Unit IV Single Phase Motors

About Single Phase Induction Motor

We use the single phase power system more widely than three phase system for domestic purposes, commercial purposes and some extent in industrial uses. Because, the single-phase system is more economical than a three-phase system and the power requirement in most of the houses, shops, offices are small, which can be easily met by a single phase system. The single phase motors are simple in construction, cheap in cost, reliable and easy to repair and maintain. Due to all these advantages, the single phase motor finds its application in vacuum cleaners, fans, washing machines, centrifugal pumps, blowers, washing machines, etc.

Construction of Single Phase Induction Motor

Like any other electrical motor asynchronous motor also have two main parts namely rotor and stator

Stator

As its name indicates stator is a stationary part of induction motor. A single phase AC supply is given to the stator of single phase induction motor.

Rotor

The rotor is a rotating part of an induction motor. The rotor connects the mechanical load through the shaft. The rotor in the single-phase induction motor is of squirrel cage rotor type.

The construction of single phase induction motor is almost similar to the squirrel cage three-phase induction motor. But in case of a single phase induction motor, the stator has two windings instead of one three-phase winding in three phase induction motor.

Stator of Single Phase Induction Motor

The stator of the single-phase induction motor has laminated stamping to reduce eddy current losses on its periphery. The slots are provided on its stamping to carry stator or main winding. Stampings are made up of silicon steel to reduce the hysteresis losses. When we apply a single phase AC supply to the stator winding, the magnetic field gets produced, and the motor rotates at speed slightly less than the synchronous speed N_s. Synchronous speed N_s is given by

$$N_s = \frac{120f}{P}$$

Where,

f = supply voltage frequency,

P = No. of poles of the motor.

The construction of the stator of the single-phase induction motor is similar to that of three phase induction motor except there are two dissimilarities in the winding part of the single phase induction motor.

1. Firstly, the single-phase induction motors are mostly provided with concentric coils. We can easily adjust the number of turns per coil can with the help of concentric coils. The MMF distribution is almost sinusoidal.
2. Except for shaded pole motor, the asynchronous motor has two stator windings namely the main winding and the auxiliary winding. These two windings are placed in space quadrature to each other.

Rotor of Single Phase Induction Motor

The construction of the rotor of the single-phase induction motor is similar to the squirrel cage three-phase induction motor. The rotor is cylindrical and has slots all over its periphery. The slots are not made parallel to each other but are a little bit skewed as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of induction motor more smooth and quieter (i.e. less noisy).

The squirrel cage rotor consists of aluminum, brass or copper bars. These aluminum or copper bars are called rotor conductors and placed in the slots on the periphery of the rotor. The copper or aluminum rings permanently short the rotor conductors called the end rings.

To provide mechanical strength, these rotor conductors are braced to the end ring and hence form a complete closed circuit resembling a cage and hence got its name as squirrel cage induction motor. As end rings permanently short the bars, the rotor electrical resistance is very small and it is not possible to add external resistance as the bars get permanently shorted. The absence of slip ring and brushes make the construction of single phase induction motor very simple and robust.

Working Principle of Single Phase Induction Motor

When we apply a single phase AC supply to the stator winding of single phase induction motor, the alternating current starts flowing through the stator or main winding. This alternating current produces an alternating flux called main flux. This main flux also links with the rotor conductors and hence cut the rotor conductors.

According to the Faraday’s law of electromagnetic induction, EMF gets induced in the rotor. As the rotor circuit is closed one so, the current starts flowing in the rotor. This current is called the rotor current. This rotor current produces its flux called rotor flux. Since this flux is produced due to the induction principle so, the motor working on this principle got its name as an induction motor. Now there are two fluxes one is main flux, and another is called rotor flux. These two fluxes produce the desired torque which is required by the motor to rotate.

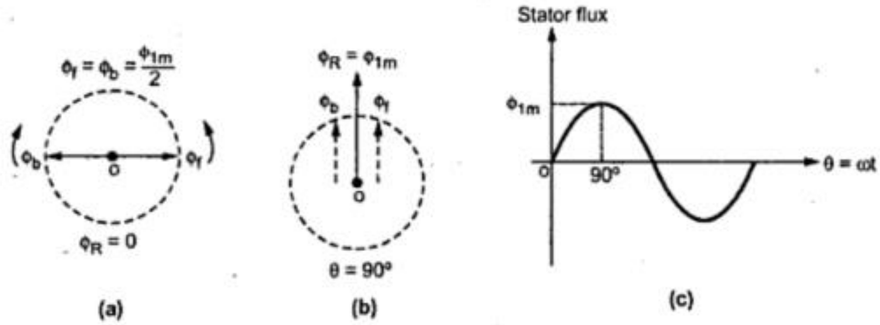
Why single phase induction motor not self starts?

Because of single phase supply, at starting condition, both the forward and backward components of flux are exactly opposite to each other. Also, both of these components of flux are equal in magnitude. So, they cancel each other and hence the net torque experienced by the rotor at the starting condition is zero. **So, the single phase induction motors are not self-starting motors.**

Double Revolving Field Theory

According to this theory, any alternating quantity can be resolved into two rotating components which rotate in opposite directions and each having magnitude as half of the maximum magnitude of the alternating quantity. In case of single phase induction motors, the stator winding produces an alternating magnetic field having maximum magnitude of $\mathbf{\Phi_{1m}}$. According to double revolving field theory, consider the two components of the stator flux, each having magnitude half of maximum magnitude of stator flux i.e. ($\mathbf{\Phi_{1m}/2}$). Both these components are rotating in opposite directions at the synchronous speed N_s which are dependent on frequency and stator poles.

Let $\mathbf{\Phi_f}$ is forward component rotating in anticlockwise direction while $\mathbf{\Phi_b}$ is the backward component rotating in clockwise direction. The resultant of these two components at any instant gives the instantaneous value of the stator flux at the instant. So resultant of these two is the original stator flux.



Stator flux and its two components

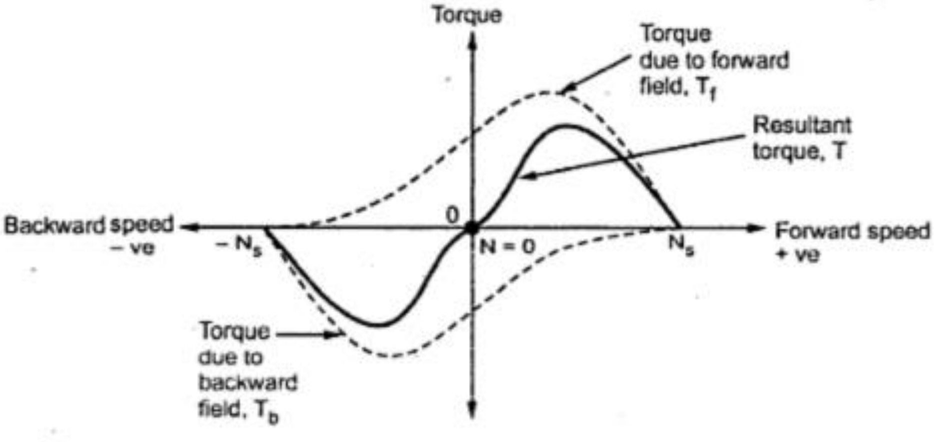
The above Fig shows the stator flux and its two components **Φ_f** and **Φ_b**. At start both the components are shown opposite to each other in the Fig. (a). Thus the resultant **$\Phi_R = 0$**. This is nothing but the instantaneous value of the stator flux at start. After 90^o, as shown in the Fig. (b), the two components are rotated in such a way that both are pointing in the same direction. Hence the resultant Φ_R is the algebraic sum of the magnitudes of the two components. So **$\Phi_R = (\Phi_{1m}/2) + (\Phi_{1m}/2) = \Phi_{1m}$**. This is nothing but the instantaneous value of the stator flux at **$\theta = 90^o$** as shown in the Fig (c). Thus continuous rotation of the two components gives the original alternating stator flux.

Both the components are rotating and hence get cut by the motor conductors. Due to cutting of flux, EMF gets induced in rotor which circulates rotor current. The rotor current produces rotor flux. This flux interacts with forward component **Φ_f** to produce a torque in one particular direction say anticlockwise direction. While rotor flux interacts with backward component **Φ_b** to produce a torque in the clockwise direction. So if anticlockwise torque is positive then clockwise torque is negative.

At start these two torques are equal in magnitude but opposite in direction. Each torque tries to rotate the rotor in its own direction. Thus net torque experienced by the rotor is zero at start. And hence the single phase induction motors are not self starting. The resultant torque is shown below

Torque speed characteristics

The two oppositely directed torques and the resultant torque can be shown effectively with the help of torque-speed characteristics. It is shown in the Fig. below.



It can be seen that at start **N = 0** and at that point resultant torque is zero. So single phase motors are not self starting.

However if the rotor is given an initial rotation in any direction, the resultant average torque increase in the direction in which rotor initially rotated. And motor starts rotating in that direction. But in practice it is not possible to give initial torque to rotor externally hence some modifications are done in the construction of single phase induction motors to make them self starting.

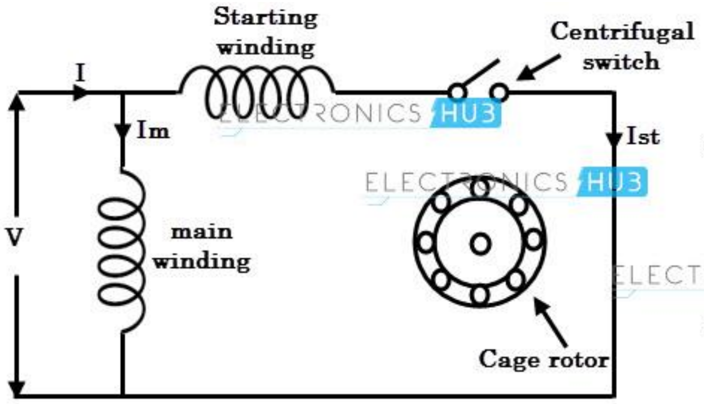
Starting Methods of Single-Phase Induction Motors

A single-phase induction motor with main stator winding has no inherent starting torque, since main winding introduces only stationary, pulsating air-gap flux wave. For the development of starting torque, rotating air-gap field at starting must be introduced. Several methods which have been developed for the starting of single-phase induction motors, may be classified as follows

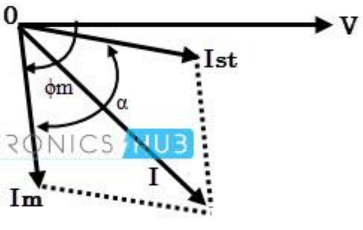
1. Split-phase motor
2. Capacitor start motor
3. Permanent capacitor run motor
4. Capacitor start capacitor run motor
5. Shaded pole motor

This is one of the most widely used types of single phase induction motors. The essential parts of the split phase motor include main winding, auxiliary winding and a centrifugal switch.

This is the simplest arrangement to set up a rotating magnetic field by providing two winding on the same stator core as shown in figure.



Circuit diagram of split phase motor



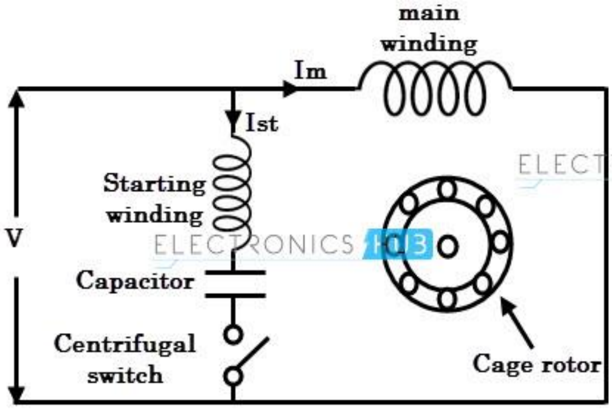
Phasor diagram

The auxiliary or starting winding carries a series resistance such that its impedance becomes highly resistive in nature. It is not wound identical to the main winding but contains fewer turns of much smaller diameter as compared to main winding. This will reduce the amount of start current lags the voltage. The main winding is inductive in nature in such that current lags the voltage by some angle. This winding is designed for the operation of 75 % of synchronous speed and above. These two windings are connected in parallel across the supply. Due to the inductive nature, current through main winding lags the supply voltage by a large angle while the current through starting winding is almost in phase with voltage due to resistive nature. Hence there exists a phase difference between these currents and thereby phase difference between the fluxes produced by these currents. The resultant of these two fluxes produce rotating magnetic field and hence the starting torque. The centrifugal switch is connected in series with the starting winding. When the motor reaches 75 to 80 percent of synchronous speed, the centrifugal switch is opened mechanically and thereby auxiliary winding is out of the circuit. Therefore, the motor runs only with main winding. Split phase motors give poor starting torque due to small phase difference between main and auxiliary currents. Also, the power factor of these motors is poor. These are mainly used for easily started loads such as blowers, fans, washing machines, grinders, etc.

