14/6/17

UNIT-I

MATHEMATICAL MODEL OF CONTROL SYSTEMS

Introduction:-

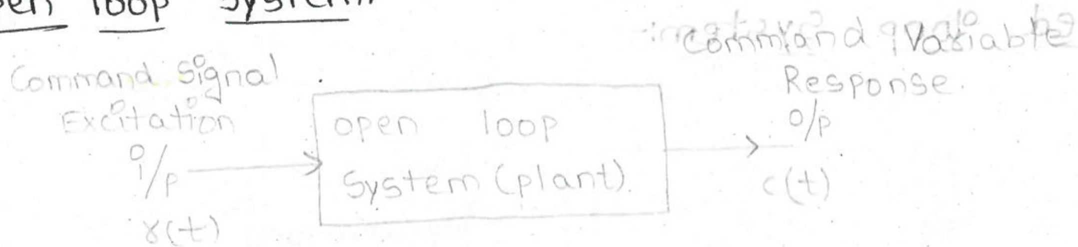
System:- when a no. of elements or Components are connected in a sequence to perform a specific function is known as a system.

Control System:- In a System when the output quantity is Controlled by varying the input quantity, then the system is called a Control System.

Types of Control Systems:-

1) open loop system
2) closed loop system

Open loop System:-



Any physical system which does not automatically correct the variations in its output is called open loop system.

(or)

control system in which the output quantity has no effect upon the input quantity are called open loop control system.

In open loop system the output can be vary by varying the input but (the) due to external disturbances the system output may in ~~outpu~~ input to Correct the output. In open loop system, the changes in output are Corrected by Changing the input manually. therefore, these are called manual control systems.

Advantages:-

→ The open loop systems are simple and economically.

→ the open loop system are easier to Constr-uct.

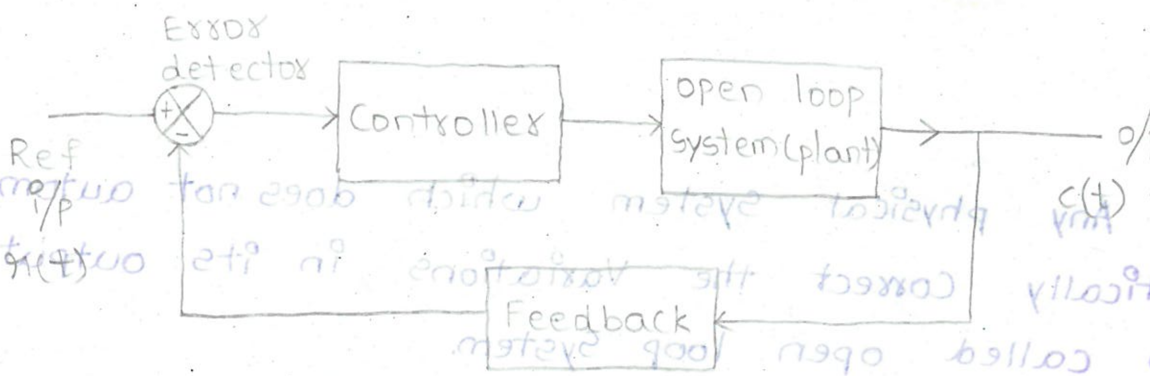
→ Generally, the open loop systems are stable.

Disadvantages:-

→ These are inaccurate and unreliable.

→ The changes in output due to external disturbances are not Corrected automatically.

closed loop system:-



Control systems in which the output has an effect on output upon the input quantity in order to maintain the desired output val-ue are called closed loop systems. The open loop system can be modified as closed loop system by providing a feedback. the feedback automatically Corrects the changes in o/p

System.

<u>Advantages:-</u>

→ These are accurate.

→ The sensitivity of the system may be made small to make the system more stable.

→ The closed loop systems are less effected by noise.

<u>Disadvantages:-</u>

→ These are complex and costly.

→ The feedback in closed loop system may lead to oscillatory response.

→ The feedback reduces the overall gain of the system.

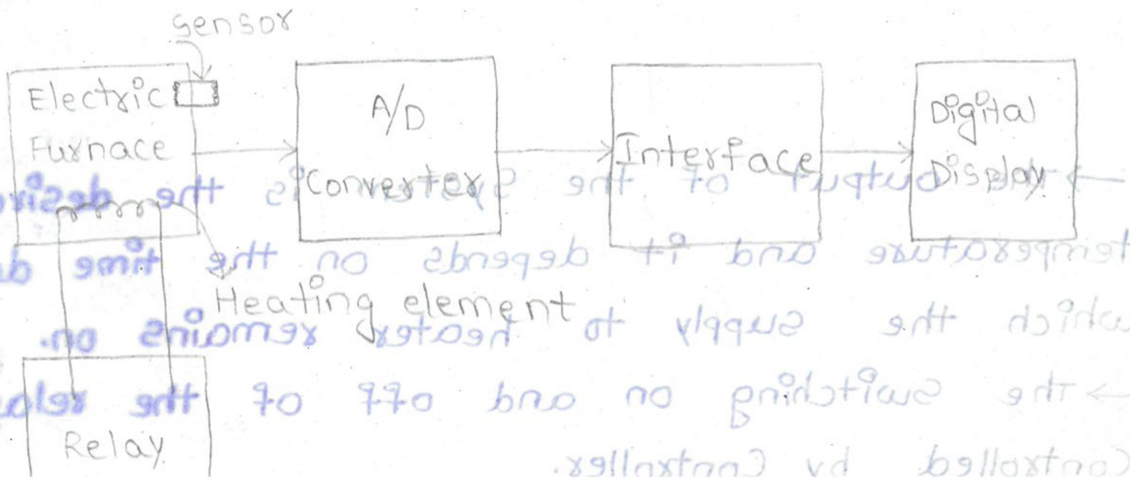
→ Stability is a major problem in a closed loop system hence, more care is needed to design a stable closed loop system.

<u>15/6/17</u>

<u>Examples of Control Systems:-</u>

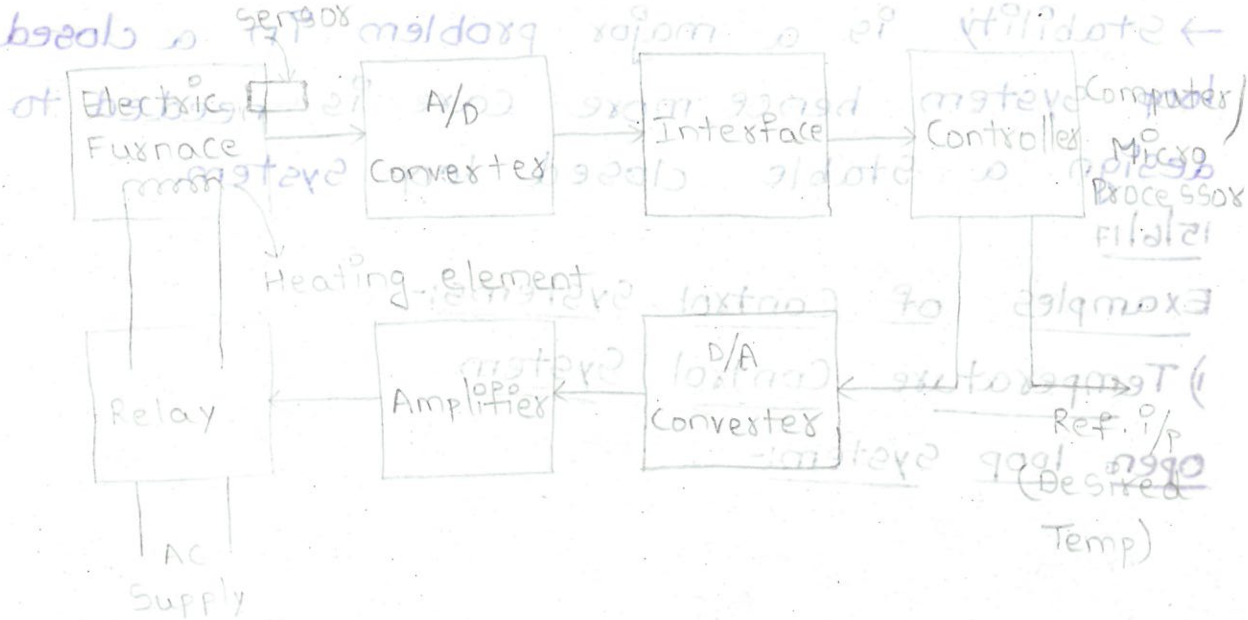
1) <u>Temperature Control System</u>

<u>open loop system:-</u>



→ The output in the System is the desired temperature and it depends on the time during which the supply to heater remains on.
→ The on and off of the supply is Controlled by time setting of the relay.
→ The temperature is measured by sensor and Converted to digital signal by A/D Converter.
→ The digital signal is given to digital display to display the temperature.
→ In this system, if there is any change in output temperature then the time setting of the relay is not altered automatically.

<u>Closed loop System</u>:-



→ The output of the System is the desired temperature and it depends on the time during which the supply to heater remains on.
→ The switching on and off of the relay is Controlled by Controller.

and Converted to Digital Signal by A/D Conver.
→The Controller Compares the actual tempera-
-ture with desired temperature.
→If it finds any difference then, it sense signal to switch on (or) off.
→The relay through D/A Converter and Amplifier. thus, the System automatically Corrects any changes in output.

2) Traffic Control System:-

open loop System:-

→ Traffic Control by traffic signals are opera-
-ted on time basis Constitutes an open loop System.

→The Sequence of Control signals are based on time slot given for each signal. The time Slot are decided based on traffic.

→the System will not measure the density of traffic before giving the signals. Since, the time Slot does not changes according to traffic density, the System is open loop System.

closed loop System:-

→Traffic Control System can be made as a closed loop system if the time slots of the signals are decided based on density of traffic.

→In closed loop traffic Control System the density of traffic is measured on all the

→ Then, the Computer decides the timings of the Control signals based on density of traffic.
→ Since, the closed loop System dynamically changes the timings the flow of vehicles will be better than open loop system.

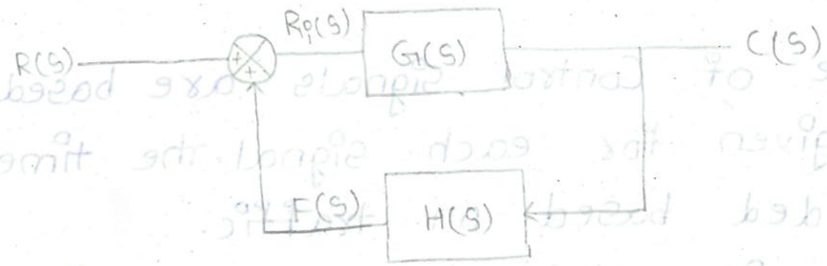
16/6/17

Feedback:- The Sample portion of output signal is fed to input signal is known as feedback.

There are two different types of feedback.

1) Regenerative feedback
2) Degenerative feedback

Regenerative Feedback:-



If a feedback signal is a positive going signal w.r.t input signal is known as Regenerative feedback.

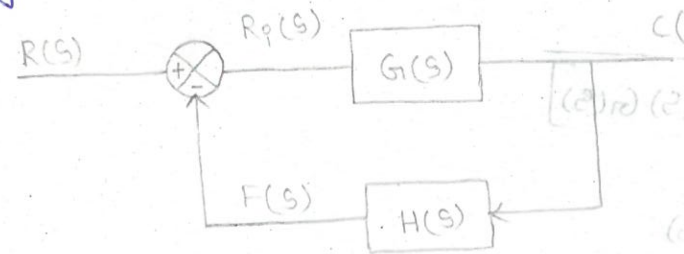
The transfer function of closed loop system is $T(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)H(S)}$

where G(S) is transfer function of open loop System and H(S) is transfer function of feedback network.

Degenerative Feedback:-

If a feedback signal is negative going

degenerative feedback.

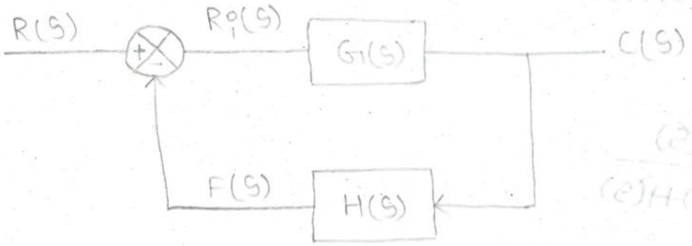


The transfer function of open loop system is

$$T(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1+G(S)H(S)}$$

Effect of feedback on gain:-

Let us Consider,



Transfer function of open loop system is

$$G(S) = \frac{C(S)}{R_i(S)} \quad \text{—— ①}$$

Transfer function of feedback system (or) network is

$$H(S) = \frac{F(S)}{C(S)} \quad \text{—— ②}$$

From the circuit, we have

$R_i(S) = R(S) - F(S)$

$\Rightarrow R(S) = R_i(S) + F(S)$

$\Rightarrow R(S) = R_i(S) + H(S)\,C(S)$ [From ②]

$\Rightarrow R(S) = R_i(S) + H(S)\,G(S)\,R_i(S)$ [From ①]

$\Rightarrow R(S) = R_i(S)\left[1 + H(S)\,G(S)\right]$ —— ③

∴ The transfer function of closed loop system

From ③

$$T(S) = \frac{C(S)}{R_i(S)\left[1 + H(S)\,G(S)\right]}$$

$$T(S) = \frac{G(S)}{1 + G(S)H(S)}$$

∴ the transfer function (gain) decreases with feedback.

