

SYLLABUS

UNIT-I :

3-phase Induction Motors Construction details of cage and wound rotor machines production of rotating magnetic field principle of operation rotor emf and rotor frequency rotor current and pf at standstill and during running conditions rotor power input, rotor copper loss and mechanical power developed and their interrelationship equivalent circuit phasor diagram

UNIT-II :

Characteristics, starting and testing methods of Induction Motors Torque equation expressions for maximum torque and starting torque torque slip characteristic double cage and deep bar rotors crawling and cogging speed control of induction motor with V/f method no load and blocked rotor tests circle diagram for predetermination of performance methods of starting starting current and torque calculations induction generator operation

UNIT – III:

Single Phase Motors Single phase induction motors Constructional features and equivalent circuit Problem of starting Double revolving field theory Starting methods, shaded pole motors, AC Series motor.

UNIT-IV:

Construction, Operation and Voltage Regulation of Synchronous generator Constructional features of non salient and salient pole type Armature windings Distributed and concentrated windings Distribution Pitch and winding factors E.M.F equation Improvements of waveform and armature reaction Voltage regulation by synchronous impedance method MMF method and Potier triangle method Phasor diagrams Two reaction analysis of salient pole machines and phasor diagram.

UNIT –V:

Parallel operation of synchronous generators Parallel operation with infinite bus and other alternators Synchronizing power Load sharing Control of real and reactive power Numerical problems.

UNIT-VI:

Synchronous motor operation, starting and performance Synchronous Motor principle and theory of operation Phasor diagram Starting torque Variation of current and power factor with excitation Synchronous condenser Mathematical analysis for power developed Hunting and its suppression

UNIT-I

Construction of Three Phase Induction Motor

The three phase induction motor is the most widely used electrical motor. Almost 80% of the mechanical power used by industries is provided by **three phase induction motors** because of its simple and rugged construction, low cost, good operating characteristics, the absence of commutator and good speed regulation. In three phase induction motor, the power is transferred from stator to rotor winding through induction. The induction motor is also called a asynchronous motor as it runs at a speed other than the synchronous speed.

Like any other electrical motor induction motor also have two main parts namely rotor and stator.

1. **Stator:** As its name indicates stator is a stationary part of induction motor. A stator winding is placed in the stator of induction motor and the three phase supply is given to it.
2. **Rotor:** The rotor is a rotating part of induction motor. The rotor is connected to the mechanical load through the shaft.

1. Squirrel cage rotor,
2. Slip ring rotor or wound rotor or phase wound rotor.

Depending upon the type of rotor construction used the **three phase induction motor** are classified as:

1. Squirrel cage induction motor,
2. Slip ring induction motor or wound induction motor or phase wound induction motor.

The construction of stator for both the kinds of three phase induction motor remains the same. The other parts, which are required to complete the induction motor, are:

1. Shaft for transmitting the torque to the load. This shaft is made up of steel.
2. Bearings for supporting the rotating shaft.
3. One of the problems with electrical motor is the production of heat during its rotation. To overcome this problem, we need a fan for cooling.
4. For receiving external electrical connection Terminal box is needed.
5. There is a small distance between rotor and stator which usually varies from 0.4 mm to 4 mm. Such a distance is called air gap.

Stator of Three Phase Induction Motor

The stator of the three-phase induction motor consists of three main parts :

1. Stator frame,
2. Stator core,
3. Stator winding or field winding.

Stator Frame



It is the outer part of the **three phase induction motor**. Its main function is to support the stator core and the field winding. It acts as a covering, and it provides protection and mechanical strength to all the inner parts of the induction motor. The frame is either made up of die-cast or fabricated steel. The frame of three phase induction motor should be strong and rigid as the air gap length of three phase induction motor is very small. Otherwise, the rotor will not remain concentric with the stator, which will give rise to an unbalanced magnetic pull.

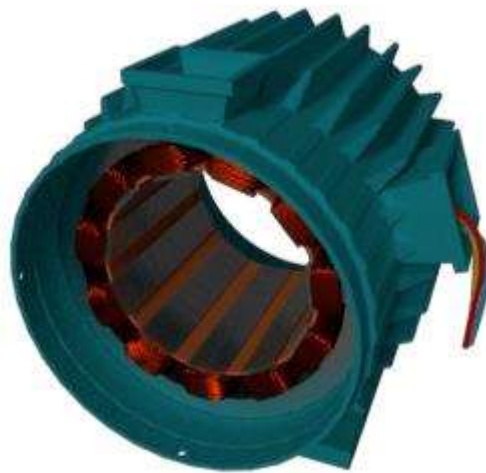
Stator Core



The main function of the stator core is to carry the alternating flux. In order to reduce the eddy current loss, the stator core is laminated. These laminated types of structure are made up of stamping which is about 0.4 to 0.5 mm thick. All the stamping are stamped together to form stator core, which is then housed in stator frame. The stamping is made up of silicon steel, which helps to reduce the hysteresis loss occurring in the motor.

Stator Winding or Field Winding

The slots on the periphery of the stator core of the three-phase induction motor carry three phase windings. We apply three phase ac supply to this three-phase winding. The three phases of the winding are connected either in star or delta depending upon which type of starting method we use. We start the squirrel cage motor mostly with star-delta stator and hence the stator of squirrel cage motor is delta connected. We start the slip ring three-phase induction motor by inserting resistances so, the stator winding of slip ring induction motor can be connected either in star or delta. The winding wound on the stator of three phase induction motor is also called field winding, and when this winding is excited by three phase ac supply, it produces a rotating



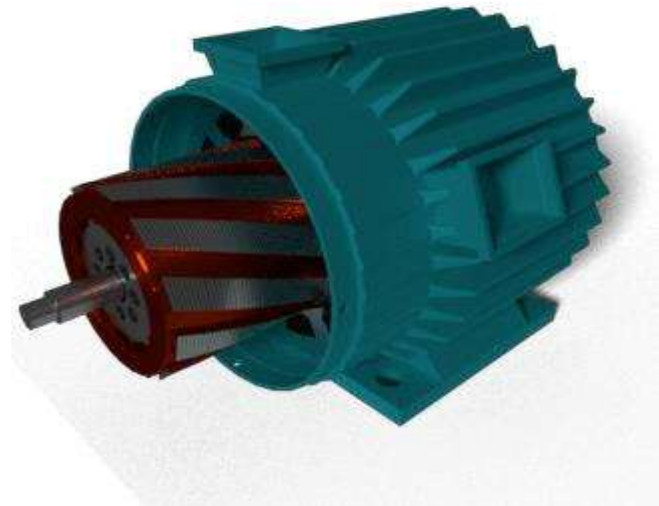
magnetic field.

Types of Three Phase Induction Motor

1. Squirrel Cage Three Phase Induction Motor

The rotor of the squirrel cage three phase induction motor is cylindrical and have slots on its periphery. The slots are not made parallel to each other but are bit skewed (skewing is not shown in the figure of squirrel cage rotor besides) as the skewing prevents magnetic locking of stator and rotor teeth and makes the working of the motor more smooth and quieter. The squirrel cage rotor consists of aluminum, brass or copper bars (copper bras rotor is shown in the figure beside). These aluminum, brass or copper bars are called rotor conductors and are placed in the slots on the periphery of the rotor. The rotor conductors are permanently shorted by the copper, or aluminum rings called the end rings. To provide mechanical strength, these rotor conductors are braced to the end ring and hence form a complete closed circuit resembling like a cage and hence got its name as squirrel cage induction motor. The squirrel cage rotor winding is made symmetrical. As end rings permanently short the bars, the rotor resistance is quite small, and it is not possible to add external resistance as the bars get permanently shorted. The absence of slip ring and brushes make the construction of Squirrel cage three-phase induction motor very simple and robust and hence widely used three phase induction motor. These motors have the advantage of adopting any

number of pole pairs. The below diagram shows a squirrel cage induction rotor having aluminum



bars short circuit by aluminum end rings.

Advantages of Squirrel Cage Induction Rotor

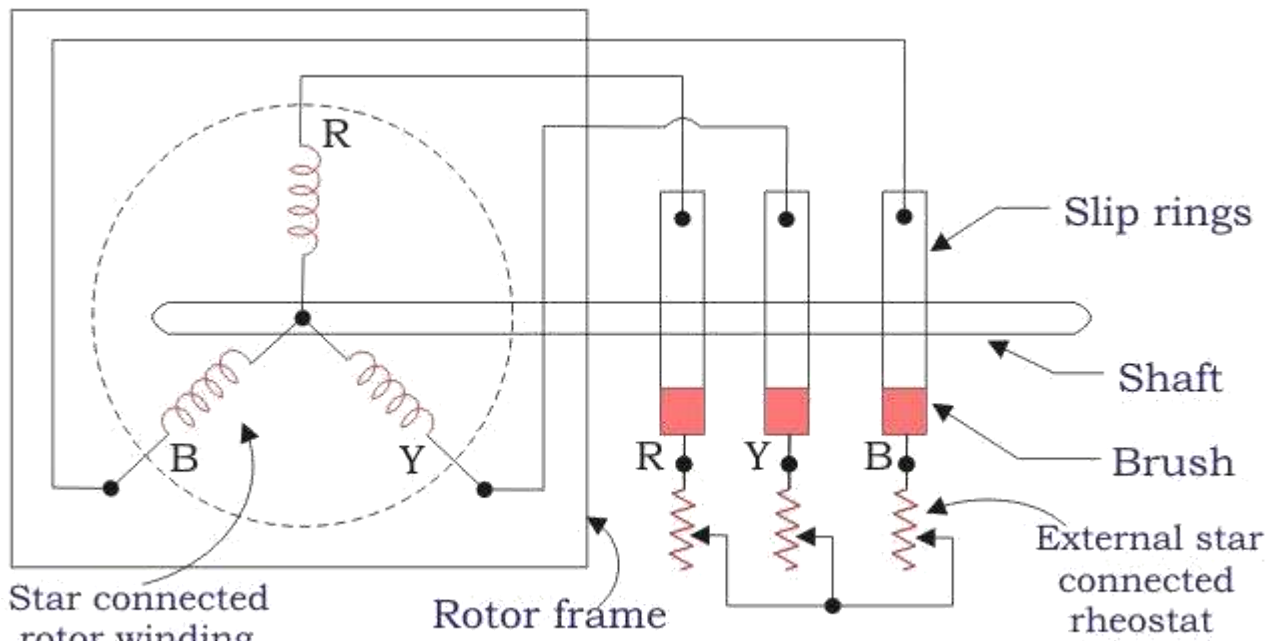
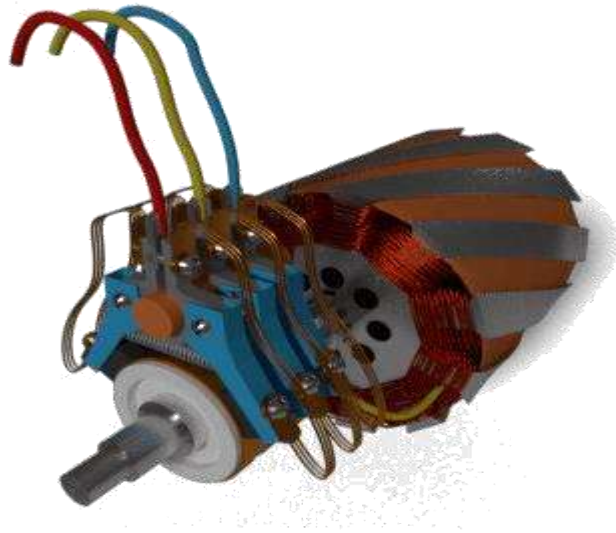
1. Its construction is very simple and rugged.
2. As there are no brushes and slip ring, these motors requires less maintenance.

Applications of Squirrel Cage Induction Rotor

We use the squirrel cage induction motors in lathes, drilling machine, fan, blower printing machines, etc

Slip Ring or Wound Rotor Three Phase Induction Motor

In this type of three phase induction motor the rotor is wound for the same number of poles as that of the stator, but it has less number of slots and has fewer turns per phase of a heavier conductor. The rotor also carries star or delta winding similar to that of the stator winding. The rotor consists of numbers of slots and rotor winding are placed inside these slots. The three end terminals are connected together to form a star connection. As its name indicates, three phase slip ring induction motor consists of slip rings connected on the same shaft as that of the rotor. The three ends of three-phase windings are permanently connected to these slip rings. The external resistance can be easily connected through the brushes and slip rings and hence used for speed controlling and improving the starting torque of three phase induction motor. The brushes are used to carry current to and from the rotor winding. These brushes are further connected to three phase star connected resistances. At starting, the resistance is connected to the rotor circuit and is gradually cut out as the rotor pick up its speed. When the motor is running the slip ring are shorted by connecting a metal collar, which connects all slip ring together, and the brushes are also removed. This reduces the wear and tear of the brushes. Due to the presence of slip rings and brushes the rotor construction becomes somewhat complicated therefore it is less used as compare to squirrel cage induction motor.



