25/7/17

UNIT - III

TIME RESPONSE ANALYSIS

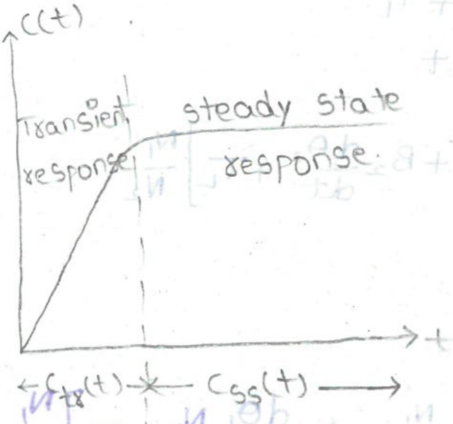
The time response of the system is the output of the system as a function of time.

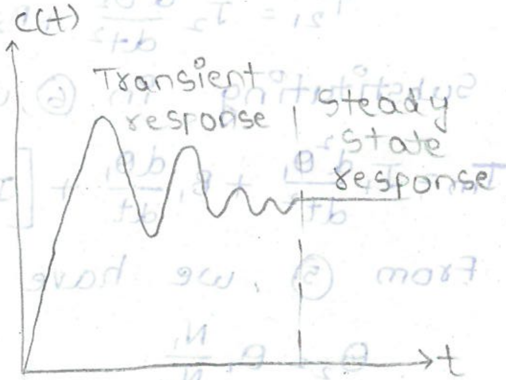
The time response of the system consists of two parts

1) Transient response
2) Steady state response

$\therefore$ Total time response = Transient Response time + Steady state Response time

$$\Rightarrow c(t) = c_{tr}(t) + c_{ss}(t)$$

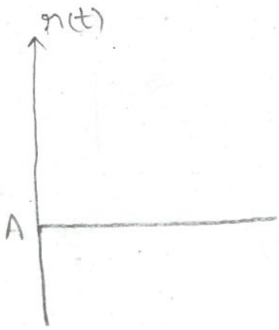




Standard Test signals

1) Step input

Step signal of size A is a signal that changes from 0 to A in '0' time



$$r(t) = A \; ; \; t \geq 0$$
$$\quad\;\; = 0 \; ; \; t < 0$$

$$R(s) = L[r(t)] = \frac{A}{s}$$

As $R(S) = \frac{1}{S}$

$$\Rightarrow C(S) = \frac{k}{S[\tau S+1]} = k\left[\frac{1}{S[\tau S+1]}\right]$$

By applying partial fractions for $\frac{1}{S[\tau S+1]}$ we get

$$\frac{1}{S[\tau S+1]} = \frac{A}{S} + \frac{B}{\tau S+1}$$

$$\Rightarrow 1 = A[\tau S+1] + BS$$

To get A put $S=0$

$$1 = A[0+1] + 0$$

$$\boxed{A=1}$$

To get B put $S = \frac{-1}{\tau}$

$$1 = A\left[\tau\left(\frac{-1}{\tau}\right)+1\right] + B\left(\frac{-1}{\tau}\right)$$

$$\boxed{B=-\tau}$$

$$\therefore C(S) = k\left[\frac{1}{S[\tau S+1]}\right] = k\left[\frac{A}{S} + \frac{B}{\tau S+1}\right]$$

$$\Rightarrow C(S) = k\left[\frac{1}{S} - \frac{\tau}{\tau S+1}\right]$$

$$\Rightarrow C(S) = k\left[\frac{1}{S} - \frac{1}{S+\frac{1}{\tau}}\right]$$

By applying Inverse Laplace Transformation, we get

$$C(t) = L^{-1}[C(S)] = k\left\{L^{-1}\left[\frac{1}{S}\right] - L^{-1}\left[\frac{1}{S+\frac{1}{\tau}}\right]\right\}$$

$$\therefore \boxed{C(t) = k\left[1-e^{-t/\tau}\right]}$$

At $t=0$; $C(0) = k[1-e^{0}] = 0$

At $t=\infty$; $C(\infty) = k[1-e^{-\infty}] = k$

increase exponentially ~~when t→∞~~ as $t \to \infty$.

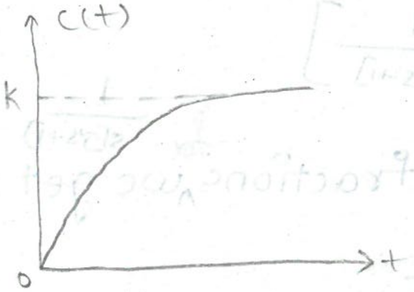
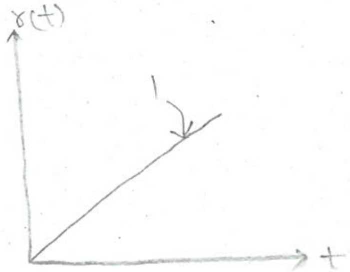


fig: step Response

28/7/17

Unit Ramp Response of First Order System

Considering Unit Ramp System.



$r(t) = t$ for $t \geq 0$

$\quad\quad 0$ for $t < 0$

$R(S) = L[r(t)] = \frac{1}{S^2}$

The transfer function of first order system is

$$\frac{C(S)}{R(S)} = \frac{k}{\tau S+1}$$

$$\Rightarrow C(S) = \frac{k}{\tau S+1} R(S)$$

$$\Rightarrow C(S) = \frac{k}{S^2[\tau S+1]} = K\left[\frac{1}{S^2[\tau S+1]}\right]$$

By applying partial fractions for $\frac{1}{S^2[\tau S+1]}$ we get

$$\frac{1}{S^2[\tau S+1]} = \frac{A}{S} + \frac{B}{S^2} + \frac{C}{\tau S+1}$$

$$\Rightarrow 1 = AS(\tau S+1) + B(\tau S+1) + CS^2 \quad \text{--- (1)}$$

Put $S=0 \Rightarrow 1 = 0+B+0 \Rightarrow \boxed{B=1}$

Put $S = \frac{-1}{\tau} \Rightarrow 1 = 0+0+C\left(\frac{-1}{\tau}\right)^2 \Rightarrow \boxed{C=\tau^2}$

Comparing s^2 terms

$$A\tau + C = 0$$

$$\Rightarrow A\tau + \tau^2 = 0$$

$$\Rightarrow A\tau = -\tau^2$$

$$\Rightarrow \boxed{A = -\tau}$$

$$\frac{k}{s^2[\tau s+1]} = k\left[\frac{A}{s} + \frac{B}{s^2} + \frac{C}{\tau s+1}\right]$$

$$C(s) = k\left[\frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau^2}{\tau s+1}\right]$$

$$\Rightarrow C(s) = k\left[\frac{-\tau}{s} + \frac{1}{s^2} + \frac{\tau}{s+\frac{1}{\tau}}\right]$$

Applying Inverse Laplace Transformation, we get

$$c(t) = k\left\{-\tau + t + \tau e^{-t/\tau}\right\}$$

$$\left[\begin{array}{l} \because L^{-1}\left[\frac{1}{s}\right] = 1 \\ L^{-1}\left[\frac{1}{s^2}\right] = t \\ L^{-1}\left[\frac{1}{s+a}\right] = e^{-at} \end{array}\right]$$

At $t=0$; $c(0) = k\{-\tau + 0 + \tau\} = 0$

At $t=\infty$; $c(\infty) = k\{-\tau + \infty + 0\} = \infty$

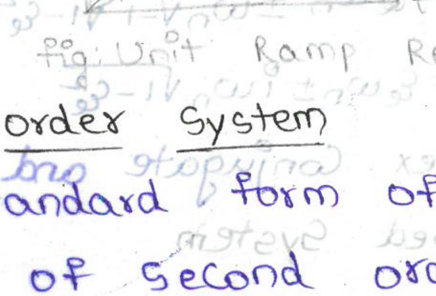


fig: Unit Ramp Response

Second order System

The standard form of closed loop transfer function of second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ξ_e = damping ratio

Case-i :- $\xi_e = 0$; Undamped System

Case-ii :- $0 < \xi_e < 1$; Underdamped System

Case-iii :- $\xi_e = 1$; Critical damped System

Case-iv :- $\xi_e > 1$; over damped System

The characteristic equation of second order System is

$$s^2 + 2\xi_e\omega_n s + \omega_n^2 = 0$$

$$\therefore s_1, s_2 = \frac{-2\xi_e\omega_n \pm \sqrt{(2\xi_e\omega_n)^2 - 4\omega_n^2}}{2}$$

$$s_1, s_2 = \frac{-2\xi_e\omega_n \pm 2\omega_n\sqrt{\xi_e^2 - 1}}{2}$$

$$s_1, s_2 = -\xi_e\omega_n \pm \omega_n\sqrt{\xi_e^2 - 1}$$

Case-i :- Undamped System

i.e., $\xi_e = 0 \Rightarrow s_1, s_2 = 0 \pm \omega_n\sqrt{-1} = \pm i\omega_n$

The roots are imaginary and unequal

Case-ii :- Underdamped System

i.e., $0 < \xi_e < 1 \Rightarrow s_1, s_2 = -\xi_e\omega_n \pm \omega_n\sqrt{(-1)(1-\xi_e^2)}$

$$= -\xi_e\omega_n \pm \omega_n\sqrt{-1}\sqrt{1-\xi_e^2}$$

$$= -\xi_e\omega_n \pm i\omega_n\sqrt{1-\xi_e^2}$$

The roots are Complex Conjugate and unequal.

Case-iii :- Critical damped System

i.e., $\xi_e = 1 \Rightarrow s_1, s_2 = -\omega_n \pm \omega_n\sqrt{1-1}$

$$= -\omega_n$$

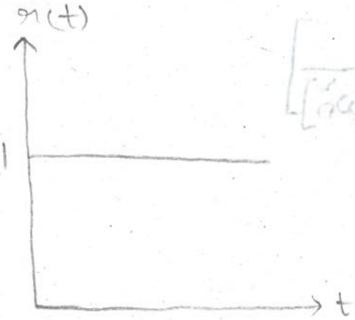
Case-iv :- Over damped System

i.e, $\xi > 1 \Rightarrow s_1, s_2 = -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1}$

The roots are real and unequal.

31/7/17

Response of Undamped Second order system for Unit step



$r(t) = 1$ for $t \geq 0$

0 for $t < 0$

$R(s) = L[r(t)] = \frac{1}{s}$

The transfer function of second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

For undamped System $\xi = 0$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2+\omega_n^2}$$

$$C(s) = R(s)\left[\frac{\omega_n^2}{s^2+\omega_n^2}\right]$$

$$C(s) = \frac{\omega_n^2}{s[s^2+\omega_n^2]}$$

From partial fractions, we have

$$\frac{1}{s[s^2+\omega_n^2]} = \frac{A}{s} + \frac{B}{s^2+\omega_n^2}$$

$$1 = A[s^2+\omega_n^2] + Bs$$

$$1 = As^2 + Bs + A\omega_n^2$$

Put $s = 0$

Put $s^2 = -\omega_n^2 \Rightarrow s = j\omega_n$

$1 = A[0] + Bj\omega_n$

$$\Rightarrow \boxed{B = \frac{1}{j\omega_n}}$$

$$C(s) = \frac{\omega_n^2}{s[s^2+\omega_n^2]} = \omega_n^2\left[\frac{A}{s} + \frac{B}{s^2+\omega_n^2}\right]$$

$$\therefore C(s) = \omega_n^2\left[\frac{1}{\omega_n^2 s} + \frac{1}{j\omega_n[s^2+\omega_n^2]}\right]$$

$$C(s) = \left[\frac{1}{s} + \frac{\omega_n}{j[s^2+\omega_n^2]}\right]$$

$$C(s) = \left[\frac{1}{s} - \frac{j\omega_n}{s^2+\omega_n^2}\right]$$

$$C(s) = \left[\frac{1}{s} - \frac{s}{s^2+\omega_n^2}\right] \qquad [\because s = j\omega_n]$$

W.K.T

$$L[1] = \frac{1}{s} \Rightarrow L^{-1}\left[\frac{1}{s}\right] = 1$$

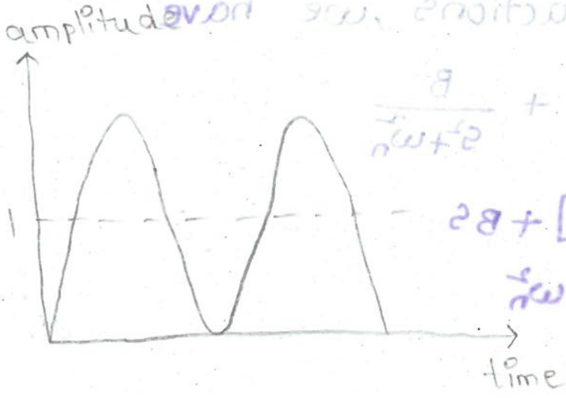
$$L[\cos at] = \frac{s}{s^2+a^2} \Rightarrow L^{-1}\left[\frac{s}{s^2+a^2}\right] = \cos at$$

$\therefore$ By applying inverse Laplace transformation, we get

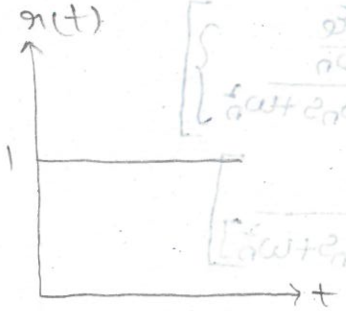
$$\boxed{c(t) = 1 - \cos\omega_n t}$$

at $t=0$; $c(t) = 1-1=0$

~~$t=0$, $c(t) = 1 - \cos 0 = \cos$ signal~~



Response of Underdamped second order System for unit step ~~unit~~:-



$r(t) = 1$ for $t \geq 0$

$\quad\;\; 0$ for $t < 0$

$R(S) = L[r(t)] = \frac{1}{S}$

The transfer function of second order system is

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$C(S) = R(S)\left[\frac{\omega_n^2}{S^2 + 2\xi\omega_n S + \omega_n^2}\right]$$

$$C(S) = \frac{\omega_n^2}{S[S^2 + 2\xi\omega_n S + \omega_n^2]}$$

From patial fractions, we have

$$\frac{1}{S[S^2 + 2\xi\omega_n S + \omega_n^2]} = \frac{A}{S} + \frac{BS + C}{S^2 + 2\xi\omega_n S + \omega_n^2}$$

$$\Rightarrow 1 = A[S^2 + 2\xi\omega_n S + \omega_n^2] + [BS + C]S$$

$$\Rightarrow 1 = AS^2 + A2\xi\omega_n S + A\omega_n^2 + BS^2 + CS$$

$$\Rightarrow 1 = S^2[A + B] + S[2A\xi\omega_n + C] + A\omega_n^2$$

Equating like terms, we get

$A\omega_n^2 = 1 \qquad A + B = 0 \qquad 2A\xi\omega_n + C = 0$

$A = \frac{1}{\omega_n^2} \qquad B = -A \qquad C = -2A\xi\omega_n$

$B = -\frac{1}{\omega_n^2} \qquad C = -2\frac{1}{\omega_n^2}\xi\omega_n$

$C = \frac{-2\xi}{\omega_n}$

$$C(s) = \omega_n^2 \left[\frac{A}{s} + \frac{Bs + C}{s^2 + 2\xi_e\omega_n s + \omega_n^2} \right]$$

$$C(s) = \omega_n^2 \left[\frac{1}{s\omega_n^2} - \left\{ \frac{\frac{s}{\omega_n^2} + \frac{2\xi_e}{\omega_n}}{s^2 + 2\xi_e\omega_n s + \omega_n^2} \right\} \right]$$

$$C(s) = \omega_n^2 \left[\frac{1}{s\omega_n^2} - \frac{s + 2\xi_e\omega_n}{\omega_n^2[s^2 + 2\xi_e\omega_n s + \omega_n^2]} \right]$$

$$C(s) = \frac{1}{s} - \frac{s + 2\xi_e\omega_n}{s^2 + 2\xi_e\omega_n s + \omega_n^2}$$

Let us add and subtract $\xi_e^2\omega_n^2$ to the denominator of second term.

$$C(s) = \frac{1}{s} - \frac{s + 2\xi_e\omega_n}{s^2 + 2\xi_e\omega_n s + \omega_n^2 + \xi_e^2\omega_n^2 - \xi_e^2\omega_n^2}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{s + 2\xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]}$$

$$\Rightarrow C(s) = \frac{1}{s} - \left\{ \frac{s + \xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} + \frac{\xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} \right\}$$

Multiplying and Dividing 3rd term with $\omega_n^2[1 - \xi_e^2]$

$$C(s) = \frac{1}{s} - \left\{ \frac{s + \xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} + \frac{\xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} \times \frac{\omega_n^2[1 - \xi_e^2]}{\omega_n^2[1 - \xi_e^2]} \right\}$$

$$C(s) = \frac{1}{s} - \left\{ \frac{s + \xi_e\omega_n}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} + \frac{\omega_n^2[1 - \xi_e^2]}{[s + \xi_e\omega_n]^2 + \omega_n^2[1 - \xi_e^2]} \times \frac{\xi_e\omega_n}{\omega_n^2[1 - \xi_e^2]} \right\}$$

W.K.T $L[1] = \frac{1}{s}$

$$L\left[e^{-at}\cos\omega t\right] = \frac{s + a}{[s + a]^2 + \omega^2}$$

$$L\left[e^{-at}\sin\omega t\right] = \frac{\omega}{[s + a]^2 + \omega^2}$$

$$c(s) = \frac{1}{s} - \left\{ \frac{s+\xi_e\omega_n}{(s+\xi_e\omega_n)^2+\omega_d^2} + \frac{\omega_d}{[s+\xi_e\omega_n]^2+\omega_d^2} \times \frac{\omega_d \xi_e \omega_n}{\omega_d^2} \right\}$$

Applying Inverse Laplace Transformation

$$c(t) = 1 - e^{-\xi_e\omega_n t}\cos\omega_d t - e^{-\xi_e\omega_n t}\sin\omega_d t \cdot \frac{\xi_e\omega_n}{\omega_d}$$

1/8/17

$$c(t) = 1 - \left\{ e^{-\xi_e\omega_n t}\cos\omega_d t + e^{-\xi_e\omega_n t}\sin\omega_d t \times \frac{\xi_e\omega_n}{\omega_d} \right\}$$

$$c(t) = 1 - e^{-\xi_e\omega_n t}\left\{ \cos\omega_d t + \sin\omega_d t \times \frac{\xi_e\omega_n}{\omega_d} \right\}$$

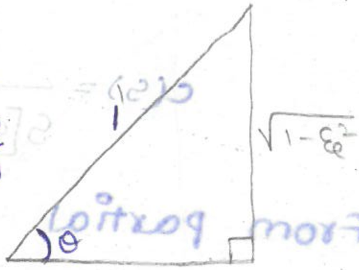
WKT $\omega_d^2 = \omega_n^2[1-\xi_e^2] \Rightarrow \omega_d = \omega_n\sqrt{1-\xi_e^2}$

$$\Rightarrow c(t) = 1 - e^{-\xi_e\omega_n t}\left\{ \cos\omega_d t + \sin\omega_d t \cdot \frac{\xi_e}{\sqrt{1-\xi_e^2}} \right\}$$

$$\Rightarrow c(t) = 1 - \frac{e^{-\xi_e\omega_n t}}{\sqrt{1-\xi_e^2}}\left\{ \sqrt{1-\xi_e^2}\cos\omega_d t + \xi_e\sin\omega_d t \right\}$$

