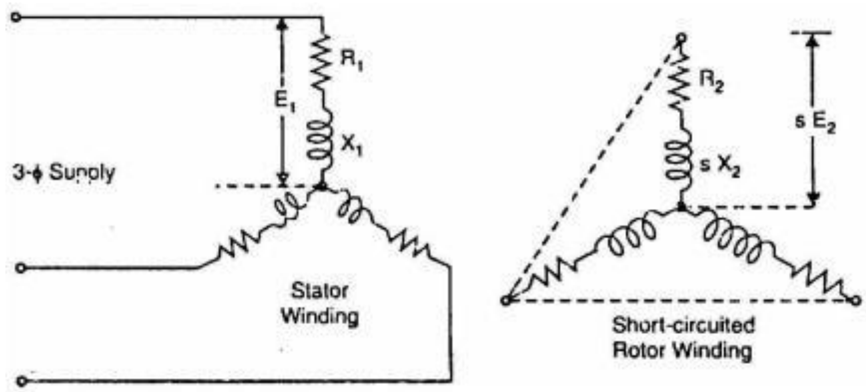


Figure 1.10 At standstill.

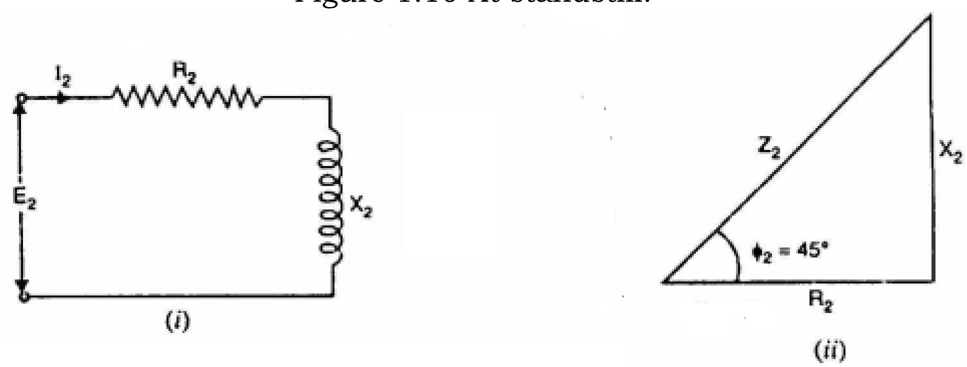


Fig. 1.11 One phase of the rotor circuit at standstill.

$$\text{Rotor current } I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{R_2{}^2+X_2{}^2}}$$

$$\text{Power factor} = \frac{R_2}{\sqrt{R_2{}^2+X_2{}^2}}$$

At running condition,

$$\text{Rotor current } I_2 = \frac{sE_2}{Z_2} = \frac{sE_2}{\sqrt{R_2{}^2+(sX_2)^2}}$$

$$\text{Power factor} = \frac{R_2}{\sqrt{R_2{}^2+(sX_2)^2}}$$

1.9 Power Flow Diagram and Losses of Induction Motor:

Power Flow Diagram of Induction Motor explains the input given to the motor, the losses occurring and the output of the motor. The input power given to an Induction motor is in the form of three-phase voltages and currents. It is given by the equation shown below:

$$P_{is} = \sqrt{3}V_L I_L \cos\varphi_i = 3V_{sp} I_{sp} \cos\varphi_i$$

Where, $\cos\varphi_i$ is the input power factor

The Power Flow Diagram of an Induction Motor is shown below.

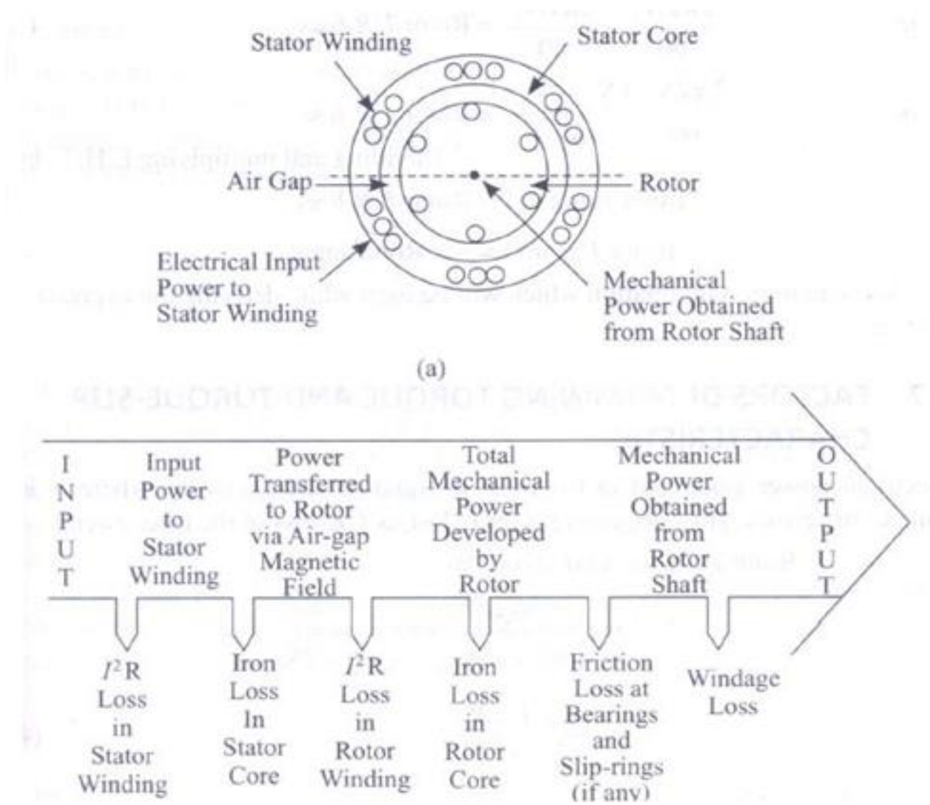


Fig 1.12 Power Flow Diagram of Induction Motor

The losses in the stator are:

i) I^2R losses in the stator winding resistances. It is also known as Stator copper losses.