Slip Ring Three Phase Induction Motor

Advantages of Slip Ring Induction Motor

- 3. It has high starting torque and low starting current.
- 4. Possibility of adding additional resistance to control speed.

Application of Slip Ring Induction Motor

Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc.

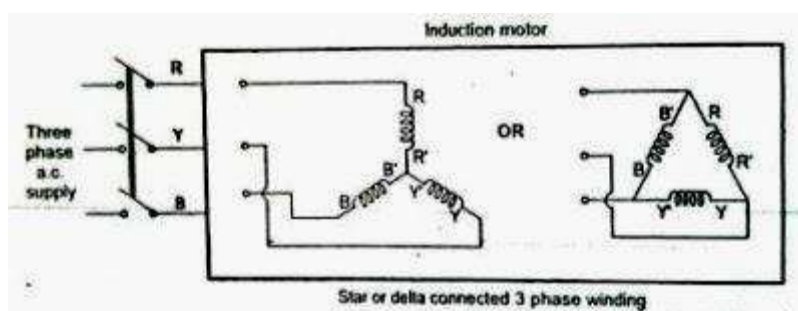
Difference between Slip Ring and Squirrel Cage Induction Motor

Slip ring or phase wound Induction motor	Squirrel cage induction motor
Construction is complicated due to presence of slip ring and brushes	Construction is very simple
The rotor winding is similar to the stator winding	The rotor consists of rotor bars which are permanently shorted with the help of end rings
We can easily add rotor resistance by using slip ring and brushes	Since the rotor bars are permanently shorted, its not possible to add external resistance
Due to presence of external resistance high starting torque can be obtained	Starting torque is low and cannot be improved
Slip ring and brushes are present	Slip ring and brushes are absent
Frequent maintenance is required due to presence of brushes	Less maintenance is required
The construction is complicated and the presence of brushes and slip ring makes the motor more costly	The construction is simple and robust and it is cheap as compared to slip ring induction motor
This motor is rarely used only 10% industry uses slip ring induction motor	Due to its simple construction and low cost. The squirrel cage induction motor is widely used
Rotor copper losses are high and hence less efficiency	Less rotor copper losses and hence high efficiency
Speed control by rotor resistance method is possible	Speed control by rotor resistance method is not possible
Slip ring induction motor are used where high starting torque is required i.e in hoists, cranes, elevator etc	Squirrel cage induction motor is used in lathes, drilling machine, fan, blower printing machines etc

Production of Rotating Magnetic Field:

The production of Rotating magnetic field in 3 phase supply is very interesting. When a 3-phase winding is energized from a 3-phase supply, a rotating magnetic field is produced. This field is such that its poles do not remain in a fixed position on the stator but go on shifting their positions around the stator. For this reason, it is called a rotating field. It can be shown that magnitude of this rotating field is constant and is equal to $1.5 f_m$ where f_m is the maximum flux due to any phase.

A three phase induction motor consists of three phase winding as its stationary part called stator. The three phase stator winding is connected in star or delta. The three phase windings are displaced from each other by 120° . The windings are supplied by a balanced three phase ac supply.



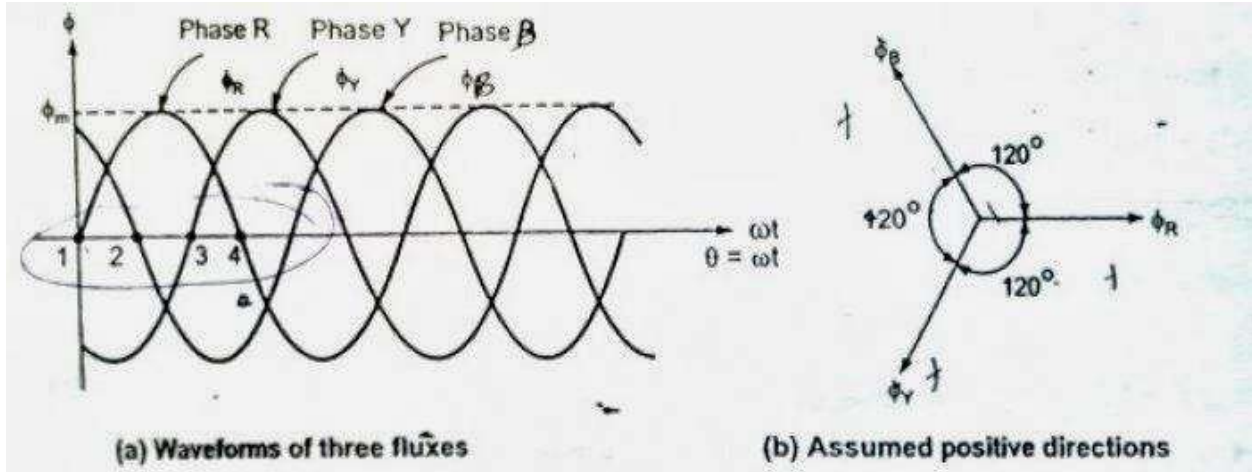
The three phase currents flow simultaneously through the windings and are displaced from each other by 120° electrical. Each alternating phase current produces its own flux which is sinusoidal. So all three fluxes are sinusoidal and are separated from each other by 120° . If the phase sequence of the windings is R-Y-B, then mathematical equations for the instantaneous values of the three fluxes Φ_R , Φ_Y , Φ_B can be written as,

$$\Phi_R = \Phi_m \sin(\omega t)$$

$$\Phi_Y = \Phi_m \sin(\omega t - 120)$$

$$\Phi_B = \Phi_m \sin(\omega t - 240)$$

As windings are identical and supply is balanced, the magnitude of each flux is Φ_m .



Case 1 : $\omega t = 0$

$$\Phi_R = \Phi_m \sin(0) = 0$$

$$\Phi_Y = \Phi_m \sin(0 - 120) = -0.866 \Phi_m$$

$$\Phi_B = \Phi_m \sin(0 - 240) = +0.866 \Phi_m$$

Case 2 : $\omega t = 60$

$$\Phi_R = \Phi_m \sin(60) = +0.866 \Phi_m$$

$$\Phi_Y = \Phi_m \sin(-60) = -0.866 \Phi_m$$

$$\Phi_B = \Phi_m \sin(-180) = 0$$

Case 3 : $\omega t = 120$

$$\Phi_R = \Phi_m \sin(120) = +0.866 \Phi_m$$

$$\Phi_Y = \Phi_m \sin(0) = 0$$

$$\Phi_B = \Phi_m \sin(-120) = -0.866 \Phi_m$$

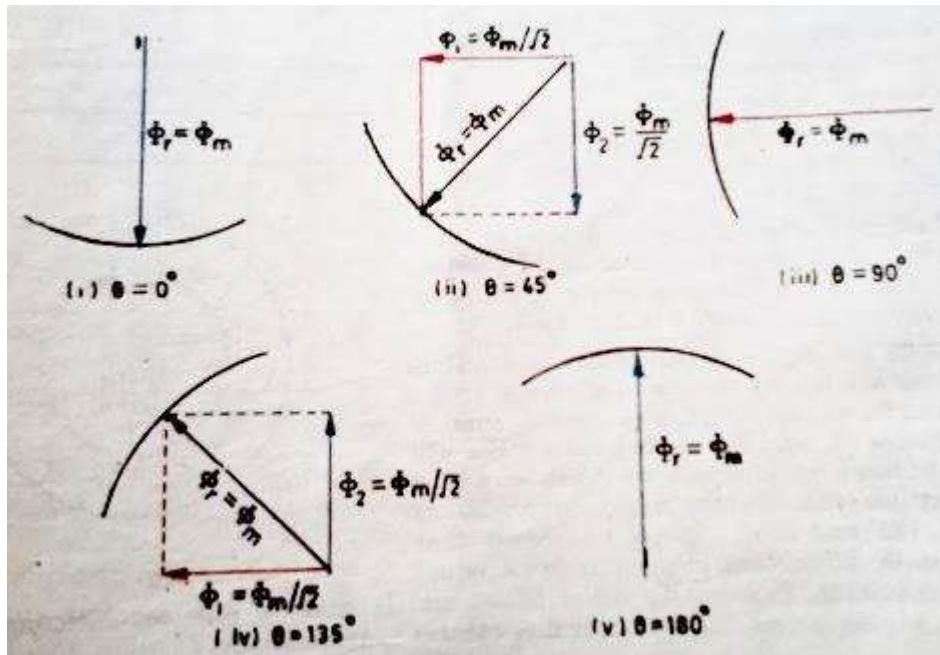
Case 4 : $\omega t = 180$

$$\Phi_R = \Phi_m \sin(180) = 0$$

$$\Phi_Y = \Phi_m \sin(60) = +.866 \Phi_m$$

$$\Phi_B = \Phi_m \sin(-60) = -0.866 \Phi_m$$

By comparing the electrical and phasor diagrams we can find the the flux rotates one complete 360 degree on the 180 degree displacement of flux.



Effect of Frequency on Rotor Parameter in the Three Phase Induction Motor

Rotor emf

□ When the three phase supply is given to the three phase induction motor, the rotating magnetic field is produced which rotates at synchronous speed.

Starting condition

- The speed of the rotor during starting condition is zero therefore the relative speed between stator rotating magnetic field and rotor speed is

$$N_s - N = N_s$$

- When the rotor conductors cut the rotating magnetic field, an emf will be induced in it.
- Let us consider that the induced emf in the rotor is E_2

Running condition

- Let us assume that the induced emf in the rotor is E_2' . The relative speed between stator rotating magnetic field and rotor speed is $N_s - N$.
- The induced emf in the rotor condition is

$$N_s \quad E_2$$

$$N_s - N \quad E_2'$$

$$E_2' = [N_s - N / N_s] E_2$$

$$E_2' = s E_2$$

□ The slip at starting is unity therefore the rotor induced emf is same as that of starting condition.

- However the rotor induced emf decreases as the speed increases or slip decreases.

Effect of slip on rotor Parameters

- The effect of slip on rotor parameter is summarized as below. As the slip decreases during running condition, all the parameter decreases during running condition.

Rotor parameter	Starting condition	Running condition
Frequency of rotor emf	f	$f' = sf$
Rotor induced emf	E_2	$E_2' = s E_2$
Rotor resistance	R_2	R_2
Rotor reactance	$X_2 = 2\pi fL$	$X_2' = s X_2 = s(2\pi fL)$
Rotor impedance	$Z_2 = \sqrt{R_2^2 + X_2^2}$	$Z_2' = \sqrt{R_2^2 + X_2'^2}$ $= \sqrt{R_2^2 + (sX_2)^2}$
Rotor current	$I_2 = E_2 / Z_2$ $I_2 = E_2 / \sqrt{R_2^2 + X_2^2}$	$I_2' = E_2' / Z_2'$ $I_2' = s E_2 / \sqrt{R_2^2 + (sX_2)^2}$

Rotor power factor	$\cos \phi_2 = R_2 / Z_2$ $= R_2 / \sqrt{R_2^2 + X_2^2}$	$\cos \phi_2' = R_2 / Z_2'$ $= R_2 / \sqrt{R_2^2 + (sX_2)^2}$

RELATION SHIP BETWEEN P2, PC, PM:

The rotor input P₂, rotor copper loss P_c and gross mechanical power developed P_m are related through the slip s. Let us derive this relationship.

Let T = Gross torque developed by motor in N-m.

We know that the torque and power are related by the relation, P = T x ω

where P = Power

and ω = angular speed

$$= (2\pi N)/60, N = \text{speed in r.p.m.}$$

Now input to the rotor P₂ is from stator side through rotating magnetic field which is rotating at synchronous speed N_s.