Constructing a right angle triangle with ξ_e and $\sqrt{1-\xi_e^2}$

$$\Rightarrow c(t) = 1 - \frac{e^{-\xi_e\omega_n t}}{\sqrt{1-\xi_e^2}}\left\{ \sin\theta\cos\omega_d t + \cos\theta\sin\omega_d t \right\}$$



$$c(t) = 1 - \frac{e^{-\xi_e\omega_n t}}{\sqrt{1-\xi_e^2}}\sin[\omega_d t + \theta];$$

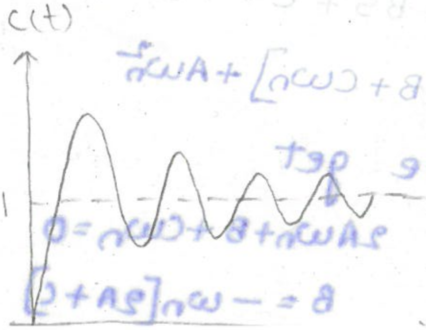
$\cos\theta = \xi_e$

$\sin\theta = \sqrt{1-\xi_e^2}$

$\tan\theta = \frac{\sqrt{1-\xi_e^2}}{\xi_e}$

where $\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$

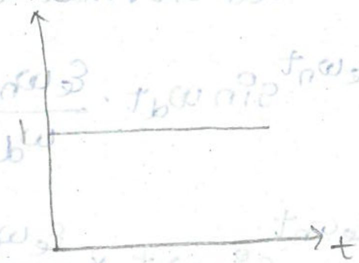
$\theta = \tan^{-1}\left\{\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right\}$



at $t=0$; $c(t)=0$

$t=\infty$; $c(t)=1$

Response of critically damped second order system for unit step



$r(t) = 1$ for $t \geq 0$

$\quad\;\; 0$ for $t < 0$

$R(s) = L[r(t)] = \frac{1}{s}$

The transfer function of second order system is

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

For critically damped system, $\xi = 1$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2}$$

$$C(s) = \frac{\omega_n^2}{s[s^2 + 2\omega_n s + \omega_n^2]}$$

$$C(s) = \frac{\omega_n^2}{s[s+\omega_n]^2}$$

From partial fractions, we have

$$\frac{1}{s[s+\omega_n]^2} = \frac{A}{s} + \frac{B}{[s+\omega_n]^2} + \frac{C}{s+\omega_n}$$

$$\Rightarrow 1 = A[s+\omega_n]^2 + Bs + C[s+\omega_n]s$$

$$\Rightarrow 1 = A[s^2 + 2s\omega_n + \omega_n^2] + Bs + Cs^2 + Cs\omega_n$$

$$\Rightarrow 1 = s^2[A+C] + s[2A\omega_n + B + C\omega_n] + A\omega_n^2$$

Equating like terms, we get

$A\omega_n^2 = 1 \qquad A + C = 0 \qquad 2A\omega_n + B + C\omega_n = 0$

$\qquad\qquad\quad C = -A \qquad B = -\omega_n[2A + C]$

$$B = \frac{-1}{\omega_n}.$$

$$C(S) = \frac{\omega_n^2}{S[S+\omega_n]^2} = \left[\frac{A}{S} + \frac{B}{(S+\omega_n)^2} + \frac{C}{S+\omega_n}\right]\omega_n^2.$$

$$\Rightarrow C(S) = \omega_n^2\left[\frac{1}{S\omega_n^2} - \frac{1}{\omega_n[S+\omega_n]^2} - \frac{1}{\omega_n^2[S+\omega_n]}\right]$$

$$\Rightarrow C(S) = \frac{1}{S} - \frac{\omega_n}{[S+\omega_n]^2} - \frac{1}{S+\omega_n}$$

WKT $L[1] = \frac{1}{S}$

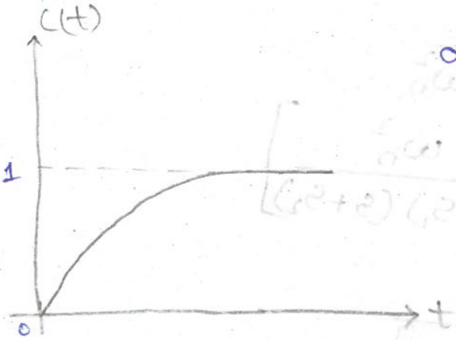
$$L[t.e^{-at}] = \frac{1}{(S+a)^2}$$

$$L[e^{-at}] = \frac{1}{S+a}$$

By applying Inverse Laplace Transformation, we get

$$C(t) = 1 - \omega_n t.e^{-\omega_n t} - e^{-\omega_n t}$$

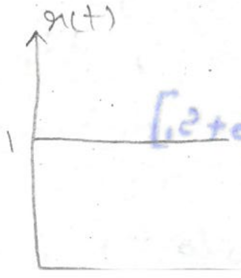
$$\Rightarrow C(t) = 1 - e^{-\omega_n t}[1+\omega_n t]$$



at $t=0$; $C(t) = 0$
$t=\infty$; $C(t) = 1$

2/8/17

Response of over damped second order system for unit step



$r(t) = 1$ for $t \geq 0$
0 for $t < 0$

$R[S] = L[r(t)] = \frac{1}{S}$

System is

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi_e\omega_n S+\omega_n^2}$$

The equation of second order system is

$$S^2+2\xi_e\omega_n S+\omega_n^2 = 0$$

$$S_a, S_b = \frac{-2\xi_e\omega_n \pm \sqrt{4\xi_e^2\omega_n^2-4\omega_n^2}}{2}$$

$$S_a, S_b = \frac{-2\xi_e\omega_n \pm 2\omega_n\sqrt{\xi_e^2-1}}{2}$$

$$S_a, S_b = -\xi_e\omega_n \pm \omega_n\sqrt{\xi_e^2-1}$$

$$S_a = -\xi_e\omega_n + \omega_n\sqrt{\xi_e^2-1}$$

$$S_b = -\xi_e\omega_n - \omega_n\sqrt{\xi_e^2-1}$$

Let $S_1 = -S_a$ & $S_2 = -S_b$

$$\Rightarrow S_1 = \xi_e\omega_n - \omega_n\sqrt{\xi_e^2-1}$$

$$S_2 = \xi_e\omega_n + \omega_n\sqrt{\xi_e^2-1}$$

$$\therefore \frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi_e\omega_n S+\omega_n^2}$$

$$\Rightarrow C(S) = R(S)\left[\frac{\omega_n^2}{(S+S_1)(S+S_2)}\right]$$

$$C(S) = \frac{\omega_n^2}{S[S+S_1][S+S_2]}$$

By applying partial fractions, we get

$$\frac{1}{S[S+S_1][S+S_2]} = \frac{A}{S} + \frac{B}{S+S_1} + \frac{C}{S+S_2}$$

$$\Rightarrow 1 = A[S+S_1][S+S_2] + BS[S+S_2] + CS[S+S_1]$$

Put $S=0 \Rightarrow 1 = AS_1S_2 \Rightarrow A = \frac{1}{S_1S_2}$

$$\Rightarrow B = \frac{-1}{s_1[s_2 - s_1]}$$

Put $s = -s_2 \Rightarrow 1 = C(-s_2)[-s_2 + s_1]$

$$\Rightarrow C = \frac{-1}{s_2[s_1 - s_2]}$$

$$\therefore C(s) = \omega_n^2 \left[\frac{1}{s[s+s_1][s+s_2]}\right] = \omega_n^2\left[\frac{A}{s} + \frac{B}{s+s_1} + \frac{C}{s+s_2}\right]$$

$$\Rightarrow C(s) = \frac{\omega_n^2}{s\, s_1 s_2} - \frac{\omega_n^2}{s_1[s_2 - s_1][s+s_1]} - \frac{\omega_n^2}{s_2[s_1-s_2][s+s_2]}$$

WKT $s_1 = \varepsilon_e \omega_n - \omega_n\sqrt{\varepsilon_e^2 - 1}$

$s_2 = \varepsilon_e \omega_n + \omega_n\sqrt{\varepsilon_e^2 - 1}$

$\therefore\ s_1 s_2 = \varepsilon_e^2 \omega_n^2 - \omega_n^2[\varepsilon_e^2 - 1]$

$\Rightarrow s_1 s_2 = \omega_n^2$

$s_2 - s_1 = 2\omega_n\sqrt{\varepsilon_e^2 - 1}$

$s_1 - s_2 = -2\omega_n\sqrt{\varepsilon_e^2 - 1}$

$$\therefore C(s) = \frac{\omega_n^2}{s\,\omega_n^2} - \frac{\omega_n^2}{s_1 2\omega_n\sqrt{\varepsilon_e^2-1}[s+s_1]} + \frac{\omega_n^2}{s_2 \cdot 2\omega_n\sqrt{\varepsilon_e^2-1}[s+s_2]}$$

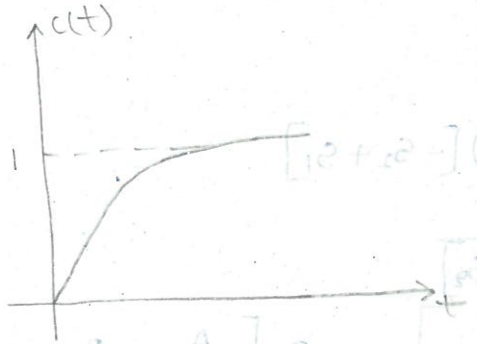
$$\Rightarrow C(s) = \frac{1}{s} - \frac{\omega_n}{2s_1\sqrt{\varepsilon_e^2-1}} \cdot \frac{1}{s+s_1} + \frac{\omega_n}{2s_2\sqrt{\varepsilon_e^2-1}} \cdot \frac{1}{s+s_2}$$

WKT $L[1] = \frac{1}{s}$ and $L[e^{-at}] = \frac{1}{s+a}$

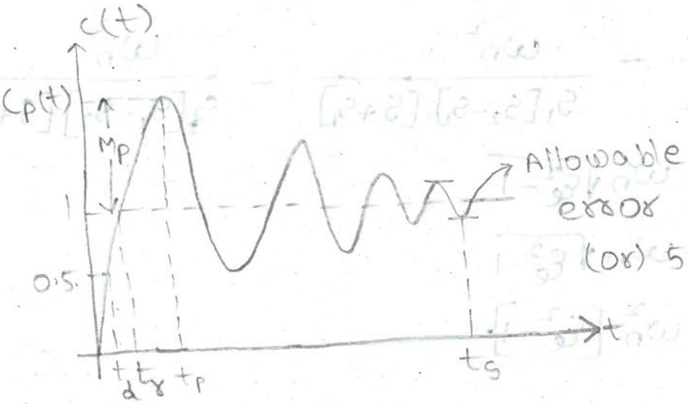
By applying Inverse Laplace Transformation, we get

$$c(t) = 1 - \frac{\omega_n}{2s_1\sqrt{\varepsilon_e^2-1}} e^{-s_1 t} + \frac{\omega_n}{2s_2\sqrt{\varepsilon_e^2-1}} e^{-s_2 t}$$

$$\Rightarrow c(t) = 1 - \frac{\omega_n}{2\sqrt{\varepsilon_e^2-1}}\left[\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right]$$



Time domain Specifications



Delay time (t_d)

It is the time taken for the response to reach 50% of the final value for the very first time.

Rise time (t_r)

For underdamped system it is the time taken for the response to rise from 0 to 100% for the very first time.

For overdamped system, 10% to 90%.

For critical damped system, 5% to 95%.

Peak time (t_p)

It is the time taken for the response to reach the peak value for the very first time.

Maximum Peak overshoot (M_p)

... ratio of maximum peak

Peak value is measured from final value.

Let $c(\infty)$ = final value of $c(t)$

$c(t_p)$ = maximum value of $c(t)$

$$\therefore M_p = \frac{c(t_p) - c(\infty)}{c(\infty)}$$

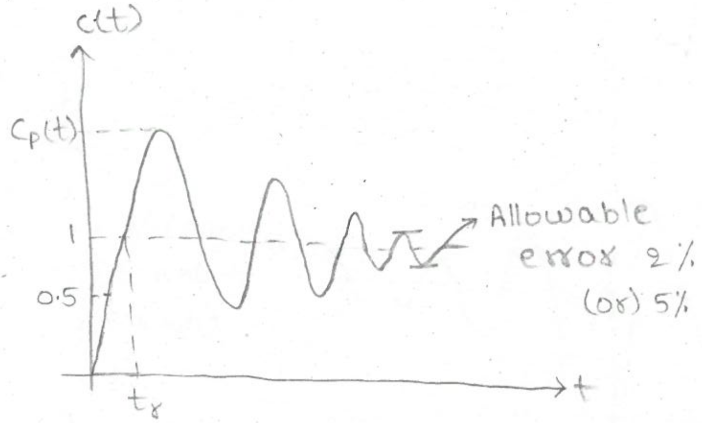
Settling time (t_s)

It is defined as time taken by the response to reach and state within a specified error. Usually the tollerable error is 2% (or) 5% of its final value.

3/8/17

Derivations of Time Domain Specifications:-

Rise time (t_r):-



The unit step response of Underdamped second order system is

$$c(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

At $t = t_r$; $c(t_r) = 1$

$$1 = 1 - \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

$$\Rightarrow \frac{e^{-\xi \omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

$\omega_d t_r + \theta = n\pi$

$$\boxed{t_r = \frac{n\pi - \theta}{\omega_d}}$$

For the first time $n=1$

$$\therefore t_r = \frac{\pi - \theta}{\omega_d}$$

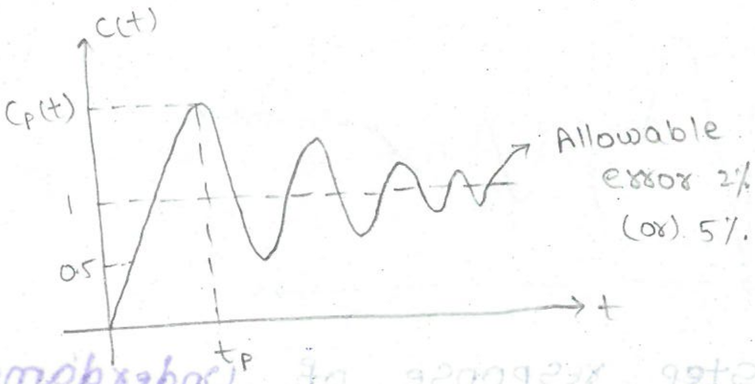
we know that

$$\theta = \tan^{-1}\left[\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right]$$

$$\omega_d = \omega_n\sqrt{1-\xi_e^2}$$

$\therefore$ Rise time $\boxed{t_r = \dfrac{\pi - \tan^{-1}\left(\dfrac{\sqrt{1-\xi_e^2}}{\xi_e}\right)}{\omega_n\sqrt{1-\xi_e^2}}}$

<u>Peak time (t_p)</u> :-



The unit step response of Underdamped second order System is

$$c(t) = 1 - \frac{e^{-\xi_e\omega_n t}}{\sqrt{1-\xi_e^2}}\sin(\omega_d t + \theta)$$

At $t = t_p$; $\frac{d}{dt}c(t) = 0$

Differentiating $c(t)$ w.r.t 't'

$$\Rightarrow \frac{d}{dt}c(t) = \frac{d}{dt}\left\{1 - \frac{e^{-\xi_e\omega_n t_p}}{\sqrt{1-\xi_e^2}}\sin(\omega_d t_p + \theta)\right\} = 0$$

$$\Rightarrow \frac{\xi_e \omega_n e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \sin(\omega_d t_p+\theta) - \frac{e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \cos(\omega_d t_p+\theta)\omega_d = 0$$

W.K.T $\quad \omega_d = \omega_n\sqrt{1-\xi_e^2}$

$$\Rightarrow \frac{\xi_e \omega_n e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \sin(\omega_d t_p+\theta) - \frac{e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \cos(\omega_d t_p+\theta)\omega_n\sqrt{1-\xi_e^2} = 0$$

$$\Rightarrow \frac{\omega_n e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \left[\xi_e \sin(\omega_d t_p+\theta) - \sqrt{1-\xi_e^2}\cos(\omega_d t_p+\theta)\right] = 0$$

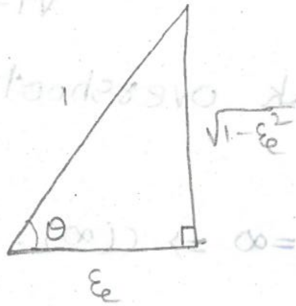
$$\xi_e \sin(\omega_d t_p+\theta) - \sqrt{1-\xi_e^2}\cos(\omega_d t_p+\theta) = 0$$

Constructing a right angle triangle with ξ_e and $\sqrt{1-\xi_e^2}$

$$\cos\theta = \xi_e$$

$$\sin\theta = \sqrt{1-\xi_e^2}$$

$$\tan\theta = \frac{\sqrt{1-\xi_e^2}}{\xi_e}$$



$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi_e^2}}{\xi_e}\right)$$

$$\Rightarrow \cos\theta \sin(\omega_d t_p+\theta) - \sin\theta\cos(\omega_d t_p+\theta) = 0$$

$$\Rightarrow \sin[\omega_d t_p + \theta - \theta] = 0$$

$$\sin \omega_d t_p = 0 = \sin n\pi$$

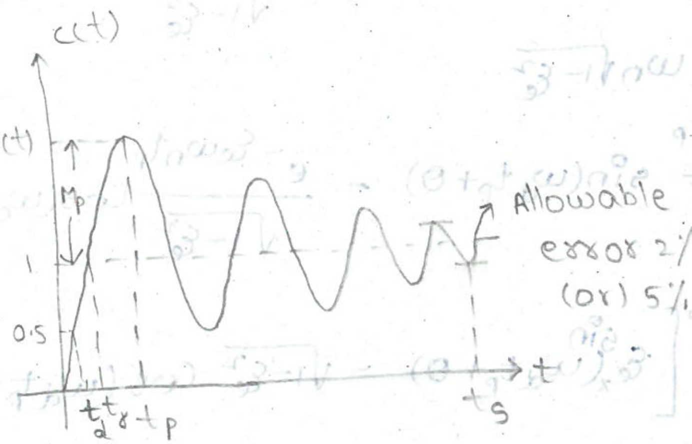
$$\omega_d t_p = n\pi$$

$$\boxed{t_p = \frac{n\pi}{\omega_d}}$$

for the first time $n=1$

$$\therefore \boxed{t_p = \frac{\pi}{\omega_d}}$$

Peak overshoot (M_p):-



The unit step response of Underdamped second order system is

$$C(t) = 1 - \frac{e^{-\xi_e \omega_n t}}{\sqrt{1-\xi_e^2}} \sin(\omega_d t + \theta)$$

$$\% \text{ Peak overshoot } (\% M_p) = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