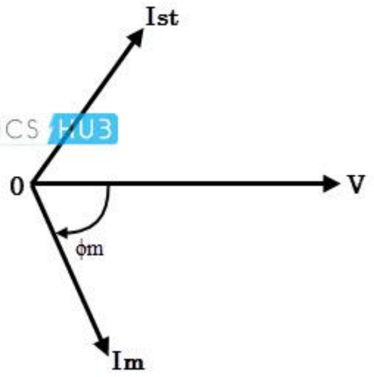
Capacitor Start Induction Motor

This motor is similar to the split phase motor, but in addition a capacitor is connected in series to auxiliary winding. This is a modified version of split phase motor.

Since the capacitor draws a leading current, the use of a capacitor increases the phase angle between the two currents (main and auxiliary) and hence the starting torque. This is the main reason for using a capacitor in single phase induction motors.



Circuit diagram of capacitor start motor



Phasor diagram

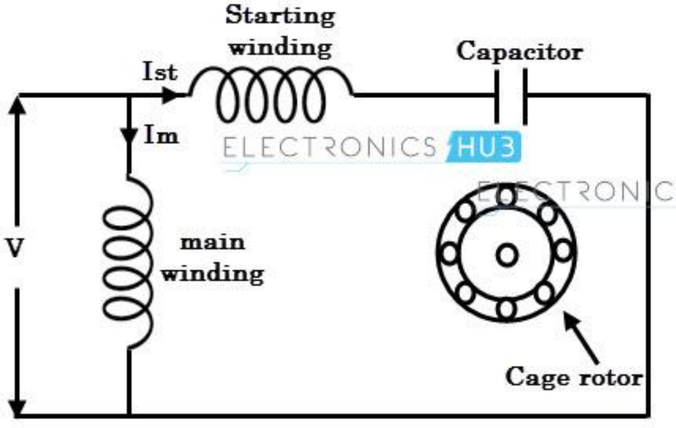
Here the capacitor is of dry-type electrolytic one which is designed only for alternating current use. Due to the inexpensive type of capacitors, these motors become more popular in wide applications.

These capacitors are designed for definite duty cycle, but not for continuous use. The schematic diagram of capacitor start motor is shown in figure above. The operation of this motor is similar to the split phase motor where the starting torque is provided by additional winding. Once the speed is picked up, the additional winding along with capacitor is removed from the circuit with the help of centrifugal switch. But, the difference is that the torque produced by this motor is higher than split phase motor due to the use of capacitor. Due to the presence of a capacitor, the current through auxiliary winding will leads the applied voltage by some angle which is more than that of split case type. Thus, the phase difference between main and auxiliary currents is increased and thereby starting torque.

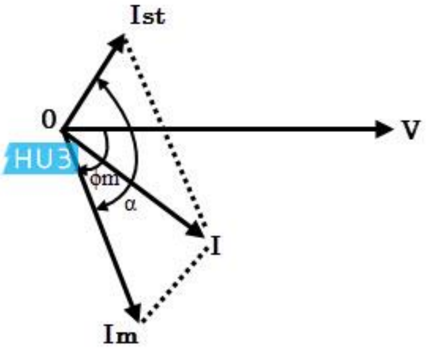
The performance of this motor is identical to the split phase motor when it runs near full load RPM. Due to the capacitor, the inrush currents are reduced in this motor. These motors have very high starting torque up to 300% full load torque. However the power factor is low at rated load and rated speed. Owing to the high starting torque, these motors are used in domestic as well as industrial applications such as water pumps, grinders, lathe machines, compressors, drilling machines, etc.

Capacitor Run Induction Motor

In this capacitor, a low capacitor is connected in series with the starting winding and is not removed from the circuit even in running condition. Due to this arrangement, centrifugal switch is not required. Here the capacitor is capable of running continuously.



Circuit diagram of capacitor run motor



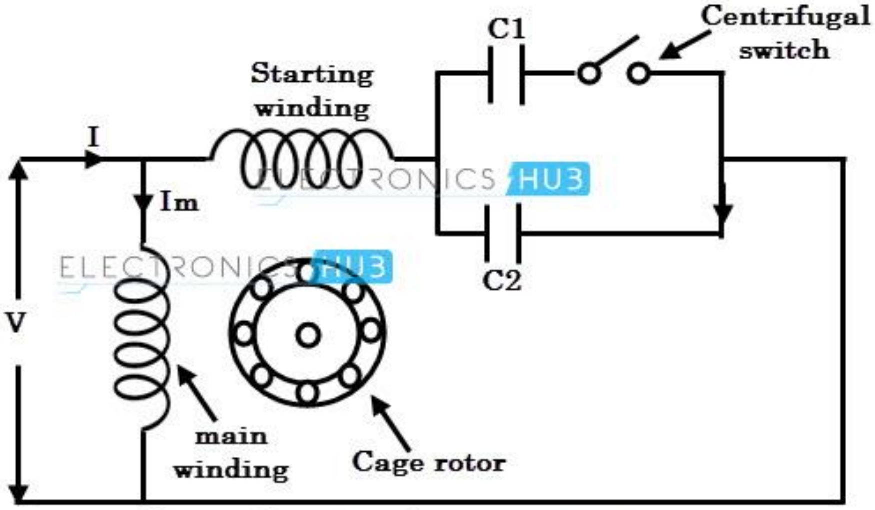
Phasor diagram

The low value capacitor produces more leading phase shift bur less total starting current as shown in phasor diagram. Hence, the starting torque produced by these motors will be considerably lower than that of capacitor start motor. The schematic circuit of this motor is shown in figure above. In this, the auxiliary winding and capacitor remains in circuit permanently and produce an approximate two phase operation at rated load point. This is the key strength of these motors. This will result better power factor and efficiency. However, the starting torque is much lower in these motors, typically about 80 percent of full load torque. Due to the continuous duty of auxiliary winding and capacitor, the rating of these components should withstand running conditions and hence permanent capacitor motor is more than equivalent split phase or capacitor start motors. These motors are used in exhaust and intake fans, unit heaters, blowers, etc.

Capacitor Start and Capacitor Run Induction Motor

These motors are also called as two-value capacitor motors. It combines the advantages of capacitor start type and permanent capacitor type induction motors.

This motor consists of two capacitors of different value of capacitance for starting and running. A high value capacitor is used for starting conditions while a low value is used for running conditions. It is to be noted that this motor uses same winding arrangement as capacitor-start motor during startup and permanent capacitor motor during running conditions. The schematic arrangement of this motor is shown in figure below.



Circuit diagram of capacitor start capacitor run motor

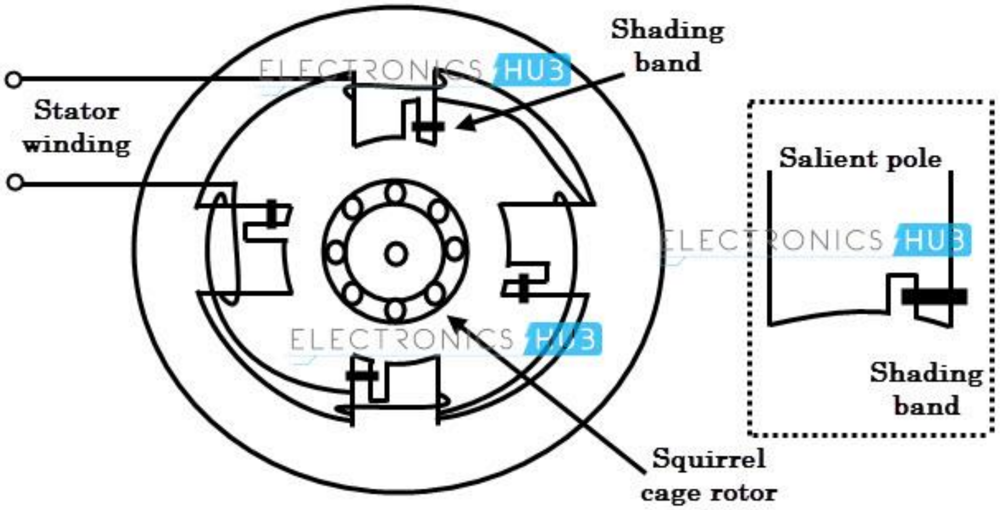
At starting, both starting and running capacitors are connected in series with the auxiliary winding. Thus the motor starting torque is more compared with other types of motors.

Once the motor reaches some speed, the centrifugal switch disconnects the starting capacitor and leaves the running capacitor in series with auxiliary winding.

Thus, both running and auxiliary windings remain during running condition, thereby improved power factor and efficiency of the motor. These are the most commonly used single phase motors due to high starting torque and better power factor. These are used in compressors, refrigerators, air conditioners, conveyors, ceiling fans, air circulators, etc

Shaded Pole Induction Motor

This motor uses entirely different technique to start the motor as compared with other motors so far we have discussed now. This motor doesn't use any auxiliary winding or even it doesn't have a rotating field, but a field that sweeps across the pole faces is enough to drive the motor. So the field moves from one side of the pole to another side of the pole. Although these motors are of small ratings, inefficient and have low starting torque, these are used in a variety of applications due to its outstanding features like ruggedness, low initial cost, small size and simple construction. A shaded pole motor consists of a stator having salient poles (or projected poles), and a rotor of squirrel cage type. In this, stator is constructed in a special way to produce moving magnetic field. Stator poles are excited with its own exciting coils by taking the supply from a single phase supply. A 4-pole shaded pole motor construction is given in below figure.

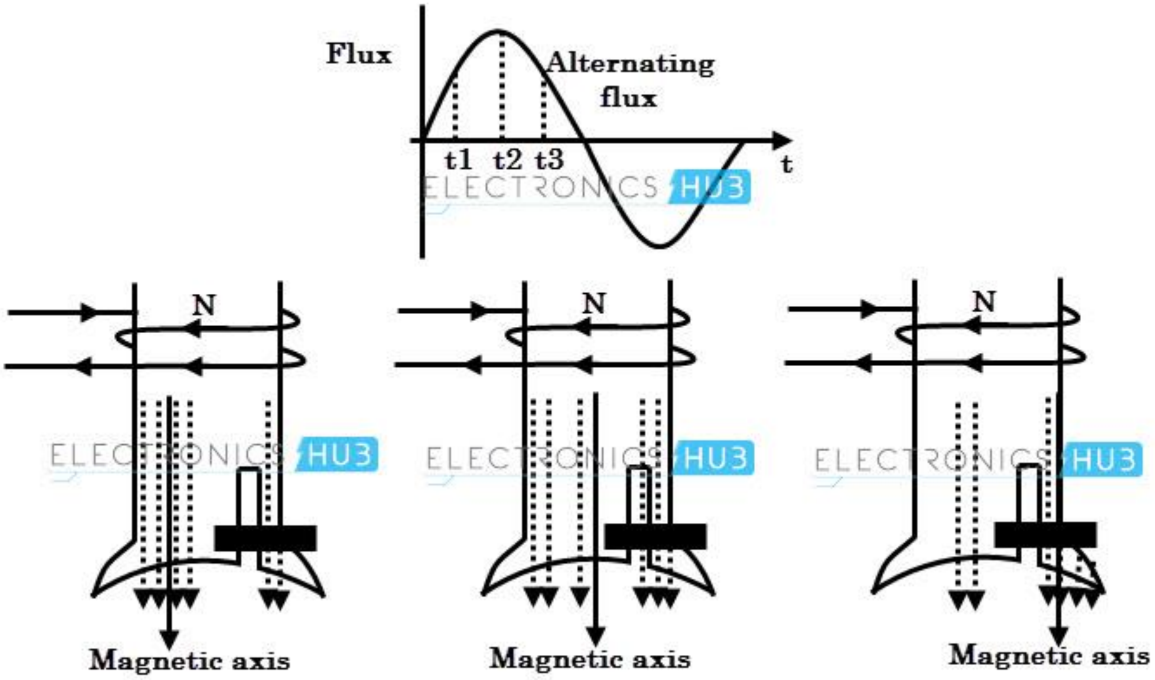


4-Pole Shaded pole motor construction

Each salient pole is divided into two parts; shaded and un-shaded. A shading portion is a slot cut across the laminations at about one third distance from one edge, and around this a heavy copper ring (also called as shading coil or copper shading band) is placed. This part where shading coil is placed is generally termed as shaded part of the pole and remaining portion is called as un-shaded part as shown in figure. Let us discuss how the sweeping action of the field takes place.