$$P_{SCL} = 3{I_{sp}}^2 R_{sp}$$

ii) Hysteresis and Eddy current losses in the stator core. These are known as

Stator core losses $\{P_{s(h+e)}\}$

The output power of the stator is given as

$$P_{os} = P_{is} - P_{sc} - P_{s(h+e)}$$

This output power of the stator is transferred to the rotor of the machine across the air gap between the stator and the rotor. It is called the air gap Pg of the machine.

Thus,

The Power output of the stator = air gap power = input power to the rotor

$$P_{os} = P_g = P_{ir}$$

The losses in the rotor are as follows:

i) I^2R losses in the rotor resistance. They are also called Rotor copper losses and represented as

$$P_{rc} = 3{I_2}^2R_2$$

ii) Hysteresis and eddy current losses in the rotor core. They are known as Rotor core losses. $\{P_{r(h+e)}\}$

iii) Friction and Windage losses P_{fw}

iv) Stray load losses P_{misc}, consisting of all losses not covered above, such as losses due to harmonic fields.

If the rotor copper losses are subtracted from rotor input power P_g, the remaining power is converted from electrical to mechanical form. This is called Developed Mechanical Power P_{md}.

Developed Mechanical power = Rotor input – Rotor copper loss

$$P_{md} = P_{ir} - P_{rc} \quad \text{or}$$

$$P_{md} = P_g - P_{rc}$$

$$P_{md} = P_g - 3{I_2}^2R_2$$

The output of the motor is given by the equation shown below:

$$P_o = P_{md} - P_{fw} - P_{misc}$$

P_o is called the shaft power or the useful power.

At rated speed, iron loss in the rotor core is negligible because iron-loss is a function of rotor frequency and the frequency of the rotor current is very small (f_r=sf). Let T be the torque in Newton- meters exerted on the rotor by the rotating magnetic field rotating at synchronous speed N_s rpm or $N_s/60$ rps.

Power transferred from stator to rotor (i.e., rotor input)

$$= 2\Pi T\ N_s/60\ W$$

This input power to the rotor is termed torque in synchronous watts.

When the rotor is rotating at a speed of N_r rpm, total mechanical power developed by the rotor is $2\Pi T\ N_r/60$ W. From the figure 1.12 we observe that the difference between power transferred to the rotor via the magnetic field of air gap and the mechanical power developed by the rotor is equal to I^2R loss in the rotor winding (assuming iron-loss in the rotor core as negligible). This is expressed as:

(Power transferred from stator to rotor) – (Mechanical power developed by the rotor) = Rotor I^2R loss

$$2\Pi T\ N_s/60 - 2\Pi T\ N_s/60 = \text{Rotor } I^2R \text{ loss}$$

$$\frac{2\Pi T\ N_s}{60\ Ns}\ (Ns - Nr) = \text{Rotor } I^2R \text{ loss} \quad \text{(dividing and multiplying L.H.S. by } N_{s)}$$

$$\text{Rotor } I^2R \text{ loss} = S \text{ X Rotor Input.}$$

This is an important relation which will be used while deriving the expression for torque.

Rotational losses:

At starting and during acceleration, the rotor core losses are high. With the increase in the speed of the induction motor these losses decreases. The friction and windage losses are zero at the start. As the speed increases the losses, also start increasing. The sum of the friction, windage and core losses are almost constant with the change in speed. These all losses are added together and are known as Rotational Losses.

It is given by the equation shown below.

$$P_{rot} = P_{fw} + P_{h+e} + P_{misc}$$

$$P_o = P_{md} - P_{rot} = P_{md} - P_{fw} - P_{h+e} - P_{misc}$$

The Rotational losses are not represented by any element of the equivalent circuit as they are purely mechanical quantity.

1.10 Efficiency of Three Phase Induction Motor:

Efficiency is defined as the ratio of the output to that of input,

$$\text{Efficiency}, \eta = \frac{\text{output}}{\text{input}}$$

Rotor efficiency of the three phase induction motor ,

$$= \frac{\text{rotor output}}{\text{rotor input}}$$

= gross mechanical power developed / rotor input

$$= \frac{P_m}{P_2}$$

Three phase induction motor efficiency,

$$= \frac{\text{power developed at shaft}}{\text{electrical input to the motor}}$$

Three phase induction motor efficiency

$$\eta = \frac{P_{out}}{P_{in}}$$

1.11 Torque Equation of Three Phase Induction Motor:

The torque produced by three phase induction motor depends upon the following three factors:

- Firstly the magnitude of rotor current
- secondly the flux which interact with the rotor of three phase induction motor and is responsible for producing emf in the rotor part of induction motor,
- lastly the power factor of rotor of the three phase induction motor.