At $t = \infty \Rightarrow c(\infty) = 1 - \frac{e^{-\infty}}{\sqrt{1-\xi_e^2}} \sin(\infty)$

$$\boxed{C(\infty) = 1}$$

At $t = t_p \Rightarrow c(t_p) = 1 - \frac{e^{-\xi_e \omega_n t_p}}{\sqrt{1-\xi_e^2}} \sin(\omega_d t_p + \theta)$

we know that $t_p = \frac{n\pi}{\omega_d}$

For the first time, $n=1$

$$t_p = \frac{\pi}{\omega_d}$$

$$c(t_p) = 1 - \frac{e^{-\xi_e \omega_n \frac{\pi}{\omega_d}}}{\sqrt{1-\xi_e^2}} \sin\left(\omega_d \frac{\pi}{\omega_d} + \theta\right)$$

W.K.T $\omega_d = \omega_n \sqrt{1-\xi_e^2}$

$$\Rightarrow c(t_p) = 1 - \frac{e^{-\xi_e \pi/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}}(-\sin\theta)$$

From right angle triangle, we have

$$\sin\theta = \sqrt{1-\xi_e^2}$$

$$c(t_p) = 1 + \frac{e^{-\xi_e \pi/\sqrt{1-\xi_e^2}}}{\sqrt{1-\xi_e^2}} \cdot \sqrt{1-\xi_e^2}$$

$$c(t_p) = 1 + e^{-\xi_e \pi/\sqrt{1-\xi_e^2}}$$

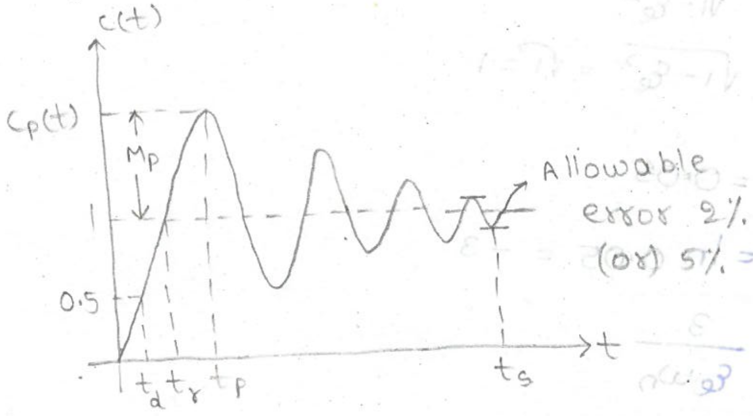
$$\therefore \quad \% M_p = \frac{c(t_p) - c(\infty)}{c(\infty)} \times 100$$

$$\% M_p = \frac{\cancel{1} + e^{-\xi_e \pi/\sqrt{1-\xi_e^2}} - \cancel{1}}{1} \times 100$$

$$\boxed{\% M_p = e^{-\xi_e \pi/\sqrt{1-\xi_e^2}} \times 100}$$

4/8/17

Settling time (t_s):-



The unit step response of underdamped second order system is

$$c(t) = 1 - \frac{e^{-\xi_e \omega_n t}}{\sqrt{1-\xi_e^2}} \sin(\omega_d t + \theta)$$

The response of second order system has two components.

2) Sinusoidal Component ($\sin(\omega_d t+\theta)$)

The decaying exponential term reduces the oscillations produced by sinusoidal term. Hence, the settling time is decided by the exponential component.

<u>For 2% tollerance</u>

At $t=t_s$; $\frac{e^{-\xi_e \omega_n t_s}}{\sqrt{1-\xi_e^2}} = 0.02$

when $\xi_e << 1$; $\sqrt{1-\xi_e^2} = \sqrt{1} = 1$

$$e^{-\xi_e \omega_n t_s} = 0.02$$

$$\Rightarrow -\xi_e \omega_n t_s = \ln 0.02 = -3.91 \approx -4$$

$$\boxed{t_s = \frac{4}{\xi_e \omega_n}}$$

<u>For 5% tollerance</u>

At $t=t_s$; $\frac{e^{-\xi_e \omega_n t_s}}{\sqrt{1-\xi_e^2}} = 0.05$

when $\xi_e << 1$; $\sqrt{1-\xi_e^2} = \sqrt{1} = 1$

$$e^{-\xi_e \omega_n t_s} = 0.05$$

$$\Rightarrow -\xi_e \omega_n t_s = \ln 0.05 = -3$$

$$\boxed{t_s = \frac{3}{\xi_e \omega_n}}$$

The time constant for second order system is

$$T = \frac{1}{\xi_e \omega_n}$$

For 2% tollerance $t_s = \frac{4}{\xi_e \omega_n} \Rightarrow \boxed{t_s = 4T}$

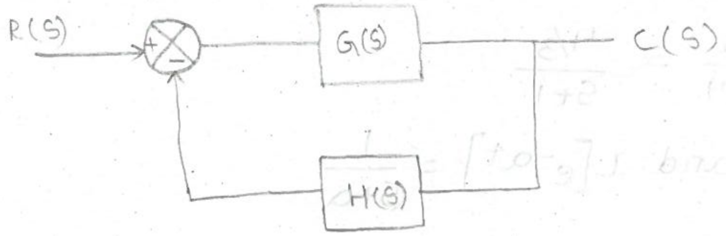
For 5% tollerance $t_s = \frac{3}{\xi_e \omega_n} \Rightarrow \boxed{t_s = 3T}$

Problems

1) Obtain the response of unity feedback system whose open loop transfer function is

$G(S) = \frac{4}{S(S+5)}$, when the input is Unit step.

Sol:- The closed loop system is



Given $G(S) = \frac{4}{S(S+5)}$

As input is unit step

$$R(S) = \frac{1}{S}$$

As the feedback is unity

$$H(S) = 1$$

The transfer function of closed loop system is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

$$\Rightarrow C(S) = R(S) \cdot \frac{G(S)}{1+G(S)}$$

$$C(S) = \frac{1}{S} \cdot \frac{\frac{4}{S[S+5]}}{1+\frac{4}{S[S+5]}}$$

$$\Rightarrow C(S) = \frac{4}{S[S(S+5)+4]} = \frac{4}{S[S^2+5S+4]}$$

$$\Rightarrow C(S) = \frac{4}{S[S^2+4S+S+4\times 1]} = \frac{4}{S[S+4][S+1]}$$

By applying partial fractions:

$\Rightarrow 4 = A[S+4][S+1] + BS[S+1] + CS[S+4]$

Put $S=0$ Put $S=-4$ Put $S=-1$

$4=4A$ $\quad 4=-4B(-3)$ $\quad 4=-C[3]$

$A=1$ $\quad B=\frac{4}{12}=\frac{1}{3}$ $\quad C=\frac{-4}{3}$

$$C(S) = \frac{A}{S} + \frac{B}{S+4} + \frac{C}{S+1}$$

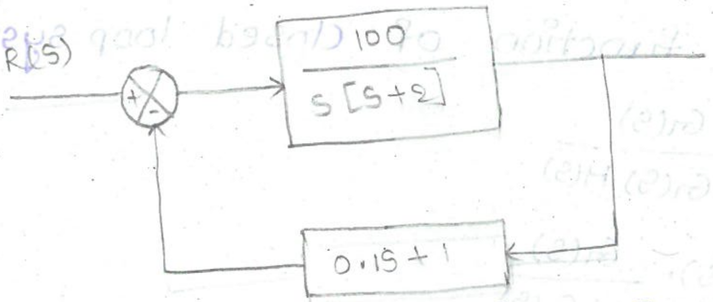
$$\Rightarrow C(S) = \frac{1}{S} + \frac{1/3}{S+4} - \frac{4/3}{S+1}$$

W.K.T $L[1]=\frac{1}{S}$ and $L[e^{-at}] = \frac{1}{S+a}$

By applying inverse Laplace transformation

$$c(t) = 1 + \frac{1}{3}e^{-4t} - \frac{4}{3}e^{-t}$$

2) A positional control system with velocity feedback is shown in figure. what is the response of system for unit step input.



Sol:- the transfer function of closed loop system is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

Given

$$G(S) = \frac{100}{S[S+2]}$$

$$H(S) = 0.1S+1$$

$$R(S) = \frac{1}{}$$

$$\frac{C(S)}{R(S)} = \frac{\frac{100}{S[S+2]}}{1 + \frac{100}{S[S+2]}[0.1S+1]}$$

$$\frac{C(S)}{R(S)} = \frac{100}{S[S+2] + 10S + 100}$$

$$C(S) = R(S) \cdot \frac{100}{S^2+12S+100}$$

$$C(S) = \frac{100}{S[S^2+12S+100]}$$

By applying partial fractions, we get

$$\frac{100}{S[S^2+12S+100]} = \frac{A}{S} + \frac{BS+C}{S^2+12S+100}$$

$$100 = A[S^2+12S+100] + (BS+C)S$$

$$100 = S^2[A+B] + S[12A+C] + 100A$$

Equating like terms

$$100 = 100A \qquad A+B=0 \qquad 12A+C=0$$

$$A = 1 \qquad B = -A \qquad C = -12A$$

$$B = -1 \qquad C = -12$$

$$\therefore C(S) = \frac{A}{S} + \frac{BS+C}{S^2+12S+100}$$

$$C(S) = \frac{1}{S} - \frac{S+12}{S^2+12S+100}$$

$$C(S) = \frac{1}{S} - \frac{S+6+6}{S^2+12S+36+64}$$

$$C(S) = \frac{1}{S} - \frac{S+6+6}{S^2+12S+6^2+8^2}$$

$$C(S) = \frac{1}{S} - \frac{S+6+6}{(S+6)^2+8^2}$$

$$\Rightarrow C(S) = \frac{1}{S} - \frac{S+6}{(S+6)^2+8^2} - \frac{8}{(S+6)^2+8^2}\cdot\frac{6}{8}$$

WKT

$$L[1] = \frac{1}{S}$$

$$L\left[\frac{S+\alpha}{(S+\alpha)^2+a^2}\right] = e^{-\alpha t}\cos at$$

$$L\left[\frac{a}{(S+\alpha)^2+a^2}\right] = e^{-\alpha t}\sin at$$

By applying inverse Laplace Transformation, we get

$$c(t) = 1 - e^{-6t}\cos 8t - e^{-6t}\sin 8t\cdot \cancel{\tfrac{3}{4}}\,\frac{6}{8}$$

$$c(t) = 1 - e^{-6t}\left[\cos 8t + \cancel{\tfrac{3}{4}}\,\frac{6}{8}\sin 8t\right] \quad -①$$

The transfer function of second order system is

$$\frac{C(S)}{R(S)} = \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2}$$

$$\Rightarrow \frac{\omega_n^2}{S^2+2\xi\omega_n S+\omega_n^2} = \frac{100}{S^2+12S+100}$$

Equating like terms.

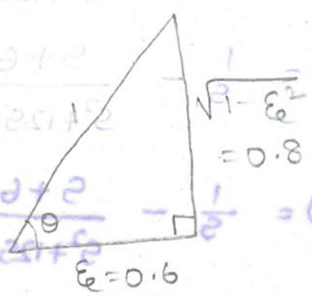
$$\omega_n^2 = 100 \qquad 2\xi\omega_n = 12$$

$$\omega_n = 10 \qquad \xi = \frac{12}{2\omega_n} = \frac{12}{20} = \frac{3}{5} = 0.6$$

From right angle triangle,

$$\cos\theta = 0.6$$

$$\sin\theta = 0.8$$



$$① \Rightarrow c(t) = 1 - e^{-6t}\left[\cos 8t + \frac{6}{8}\sin 8t\right]$$

$$c(t) = 1 - e^{-6t}\left[\sin\theta \cos 8t + \cos\theta \sin 8t\right]\frac{10}{8}$$

$$c(t) = 1 - \frac{5}{4} e^{-6t}\left[\sin(8t+\theta)\right]$$

22/8/17

Type number of Control Systems

The type number is specified for the loop transfer function $G(s)H(s)$. The no. of poles of loop transfer function lying at origin decides the type number of Control systems. If N is the no. of poles at origin then type number is N. The loop transfer function can be expressed as

$$G(s)H(s) = k\frac{P(s)}{Q(s)} = k\frac{(s+z_1)(s+z_2)\text{------}}{s^N(s+P_1)(s+P_2)\text{-----}}$$

where z_1, z_2, z_3 ---- are zeroes of transfer function.

P_1, P_2, P_3 ---- are poles of transfer function.

N is no. of poles at origin

k is constant

If $N=0$; the system is Type zero

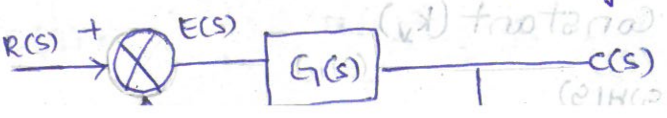
If $N=1$; the system is Type one

so on ------

Steady State Error

The steady state error is the value of error signal $e(t)$ when $t \to \infty$.

Consider a closed loop system



The error signal E(S) is given as

$$E(S) = R(S) - C(S)H(S)$$

but $C(S) = E(S)G(S)$

$$\Rightarrow E(S) = R(S) - E(S)\,G(S)\,H(S)$$

$$\Rightarrow E(S)\left[1 + G(S)H(S)\right] = R(S)$$

$$E(S) = \frac{R(S)}{1+G(S)H(S)}$$

Let e(t) be the error signal in the time domain

$$\therefore e(t) = L^{-1}\left[E(S)\right]$$

$$= L^{-1}\left\{\frac{R(S)}{1+G(S)H(S)}\right\}$$

$\therefore$ steady state error $e_{ss} = \underset{t\to\infty}{Lt}\ e(t)$

The final value theorem of Laplace transformation states that if $F(S) = L[f(t)]$ then

$$\boxed{\underset{t\to\infty}{Lt}\ f(t) = \underset{S\to 0}{Lt}\ SF(S)}$$

From final value theorem the steady state error

$$e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{S\to 0}{Lt}\ SE(S)$$

$$\boxed{\therefore\ e_{ss} = \underset{S\to 0}{Lt}\ S\,\frac{R(S)}{1+G(S)H(S)}}$$

<u>Static Error Constants</u>

1) Positional error constant (k_p)

$$k_p = \underset{S\to 0}{Lt}\ G(S)H(S)$$

2) velocity error constant (k_v)

$$k_v = \underset{}{Lt}\ SG(S)H(S)$$

$$k_a = \underset{S \to 0}{Lt}\; S^2 G(S)H(S)$$

Steady state error when the input is Unit step:-

we know that

Steady state error $e_{ss} = \underset{S \to 0}{Lt}\; S \dfrac{R(S)}{1 + G(S)H(S)}$

As input is Unit step; $R(S) = \frac{1}{S}$

$$\Rightarrow e_{ss} = \underset{S \to 0}{Lt} \frac{S \cdot \frac{1}{S}}{1 + G(S)H(S)}$$

$$\Rightarrow e_{ss} = \underset{S \to 0}{Lt} \frac{1}{1 + G(S)H(S)}$$

$$\Rightarrow e_{ss} = \frac{1}{1 + \underset{S \to 0}{Lt}\, G(S)H(S)}$$

$$\Rightarrow e_{ss} = \frac{1}{1 + k_p}$$

$$\therefore k_p = \underset{S \to 0}{Lt}\, G(S)H(S) = \underset{S \to 0}{Lt}\; k \frac{(S+Z_1)(S+Z_2) - - - -}{S^N (S+P_1)(S+P_2) - - - -}$$

Type zero System

For Type zero System $N = 0$

$$\therefore S^N = S^0 = 1$$

$$\therefore k_p = \underset{S \to 0}{Lt}\; k \frac{(S+Z_1)(S+Z_2) - - - -}{(S+P_1)(S+P_2) - - - - -}$$

$$\Rightarrow k_p = k \frac{Z_1 Z_2 Z_3 - - - - -}{P_1 P_2 P_3 - - - - - -} = \text{Constant}$$

$\therefore$ Steady state error $e_{ss} = \dfrac{1}{1+k_p} = \dfrac{1}{1+\text{Constant}}$

$$e_{ss} = \text{Constant}$$

Hence in Type zero system if the input is unit step then e_{ss} is Constant.

$\therefore S^N = S^1 = S$

$$\therefore k_p = \underset{S\to 0}{Lt}\ k \frac{(S+Z_1)(S+Z_2)\ldots}{S(S+P_1)(S+P_2)\ldots}$$

$$\Rightarrow k_p = k\ \frac{Z_1 Z_2 Z_3 \ldots}{0 \cdot P_1 P_2 P_3 \ldots} = \infty$$

Steady state error $e_{ss} = \frac{1}{1+k_p} = \frac{1}{1+\infty}$

$$e_{ss} = 0$$

For Type one system if the input is Unit step then $e_{ss}=0$

<u>steady state error when the input is Unit Ramp:-</u>

we know that

Steady state error $e_{ss} = \underset{S\to 0}{Lt} \frac{S R(S)}{1+G(S)H(S)}$

As the input is Unit Ramp; $R(S) = \frac{1}{S^2}$

$$\Rightarrow e_{ss} = \underset{S\to 0}{Lt} \frac{S \cdot \frac{1}{S^2}}{1+G(S)H(S)}$$

$$e_{ss} = \underset{S\to 0}{Lt} \frac{1}{S[1+G(S)H(S)]}$$

$$e_{ss} = \underset{S\to 0}{Lt} \frac{1}{S+S\,G(S)H(S)}$$

$$e_{ss} = \frac{1}{0+\underset{S\to 0}{Lt} S\,G(S)H(S)}$$

$$e_{ss} = \frac{1}{k_v}$$

$$\therefore k_v = \underset{S\to 0}{Lt} S\,G(S)H(S) = \underset{S\to 0}{Lt} S\ k \frac{(S+Z_1)(S+Z_2)\ldots}{S^N(S+P_1)(S+P_2)\ldots}$$

<u>Type zero System</u>

For Type Zero System N=0

$$k_v = \underset{S\to 0}{Lt}\ Sk\ \frac{(S+Z_1)(S+Z_2)----}{(S+P_1)(S+P_2)-----}$$

$$k_v = 0k\ \frac{Z_1Z_2Z_3----}{P_1P_2P_3-----} = 0$$

$$e_{ss} = \frac{1}{k_v} = \frac{1}{0} = \infty.$$

For type zero system if the input is Unit ramp then $e_{ss} = \infty$.

Type One system

For Type One System $N = 1$

$\therefore S^N = S^1 = S.$

$$k_v = \underset{S\to 0}{Lt}\ Sk\ \frac{(S+Z_1)(S+Z_2)----}{S(S+P_1)(S+P_2)-----}$$

$$k_v = \underset{S}{Lt}\ 0k\ \frac{Z_1Z_2Z_3----}{0.P_1P_2P_3-----} = 0$$

$$e_{ss} = \frac{1}{k_v} = \frac{1}{0} = \infty.$$

For Type one System if the input is Unit ramp then $e_{ss} = \infty$.