When an alternating supply is given to the stator coils, an alternating flux will be produced. The distribution of flux in the pole face area is influenced by the presence of copper shading band.

Let us consider the three instants, t1, t2, and t3 of alternating flux for an half cycle of the flux as shown in figure.



(a).t= t1 (b).t= t2 (c).t= t3

1. At instant t= t1, the rate of change of flux (rising) is very high. Due to this flux, an EMF is induced in the copper shading band and as the copper shading band is shorted, current circulates through it. This causes current to create its own field. According to Lenz's law, the current through copper shading band opposes the cause, i.e., rise of supply current (and hence rise of main flux). Therefore the flux produced by shading ring opposes the main flux. So there is weakening of flux in the shaded part while crowding of flux in un-shaded part. So the axis of overall flux shifts to non-shaded part of the pole as shown in the figure.

2. At instant t=t2, the rate of rise of flux is almost zero, and hence very little EMF is induced in the shaded band. It results negligible shaded ring flux and hence there is no much affect on distribution of main flux. Therefore, the distribution of flux is uniform and the overall flux axis lies at the center of the pole as shown in figure.
3. At instant t=t3, the rate of change of flux (decreasing) is very high, and induces EMF in copper shading band. The flux produced by the shading ring is now opposes the cause according to Lenz’s law. Here, the cause is decreasing flux, and opposing means its direction is same as that of main flux. Hence, this flux strengthens the main flux. So there will be crowding of flux in the shaded part compared to the non shaded part. Due to this overall flux axis shifts to the middle of shaded part. This sequence will repeat for negative cycle too and consequently it produce moving magnetic field for every cycle from non shaded part of the pole to shaded part of the pole. Due to this field, motor produces the starting torque. This starting torque is low about 40 to 50 percent of full load torque. Therefore, these motors are used in low starting torque applications such as fans, toy motors, blowers, hair dryers, photocopy machines, film projectors, advertising displays, etc.

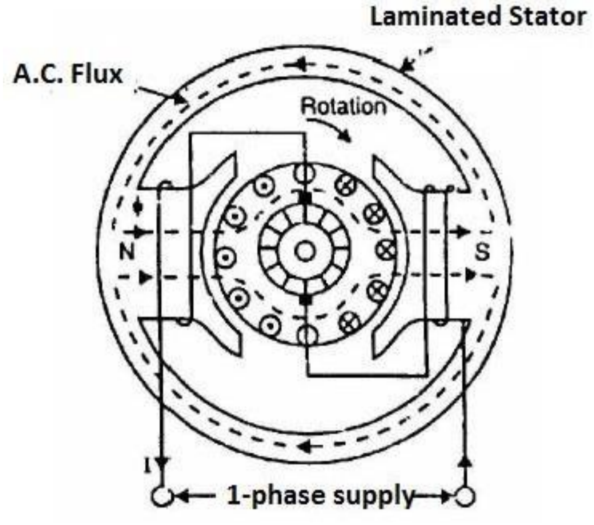
A.C. Series Motor (or) Universal Motor

A DC series motor will rotate in the same direction regardless of the polarity of the supply. One can expect that a DC series motor would also operate on a single-phase supply. It is then called an AC series motor. However, some changes must be made in a DC motor that is to operate satisfactorily on AC supply. The changes effected are:

- ✓ The entire magnetic circuit is laminated in order to reduce the eddy current loss. Hence an AC series motor requires a more expensive construction than a DC series motor.
- ✓ The series field winding uses as few turns as possible to reduce the reactance of the field winding to a minimum. This reduces the voltage drop across the field winding.
- ✓ A high field flux is obtained by using a low-reluctance magnetic circuit.
- ✓ There is considerable sparking between the brushes and the commutator when the motor is used on AC supply. It is because the alternating flux establishes high currents in the coils short-circuited by the brushes. When the short-circuited coils break contact from the commutator, excessive sparking is produced. This can be eliminated by using high- resistance leads to connect the coils to the commutator segments.

Construction

The construction of an AC series motor is very similar to a DC series motor except that above modifications are incorporated. Such a motor can be operated either on AC or DC supply and the resulting torque-speed curve is about the same in each case. For this reason, it is sometimes called a universal motor.



Operation

When the motor is connected to an AC supply, the same alternating current flows through the field and armature windings. The field winding produces an alternating flux that reacts with the current flowing in the armature to produce a torque. Since both armature current and flux reverse simultaneously, the torque always acts in the same direction. It may be noted that no rotating flux is produced in this type of machines; the principle of operation is the same as that of a DC series motor.

Characteristics

The operating characteristics of an AC series motor are similar to those of a DC series motor.

- ✓ The speed increases to a high value with a decrease in load. In very small series motors, the losses are usually large enough at no load that limits the speed to a definite value (1500 - 15,000 RPM).
- ✓ The motor torque is high for large armature currents, thus giving a high starting torque.
- ✓ At full-load, the power factor is about 90%. However, at starting or when carrying an overload, the power factor is lower.

Applications

The fractional horsepower AC series motors have high speed (and corresponding small size) and large starting torque. They can, therefore, be used to drive

- ✓ High-speed vacuum cleaners
- ✓ Sewing machines
- ✓ Electric shavers
- ✓ Drills
- ✓ Machine tools etc.

Equivalent circuit of single phase induction motor :-

The double revolving field theory can be effectively used to obtain the equivalent circuit of a single phase induction motor. The method consist of determining the values of both the field Clock-wise and anticlockwise at any given slip. when the two fields are known, the torque produced by each can be obtained. The difference between these two torque is the net torque acting on the rotor.

Imagine that the single phase IM is made up of one stator winding and two imaginary rotor windings. one rotor is rotating in forward direction i.e in the direction of RMF with slip s while other is rotating backward i.e, in the direction of oppositely directed RMF with slip 2-s.

To develop equivalent circuit with & without core loss

1) without core loss :-

Let the stator impedance be Z Ω

$$Z = R_1 + jX_1$$

where R_1 - Stator resistance

X_1 - Stator reactance

X_2 - Rotor reactance referred to stator

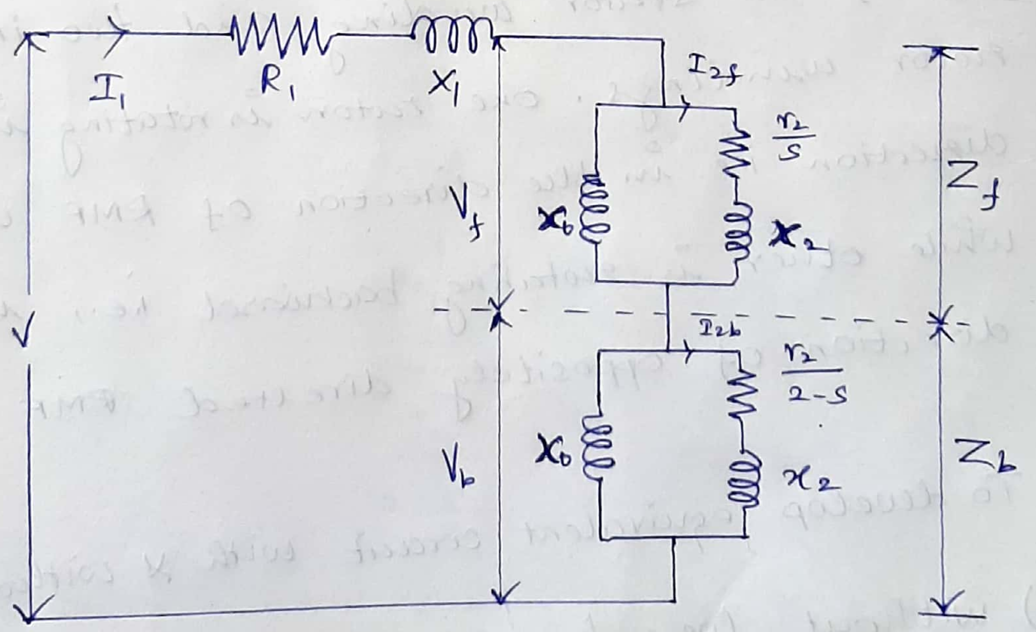
R_2 - Rotor resistance " " "

hence the impedance of each rotor is

$r_2 + jx_2$ where

$$x_2 = \frac{X_2}{2}$$

The resistance of forward field rotor is $\frac{r_2}{s}$ while the resistance of backward field rotor is $\frac{r_2}{2-s}$. The r_2 value is half of the actual



Equivalent ckt without core loss

rotor resistance referred to stator. As the core loss is neglected, R_0 is not existing in the

(2)

(PTO)

equivalent circuit. The x_0 is half of the actual magnetising reactance of the motor. So the equivalent circuit referred to stator is shown in the figure above.

Now the impedance of the forward field rotor is Z_f which is parallel combination of $(0 + jx_0)$ and $\left(\frac{r_2}{s}\right) + jx_2$.

$$Z_f = \frac{jx_0\left[\left(\frac{r_2}{s}\right) + jx_2\right]}{\frac{r_2}{s} + j(x_0 + x_2)}$$

while the impedance of the backward field rotor is Z_b which is parallel combination of $(0 + jx_0)$ and $\left(\frac{r_2}{2-s}\right) + jx_2$.

$$Z_b = \frac{jx_0\left[\left(\frac{r_2}{2-s}\right) + jX_2\right]}{\frac{r_2}{2-s} + j(x_0 + x_2)}$$

under the standstill condition, $s = 1$ & $2 - s = 1$

hence $\boxed{Z_f = Z_b}$ & hence $V_f = V_b$. But in the running Condition, V_f becomes almost 90 to 95 %. Of the applied voltage.