Combining all these factors together we get the equation of torque as

$$T \propto \emptyset I_2 cos\theta_2$$

where, T → the torque produced by induction motor,
φ → flux responsible of producing induced emf,
I_2 → rotor current,
$\cos\theta_2$ → the power factor of rotor circuit.

The flux φ produced by the stator is proportional to stator emf E_1. i.e., $\varphi \propto E_1$
We know that transformation ratio K is defined as the ratio of secondary voltage (rotor voltage) to that of primary voltage (stator voltage).

$$K = \frac{E_2}{E_1}$$

$$or, K = \frac{E_2}{\emptyset}$$

$$or, E_2 \propto \emptyset$$

Rotor current I_2 is defined as the ratio of rotor induced emf under running condition, sE_2 to total impedance, Z_2 of rotor side, $i.e., I_2 = \frac{sE_2}{Z_2}$ and total impedance Z_2 on rotor side is given by ,

$$Z_2 = \sqrt{{R_2}^2 + (sX_2)^2}$$

Putting this value in above equation we get, $I_2 = \frac{sE_2}{\sqrt{{R_2}^2+(sX_2)^2}}$

We know that power factor is defined as ratio of resistance to that of impedance. The power factor of the rotor circuit is $cos\theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{{R_2}^2+(sX_2)^2}}$

Putting the value of flux φ, rotor current I_2, power factor $\cos\theta_2$ in the equation of torque we get,

$$T \propto E_2 \frac{sE_2}{\sqrt{{R_2}^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{{R_2}^2 + (sX_2)^2}}$$

Combining similar term we get, $T \propto s{E_2}^2 \frac{R_2}{{R_2}^2+(sX_2)^2}$

Removing proportionality constant we get, $T = Ks{E_2}^2 \frac{R_2}{{R_2}^2+(sX_2)^2}$

This constant $K = \frac{3}{2\pi n_s}$

Where n_s is synchronous speed in r. p. s,

$n_s = N_s / 60$.

So, finally the equation of torque becomes,

$$\boldsymbol{T = s{E_2}^2 \times \frac{R_2}{{R_2}^2 + (sX_2)^2} \times \frac{3}{2\pi n_s} \;\; N-m}$$

Derivation of K in torque equation:

In case of three phase induction motor, there occur copper losses in rotor. These rotor copper losses are expressed as $P_c = 3{I_2}^2R_2$

We know that rotor current, $I_2 = \frac{sE_2}{\sqrt{{R_2}^2+(sX_2)^2}}$

Substitute this value of I_2 in the equation of rotor copper losses, P_c. So, we get

$$P_c = 3R_2\left(\frac{sE_2}{\sqrt{{R_2}^2 + (sX_2)^2}}\right)^2$$

The ratio of $P_2 : P_c : P_m = 1 : s : (1 - s)$ Where P_2 is the rotor input, P_c is the rotor copper losses, P_m is the mechanical power developed.

$$\frac{P_c}{P_m} = \frac{s}{1-s}$$

$$or\ P_m = \frac{(1-s)}{s}P_c$$

Substitute the value of Pc in above equation we get, $P_m = \frac{1}{s} \times \frac{(1-s)3R_2s^2{E_2}^2}{{R_2}^2+(sX_2)^2}$

On simplifying we get, $P_m = \frac{(1-s)3R_2sE_2{}^2}{R_2{}^2+(sX_2)^2}$

The mechanical power developed $P_m = T\omega$, $\omega = \frac{2\pi N}{60}$

$$or\ P_m = T\frac{2\pi N}{60}$$

Substituting the value of P_m

$$\frac{1}{s}\times\frac{(1-s)3R_2s^2E_2{}^2}{R_2{}^2+(sX_2)^2} = T\frac{2\pi N}{60}$$

$$or\ T = \frac{1}{s}\times\frac{(1-s)3R_2s^2E_2{}^2}{R_2{}^2+(sX_2)^2}\times\frac{60}{2\pi N}$$

We know that the rotor speed $N = N_s (1 - s)$

Substituting this value of rotor speed in above equation we get,

$$T = \frac{1}{s}\times\frac{(1-s)3R_2s^2E_2{}^2}{R_2{}^2+(sX_2)^2}\times\frac{60}{2\pi N_s(1-s)}$$

N_s is speed in revolution per minute (rpm) and n_s is speed in revolution per sec (rps) and the relation between the two is $\frac{N_s}{60}=n_s$

Substitute this value of N_s in above equation and simplifying it we get

$$Torque, T = \frac{sE_2{}^2R_2}{R_2{}^2+(sX_2)^2}\times\frac{3}{2\pi n_s}$$

$$or\ T = KsE_2{}^2\frac{R_2}{R_2{}^2+(sX_2)^2}$$

Comparing both the equations, we get, constant $K = 3 / 2\pi n_s$

1.12 Equation of Starting Torque of Three Phase Induction Motor:

Starting torque is the torque produced by induction motor when it is started. We know that at start the rotor speed, N is zero.

$$So, slip\ s = \frac{N_s - N}{N_s}\ becomes\ 1.$$

So, the equation of starting torque is easily obtained by simply putting the value of s = 1 in the equation of torque of the three phase induction motor,

$$Tst = \frac{{E_2}^2 R_2}{{R_2}^2 + {X_2}^2} \times \frac{3}{2\pi n_s} \; N-m$$

Starting Torque of a squirrel-cage motor:

- The resistance of a squirrel cage motor is fixed and small as compared to its reactance because at starting the frequency of the rotor current is equal to supply frequency.
- Hence the starting current I_2 of rotor though very large in magnitude lags by very large angle behind E_2 so the starting torque is very poor for squirrel cage Induction Motor.
- It is roughly 1.5 times the full-load torque although starting current is 5 to 7 times full –load current.
- Hence such motors are not useful where the motor has to start against heavy loads.