26/8/17 for type 2 System S^N, S^2

Steady state error when the input is Unit Parabolic:-

we know that $r(t) = \frac{t^2}{2}$

$$e_{ss} = \underset{S\to 0}{Lt}\ \frac{S\,R(S)}{1+G(S)H(S)}$$

$R(S) = \frac{2}{S^3.2} = \frac{1}{S^3}$

As the input is Unit Parabolic $R(S) = \frac{1}{S^3}$

$$e_{ss} = \underset{S\to 0}{Lt}\ \frac{S.\frac{1}{S^3}}{1+G(S)H(S)}$$

$e_{ss} = Lt\ \ 1$

$$e_{ss} = \frac{1}{0 + \lim_{s\to 0} s^2 G(s)H(s)}$$

$$\boxed{e_{ss} = \frac{1}{k_a}}$$

where $k_a = \lim_{s\to 0} s^2 G(s)H(s)$

$$k_a = \lim_{s\to 0} s^2 k \frac{(s+z_1)(s+z_2)(s+z_3)-----}{s^N(s+p_1)(s+p_2)(s+p_3)-----}$$

For type zero system

For type zero system put $N=0$

$$\Rightarrow s^N = s^0 = 1$$

$$k_a = \lim_{s\to 0} s^2 k \frac{(s+z_1)(s+z_2)----}{s^N(s+p_1)(s+p_2)----}$$

$$k_a = 0 \cdot k \frac{z_1 z_2}{0 \cdot p_1 p_2}$$

$$k_a = 0$$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$

For type zero system as input is unit parabolic the steady state error ess is ∞.

For type One System

For type one system Put $N=1$

$$\Rightarrow s^N = s^1 = s.$$

$$k_a = \lim_{s\to 0} s^2 k \frac{(s+z_1)(s+z_2)----}{s^N(s+p_1)(s+p_2)----}$$

$$k_a = 0 \cdot k \frac{z_1 z_2}{0 p_1 p_2}$$

$$k_a = 0$$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{0} = \infty$$

system as input is Unit

For type Two system

Put $N=2$

$$s^N = s^2$$

$$k_a = \underset{s\to 0}{Lt}\; s^2 k \frac{(s+z_1)(s+z_2)\ldots}{s^N (s+P_1)(s+P_2)\ldots}$$

$$k_a = k \frac{z_1 z_2 \ldots}{P_1 P_2 \ldots}$$

$$k_a = \text{Constant}$$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{\text{Constant}}$$

$$e_{ss} = \text{Constant}$$

For type two system Unit Parabolic input the e_{ss} is Constant.

For type three System

$$N=3$$

$$s^N = s^3$$

$$k_a = \underset{s\to 0}{Lt}\; s^2 k \frac{(s+z_1)(s+z_2)\ldots}{s^3 (s+P_1)(s+P_2)\ldots}$$

$$k_a = k \frac{z_1 z_2}{0 \cdot P_1 P_2 \ldots}$$

$$k_a = \infty.$$

$$e_{ss} = \frac{1}{k_a} = \frac{1}{\infty} = 0$$

For type three system the input of Unit parabolic, the e_{ss} is 0.

3:- Concept of Stability and Root Locus Technique

→ Every system is designed by impulse response.

→ $c(t)_{\text{(impulse)}} = L^{-1}[F(s)]$

$R(s) \to \boxed{F(s)} \to C(s)$

The system response is given by ILT of transfer function.

$C(s) = R(s)\,F(s)$

$R(s) = 1$ (impulse)

$C(s) = F(s)$

$c(t) = L^{-1}[F(s)]$

* Convalution is used to determine the relation ship b/w i/p and o/p signals of the system.

* Correlation is used to determine the similarity b/w 2 signals.

→ Auto correlation $R_{xx}(\tau)$

→ Cross correlation $(R_{xy}(\tau))$

* Determining the similarity b/w 2 same signals is called auto correlation.

* Determining the similarity b/w 2 different signals is called cross correlation.

→ The stability of the closed loop control system is by using the characteristic equation.

$$1 + G(s)\,H(s) = 0$$

To determine the stability of the system by using analytical method is known as "Routh Hurwitz criterion"

→ To determine the stability of the system by using graphical method is known as "Root Locus technique".

~~Routh- Hurwitz criterion :-~~

Stability :-

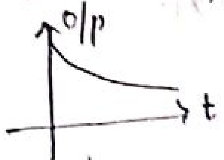
* A linear time invarient system is said to be stable if it produces the following response

i) stable

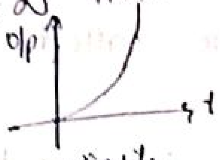
ii) finite value

iii) Predetermined value.

* A system is said to be stable if it produces bounded output for a bounded input.
* BIBO signals are step, impulse.
* If $t \to \infty$, o/p = 0 then it is stable.
* If $t \to \infty$, o/p $\to \infty$ then it is unstable.

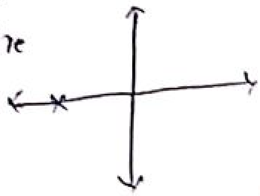


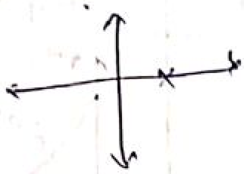
stable

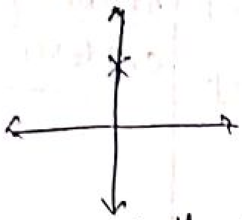


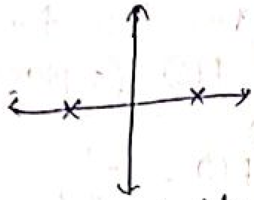
unstable.

* The


Stable.


Unstable


Marginally stable.


Unstable

* G(S) H(S) - open loop transfer function.
* Impulse f'n is called as shocking pulse

Routh - Hurwitz criterion :-

Location of poles on s-plane for stability :-

The relationship b/w i/p and o/p of a closed loop control system is expressed by nth order differential equations. It can be expressed as

$$a_n \frac{d^n x_0}{dt^n} + a_{n-1} \frac{d^{n-1} x_0}{dt^{n-1}} + \cdots + a_0 x_0 = b_m \frac{d^m x_i}{dt^m} + b_{m-1} \frac{d^{m-1} x_i}{dt^{m-1}} + \cdots + b_0 x_i.$$

$x_0 \to$ o/p of the system

$x_i \to$ i/p of the system

a_n, b_m are system physical coefficients

The above eq. can be expressed as

$$a_n s^n x_0 + a_{n-1} s^{n-1} x_0 + \cdots + a_0 x_0 = b_m s^m x_i + b_{m-1} s^{m-1} x_i + \cdots + b_0 x_i$$

we assume that F(s) is the transfer function of the closed loop control system.

$$\therefore F(S) = \frac{\text{system o/p}}{\text{system i/p}} = \frac{x_0}{x_i}$$

$$x_0 \left[a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0\right] = x_i \left[b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0\right]$$

$$\frac{x_0}{x_i} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0} = F(s)$$

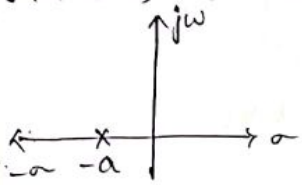
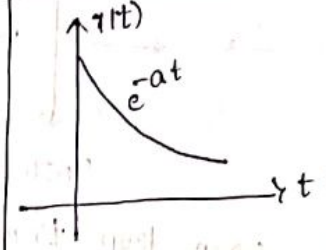
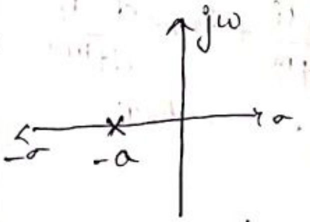
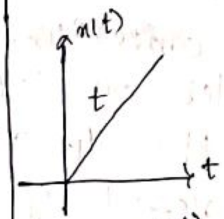
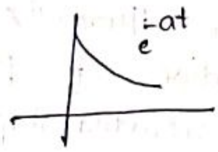
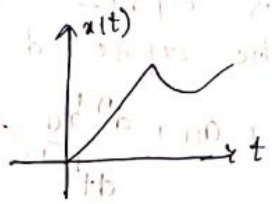
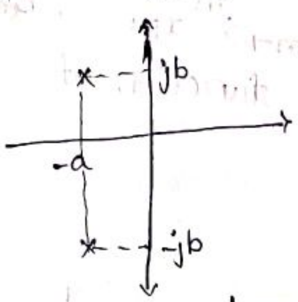
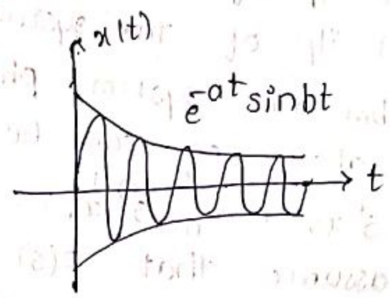
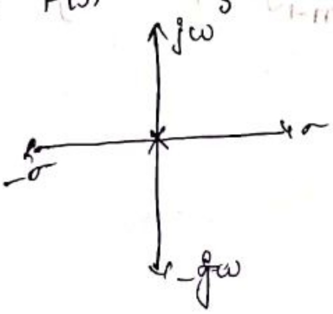
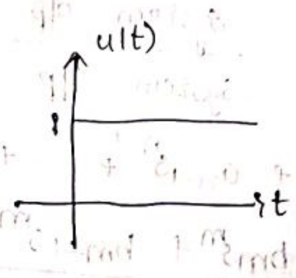
The above eq. is a polynomial

The above eq. F(s)' can be expressed as

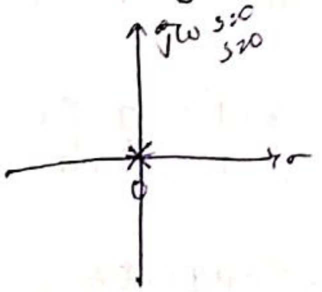
$$P(S) = \frac{(s+z_1)(s+z_2)+\cdots\cdot(s+z_m)}{(s+p_1)(s+p_2)+\cdots(s+p_n)}$$

The roots present in numerator polynomial are called zeros (o) and the roots present in denominator polynomial are called poles (x).

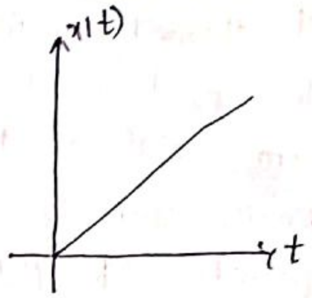
· Transfer function F

Transfer function F(S) (s-plane) $s=\sigma+j\omega$	stability criterion	impulse response
1) $F(S) = \frac{1}{S+a}$ $S+a=0 \Rightarrow S=-a$ 	stable	
2) $F(S) = \frac{1}{(s+a)^2}$ $= \frac{1}{(s+a)(s+a)}$ 	stable	  
3) $F(S) = \frac{1}{(s+a)^2+b^2}$ 	stable	
4) $F(S) = \frac{1}{S}$ 	Stable	

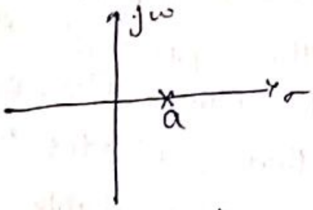
5) $F(s) = \frac{1}{s^2}$



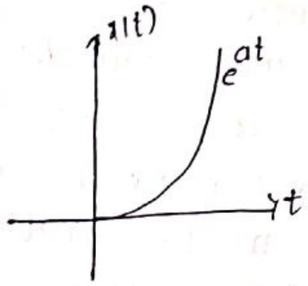
Unstable



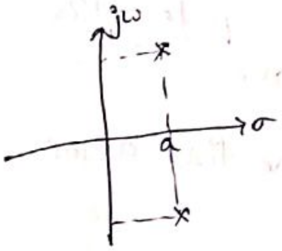
6) $F(s) = \frac{1}{s-a}$



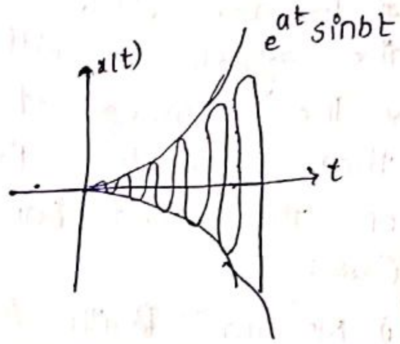
unstable.



7) $F(s) = \frac{1}{(s-a)^2+b^2}$



unstable



1. Consider the characteristic polynomial with all +ve coefficients determine the system is stable or not for

$s^3 + s^2 + 2s + 8 = 0$

$$\begin{array}{r|rrrr} & 1 & 1 & 2 & 8 \\ -2 & 0 & -2 & 2 & -8 \\ \hline & 1 & -1 & 4 & 0 \end{array}$$

$(s+2)(s^2 - s + 4) = 0$

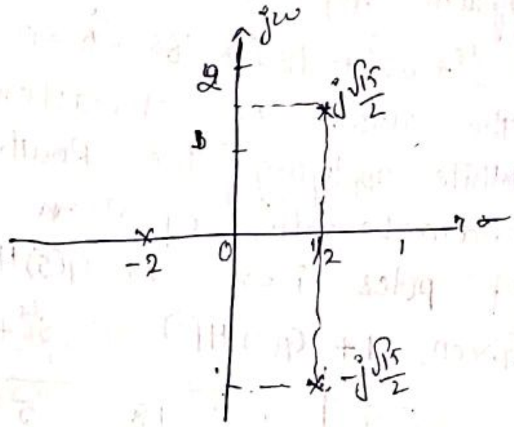
$s+2 = 0$; $s^2 - s + 4 = 0$

$s = \frac{1 \pm \sqrt{1-4(1)(4)}}{2}$

$s = \frac{1 \pm \sqrt{1-16}}{2}$

$= \frac{1 \pm \sqrt{-15}}{2}$

$s = -2$; $s = \frac{1 \pm j\sqrt{15}}{2}$



One pole lies on the left side of s-plane and other two poles lies on the right side of s-plane.
Hence, the system is unstable.

Scanned by CamScanner

Routh Hurwitz criterion :-

* This method is analytical method to determine the system either stable or unstable.

* Routh stability criterion is based on ordering the coefficients of polynomial into a schedule. is known as Routh Array

* While constructing the routh Hurwitz criterion one may can' be happen out of 3 cases

* In constructing Routh Hurwitz criterion the first column elements are positive then the system is said to be stable. Otherwise, the first column elements

* In are either zero (or) negative that indicates the system is either unstable (or) marginally stable

* The number of sign changes in the first column that indicates that much number of poles lies on the right hand side of the s-plane.

Case :-

1) Normal Routh Array (All elements in the first column are positive / non-zero)

2) All elements entire in the row is having zeroes

3) First element in the row is zero

Model-1:-

Using Routh criterion, determine the stability of the system represented by characteristic equation $s^4+8s^3+18s^2+16s+5=0$. Comment on the location of the roots of characteristic equation.

While applying the Routh stability criterion, we require the characteristic equation for determine the location of poles i.e., $1+G(s)H(s)=0$

Given, $1+G(s)H(s) = s^4+8s^3+18s^2+16s+5=0$

s^4	1	18	5
s^3	8	16	0
s^2	16	5	0
s^1	13.5	0	0
s^0	5	0	0

$\frac{8\times18-16\times1}{8}$ $\quad$ $\frac{8\times5-1\times0}{8}$

$\frac{16\times16-8\times5}{16}$ $\quad$ $\frac{16\times0-0\times8}{16}$

$\frac{13.5\times5-0\times16}{13.5}$ $\quad$ $\frac{13.5\times0-16\times0}{13.5}$

$5\times0+$

Scanned by CamScanner

All elements in the first column are positive.
The system is said to be stable.
The given problem is 4th order polynomial which is having 4 roots that lies on left hand side of the s-plane.

By Routh stability criterion, determine the stability of the system represented by the characteristic eqn location
$9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$. Comment on the
of roots of characteristic eqn.
While applying the Routh stability criterion, we require the characteristic eqn i.e., $1 + G(s)H(s) = 0$

Given, $1 + G(s)H(s) = 9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$.

s^5	9	10	-9
s^4	-20	-1	-10
s^3	9.55	-13.5	0
s^2	-29.2	-10	0
s^1	-16.77	0	0
s^0	-10	0	0

$\frac{-20\times10 - 9\times(-1)}{-20}$ $\quad$ $\frac{-20\times(-9) - 9\times(-10)}{-20}$

$\frac{-200+9}{-20}$ $\quad$ $\frac{180+90}{-20}$

$\frac{9.55\times(-1) - (-20)(-13.5)}{9.55}$

$\frac{-9.55 - \cancel{270}}{9.55} =$

$\frac{9.55(-10) - 0}{9.55}$

$\frac{-29.2(-13.5) + 9.55(10)}{-29.2}$

The given problem is 5th order polynomial which is having 5 roots.
Out of 5 roots, 3 roots lie on the right-hand side of the s-plane due to 3 sign changes in the routh hurwitz criterion
Hence, the system is unstable.

Model-2 :-
Construct Routh array and determine the stability of the sy'm whose characteristic eqn
$s^6 + 2s^5 + 8s^4 + 12s^3 + 20s^2 + 16s + 16 = 0$. Also determine the no. of roots lie on right of the s-plane, left half of the s-plane and on imaginary axis.

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0
s^2	0	0	0	0
s^1	0	0	0	0
s^0	0	0	0	0

$\frac{2\times 8 - 12\times 1}{2}$ $\frac{2\times 20 - 16\times 1}{2}$

$\frac{2\times 16 - 0}{2}$

Consider an auxiliary eq'n

$A = 2s^4 + 12s^2 + 16 = 0$

$A : s^4 + 6s^2 + 8 = 0$

Diff w.r.to 's'

$\frac{dA}{ds} = 4s^3 + 12s$

s^6	1	8	20	16
s^5	2	12	16	0
s^4	2	12	16	0
s^3	0	0	0	0
s^3	4	12	0	0
s^2	6	16	0	0
s^1	1.33	0	0	0
s^0	6	0	0	0

$\frac{4\times 12 - 12\times 2}{4}$ $\frac{4\times 16 - 0}{4}$

$\frac{12\times 6 - 16\times 4}{6}$

While observing the Routh-Hurwitz criterion, there is no sign changes in the 1st column. No root will be present in the right half of the s-plane.

Put $s^2 = x$ in auxiliary equation.

$x^2 + 6x + 8 = 0$

$(x+4)(x+2) = 0$

$x = -4, -2$

$s^2 = -4$ $\quad s^2 = -2$

$s = \sqrt{-4}$ $\quad s = \sqrt{-2}$

$s = \pm 2j$ $\quad s = \pm\sqrt{2}j$

The given ~~sys~~ characteristic eq'n is 6th order polynomial is having 6 roots. Out of 6 roots, 4 roots lie on imaginary axis and remaining 2 roots lie on left hand side of s-plane.

∴ The system is marginally stable.

The characteristic polynomial of the system is $s^7 + 9s^6 + 24s^5 + 24s^4 + 24s^3 + 24s^2 + 23s + 15 = 0$. Determine the location of roots on s-plane and hence the stability of the system.

s^7	1	24	24	23
s^6	9	24	24	15
s^5	21.3	21.3	21.3	0
s^4	15	15	15	0
s^3	0	0	0	0
s^3	4	2	0	0
s^2	7.5	15	0	0
s^1	-6	0	0	0
s^0	+15	0	0	0

$\frac{24\times9 - 24\times1}{9}$

$\frac{24\times23 - 15\times24}{24}$

$\frac{552 - 360}{24}$

Consider an auxiliary eq'n.