$Z_q = Z_1 + Z_f + Z_b \Rightarrow$ Equivalent Impedance

let I_{2f} = current through forward rotor referred to stator

I_{2b} = current through backward rotor referred to stator

$$I_{2f} = \frac{V_f}{\left[\frac{r_2}{s} + jx_2\right]} \quad \text{where } V_f = I_1 \times Z_f$$

and
$$I_{2b} = \frac{V_b}{\left[\frac{r_2}{2-s} + jx_2\right]} \quad \text{where } V_b = I_1 \times Z_b$$

$P_f \Rightarrow$ Power input to forward field rotor

$\Rightarrow (I_{2f})^2 \left(\frac{r_2}{s}\right)$ watts

$P_b \Rightarrow$ Power input to backward field rotor

$\Rightarrow (I_{2b})^2 \left(\frac{r_2}{2-s}\right)$ watts

$P_m = (1-s)$ [Net power i/p] $\Rightarrow (1-s)(P_f - P_b)$ watts

P_{out} = P_m - Mechanical loss - Core loss

$$T_f = \text{forward torque} = \frac{P_f}{\left(\frac{2\pi N}{60}\right)} \text{ N-m}$$

$$T_b = \text{Backward torque} = \frac{P_b}{\left(\frac{2\pi N}{60}\right)} \text{ N-m}$$

$$T = \text{Net torque} = T_f - T_b$$

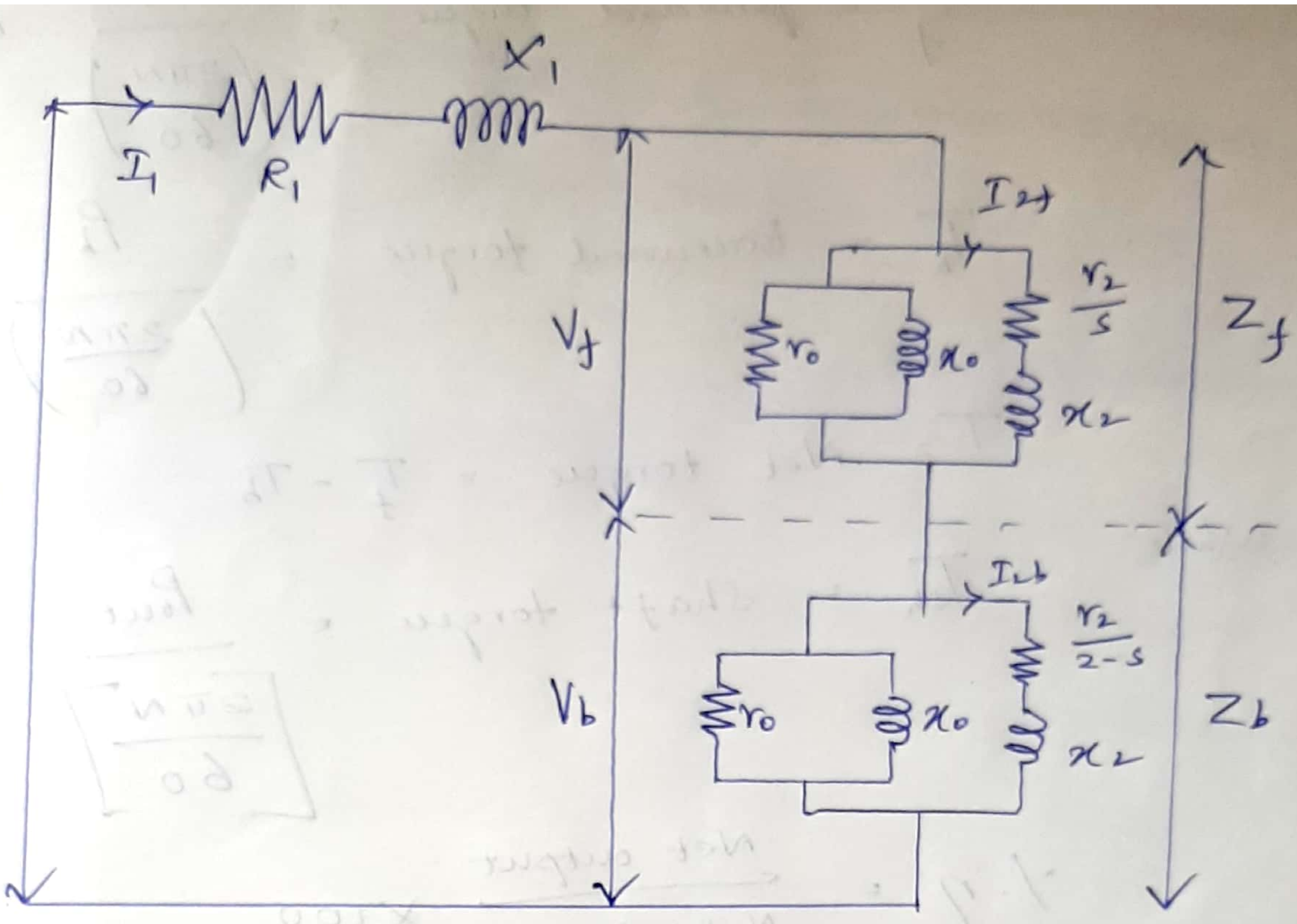
while $$T_{sh} = \text{Shaft torque} = \frac{P_{out}}{\left[\frac{2\pi N}{60}\right]} \text{ Nm}$$

$$\% \eta = \frac{\text{Net output}}{\text{Net input}} \times 100$$

<u>with Core loss :-</u>

If the core loss is to be Considered then it is necessary to Connect a resistance r_0 in parallel with x_0, in an exciting branch of each rotor. r_0 is half the value of actual core loss resistance. Thus the equivalent circuit with core loss can be shown as in the fig.

(P. T. O)



equivalent Circuit with Core loss

Let Z_{of} ⇒ equivalent impedance of exciting branch in forward rotor ⇒ $r_0 \parallel (jx_0)$

& Z_{ob} ⇒ " " " in backward rotor

⇒ $r_0 \parallel (jx_0)$

$$Z_f = Z_{of} \parallel \left[\frac{r_2}{s} + jx_2\right]$$

all other expressions remains same as stated earlier in case of without core loss.

CHAPTER 3

SINGLE PHASE MOTORS

Introduction

Single phase motors are the most familiar of all electric motors because they are extensively used in home appliances, shops, offices etc. It is true that single phase motors are less efficient substitute for 3-phase motors but 3-phase power is normally not available except in large commercial and industrial establishments. Since electric power was originally generated and distributed for lighting only, millions of homes were given single-phase supply. This led to the development of single-phase motors. Even where 3-phase mains are present, the single-phase supply may be obtained by using one of the three lines and the neutral. Single-phase induction motors are usually two-pole or four-pole, rated at 2 hp or less, while slower and larger motor can be manufactured for special purposes. They are widely used in domestic appliances and for a very large number of low power drives in industry. The single phase induction motor resembles, three-phase, squirrel-cage motor except that, single phase induction motor has no starting torque and some special arrangement have to be made to make it as self starting. In this chapter, we shall focus our attention on the construction, working and characteristics of commonly used single-phase motors.

Types of Single-Phase Motors

Single-phase motors are generally built in the fractional-horsepower range and may be classified into the following four basic types:

1. Single-phase induction motors

(i) split-phase type (ii) capacitor start type (iii) capacitor start capacitor run type (v) shaded-pole type

2. A.C. series motor or universal motor

3. Repulsion motors

(i) Repulsion-start induction-run motor (ii) Repulsion-induction motor

4. Synchronous motors

(i) Reluctance motor (ii) Hysteresis motor

Single-Phase Induction Motors

A single phase induction motor is very similar to a 3-phase squirrel cage induction motor. Unlike a 3-phase induction motor, a single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. his method of starting is generally not convenient for large motors. Figure 3.2 shows picture of single phase induction motor.

Figure 3.1 Single phase induction motor.

Construction of single phase induction motor

The construction parts on of single phase induction motor consist of main two parts: stationary stator and revolving rotor. The stator separate from rotor by small air gap have ranges from 0.4 mm to 4 mm depends to size of motor.

Stator

The single-phase motor stator has a laminated iron core with two windings arranged perpendicularly, One is the main and the other is the auxiliary winding or starting winding as showing in the figure 3.2. It consists of a steel frame which encloses a hollow, cylindrical core made up of thin laminations of silicon steel to reduce hysteresis and eddy current losses. A number of evenly spaced slots are provided on the inner periphery of the laminations.

Figure 3.2 Stator of single phase induction motor.

Rotor

The rotor, mounted on a shaft, is a hollow laminated core having slots on its outer periphery. The winding placed in these slots (called rotor winding) may be one of the following two types:

(i) Squirrel cage rotor:

It consists of a laminated cylindrical core having parallel slots on its outer periphery. One copper or aluminum bar is placed in each slot. All these bars are joined at each end by metal rings called end rings [See Fig. 3.3]. This forms a permanently short-circuited winding which is indestructible. The entire construction (bars and end rings) resembles a squirrel cage and hence the name. The rotor is not connected electrically to the supply but has current induced in it by transformer action from the stator. Those induction motors which employ squirrel cage rotor are called squirrel cage induction motors. Most of single phase induction motors use squirrel cage rotor as it has a remarkably simple and robust construction enabling it to operate in the most adverse circumstances. However, it suffers from the disadvantage of a low starting torque. It is because the rotor bars are permanently short-circuited and it is not possible to add any external resistance to the rotor circuit to have a large starting torque. In this type of rotor the bars conductor are skew to reduce the noise.

Figure 3.3 Squirrel cage rotor.

(ii) Wound rotor:

It consists of a laminated cylindrical core and carries a single phase winding, similar to the one on the stator. The open ends of the rotor winding are brought out and joined to three insulated slip rings mounted on the rotor shaft with one brush resting on each slip ring. The two brushes are connected to a single phase star-connected rheostat as shown in Figure 3.4. At starting, the external resistances are included in the rotor circuit to give a large starting torque. These resistances are gradually reduced to zero as the motor runs up to speed. The external resistances are used during starting period only. When the motor attains normal speed, the two brushes are short-circuited so that the wound rotor runs like a squirrel cage rotor.

Figure 3.4 wound rotor of single phase induction motor.

3.5 principle of Work

A single-phase induction motor is not self starting but requires some starting means. The single-phase stator winding produces a magnetic field that pulsates in strength in a sinusoidal manner. The field polarity reverses after each half cycle but the field does not rotate. Consequently, the alternating flux cannot produce rotation in a stationary squirrel-cage rotor. However, if the rotor of a single-phase motor is rotated in one direction by some mechanical means, it will continue to run in the direction of rotation. As a matter of fact, the rotor quickly accelerates until it reaches a speed slightly below the synchronous speed. Once the motor is running at this speed, it will continue to rotate even though single-phase current is flowing through the stator winding. This method of starting is generally not convenient for large motors. Figure shows single-phase induction motor having a squirrel cage rotor and a single phase distributed stator winding. Such a motor inherently docs not develop any starting torque and, therefore, will not start to rotate if the stator winding is connected to single-phase A.C. supply. However, if the rotor is started by auxiliary means, the motor will quickly attain me final speed. This strange behavior of single-phase induction motor can be explained on the basis of double-field revolving theory.

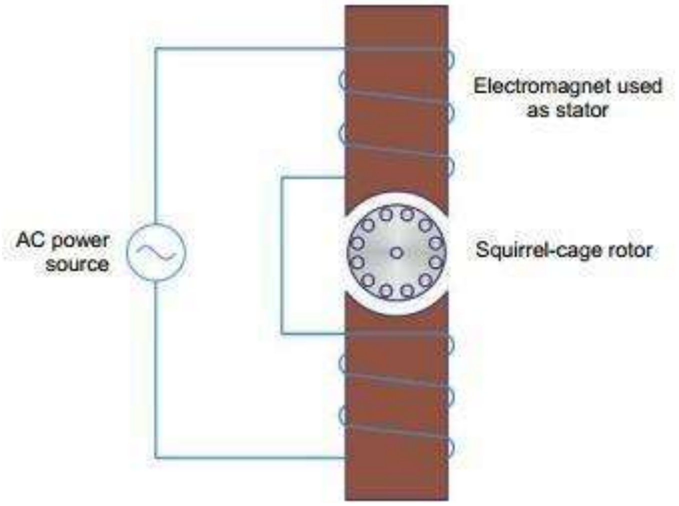


Figure 3.5 single-phase induction motor.

Operation of Single phase induction motor

(i) When stator winding is energized from a.c. supply, a rotating magnetic field (RMF) is set up which rotates round the stator at synchronous speed Ns (= 120 f/P), when f = frequency and P No. of poles .

(ii) The rotating field passes through the air gap and cuts the rotor conductors, which as yet, are stationary . Due to the relative speed between the rotating flux and the stationary rotor, electrical motive force (EMF) are induced in the rotor conductors. Since the rotor circuit is short-circuited, currents start flowing in the rotor conductors (Figure 3.6). The flux from the stator will cut the coil in the rotor and since the rotor coils are short circuited, according to Faraday's law of electromagnetic induction, current will start flowing in the coil of the rotor.

(iii) The current-carrying rotor conductors are placed in the magnetic field produced by the stator. Consequently, mechanical force acts on the rotor conductors. The sum of the mechanical forces on all the rotor conductors produces a torque which tends to move the rotor in the same direction as the rotating field with speed N =Ns (1-S) when S= slip and N = rotor speed (Figure 3.6).