Starting Torque of a slip-ring Induction Motor:

- The starting torque can be increased by improving its power factor by adding external resistance in the rotor circuit from the star connected rheostat, the rheostat resistance being progressively cut out as the motor gathers speed.
- Addition of external resistance, however, increases the rotor impedance and so reduces the rotor current.
- The effect of improved power factor predominates the current –decreasing effect of impedance.
- Hence starting torque is increased.
- But after a certain point, the effect of increased impedance predominates the effect of improved power factor and so the torque starts decreasing.

1.13 Condition for Maximum Starting torque:

We know

$$Tst = \frac{{E_2}^2 R_2}{{R_2}^2 + {X_2}^2} \times \frac{3}{2\pi n_s} \; N-m$$

The rotor inductive reactance and synchronous speed of induction motor remain constant. The supply voltage to the three phase induction motor is usually rated and remains constant so the stator emf also remains the constant. The transformation ratio is defined as the ratio of rotor emf to that of stator emf. So if stator emf remains constant then rotor emf also remains constant.

If we want to find the maximum value of some quantity then we have to differentiate that quantity with respect to some variable parameter and then put it equal to zero. In this case we have to find the condition for maximum starting torque so we have to differentiate starting torque with respect to some variable quantity which is resistance in this case as all other parameters in the equation of torque remains constant.

$$\frac{dTst}{dR_2} = 0$$

$$\frac{d}{dR_2}\left(\frac{{E_2}^2 R_2}{{R_2}^2 + {X_2}^2} \times \frac{3}{2\pi n_s}\right) = 0$$

$$K_2 \frac{\left({R_2}^2 + {X_2}^2\right)(1) - R_2(2R_2)}{\left({R_2}^2 + {X_2}^2\right)^2} = 0$$

$${R_2}^2 + {X_2}^2 - 2{R_2}^2 = 0$$

$${X_2}^2 = {R_2}^2$$

$$R_2 = X_2$$

This is the condition for maximum starting torque which occurs when rotor resistance per phase is equal to rotor reactance per phase.

1.14 Maximum Torque Condition for Three Phase Induction Motor:

In the equation of torque,

$$T = \frac{s{E_2}^2 R_2}{{R_2}^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}\ N - m$$

The rotor resistance, rotor inductive reactance and synchronous speed of induction motor remain constant. The supply voltage to the three phase induction motor is usually rated and remains constant so the stator emf also remains the constant. The transformation ratio is defined as the ratio of rotor emf to that of stator emf. So if stator emf remains constant then rotor emf also remains constant.

In this case we have to find the condition for maximum torque so we have to differentiate torque with respect to some variable quantity which is slip; s in this case as all other parameters in the equation of torque remains constant.

So, for torque to be maximum,

$$\frac{dT}{ds} = 0$$

$$T = KsE_2{}^2 \frac{R_2}{R_2{}^2 + (sX_2)^2}$$

Now differentiate the above equation by using division rule of differentiation. On differentiating and after putting the terms equal to zero we get,

$$s^2 = \frac{R_2^2}{X_2^2}$$

Neglecting the negative value of slip we get

$$s = \frac{R_2}{X_2}$$

So, when slip s = R_2 / X_2, the torque will be maximum and this slip is called maximum slip S_m and it is defined as the ratio of rotor resistance to that of rotor reactance.

NOTE: At starting s= 1, so the maximum starting torque occur when rotor resistance is equal to rotor reactance.

1.15 Equation of Maximum Torque:

The equation of torque is $T = \frac{sE_2{}^2R_2}{R_2{}^2+(sX_2)^2}$

The torque will be maximum when slip s = R_2 / X_2

Substituting the value of this slip in above equation we get the maximum value of torque as,

$$T_{max} = K \frac{{E_2}^2}{2X_2} \ N - m$$

Conclusion:

From the above equation it is concluded that

1. The maximum torque is directly proportional to square of rotor induced emf at the standstill.
2. The maximum torque is inversely proportional to rotor reactance.
3. The maximum torque is independent of rotor resistance.
4. The slip at which maximum torque occur depends upon rotor resistance, R_2. So, by varying the rotor resistance, maximum torque can be obtained at any required slip.

1.16 Torque Slip Characteristics of Three Phase Induction Motor:

The torque slip curve for an induction motor gives us the information about the variation of torque with the slip. The variation of slip can be obtained with the variation of speed that is when speed varies the slip will also vary and the torque corresponding to that speed will also vary.