Effect of feedback on stability:-

The transfer function of closed loop system is

$$T(S) = \frac{G(S)}{1 + G(S)H(S)}$$

As $G(S)H(S) >> 1$

$$\therefore T(S) = \frac{\cancel{G}(S)}{\cancel{G}(S)H(S)}$$

$$T(S) = \frac{1}{H(S)}$$

∴ T(S) depends on H(S) but doesn't depends on G(S). Thus, due to feedback stability increases.

17/6/17

Effect of feedback on sensitivity:-

Sensitivity is defined as percentage change in transfer function with feedback to percentage change in transfer function without feedback.

$$\therefore \text{Sensitivity} = \frac{\%\ \text{change in } T(S)}{\%\ \text{change in } G(S)}$$

$$\Rightarrow \text{Sensitivity} = \frac{\delta T(S)/T(S)}{\delta G(S)/G(S)}$$

$$= \frac{\delta T(S)}{\delta G(S)} \times \frac{G(S)}{T(S)}$$

Partial differentiating $T(S)$ w.r.t $G(S)$

$$\frac{\partial}{\partial G(S)}[T(S)] = \frac{\partial}{\partial G(S)}\left[\frac{G(S)}{1+G(S)H(S)}\right]$$

$$\frac{\partial T(S)}{\partial G(S)} = \frac{[1+G(S)H(S)](1) - G(S)H(S)}{[1+G(S)H(S)]^2}$$

$$\Rightarrow \frac{\partial T(S)}{\partial G(S)} = \frac{1}{[1+G(S)H(S)]^2}$$

$$\therefore \text{Sensitivity} = \frac{\partial T(S)}{\partial G(S)} \times \frac{G(S)}{T(S)}$$

$$= \frac{1}{[1+G(S)H(S)]^2} \times \frac{G(S)}{T(S)}$$

$$= \frac{1}{[1+G(S)H(S)]^{\cancel{2}}} \times \frac{\cancel{G(S)}}{\frac{\cancel{G(S)}}{\cancel{[1+G(S)H(S)]}}}$$

$$\therefore \text{Sensitivity} = \frac{1}{1+G(S)H(S)}$$

Controlled Variable and Manipulated Variable:-

→ A Control System is an interconnection of components that gives the desired response.

→ The primary objective of any control system is to maintain the output of system to desired value.

→ The output variable to be regulated is called as Controlled Variable.

→ The desired value of the Controlled variable ~~as~~ is called as reference variable.

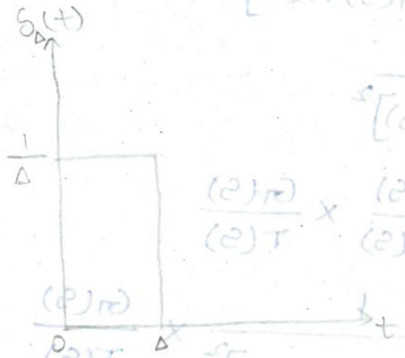
→ The system has input variables which can be manipulated to modify the Controlled as Manipulated Variables.

22/6/17

Mathematical methods of Control Systems:-

1) Impulse Response Method:-

Let us consider the pulse signal



$$\delta_\Delta(t) = \frac{1}{\Delta} \quad \text{for} \quad 0 < t < \Delta$$
$$= 0 \quad \text{otherwise}$$

If $\Delta \to 0$ then pulse signal is said to be unit Impulse $\delta(t)$

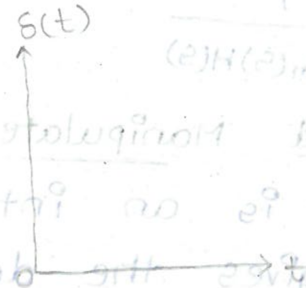


fig: Unit Impulse function

$$\delta(t) = 0 \quad \text{for} \quad t \neq 0$$
$$= 1 \quad \text{for} \quad t = 0$$

Impulse function with amplitude A is given as

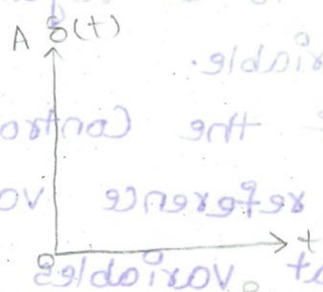


fig: Impulse function with amplitude A

The shifted impulse is given as

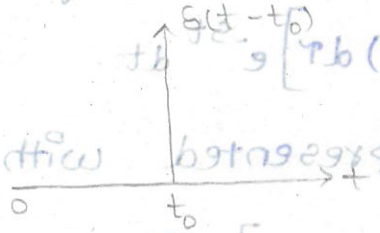
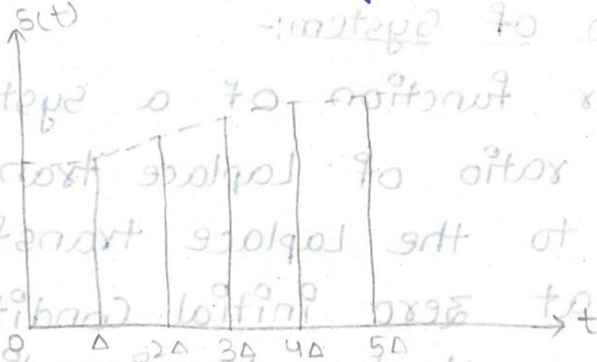


fig. shifted Impulse

$\delta(t-t_0) = 0$ for $t \neq t_0$

Scale Impulse Function

A Impulse function is said to be scale impulse function when the magnitude of the impulse is not unity.



Let the scale impulse function is represe-nted by $x(t)$. therefore,

$$x(t) = \int_0^{\infty} x(\tau)\, \delta(t-\tau)\, d\tau \quad \text{--- (1)}$$

consider an SISO system

The output $y(t)$ is the transfer function of input signal $x(t)$.

$$\therefore\ y(t) = T[x(t)]$$

Applying Laplace transformation, we get

$$Y(S) = \int_0^\infty \left[\int_0^\infty r(\tau)\, \delta(t-\tau)\, d\tau \right] e^{-St}\, dt$$

For simplicity, δ is represented with 'g'

$$\Rightarrow Y(S) = \int_0^\infty \left[\int_0^\infty r(\tau)\, g(t-\tau)\, d\tau \right] e^{-St}\, dt$$

$$\Rightarrow Y(S) = \int_0^\infty \left[\int_0^\infty g(t-\tau)\, e^{-S(t-\tau)}\, dt \right] e^{-S\tau}\, r(\tau)\, dt\, d\tau$$

$$\Rightarrow Y(S) = \int_0^\infty g(t-\tau)\, e^{-S(t-\tau)}\, dt \int_0^\infty r(\tau)\, e^{-S\tau}\, d\tau$$

$$\Rightarrow \boxed{Y(S) = G(S)\, R(S)}$$

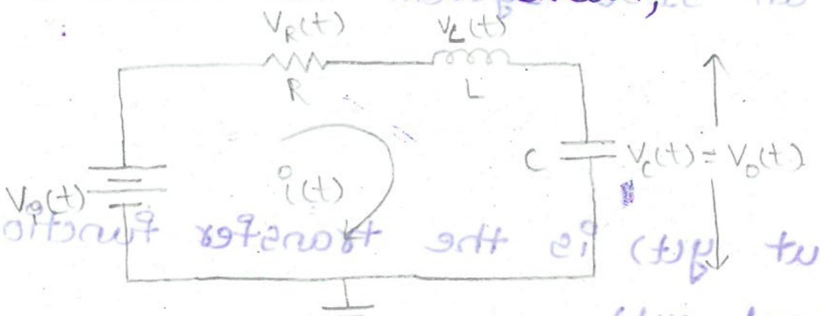
2) <u>Transfer function of System</u>:-

The transfer function of a system is defined as the ratio of Laplace transformation of the output to the Laplace transformation of the input at zero initial conditions.

$$\therefore \text{Transfer function} = \frac{\text{Laplace transformation of o/p}}{\text{Laplace transformation of i/p}}$$

$$\Rightarrow \boxed{G(S) = \frac{Y(S)}{R(S)}}$$

Consider an RLC circuit,



Apply KVL to the above circuit,

$$V_i(t) = V_R(t) + V_L(t) + V_C(t)$$

Applying Laplace transformation, we get

$$\Rightarrow V_i(S) = R I(S) + LS\, I(S) + \frac{1}{CS} I(S)$$

$$\Rightarrow V_i(S) = I(S)\left[R + LS + \frac{1}{CS}\right] \quad ①$$

the output of the circuit is calculated across the capacitor

$$\therefore V_0(t) = V_c(t) = \frac{1}{C}\int i(t)\,dt$$

Applying Laplace transformation, we get

$$\Rightarrow V_0(S) = \frac{1}{CS} I(S) \quad ②$$

therefore, the transfer function

$$G(S) = \frac{V_0(S)}{V_i(S)}$$

$$G(S) = \frac{\frac{1}{CS}\cancel{I(S)}}{\cancel{I(S)}\left[R + LS + \frac{1}{CS}\right]}$$

$$G(S) = \frac{1/CS}{R + LS + \frac{1}{CS}}$$

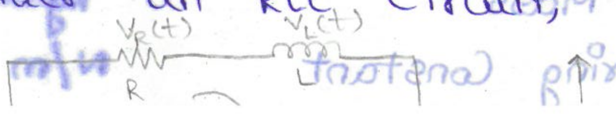
$$G(S) = \frac{1}{RCS + LCS^2 + 1}$$

3) State Variable method:-

It says that the n^{th} order differential equation can be divided into n 1^{st} order differential equations.

The number of 1^{st} order differential equations division will be equal to number of storage elements available in the circuit.

Consider an RLC Circuit,



Apply KVL, to the above circuit

$$V_i(t) = V_R(t) + V_L(t) + V_c(t)$$

$$V_i(t) = Ri(t) + L\frac{d}{dt}[i(t)] + \frac{1}{C}\int i(t)\,dt$$

$$\Rightarrow \frac{V_i(t) - Ri(t) - \frac{1}{C}\int i(t)\,dt}{L} = \frac{d}{dt}i(t)$$

$$\Rightarrow \frac{d}{dt}i(t) = \frac{V_i(t)}{L} - \frac{R}{L}i(t) - \frac{1}{LC}\int i(t)\,dt \text{ —— ①}$$

$$\Rightarrow \frac{d}{dt}i(t) = \frac{V_i(t)}{L} - \frac{R}{L}i(t) - \frac{1}{L}V_c(t) \text{ —— ①}$$

From the circuit, we have

$$V_o(t) = V_c(t) = \frac{1}{C}\int i(t)\,dt$$

Applying differentiation on b.s

$$\frac{d}{dt}V_o(t) = \frac{1}{C}i(t) \text{ —— ②}$$

Eq-① & ② can be solved by using matrix method

$$\begin{bmatrix} V_o'(t) \\ i'(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} V_i(t) \\ i(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{L} \end{bmatrix} V_c(t)$$

Mathematical models of Control Systems:-

1) Mechanical Translation System:-

Parameter	Definitions	Units
1) x	Displacement	meters
2) velocity v, $v = \frac{dx}{dt}$	velocity	meters/sec
3) $a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$	Accelaration	meters/sec^2
4) M	Mass	kg
5) k	Spring Constant	N/m

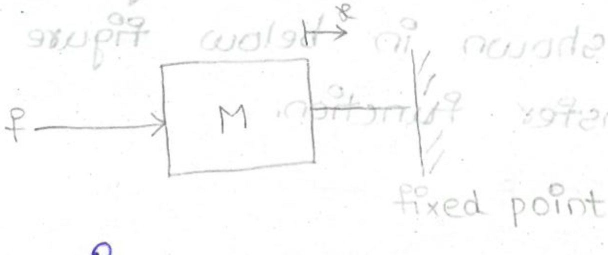
7) f_M	Opposing force due to mass	Newton
8) f_B	Opposing force due to friction	Newton
9) f_K	Opposing force due to spring	Newton
10) F	Force	Newton

23/6/17

A mechanical translation system consists of three elements

1) Mass
2) Dashpot
3) Spring

Mass:- weight of mechanical translation system is represented by mass element. when a force is applied to given mass element, then opposite force f_M develops and it is directly proportional to accelaration.

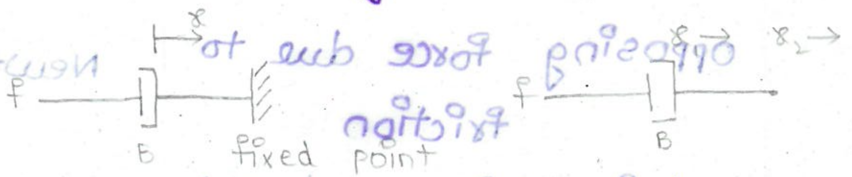


$$f_M \propto a$$

$$\Rightarrow f_M \propto \frac{dv}{dt} \propto \frac{d^2x}{dt^2}$$

$$\Rightarrow \boxed{f_M = M\frac{d^2x}{dt^2}}$$

Dashpot:- Friction existing on mechanical translatithen opposite force f_B develops and it is directly proportional to velocity.



$f_B \propto v \propto \frac{dx}{dt}$ $\qquad$ $f_B \propto v \propto \frac{d}{dt}(x_1 - x_2)$

$\boxed{f_B = B\cdot\frac{dx}{dt}}$ $\qquad$ $\Rightarrow f_B = B\frac{d}{dt}(x_1 - x_2)$

Spring :- (Elastic) (veloci)

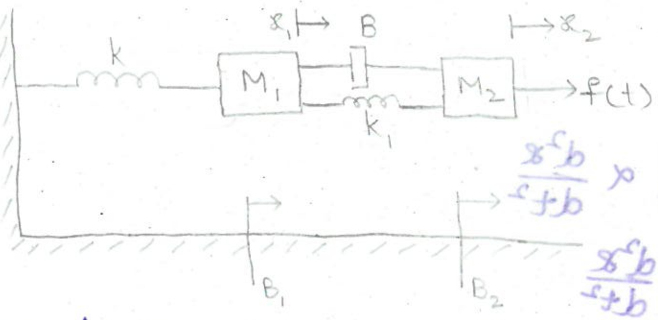
Elastic deformation of the body is represented by spring. when a force is applied to given spring element then opposite force f_K develops which is directly proportional to displacement.