$A = 15s^4 + 15s^2 + 15 = 0$

$A = s^4 + s^2 + 1 = 0$

Diff. w.r.to 's'

$\frac{dA}{ds} = 4s^3 + 2s$

$\frac{15\times4 - 2\times15}{4} = \frac{60-30}{4}$

while observing the Routh-Hurwitz criterion, there is a sign changes in the first column. i.e., 2 roots will be present in the right hand side of s-plane.

put $s^2 = x$ in auxiliary eq'n.

$x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1-4(1)(1)}}{2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$

$s^2 = \frac{-1 + j\sqrt{3}}{2}$ $\quad s^2 = \frac{-1 - j\sqrt{3}}{2}$

$s = \pm\sqrt{\frac{-1 + j\sqrt{3}}{2}}$ $\quad s = \pm\sqrt{\frac{-1 - j\sqrt{3}}{2}}$

4-roots lie on imaginary axis.

Scanned by CamScanner

and the remaining one root lie on the left hand side of the s-plane.

Hence the system is unstable.

Model-3//

Construct Routh Array and determine the stability of the system represented by the characteristic eq^n $s^5+s^4+2s^3+2s^2+3s+5=0$. Comment on the location of roots of characteristic eq^n.

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	0
s^3	ϵ	-2	0
s^2	$\frac{2\epsilon+2}{\epsilon}$	5	0
s^1	$\frac{-5\epsilon^2-4\epsilon-4}{2\epsilon+2}$	0	0
s^0	5	0	0

$\frac{1\times2-2\times1}{1}$ $\quad$ $\frac{1\times3-5\times1}{1}$

Replace zero by ϵ

$$\frac{\left(\frac{2\epsilon+2}{\epsilon}\right)(-2)-5\epsilon}{\left(\frac{2\epsilon+2}{\epsilon}\right)}$$

$2\ 4\epsilon-$

$$\frac{\left(\frac{-4\epsilon-4}{\epsilon}\right)-5\epsilon}{\frac{2\epsilon+2}{\epsilon}}$$

$$\frac{\frac{-4\epsilon-4-5\epsilon^2}{2\epsilon+2}}{\epsilon}$$

$$\frac{-4\epsilon^2-4\epsilon-5\epsilon^3}{2\epsilon+2}$$

$$\frac{-4\epsilon-4-5\epsilon^2}{2\epsilon+2}$$

Substitute $\epsilon=0$

s^5	1	2	3
s^4	1	2	5
s^3	0	-2	0
s^2	∞	5	0
s^1	-2	0	0
s^0	5	0	0

out of 5 roots, 2 roots lie on right-hand side of the s-plane and 1 root lie on imaginary axis and other remaining 2 roots lie on left hand side of the s-plane.

Hence, the system is unstable.

** Determine the range of 'K' for stability of unity feed back system whose open loop transfer f'n is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Given, $G(s) = \frac{K}{s(s+1)(s+2)}$

$H(s) = 1$

The characteristic eq'n is $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s+1)(s+2)}(1) = 0$$

$$s(s+1)(s+2) + K = 0$$

$$(s^2+s)(s+2) + K = 0$$

$$s^3 + 2s^2 + s^2 + 2s + K = 0$$

$$CE = s^3 + 3s^2 + 2s + K = 0$$

s^3	1	2
s^2	3	K
s^1	$\frac{6-K}{3}$	0
s^0	K	0

$\frac{3\times2 - K\times1}{3}$

$K > 0$

$\frac{6-K}{3} > 0$

$6-K > 0$

$6 > K.$

$$\frac{\left(\frac{6-K}{3}\right)K}{\frac{6-K}{3}}$$

$$\frac{(6-K)K}{3\left(\frac{6-K}{3}\right)} \quad \frac{6K-K^2}{6-K}$$

To maintain the stability of the system the values of K must be

$0 < K < 6$

* The open loop transfer function of a unity feed back control system is given by $G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$. Discuss the stability by applying the Routh criterion. Discuss the location of of the system and also determine the location of poles.

Given, $G(s) = \frac{K}{(s+2)(s+4)(s^2+6s+25)}$

$H(s) = 1$

CE $1 + G(s)H(s) = 0$

$$1 + \frac{K}{(s+2)(s+4)(s^2+6s+25)} = 0$$

$$(s+2)(s+4)(s^2+6s+25)+K=0$$

$$(s^2+6s+8)(s^2+6s+25)+K=0$$

$$s^4+6s^3+25s^2+6s^3+36s^2+150s+8s^2+48s+200+K=0$$

$$s^4+12s^3+69s^2+198s+200+K=0$$

s^4	1	69	$(200+K)$
s^3	12	198	0
s^2	52.5	$\frac{2400-K}{12}$	0
s^1	$\frac{7995+K}{52.5}$	K	0

$$\frac{12\times 69-198}{12} =$$

$$\frac{12\times 200-K}{12} = \frac{2400-K}{12}$$

$$\frac{52.5\times 198 - 12\left(\frac{2400-K}{12}\right)}{52.5}$$

$$\frac{10395-2400+K}{52.5}$$

$$10395-\frac{(2400+12K)}{52.5} \quad \frac{7995+K}{52.5}$$

$$10395-\frac{7995-12K}{52.5}$$

$$52.5K$$

$$\frac{\left(\frac{7995+K}{52.5}\right)\left(\frac{2400-K}{12}\right)-52}{\left(\frac{7995+K}{52.5}\right)}$$

s^4	1	69	200+K
s^3	12	198	0
s^3	1	16.5	0
s^2	52.5	200+K	0
s^1	$\frac{666.25-K}{52.5}$	0	0
s^0	200+K	0	0

$200+K>0$

$K>-200$

$666.25-K>0$

$666.25>K$

$K<666.25$

$$\frac{12(200+K)}{12}$$

$$\frac{866.25-200-K}{52.5}$$

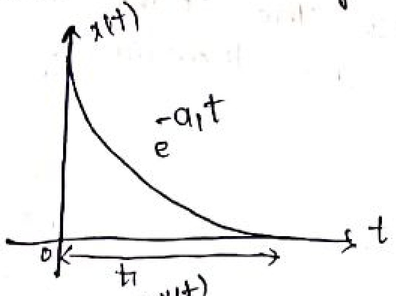
$$\left(\frac{666.25-K}{52.5}\right)(200+K)$$

To maintain the stability of the system, the range of K should be

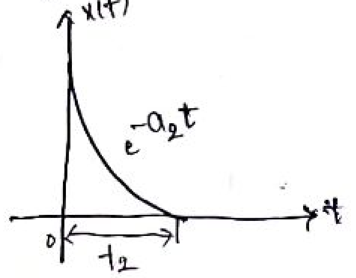
$$-200<K<666.25$$

Relative stability Analysis:-

$$F_1(s) = \frac{1}{s+a_1}$$



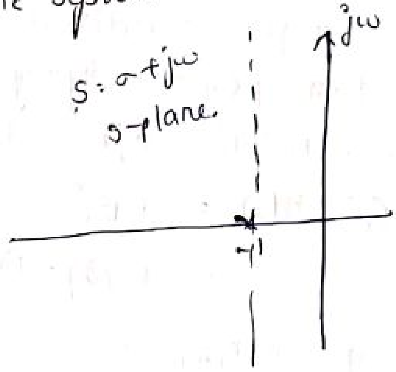
$$F_2(s) = \frac{1}{s+a_2}$$



Root Locus

Consider two systems are stable systems out of these 2 systems, system 2 is more stable / relatively stable than system 1 because system 2 is fast response from the shocking pulse i.e., $t_2 << t_1$

i) Determine the given system is more stable or relatively stable when $s = -1$ of the system $s^3+7s^2+25s+39=0$



Given,

$s^3+7s^2+25s+39=0$

$s=-1$

$s+1=z$

$s=z-1$

$(z-1)^3+7(z-1)^2+25(z-1)+39=0$

$z^3-3z^2+3z-1+7(z^2-2z+1)+25z-25+39=0$

$z^3-3z^2+3z-1+7z^2-14z+7+25z+14=0$

$z^3+4z^2+14z+20=0$

$39 - 25 = 14$

$56 - 20 = 36$

$\frac{20-56}{4}$

$\frac{14-5}{1}$

z^3	1	14
z^2	4	20
z^2	1	5
z^1	9	0
z^0	5	0

The first column in the Routh-Hurwitz criterion are +ve. Hence the system is stable. Hence the z-plane is more stable than s-plane. i.e., $s=-1$

Root Locus Technique:

Root locus is defined as the to obtain the locus of the point when the system gain 'k' is varied from 0 to ∞.

Angle condition and Magnitude condition:

To determine the system is stable or unstable or marginally stable by using characteristic eq^n

Scanned by CamScanner

i.e., $1 + G(s)H(s) = 0 \Rightarrow \underbrace{G(s)H(s) = -1}_{OLTS}$

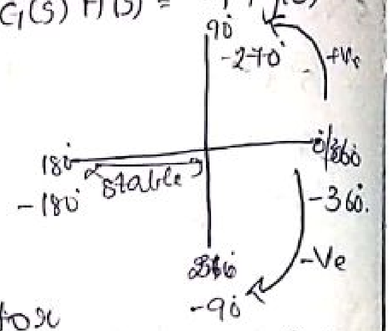
Angle condition:-

$\angle G(s)H(s) = \pm 180°$

$|G(s)H(s)| = 1$

The angle condition is used for checking whether certain points lying on root locus or not

Characteristic eq'n $1 + G(s)H(s) = 0 \Rightarrow G(s)H(s) = -1 + j(0)$



$\angle G(s)H(s) = \pm 180°$

$= \pm(2q+1)180°$

$q = 0, 1, 2, 3, 4 \ldots$

Magnitude Condition:-

The magnitude condition is used for finding the value of system gain 'k' at any point on the root locus.

$|G(s)H(s)| = 1$

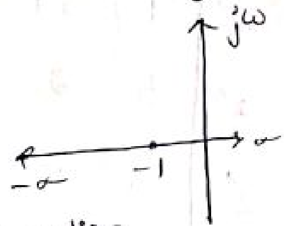
Construction rules for root locus:-

Rule 1:

The root locus is symmetric about real axis

$1 + G(s)H(s) = 0$

$\underbrace{G(s)H(s)}_{OLCS} = -1$



Rule 2:

P = no. of poles in open loop transfer function.

Z = no of zeroes in open loop transfer function.

P > Z :-

P = no. of branches of i.e., root locus

Z = no. of branches terminating at zeroes.

P − Z = no. of branches terminating at infinity.

P < Z

System is stable but practically does not exist

Rule - 3 :-

The point is on real axis and it is said to be root locus the right side of that point is sum of poles and zeroes that is to be odd.

$$G(s)H(s) = \frac{K(s+2)(s+6)}{s(s+4)(s+8)}$$

1. $1 + G(s)H(s) = 0 \Rightarrow G(s)H(s) = -1$

2. Poles are located at $s = 0, -4, -8$ (3)

Zeroes are located at $s = -2, -6$ (2)

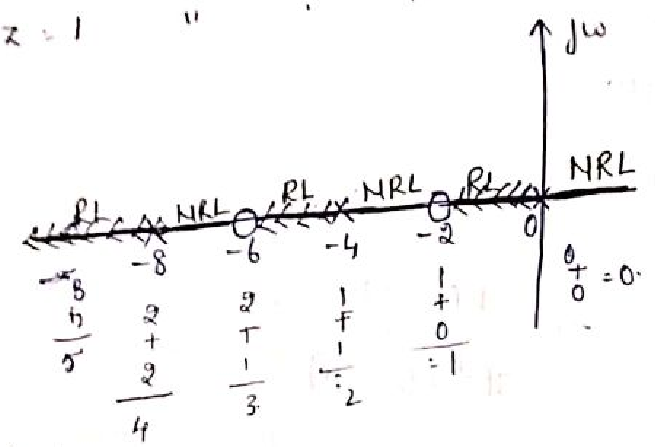
p 7z.

$P = 3$ branches of root locus 372.

$z = 2$ branches terminating at '0'

$P - z = 1$ " " " '∞'.

3.



Note :-
Poles are terminating either zeroes or infinity

Rule 4 :-
Angle of Asymptotes :-
The $P-z$ branches are terminating at infinity on a straight line is known as real axis with a angle of asymptotes.

$$\theta = \frac{\pm(2q+1)180^\circ}{P-z}, \quad q = 0, 1, 2, 3 \cdots \infty$$

Ex :- $s(s+4)(s^2+2s+5) + K(s+1) = 0$

$G(s)H(s) = -1$

$$1 + \frac{K(s+1)}{s(s+4)(s^2+2s+5)} = 0$$

Zeroes = 1

poles = 4

$P - z = 3$, $q = 0, 1, 2, 3$

$$\theta_1 = \frac{\pm(2q+1)180^\circ}{3}, \quad q = 0, \ \theta_1 = 60^\circ$$

$q = 1, \ \theta_2 = 180^\circ$

$q = 2, \ \theta_3 = 300^\circ$

$q = 3, \ \theta_4 = 420^\circ$

→ The angle b/w 2 symptotes $= \dfrac{2\pi}{P-z}$

5 Centroid :- The point of intersection of asymptotes on real axis it may (or) maynot be part of root locus

$$C = \frac{\Sigma\begin{pmatrix}\text{real part}\\ \text{of poles}\end{pmatrix} - \Sigma\begin{pmatrix}\text{real part}\\ \text{of zeroes}\end{pmatrix}}{P-Z}$$

Determine the centroid for given characteristic equation

$s^3 + 5s^2 + (k+6)s + k = 0$

$s^3 + 5s^2 + (s+1)k + 6s = 0$

$s^3 + 5s^2 + 6s + (s+1)k = 0$

$$1 + \frac{k(s+1)}{s^3+5s^2+6s} = 0$$

$$\frac{k(s+1)}{s^3+5s^2+6s} = -1 \Rightarrow \frac{k(s+1)}{s(s^2+5s+6)} = -1$$

zeroes at $s = -1$

poles at $s = 0, -2, -3$

$P > Z$

$$C = \frac{(0-2-3) - (-1)}{3-1} = \frac{-5+1}{2} = \frac{-4}{2} = -2$$

6. Break away (or) Breaking point :-

This is the point where multiple roots of characteristic equation will occur.

Break away point (B.A) is lies between 2 poles.

Break in point (B.I) is lies b/w 2 zeroes.

Between one pole and one zero there will a possible either break in or break away.

① Construct characteristic equation.

② represents in terms of 'k'.

③ Differentiate 'k' w.r.to 's'. $\left(\frac{dk}{ds} = 0\right)$ $\left(\frac{dk}{ds} = \sqrt{P}\right)$

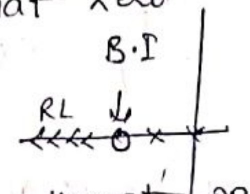
B.A B.I

④ Substitute 's' values in step ②.

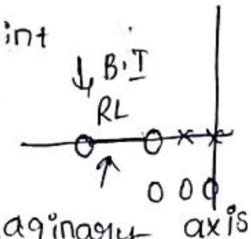
Note :-

1. Poles are always $\geq$ zeroes. $(P \geq Z)$

2. whenever a zero lies on x-axis (real) and leftside of that zero there is no pole and no zero and it is a part of root locus. After that zero we will get break in point.



3. whenever 2 zeroes are placed adjacent on real axis and it is a part of root locus there is a possible to get break in point



7) Intersection of Root locus with imaginary axis :-
The roots of the auxiliary equation i.e., $A(s)$ at $k = k_{marginal}$ will give the intersection of root locus with imaginary axis.

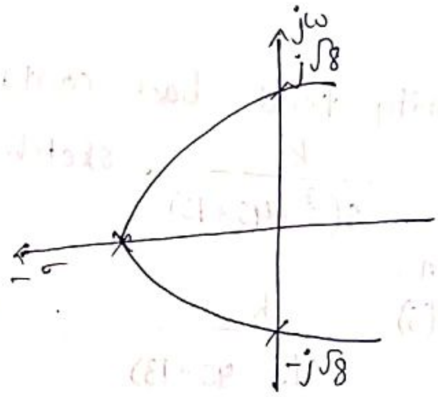
$$G(s)H(s) = \frac{K}{s(s+2)(s+4)}$$

CE is $1 + G(s)H(s) = 0$

$$\Rightarrow 1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\Rightarrow s(s+2)(s+4) + K = 0$$

$$\Rightarrow s^3 + 6s^2 + 8s + K = 0$$



s^3	1	8
s^2	6	K
s^1	$\frac{48-K}{6}$	0
s^0	K	0

$K > 0$; $48 - K > 0$

$48 > K$

$K < 48$

$0 < K < 48$

$K = K_{mar} = 48$

AE for given OLTF is $6s^2 + K = 0$

$6s^2 = -K$

$6s^2 = -48$

$s^2 = -8$

$s = \pm j\sqrt{8}$

8) Angle of departure & Angle of Arrival :

Angle of departure is used to determine the angles of complex poles.

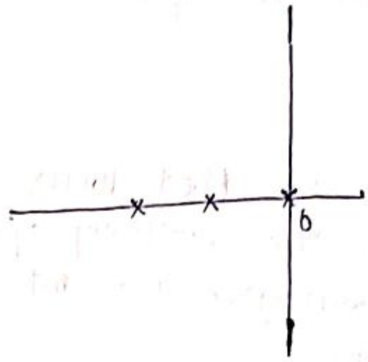
$$\phi_D = 180 + \phi$$

$$\boxed{\phi = \Sigma\phi_{zeros} - \Sigma\phi_{poles}}$$

Angle of arrival is used to determine the angles of complex zeroes.

$$\phi_A = 180 - \phi$$

A



A unity feed back control system has OLTS $G(s) = \frac{k}{s(s^2+4s+13)}$. sketch the rootlocus.

Given,

$G(s) = \frac{K}{s(s^2+4s+13)}$; $H(s) = 1$

$G(s)H(s) = \frac{K}{s(s^2+4s+13)}$

$G(s)H(s) = -1$

$\Rightarrow \frac{K}{s(s^2+4s+13)} = -1$

$\Rightarrow k = -s(s^2+4s+13)$

$k = -s^3 - 4s^2 - 13s$

$s^3 + 4s^2 + 13s + k = 0$

$s = \frac{-4 \pm \sqrt{16-4(13)}}{2} = \frac{-4 \pm \sqrt{-36}}{2} = \frac{-4 \pm j6}{2}$

$= -2 \pm 3i$

Scanned by CamScanner

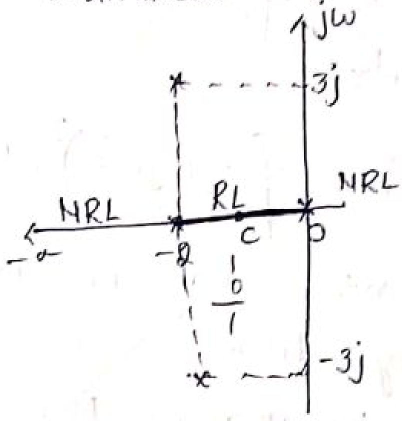
poles are at $s_1=0$, $s_2=-2+3j$, $s_3=-2-3j$

$P>Z$. $(P=3,\ Z=0)$

$P=3=$ no. of branches.

$Z=0 \Rightarrow$ no branches will terminate at zero.

$P-Z=3 \Rightarrow$ no. of branches will terminate at infinity.