The curve can be described in three modes of operation:

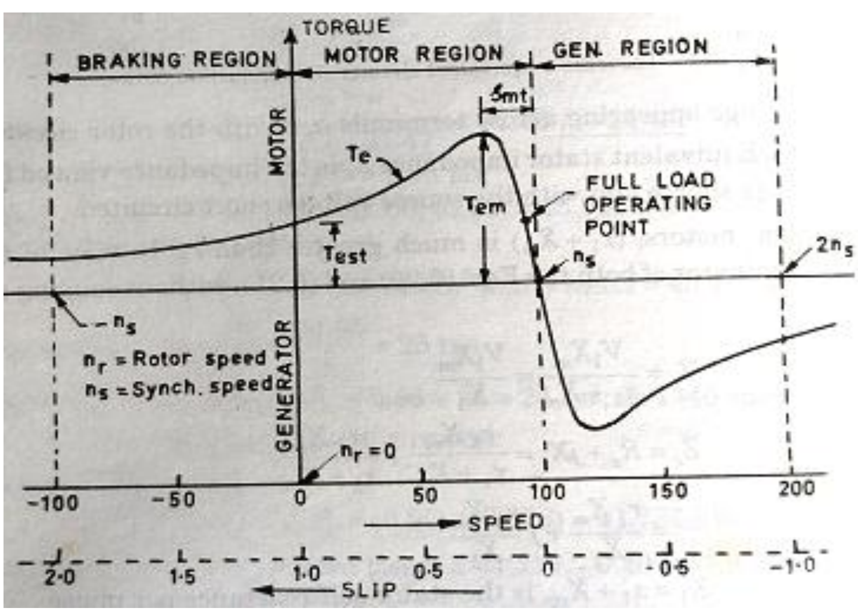


Figure 1.13 Torque slip curve of 3-phase Induction Motor

Motoring Mode:

In this mode of operation, supply is given to the stator side and the motor always rotates below the synchronous speed. The induction motor torque varies from zero to full load torque as the slip varies. The slip varies from zero to one. It is zero at no load and one at standstill. From the curve it is seen that the torque is directly proportional to the slip. That is, more is the slip; more will be the torque produced and vice-versa. The linear relationship simplifies the calculation of motor parameter to great extent.

Generating Mode:

In this mode of operation induction motor runs above the synchronous speed and it should be driven by a prime mover. The stator winding is connected to a three phase supply in which it supplies electrical energy. Actually, in this case, the torque and slip both are negative so the motor receives mechanical energy and delivers electrical energy. Induction motor is not much used as generator because it requires reactive power for its operation. That is, reactive power should be supplied from outside and if it runs below the synchronous speed by any means, it consumes electrical energy rather than giving it at the output. So, as far as possible, induction generators are generally avoided.

Braking Mode:

In the braking mode, any two leads of the supply voltage are inter-changed so that the motor starts to rotate in the reverse direction and as a result the motor stops. This method of braking is known as plugging. This method is used when it is required to stop the motor within a very short period of time. The kinetic energy stored in the revolving load is dissipated as heat. Also, motor is still receiving power from the stator which is also dissipated as heat. So as a result of which motor develops enormous heat energy. For this stator is disconnected from the supply before motor enters the braking mode.

If load which the motor drives accelerates the motor in the same direction as the motor is rotating, the speed of the motor may increase more than synchronous speed. In this case, it acts as an induction generator which supplies electrical energy to the mains which tends to slow down the motor to its synchronous speed, in this case the motor stops. This type of braking principle is called dynamic or regenerative braking.

1.17 Effect of change in supply voltage on Torque and Speed:

The torque equation is

$$T = \frac{K\emptyset s E_2 R_2}{{R_2}^2 + (sX_2)^2}$$

We know $\boldsymbol{E_2} \propto \emptyset \propto \boldsymbol{V}$ where V is supply voltage

So $T \propto sV^2$

- So the torque at any speed is proportional to the square of the applied voltage.
- If stator voltage decreases then torque also decreases.
- Hence to maintain the same torque, slip increases and speed falls.

1.18 Effect of change in supply frequency on Torque and Speed:

- Hardly changes in frequency takes place on a large distribution system except during a major disturbance.
- The major effect of change in supply frequency is on motor speed.
- If frequency drops by 10% then motor speed also drops by 10%.

$$f = \frac{pN}{120}$$

$$f \propto N$$

1.19 Effect of Variation of Rotor Resistance on the Torque-slip Characteristic:

The torque equation for an induction motor at constant input voltage is rewritten as

$$T = \frac{KsR_2}{R_2 + s^2{X^2}_2}$$

- Let the values of R_2 and X_2 for a particular induction motor are 1Ω and 10 Ω respectively. Let us also assume that it is possible to increase the rotor circuit resistance by some external means.

- In slip-ring induction motors, external resistance can be connected across the rotor terminals with the help of brush and slip-ring arrangement.

- Let us assume that the total rotor resistance is made 1, 2, 6 and 10 Ω respectively.

- The rotor standstill reactance X_2= 10 Ω will remain constant since X_2 is fixed by the design of the rotor.

- If we calculate the value of torque, T, for different values of slip, s, we shall get a number of points on the torque-slip characteristic.

- Table 1.1 gives the magnitudes of torque at different values of slip for R_2=1, X_2=10; R_2=2, X_2=10; R_2=6, X_2=10; and R_2=10, X_2=10;

- For studying the nature of torque-slip characteristics, a suitable value of K can be taken. Let us take K= 100.