$f_K \propto x$

$\Rightarrow f_K = Kx$

Problems

1) Write the differential equations governing mechanical system shown in below figure and determine its transfer function.



Sol:- Free body diagram of M_1

From newton's second law, we have

$$f_{M_1} + f_k + f_{B_1} + f_B + f_{K_1} = 0$$

$$M_1 \frac{d^2x_1}{dt^2} + Kx_1 + B_1\frac{dx_1}{dt} + B\frac{d}{dt}[x_1 - x_2] + K_1[x_1 - x_2] = 0$$

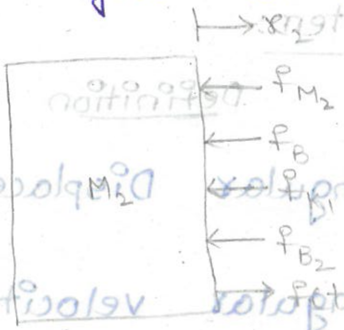
Applying Laplace transformation

$$M_1S^2X_1(S) + KX_1(S) + B_1SX_1(S) + BS[X_1(S) - X_2(S)] + K_1[X_1(S) - X_2(S)] = 0$$

$$\Rightarrow X_1(S)[M_1S^2 + K + B_1S + BS + K_1] - X_2(S)[BS + K_1] = 0$$

$$\Rightarrow \boxed{X_1(S) = \frac{[BS+K_1]X_2(S)}{M_1S^2 + S(B_1+B) + (K+K_1)}}$$

Free body diagram of M_2



From newton's second law, we have

$$f_{M_2} + f_B + f_{K_1} + f_{B_2} = f(t)$$

$$\Rightarrow M_2\frac{d^2x_2}{dt^2} + B\frac{d}{dt}[x_2 - x_1] + K_1[x_2 - x_1] + B_2\frac{dx_2}{dt} = f(t)$$

Applying Laplace transformation

$$M_2S^2X_2(S) + BS[X_2(S) - X_1(S)] + K_1[X_2(S) - X_1(S)] + B_2SX_2(S) = F(S)$$

$$\Rightarrow X_2(S)[M_2S^2 + BS + K_1 + B_2S] - X_1(S)[BS + K_1] = F(S)$$

Substitute $X_1(S)$ value in above equation

$$X_2(S)[M_2S^2 + BS + K_1 + B_2S] - \frac{[BS+K_1]X_2(S)}{M_1S^2 + S(B_1+B) + (K+K_1)} \cdot [BS+K_1]$$

$$\frac{X_2(S)\left[M_2S^2+BS+k_1+B_2S\right]\left[M_1S^2+S(B_1+B)+(k+k_1)\right]-\left[BS+k_1\right]^2X_2(S)}{M_1S^2+S(B_1+B)+(k+k_1)}=F(S)$$

$$X_2(S)\left[M_2S^2+BS+k_1+B_2S\right]\left[M_1S^2+S(B_1+B)+(k+k_1)\right]-\left[BS+k_1\right]^2X_2(S)$$
$$=F(S)\left[M_1S^2+S(B_1+B)+(k+k_1)\right]$$

$$X_2(S)\left\{\left[M_2S^2+BS+k_1+B_2S\right]\left[M_1S^2+S(B_1+B)+(k+k_1)\right]-(BS+k_1)^2\right\}$$
$$=F(S)\left[M_1S^2+S(B_1+B)+(k+k_1)\right]$$

$$\frac{X_2(S)}{F(S)}=\frac{M_1S^2+S(B_1+B)+(k+k_1)}{\left[M_2S^2+BS+k_1+B_2S\right]\left[M_1S^2+S(B+B_1)+(k+k_1)\right]-(BS+k_1)^2}$$

24/6/17

Mechanical Rotation System:-

Parameter	Definition
1) θ	Angular Displacement
2) $\frac{d\theta}{dt}$	Angular velocity
3) $\frac{d^2\theta}{dt^2}$	Angular Accelaration
4) T	Applied Torque
5) J	Moment of Inertia
6) B	Dashpot
7) k	Spring Constant

Mechanical rotation System Consists of three basic elements.

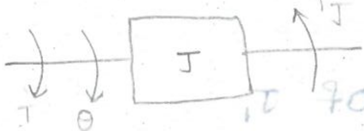
1) Moment of Inertia (J)

2) Dash-Pot (B)

3) Spring Constant (k)

Moment of Inertia (J):-

when a torque is applied to moment of inertia with some angular displacement θ, then an opposing torque is developed and it is proportional to angular accelaration.

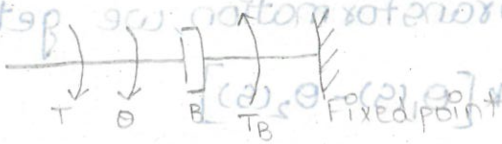


$$T_J \propto \frac{d^2\theta}{dt^2}$$

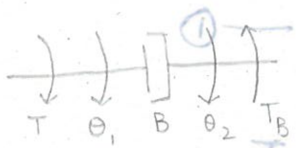
$$T_J = J\,\frac{d^2\theta}{dt^2}$$

Dash-pot (B):-

when a torque is applied to given dash-pot then, an opposite torque is developed which is proportionate to angular velocity.



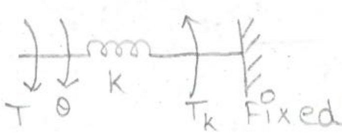
$$T_B \propto \frac{d\theta}{dt} \Rightarrow T_B = B\frac{d\theta}{dt}$$



$$T_B \propto \frac{d}{dt}[\theta_1 - \theta_2] \Rightarrow T_B = B\frac{d}{dt}[\theta_1 - \theta_2]$$

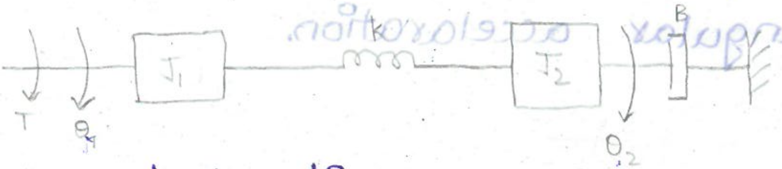
Spring Constant (k):-

when a torque is applied to given spring, then an opposite torque is developed which is proportionate to angular ~~velo~~ displacement.

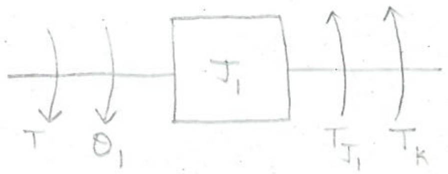


Problems

1) write the differential equations governing the mechanical rotation system shown in figure and find its transfer function.



Sol:- Free body diagram of J_1



$\therefore$ From Newton's second Law

$$T = T_{J_1} + T_k$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + k[\theta_1 - \theta_2]$$

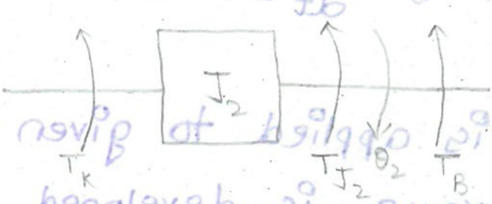
Applying Laplace Transformation, we get

$$T(s) = J_1 s^2 \theta_1(s) + k[\theta_1(s) - \theta_2(s)]$$

$$T(s) = \theta_1(s)[J_1 s^2 + k] - k\theta_2(s)$$

$$\Rightarrow \quad \theta_1(s) = \frac{T(s) + k\theta_2(s)}{[J_1 s^2 + k]} \quad \text{---} (1)$$

Free body diagram of J_2



$\therefore$ From Newton's second Law

$$T_k + T_{J_2} + T_B = 0$$

$$k[\theta_2 - \theta_1] + J_2 \frac{d^2\theta_2}{dt^2} + B\frac{d\theta_2}{dt} = 0$$

$$k[\theta_2(s) - \theta_1(s)] + J_2 s^2 \theta_2(s) + Bs\theta_2(s) = 0$$

$$\theta_2(s)[k + J_2 s^2 + Bs] - k\theta_1(s) = 0$$

$$\theta_2(s) = \frac{k\theta_1(s)}{k + J_2 s^2 + Bs}$$

$$\theta_2(s) = \frac{k\theta_1(s)}{J_2 s^2 + Bs + k}$$

From (1)

$$\theta_2(s) = \frac{k}{J_2 s^2 + Bs + k} \times \frac{T(s) + k\theta_2(s)}{J_1 s^2 + k}$$

$$\theta_2(s)[J_2 s^2 + Bs + k][J_1 s^2 + k] = kT(s) + k^2\theta_2(s)$$

$$\theta_2(s)\left\{[J_2 s^2 + Bs + k][J_1 s^2 + k] - k^2\right\} = kT(s)$$

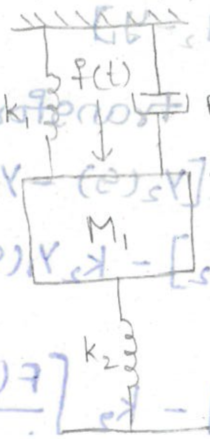
$$\frac{\theta_2(s)}{T(s)} = \frac{k}{[(J_2 s^2 + Bs + k)(J_1 s^2 + k)] - k^2}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{k}{J_1 J_2 s^4 + J_1 B s^3 + J_1 k s^2 + k J_2 s^2 + kBs + \cancel{k^2} - \cancel{k^2}}$$

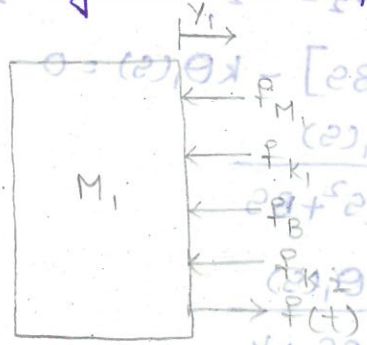
$$\frac{\theta_2(s)}{T(s)} = \frac{k}{J_1 J_2 s^4 + J_1 B s^3 + [J_1 + J_2] k s^2 + Bks}$$

27/6/17

2) Determine the transfer function of the given system.



Sol:- Free body diagram of M_1



From Newton's second law, we have

$$f(t) = f_{M_1} + f_{K_1} + f_B + f_{K_2}$$

$$f(t) = M_1 \frac{d^2 y_1}{dt^2} + k_1 y_1 + B\frac{dy_1}{dt} + k_2[y_1 - y_2]$$

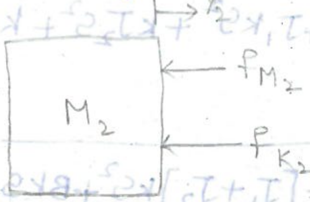
Applying Laplace transformation, we get

$$F(S) = M_1 S^2 Y_1(S) + k_1 Y_1(S) + BS\,Y_1(S) + k_2[Y_1(S) - Y_2(S)]$$

$$F(S) = Y_1(S)\left[M_1 S^2 + k_1 + BS + k_2\right] - k_2 Y_2(S)$$

$$Y_1(S) = \frac{F(S) + k_2 Y_2(S)}{M_1 S^2 + BS + k_1 + k_2} \quad \text{—— (1)}$$

Free body diagram of M_2



From Newton's second law, we have

$$f_{M_2} + f_{K_2} = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + k_2[y_2 - y_1]$$

Applying Laplace transformation, we get

$$M_2 S^2 Y_2(S) + k_2[Y_2(S) - Y_1(S)] = 0$$

$$Y_2(S)\left[M_2 S^2 + k_2\right] - k_2 Y_1(S) = 0$$

From (1)

$$\Rightarrow Y_2(S)\left[M_2 S^2 + k_2\right] - k_2\left[\frac{F(S) + k_2 Y_2(S)}{\cdots}\right] = 0$$

$$\Rightarrow Y_2(S)\left[M_2S^2+k_2\right]\left[M_1S^2+BS+k_1+k_2\right]-k_2^2Y(S)=k_2F(S)$$

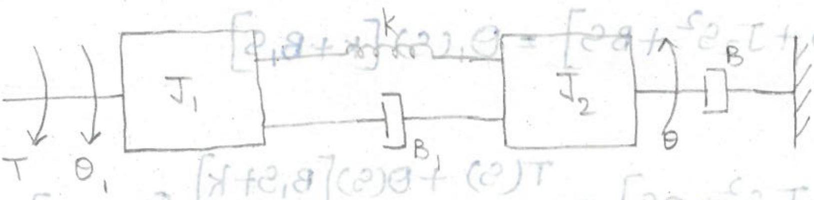
∴ Transfer Function

$$\frac{Y_2(S)}{F(S)}=\frac{k_2}{\left[M_2S^2+k_2\right]\left[M_1S^2+BS+k_1+k_2\right]-k_2^2}$$

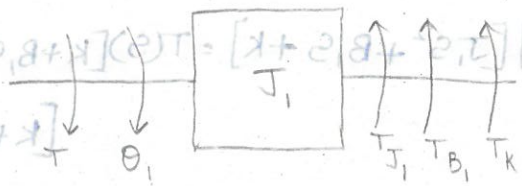
$$\Rightarrow \frac{Y_2(S)}{F(S)}=\frac{k_2}{M_1M_2S^4+BM_2S^3+k_1M_2S^2+k_2M_2S^2+k_2M_1S^2+Bk_2S+k_1k_2+\cancel{k_2^2}-\cancel{k_2^2}}$$

$$\Rightarrow \frac{Y_2(S)}{F(S)}=\frac{k_2}{M_1M_2S^4+BM_2S^3+\left[k_1M_2+k_2(M_1+M_2)\right]S^2+Bk_2S+k_1k_2}$$