Angle of asymptotes :-

$$\theta = \frac{\pm(2q+1)180^\circ}{P-Z}$$

$P-Z = 3 \Rightarrow q = 0,1,2,3$

$\theta_1 = \pm 60^\circ$ $\qquad$ $\theta_3 = \pm 300^\circ$

$\theta_2 = \pm 180^\circ$ $\qquad$ $\theta_4 = \pm 420^\circ$

Centroid, $C = \dfrac{\Sigma\left(\text{real part of poles}\right) - \Sigma\left(\text{real part of zeroes}\right)}{P-Z}$

$$C = -4/3 = -1.33$$

CE is $1+G(s)H(s)=0$

$$1+\frac{K}{s(s^2+4s+13)}=0$$

$$s(s^2+4s+13)+K=0$$

$$K = -s^3-4s^2-13s$$

$\Rightarrow \dfrac{dK}{ds}=0$

$\Rightarrow -3s^2-8s-13=0$

$\Rightarrow 3s^2+8s+13=0$

$\Rightarrow s = \dfrac{-8\pm\sqrt{64-4(3)(13)}}{6} = \dfrac{-8\pm\sqrt{-92}}{6}$

$= \dfrac{-8\pm\sqrt{64-156}}{6} = \dfrac{-8\pm j2\sqrt{23}}{6}$

$= \dfrac{-4\pm j\sqrt{23}}{3}$

$s_1 = \frac{-4+j\sqrt{23}}{3}$, $s_2 = \frac{-4-j\sqrt{23}}{3}$

sub. s_1, s_2 in K.

$s = s_1 \Rightarrow k_1 = -\left[\left(\frac{-4+j\sqrt{23}}{3}\right)^3 + 4\left(\frac{-4+j\sqrt{23}}{3}\right)^2 + 13\left(\frac{-4+j\sqrt{23}}{3}\right)\right]$

$s = s_2 \Rightarrow k_2$

$s = s_2 \Rightarrow k_2 = -\left[\left(\frac{-4-j\sqrt{23}}{3}\right)^3 + 4\left(\frac{-4-j\sqrt{23}}{3}\right)^2 + 13\left(\frac{-4-j\sqrt{23}}{3}\right)\right]$

k_1 is having imaginary term and k_2 is having imaginary term.

Neither Breakaway nor Break in for given system.

7) CE is $s^3 + 4s^2 + 13s + K = 0$

sub. $s = j\omega$ in characteristic equation.

$(j\omega)^3 + 4(j\omega)^2 + 13(j\omega) + K = 0$

$j^3\omega^3 + 4(-\omega^2) + 13j\omega + K = 0$

$\Rightarrow -j\omega^3 - 4\omega^2 + 13j\omega + K = 0$

$\Rightarrow (-4\omega^2 + K) + j(13\omega - \omega^3) = 0$

$-4\omega^2 + K = 0$

$4\omega^2 = K$

$K = 4(13)$

$K = 52$

$13\omega - \omega^3 = 0$

$13\omega = \omega^{3}$

$\omega^2 = 13$

$\omega = \pm\sqrt{13}$

$\omega \rightarrow$ the root locus crosses the imaginary axis at $\sqrt{13}$.

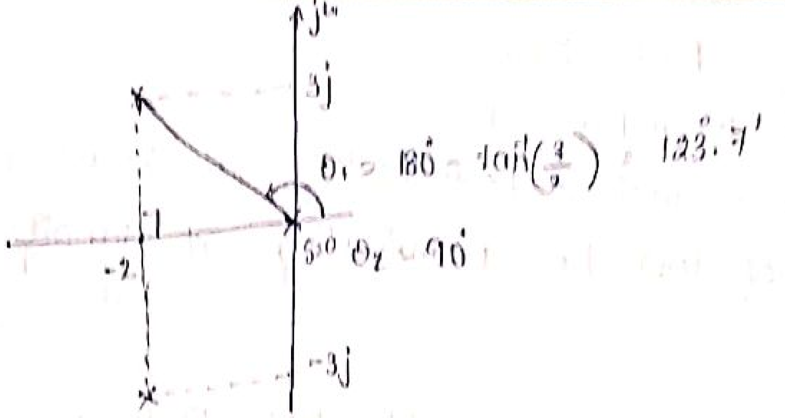
$K \rightarrow$ system gain.

It is not possible to determine the angle of arrival b/c no complex zeroes.

So, angle of departure exists.

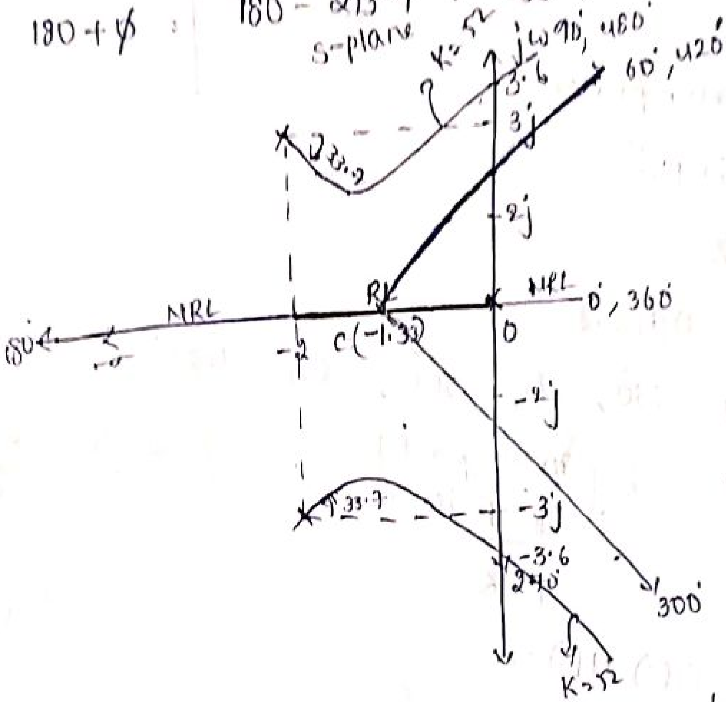
$\phi_D = 180 + \phi$

$\phi = \Sigma\phi_{zeroes} - \Sigma\phi_{poles}$



$\phi = -(123.7 + 90) = -213.7$

$\phi_0 = 180 + \phi = 180 - 213.7 = -33.7°$



* Sketch the root locus of the system whose open loop transfer function is $G(s) = \frac{K}{s(s+2)(s+4)}$. Find the value of K so that the damping ratio of closed loop system is 0.5.

Given, $G(s) = \frac{K}{s(s+2)(s+4)}$, Let $H(s) = 1$.

1. $G(s)H(s) = -1$, root locus is symmetrical on real axis.

$$\frac{K}{s(s+2)(s+4)} = -1$$

2. $$G(s)\,H(s) = \frac{K}{s(s+2)(s+4)}$$

Poles at $s = 0, -2, -4$

Zeroes ~~at~~ 0.

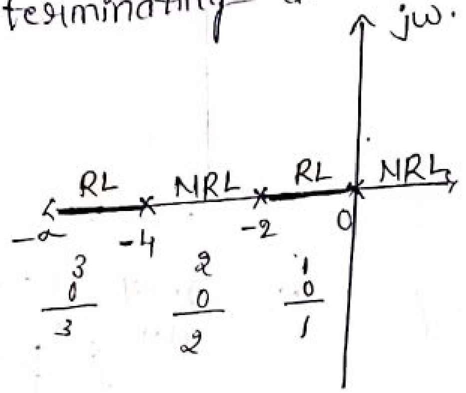
$P > z$, $\quad P = 3, \; z = 0$

P = no. of branches = 3

z = no. of branches terminating at zeroes = 0.

$P - z$ = no. of branches terminating at infinity = 3

3)



4) Angle of asymptotes:

$$\theta = \frac{\pm(2q+1)180^\circ}{P-z}$$

$P - z = 3, \quad q = 0, 1, 2, 3$

$\theta_1 = 60^\circ, \quad \theta_2 = 180^\circ, \quad \theta_3 = 300^\circ, \quad \theta_4 = 420^\circ$

5) Centroid, $c = \dfrac{\Sigma\begin{pmatrix}\text{real part}\\ \text{of poles}\end{pmatrix}}{P-z} = \dfrac{0-2-4}{3} = \dfrac{-6}{3} = -2$

6) CE is $1 + G(s)H(s) = 0$

$$\Rightarrow \quad 1 + \frac{K}{s(s+2)(s+4)} = 0$$

$$\Rightarrow \quad s(s+2)(s+4) + K = 0$$

$$\Rightarrow \quad (s^2+2s)(s+4) + K = 0$$

$$\Rightarrow \quad s^3 + 4s^2 + 2s^2 + 8s + K = 0$$

$$\Rightarrow \quad s^3 + 6s^2 + 8s + K = 0$$

$$\Rightarrow \quad K = -(s^3 + 6s^2 + 8s)$$

$$\frac{dK}{ds} = -(3s^2 + 12s + 8)$$

$$\Rightarrow \frac{dK}{ds} = 0 \Rightarrow 3s^2 + 12s + 8 = 0$$

$$\Rightarrow \quad s = \frac{-12 \pm \sqrt{144 - 4(3)(8)}}{6}$$

$$= \frac{-12 \pm \sqrt{144-96}}{6} = \frac{-12 \pm \sqrt{48}}{6}$$

Scanned by CamScanner

$S = \dfrac{-12 \pm 4\sqrt{3}}{6} = \dfrac{-6 \pm 2\sqrt{3}}{3} = -2 \pm \dfrac{2}{\sqrt{3}}$

$= -0.845 \rightarrow$ BA Point

$S_1 = \dfrac{-6+2\sqrt{3}}{3}$, $S_2 = \dfrac{-6-2\sqrt{3}}{3} = -3.15$

Sub. S_1, S_2 in K'.

$S = S_1 \Rightarrow K_1 = -\left[\left(\dfrac{-6+2\sqrt{3}}{3}\right)^3 + 6\left(\dfrac{-6+2\sqrt{3}}{3}\right)^2 + 8\left(\dfrac{-6+2\sqrt{3}}{3}\right)\right]$

$K_1 = -\left[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)\right]$

$K_1 = -(-0.603 + 4.284 - 6.76)$

$K_1 = 3.07$ ✓, +ve, real (Valid break away)

$K_2 = -3.07$ −ve, real (invalid break away)

7) CE is $s^3 + 6s^2 + 8s + k = 0$

sub. $s = j\omega$.

$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$

$-j\omega^3 - 6\omega^2 + 8j\omega + K = 0$

$(K - 6\omega^2) + j(8\omega - \omega^3) = 0$

$K - 6\omega^2 = 0$ $\quad 8\omega - \omega^3 = 0$

$K = 6\omega^2$ $\quad 8\omega = \omega^{3\,2}$

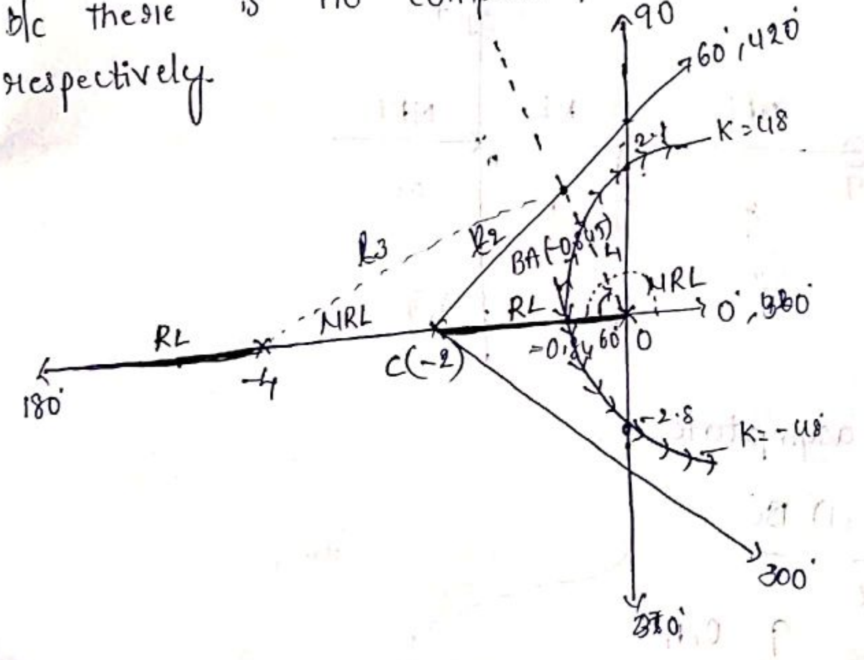
$K = 6(8)$ $\quad \omega^2 = 8$

$K = 48$ $\quad \omega = \pm\sqrt{8}$

$\omega = \pm 2.82$

8) Angle of departure, angle of arrival is not possible b/c there is no complex poles and no complex zeroes respectively.

No-Zeros = 1



Given, damping ratio is 0.5 (ξ) $(0<\xi<1)$

But $\cos\theta = \xi$

$\theta = \cos^{-1}(\xi) = \cos^{-1}(0.5)$

$\theta = 60^\circ$

$K = \frac{l_1 l_2 l_3}{1}$

* The unity fb system has OLTS $G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$

Sketch the root locus.

Given, $G(s) = \frac{K(s+9)}{s(s^2+4s+11)}$, $H(s) = 1$

1. $G(s)H(s) = -1$

$$\frac{K(s+9)}{s(s^2+4s+11)} = -1$$

2 Zeroes at $s = -9$

Poles at $s_1 = 0$, $s_2 = -2+j\sqrt{7}$ $\overset{j2.6}{}$ $s_3 = -2-j\sqrt{7}$

$$s = \frac{-4 \pm \sqrt{16-4(1)(11)}}{2} = \frac{-4 \pm \sqrt{-28}}{2} = \frac{-4 \pm j2\sqrt{7}}{2}$$

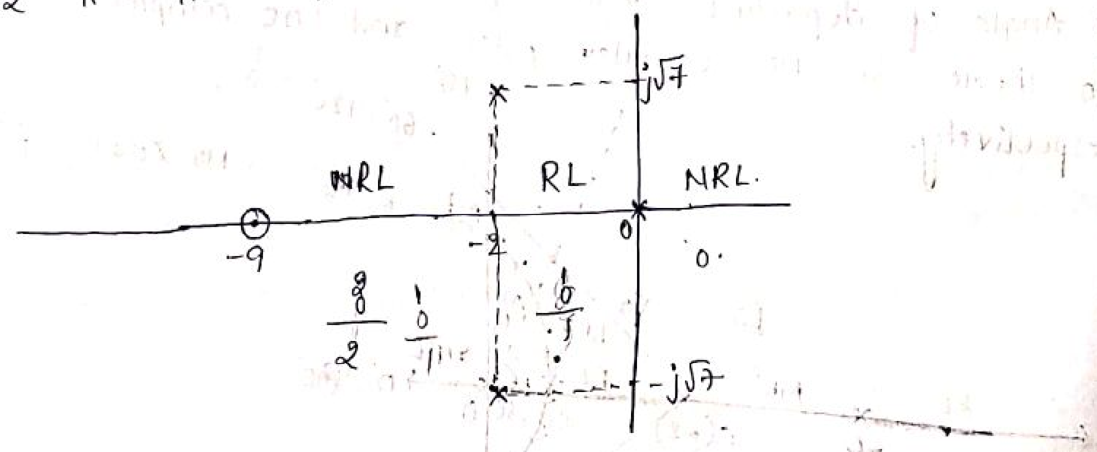
$P>Z$, $P = 3$, $Z = 1$.

P = no. of branches = 3

Z = 1 " " " terminating at zero.

P-Z = 2 " " " " " infinity.

3)



4) Angle of asymptotes:

$$\theta = \frac{\pm(2q+1)180^\circ}{P-Z}$$

$P-Z = 2$, $q = 0,1,2$

$\theta_1 = \frac{180°}{2} = 90°$

$\theta_2 = \frac{3 \times 180°}{2} = 270°$

$\theta_3 = \frac{5 \times 180°}{2} = 450°$

5) centroid, $C = \frac{\Sigma \text{ real part of poles} - \Sigma \text{ real part of zeros}}{p-z}$

$$C = \frac{(0-2-2) - (-9)}{2}$$

$$C = \frac{-4+9}{2} = \frac{5}{2} = 2.5$$

6) CE is $1 + G(s)H(s) = 0$

$$1 + \frac{K(s+9)}{s(s^2+4s+11)} = 0$$

$$s(s^2+4s+11) + K(s+9) = 0$$

$$s^3 + 4s^2 + 11s + Ks + 9k = 0$$

$$s^3 + 4s^2 + s(K+11) + 9k = 0$$

Since, the point lies b/w one zero and one pole, there may exist either break away (or) break in.

7) CE is $s^3 + 4s^2 + s(11+K) + 9k = 0$

sub. $s = j\omega$

$\Rightarrow (j\omega)^3 + 4(j\omega)^2 + (j\omega)(11+K) + 9k = 0$

$\Rightarrow -j\omega^3 - 4\omega^2 + 11j\omega + Kj\omega + 9k = 0$

$\Rightarrow (9k - 4\omega^2) + j(11\omega + K\omega - \omega^3) = 0$

$\Rightarrow 9k - 4\omega^2 = 0$; $11\omega + K\omega - \omega^3 = 0$

$9k = 4\omega^2$; $\omega^3 - 11\omega - K\omega = 0$

$\omega^2 = \frac{9k}{4}$; $\omega^{\cancel{3}2} = \cancel{\omega}(K+11)$

$\omega^2 = 19.8$; $\omega^2 = K+11$

$\omega = \pm 4.44$

$\frac{9k}{4} = K+11$

$9k = 4K + 44$

$5K = 44$

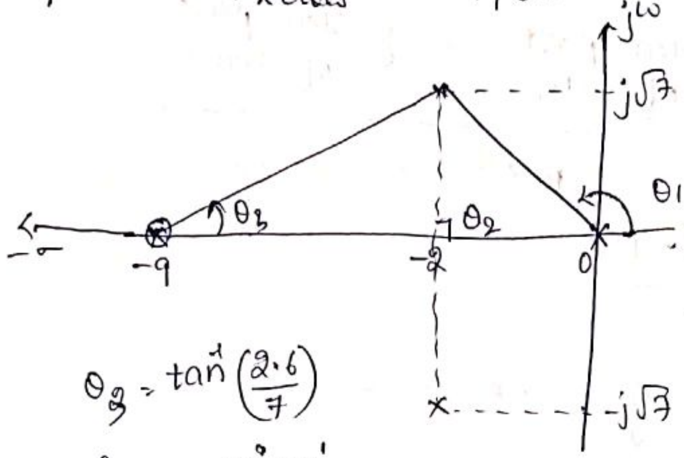
$k = 44/5 = 8.8$

Scanned by CamScanner

Since, there exists complex poles. So, angle of departure exists

$\phi_D = 180° + \phi$

$\phi = \Sigma \phi_{zeroes} - \Sigma \phi_{poles}$



$\theta_1 = 180° - \tan^{-1}\left(\frac{\sqrt{7}}{2}\right) = 180° - 52.\dots$
$127.57'$

$\theta_2 = 90°$

$\theta_3 = \tan^{-1}\left(\frac{2.6}{7}\right)$

$\theta_3 = 20°.37'$

$\phi = 20°.37' - (127.57' + 90°) = \cancel{-107.2}\ -197.2$

$\phi_D = 180° - 197.2 = \cancel{72.8}\ -17.2$

* Sketch the root locus for the unity feed back system whose open loop transfer function is $G(s) = \frac{K}{s(s^2+6s+10)}$

1. Given, $G(s) = \frac{K}{s(s^2+6s+10)}$; $H(s) = 1$

$G(s)H(s) = -1$

$\frac{K}{s(s^2+6s+10)} = -1$

2. Poles at $s = 0$;

$s = \frac{-6 \pm \sqrt{36-4(1)(10)}}{2} = \frac{-6 \pm \sqrt{-4}}{2} = \frac{-6 \pm 2j}{2} = -3 \pm j$

$s_1 = 0$; $s_2 = -3 + j$; $s_3 = -3 - j$

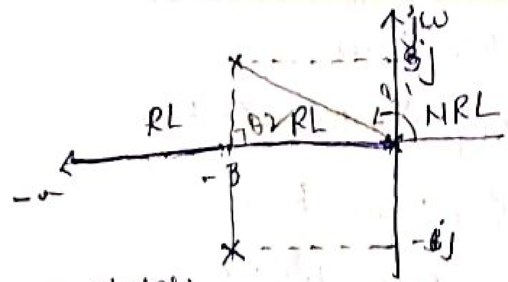
$P > Z$, $P = 3$, $Z = 0$.