So torque developed by the rotor can be expressed interms of power input and angular speed at which power is inputted i.e. ω_s as,

$$P_2 = T \times \omega_s \quad \text{where } \omega_s = (2\pi N_s)/60 \text{ rad/sec}$$

$$P_2 = T \times (2\pi N_s)/60 \quad \text{where } N_s \text{ is in r.p.m.} \quad \dots\dots\dots(1)$$

The rotor tries to deliver this torque to the load. So rotor output is gross mechanical power developed P_m and torque T. But rotor gives output at speed N and not N_s. So from output side P_m and T can be related through angular speed ω and not ω_s.

$$P_m = T \times \omega \quad \text{where } \omega = (2\pi N)/60$$

$$P_m = T \times (2\pi N)/60 \quad \dots\dots\dots(2)$$

The difference between P₂ and P_m is rotor copper loss P_c.

$$P_c = P_2 - P_m = T \times (2\pi N_s/60) - T \times (2\pi N/60)$$

$$P_c = T \times (2\pi/60)(N_s - N) = \text{rotor copper loss} \quad \dots\dots\dots(3)$$

Dividing (3) by (1),

$$\frac{P_c}{P_2} = \frac{T \times \frac{2\pi}{60} (N_s - N)}{T \times \frac{2\pi}{60} \times N_s} = \frac{N_s - N}{N_s}$$

$$P_c/P_2 = s \text{ as } (N_s - N)/N_s = \text{slip } s$$

Rotor copper loss P_c = s x Rotor input P₂

Thus total rotor copper loss is slip times the rotor input.

Now

$$P_2 - P_c = P_m$$

$$P_2 - sP_2 = P_m$$

$$(1 - s)P_2 = P_m$$

Thus gross mechanical power developed is (1 - s) times the rotor input

The relationship can be expressed in the ratio from as,

P₂ : P_c : P_m is 1 : s : 1 - s

The ratio of any two quantities on left hand side is same as the ratio of corresponding two sides on the right hand side.

For example,	$\frac{P_c}{P_m} = \frac{s}{1-s}, \quad \frac{P_2}{P_c} = \frac{1}{s} \quad \text{and so on.}$
--------------	--

This relationship is very important and very frequently required to solve the problems on the power flow diagram.

Key Point : The torque produced by rotor is gross mechanical torque and due to mechanical losses entire torque can not be available to drive load. The load torque is net output torque called shaft torque or useful torque and is denoted as T_{sh} . It is related to P_{out} as,

$$T_{sh} = \frac{P_{out}}{\omega} = \frac{P_{out}}{\left(\frac{2\pi N}{60}\right)}$$

and $T_{sh} < T$ due to mechanical losses.

Derivation of k in Torque Equation

We have seen earlier that

$$T = (k s E_2^2 R_2) / (R_2^2 + (s X_2)^2)$$

and it mentioned that $k = 3 / (2\pi n_s)$. Let us see its proof.

The rotor copper losses can be expressed as,

$$P_c = 3 \times I_{2r}^2 \times R_2$$

but $I_{2r} = (s E_2) / \sqrt{(R_2^2 + (s X_2)^2)}$, hence substituting above

$$P_c = 3 \times \left[\frac{s E_2}{\sqrt{R_2^2 + (s X_2)^2}} \right]^2 \times R_2$$

$$\therefore P_c = \frac{3 s^2 E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Now as per $P_2 : P_c : P_m$ is $1 : s : 1-s$,

$$P_c / P_m = s / (1-s)$$

$$\begin{aligned} \text{Now } P_m &= T \times \omega \\ &= T \times (2\pi N / 60) \end{aligned}$$

$$\therefore T \times \frac{2\pi N}{60} = \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

$$\therefore T = \frac{60}{2\pi N} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (s X_2)^2}$$

Now $N = N_s (1-s)$ from definition of slip, substituting in above,

$$\therefore T = \frac{60}{2\pi N_s (1-s)} \times \frac{(1-s) 3 s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

$$= \frac{3}{2\pi \left(\frac{N_s}{60}\right)} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

but $N_s/60 = n_s$ in r.p.m.

So substituting in the above equation,

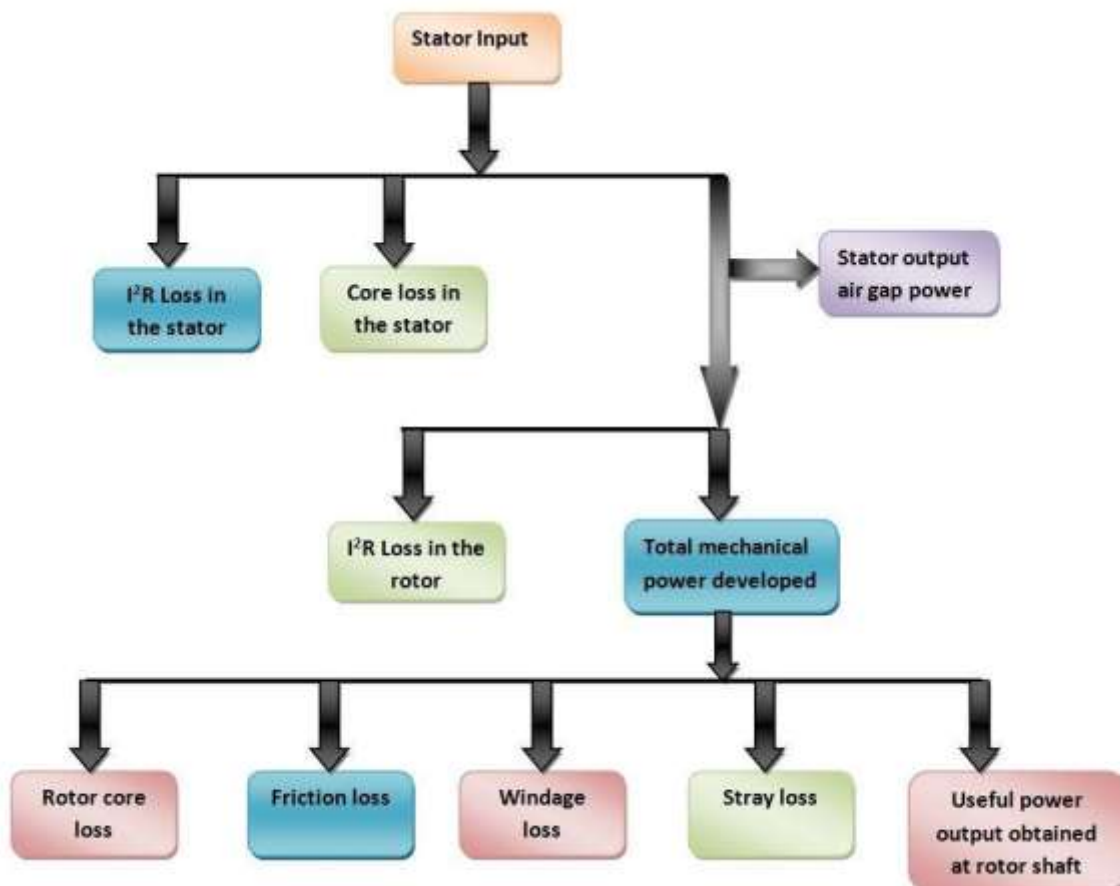
$$T = \frac{3}{2\pi n_s} \times \frac{s E_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Comparing the two torque equations we can write,

$$k = \frac{3}{2\pi n_s} \text{ where } n_s \text{ is in r.p.s.}$$

Power Flow Diagram and Losses of Induction Motor:

Power Flow Diagram of Induction Motor explains the input given to the motor, the losses occurring and the output of the motor. The input power given to an Induction motor is in the form of three-phase voltage and currents. The Power Flow Diagram of an Induction Motor is shown below.



The power flow is given by the equation shown below.

$$P_{is} = \sqrt{3}V_L I_L \cos\phi_i = 3V_{sp} I_{sp} \cos\phi_i$$

Where $\cos\phi_i$ is the input power factor

The **losses** in the stator are

IR losses in the stator winding resistances. It is also known as **Stator copper losses**.

$$P_{SCL} = 3I_{sp}^2 R_{sp}$$

Hysteresis and **Eddy current** losses in the stator core. These are known as **Stator core losses**.

$$P_{s(h+e)}$$

The output power of the stator is given as

$$P_{os} = P_{is} - P_{sc} - P_{s(h+e)}$$

This output power of the stator is transferred to the rotor of the machine across the air gap between the stator and the rotor. It is called the **air gap P_g** of the machine.

Thus,

The Power output of the stator = air gap power = input power to the rotor

$$P_{os} = P_g = P_{ir}$$

The losses in the rotor are as follows.

IR losses in the rotor resistance. They are also called **Rotor copper losses** and represented as

$$P_{rc} = 3I_2^2 R_2$$

Hysteresis and eddy current losses in the rotor core. They are known as **Rotor core losses**.

$$P_{r(h+e)}$$

Friction and Windage losses P_{fw}

Stray load losses P_{misc}, consisting of all losses not covered above, such as losses due to harmonic fields.

If the rotor copper losses are subtracted from rotor input power P_r , the remaining power is converted from electrical to mechanical form. This is called **Developed Mechanical Power P_{md}** .

Developed Mechanical power = Rotor input – Rotor copper loss

$$P_{md} = P_{ir} - P_{rc} \quad \text{or}$$

$$P_{md} = P_g - P_{rc}$$

$$P_{md} = P_g - 3 I_2^2 R_2$$

The output of the motor is given by the equation shown below.

$$P_o = P_{md} - P_{fw} - P_{misc}$$

P_o is called the **shaft power** or the **useful power**.

Rotational losses

At starting and during acceleration, the rotor core losses are high. With the increase in the speed of the induction motor these losses decrease. The friction and windage losses are zero at the start. As the speed increases the losses, also start increasing. The sum of the friction, windage and core losses are almost constant with the change in speed. These all losses are added together and are known as **Rotational Losses**.

It is given by the equation shown below.

$$P_{rot} = P_{fw} + P_{h+e} + P_{misc}$$

$$P_o = P_{md} - P_{rot} = P_{md} - P_{fw} - P_{h+e} - P_{misc}$$

The **Rotational losses** are not represented by any element of the equivalent circuit as they are purely mechanical quantity.

Phasor Diagram of Three Phase Induction Motor:

In a 3-phase induction motor, the stator winding is connected to 3-phase supply and the rotor winding is short-circuited. The energy is transferred magnetically from the stator winding to the short-circuited, rotor winding. Therefore, an induction motor may be considered to be a transformer with a rotating secondary (short-circuited). The stator winding corresponds to transformer primary and the rotor winding corresponds to transformer secondary. In view of the similarity of the flux and voltage conditions to those in a transformer, one can expect that the equivalent circuit of an induction motor will be similar to that of a transformer. Fig. 3.8 shows the equivalent circuit per phase for an induction motor. Let discuss the stator and rotor circuits separately.

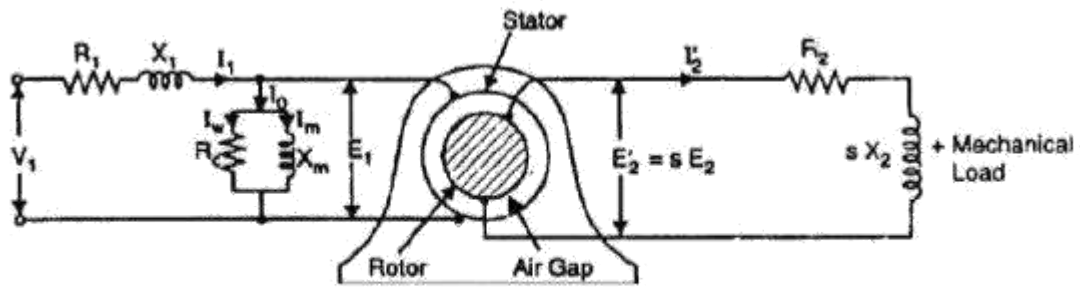


Fig 3.8

Stator circuit. In the stator, the events are very similar to those in the transformer primary. The applied voltage per phase to the stator is V_1 and R_1 and X_1 are the stator resistance and leakage reactance per phase respectively. The applied voltage V_1 produces a magnetic flux which links the stator winding (i.e., primary) as well as the rotor winding (i.e., secondary). As a result, self-induced e.m.f. E_1 is induced in the stator winding and mutually induced e.m.f.