3) write the differential equations governing mechanical rotation system and find its transfer function:



Sol:- Free body diagram of J_1



From Newton's second law, we have

$$T=T_{J_1}+T_{B_1}+T_K$$

$$\Rightarrow T=J_1\frac{d^2\theta_1}{dt^2}+B_1\frac{d}{dt}\left[\theta_1-\theta\right]+k\left[\theta_1-\theta\right]$$

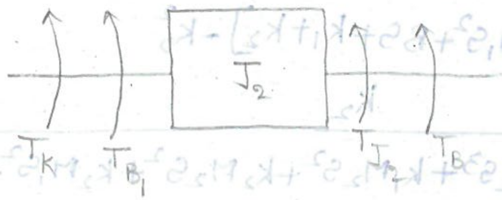
Applying Laplace Transformation, we get

$$T(S)=J_1S^2\theta_1(S)+B_1S\left[\theta_1(S)-\theta(S)\right]+k\left[\theta_1(S)-\theta(S)\right]$$

$$\Rightarrow T(S)=\theta_1(S)\left[J_1S^2+B_1S+k\right]-\theta(S)\left[\ldots\right]$$

$$\theta_1(S) = \frac{T(S) + \theta(S)[B_1 S + k]}{J_1 S^2 + B_1 S + k} \quad \text{——— (1)}$$

Free body diagram of J_2



From Newtons Second law, we have

$$T_K + T_{B_1} + T_{J_2} + T_B = 0$$

$$\Rightarrow k[\theta - \theta_1] + B_1 \frac{d}{dt}[\theta - \theta_1] + J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} = 0$$

Applying Laplace transformation, we get

$$k[\theta(S) - \theta_1(S)] + B_1 S[\theta(S) - \theta_1(S)] + J_2 S^2 \theta(S) + BS\theta(S) = 0$$

$$\theta(S)[k + B_1 S + J_2 S^2 + BS] - \theta_1(S)[k + B_1 S] = 0$$

$$\theta(S)[k + B_1 S + J_2 S^2 + BS] = \theta_1(S)[k + B_1 S]$$

From (1)

$$\theta(S)[k + B_1 S + J_2 S^2 + BS] = \frac{T(S) + \theta(S)[B_1 S + k]}{J_1 S^2 + B_1 S + k}[k + B_1 S]$$

$$\theta(S)[k + B_1 S + J_2 S^2 + BS][J_1 S^2 + B_1 S + k] = T(S)[k + B_1 S] + \theta(S)[k + B_1 S]^2$$

$\therefore$ Transfer function

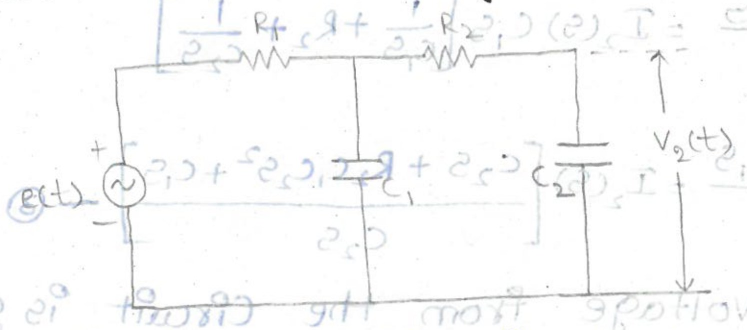
$$\frac{\theta(S)}{T(S)} = \frac{k + B_1 S}{[k + B_1 S + J_2 S^2 + BS][J_1 S^2 + B_1 S + k] - [k + B_1 S]^2}$$

28/6/17

Electrical Systems:-

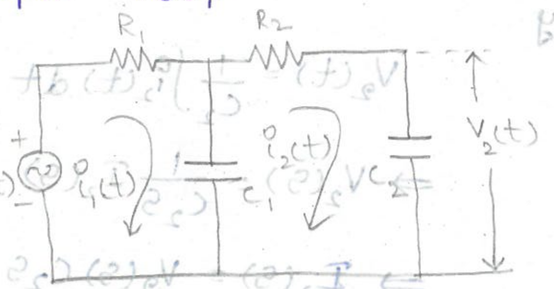
Problem

1) obtain the transfer function of electrical network shown in figure.



Sol:- Apply KVL to the input loop

~~e(t) = I_1R_1 +~~



$$e(t) = i_1(t) \cdot R_1 + \frac{1}{C_1}\int [i_1(t) - i_2(t)]\, dt$$

Applying Laplace Transformation, we get

$$E(S) = R_1 I_1(S) + \frac{1}{C_1 S}[I_1(S) - I_2(S)]$$

$$\Rightarrow E(S) = I_1(S)\left[R_1 + \frac{1}{C_1 S}\right] - \frac{1}{C_1 S} I_2(S)$$

$$\Rightarrow I_1(S) = \frac{E(S) + \frac{1}{C_1 S} I_2(S)}{R_1 + \frac{1}{C_1 S}}$$

$$\Rightarrow I_1(S) = \frac{I_2(S) + E(S) C_1 S}{1 + R_1 C_1 S} \quad \text{—— (1)}$$

Apply KVL to the output loop

$$\frac{1}{C_1}\int [i_2(t) - i_1(t)]\, dt + R_2 i_2(t) + \frac{1}{C_2}\int i_2(t)\, dt = 0$$

Applying Laplace Transformation, we get

$$\Rightarrow I_2(S)\left[\frac{1}{C_1S}+R_2+\frac{1}{C_2S}\right]=\frac{1}{C_1S}\cdot I_1(S)$$

$$\Rightarrow I_1(S)=I_2(S)\,C_1S\left[\frac{1}{C_1S}+R_2+\frac{1}{C_2S}\right] \quad \text{--- (2)}$$

eq-(1) = eq-(2)

$$\therefore \frac{I_2(S)+E(S)C_1S}{1+R_1C_1S}=I_2(S)\,C_1S\left[\frac{1}{C_1S}+R_2+\frac{1}{C_2S}\right]$$

$$\frac{I_2(S)+E(S)C_1S}{1+R_1C_1S}=I_2(S)\left[\frac{C_2S+R_2C_1C_2S^2+C_1S}{C_2S}\right] \quad \text{--- (3)}$$

The output voltage from the circuit is given by

$$V_2(t)=\frac{1}{C_2}\int i_2(t)\,dt$$

$$\Rightarrow V_2(S)=\frac{1}{C_2S}I_2(S)$$

$$\Rightarrow I_2(S)=V_2(S)\,C_2S \quad \text{--- (4)}$$

Substitute eq-(4) in eq-(3)

$$\frac{V_2(S)C_2S+E(S)C_1S}{1+R_1C_1S}=V_2(S)C_2S\left[\frac{C_2S+R_2C_1C_2S^2+C_1S}{C_2S}\right]$$

$$\Rightarrow V_2(S)C_2S+E(S)C_1S=V_2(S)\left[C_2S+R_2C_1C_2S^2+C_1S\right]\left[1+R_1C_1S\right]$$

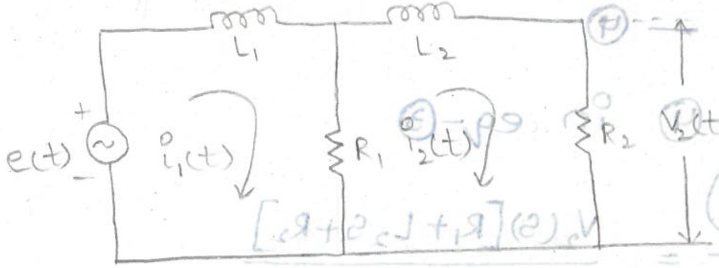
$$\Rightarrow E(S)C_1S=V_2(S)\left\{\left[C_2S+R_2C_1C_2S^2+C_1S\right]\left[1+R_1C_1S\right]-C_2S\right\}$$

$\therefore$ The transfer function of the given circuit is

$$\frac{V_2(S)}{E(S)}=\frac{C_1S}{\left[C_2S+R_2C_1C_2S^2+C_1S\right]\left[1+R_1C_1S\right]-C_2S}$$

2) Obtain the transfer function of electrical network shown in figure.

Sol:-



Apply KVL to the input loop

$$e(t) = L_1 \frac{d}{dt} i_1(t) + R_1[i_1(t) - i_2(t)]$$

Applying Laplace Transformation, we get

$$E(S) = L_1 S\, I_1(S) + R_1[I_1(S) - I_2(S)]$$

$$E(S) = I_1(S)[L_1S + R_1] - R_1 I_2(S).$$

$$\Rightarrow I_1(S) = \frac{E(S) + R_1 I_2(S)}{L_1 S + R_1} \quad \text{---} (1)$$

Apply KVL to the output loop

$$R_1[i_2(t) - i_1(t)] + L_2 \frac{d}{dt} i_2(t) + R_2 i_2(t) = 0$$

Applying Laplace Transformation, we get

$$R_1[I_2(S) - I_1(S)] + L_2 S I_2(S) + R_2 I_2(S) = 0.$$

$$I_2(S)[R_1 + L_2S + R_2] - R_1 I_1(S) = 0$$

$$I_2(S)[R_1 + L_2S + R_2] = R_1 I_1(S)$$

$$I_1(S) = \frac{I_2(S)[R_1 + L_2S + R_2]}{R_1} \quad \text{---} (2)$$

$\therefore$ eq-(1) = eq-(2)

$$\frac{E(S) + R_1 I_2(S)}{L_1S + R_1} = \frac{I_2(S)[R_1 + L_2S + R_2]}{R_1} \quad \text{---} (3)$$

The output voltage from the circuit is given by.

$$V_2(t) = R_2 i_2(t)$$

$$I_2(S) = \frac{V_2(S)}{R_2} \quad — ④$$

Substitute eq-④ in eq-③

$$\frac{E(S)+R_1\left(\frac{V_2(S)}{R_2}\right)}{L_1S+R_1} = \frac{V_2(S)\left[R_1+L_2S+R_2\right]}{R_1R_2}$$

$$\frac{E(S)+R_1\left(\frac{V_2(S)}{R_2}\right)}{L_1S+R_1} = V_2(S)\left[\frac{1}{R_2}+\frac{L_2S}{R_1R_2}+\frac{1}{R_1}\right]$$

$$E(S)+\frac{R_1V_2(S)}{R_1R_2} = V_2(S)\left[\frac{1}{R_2}+\frac{L_2S}{R_1R_2}+\frac{1}{R_1}\right]\left[L_1S+R_1\right].$$

~~E(S)R1R2~~

$$E(S) = V_2(S)\left\{\left[\frac{1}{R_2}+\frac{L_2S}{R_1R_2}+\frac{1}{R_1}\right]\left[L_1S+R_1\right] - \frac{R_1}{\cancel{R_1}R_2}\right\}$$

∴ The transfer function of the given circuit is.

$$\frac{V_2(S)}{E(S)} = \frac{1}{\left[\frac{1}{R_2}+\frac{L_2S}{R_1R_2}+\frac{1}{R_1}\right]\left[L_1S+R_1\right]-\frac{R_1}{R_2}}$$

29/6/17

Analogous Systems

The systems with identical mathematical expressi-ons are known as Analogous Systems.

1) Electrical Analogous Systems with Mechanical Translation Systems.

a) Force voltage Analogy

b) Force Current Analogy

2) Electrical Systems with Mechanical Rotation Systems.

a) Torque voltage Analogy

Force voltage Analogy:-

Consider a mechanical Translation System

$$f(t) = f_M + f_B + f_K$$

$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx \quad \text{--- (1)}$$

Replace displacement with velocity

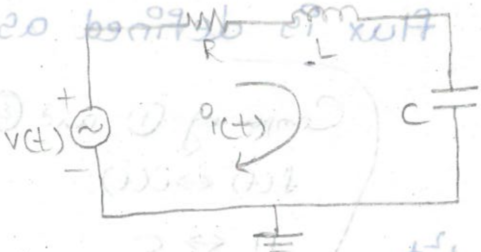
i.e $\frac{d^2x}{dt^2} = \frac{dv}{dt}$

$\frac{dx}{dt} = v$

$x = \int v\,dt$

$$\therefore f(t) = M\frac{dv}{dt} + Bv + k\int v\,dt \quad \text{--- (1)}$$

Consider Series RLC Circuit



Apply KVL, we get

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt \quad \text{--- (2)}$$

we know that, rate of change of charge is Current

$$\therefore i(t) = \frac{dq}{dt}$$

$$\Rightarrow V(t) = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}\cdot q$$

$$\Rightarrow V(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\cdot q \quad \text{--- (2)}$$

Comparing (1) and (2) we get

$f(t) \leftrightarrow V(t)$ | $B \leftrightarrow R$ | $K \leftrightarrow \frac{1}{C}$

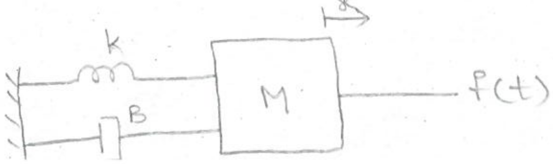
$M \leftrightarrow L$

By Comparing (1) & (2), we get

$$M \leftrightarrow L;\ B \leftrightarrow R;\ k \leftrightarrow \frac{1}{C};\ x \leftrightarrow q$$

Force Current Analogy:-

Consider a mechanical Translation System

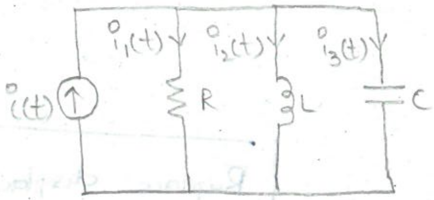


$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

Replacing displacement with velocity

$$\therefore f(t) = M\frac{dv}{dt} + Bv + K\int v\,dt \quad \text{--- (1)}$$

Consider shunt RLC circuit.