P = no. of branches.

Z = 0 " " " terminating at zero.

P-Z = no. of branches terminating at infinity

3)



4) Angle of asymptotes:

$$\theta = \frac{\pm(2q+1)180^\circ}{p-z} \quad ; \quad p-z=3 \quad q=0,1,2,3$$

$\theta_1 = 60^\circ$; $\theta_2 = 180^\circ$; $\theta_3 = 300^\circ$; $\theta_4 = 420^\circ$

5) $C = \frac{(0-3-3)}{3} = \frac{-6}{3} = -2$

6) CE is $1 + G(s)H(s) = 0$

$$1 + \frac{K}{s(s^2+6s+10)} = 0$$

$\Rightarrow s(s^2+6s+10) + K = 0$

$\Rightarrow s^3 + 6s^2 + 10s + K = 0$

$K = -(s^3 + 6s^2 + 10s)$

$\Rightarrow \frac{dK}{ds} = 0 \Rightarrow -(3s^2+12s+10) = 0$

$\Rightarrow 3s^2+12s+10 = 0$

$\Rightarrow s = \frac{-12 \pm \sqrt{144-4(3)(10)}}{6}$

$= \frac{-12 \pm \sqrt{24}}{6} = \frac{-12 \pm 4.8}{6}$

$s_1 = -1.2$; $s_2 = -2.8$

sub. s_1, s_2 in K.

$s = s_1 \Rightarrow K_1 = -[(-1.2)^3 + 6(-1.2)^2 + 10(-1.2)] = \cancel{4.288}$ 5.08

$s = s_2 \Rightarrow K_2 = -[(-2.8)^3 + 6(-2.8)^2 + 10(-2.8)] = 2.912$

The value which is higher is break away.

s_1 is break away point

s_2 is break in point

7) CE is $s^3 + 6s^2 + 10s + K = 0$

$s = j\omega \Rightarrow (j\omega)^3 + 6(j\omega)^2 + 10(j\omega) + K = 0$

$-j\omega^3 - 6\omega^2 + 10j\omega + K = 0$

$(K - 6\omega^2) + j(10\omega - \omega^3) = 0$

$K - 6\omega^2 = 0$; $10\omega - \omega^3 = 0$

$10 - \omega^2 = 0$

$K - 6 \times 10 = 0$; $\omega^2 = 10$

$K = 60$; $\omega = \pm 3.16$

8) Since, there exists complex poles. So, angle of departure exists

$\phi_D = 180° + \phi$

$\theta_1 = 180° - \tan^{-1}\left(\frac{1}{3}\right) = 161.6°$

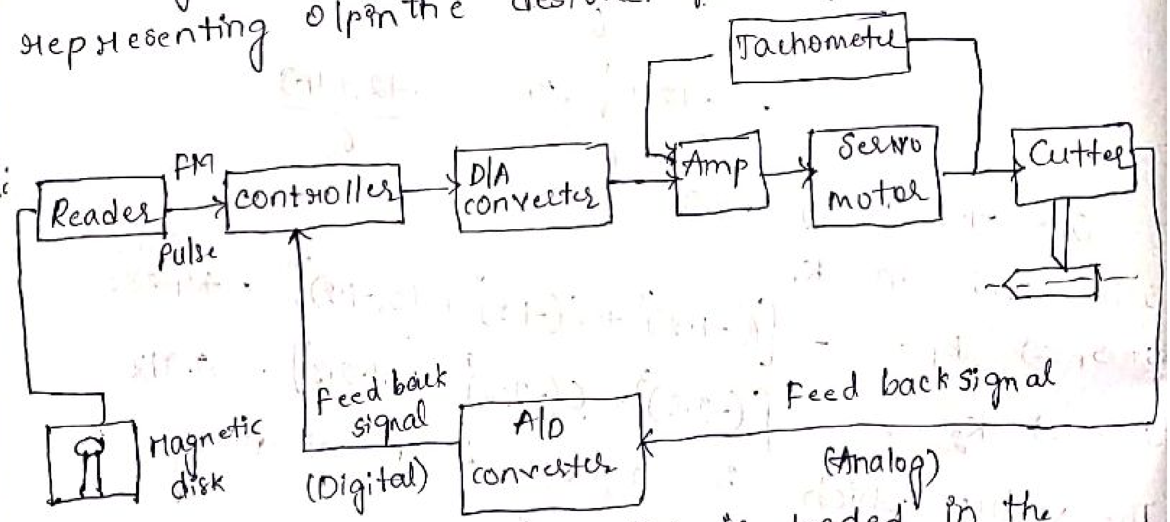
$\theta_2 = 90°$

$\phi = -(161.6 + 90) = -251.6°$

$\phi_D = -71.6°$

Closed loop System:-

* A magnetic disk is prepared in the binary form representing o/p in the desired part p.



* To start the system, the disc is loaded in the reader. The controller ~~component~~ compares the frequency modulated i/p pulse signal with the feedback pulse signal

* The controller carries out mathematically operation

on the difference in the pulse signal & generates an error signal

* The amplified analog signal rotates the servomotor to position tool on job. The transducer attached to cutterhead convert the motion into an electrical signal

* Then electrical signal – digital phase signal by A/D converter; this sgl is compared with the i/p pulse signal.

* If there is any difference b/w these 2, the controller sends a sgl to the servometer to reduce it

* Thus the system automatically corrects any deviation in the desired o/p tool position.

Advantages :-

That complex parts can be produced with uniform tolerances at the maximum milling speed.

28/8/17

UNIT-IV

Stability & Root Locus

Stability:-

The term stability refers to the stable working condition of a control system. In the stable system, the output is protectable and finite and stable for a given input. If the system output is stable for all the variations of its parameters, then the system is called absolutely stable system.

If a system output is stable for a limited range of variations of its parameters, then the system is called conditionally stable system.

ROUTH HURWITZ Criteria:-

(The Routh Hurwitz Criteria can be stated as follows)

The necessary condition for stability is all the coefficients of the polynomial be positive.

If some of the coefficients are zero (or) negative then, it can be concluded that the system is not stable.

When all the coefficients are positive, the system need not to be stable because eventhough the coefficients are positive some of the roots may lie on right half of be positive and the roots should lie on left half of s-plane.

Let us consider the characteristic polynomial be

$$a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s^1 + a_n s^0 = 0$$

s^n : a_0 a_2 a_4 ----

s^{n-1} : a_1 a_3 a_5 -----

s^{n-2} : b_1 b_2 b_3 -----

s^{n-3} : c_1 c_2 c_3 ----

⋮

s^1 : g_0

s^0 : h_0

where $b_1 = \dfrac{-\begin{vmatrix} a_0 & a_2 \\ a_1 & a_3 \end{vmatrix}}{a_1}$ $b_2 = \dfrac{-\begin{vmatrix} a_0 & a_4 \\ a_1 & a_5 \end{vmatrix}}{a_1}$

$$b_3 = \dfrac{-\begin{vmatrix} a_0 & a_6 \\ a_1 & a_7 \end{vmatrix}}{a_1}$$

$$C_1 = \dfrac{-\begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix}}{b_1} \qquad C_2 = \dfrac{-\begin{vmatrix} a_1 & a_5 \\ b_1 & b_3 \end{vmatrix}}{b_1}$$

The routh hurwitz Criteria can be stated as follows

The necessary and sufficient Condition for stability is that, all the elements in the first coloumn of routh array be positive, if this Condition is not satisfied then, the system is unstable and the no. of sign right half of s-plane.

29/8/17

In the process of Constructing routh array, the missing terms are Considered as zero,

In the Construction of routh array one may Come across the following three Cases.

Case-i Normal Routh Array

Case-ii A row of all zeroes.

case-iii First element of row is zero but other elements are non-zeroes.

Problems

1) Using Routh Criteria, determine the stability of the System represented by the characteristic equation $s^4+8s^3+18s^2+16s+5=0$ Comment on the location of ~~rout~~ roots of characteristic equation.

Sol:- Given characteristic equation

$s^4+8s^3+18s^2+16s+5$ as it is of 4^{th} order.

It Contains 4 roots.

Routh Array

s^4 :	1	18	5	Row 1
s^3 :	8	16	0	Row 2

Dividing row 2 with 8

s^3 :	1	2	0	Row 2
s^2 :	$\frac{18\times1-2\times1}{1}$	$\frac{5\times1-0\times1}{1}$		
s^2 :	16	5		Row 3

$s^0 : \frac{1.7\times5 - 16\times0}{1.7} = \frac{8.5}{1.7} = 5$

s^4 :	1	18	5	Row 1
s^3 :	1	2		Row 2
s^2 :	16	5		Row 3
s^1 :	1.7			Row 4
s^0 :	5			Row 5

As all the elements of first coloumn are positive hence the system is stable and all the four roots lie on left half of s-plane.

2) Construct Routh Array and determine the stability of the system whose characteristic equation is $s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$. Determine the no.of roots lying on right half of s-plane, left half of s-plane and on imaginary axis.

Sol:- Given characteristic equation

$s^6+2s^5+8s^4+12s^3+20s^2+16s+16=0$

Routh Array

s^6 :	1	8	20	16
s^5 :	2	12	16	0

Dividing s^5 row with 2.

s^5 :	1	6	8	0
s^4 :	$\frac{8-6}{1}$	$\frac{20-8}{1}$	$\frac{16-0}{1}$	
s^4 :	2	12	16	

Dividing s^4 row with 2

4

The auxillary equation is $s^4+6s^2+8=0$

Cons Differentiate w.r.t 's'

$$4s^3+12s=0$$

s^3 : 4 12

Dividing s^3 row with 4

s^3 : 1 3

s^2 : 3 8

s^1 : $\frac{1}{3}=0.3$

s^0 : $\frac{2.4}{0.3}=8$

s^6	1	8	20	16
s^5	1	6	8	
s^4	1	6	8	
s^3	1	3		
s^2	3	8		

s^3	0	0
s^3	1	3
s^2	3	8
s^1	0.3	
s^0	8	

30/8/17

As there is no sign change in the elements of first coloumn none of the roots lie on right half of s-plane, but the row with all zeroes indicate the possibility of roots on imaginary axis.

∴ The auxillary equation is $s^4+6s^2+8=0$

$$x^2+4x+2x+8=0$$
$$x(x+4)+2(x+4)=0$$
$$(x+4)(x+2)=0$$
$$x=-4,\ x=-2$$

as $s^2=x$

$$s^2=-4,\ s^2=-2$$
$$s=\pm\sqrt{-4}\ ;\ s=\pm\sqrt{-2}$$
$$s=\pm j\sqrt{4}\ ;\ s=\pm j\sqrt{2}$$

$\therefore$ the roots of auxillary equation are

$$s=-j\sqrt{4},\ -j\sqrt{2},\ j\sqrt{2},\ j\sqrt{4}$$

$\therefore$ the roots of auxillary equation lie on imaginary axis.

Hence four roots lie on imaginary axis and two roots lie on left half of s-plane. Therefore, the system is limitedly stable.

3) Construct routh array and determine the stability of a system represented by the characteristic equation $s^5+s^4+2s^3+2s^2+3s+5=0$ Comment the location of roots of characteristic equation.

<u>Sol</u>:- Given characteristic equation is

$$s^5+s^4+2s^3+2s^2+3s+5=0$$

<u>Routh</u> <u>Array</u>:-

s^5 :	1	2	3
s^4 :	1	2	5
s^3 :	0	-2	

$$s^2 : \quad \frac{2\varepsilon+2}{\varepsilon} \qquad 5$$

$$s^1 : \quad \frac{\frac{-4\varepsilon-4}{\varepsilon} - 5\varepsilon}{\frac{2\varepsilon+2}{\varepsilon}} = \frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2}$$

$$s^0 : \quad \frac{5\left[\cancel{\frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2}}\right]}{\cancel{\frac{-4\varepsilon-4-5\varepsilon^2}{2\varepsilon+2}}} = 5$$

Replacing ε with zero.

s^5 :	1	2	3
s^4 :	1	2	5
s^3 :	0	-2	
s^2 :	∞	5	
s^1 :	-2		
s^0 :	5		

By observing the elements of first coloumn, it is found that there are two sign changes.

$\therefore$ Two roots lie on right half of s-plane and remaining three roots lie on left half of s-plane.

$\therefore$ The system is not stable.

4) Find routh stability criteria determine the stability of the system represented by the characteristic equation $9s^5 - 20s^4 + 10s^3 - s^2 - 9s - 10 = 0$ Comment on the location of the roots of characteristic equation.

-eristic equation it is found that some of the coefficients are negative. Hence, some roots lie on right half of s-plane. Therefore, the system is not stable.

The routh array is constructed to find the no. of roots lying on right half of s-plane.

<u>Routh</u> <u>Array</u>

s^5 : 9 10 −9

s^4 : −20 −1 −10

s^3 : $\frac{-200+9}{-20}$ $\frac{180+90}{-20}$

s^3 : 9.5 −13.5

s^2 : $\frac{-9.5 + -270}{9.5}$ $\frac{-95}{9.5}$

s^2 : −29.42 −10

s^1 : $\frac{397.17+95}{-29.42} = -16.72$

s^0 : $\frac{167.2}{-16.72} = -10.$

s^5 :	9	10	−9
s^4 :	−20	−1	−10
s^3 :	9.5	−13.5	
s^2 :	−29.42	−10	
s^1 :	−16.72		
s^0 :	−10		

By observing the elements of first coloumn, there are three sign changes. Therefore, three roots lie on right half of s-plane and

5/9/17

Root Locus:-

Problems

1) A unity feedback control system has an open loop transfer function $G(s) = \frac{k}{s[s^2+4s+13]}$. Sketch the root locus.

Sol:- Given open loop transfer function is

$$G(s) = \frac{k}{s[s^2+4s+13]}$$

Step-1:- To locate poles and zeroes.

For a given transfer function there are no zeroes.

The poles of given transfer function are obtained by getting the roots of the denominator.

$$s[s^2+4s+13] = 0$$

$$\Rightarrow s=0, \; s^2+4s+13=0$$

$$\Rightarrow s=0, \; s = \frac{-4 \pm \sqrt{16-52}}{2}$$

$$\Rightarrow s=0, \; s = \frac{-4 \pm \sqrt{-36}}{2}$$

$$\Rightarrow s=0, \; s = \frac{-4 \pm j6}{2}$$

$$\Rightarrow s=0; \; -2+j3; \; -2-j3$$

The poles are marked as 'x' and zeroes are marked as 'o'.

Step-2:- To find the root locus on real axis

There is only one pole on real axis at origin. Hence if we choose any test point of the test point there exists only one pole which is an odd number. Hence the entire negative real axis will lie on root locus.

Step-3:- To find the angles of asymptotes and centroid

The no.of asymptotes will be equal to $n-m$

where n is no.of poles i.e., $n=3$

m is no.of zeroes i.e., $m=0$

$\therefore$ The no.of asymptotes $= n-m = 3-0 = 3$

Angles of asymptotes $= \pm \frac{180^\circ(2q+1)}{n-m}$

where $q = 0,1,2 \ldots\ldots n-m$

$\Rightarrow q = 0,1,2,3$

when $q=0$

Angle of asymptotes $= \pm \frac{180^\circ}{3} = \pm 60^\circ$

when $q=1$

Angle of asymptotes $= \pm \frac{180^\circ \times 3}{3} = \pm 180^\circ$

when $q=2$

Angle of asymptotes $= \pm \frac{180^\circ \times 5}{3} = \pm 300^\circ = \mp 60^\circ$

when $q=3$

Angle of asymptotes $= \pm \frac{180^\circ \times 7}{3} = \pm 420^\circ = \pm 60^\circ$.

Centroid (G) $= \frac{\text{Sum of poles} - \text{Sum of Zeroes}}{n-m}$

$\Rightarrow$ Centroid $= \frac{-2+j3-2-j3}{3} = \frac{-4}{3} = -1.3$

6/9/17

Step-4:- To find break away and break in point

the closed loop transfer function is

C(S) ...

Considering $H(S)=1$

$$\therefore \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$$

$$\therefore \frac{C(S)}{R(S)} = \frac{\frac{k}{S[S^2+4S+13]}}{1+\frac{k}{S[S^2+4S+13]}} = \frac{k}{S[S^2+4S+13]+k}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{k}{S^3+4S^2+13S+k}$$

$\therefore$ the characteristic equation is

$$S^3+4S^2+13S+k=0$$

$$\Rightarrow k = -[S^3+4S^2+13S] \quad \text{—— (1)}$$

Differentiate k w.r.t 'S'

$$\frac{dk}{dS} = -[3S^2+8S+13]$$

Put $\frac{dk}{dS}=0$

$$3S^2+8S+13=0$$

$$S = \frac{-8 \pm \sqrt{64-156}}{6}$$

$$\Rightarrow S = \frac{-8 \pm \sqrt{-92}}{6}$$

$$\Rightarrow S = \frac{-8 \pm j9.5}{6}$$

$$S = -1.3 \pm j1.5$$

$$\Rightarrow S = -1.3-j1.5 \,;\, -1.3+j1.5$$

when $S = -1.3-j1.5$

$$(1) \Rightarrow k = -[S^3+4S^2+13S]$$

$$\Rightarrow k = -\{(-1.3-j1.5)^3+4(-1.3-j1.5)^2+13(-1.3-j1.5)\}$$

$① \Rightarrow k = -[s^3 + 4s^2 + 13s]$

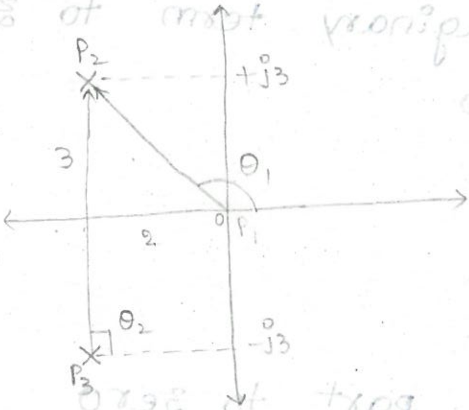
$\Rightarrow k = -[(-1.3 + j1.5)^3 + 4(-1.3 + j1.5)^2 + 13(-1.3 + j1.5)]$

$\Rightarrow k \neq \text{real}$

Since the values of k for $s = -1.3 \pm j1.5$ are not real and positive. The root locus has neither break away or break in points.