$E'_2 (= s E_2 = s K E_2$ where K is transformation ratio) is induced in the rotor winding. The flow of stator current I_1 causes voltage drops in R_1 and X_1 .

$$V_1 = - E_1 + I_1(R_1 + jX_1) \dots\dots \text{phasor sum}$$

When the motor is at no-load, the stator winding draws a current I_0 . It has two components viz.,

(i) which supplies the no-load motor losses and (ii) magnetizing component I_m which sets up magnetic flux in the core and the air gap. The parallel combination of R_c and X_m , therefore, represents the no-load motor losses and the production of magnetic flux respectively.

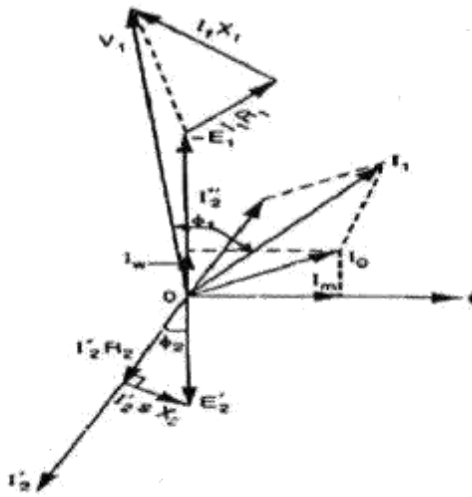
$$I_0 = I_w + I_m$$

Rotor circuit. Here R_2 and X_2 represent the rotor resistance and standstill rotor reactance per phase respectively. At any slip s , the rotor reactance will be sX_2 . The induced voltage/phase in the rotor is $E'_2 = s E_2 = s K E_1$. Since the rotor winding is short-circuited, the whole of e.m.f. E'_2 is used up in circulating the rotor current I'_2 .

$$E'_2 = I'_2 (R_2 + jsX_2)$$

The rotor current I'_2 is reflected as $I''_2 (= K I'_2)$ in the stator. The phasor sum of I''_2 and I_0 gives the stator current I_1 .

It is important to note that input to the primary and output from the secondary of a transformer are electrical. However, in an induction motor, the inputs to the stator and rotor are electrical but the output from the rotor is mechanical. To facilitate calculations, it is desirable and necessary to replace the mechanical load by an equivalent electrical load. We then have the transformer equivalent circuit of the induction motor.



It may be noted that even though the frequencies of stator and rotor currents are different, yet the magnetic fields due to them rotate at synchronous speed N_s . The stator currents produce a magnetic flux which rotates at a speed N_s . At slip s , the speed of rotation of the rotor field relative to the rotor surface in the direction of rotation of the rotor is

$$= \frac{120 f'}{P} = \frac{120 s f}{P} = s N_s$$

But the rotor is revolving at a speed of N relative to the stator core. Therefore, the speed of rotor field relative to stator core

$$= sN_s + N = (N_s - N) + N = N_s$$

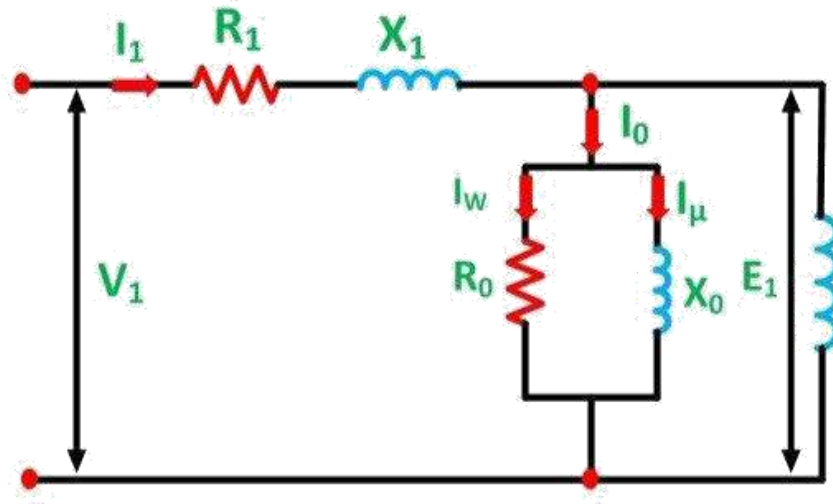
Thus no matter what the value of slip s , the stator and rotor magnetic fields are synchronous with each other when seen by an observer stationed in space. Consequently, the 3-phase induction motor can be regarded as being equivalent to a transformer having an air-gap separating the iron portions of the magnetic circuit carrying the primary and secondary windings. Fig. 3.9 shows the phasordiagram of induction motor.

Equivalent Circuit of an Induction Motor

Equivalent Circuit of an Induction motor enables the performance characteristics which are evaluated for steady state conditions. An induction motor is based on the principle of induction of voltages and currents. The voltage and current is induced in the rotor circuit from the stator circuit for the operation. The equivalent circuit of an induction motor is similar to that of the transformer.

Stator Circuit Model

The stator circuit model of an induction motor consists of a stator phase winding resistance R_1 , stator phase winding leakage reactance X_1 as shown in the circuit diagram below.



The no load current I_0 is simulated by a pure inductive reactor X_0 taking the magnetizing component I_μ and a noninductive resistor R_0 carrying the core loss current I_ω . Thus,

$$I_0 = I_\mu + I_\omega \dots \dots \dots (1)$$

The total magnetizing current I_0 is considerably larger in the case of the induction motor as compared to that of a transformer. This is because of the higher reluctance caused by the air gap of the induction motor. As we know that, in a transformer the no load current varies from 2 to 5% of the rated current, whereas in an induction motor the no load current is about 25 to 40% of the rated current depending upon the size of the motor. The value of the magnetizing reactance X_0 is also very small in an induction motor.

Rotor Circuit Model

When a three phase supply is applied to the stator windings, a voltage is induced in the rotor windings of the machine. The greater will be the relative motion of the rotor and the stator magnetic fields, the greater will be the resulting rotor voltage. The largest relative motion occurs at the standstill condition. This condition is also known as the locked rotor or blocked rotor condition. If the induced rotor voltage at this condition is E_{20} then the induced voltage at any slip is given by the equation shown below.

$$E_{2s} = sE_{20} \dots \dots \dots (2)$$

The rotor resistance is constant and is independent of the slip. The reactance of the induction motor depends upon the inductance of the rotor and the frequency of the voltage and current in the rotor.

If L_2 is the inductance of rotor, the rotor reactance is given by the equation shown below.

$$X_2 = 2\pi f_2 L_2$$

But, as we know

$$f_2 = sf_1$$

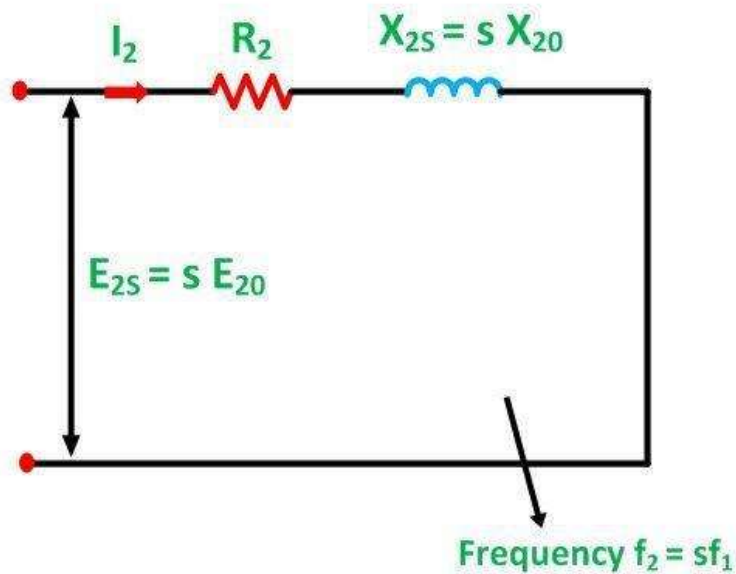
Therefore,

$$X_2 = 2\pi sf_1 L_2 = s (2\pi f_1 L_2) \quad \text{or}$$

$$X_2 = sX_{20} \dots \dots \dots (3)$$

Where, X_{20} is the standstill reactance of the rotor.

The rotor circuit is shown below.



The rotor impedance is given by the equation below.

$$Z_{2s} = R_2 + jX_{2s} \quad \text{or}$$

$$Z_{2s} = R_2 + jsX_{20} \dots \dots \dots (4)$$

The rotor current per phase is given by the equation shown below.

$$I_{2s} = \frac{E_{2s}}{Z_{2s}}$$

$$I_{2s} = \frac{sE_{20}}{R_2 + jsX_{20}} \dots \dots \dots (5)$$

Here, I_2 is the slip frequency current produced by a slip frequency induced voltage sE_{20} acting in the rotor circuit having an impedance per phase of $(R_2 + jsX_{20})$.

Now, dividing the equation (5) by slip s we get the following equation.

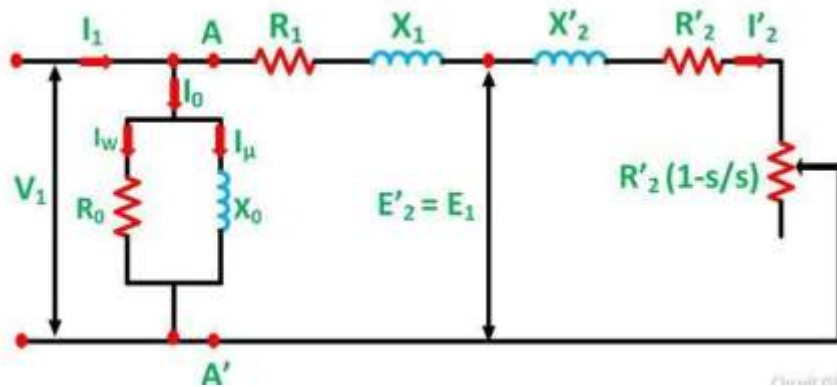
$$I_{2s} = \frac{E_{20}}{\frac{R_2}{s} + jX_{20}} \dots\dots\dots (6)$$

The R_2 is a constant resistance and a variable leakage reactance sX_{20} . Similarly, the rotor circuit shown below has a constant leakage reactance X_{20} and a variable resistance R_2/s .

The equation (6) above explains the secondary circuit of an imaginary transformer, with a constant voltage ratio and with the same frequency of both sides. This imaginary stationary rotor carries the same current as the actual rotating rotor. This makes possible to transfer the secondary rotor impedance to the primary stator side.

Approximate Equivalent Circuit of an Induction Motor

The equivalent circuit is further simplified by shifting the shunt impedance branches R_0 and X_0 to the input terminals as shown in the circuit diagram below.



The approximate circuit is based on the assumption that $V_1 = E_1 = E'_2$. In the above circuit, the only component that depends on the slip is the resistance. All the other quantities are constant. The following equations can be written at any given slip s as follows:-

Impedance beyond AA' is given as

$$Z_{AA'} = \left(R_1 + \frac{R'_2}{s} \right) + j(X_1 + X'_2) \dots\dots\dots (7)$$

$$I'_2 = \frac{V_1}{Z_{AA'}} \dots\dots\dots (8)$$

Putting the value of $Z_{AA'}$ from the equation (7) in the equation (8) we get

$$I'_2 = \frac{V_1}{\left(R_1 + \frac{R'_2}{s} \right) + j(X_1 + X'_2)} \dots\dots\dots (9)$$

Therefore,

$$I'_2 = |I'_2| \frac{V_1}{\sqrt{\left(R_1 + \frac{R'_2}{s}\right)^2 + j(X_1 + X'_2)^2}} \dots\dots\dots (10)$$

Hence,

$$I'_2 = I'_2 \cos\phi_2 - jI'_2 \sin\phi_2 \dots\dots\dots (11)$$

Where,

$$\tan \phi_2 = \frac{X_1 + X'_2}{R_1 + \frac{R'_2}{s}} \dots\dots\dots (12) \quad \text{and}$$

$$\cos \phi_2 = \frac{R_1 + (R'_2/s)}{|Z_{AA'}|} \dots\dots\dots (13)$$

No load current I_0 is

$$I_0 = I_\mu + I_\omega$$

$$I_0 = \frac{V_1}{R_0} + \frac{V_1}{jX_0}$$

$$I_0 = V_1 \left(\frac{1}{R_0} - j \frac{1}{X_0} \right) \dots\dots\dots (14)$$

Total stator current is given by the equation shown below.

$$I_1 = I'_2 + I_0$$

Total core losses are given by the equation shown below.

$$P_{h+e} = 3V_1 I_0 \cos\phi_0 \dots\dots\dots (15)$$

$$\text{Stator input} = 3V_1 I_1 \cos\phi_1$$

$$\text{Stator input} = 3V_1 I'_2 \cos\phi_2 + P_{h+e}$$

$$\text{Stator input} = 3 I'^2_2 \left(R_1 + \frac{R'_2}{s} \right) + P_{h+e}$$

Air gap power per phase is given as

$$P_g = V_1 I'_2 \cos\phi_2 = I_2'^2 \frac{R'_2}{s} = \frac{V_1^2 (R'_2/s)}{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2}$$

Developed torque is given by the equation shown below.

$$T_d = \frac{P_g}{\omega_s} \quad \text{or}$$
$$T_d = \frac{V_1^2 (R'_2/s)}{\omega_s \left[\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2 \right]} \dots\dots\dots (16)$$

The above equation is the torque equation of an induction motor. The approximate equivalent circuit model is the standard for all performance calculation of an induction motor.