$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

$$\Rightarrow i(t) = \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt + C\frac{dV(t)}{dt} \quad \text{--- (2)}$$

The rate of change of flux is defined as voltage.

$$\therefore V(t) = \frac{d\phi}{dt}$$

$$\Rightarrow i(t) = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\cdot\phi + C\cdot\frac{d^2\phi}{dt^2}$$

$$\Rightarrow i(t) = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi \quad \text{(2)}$$

Comparing (1) and (2) we get

$f(t) \leftrightarrow i(t)$

$M \leftrightarrow C$

$B \leftrightarrow \frac{1}{R}$

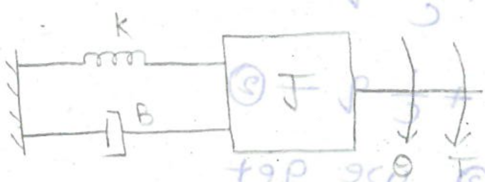
$K \leftrightarrow \frac{1}{L}$

By Comparing (1) & (2), we get

$$M \leftrightarrow C \,;\; B \leftrightarrow \frac{1}{R} \,;\; K \leftrightarrow \frac{1}{L} \,;\; x \leftrightarrow \phi$$

Torque voltage Analogy:-

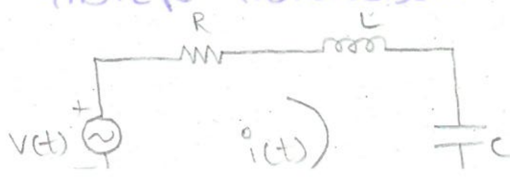
Consider a mechanical Rotational System



$$T = T_J + T_B + T_K$$

$$\Rightarrow T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\cdot\theta \quad \text{--- (1)}$$

Consider Series RLC Circuit.



Apply KVL, we get

$$V(t) = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)\,dt$$

we know that, rate of change of charge is Current

$$i(t) = \frac{dq}{dt}$$

$$\Rightarrow V(t) = R\frac{dq}{dt} + L\frac{d^2q}{dt^2} + \frac{1}{C}\cdot q$$

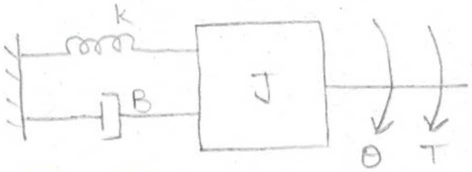
$$\Rightarrow V(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}\cdot q \quad \text{--- (2)}$$

Comparing (1) & (2), we get

$J \leftrightarrow L$; $B \leftrightarrow R$; $K \leftrightarrow \frac{1}{C}$; $\theta \leftrightarrow q$.

Torque Current Analogy:-

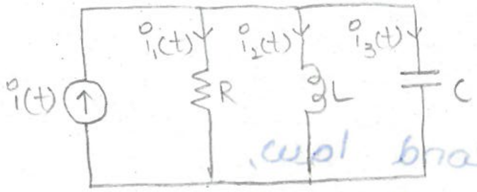
Consider a mechanical Rotational system.



$$T = T_J + T_B + T_K.$$

$$T = J\frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K\cdot\theta \quad \text{--- (1)}$$

Consider Shunt RLC Circuit



$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

$$\Rightarrow i(t) = \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt + C\cdot\frac{dV(t)}{dt}$$

The rate of change of flux is defined as voltage.

$$\therefore V(t) = \frac{d\phi}{dt}.$$

$$\Rightarrow i(t) = \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\cdot\phi + C\cdot\frac{d^2\phi}{}$$

$$\Rightarrow i(t) = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi \quad \text{—} (2)$$

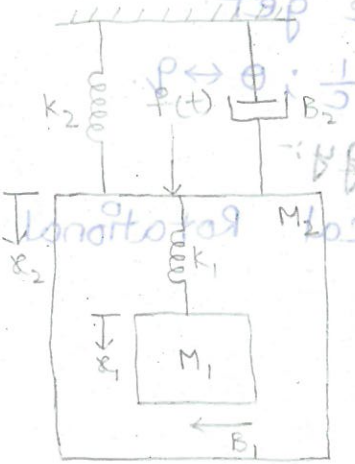
Comparing (1) & (2)

$$J \leftrightarrow C \; ; \; B \leftrightarrow \frac{1}{R} \; ; \; k \leftrightarrow \frac{1}{L} \; ; \; \theta \leftrightarrow \phi$$

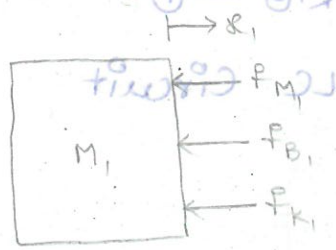
3/7/17

Problems

1) write the differential equations governing the mechanical system shown in figure. Draw the force voltage and force current electrical analogous circuits shown in figure, verify by using mesh and nodal analysis.



Sol:- Free body diagram of M_1

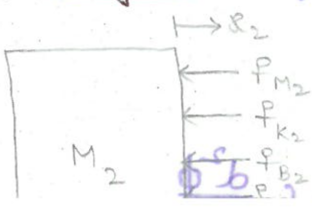


From Newton's second law,

$$f_{M_1} + f_{B_1} + f_{K_1} = 0$$

$$\Rightarrow M_1\frac{d^2x_1}{dt^2} + B_1\frac{d(x_1-x_2)}{dt} + K_1[x_1-x_2] = 0 \quad \text{—} (1)$$

Free body diagram of M_2.



From Newton's second Law,

$$f(t) = f_{M_2} + f_{B_2} + f_{k_2} + f_{B_1} + f_{k_1}$$

$$\Rightarrow f(t) = M_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_2 x_2 + B_1 \frac{d}{dt}[x_2 - x_1] + k_1[x_2 - x_1] \quad \text{--- (2)}$$

Force Voltage Analogy (or) Force voltage Electrical Analogous Circuit

$M_1 \leftrightarrow L_1 \quad B_1 \leftrightarrow R_1 \quad k_1 \leftrightarrow \frac{1}{C_1} \quad x \leftrightarrow q \qquad f(t) = e(t)$

$M_2 \leftrightarrow L_2 \quad B_2 \leftrightarrow R_2 \quad k_2 \leftrightarrow \frac{1}{C_2} \quad \frac{dx}{dt} = \frac{dq}{dt} = v = i$

$x = \int i\,dt$

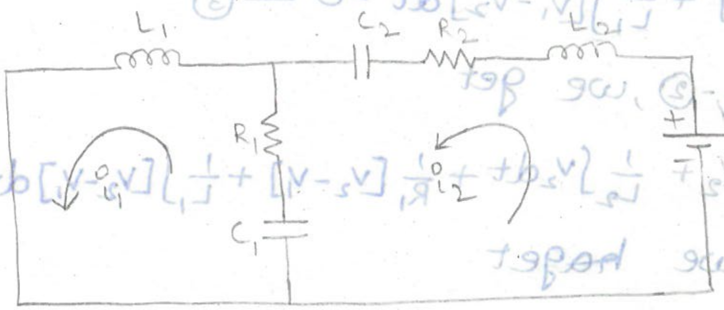
Considering eq-(1)

$$L_1 \frac{di_1}{dt} + R_1[i_1 - i_2] + \frac{1}{C_1}\int [i_1 - i_2]\,dt = 0 \quad \text{--- (3)}$$

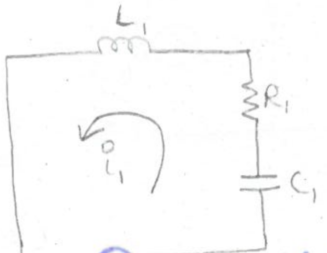
Considering eq-(2)

$$e(t) = L_2 \frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + R_1[i_2 - i_1] + \frac{1}{C_1}\int [i_2 - i_1]\,dt \quad \text{--- (4)}$$

From eq-(3) and eq-(4) we get

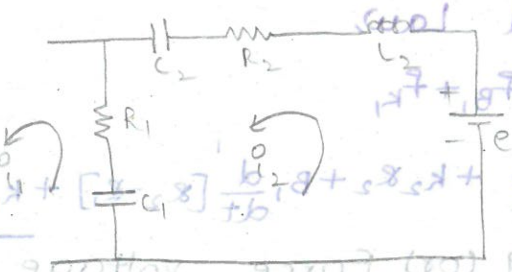


Mesh Analysis:- Considering (1) loop.



$$L_1 \frac{di_1}{dt} + R_1[i_1 - i_2] + \frac{1}{C_1}\int [i_1 - i_2]\,dt = 0 \quad \text{--- (5)}$$

Considering 2nd loop



$$e(t) = L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2\,dt + R_1(i_2 - i_1) + \frac{1}{C_1}\int [i_2 - i_1]\,dt \quad \text{—⑥}$$

As eq-③ = eq-⑤ ; eq-④ = eq-⑥ thus Verified.

<u>Force Current Electrical Analogous Circuit</u>

$M_1 \leftrightarrow C_1$ $\quad B_1 \leftrightarrow \frac{1}{R_1}$ $\quad k_1 \leftrightarrow \frac{1}{L_1}$ $\quad x \leftrightarrow \phi$

$M_2 \leftrightarrow C_2$ $\quad B_2 \leftrightarrow \frac{1}{R_2}$ $\quad k_2 \leftrightarrow \frac{1}{L_2}$ $\quad \frac{d\phi}{dt} = V$ $\quad f(t) = i(t)$

$$\frac{dx}{dt} = \frac{d\phi}{dt} = V$$

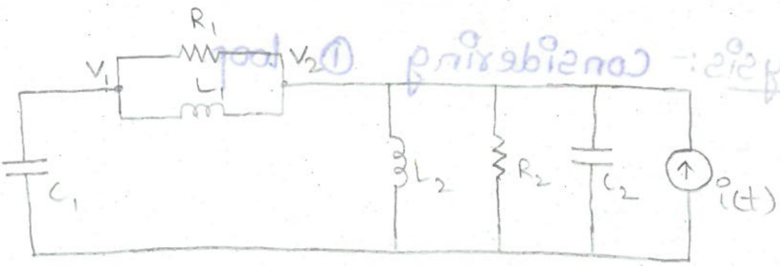
Considering eq-①, we get

$$C_1\frac{dV_1}{dt} + \frac{1}{R_1}[V_1 - V_2] + \frac{1}{L_1}\int [V_1 - V_2]\,dt = 0 \quad \text{— ③}$$

Considering eq-②, we get

$$i(t) = C_2\frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_1}[V_2 - V_1] + \frac{1}{L_1}\int [V_2 - V_1]\,dt \quad \text{— ④}$$

From ③ & ④, we ~~ha~~get



Considering Node V_1, we get

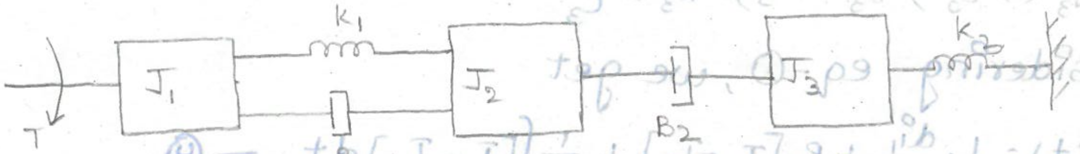
$$C_1\frac{dV_1}{dt} + \frac{1}{R_1}[V_1 - V_2] + \frac{1}{L_1}\int [V_1 - V_2]\,dt = 0. \quad \text{— ⑤}$$

Considering Node V_2, we get

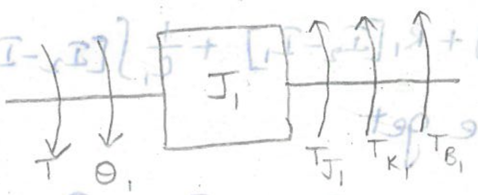
$$i(t) = C_2\frac{dV_2}{dt} + \frac{1}{R_2}V_2 + \frac{1}{L_2}\int V_2\,dt + \frac{1}{R_1}[V_2 - V_1] + \frac{1}{L_1}\int [V_2 - V_1]\,dt \quad \text{— ⑥}$$

6/7/17

2) write the differential equations governing mechanical rotation system shown in figure. Draw the Torque voltage and Torque Current analog-ous Circuits.