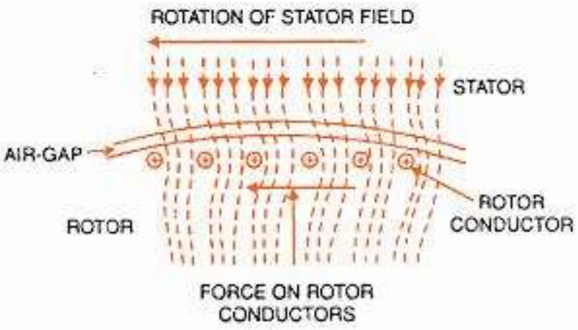


Figure 3.6 Transmission of Rotate magnetic field

Cross-field theory

The principle of operation of a single-phase induction motor can be explained from the cross-field theory. As soon as the rotor begins to turn, a speed emf E is induced in the rotor conductors, as they cut the stator flux F_S. This voltage increases as the rotor speed increases. It causes current Ir to flow in the rotor bars facing the stator poles as shown in figure 3.3 . These currents produce an ac flux F_R which act at right angle to the stator flux F_S. Equally important is the fact that F_R does not reach its maximum value at the same time as F_S does, in effect, F_R lags almost 90^0 behind F_S, owing to the inductance of the rotor The combined action of F_S and F_R produces a revolving magnetic field, similar to that in a three-phase motor. The value of F_R increases with increasing speed, becoming almost equal to F_S at synchronous speed. The flux rotates counterclockwise in the same direction as the rotor and it rotates at synchronous speed irrespective of the actual speed of the rotor. As the motor approaches synchronous speed, F_R becomes almost equal to F_S and a nearly perfect revolving field is produces.

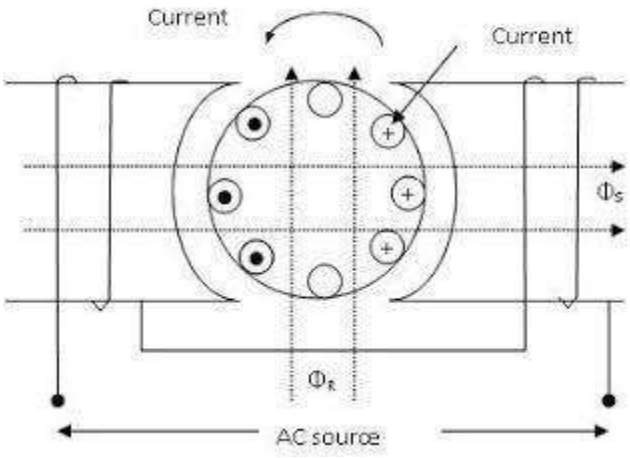


Figure 3.3 Current induced in the rotor bars due to rotation.

Double-field revolving theory

When the stator winding (distributed one as stated earlier) carries a sinusoidal current (being fed from a single-phase supply), a sinusoidal space distributed mmf, whose peak or maximum value pulsates (alternates) with time, is produced in the air gap. This sinusoidal varying flux (φ) is the sum of two rotating fluxes or fields, the magnitude of which is equal to half the value of the alternating flux (φ / 2), and both the fluxes rotating synchronously at the speed, in opposite directions. The first set of figures (Figure 3.8a (i-iv)) show the resultant sum of the two rotating fluxes or fields, as the time axis (angle) is changing from $\theta = 0°$ to π °(180) . Figure 3.8b shows the alternating or pulsating flux (resultant) varying with time or angle.

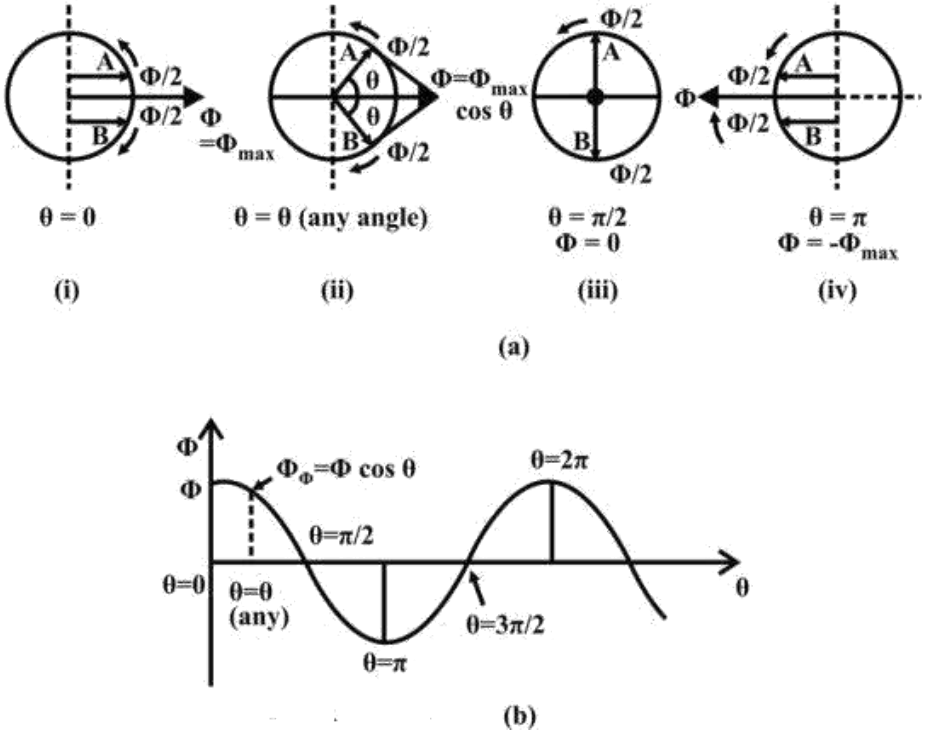


Figure 3.8 Double field revolving.

The flux or field rotating at synchronous speed, say, in the anticlockwise direction, i.e. the same direction, as that of the motor (rotor) taken as positive induces EMF (voltage) in the rotor conductors. The rotor is a squirrel cage one, with bars short circuited via end rings. The current flows in the rotor conductors, and the electromagnetic torque is produced in the same direction as given above, which is termed as positive (+ve). The other part of flux or field rotates at the same speed in the opposite (clockwise) direction, taken as negative. So, the torque produced by this field is negative (-ve), as it is in the clockwise direction, same as that of the direction of rotation of this field. Two torques are in the opposite direction, and the resultant (total) torque is the difference of the two torques produced. Let the flux φ1 rotate in anti clockwise direction and flux φ2 in clockwise direction. The flux φ1 will result in the production of torque T1 in the anti clockwise direction and flux φ2 will result in the production of torque T2 In the clockwise direction. Thus the point of zero slip for one field corresponds to 200% slip for the other as explained later. The value of 100% slip (standstill condition) is the same for both the fields. This fact is illustrated in Figure 3.9. At standstill, these two torques are equal and opposite and the net torque developed is zero. Therefore, single-phase induction motor is not self-starting. Note that each rotating field tends to drive the rotor in the direction in which the field rotates.

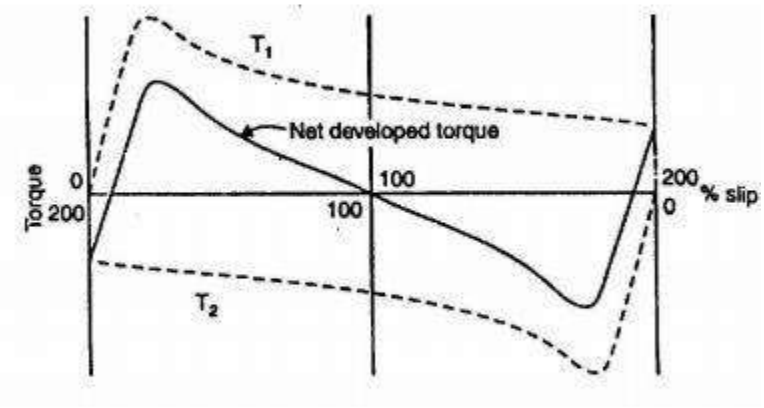


Figure 3.9 Speed Torque characteristics.

Now assume that the rotor is started by spinning the rotor or by using auxiliary circuit, in say clockwise direction. The flux rotating in the clockwise direction is the forward rotating flux (φf) and that in the other direction is the backward rotating flux (φb). The slip w.r.t. the forward flux will be

$$Sf = \frac{Ns - N}{Ns} = 1 - \frac{N}{Ns} \; or \; \frac{N}{Ns} = 1 - S$$

The rotor rotates opposite to the rotation of the backward flux. Therefore, the slip w.r.t. the backward flux will be

$$Sb = \frac{Ns - (-N)}{Ns} = \frac{Ns + N}{Ns} = 1 + \frac{N}{Ns} = 1 + (1 - S) = 2 - S$$

Thus fur forward rotating flux, slip is s (less than unity) and for backward rotating flux, the slip is 2 − s (greater than unity). Since for usual rotor resistance/reactance ratios, the torques at slips of less than unity arc greater than those at slips of more than unity, the resultant torque will be in the direction of the rotation of the forward flux. Thus if the motor is once started, it will develop net torque in the direction in which it has been started and will function as a motor.

Generation of Rotate magnetic field (RMF)

A rotating magnetic field is probably most easily seen in a two-phase stator. The stator of a two-phase induction motor is made up of two winding(main winding and auxiliary winding). They are placed at right angles to each other around the stator. The simplified drawing in figure 3.10 illustrates a two-phase stator.

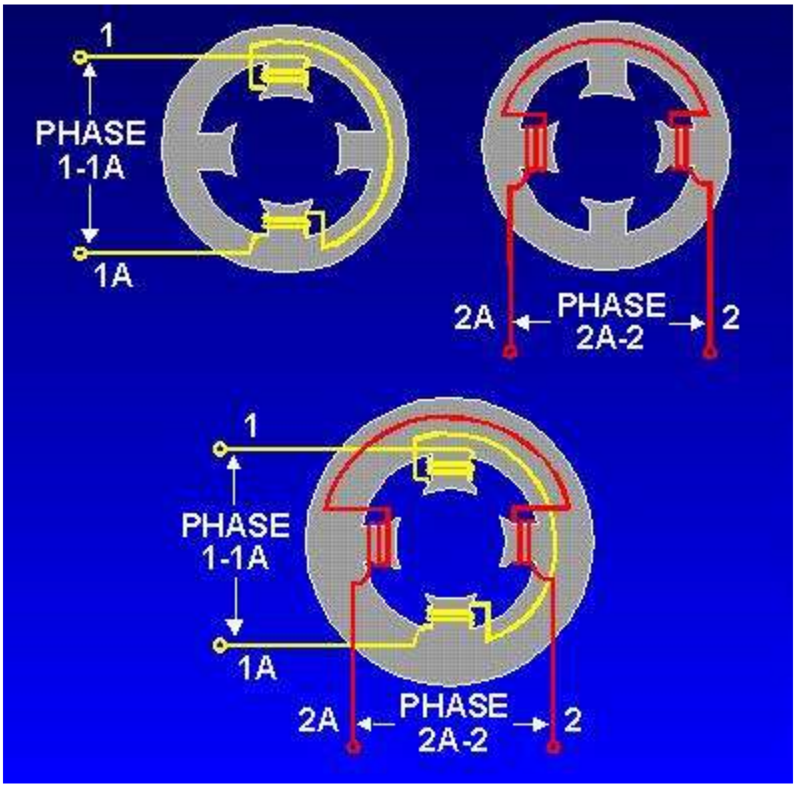


Figure 3.10 Two phase motor stator.

If the voltages applied to phases 1-1A and 2-2A are 90° out of phase, the currents that flow in the phases are displaced from each other by 90°. Since the magnetic fields generated in the coils are in phase with their respective currents, the magnetic fields are also 90° out of phase with each other. These two out-of-phase magnetic fields, whose coil axes are at right angles to each other, add together at every instant during their cycle. They produce a resultant field that rotates one revolution for each cycle of ac. To analyze the rotating magnetic field in a two-phase stator. The arrow represents the rotor. For each point set up on the voltage chart, consider that current flows in a direction that will cause the magnetic polarity indicated at each pole piece. Note that from one point to the next, the polarities are rotating from one pole to the next in a clockwise manner. One complete cycle of input voltage produces a 360-degree rotation of the pole polarities. Let's see how this result is obtained. Figure 3.11. - Two-phase rotating field.