Unit :- 1 :-

Three phase induction Motor

Construction of 3-phase induction motor :-

* The Basic parts of 3-phase induction motor are

1. stator
2. Rotor

* stator is a stationary part of 3-phase induction motor and it carries a winding called stator winding.

* The stator winding is connected in either star or delta connection.

* Rotor is the rotating part of 3-phase induction motor and it is also having winding called rotor winding.

* Based on rotor construction 3-phase induction motor are classified into two types :

1. squirrel cage induction motor
2. slip ring induction motor or wound type induction motor

* Whenever supply is given to stator of 3-phase induction motor it produces a magnetic field,

named as rotating magnetic field (RMF).

and it runs with a speed of synchronous speed

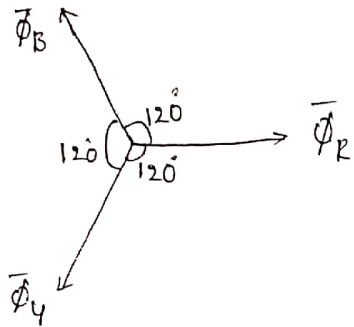
$$N_s = \frac{120f}{P}$$

f = supply frequency or stator frequency

P = no. of poles.

* The principle of induction motor is mutual induction principle. When we give supply to stator it produces flux (RMF). This flux will interact with rotor winding and emf will also produce in rotor.

Production of RMF:-



$$\bar{\Phi}_R = \phi_m \sin \omega t$$

$$\bar{\Phi}_Y = \phi_m \sin(\omega t - 120^\circ)$$

$$\bar{\Phi}_B = \phi_m \sin(\omega t + 120^\circ)$$

$$\bar{\Phi}_R = \bar{\Phi}_R + \bar{\Phi}_Y + \bar{\Phi}_B$$

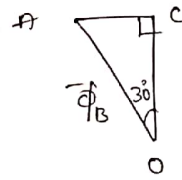
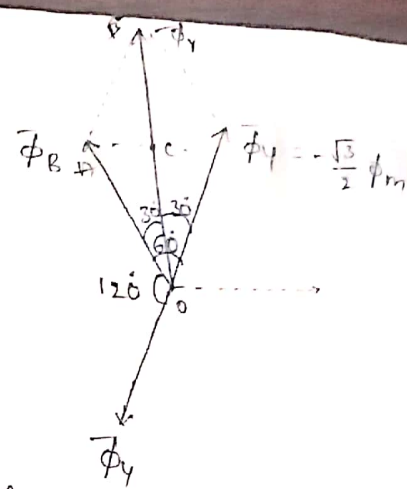
$$\bar{\Phi}_R = \phi_m [\sin \omega t + \sin(\omega t - 120^\circ) + \sin(\omega t + 120^\circ)]$$

At $\omega t = 0^\circ$:-

$$\bar{\Phi}_R = 0$$

$$\bar{\Phi}_Y = -\frac{\sqrt{3}}{2} \phi_m$$

$$\bar{\Phi}_B = \frac{\sqrt{3}}{2} \phi_m$$



$$\cos 30^\circ = \frac{OC}{OA}$$

$$OC = OA \times \cos 30^\circ$$

$$OB = 2 \times OC$$

$$\bar{\Phi}_R = 2 \times OA \times \cos 30^\circ$$

$$\bar{\Phi}_R = 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2}$$

$$\bar{\Phi}_R = \frac{3}{2} \phi_m$$

$$\bar{\Phi}_R = \frac{3}{2} \phi_m$$

$$\bar{\Phi}_R = 1.5 \phi_m$$

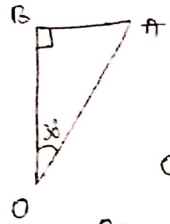
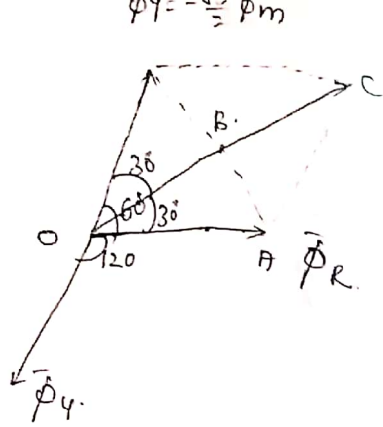
* Resultant flux is equal to 1.5 times of maximum flux.

$$2. \text{ At } \omega t = 60^\circ :-$$

$$\bar{\phi}_R = \frac{\sqrt{3}}{2} \phi_m$$

$$\bar{\phi}_Y = -\frac{\sqrt{3}}{2} \phi_m$$

$$\bar{\phi}_B = 0$$



$$\cos 30^\circ = \frac{OB}{OA}$$

$$OB = \cos 30^\circ \times OA$$

$$OC = 2 \times OB$$

$$\begin{aligned} \bar{\phi}_R &= 2 \times OA \times \cos 30^\circ \\ &= 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$\bar{\phi}_R = \frac{3}{2} \phi_m$$

$$\boxed{\begin{aligned} \bar{\phi}_R &= \frac{3}{2} \phi_m \\ \bar{\phi}_R &= 1.5 \phi_m \end{aligned}}$$

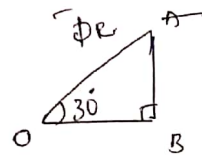
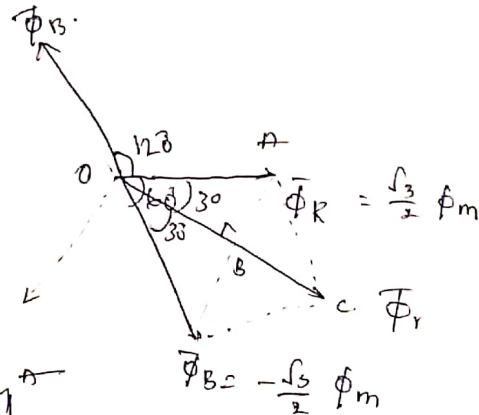
* Resultant flux is always equal to 1.5 times of maximum flux

$$3. \text{ At } \omega t = 120^\circ :-$$

$$\bar{\phi}_R = \phi_m \sin 120^\circ = \frac{\sqrt{3}}{2} \phi_m$$

$$\bar{\phi}_Y = 0$$

$$\bar{\phi}_B = \phi_m \sin 240^\circ = -\frac{\sqrt{3}}{2} \phi_m$$



$$\cos 30^\circ = \frac{OB}{OA}$$

$$OB = OA \cos 30^\circ$$

$$\begin{aligned} OC &= 2 \times OB = 2 \times OA \times \cos 30^\circ \\ &= 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2} \end{aligned}$$

$$\boxed{\begin{aligned} \bar{\phi}_{RY} &= \frac{3}{2} \phi_m \\ \bar{\phi}_R &= 1.5 \phi_m \end{aligned}}$$

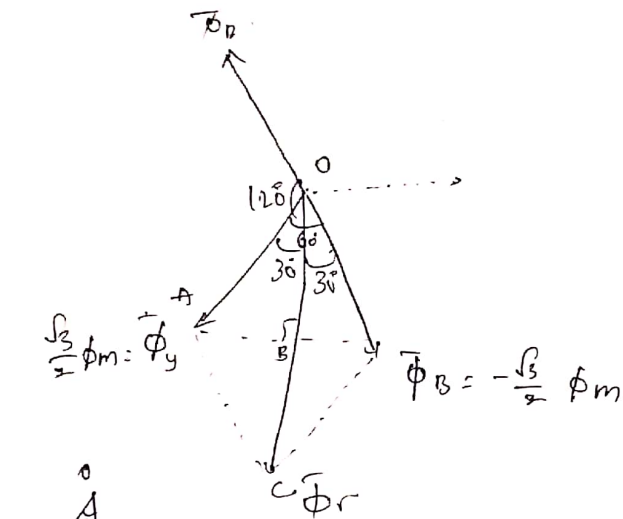
* Resultant flux is equal to 1.5 times of maximum flux

4. At $\omega t = 180^\circ$

$$\vec{\phi}_R = \phi_m \sin 180^\circ = 0$$

$$\vec{\phi}_Y = \phi_m \sin (180^\circ - 120^\circ) = \frac{\sqrt{3}}{2} \phi_m$$

$$\vec{\phi}_B = \phi_m \sin (180^\circ + 120^\circ) = -\frac{\sqrt{3}}{2} \phi_m$$



$$\cos 30^\circ = \frac{OB}{OA}$$

$$OB = OA \times \cos 30^\circ$$

$$OC = 2 \times OB$$

$$= 2 \times OA \times \cos 30^\circ$$

$$= 2 \times \frac{\sqrt{3}}{2} \phi_m \times \frac{\sqrt{3}}{2}$$

$$\phi_r = \frac{3}{2} \phi_m$$

$$\phi_r = 1.5 \phi_m$$

Resultant flux is equal to 1.5 times of maximum flux.

Importance of 3-phase induction motor over other motors.

1. Cost of the 3-phase induction motor (squirrel cage induction motor) is less compared to other motors of same power rating.
2. Squirrel cage induction motor has rugged & strong mechanical construction compared to other motors.
3. 3-phase induction motors are used in any environment like ~~ex.~~ explosive environment also.
4. 3-phase induction motor has high efficiency compared to other motors.
5. 3-phase induction motor need low maintenance or less running cost.
6. 3-phase induction motor have wide range of applications.

Disadvantages:-

1. Smooth speed control is not possible in 3-phase induction motor
2. 3-phase induction motor has less starting torque as compared to DC motor.

power stages in 3-phase induction motor:-

Let P_1 be the power input to the stator winding.

P_1 = input power to the motor

$$P_1 = \sqrt{3} V_L I_L \cos \phi$$

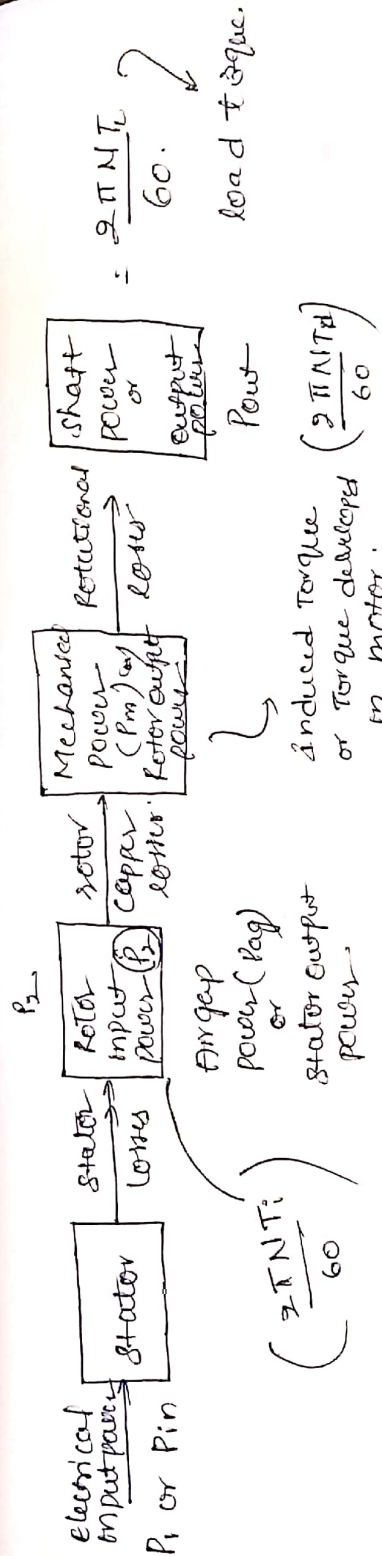
V_L = input voltage applied to stator winding or source voltage

I_L = current flowing thru stator winding or source current.