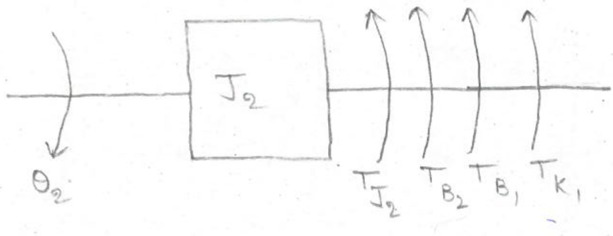
Sol:- Free body diagram of J_1:-



$$T = T_{J_1} + T_{B_1} + T_{K_1}$$

$$T = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d}{dt}[\theta_1 - \theta_2] + k[\theta_1 - \theta_2] \quad \text{— (1)}$$

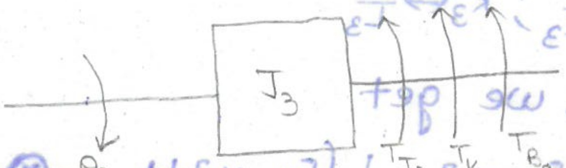
Free body diagram of J_2:-



$$0 = T_{J_2} + T_{B_2} + T_{B_1} + T_{K_1}$$

$$0 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d}{dt}[\theta_2 - \theta_3] + B_1 \frac{d}{dt}[\theta_2 - \theta_1] + k_1[\theta_2 - \theta_1] \quad \text{— (2)}$$

Free body diagram of J_3:-



$$0 = J_3 \frac{d^2\theta_3}{dt^2} + k_3\theta_3 + B_2 \frac{d}{dt}[\theta_3 - \theta_2] \quad \text{--- ③}$$

Torque voltage Analogous Circuit:-

$J_1 \leftrightarrow L_1$; $B_1 \leftrightarrow R_1$; $k_1 \leftrightarrow \frac{1}{C_1}$ $\quad \theta \leftrightarrow q$

$J_2 \leftrightarrow L_2$; $B_2 \leftrightarrow R_2$; $\quad T - V(t)$

$J_3 \leftrightarrow L_3$; $B_3 \leftrightarrow R_3$; $k_3 \leftrightarrow \frac{1}{C_3}$

Considering eq-①, we get

$$V(t) = L_1 \frac{di_1}{dt} + R_1[I_1 - I_2] + \frac{1}{C_1}\int [I_1 - I_2]\,dt \quad \text{--- ④}$$

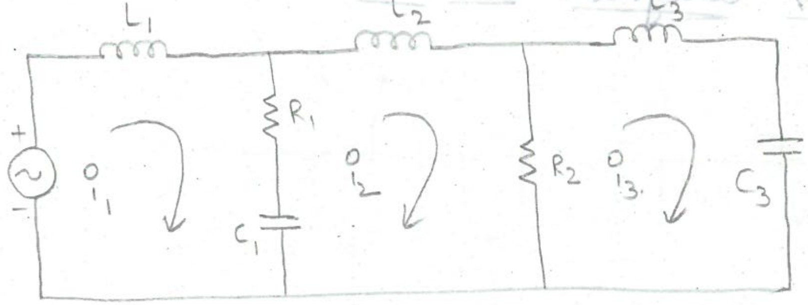
Considering eq-②, we get

$$0 = L_2 \frac{di_2}{dt} + R_2[I_2 - I_3] + R_1[I_2 - I_1] + \frac{1}{C_1}\int [I_2 - I_1]\,dt \quad \text{--- ⑤}$$

Considering eq-③, we get

$$0 = L_3 \frac{di_3}{dt} + \frac{1}{C_3}\int I_3\,dt + R_2[I_3 - I_2] \quad \text{--- ⑥}$$

By Considering 4,5,6 equations we get Torque voltage Analogous Circuit



Torque Current Analogous Circuit:-

$J_1 \leftrightarrow C_1$; $B_1 \leftrightarrow \frac{1}{R_1}$; $k_1 \leftrightarrow \frac{1}{L_1}$ $\quad T = i(t)$.

$J_2 \leftrightarrow C_2$; $B_2 \leftrightarrow \frac{1}{R_2}$;

$J_3 \leftrightarrow C_3$; $B_3 \leftrightarrow \frac{1}{R_3}$; $k_3 \leftrightarrow \frac{1}{L_3}$

Considering eq-①, we get

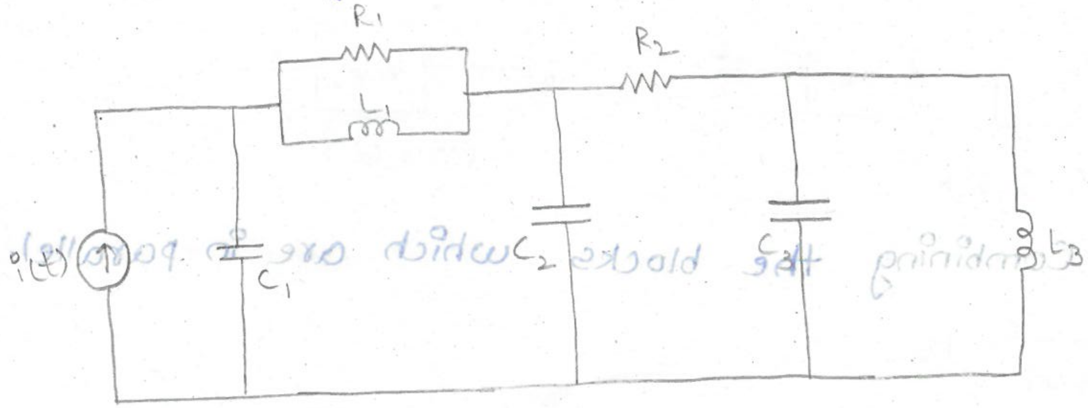
$$i(t) = C_1 \frac{dV_1}{dt} + \frac{1}{R_1}[V_1 - V_2] + \frac{1}{L_1}\int [V_1 - V_2]\,dt \quad \text{--- ⑦}$$

$$0 = C_2 \frac{dV_2}{dt} + \frac{1}{R_2}[V_2 - V_3] + \frac{1}{R_1}[V_2 - V_1] + \frac{1}{L_1}\int [V_2 - V_1]\, dt \quad \text{— (8)}$$

Considering eq-(3), we get

$$0 = C_3 \frac{dV_3}{dt} + \frac{1}{L_3}\int V_3\, dt + \frac{1}{R_2}[V_3 - V_2] \quad \text{— (9)}$$

By Considering 7,8,9 equations, we get Torque Current Analogous Circuit

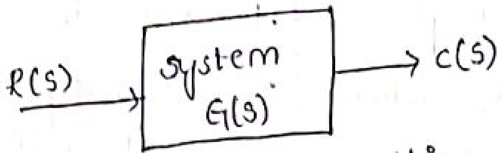


Control Systems → Magoor Kani

A control system manages, commands, directs (or) regulates the behaviour of other devices (or) systems using control loops.

A control system is a system which provides the desired response by controlling the output.

The simple block diagram of control system is shown by



$$G(s) = \frac{C(s)}{R(s)}$$

Ex :- Traffic light control machine, AC's, refrigerators.

Classification of control Systems:

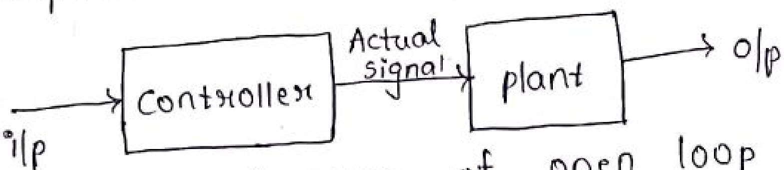
→ Continuous time & discrete time control systems

→ single input single output (SISO)
Multi input Multi output (MIMO) control systems

→ Open loop and closed loop Control Systems

* Control systems can be classified as open loop and closed loop controlled systems based on the feed back path.

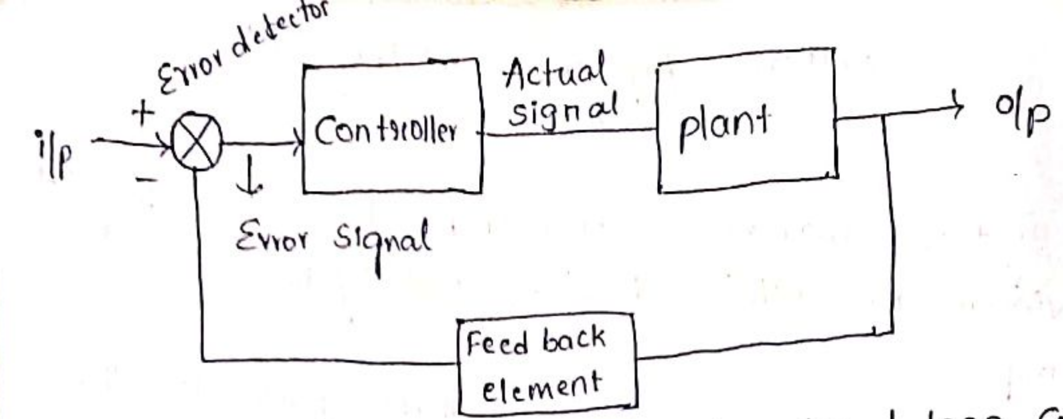
→ In open loop control system output is not fed back to the input. So, the control action is independent of the desired output



The block diagram of open loop control system is shown above

→ In closed loop control system output is fed back to the input. So, the control action is dependent on the desired output.

The block diagram of closed loop control system is as shown:



Differences b/w open loop and closed loop Control system

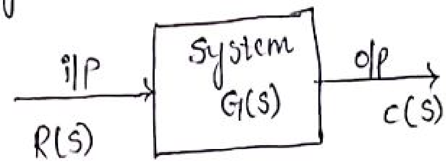
Open loop CS	closed loop CS
1. There is no feed back element in open loop.	1. There is feed back element in closed loop.
2. Accuracy is less when compared with closed loop.	2. Accuracy is more when compared with open loop.
3. The control action is independent on the o/p.	3. The control action is dependent on the o/p.
4. Cost is less.	4. cost is more.
5. There is less complexity.	5. Complexity is more when compared with openloop.

Control Systems:

System:
It is a combination of elements / components that are arranged in sequence to perform a specific function, the formation of this group is called a system.

Control System:
To achieve the desired output of the system by varying the input of the system is called controlled system.

Input of the control systems are command signal (or) exitation signal. It is represented by R(S).
Output of the control systems are controlled signal (or) response signal. It is represented by C(S).

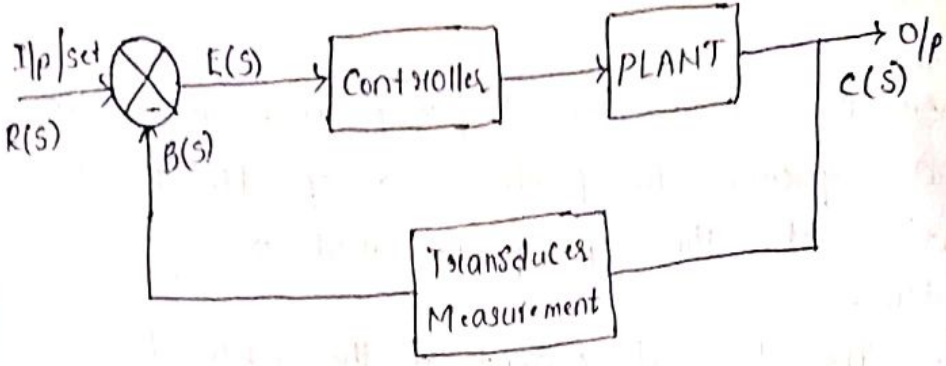


Control Systems

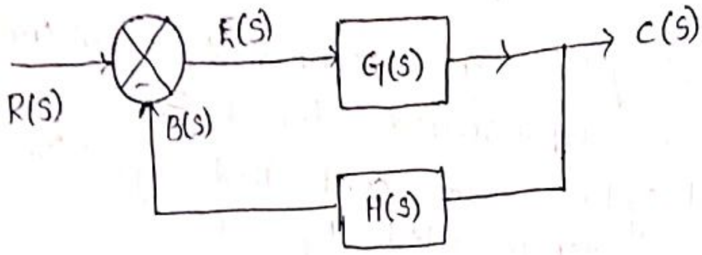
- Open loop control system (OLCS) (or) conditional controlled system
- closed loop closed system (CLCS)

* OLCS are more stable than CLCS when condition is applied.
* CLCS are more stable than OLCS without condition

Closed loop control systems:
In closed loop control system the changes in the o/p are measured through feed back and compared with the i/p (or) set point to acheive the controlled objective.



Canonical form :-



Transfer Function for CLCS :-

$$E(S) = R(S) - B(S)$$

$$E(S) \cdot G(S) = C(S)$$

$$E(S) = \frac{C(S)}{G(S)}$$

$$\frac{C(S)}{G(S)} = R(S) - B(S) \qquad B(S) = H(S)\,C(S)$$

$$C(S) = G(S)\left[R(S) - B(S)\right]$$

$$C(S) = G(S)\,R(S) - G(S)\,B(S)$$

$$C(S) = G(S)\,R(S) - G(S) \cdot H(S)\,C(S)$$

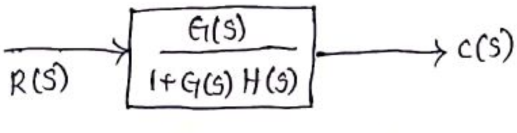
$$C(S) + G(S)\,H(S)\,C(S) = G(S)\,R(S)$$

$$C(S)\left[1 + G(S)\,H(S)\right] = G(S)\,R(S)$$

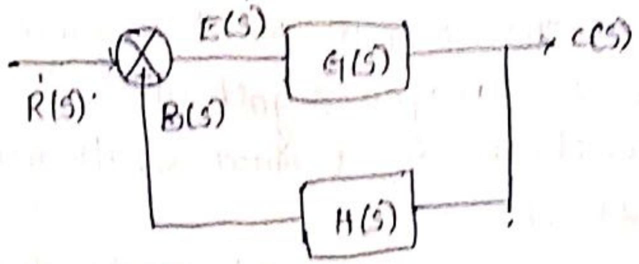
$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\,H(S)}$$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)\,H(S)}$$

Mathematical form :-



Feedback with positive :-



$$E(S) = R(S) + B(S)$$

$$E(S)\,G(S) = C(S)$$

$$E(S) = \frac{C(S)}{G(S)}$$

$$\frac{C(S)}{G(S)} = R(S) + B(S)$$

$$C(S) = G(S)\left[R(S) + B(S)\right]$$

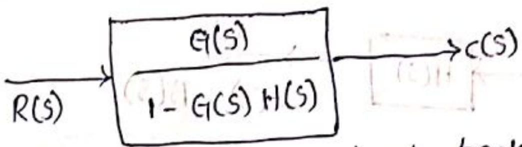
$$C(S) = G(S)R(S) + G(S)B(S)$$

$$C(S) = G(S)R(S) + G(S)H(S)C(S)$$

$$C(S)\left[1 - G(S)H(S)\right] = G(S)R(S)$$

$$TF = F(S) = \frac{C(S)}{R(S)} = \frac{G(S)}{1 - G(S)H(S)}$$

Mathematical form:



→ By comparing the +Ve feed back amplifiers and -Ve feed back amplifiers, the gain is more in +Ve feed back amplifier rather than -Ve feed back amplifier

→ -Ve feed back amplifiers are more stable than +Ve feed back amplifiers.