- Thus

$$T = \frac{100sR_2}{R_2 + s^2X^2{}_2}$$

- The values of torque calculated at different values of slip for various combinations of R_2 and X_2 are shown in table 1.1.

SLIP	TORQUE FOR $R_2 = 1, X_{20} = 10$	TORQUE FOR $R_2 = 2, X_{20} = 10$	TORQUE FOR $R_2 = 6, X_{20} = 10$	TORQUE FOR $R_2 = 10, X_{20} = 10$
0.01	0.99	0.49	0.16	0.09
0.02	1.92	0.99	0.33	0.199
0.05	4.0	2.35	0.82	0.49
0.1	5.0	4.0	1.62	0.99
0.15	4.6	4.8	2.35	1.46
0.2	4.0	5.0	3.0	1.92
0.3	3.0	4.61	4.0	2.75
0.4	2.35	4.0	4.61	3.45
0.5	1.92	3.45	4.92	4.0
0.6	1.62	3.0	5.0	4.41
0.9	1.09	2.11	4.61	4.97
1.0	0.99	1.92	4.41	5.0
0.7	1.23	2.35	4.8	4.87

Table 1.1 Calculated Values of Torque at Different Slips Having Variable Rotor Resistance

- Torque-slip characteristics for different values of rotor circuit resistance as per table 1.1 have been drawn as shown in figure 1.14.

- Torque-slip characteristics for four different values of rotor circuit resistance have been drawn together for the sake of comparison.

From figure 1.14 the following observations can be made:

- Starting torque increases with increase in value of rotor resistance.

- Maximum torque remains constant and is independent of the value of rotor resistance.

- The slip at which maximum torque occurs varies with the variation of rotor resistance.

- Maximum torque is developed at starting when rotor resistance is equal to the standstill rotor reactance, i.e., when R_2 is equal to X_2.

- Torque is maximum when the rotor reactance X_{2r}= (sX_2) is equal to the rotor resistance R_2 (in graph A, for example, maximum torque occurs when s=0.1. Thus the relationship R_2= sX_2 holds well. For R_2=10 and X_2=10, maximum torque occurs when s=1).

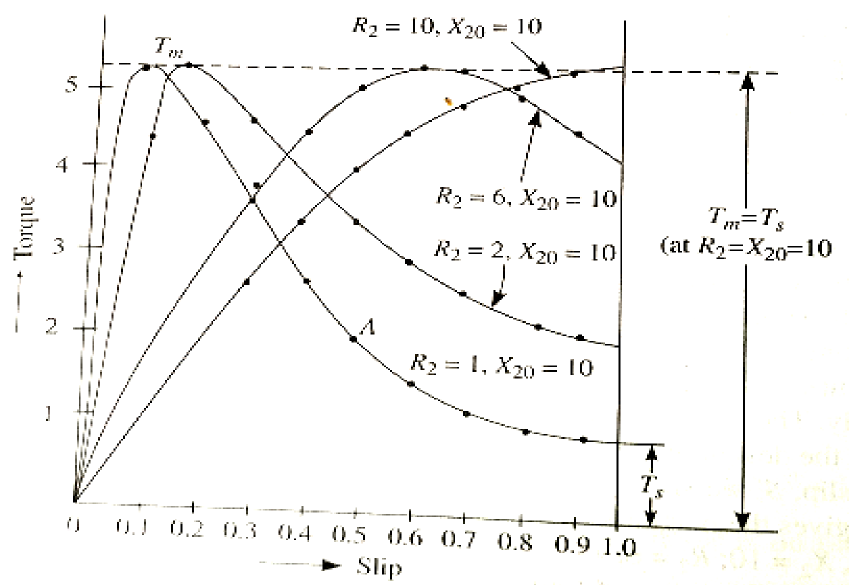


Figure 1.14 Effect of variation of rotor circuit resistance on torque slip characteristic of an Induction Motor

1.20 Relation between full load torque (T_f) and Maximum Torque(T_{max}):

Let s_f be the slip of full-load torque

$$T_f \propto \frac{s_f R_2}{{R_2}^2 + (s_f X_2)^2}$$

$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_f}{T_{max}} = \frac{2 s_f R_2 X_2}{{R_2}^2 + (s_f X_2)^2}$$

Dividing numerator and denominator by X_2^2 we get

$$\frac{T_f}{T_{max}} = \frac{\frac{2 s_f R_2}{X_2}}{(\frac{R_2}{X_2})^2 + {s_f}^2}$$

$$\text{Let } \frac{R_2}{X_2} = a$$

$$\frac{T_f}{T_{max}} = \frac{2 a s_f}{a^2 + {s_f}^2}$$

In fact $a = s_m$ → slip corresponding to maximum torque

So

$$\frac{T_f}{T_{max}} = \frac{2 s_m s_f}{{s_m}^2 + {s_f}^2}$$

1.21 Relation between Starting Torque (Tst) and Maximum Torque(T_{max}):

$$Tst \propto \frac{R_2}{{R_2}^2 + {X_2}^2}$$

$$T_{max} \propto \frac{1}{2X_2}$$

$$\frac{T_{st}}{T_{max}} = \frac{2R_2X_2}{{R_2}^2 + {X_2}^2}$$

Dividing numerator and denominator by X_2^2 on both sides

$$\frac{T_{st}}{T_{max}} = \frac{\frac{2R_2}{X_2}}{(\frac{R_2}{X_2})^2 + 1}$$

So $\frac{T_{st}}{T_{max}} = \frac{2a}{1+a^2}$

1.22 Equivalent Circuit for an Induction Motor:

Induction motor is a well-known device which works on the principle of transformer. That is, when an EMF is supplied to its stator, then as a result of electromagnetic induction, a voltage is induced in its rotor. So an induction motor is said to be a transformer with rotating secondary. Here, primary of transformer resembles stator winding of an induction motor and secondary resembles rotor.