$\cos \phi$ = input power factor.

Stator winding has iron loss and copper loss, they both collectively called stator loss.

- input - stator loss = Stator output power or
- = Rotor input power
- = air gap power.



* Rotor has only copper losses

Initial torque > Induced torque > load torque

$$T_i > T_{ind} > T_L$$

Torque in rotor

Torque

at mechanical power

Torque at load

T_{ind} = induced torque or developed torque

T_L = load torque

T_i = initial torque

Slip speed:-

The difference between the ^{synchronous} speed and rotor speed is nothing but slip speed.

$$\therefore \text{Slip speed} = s = N_s - N_r$$

$$\% s = \frac{N_s - N_r}{N_s} \times 100$$

% s is called slip.

N_s = Synchronous speed

N_r = rotor speed or motor speed.

* When the 3-phase induction motor is not in running condition ($N_r = 0$) motor speed is 0.
 \hookrightarrow stand still condition.

hence the value of slip is

$$\% s = \frac{N_s - 0}{N_s} \times 100$$

$$s = 1 \text{ or } 100\%$$

* When the 3-phase induction motor rotates with synchronous speed ($N_r = N_s$),

then the value of slip is

$$\% s = \frac{N_s - N_s}{N_s} \times 100$$

$$s = 0 \text{ or } 0\%$$

\therefore slip always lies b/w 0 and 1

$$* 0 \leq s \leq 1$$

parameters affected by slip:-

(rotor parameters)

1. Voltage.
2. Current
3. reactance
4. frequency.

www.Jntufastupdates.com

$$1) * E_{2r} = s E_2$$

E_2 - rotor induced emf at stand still condition.

E_{2r} = rotor induced emf at running condition.

3) \therefore Rotor impedance at stand still condition = Z_2

$$Z_2 = R_2 + jX_2 \quad \left| \begin{array}{l} R_2 \\ X_2 \end{array} \right.$$

$$|Z_2| = \sqrt{R_2^2 + X_2^2}$$

Z_{2r} = Rotor impedance at running condition.

$$Z_{2r} = R_2 + jX_{2r} \quad \left| \begin{array}{l} R_2 \\ X_{2r} = sX_2 \end{array} \right.$$

$$Z_{2r} = R_2 + j s X_2$$

$$X_{2r} = s X_2$$

$$|Z_{2r}| = \sqrt{R_2^2 + (sX_2)^2}$$

2) I_2 = Rotor current at stand still condition

$$I = \frac{\text{voltage}}{\text{impedance}}$$

$$\therefore I_2 = \frac{E_2}{Z_2}$$

$$I_{2s} = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

I_{2r} = Rotor current at running condition

$$I_{2r} = \frac{E_{2r}}{Z_{2r}}$$

$$I_{2r} = \frac{s E_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

* power factor = $\cos \phi$

$$\cos \phi_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

power factor at running condition

pf at stand still condition

4) f_1 = supply frequency or stator frequency
 f_2 = Rotor frequency

At stand still condition

$$f_2 = f_1$$

At running condition

f_{2r} = Rotor frequency under running condition

$$f_{2r} = s f_1$$

$$* N_s = \frac{120f}{p}$$

$$N_s - N_r = \frac{120f'}{p}$$

$$\frac{f'}{f} = \frac{N_s - N_r}{N_s} = s$$

$$f' = s f$$

$f_2 = f_{2r}$; $f = f_1$

$$E_2 \propto N_s$$

$$E_{2r} \propto N_s - N_r$$

$$\frac{E_{2r}}{E_2} = \frac{N_s - N_r}{N_s}$$

$$E_{2r} = s E_2$$

Differences b/w squirrel cage and slip ring induction motor.

<u>Cage Rotor</u>	<u>Wound Rotor</u>
1. The rotor of the motor is squirrel cage type	1. The rotor of the motor is constructed as a slip ring type.
2. Cage rotor is simple design.	2. phase wound rotor is complicated
3. The rotor bar is permanently shorted at the end of the ring	3. Terminals are extended from rotor.
4. The rotor resistance starter is not used.	4. The rotor resistance starter can be used
5. Starting torque is low	5. Starting torque is high.
6. Less maintenance and cheap	6. Sliprings and brush assembly is there. Frequent maintenance is required
7. used in lathe machines, fan, blowers	7. use in hoist, cranes, elevator

power stages in 3-phase induction motor:-

Let us consider, P_{in} = input power to stator

R_1 = Resistance of stator winding.

R_2 = Resistance of rotor winding.

I_L = Current flowing through stator winding.

Stator copper loss = $3 I_L^2 R_1$

Stator losses = Stator iron loss + Stator copper loss

Air gap power P_{ag} = $P_{in} - \left[\begin{matrix} \text{stator iron loss} \\ + 3 I_L^2 R_1 \end{matrix} \right]$

Stator output power or Rotor input power

I_r = Current flowing through rotor winding.

rotor copper loss = $3 I_r^2 R_2$

mechanical power (P_m) = Rotor input power - $3 I_r^2 R_2$

output power or shaft power = mechanical power - Rotational losses

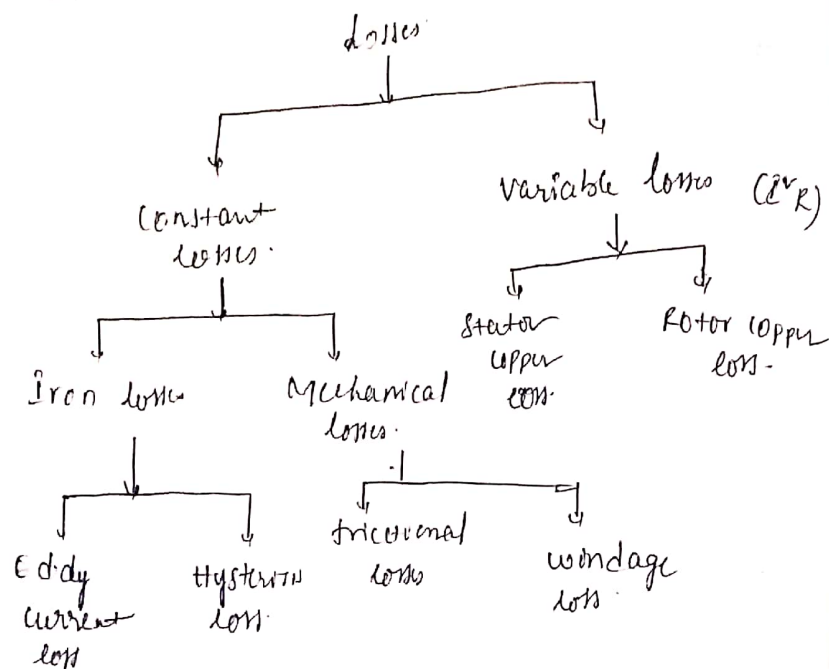
P_{out} = $P_m - \left[\begin{matrix} \text{friction and} \\ \text{windage loss} \end{matrix} \right]$

load torque =

Efficiency of 3-phase induction motor:

$$\eta = \frac{\text{shaft output power}}{\text{stator input power}} \times 100.$$

Losses in 3- ϕ induction motor:-



Relation between Rotor input power, Rotor copper loss and mechanical power:-

$$P_2 = \text{Rotor input power} = \frac{2\pi N_s T}{60} \quad \text{--- (1)}$$

$$P_m = \text{mechanical power} = \frac{2\pi N_r T}{60}$$

$$P_2 = P_m + \text{Rotor copper loss}$$

$$P_2 - P_m = \frac{2\pi N_s T}{60} - \frac{2\pi N_r T}{60}$$

$$P_2 - P_m = \frac{2\pi T}{60} [N_s - N_r] \quad \text{--- (2)}$$

$$\text{Rotor copper loss} = \frac{2\pi T}{60} [N_s - N_r]$$

$$\frac{\text{Rotor copper loss}}{P_2} = \frac{\frac{2\pi T}{60} [N_s - N_r]}{\frac{2\pi N_s T}{60}}$$

$$\frac{\text{Rotor copper loss}}{P_2} = \frac{N_s - N_r}{N_s}$$

$$\boxed{\text{Rotor copper loss} = s \times P_2} \quad \text{--- (3)}$$

from (1) $P_2 = P_m + sP_2$

$$P_m = P_2 - sP_2$$

$$\boxed{P_m = (1-s) P_2}$$

1) The power input to 3- ϕ induction motor, 60 kW. The total stator loss is 1 kW. Find the rotor copper loss per phase if the motor is running with a slip of 3%.

$$\text{Stator input power} = P_{in} = 60 \text{ kW}$$

$$\text{Total stator losses} = 1 \text{ kW}$$

$$s = 3\% = 0.03$$

$$\text{Rotor input power} = \text{Stator input} - \text{Total stator loss}$$

$$= 60 - 1$$

$$P_2 = 59 \text{ kW}$$

$$\text{Rotor copper loss} = s \times P_2$$

$$\text{per 3-}\phi = 0.03 \times 59 \text{ kW}$$

$$= 1.770 \text{ W}$$

$$\therefore \text{Rotor copper loss per phase} = \frac{1.770}{3}$$

$$= 590 \text{ W}$$

∴

2) A 15 kW, 400 V, 3- ϕ , 6 pole 50 Hz induction motor runs at a speed of 970 rpm at full load. The total mechanical loss is 520 W. Calculate

i) Rotor copper loss

ii) efficiency of the motor at full load if the stator loss is 750 W.

$$\text{Shaft power } P_{out} = 15 \text{ kW}$$

$$\text{line voltage } V_L = 400 \text{ V}$$

$$p = 6$$

$$\text{frequency} = 50 \text{ Hz}$$

$$\text{Mechanical loss} = 520 \text{ W}$$

$$\text{Stator loss} = 750 \text{ W}$$

$$N_r = 970 \text{ rpm}$$

$$N_s = \frac{120f}{p} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$s = \frac{N_s - N_r}{N_s} = \frac{1000 - 970}{1000}$$

$$= 0.03$$

$$P_{\text{mechanical power}} P_m = P_{out} + \text{Mechanical loss}$$

$$= 15 \text{ kW} + 0.52$$

$$P_m = 15.52 \text{ kW} \quad 32$$

$$P_m = (1-s) P_2$$

$$P_2 = \frac{P_m}{1-s} = \frac{15.52}{0.03}$$

$$P_2 = 517.33 \text{ kW} \approx 16 \text{ kW}$$

Rotor cu loss

$$\text{Rotor copper loss} = s \times P_2$$

$$= 0.03 \times 517.33 \text{ kW}$$

$$\text{Rotor cu loss} = 0.48 \text{ kW}$$

$$\text{Rotor copper loss} = 480 \text{ W}$$

Loss

$$\text{input power } P_m = P_2 + \text{stator loss}$$

$$= 16 + 0.75$$

$$P_m = 16.75 \text{ kW}$$

$$\text{Efficiency } \eta = \frac{P_{out}}{P_m} \times 100$$

$$= \frac{15.52}{16.75} \times 100 = 92.65\%$$

$$\eta = 92.65\%$$

$$= 89.5\%$$

4

3) The rotor emf of a 3 phase, 6 pole, 400V 50 Hz induction motor alternates at 3 Hz. Find

i, speed of motor

ii, slip

iii, Rotor copper losses per phase if the power input to the motor is 111.9 kW.

no. of poles, $p = 6$

line voltage $V_t = 400 \text{ V}$

stator frequency = 50 Hz

rotor frequency = 3 Hz

$$P_2 = 111.9 \text{ kW}$$

$$N_s = \frac{120f}{p}$$

$$= \frac{120 \times 50}{6}$$

$$N_s = 1000 \text{ rpm}$$

rotor frequency = $s \times$

supply frequency

$$f_r = s \times f_s$$

$$s = \frac{f_r}{f_s} = \frac{3}{50}$$

slip, $s = 0.06$

$$s = \frac{N_s - N_r}{N_s}$$

$$N_r = (1-s) N_s$$

$$i) N_r = 940 \text{ rpm}$$

$$ii) s = 0.06$$

iii) Rotor copper loss

$$= s \times \text{Rotor input power}$$

$$= 6.714 \text{ kW}$$

rotor copper loss

$$\text{per phase} = \frac{6.714}{3}$$

$$= 2.238 \text{ kW}$$

4.