Open Loop Control System (OLCS) :-

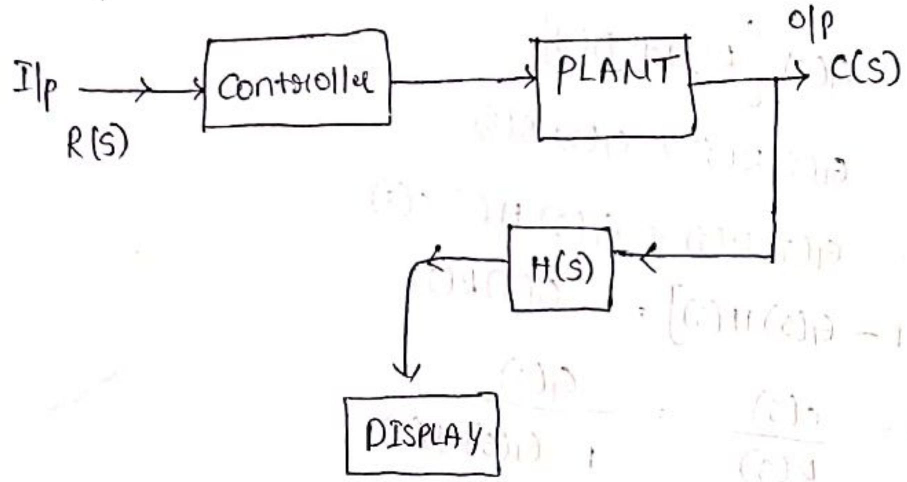
They are conditional controlled systems formulated under the condition that the system is not subjected to any type of disturbances.

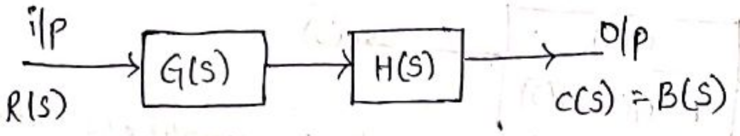
In this configuration, the feed back (or) measurement is not connected to the forward path (or) controller

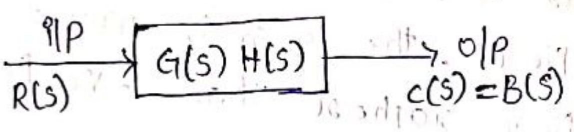
→ Feedback is in open loop systems except for displaying the information about the output which is same as the setpoint has no major significance. This insignificance of feedback is termed as elimination (or) removal of feedback.

→ Open lo Performance analysis is not applicable for open loop control systems because they are highly stable systems and they are not subjected to any type of disturbances

→ Representation of openloop Control System is







$$R(S)\,G(S)\,H(S) = C(S)$$

$$G(S)H(S) = \frac{C(S)}{R(S)}$$

$$T.F = F(S) = \frac{C(S)}{R(S)} = G(S)H(S)$$

Differences b/w OLCS and CLCS

OLCS	CLCS
1) It is easy to design.	1) It is complex to design.
2) It requires less cost	2) Cost is more in CLCS.
3) Accuracy is less in OLCS	3) Accuracy is more in CLCS.
4) Gain is more.	4) Gain is less.
5) under condition, OLCS are more stable	5) under condition, CLCS are ~~un~~ less stable.
6) without condition, OLCS are unstable	6) without condition, CLCS are more stable

Examples of OLCS & CLCS :-

↳ Temperature control system

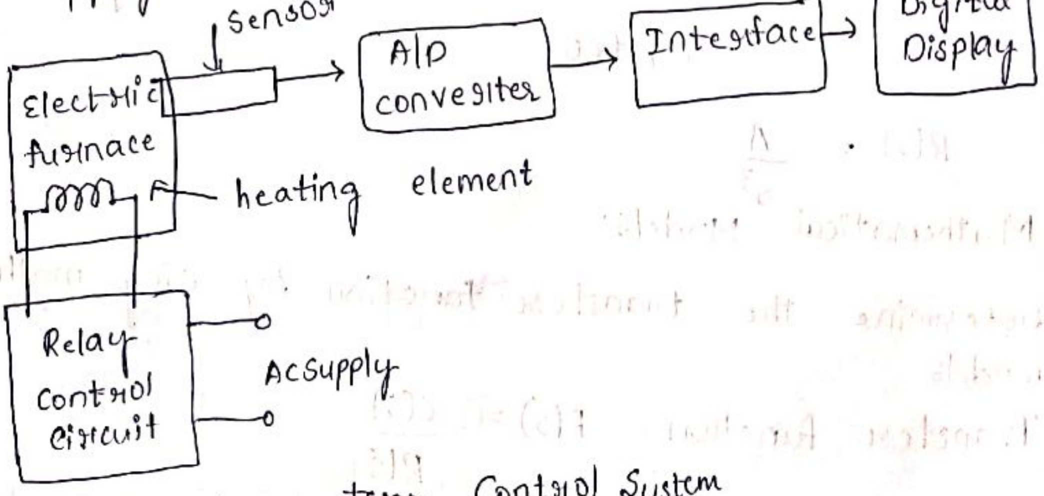
→ Traffic control system

→ Numerical control system

Temperature Control System:

Open loop System:

The electric furnace shown in fig 1.3 is an open loop system. The o/p in the system is the desired temp. The temp. of the system is raised by heat generated by the heating element. The o/p temperature depends on the time during which the supply to heater remains ON.

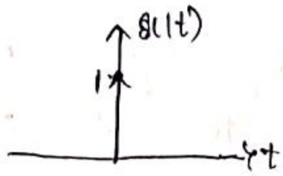


Open loop temp. Control System

Weighting Functions :-

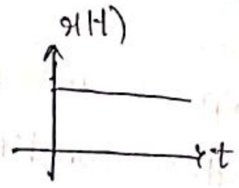
1. ~~Ramp~~ Impulse Signal :-

$$r(t) = \begin{cases} 1, & t=0 \\ 0, & \text{else} \end{cases}$$



$R(s) = 1$

2. Step Signal :-



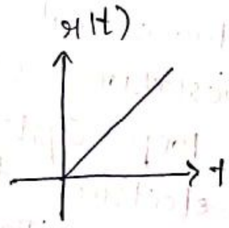
$$\cancel{r(t) = \begin{cases} 1, & t>0 \\ 0, & \text{else} \end{cases}}$$

$$\cancel{r(t) = \begin{cases} A\,u(t), & t>0 \\ 0, & \text{else} \end{cases}}$$

$$r(t) = A\,u(t) = \begin{cases} 1, & t>0 \\ 0, & \text{else} \end{cases}$$

$$R(s) = \frac{A}{s}$$

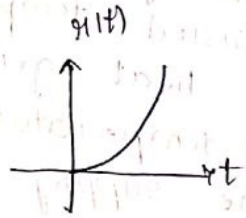
3. Ramp Signal :-



$$r(t) = At \neq 0, \; t>0$$
$$= 0, \; t<0$$

$$R(s) = \frac{A}{s^2}$$

4) Parabolic signal :-



$$r(t) = \frac{At^2}{2} \neq 0, \; t>0$$
$$0, \; t\leq 0$$

$$R(s) = \frac{A}{s^3}$$

Mathematical Models:

Determine the transfer function by using mathematical models:

Transfer function : $F(s) = \dfrac{C(s)}{R(s)}$

$$C(s) = R(s)\,F(s)$$

$R(s) = 1 \qquad C(s) = F(s)$

Scanned by CamScanner

Transfer function is defined as the laplace transform of impulse response. is known as weighting function.

Mathematical Model of a control System:

→ Mathematical Model of a C.S is a combination of various components are connected in sequence of a system to serve an objective.

→ The input & output relation of a system is represented by differential equations.

→ To oblain the response of the system (or) the response of the system with the help of studying the differential equations.

by solving

These are classified into 2 types.

- Electrical system
  - I/p — O/p
  - → I — V
  - → V — I
  - → passive elements R, L, C
- Mechanical system
  - Translational system
    - I/p - Force (F)
    - O/p - Velocity (v)
    - O/p - displacement (x)
    - → Mass element $\boxed{M}$
    - → spring element K
    - → dash pot/damper element B
  - Transrotational system
    - I/p - Torque (T)
    - O/p → ω - (angular velocity)
    - → θ - (angular displacement)
    - → Inertia element (J) $\boxed{J}$
    - → Torsional spring element K
    - → Torsional damper element B

Electrical System:-

→ It consists of I, V, R, L, C

$R \to V = IR \to I = V/R$

$L \to V = L\frac{di}{dt} \to I = \frac{1}{L}\int v\,dt$

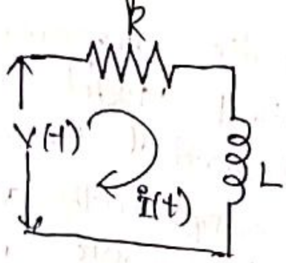
$C \to V = \frac{1}{C}\int i\,dt \to I = C\frac{dv}{dt}$

→ Inductor stores the energy in the form of current

→ Capacitor stores the energy in the form of voltage

Scanned by CamScanner

Determine the T.F for given ckt diagram:-



$$\cancel{V(t) = I R + L\frac{dI}{dt}}$$

$$V(t) = i(t) R + L\frac{di(t)}{dt}$$

Apply LS

$$V(s) = I(s) R + L s I(s)$$

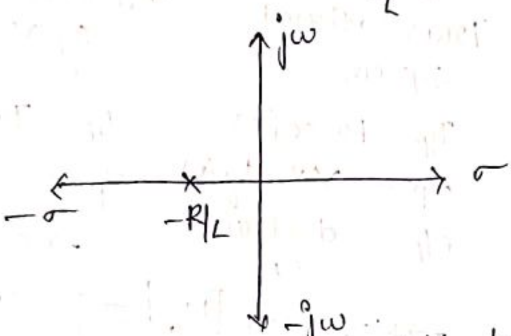
$$V(s) = I(s)(R + Ls)$$

$$\frac{I(s)}{V(s)} = \frac{1}{R+Ls}$$

$$\frac{I(s)}{V(s)} = \frac{1}{s\left(L+\frac{R}{s}\right)} = \frac{1/L}{s+R/L}$$

poles are located at $s+\frac{R}{L}=0 \Rightarrow s=-R/L$

Roc



The pole is located at left hand side of the s-plane. Hence the system is stable

Mechanical Translational System:-

We obtain the response of the mechanical translational systems. we will require 3 elements. They are

Mass element –

Spring element – k (Elasticity is present)

damper element – B (Friction is present)

displacement – x

I/p – Force – F

o/p – Velocity – v

displacement – X.

Velocity $V = \frac{dx}{dt}$

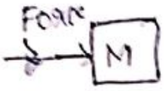
acceleration $a = \frac{d^2x}{dt^2}$

Note: $a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$

All these 3 elements will exhibits Newton's 2nd law.

Scanned by CamScanner

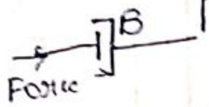
Mass element :-
If force applied on mass element it will exhibits opposite force from the mass element i.e, acceleration



$\mapsto$ acceleration.

$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$

Damper element :-
If force applied on damper element it will exhibits opposite force which is in the form of velocity.



$\mapsto$ velocity (V)

$V = \frac{dx}{dt}$

Spring element :-
If force applied on spring element it will exhibits opposite force which is in the form of displacement.

$\mapsto$ displacement

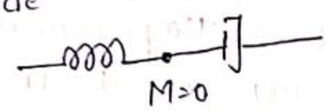
x

$x = \int v\,dt$

→ The sum of the forces acting on the body is equal to zero.

Note :-
If 2 elements are connected in series consider a dummy node in b/w these elements.

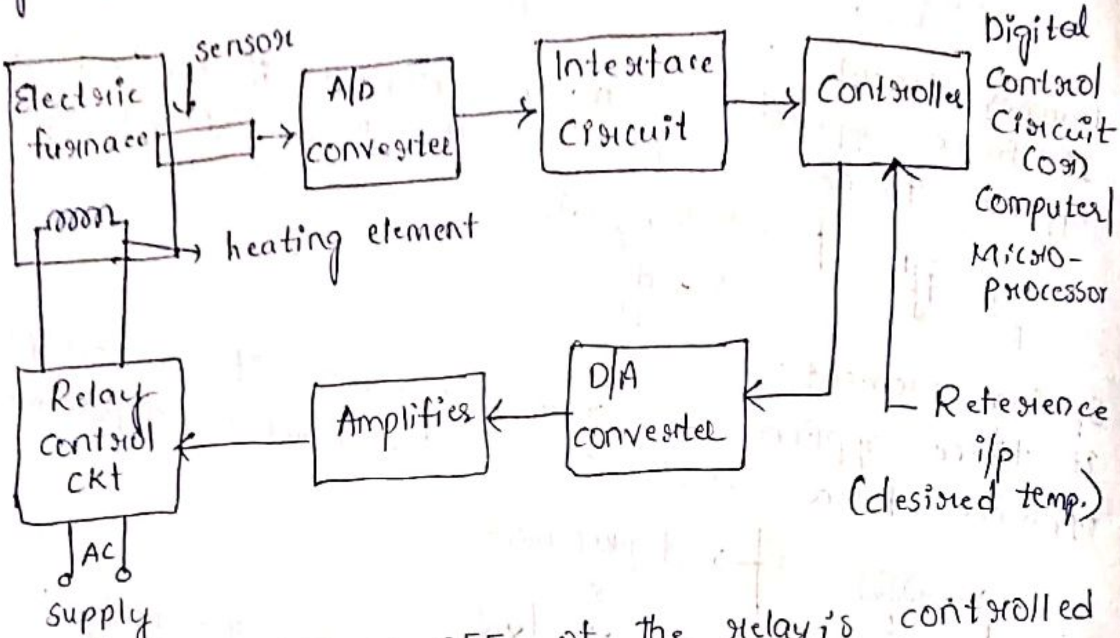
Ex :-



⇒ Examples of CS :-
Temperature Control System :-
The ON and OFF of the supply is governed by the time setting of the relay. The temperature is measured by a sensor, which gives an analog voltage corresponding to the temperature of the furnace. The analog signal is converted to digital signal by an Analog - to Digital converter (AD conver) The digital signal is given to the digital display device to display the temperature. In this system if there is any change in output temperature then the time setting of the relay is not altered automatically.