The induction motor always runs below the synchronous or full load speed and the relative difference between the synchronous speed and speed of rotation is known as slip which is denoted by s:

$$s = \frac{(N_s - N)}{N_s}$$

Where, N_s is synchronous speed of rotation which is given by:

$$N_s = \frac{120f}{P}$$

Where, f is the frequency of the supply.

P is the number of poles of the machine.

N is the speed of rotation.

The equivalent circuit of any machine shows the various parameters of the machine such as its ohmic losses and also other losses. The losses are modeled just by inductor and resistor. The copper losses are occurred in the windings so the winding resistance is taken into account. The winding also has inductance for which there is a voltage drop due to inductive reactance.

Exact Equivalent Circuit:

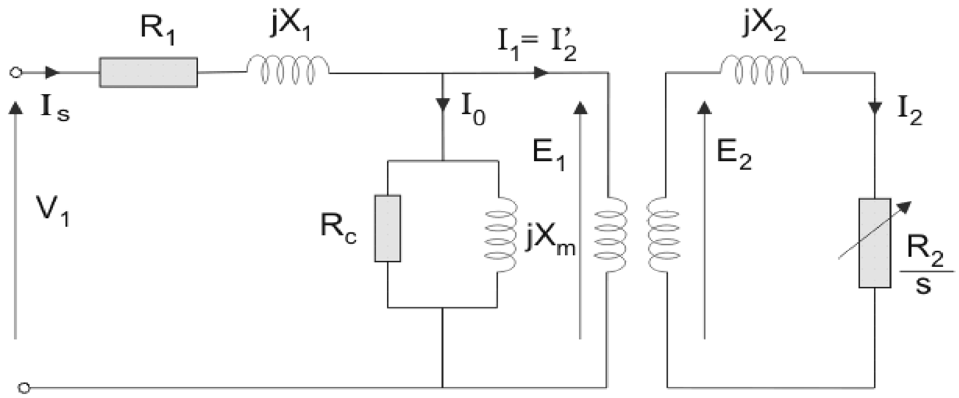


Figure 1.15 Exact Equivalent Circuit

Here, R_1 is the winding resistance of the stator. X_1 is the inductance of the stator winding. R_c is the core loss component. X_M is the magnetizing reactance of the winding. R_2/s is the power of the rotor, which includes output mechanical power and copper loss of rotor. If we draw the circuit with referred to the stator then the circuit will look like

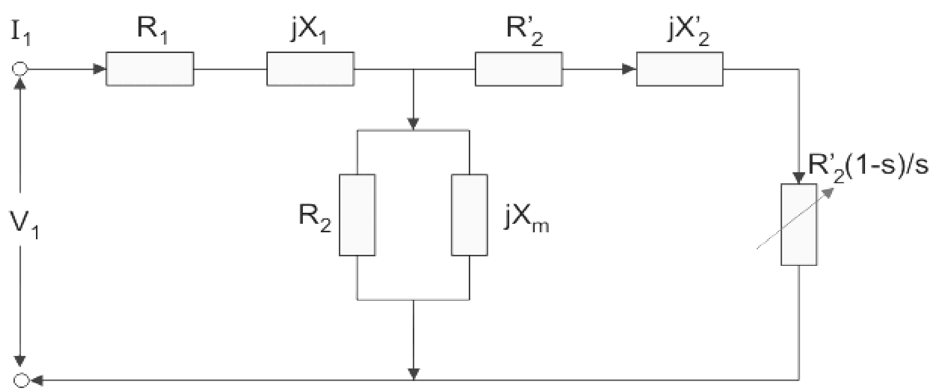


Figure 1.16 Equivalent circuit referred to Stator side

Here all the other parameters are same except- R_2' is the rotor winding resistance with referred to stator winding. X_2' is the rotor winding inductance with referred to stator winding. R_2 (1 - s) / s is the resistance which shows the power which is converted to mechanical power output or useful power. The power dissipated in that resistor is the useful power output or shaft power.

The rotor current is given by

$$I_2 = \frac{sE_2}{\sqrt{{R_2}^2 + s^2{X_2}^2}}$$

$$I_2 = \frac{E_2}{\sqrt{(\frac{R_2}{s})^2 + {X_2}^2}}$$

The rotor circuit can, therefore, be represented by a resistance R_2/s and a reactance X_2 connected in series across a voltage source E_2 which causes a current I_2 flowing through the circuit.

To show the mechanical power conversion in the rotor circuit, the resistance R_2/s of the rotor circuit can be represented as two separate resistances, viz., R_2 and $R_2\frac{(1-s)}{s}$ as

$$\frac{R_2}{s} = \frac{R_2}{s} - R_2 + R_2 \qquad \text{(By subtracting and adding } R_2\text{)}$$

$$\frac{R_2}{s} = R_2\left(\frac{1-s}{s}\right) + R_2$$

Power Relation of Equivalent Circuit:

1. Input power to stator- 3 V_1I_1CosΘ. Where, V_1 is the stator voltage applied. I_1 is the current drawn by the stator winding. CosΘ is the stator power stator.
2. Rotor input=Power input- Stator copper and iron losses.
3. Rotor Copper loss = Slip × power input to the rotor.
4. Mechanical Power Developed = (1 - s) × Rotor input power.