4) A 3- ϕ , 6 pole, 400V, 50Hz induction motor takes a line current of 40A at 0.8 power factor and runs at 950 rpm.

Find
 i) Shaft power
 ii) efficiency, if the frictional & stator losses are 4kW & 3kW.

NO of pole $p = 6$
 $V_t = 400V$
 $f = 50Hz$
 $I_L = 40A$
 $N_r = 950rpm$
 $pf = 0.8$

frictional loss = 4kW
 Stator loss = 3kW

Power input to stator
 $P_m = \sqrt{3} V_L I_L \cos\phi$
 $= \sqrt{3} \times 400 \times 40 \times 0.8$
 $P_m = 22.17 \text{ kW}$

Rotor input power
 $P_2 = P_m - \text{stator loss}$
 $= 22.17 - 3$
 $P_2 = 19.17 \text{ kW}$

$$N_s = \frac{120 f}{p} = \frac{120 \times 50}{6}$$

$$N_s = 1000 \text{ rpm}$$

$$s = \frac{N_s - N_r}{N_s} = 0.05$$

$$P_m = (1-s) P_2$$

 $P_m = 18.21 \text{ kW}$

$$P_{out} = \text{Shaft power} - \text{frictional loss}$$

$$P_{out} = 18.21 - 4$$

$$\text{Shaft power} = 14.21 \text{ kW}$$

$$\eta = \frac{P_{out}}{P_m} \times 100$$

$$\eta = 64.1\%$$

5) A 3- ϕ induction motor has an output power of 10 kW at 1450 rpm. The mechanical & stator losses are 700W & 900W.

Calculate
 i) input power
 ii) efficiency
 iii) line current, the supply voltage is 440V, supply frequency = 50Hz, pf = 0.72
 It has 4 poles.

$P_{out} = 10 \text{ kW}$
 $N_r = 1450 \text{ rpm}$
 Mechanical loss = 0.7 kW
 Stator loss = 0.9 kW
 $V_t = 440V$
 $f = 50Hz$, $pf = 0.72$
 $p = 4$

$$N_s = \frac{120 f}{p} = \frac{120 \times 50}{4}$$

$$N_s = 1500 \text{ rpm}$$

$$\text{Slip } s = \frac{N_s - N_r}{N_s} = \frac{1500 - 1450}{1500}$$

$$s = 0.033$$

$P_m = \text{mechanical power}$
 $P_2 = \text{Rotor input power}$

$$P_m = (1-s) P_2$$

$$P_m = (1-0.033) P_2$$

$$P_m = P_{out} + \text{Mechanical loss}$$

$$P_m = 10 + 0.7 = 10.7 \text{ kW}$$

$$\therefore P_2 = \frac{10.7}{1-0.033}$$

$$P_2 = 11.06 \text{ kW}$$

R ϕ

$$\text{Input power} = P_m$$

$$P_m = P_2 + \text{Stator losses}$$

$$= 11.06 \text{ kW} + 0.9 \text{ kW}$$

$$P_m = 11.96 \text{ kW}$$

$$\therefore \text{i), Input power} = 11.96 \text{ kW}$$

$$\text{ii), efficiency} = \eta$$

$$\eta = \frac{P_{out}}{P_m} \times 100$$

$$= \frac{10}{11.96} \times 100$$

$$\eta = 83.57\%$$

$$\text{iii) Input power} = P_m$$

$$P_m = \sqrt{3} V_L I_L \cos \phi$$

$$I_L = \frac{P_m}{\sqrt{3} V_L \cos \phi}$$

$$= \frac{11.96 \times 10^3}{\sqrt{3} \times 440 \times 0.72}$$

$$I_L = 21.79 \text{ A}$$

$$\text{Line current} = I_L$$

$$I_L = 21.79 \text{ A}$$

6) A 20 hp, 4 pole, 50 Hz, 3- ϕ induction motor has friction and windage loss 2% of output. The full load slip is 3%.

Calculate

i) Rotor copper loss.

ii) Rotor input power.

$$1 \text{ hp} = 746 \text{ W}$$

$$P = 20 \times 4$$

$$f = 50 \text{ Hz}$$

$$\delta = 0.03$$

$$P_{out} = 20 \times 746$$

$$P_{out} = 14.92 \text{ kW}$$

$$\text{mechanical} = \frac{2 \times 14.92}{100}$$

$$\text{losses} = 0.2984 \text{ kW}$$

$$= 298.4 \text{ W}$$

$$= 0.2984 \text{ kW}$$

\therefore mechanical

$$P_m = P_{out} + 0.2984$$

$$P_m = 15.218 \text{ kW}$$

$$P_m = 15.218 \text{ kW}$$

$$P_m = (1 - \delta) P_2$$

$$P_2 = \frac{P_m}{1 - \delta}$$

$$P_2 = 15.68 \text{ kW}$$

$$\text{ii) Rotor input power}$$

$$P_2 = 15.68$$

$$\text{Rotor copper loss}$$

$$= S \times P_2$$

$$= 0.03 \times 15.68$$

$$= 0.47 \text{ kW}$$

$$= 0.47 \text{ kW}$$

$$\frac{2 \times 14.92}{100} = 0.2984$$

$$= 298.4 \text{ W}$$

$$= 0.2984 \text{ kW}$$

$$I_r = s I$$

$$I_2 = \frac{E_2}{\sqrt{R_2^2 + X_2^2}}$$

$$E_{2r} = s E_2$$

$$Z_{2r} = \sqrt{R_2^2 + (sX_2)^2}$$

$$I_{2r} = \frac{E_{2r}}{Z_{2r}}$$

$$\cos \phi_2 = \frac{R_2}{Z_2}$$

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}}$$

$$N_s = \frac{120 f}{P}$$

$$N_r = N_s(1-s)$$

$$\frac{N_s - N_r}{N_s} = s$$

$$N_s - N_r = s N_s$$

$$N_r = (1-s) N_s$$

$$N_s = \frac{N_r}{(1-s)}$$

7) A 3-phase induction motor has 4 poles and supplied from 50 Hz frequency. Calculate

- i) Synchronous speed.
- ii) Speed of the motor when slip is 4%.
- iii) Rotor frequency when rotor ~~rotor~~ rotates at 600 rpm

$$\text{poles} = p = 4$$

$$f = 50 \text{ Hz}$$

$$s = 4\% = 0.04$$

$$i) N_s = \frac{120 f}{P}$$

$$N_s = 1500$$

$$ii) N_r = N_s(1-s)$$

$$N_r = 1500(1-0.04)$$

$$N_r = 1440$$

$$1440 \rightarrow 0.04$$

$$600 \rightarrow s'$$

$$s' \times 1440 = 600 \times 0.04$$

$$s' = \frac{600 \times 0.04}{1440}$$

$$s' = 0.016$$

$$I_r = f \times s'$$

$$I_r = 50 \times 0.016$$

$$= 0.83$$

$$N_r = N_s(1-s)$$

$$s = 1 - \frac{N_r}{N_s}$$

$$= 1 - \frac{600}{1500}$$

$$= 0.6$$

$$f = 0.6 \times 50$$

$$= 30 \text{ Hz}$$

$$s = 0.6$$

3) A 3- ϕ induction motor having an induced emf of 80V on rotor at stand still condition on open circuit. The rotor has a resistance & reactance ϕ per phase of 1Ω & 4Ω respectively. Calculate

- i) Rotor current per phase.
- ii) power factor, when slip rings are short circuited.
- iii) slip rings are connected to a star connected stator of 3Ω per phase.

$$E_2(l) = 80V$$

$$\left. \begin{aligned} R_2 &= 1\Omega \\ X_2 &= 4\Omega \end{aligned} \right\} \text{ per phase.}$$

$$E_2(\text{ph}) = \frac{E_2(l)}{\sqrt{3}} = \frac{80}{\sqrt{3}}$$

$$E_2(\text{ph}) = 46.18V$$

$$i_2 = \frac{E_2}{R_{2L}}$$

$$Z_2 = 1 + j4$$

$$Z_2 = \frac{46.18 \angle 0^\circ}{1 + j4}$$

$$= \frac{46.18 \angle 0^\circ}{4.123 \angle 75.9^\circ}$$

$$i_2 = (11.20 \angle -75.9^\circ) A$$

$$\cos \phi_2 = \frac{R_2}{Z_2}$$

$$= \frac{1}{4.123}$$

$$= 0.24 \text{ lag.}$$

$$i_1) R_{ext} = 3\Omega$$

$$R_2 = R_{ext} + 1 = 4\Omega$$

$$Z_2 = [4 + j4] \Omega = 5.66 \angle 45^\circ$$

$$I_2 = \frac{46.18}{4 + j4} = 8.16 \angle -45^\circ A$$

$$\cos \phi_2 = \frac{R_2}{Z_2}$$

$$= \frac{4}{5.66}$$

$$= 0.707 \text{ lag.}$$

9) The star connected rotor of an induction motor has a stand still impedance of $(0.4 + j4)\Omega$ per phase. And Rheostat impedance of $(6 + j2)\Omega$ per phase. The motor has an induced emf 50V at stand still condition

$$E_2(l) = 80V$$

$$z_2 = (0.4 + j4)\Omega$$

$$I_2 = \frac{E_2(ph)}{z_2}$$

$$E_2(ph) = \frac{E_2(l)}{\sqrt{3}} = 46.18V$$

$$I_2 = \frac{46.18}{z_2}$$

$$z_2 = 4.01 \angle 84.28^\circ \Omega$$

$$I_2 = \frac{46.18 \angle 0^\circ}{4.01 \angle 84.28^\circ}$$

$$I_2 = 11.51 \angle -84.28^\circ A$$

$$\cos \phi_2 = \frac{R_2}{z_2}$$

$$= \frac{0.4}{4.01}$$

$$\cos \phi_2 = 0.099 \text{ lag}$$

$$= 0.1 \text{ lag}$$

$$\text{iii } z_2 = (0.4 + j4)\Omega$$

$$z_{ext} = (6 + j2)\Omega$$

$$z_2' = (6.4 + j6)\Omega$$

$$z_2' = 8.72 \angle 43.15^\circ \Omega$$

$$\cos \phi_2' = \frac{6.4}{8.72}$$

$$= 0.72 \text{ lag}$$

$$I_2 = \frac{46.18}{z_2'}$$

$$= \frac{46.18 \angle 0^\circ}{8.72 \angle 43.15^\circ}$$

$$I_2 = 5.26 A \angle -43.15^\circ$$

10) The star connected rotor of an induction motor as a stand still impedance of $(0.4 + j4)\Omega$ ohms per phase. And Rheostat impedance of $(6 + j2)\Omega$ per phase. The motor has an induced emf of 80V at stand still between slip rings at stand still. Find

i) Rotor current

ii) Rotor power factor when slip rings are short circuited and motor running at 3% slip.

$$E_2(l) = 80V$$

$$z_2 = (0.4 + j4)\Omega$$

$$z_{ext} = (6 + j2)\Omega$$

$$s = 3\% \text{ or } 0.03$$

$$E_2(ph) = \frac{E_2(l)}{\sqrt{3}} = \frac{80}{\sqrt{3}}$$

$$E_2(ph) = 46.18$$

$$I_{2r} = \frac{s \times E_2}{\sqrt{R_2 + s^2(X_2^2)}}$$

$$= \frac{0.03 \times 46.18}{\sqrt{0.4 + (4 \times 0.03^2)}}$$

Here slip rings are short circuited we don't add z_{ext} to motor.