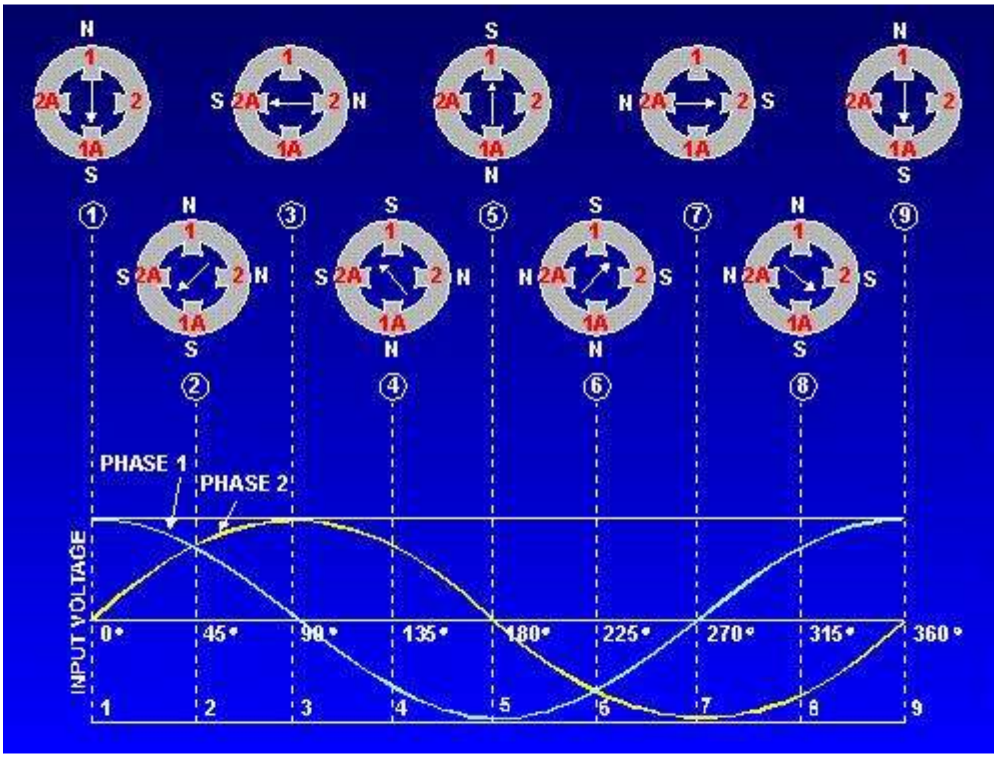


Figure 3.11 Two-phase rotating field.

The waveforms in figure 3.11 are of the two input phases, displaced 90° because of the way they were generated in a two-phase alternator. The waveforms are numbered to match their associated phase. Although not shown in this figure, the windings for the poles 1-1A and 2-2A would be as shown in the previous figure.

(i) When $\theta = 0^\circ$, magnitude of the flux set up by phase-1 will be 0 and the magnitude of the flux by phase 2 will be maximum but in negative direction. Hence the magnitude of the resultant flux Φ_r will be equal to Φ_m.

(ii) $\theta=45^\circ$

Flux by phase-1 Φ_1 = sqrt.2 * Φ_m.

Flux by phase-2 Φ_2 = sqrt.2 * Φ_m.

Hence resultant flux $\Phi_r = \Phi_m$.

But the resultant has shifted 45 degrees clockwise.

(iii) $\theta = 90°$

Flux by phase-1 $\Phi_1 = \Phi_m$.

Flux by phase-2 $\Phi_2 = 0$.

Hence resultant flux $\Phi_r = \Phi_m$.

But the resultant has further shifted 45 degrees clockwise OR resultant has shifted 90 degrees from its initial position.

(iv) $\theta = 135°$

Flux by phase-1 $\Phi_1 = \Phi_m$.

Flux by phase-2 $\Phi_2 = \Phi_m$.

Hence resultant flux $\Phi_r = \Phi_m$.

But the resultant has further shifted 45 degrees clockwise OR resultant has shifted 135 degrees from its initial position.

(iv) $\theta = 180°$

Flux by phase-1 $\Phi_1 = 0$.

Flux by phase-2 $\Phi_2 = \Phi_m$.

Hence resultant flux $\Phi_r = \Phi_m$.

When the two-phase voltages have completed one full cycle (position 9), the resultant magnetic field has rotated through 360°. Thus, by placing two windings at right angles to each other and exciting these windings with voltages 90° out of phase, a rotating magnetic field results.

The speed of the rotating magnetic flux is called as synchronous speed (Ns) and it is given by

$$Ns = \frac{120\,f}{P} \quad (\mathrm{RPM})$$

where, f =frequency of the supply and P = number of poles.

Why Single Phase Induction Motor is not Self Starting?

According to double field revolving theory, any alternating quantity can be resolved into two components, each component have magnitude equal to the half of the maximum magnitude of the alternating quantity and both these component rotates in opposite direction to each other. For example – a flux, φ can be resolved into two components

$$\frac{\phi_m}{2} \; and \; -\frac{\phi_m}{2}$$

Each of these components rotates in opposite direction i. e if one φm / 2 is rotating in clockwise direction then the other φm / 2 rotates in anticlockwise direction.

When a single phase ac supply is given to the stator winding of single phase induction motor, it produces its flux of magnitude, φm. According to the double field revolving theory, this alternating flux, φm is divided into two components of magnitude φm /2. Each of these components will rotate in opposite direction, with the synchronous speed, Ns. Let us call these two components of flux as forward component of flux, φf and backward component of flux, cb. The resultant of these two component of flux at any instant of time, gives the value of instantaneous stator flux at that particular instant.

$$i.e.\, \phi_r = \frac{\phi_m}{2} + \frac{\phi_m}{2} \; or \; \phi_r = \phi_f + \phi_b$$

Now at starting, both the forward and backward components of flux are exactly opposite to each other. Also both of these components of flux are equal in magnitude. So, they cancel each other and hence the net torque experienced by the rotor at starting is zero. So, the single phase induction motors are not self starting motors.

Making Single-Phase Induction Motor Self-Starting

The single-phase induction motor is not self starting and it is undesirable to resort to mechanical spinning of the shaft or pulling a belt to start it. To make a single-phase induction motor self-starting, we should somehow produce a revolving stator magnetic field. This may be achieved by converting a single-phase supply into two-phase supply through the use of an additional winding. When the motor attains sufficient speed, the starting means (i.e., additional winding) may be removed depending upon the type of the motor. As a matter of fact, single-phase induction motors are classified and named according to the method employed to make them self-starting.

(i) Split-phase motors-started by two phase motor action through the use of an auxiliary or starting winding.

(ii) Capacitor start motors-started by two-phase motor action through the use of an auxiliary winding and a capacitor.

(iii) Capacitor start Capacitor run motors-started by two-phase motor action through the use of an auxiliary winding and two capacitors.

(v) Shaded-pole motors-started by the motion of the magnetic field produced by means of a shading coil around a portion of the pole structure.

Split-phase induction motors

The stator of a split-phase induction motor is provided with an auxiliary or starting winding S in addition to the main or running winding M. The starting winding is located 90° electrical from the main winding and the picture of split phase induction motor [See Fig3.12 (i))] and operates only during the brief period when the motor starts up. The two windings are so resigned that the starting winding S has a high resistance and relatively small reactance while the main winding M has relatively low resistance and large reactance to be as inductance (the current delay with voltage) to make shifting current as shown in the schematic connections in Figure 3.12 (ii)). Consequently, the currents flowing in the two windings have reasonable phase difference c (25° to 30°) as shown in the pharos diagram this shifting in current its necessary for starting torque in Figure 3.12 (iii)). Figure 3.12 (iv) shows typical torque speed characteristics.

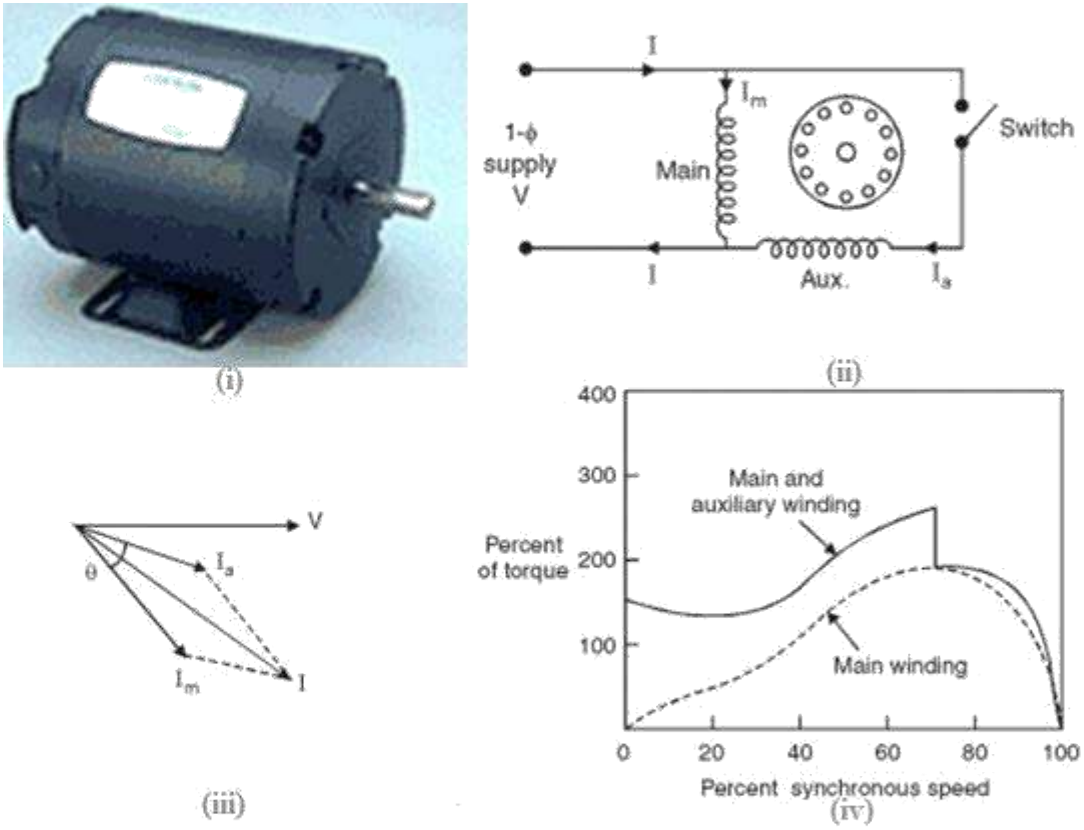


Figure 3.12 Split-phase induction motors.

Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current Im while the starting winding carries current Is.

(ii) Since main winding is made highly inductive while the starting winding highly resistive, the currents Im and Is have a reasonable phase angle a (25° to 30°) between them. Consequently, a weak revolving field approximating to that of a 2-phase machine is produced which starts the motor.

(iii) When the motor reaches about 80% of synchronous speed, the centrifugal switch opens the circuit of the starting winding. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed. The normal speed of the motor is below the synchronous speed and depends upon the load on the motor.

Characteristics

(i) The sinning torque is 2 times the full-loud torque mid (lie starting current is 6 to 8 times the full-load current.

(ii) Due to their low cost, split-phase induction motors are most popular single phase motors in the market.

(iii) Since the starting winding is made of fine wire, the current density is high and the winding heats up quickly. If the starting period exceeds 5 seconds, the winding may burn out unless the motor is protected by built-in-thermal relay. This motor is, therefore, suitable where starting periods are not frequent.

(iv) An important characteristic of these motors is that they are essentially constant-speed motors. The speed variation is 2-5% from no-load to full load.