<u>Step-5:-</u> To find angle of departure

Let us Consider the Complex pole P_2, draw Vectors from all other poles to P_2



Let the angles of these vectors be θ_1 & θ_2

$\therefore \theta_1 = 180° - \tan^{-1}\left[\frac{3}{2}\right]$

$\Rightarrow \theta_1 = 180° - 56.3°$

$\Rightarrow \theta_1 = 123.6°$

$\therefore \theta_2 = 90°$

$\therefore$ Angle of departure from Complex pole P_2 is

$P_2 = 180° - [\theta_1 + \theta_2]$

$P_2 = 180° - [123.6° + 90]$

$P_2 = -33.6°$

The angle of departure from Complex pole

$\therefore$ Angle of departure at $P_3 = +33.6^\circ$

Step-6:- To find crossing point on imaginary axis.

The characteristic equation is

$$s^3+4s^2+13s+k=0$$

put $s=j\omega$

$$\Rightarrow (j\omega)^3+4(j\omega)^2+13(j\omega)+k=0$$

$$\Rightarrow -j\omega^3-4\omega^2+13j\omega+k=0$$

$$\Rightarrow k-4\omega^2+j(13\omega-\omega^3)=0$$

On equating imaginary term to zero

$$13\omega-\omega^3=0$$

$$\Rightarrow 13=\omega^2$$

$$\Rightarrow \omega=\pm\sqrt{13}$$

$$\Rightarrow \boxed{\omega=\pm 3.6}$$

On equating real part to zero

$$k-4\omega^2=0$$

$$\Rightarrow k=4\omega^2$$

$$\Rightarrow k=4(3.6)^2=51.84\approx 52$$

The crossing point of root locus is $\pm j3.6$ and the value of k at its crossing point is 52.

7/9/17

2) Sketch the root locus of the system whose transfer function is $G(s)=\dfrac{k}{s[s+2][s+4]}$. Find the value of k so that the damping ratio of closed loop system is 0.5.

$$G(s) = \frac{k}{s[s+2][s+4]}$$

Step-1:- To locate poles and zeroes

No. of zeroes $m = 0$

The poles of the transfer function are the roots of equation

$$s[s+2][s+4] = 0$$

$$\Rightarrow s = 0, -2, -4$$

$\therefore P_1 = 0, \; P_2 = -2, \; P_3 = -4$

Step-2:- To find the root locus on real axis.

Test point b/w 0 and -2

To the right of this test point the total no. of real poles and zeroes is one which is an odd number. Hence real axis between $s=0$ and $s=-2$ lies on root locus.

Test point b/w -2 and -4

To the right of this test point, the total no. of real poles and zeroes is two which is an even number. Hence negative real axis b/w $s=-2$ and $s=-4$ does not lie on root locus.

Test point ~~afte~~ to the left of $s=-4$

To the right of this test point, the total no. of real poles and zeroes is three which is an odd number. Hence negative real axis from $s=-4$ to $-\infty$ lies on root locus.

Step-3:- To find angle of asymptotes and centroid.

where $q = 0, 1, 2, \ldots\ldots n-m$

$n - m = 3 - 0 = 3.$

$\Rightarrow q = 0, 1, 2, 3$

when $q = 0$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ [2q+1]}{n-m}$$

$$= \frac{\pm 180^\circ}{3} = \pm 60^\circ$$

when $q = 1$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ [2q+1]}{n-m} = \frac{\pm 180^\circ \times 3}{3}$$

$$= \pm 180^\circ$$

when $q = 2$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ [2q+1]}{n-m} = \frac{\pm 180^\circ \times 5}{3}$$

$$= \pm 300^\circ = \mp 60^\circ$$

when $q = 3$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ [2q+1]}{n-m} = \frac{\pm 180^\circ \times 7}{3}$$

$$= \pm 420^\circ = \pm 60^\circ$$

$$\text{Centroid } (G) = \frac{\text{Sum of poles} - \text{Sum of zeroes}}{n-m}$$

$$G = \frac{0 - 2 - 4}{3} = -2.$$

Step-4:- To find break away and break in points

The closed loop transfer function is

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)} = \frac{G(S)}{1 + G(S)} \quad [\because H(S) = 1]$$

$$\frac{C(S)}{} = \frac{K}{S[S+2][S+4]}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s[s+2][s+4]+k}$$

$$\frac{C(s)}{R(s)} = \frac{k}{[s^2+2s][s+4]+k}$$

$$\frac{C(s)}{R(s)} = \frac{k}{s^3+6s^2+8s+k}$$

The characteristic equation is given by

$$s^3+6s^2+8s+k=0$$

$$k=-[s^3+6s^2+8s] \quad \text{—— ①}$$

Differentiate k w.r.t 's'

$$\frac{dk}{ds} = -[3s^2+12s+8]$$

Put $\frac{dk}{ds}=0$

$$\therefore\ 3s^2+12s+8=0$$

$$s=\frac{-12 \pm \sqrt{144-96}}{6} = -2 \pm \overset{1.15}{\cancel{0.9}}$$

$$s=-2+\overset{1.15}{\cancel{0.9}}\ ;\ -2-\overset{1.15}{\cancel{0.9}}$$

$$s=\cancel{-1.1\ ;\ -2.9} \quad -0.85\ ;\ -3.15$$

when $s=-0.85$

$$① \Rightarrow \quad k=-[s^3+6s^2+8s]$$

$$k=-[(-0.85)^3+6(-0.85)^2+8(-0.85)]$$

$$k=3.07$$

when $s=-3.15$

$$① \Rightarrow \quad k=-[(-3.15)^3+6(-3.15)^2+8(-3.15)]$$

$$k=-3.07$$

since k is positive and real for $s=-0.85$

Step-5:- To find angle of departure

Since there are no complex pole or zero we will not find angle of departure.

Step-6:- To find crossing point on imaginary axis.

The characteristic equation is

$$s^3+6s^2+8s+k=0$$

Put $s=j\omega$

$$\therefore (j\omega)^3+6(j\omega)^2+8(j\omega)+k=0$$

$$-j\omega^3-6\omega^2+j8\omega+k=0$$

$$k-6\omega^2+j(8\omega-\omega^3)=0$$

Equating imaginary values to '0'.

$$8\omega-\omega^3=0$$

$$\Rightarrow 8-\omega^2=0$$

$$\Rightarrow 8=\omega^2$$

$$\Rightarrow \omega=\pm\sqrt{8}$$

$$\Rightarrow \omega=\pm 2.8$$

Equating real values to zero

$$k-6\omega^2=0$$

$$\Rightarrow k=6\omega^2$$

$$\Rightarrow k=6\times 8$$

$$k=48$$

The crossing of root locus is $\pm j2.8$ and the value of k at this crossing point is 48.

9/9/17

To find value of k for $\xi=0.5$

Let $\alpha=\cos^{-1}0.5=60^\circ$

Draw a line OP such that the angle between line OP and negative real axis is 60°. The meeting point of line OP and root locus be

$$k_{s_d} = \frac{\text{Product of length of vector from all Poles to the point, } s=s_d}{\text{Product of length of vector from all Zeroes to the point, } s=s_d}$$

$$k_{s_d} = \frac{1.3 \times 1.8 \times 3.5}{1} = 8.19$$

3) The open loop transfer function of unity feedback system is given by $G(s) = \frac{k(s+9)}{s[s^2+4s+11]}$

Sketch the root locus of the system.

Sol:- Given the open loop transfer function is

$$G(s) = \frac{k(s+9)}{s[s^2+4s+11]}$$

Step-1:- To locate poles and zeroes

The zeroes from in given transfer function are obtained from $s+9=0$

$$\Rightarrow s = -9$$

$$\therefore Z = -9$$

The poles of the transfer function is obtained from $s[s^2+4s+11]=0$

$$s=0;\ \frac{-4 \pm \sqrt{16-44}}{2}$$

$$s=0;\ \frac{-4 \pm j5.29}{2}$$

$$s=0;\ -2+j2.64;\ -2-j2.64$$

$$\therefore P_1 = 0;\ P_2 = -2-j2.64;\ P_3 = -2+j2.64.$$

$\therefore$ no. of poles $n=3$

no. of zeroes $m=1$

Step-2:- To find the root locus on real axis

choose a test point in between 0 and

is one which is an odd number. Hence the negative real axis in between 0 and -9 lies on root locus.

choose a test point to the left of s=-9, to the right of this test point, the total no. of poles and zeroes is two which is an even number. Hence the negative real axis from -9 to -∞ does not lie on root locus.

9/9/17

Step-3:- To find angle of asymptotes and centroid

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ(2q+1)}{n-m}$$

where n = no. of poles = 3

m = no. of zeroes = 1

$q = 0, 1, 2 \ldots\ldots n-m$

$q = 0, 1, 2$

when $q=0$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ(1)}{2} = \pm 90^\circ$$

when $q=1$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ(3)}{2} = \pm 270^\circ = \mp 90^\circ$$

when $q=2$

$$\text{Angle of asymptotes} = \frac{\pm 180^\circ(5)}{2} = \pm 450^\circ = \pm 90^\circ$$

$$\text{Centroid } (G) = \frac{\text{Sum of Poles} - \text{Sum of Zeroes}}{n-m}$$

$$G = \frac{0 - 2 - \cancel{j2.64} - 2 + \cancel{j2.64} - (-9)}{2}$$

$$G = \frac{5}{2} = 2.5$$

Step-4:- To find break away and break in points

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

Let $H(S)=1$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{\frac{K(S+9)}{S[S^2+4S+11]}}{1+\frac{K(S+9)}{S[S^2+4S+11]}}$$

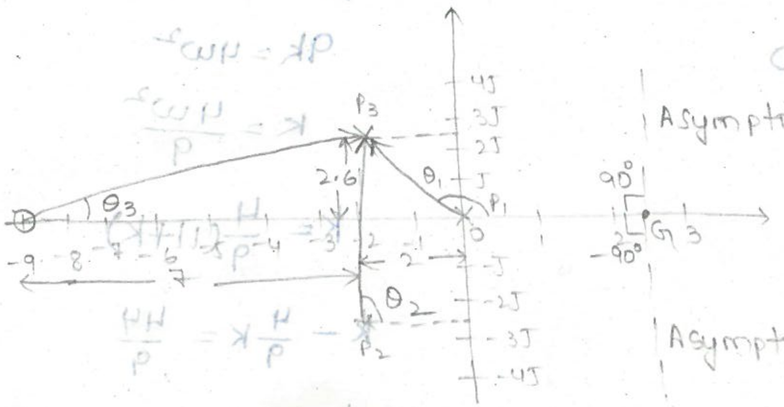
$$\Rightarrow \frac{C(S)}{R(S)} = \frac{K(S+9)}{S[S^2+4S+11]+K(S+9)}$$

$$\Rightarrow \frac{C(S)}{R(S)} = \frac{K(S+9)}{S^3+4S^2+11S+KS+9K}$$

By ~~seeing~~ observing the location of poles and zeroes it can be concluded that there will not be any ~~br~~ break away or break in points.

<u>Step-5</u>:- To find angle of departure

Let us consider the complex pole P_3, draw vectors from all other poles and zeroes to the pole P_3. Let the angles be θ_1, θ_2 and θ_3.



Angle of departure $= 180^\circ$ − (sum of angles at Poles) + (sum of angles at zeroes)

$$= 180^\circ - (\theta_1+\theta_2)+\theta_3$$

$\theta_1 = 180^\circ - \tan^{-1}[2.6]$

$\theta_3 = \tan^{-1}\left[\frac{2.6}{7}\right] = 20.37^0$

$\therefore$ Angle of departure $= 180^0 - [127.1^0 + 90^0] + 20.37^0$

$= -16.7^0 \approx \cancel{17} - 17^0$

As the angle of departure at Complex pole P_2 is negative to the angle of departure at Complex pole P_3.

$\therefore$ Angle of departure at pole P_2 is $+17^0$

<u>Step-6</u>:- To find Crossing point on imaginary axis

The characteristic equation is

$$s^3 + 4s^2 + 11s + ks + 9k = 0$$

Put $s = j\omega$

$$\therefore (j\omega)^3 + 4(j\omega)^2 + 11(j\omega) + k(j\omega) + 9k = 0$$

$$-j\omega^3 - 4\omega^2 + j11\omega + kj\omega + 9k = 0$$

$$(9k - 4\omega^2) + j(11\omega + k\omega - \omega^3) = 0$$

Equating imaginary values to '0'

$11\omega + k\omega - \omega^3 = 0$

$11 + k - \omega^2 = 0$

$\Rightarrow 11 + k = \omega^2$

$\omega^2 = 11 + 8.8$

$\omega^2 = 19.8$

$\omega = \pm\sqrt{19.8}$

$\omega = \pm 4.4$

Equating real values to zero.

$9k - 4\omega^2 = 0$

$9k = 4\omega^2$

$k = \frac{4\omega^2}{9}$

$k = \frac{4}{9}(11 + k)$

$k - \frac{4}{9}k = \frac{44}{9}$

$\frac{9k - 4k}{9} = \frac{44}{9}$

$\frac{5k}{9} = \frac{44}{9}$

$5k = 44$

$k = \frac{44}{5} = 8.8$

4) Sketch the root locus for unity feedback system whose open loop transfer function is $G(s) = \frac{k}{s[s^2+6s+10]}$

Sol:- Given open loop transfer function is

$$G(s) = \frac{k}{s[s^2+6s+10]}$$

Step-1:- To find Poles and zero's

There are no zeroes that is $m=0$

The poles of the transfer function are obtain from $s[s^2+6s+10]=0$.

$$s=0,\ s^2+6s+10=0$$

$$s=0,\ \frac{-6 \pm \sqrt{36-40}}{2}$$

$$s=0,\ \frac{-6 \pm j2}{2}$$

$$s=0, -3-j ; -3+j$$

$$P_1=0 ; P_2=-3-j ; P_3=-3+j$$

∴ No. of poles $n=3$

Step-2:- To find the root locus on real axis

There is only one pole on real axis at origin. Take a test point to the left of $s=0$, to the right of that point the total no. of poles and zeroes is 1 which is an odd number. Hence the entire negative real axis from 0 to $-\infty$ lies on root locus.

Step-3:- To find angle of asymptotes and Centroid

$$\text{Angle of Asymptotes} = \frac{\pm 180°(2q+1)}{n-m}$$

where $q = 0, 1, 2 \text{ ------ } n-m$

$$\Rightarrow n-m = 3-0 = 3$$

$\Rightarrow n-m \ldots$

Angle of asymptotes $= \frac{\pm 180^\circ}{3} = \pm 60^\circ$

when $q = 1$

Angle of asymptotes $= \frac{\pm 180^\circ \times 3}{3} = \pm 180^\circ$

when $q = 2$

Angle of asymptotes $= \frac{\pm 180^\circ \times 5}{3} = \pm 300^\circ = \mp 60^\circ$

when $q = 3$

Angle of asymptotes $= \frac{\pm 180^\circ \times 7}{3} = \pm 420^\circ = \pm 60^\circ$

Centroid $(G) = \frac{0 - 3 - \cancel{j} - 3 + \cancel{j}}{3}$

$G = -2$

Step-4:- To find break away and break in points

The closed loop transfer function is given as

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S} \quad (\because H(S) = 1)$$

$$\frac{C(S)}{R(S)} = \frac{\frac{k}{S[S^2+6S+10]}}{1 + \frac{k}{S[S^2+6S+10]}}$$

$$\frac{C(S)}{R(S)} = \frac{k}{S^3 + 6S^2 + 10S + k}$$

The characteristic equation is $S^3 + 6S^2 + 10S + k = 0$

$$k = -[S^3 + 6S^2 + 10S]$$

Differentiate 'S' w.r.t to k.

$$\frac{dk}{dS} = -[3S^2 + 12S + 10]$$

$\therefore$ As $\frac{dk}{dS} = 0 \Rightarrow 3S^2 + 12S + 10 = 0$

$\rightarrow S = -12 \pm \sqrt{144 - 120}$

$$s = \frac{-12 \pm \sqrt{24}}{6}$$

$$s = \frac{-12 \pm 4.9}{6}$$

$$s = \frac{-16.9}{6}$$

$$s = -2.8 ; -1.2$$

when $s = -2.8$

$$k = -\left[s^3 + 6s^2 + 10s\right]$$

$$k = -\left[(-2.8)^3 + 6(-2.8)^2 + 10(-2.8)\right]$$

$k = 2.9$ Real & positive

when $s = -1.2$

$$k = -\left[s^3 + 6s^2 + 10s\right]$$

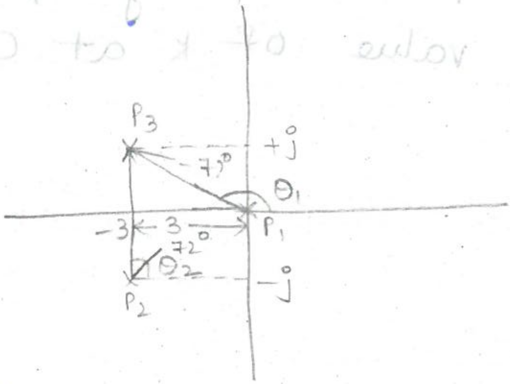
$$k = -\left[(-1.2)^3 + 6(-1.2)^2 + 10(-1.2)\right]$$

$k = 5.0$ Real & positive

Step 5:- To find angle of departure

Consider the complex pole P_3 draw vectors from all other poles to pole P_3.

Consider let the angles be θ_1 and θ_2



$$\theta_1 = 180^\circ - \tan^{-1}(1/3)$$

$$\theta_1 = 161.56 \approx 162^\circ$$

$$\theta_2 = 90^\circ$$

$P_3 = 180^\circ - (\theta_1 + \theta_2)$

$P_3 = -72^\circ$

The angle of departure at Complex pole P_2 will be opposite to angle of departure at pole P_3.

$\therefore$ Angle of departure at $P_2 = 72^\circ$.

Step-6:- To find Crossing point on imaginary axis

The characteristic equation be $s^3+6s^2+10s+k=0$

Put $s=j\omega$

$(j\omega)^3 + 6(j\omega)^2 + 10(j\omega) + k = 0$

$-j\omega^3 - 6\omega^2 + 10j\omega + k = 0$

$k - 6\omega^2 + j(10\omega - \omega^3) = 0$

Equating real terms to zero	Equating imaginary terms to zero
$k - 6\omega^2 = 0$	$10\omega - \omega^3 = 0$
$k = 6\omega^2$	$10 - \omega^2 = 0$
$k = 6(10)$	$10 = \omega^2$
$k = 60$	$\omega = \pm\sqrt{10}$
	$\omega = \pm 3.16$

The root locus process imaginary axis at $\pm j3.16$ and the value of k at Crossing point is 60.