$$I_{2r} = 3.31 \text{ A}$$

$$\cos \phi_{2r} = \frac{R_2}{Z_{2r}}$$

$$= \frac{0.4}{(0.4 + (0.03 \times 4)j)}$$

$$\cos \phi_L = 0.95 \text{ lag.}$$

Case ii b-

Supply is connected to external impedance and I_{2r} & $\cos \phi_L$

$$Z_2' = (0.4 + 4j) + (6 + 2j) = (6.4 + 6j) \Omega / \text{ph}$$

$$I_{2r} = \frac{S \times E_2}{R_2 + (sX_2)j} = \frac{0.03 \times 46.18}{6.4 + (6 \times 0.03)j}$$

$$I_{2r} = 0.216 \text{ A}$$

$$\cos \phi_{2r} = \frac{R_L}{Z_{2r}} = \frac{6.4}{6.4 + (6 \times 0.03)j}$$

$$= 0.92 \text{ lag.}$$

ii) A 100 kW, 3.3 kV, 50 Hz, 3 phase star connected induction motor has synchronous speed of 500 rpm. The full load slip is 1.8%. and full load power factor is 0.85 lag. Stator copper losses and iron loss are 2400 W & 3500 W. Rotational loss is 1.2 kW. Find.

i) Find rotor copper loss.

ii) line current.

iii) full load efficiency.

$$P_{out} = 100 \text{ kW}$$

$$V_L = 3.3 \text{ kV}$$

$$f = 50 \text{ Hz}$$

$$\text{Total stator loss} = P_{iron} + P_{copper} = 2.4 \text{ kW} + 3.5 \text{ kW} = 5.9 \text{ kW}$$

$$N_s = 500 \text{ rpm}$$

$$\text{pf} = 0.85 \text{ lag.}$$

$$\text{Rotational loss} = 1.2 \text{ kW}$$

$$s = 1.8\% = 0.018$$

$$P_m = P_{out} + \text{rotational loss} =$$

$$= 100 \text{ kW} + 1.2 \text{ kW}$$

$$P_m = 101.2 \text{ kW}$$

$$P_2 = \frac{P_m}{1-s}$$

$$= \frac{101.2}{1-0.018}$$

$$= 103.05 \text{ k.w.}$$

$$P_{in} = P_2 + \text{stator loss}$$

$$= 103.05 + 5.91$$

$$= 108.95 \text{ k.w.}$$

$$P_r \text{ Rotor (w loss)}$$

$$= P_2 \times s$$

$$= 1.85 \text{ k.w.}$$

iii, line current

$$I_L = \frac{P_{in}}{\sqrt{3} V_L}$$

$$= \frac{108.95}{\sqrt{3} \times 3.3}$$

$$= 83.01 \text{ A}$$

$$P_m = \sqrt{3} V_L I_L \cos \phi$$

$$I_L = \frac{108.95}{\sqrt{3} \times 3.3 \times 0.88}$$

$$I = 22.415 \text{ A}$$

www.Jntufastupdates.com

iii, efficiency = η

$$\eta = \frac{P_{out}}{P_m} \times 100$$

$$= \frac{100 \text{ k.w.}}{108.95 \text{ k.w.}} \times 100$$

$$\eta = 91.78\%$$

Rotor (w loss)

$$= P_m \times s$$

$$= 1.85 \text{ k.w.}$$

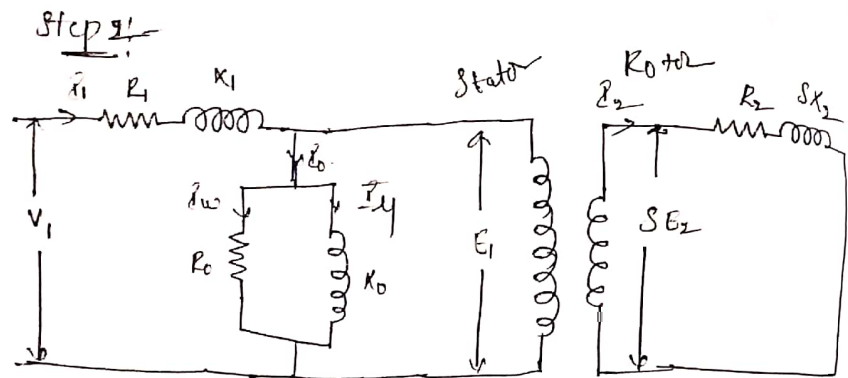
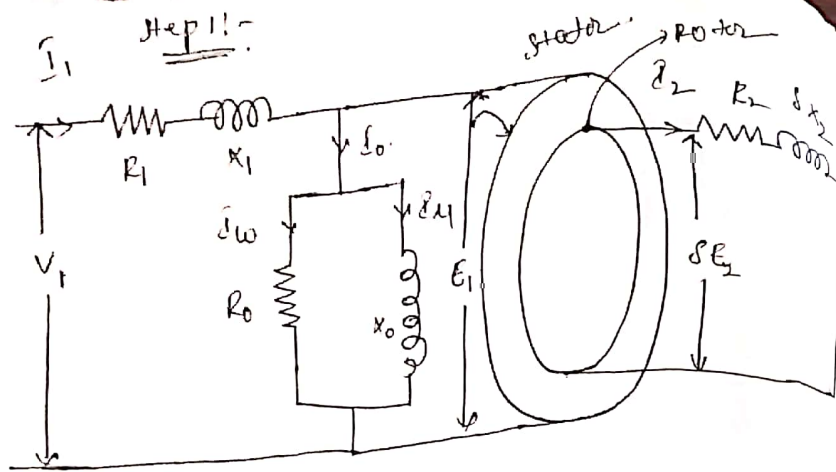
Equivalent circuit of 3-phase

induction motor:-

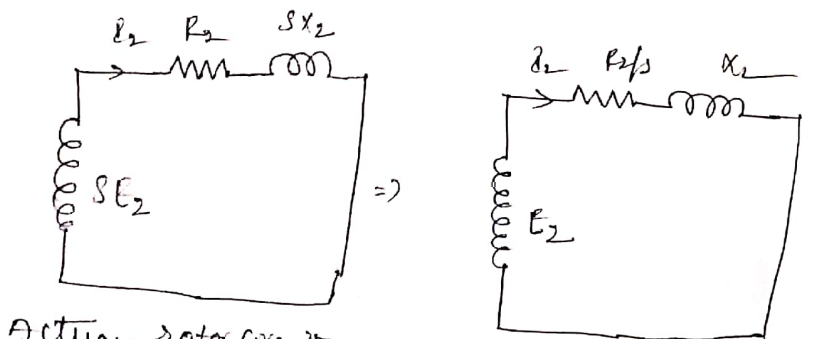
- let us consider,
- R_1 = Stator resistance.
 - R_2 = Rotor resistance.
 - X_1 = Stator reactance.
 - X_2 = Rotor reactance.
 - R_0 = shunt branch resistance
 - X_0 = shunt branch reactance
 - E_1 = induced emf in stator winding.
 - E_2 = induced emf in rotor winding.

* Equivalent circuit of 3-phase induction motor is similar to Equivalent circuit of Transformer

* 3-phase induction is also called as Rotating Transformer. Because of principle of 3- ϕ induction motor and transformer is same. i.e. mutual induction principle.



Step 3:- Modification of rotor circuit.



Actual rotor circuit

$$I_2 = \frac{SE_2}{\sqrt{R_2^2 + (X_2)^2}}$$

Equivalent rotor circuit

$$I_2 = \frac{E_2}{\sqrt{\left(\frac{R_2}{s}\right)^2 + X_2^2}}$$

$P_2 = \text{Rotor copper loss} + \text{mechanical power} (P_m)$

$$P_2 = I_2^2 R_2 + P_m$$

Rotor copper loss = $s \times P_2$

$$P_2 = \frac{\text{Rotor copper loss}}{s} = \frac{I_2^2 R_2}{s}$$

$P_m = P_2 - \text{Rotor copper loss}$

$$P_m = \text{Rotor copper loss} \left[\frac{1}{s} - 1 \right]$$

$$P_m = I_2^2 R_2 \left[\frac{1-s}{s} \right]$$

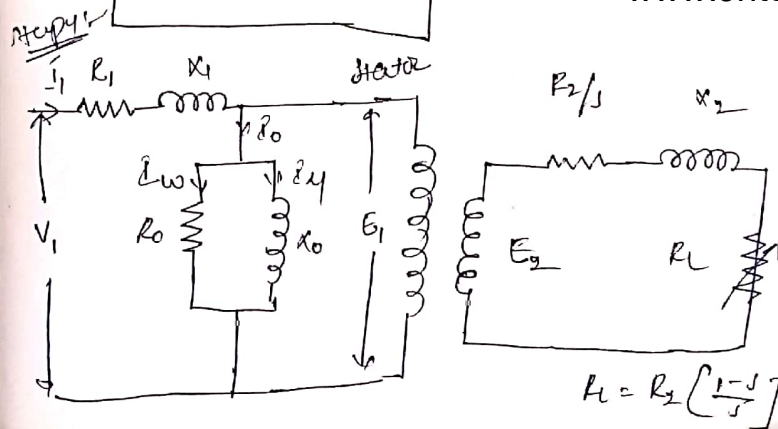
$$\frac{I_2^2 R_2}{s} = I_2^2 R_2 + I_2^2 R_2 \left[\frac{1-s}{s} \right]$$

$$\boxed{\frac{R_2}{s} = R_2 + R_L}$$

$R_L = \text{Equivalent mechanical load}$

$$\boxed{R_L = R_2 \left[\frac{1-s}{s} \right]}$$

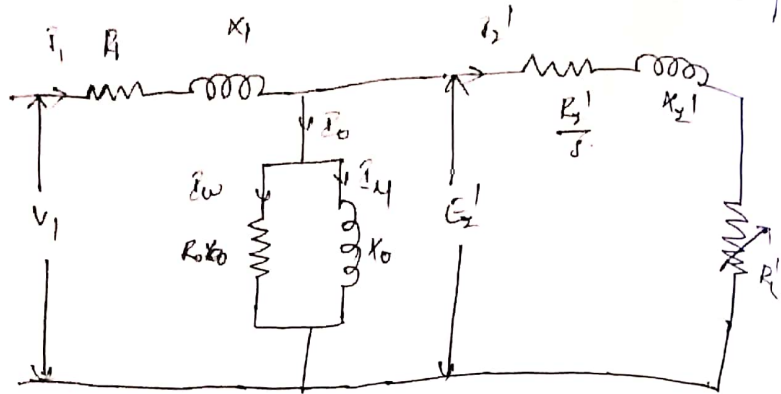
www.Jntufastupdates.com



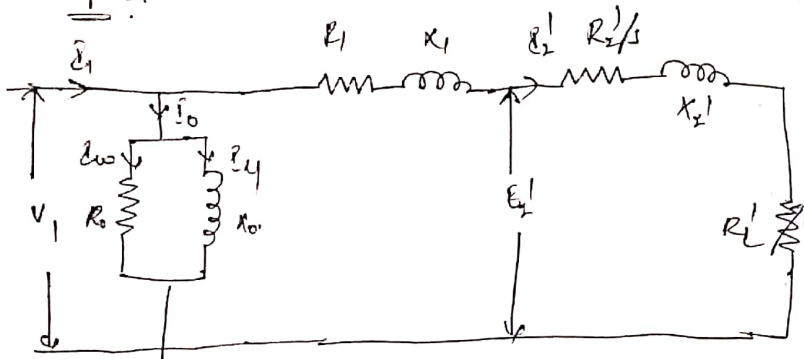
$$R_L = R_2 \left[\frac{1-s}{s} \right]$$

Step 5:- Transfer the rotor parameters to the stator side.

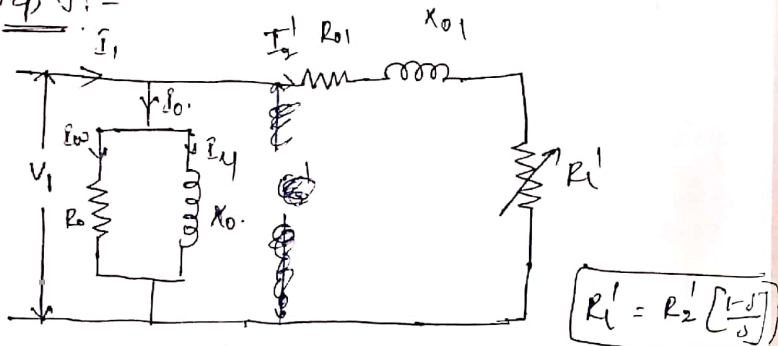
$$R_L' = R_2' \left[\frac{1-s}{s} \right]$$



Step 6:-



Step 7:-



$$R_L' = R_2' \left[\frac{1-s}{s} \right]$$

$$R_{01} = R_1 + \frac{R_2'}{s}$$

$$X_{01} = X_1 + X_2'$$