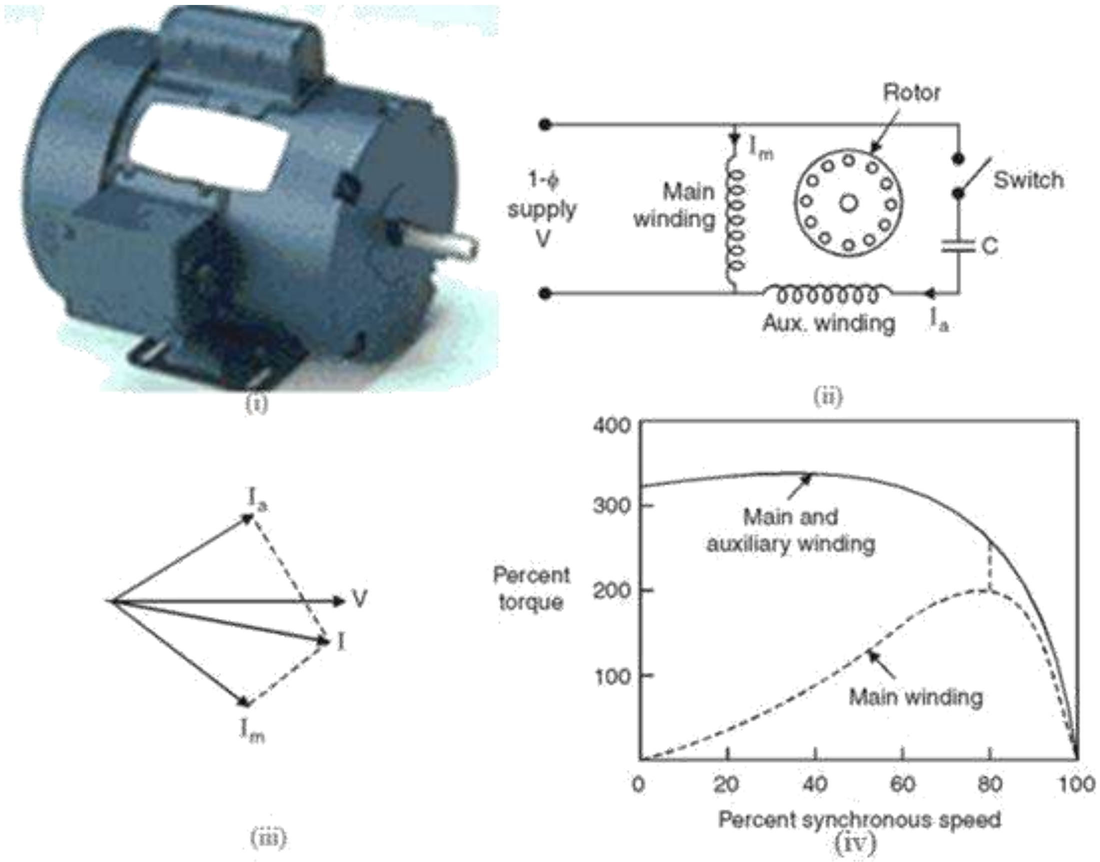
(v) These motors are suitable where a moderate starting torque is required and where starting periods are infrequent e.g., to drive:

(a) fans (b) washing machines (c) oil burners (d) small machine tools etc.

The power rating of such motors generally lies between 60 W and 250 W.

Capacitor induction Motor

The capacitor-start motor is identical to a split-phase motor except that the starting winding has as many turns as the main winding. The picture of capacitor start induction motor is shows in Figure 3.13 (i). Moreover, a capacitor C (3-20 μF) is connected in series with the starting winding as shown in Figure 3.13 (ii)). The value of capacitor is so chosen that Is leads Im by about 80° which is considerably greater than 25° found in split - phase motor [See Figure 3.13 (iii))]. Figure 3.13(iv) shows typical torque speed characteristic.



3.13 Capacitor-Start Motor.

Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current Im while the starting winding carries current Is.

(ii) Due to cap acitance the currents Im and Is have a reasonable phase angle a (80°) between them.

(iii) When starting torque is much more than that of a split-phase motor Again, the starting winding is opened by the centrifugal switch when the motor attains about 80% of synchronous speed. The motor then operates as a single-phase induction motor and continues to accelerate till it reaches the normal speed.

Characteristics

(i) Although starting characteristics of a capacitor-start motor are better than those of a split-phase motor, both machines possess the same running characteristics because the main windings are identical.

(ii) The phase angle between the two currents is about 80° compared to about 25° in a split-phase motor. Consequently, for the same starting torque, the current in the starting winding is only about half that in a split-phase motor. Therefore, the starting winding of a capacitor start motor heats up less quickly and is well suited to applications involving either frequent or prolonged starting periods.

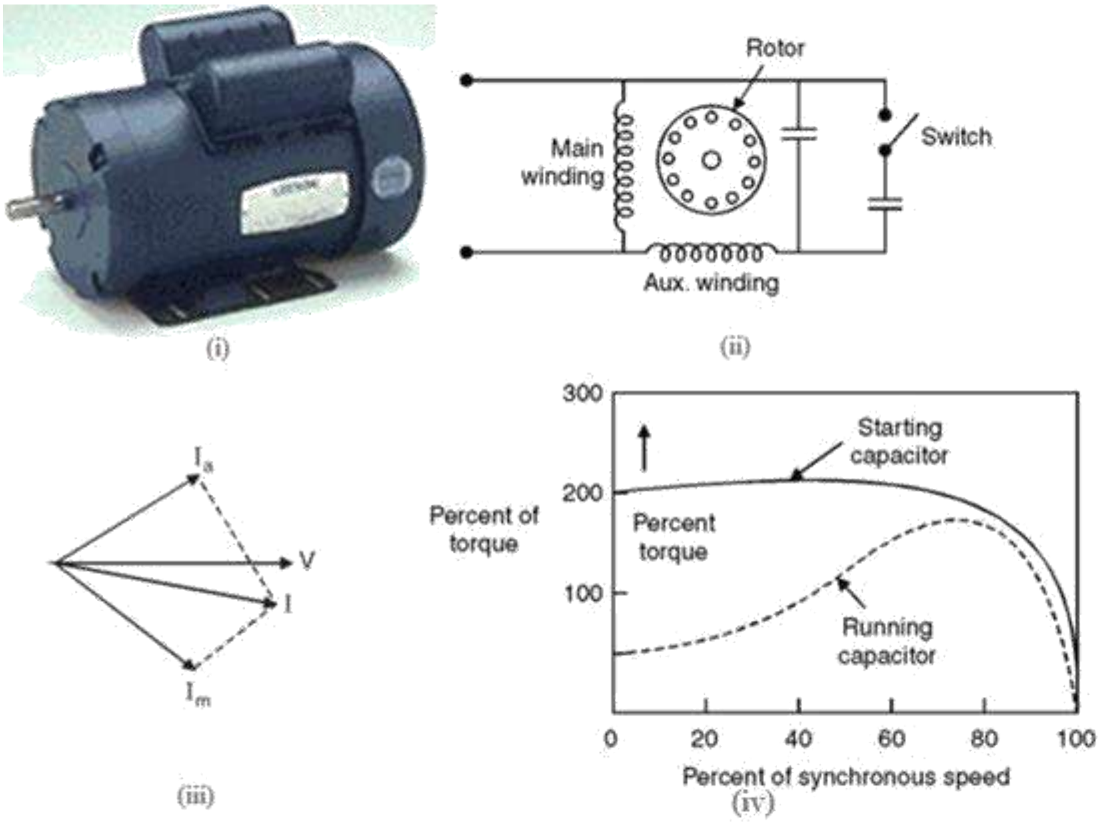
(iii) Capacitor-start motors are used where high starting torque is required and where the starting period may be long e.g., to drive:

(a) compressors (b) large fans (c) pumps (d) high inertia loads The

power rating of such motors lies between 120 W and 3-5 kW.

Capacitor start Capacitor run induction motors

This motor is identical to a capacitor-start motor except that starting winding is not opened after starting so that both the windings remain connected to the supply when running as well as at starting. Two designs are generally used. Figure 3.14 (i) shows picture of capacitor start capacitor run induction motor. This design eliminates the need of a centrifugal switch and at the same time improves the power factor and efficiency of the motor. In the other design, two capacitors C1 and C2 are used in the starting winding as shown in Figure 3.14 (ii). The value of capacitor is so chosen that Is leads Im by about 80° [See Figure 14 (iii))]. The smaller capacitor C1 required for optimum running conditions is permanently connected in series with the starting winding. The much larger capacitor C2 is connected in parallel with C1 for optimum starting and remains in the circuit during starting. The starting capacitor C2 is disconnected when the motor approaches about 80% of synchronous speed. The motor then runs as two-phase induction motor. Figure 3.14 (iv) shows typical torque speed characteristic.



3.14 Capacitor start Capacitor run induction motors.

Operation

(i) When the two stator windings are energized from a single-phase supply, the main winding carries current Im while the starting winding carries current Is.

(ii) Due to capacitance C1 the currents Im and Is have a reasonable phase angle a (80°) between them.

(iii) When The starting capacitor C2 is disconnected when the motor approaches about 80% of synchronous speed. The motor then runs as two-phase induction motor.

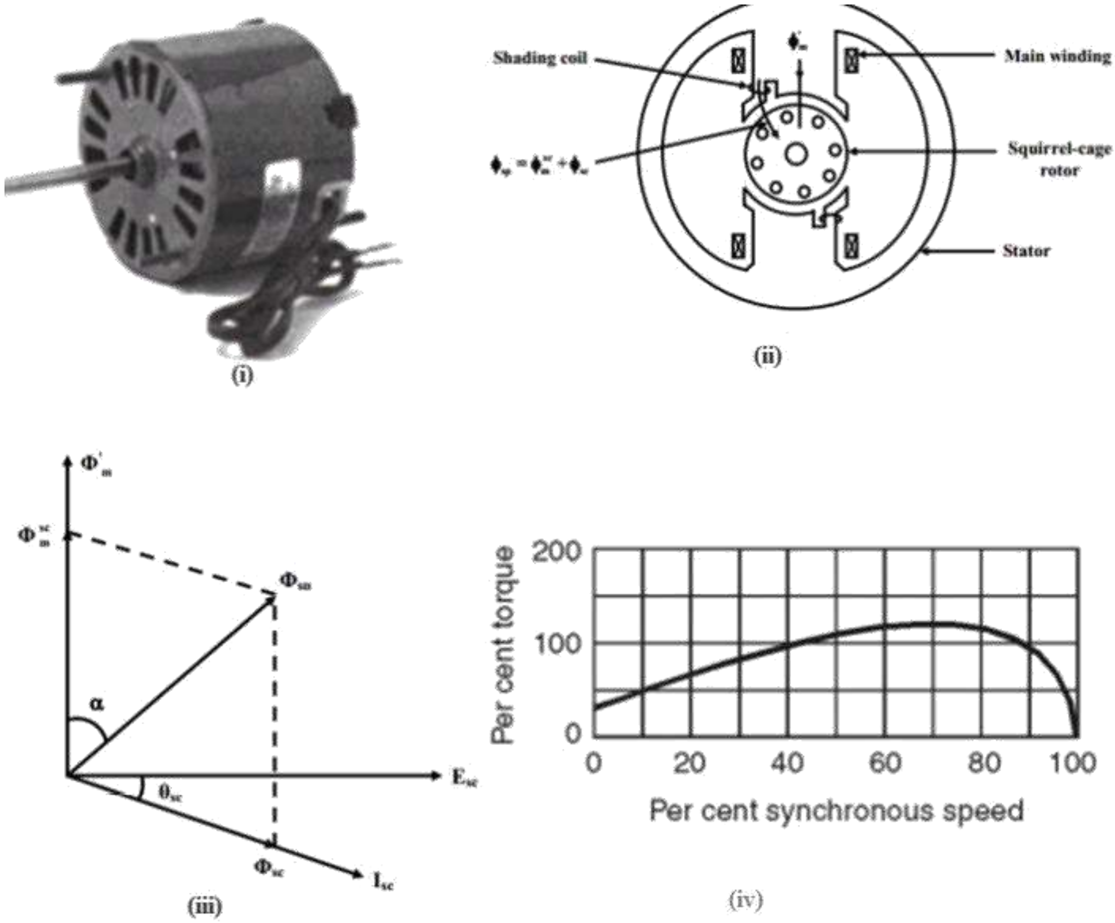
Characteristics

(i) The starting winding and the capacitor can be designed for perfect 2-phase operation at any load. The motor then produces a constant torque and not a pulsating torque as in other single-phase motors.

(ii) Because of constant torque, the motor is vibration free and can be used in:
(a) hospitals (b) studios and (c) other places where silence is important.