1.23 Crawling and Cogging of Induction Motor:

The important characteristics normally shown by a squirrel cage induction motors are crawling and cogging. These characteristics are the result of improper functioning of the motor that means either motor is running at very slow speed or it is not taking the load.

Crawling of Induction Motor:

- Certain combinations of S_1 and S_2 cause accentuation of certain space harmonics of the mmf wave, e.g. fifth and seventh harmonics which correspond to poles five and seven times that of the fundamental.
- Since the space-phase difference between fundamental poles of the winding phase is (0^0, 120^0, 240^0), this (space-phase) difference is (0^0,240^0, 120^0), for the fifth harmonic poles and (0^0,120^0,240^0), for the seventh.
- Hence the fifth harmonic poles rotate backwards with synchronous speed of $n_s/5$ and the seventh harmonic poles rotate forward at $n_s/7$.
- Theses harmonic mmfs produce their own asynchronous (induction) torques of the same general torque-slip shape as that of the fundamental.
- Figure 1.17 below shows the superimposition of the fundamental, fifth and seventh harmonic torque-slip curves.
- A marked saddle effect is observed with stable region of operation (negative torque-slip slope) around $1/7^{th}$ normal speed (s=6/7) .
- In figure 1.17 the load torque curve intersects the motor torque curve at the point M resulting in stable operation.
- This phenomenon is known as crawling (running stably at low speed).
- Certain slot combinations, e.g. S_1=24 and S_2=18 cause the stator mmf to possess a reversed 11^{th} and a forward 13^{th} harmonic mmf while the rotor has a reversed 13^{th} and a forward 15^{th}.
- The stator 13^{th} harmonic mmf rotates at speed $+n_s/13$ with respect to the stator and the rotor mmf of the 13^{th} harmonic rotates at $-(n_s-n)/13$ with respect to the rotor when the rotor is running at speed n.
- These two mmf's lock into each other to produce a synchronous torque when

$$\frac{n_s}{13} = n - \frac{n_s - n}{13}$$

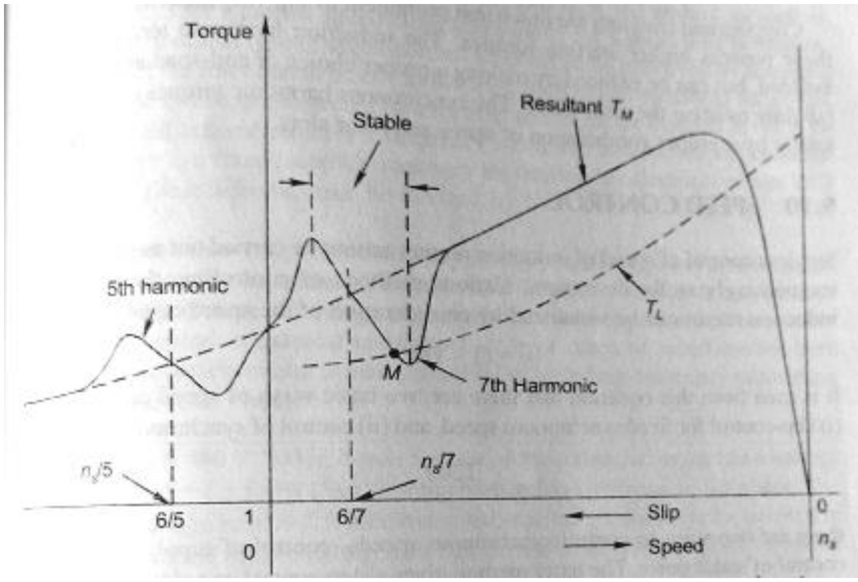


Figure 1.17 Torque-slip Characteristic of a 3-phase Induction motor showing the effect of harmonic asynchronous (induction) torques

Cogging of Induction Motor:

- A squirrel-cage rotor may exhibit a peculiar behavior in starting for certain relationships between the number of poles and the stator and rotor slots.
- With the number of stator slots S_1 equal to or an integral multiple of rotor slots S_2, the variation of reluctance as a function of space will be quite pronounced resulting in strong alignment forces at the instant of starting.
- These forces may create an aligning torque stronger than the accelerating torque with consequent failure of the motor to start.
- This phenomenon is known as cogging.
- Such combination of stator and rotor slots must, therefore, be avoided in machine design.

Methods to overcome cogging:

This problem can be easily solved by adopting several measures. These solutions are as follows:

- The number of slots in rotor should not be equal to the number of slots in the rotor.
- Skewing of the rotor slots, that means the stack of the rotor is arrange in such a way that it angled with the axis of the rotation.

Cogging and crawling are much less prominent in slip-ring induction machines as these possess higher starting torques. The induction harmonic torque cannot be avoided, but can be reduced by making a proper choice of coil-span and by skewing (slightly twisting the rotor teeth). The synchronous harmonic torques can be avoided totally by a proper combination of stator and rotor slots.