3.8.4 Shaded-pole induction motors

A picture of shaded pole induction motor are shows in Figure 3.15 (i). A typical shaded-pole motor with a cage rotor is shown in Figure 3.15 (ii). This is a single phase induction motor, with main winding in the stator. A small portion of each pole is covered with a short-circuited, single-turn copper coil called the shading coil. The sinusoidal varying flux created by ac (single-phase) excitation of the main winding induces in the shading coil. As a result, induced currents flow in the shading coil producing their own flux in the shaded portion of the pole. as shown in Figure 3.15 (iii) and lags the flux φ_m' of the remaining pole by the angle α . The two sinusoidal varying fluxes φ_m' and φ_{sp}' are displaced in space as well as have a time phase difference (α), thereby producing forward and backward rotating fields, which produce a net torque. It may be noted that the motor is self-starting unlike a single-phase single-winding motor. It is seen from the phasor diagram (Figure 3.15 (iii) that the net flux in the shaded portion of the pole (φ_{sp}) lags the flux (φ_m') in the unshaded portion of the pole resulting in a net torque, which causes the rotor to rotate from the unshaded to the shaded portion of the pole. The motor thus has a definite direction of rotation, which cannot be reversed. Atypical torque speed characteristic are shows in Figure 3.15 (iv) .


(i) (ii) (iii) (iv)

3.15 Shaded-pole induction motors.

Operation

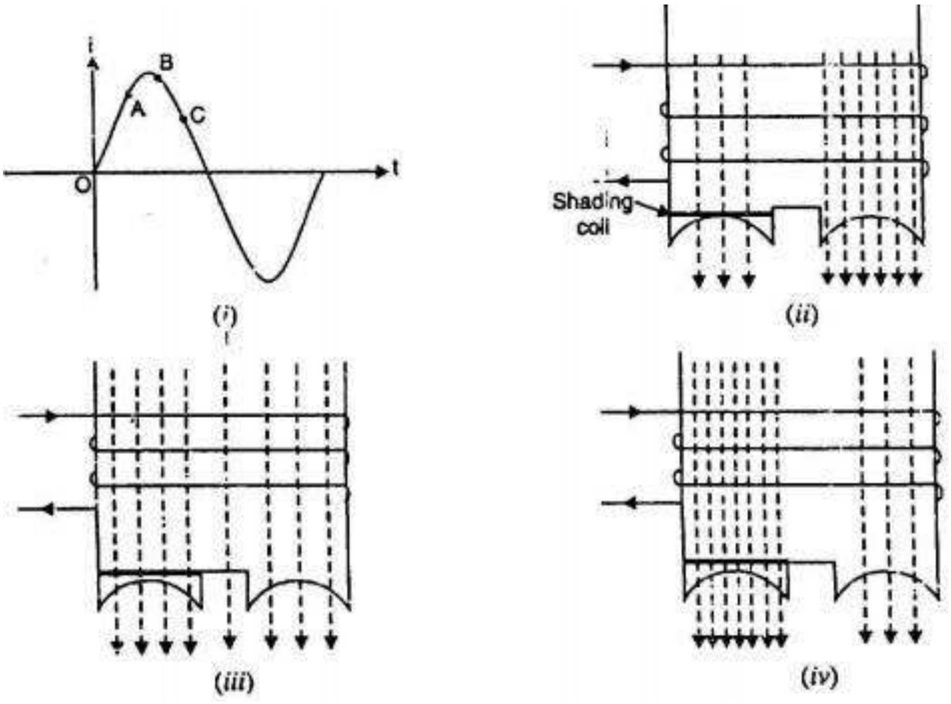
The operation of the motor can be understood by referring to Figure (3.16) which shows one pole of the motor with a shading coil.

(i) During the portion OA of the alternating-current cycle [See Figure (3.16)], the flux begins to increase and an EMF. is induced in the shading coil. The resulting current in the shading coil will be in such a direction so as to oppose the change in flux. Thus the flux in the shaded portion of the pole is weakened while that in the unshaded portion is strengthened as shown in Figure (3.16 (ii)).

(ii) During the portion AB of the alternating-current cycle, the flux has reached almost maximum value and is not changing. Consequently, the flux distribution across the pole is uniform [See Figure (3.16 (iii))] since no current is flowing in the shading coil. As the flux decreases (portion BC of the alternating current cycle), current is induced in the shading coil so as to oppose the decrease in current. Thus the flux in the shaded portion of the pole is strengthened while that in the un shaded portion is weakened as shown in Figure (3.16 (iv)).

(iii) (iii) The effect of the shading coil is to cause the field flux to shift across the pole face from the unshaded to the shaded portion. This shifting flux is like a rotating weak field moving in the direction from unshaded portion to the shaded portion of the pole.

(iv) The rotor is of the squirrel-cage type and is under the influence of this moving field. Consequently, a small starting torque is developed. As soon as this torque starts to revolve the rotor, additional torque is produced by single-phase induction-motor action. The motor accelerates to a speed slightly below the synchronous speed and runs as a single-phase induction motor.



(i) (ii) (iii) (iv)

3.16 one pole of the motor with a shading coil.

Characteristics

(i) The salient features of this motor are extremely simple construction and absence of centrifugal switch.

(ii) The motor efficiency is poor, but it is cheap.

(iii) Since starting torque, efficiency and power factor are very low, these motors are only suitable for low power applications e.g., to drive:

(a) small fans (6) toys (c) hair driers (d) desk fans etc.

Equivalent circuit of single phase induction motor

When the stator of single phase induction motor is connected to single – phase supply, the stator current produces a pulsating flux. According to the double – revolving field theory, the pulsating air – gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor. Since the magnitude of each rotating flux is one – half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single – phase induction motor may be considered as consisting of two motors having a common stator winding and two imaginary rotors, which rotate in opposite directions.

At standstill condition

The equivalent circuit of single – phase induction motor is shown in Figure 3.13

Where :

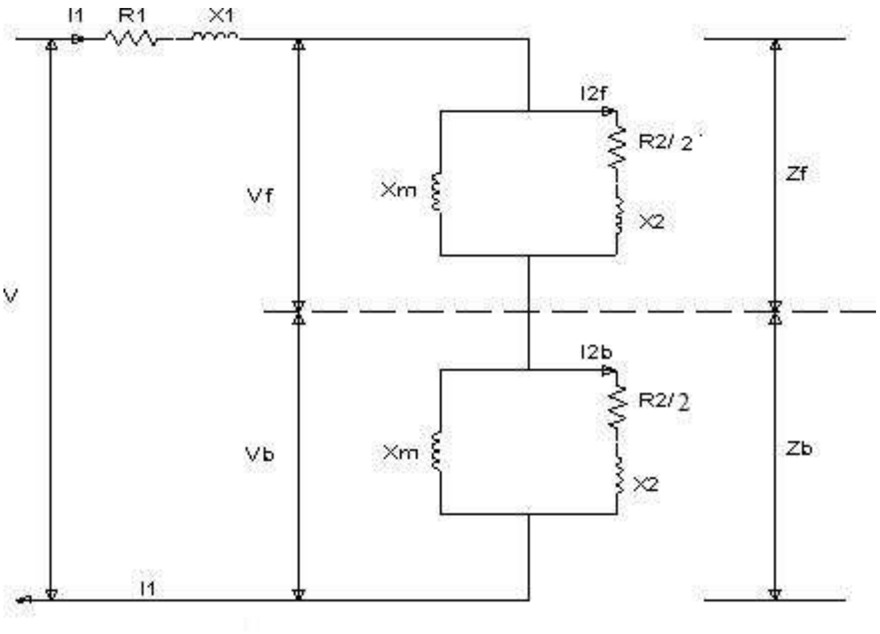
R_1 = resistance of stator winding

X_1 = leakage reactance of stator winding

X_m = total magnetizing reactance

R_2 = resistance of rotor referred to the stator

X_2 = leakage reactance of rotor referred to the stator



3.13 equivalent circuit of single phase induction motor at standstill.

At standstill,

$\varphi_f = \varphi_b$ Therefore, $V_f = V_b$

$V_b = I_1 Z_f$

$V_b = I_1 Z_b$

$Z_f = Z_b$

Z_f= impedance of forward parallel branch

Z_b = impedance of backward parallel branch

$I_1 = \frac{v}{zt}$

Where $Z_t = Z_1 + Z_f + Z_b$

$Z_1 = R_1 + X_1$

$Z_f = Z_b$

$$Zf = \frac{JXm\,.\,(\frac{R2}{2} + j\frac{X2}{2})}{\frac{R2}{2} + j(Xm + \frac{X2}{2})}$$

The torque of the backward field is in opposite direction to that of the forward field, and therefore the total air – gap power in a single phase induction motor is

$P_g = P_f - P_b$

Where Pf = air – gap power for forward field

$P_f = I^2 R_f$

P_b = air – gap power for backward field

$P_b = I^2 R_b$

The torque produced by the forward field is

$$Tf = \frac{Pf}{w} = \frac{Pf}{2\pi n}$$

The torque produced by the backward field

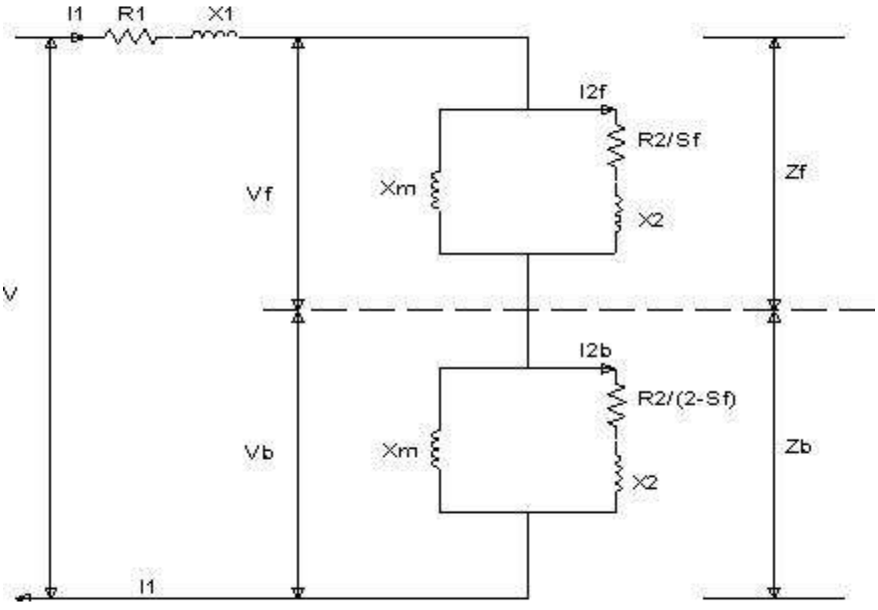
$$Tb = \frac{Pb}{ws} = \frac{Pb}{2\pi n}$$

The resultant electromagnetic or induced torque T_{in} is the difference between the torque Tf and Tb

$$T_{in} = Tf - Tb$$

At running condition

Now consider that the motor is pinning at some speed in the direction of the forward revolving field, the slip being s. The rotor current produced by the forward field will have a frequency sf where f is the stator frequency. Also, the rotor current produced by the backward field will have a frequency of (2 − s)f. Figure 3.18 shows the equivalent circuit of a single-phase induction motor when the rotor is rotating at slip s. It is clear, from the equivalent circuit that under running conditions, Ef becomes much greater than Eb because the term R'2/2s increases very much as s tends towards zero. Conversely, E^ falls because the term R'2/2(2 − s) decreases since (2 − s) tends toward 2. Consequently, the forward field increases, increasing the driving torque while the backward field decreases reducing the opposing torque.



equivalent circuit of single phase induction motor at operation without core loss.

$Z_t = Z_1 + Z_f + Z_b$

$Z_1 = R_1 + X_1$

$$Zf = \frac{JXm\,.\,(\frac{R2}{2S} + j\frac{X2}{2})}{\frac{R2}{2S} + j(\,Xm + \frac{X2}{2})}$$

$$Zb = \frac{JXm\,.\,(\frac{R2}{2(2-S)} + j\frac{X2}{2})}{\frac{R2}{2(2-S)} + j(\,Xm + \frac{X2}{2})}$$

The total copper loss is the sum of rotor copper loss due to the forward field and the rotor copper loss due to the backward field.

$Pcr = Pcr_f + Pcr_b$

Where

$Pcr = Slip * P_g$

$Pcr = S\,Pg_f -+ (2-S)\,Pg_b$

The power converted from electrical to mechanical form in a single phase induction motor is given by

$Pmech = (1-S)P_g$

Shaft output power

Pout =Pmech – friction loss – windage loss