Closed loop System:

The electric furnace shown in fig. is a closed loop system. The o/p of the system is the desired temperature and it depends on the time during which the supply to heater remains ON.



The switching ON and OFF of the relay is controlled by a controller which is a digital system (or) computer. The desired temperature is i/p to the system through keyboard or as a signal corresponding to desired temp. via ports. The actual temp. is sensed by sensor and converted to digital signal by the A/D converter. The computer reads the actual temperature and compares with desired temperature. If it finds any difference then it sends signal to switch ON ~~and~~ or OFF the relay through D/A converter and amplifier. Thus the system automatically corrects any changes in o/p. Hence it is a closed loop system

Traffic Control System:

Open loop System:

Traffic control by means of traffic signals operated on a time basis constitutes an open-loop control system. The sequence of control signals are based on a time slot given for each signal. The time slots are decided based on a traffic study. The system will not measure the density of the traffic before giving the signals. Since the timeslot does not changes according to traffic density, the system is open loop system.

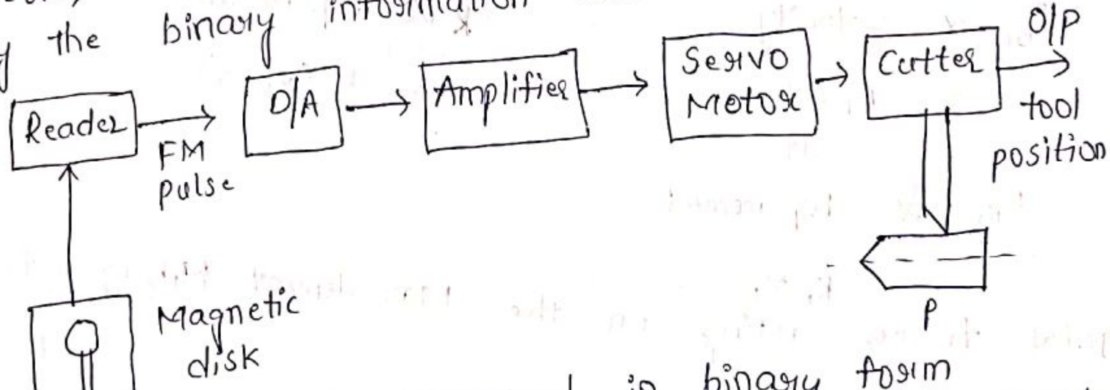
Closed loop System:

Traffic control system can be made as a closed
loop system if the time slots of the signals are
decided based on the density of traffic. In closed loop
traffic control system, the density of the traffic is
measured on all the sides and the information
is fed to a computer. The timings of the control
systems signals are decided by the computer
based on the density of traffic. Since the closed
loop system dynamically changes the timings, the
flow of vehicles will be better than open loop
system.

Numerical control System:-

Open loop system:

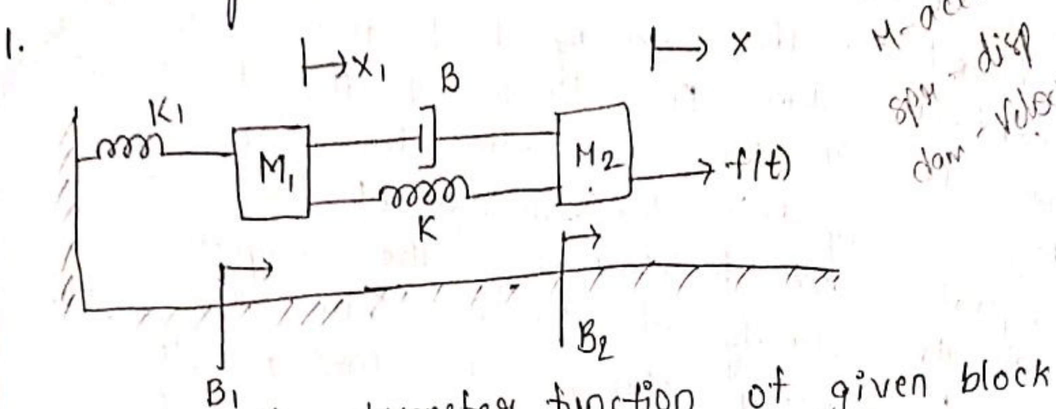
Numerical control is a method of controlling the
motion of machine components using of numbers.
Here, the position of work head tool is controlled
by the binary information contained in a disk.



A magnetic disk is prepared in binary form
representing the desired part P (P is the metal part
to be machined). The tool will operate on the
desired part P. To start the system, the disk is fed
through the reader to the D/A converter. The
D/A converter converts the FM o/p of the reader
to a analog signal. It is amplified and fed to
servomotor which positions the cutter on the
desired part P. The position of the cutter head
is controlled by the angular motion of the
servomotor. This is an open loop system since no
feedback path exists b/w the o/p and i/p. The system

positions the tool for a given i/p command. Any deviation in the desired position is not checked and corrected automatically.

1.



Determine the transfer function of given block diagram.
No. of differential equations is dependent on no of mass elements.

At node M_1 :-

$F_{M_1} \propto$ acceleration

$$= M_1 \frac{d^2x_1}{dt^2}$$

$F_B \propto$ velocity

$$= B\frac{d}{dt}(x_1 - x)$$

$F_{B_1} \propto$ Velocity

$$= B_1 \frac{dx_1}{dt}$$

$F_K \propto$ displacement

$$= K(x_1 - x)$$

$F_{K_1} \propto$ displacement

$$= K_1 x_1$$

Total forces acting on the Mass element M_1 is equal to zero.

$$F_{M_1} + F_{K_1} + F_{B_1} + F_B + F_K = 0$$

$$\Rightarrow M_1 \frac{d^2x_1}{dt^2} + k_1x_1 + B_1\frac{dx_1}{dt} + B\frac{d}{dt}(x_1 - x) + k(x_1 - x) = 0$$

Apply Laplace Transform on both sides.

$$\Rightarrow M_1 s^2 x_1(s) + k_1 x_1(s) + B_1 s x_1(s) + B[s x_1(s) - s x(s)] + k[x_1(s) - x(s)] = 0$$

$$\Rightarrow x_1(s)[M_1 s^2 + k_1 + B_1 s + Bs + k] + x(s)[-Bs - k] = 0$$

$$\Rightarrow x_1(s)[M_1 s^2 + k_1 + B_1 s + Bs + k] - x(s)(Bs + k) = 0 \quad (1)$$

At node M_2 :

$F_{M_2} \propto$ acceleration

$= M_2 \frac{d^2x}{dt^2}$

$F_{B_2} \propto$ velocity

$= B_2 \frac{dx}{dt}$

$F_B \propto$ velocity

$= B \frac{d}{dt}(x - x_1)$

$F_K \propto$ displacement

$= K(x - x_1)$

At Mass M_2, applied force is equal to sum of opposite forces.

$$f(t) = M_2 \frac{d^2x}{dt^2} + B_2 \frac{dx}{dt} + B \frac{d}{dt}(x - x_1) + K(x - x_1)$$

Apply L/T on b.s

$$F(S) = M_2 s^2 x(S) + B_2 s X(S) + B[sX(S) - sX_1(S)] + K[X(S) - X_1(S)]$$

$$F(S) = X(S)\left[M_2 s^2 + B_2 s + Bs + K\right] - X_1(S)\left[Bs + K\right] \quad \text{—②}$$

From eq ①

$$\Rightarrow X_1(S)\left[M_1 s^2 + K_1 + B_1 s + Bs + K\right] = X(S)\left[Bs + K\right]$$

$$\Rightarrow X_1(S) = \frac{X(S)(Bs + K)}{M_1 s^2 + K_1 + B_1 s + Bs + K} \quad \text{—③}$$

sub. eq ③ in eq ②

$$F(S) = X(S)\left[M_2 s^2 + B_2 s + Bs + K\right] - \frac{(Bs + K)^2 X(S)}{M_1 s^2 + K_1 + (B_1 + B)s + K}$$

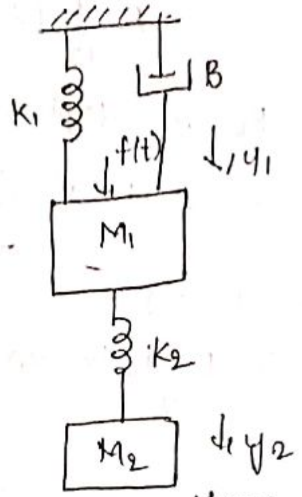
$$= X(S)\left\{M_2 s^2 + s(B + B_2) + K - \frac{(Bs + K)^2}{M_1 s^2 + K_1 + (B_1 + B)s + K}\right\}$$

$$F(S) = X(S)\left\{\frac{\left[M_2 s^2 + s(B + B_2) + K\right]\left[M_1 s^2 + K_1 + (B + B_1)s + K\right] - (Bs + K)^2}{M_1 s^2 + K_1 + (B_1 + B)s + K}\right\}$$

$$\Rightarrow \frac{X(S)}{F(S)} = \frac{M_1 s^2 + K_1 + (B_1 + B)s + K}{\left[M_2 s^2 + s(B + B_2) + K\right]\left[M_1 s^2 + K_1 + (B + B_1)s + K\right] - (Bs + K)^2}$$

Scanned by CamScanner

2. Write the eqns of motion in s-domain for the system shown. Determine the transfer f'n of the system.



No. of differential equations is dependent on no. of mass elements.

At node M_1 :-

$F_{M_1} \propto$ acceleration

$= M_1 \dfrac{d^2 Y_1}{dt^2}$

$F_{K_1} \propto$ displacement

$= K_1 Y_1$.

$F_B \propto$ Velocity

$= B \dfrac{dY_1}{dt}$

$F_{K_2} \propto$ displacement

$= K_2 (Y_1 - Y_2)$

Applied force is equal to sum of opposite forces.

$\Rightarrow \quad F_{M_1} + F_{K_1} + F_B + F_{K_2} = f(t)$

$\Rightarrow \quad f(t) = M_1 \dfrac{d^2 Y_1}{dt^2} + K_1 Y_1 + B \dfrac{dY_1}{dt} + K_2 (Y_1 - Y_2)$

Apply LT on b.s

$F(s) = M_1 s^2 Y_1(s) + K_1 Y_1(s) + B s Y_1(s) + K_2 [Y_1(s) - Y_2(s)]$

$F(s) = Y_1(s) [M_1 s^2 + K_1 + Bs + K_2] - K_2 Y_2(s) \quad \text{—①}$

At node M_2:

$F_{M_2} \propto$ acceleration

$= M_2 \dfrac{d^2 Y_2}{dt^2}$

$F_{K_2} \propto$ displacement

$= K_2 (Y_2 - Y_1)$

Total force acting on the Mass M_2 is equal to zero.

$$F_{M_2} + F_{K_2} = 0$$

$$M_2 \frac{d^2 y_2}{dt^2} + K_2(y_2 - y_1) = 0$$

Apply L.T on b.s

$$M_2 s^2 Y_2(s) + K_2[Y_2(s) - Y_1(s)] = 0$$

$$[M_2 s^2 + K_2]\, Y_2(s) - K_2 Y_1(s) = 0 \quad \text{--- (2)}$$

From eq (2)

$$Y_1(s) = \frac{[M_2 s^2 + K_2]\, Y_2(s)}{K_2} \quad \text{--- (3)}$$

sub. eq (3) in eq (1)

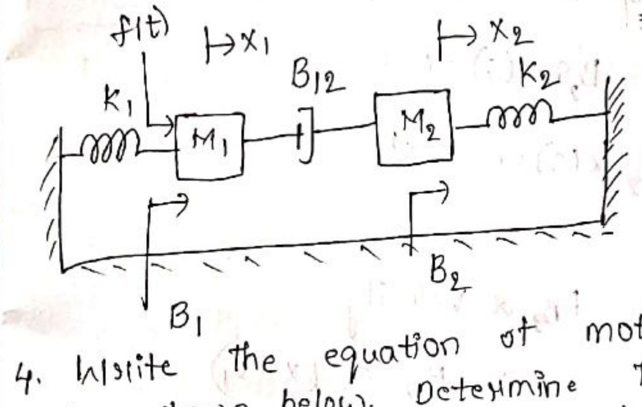
$$F(s) = \frac{[M_2 s^2 + K_2]\, Y_2(s)}{K_2}[M_1 s^2 + K_1 + Bs + K_2] - K_2 Y_2(s)$$

$$= Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + (K_1 + K_2 + Bs))}{K_2} - K_2\right]$$

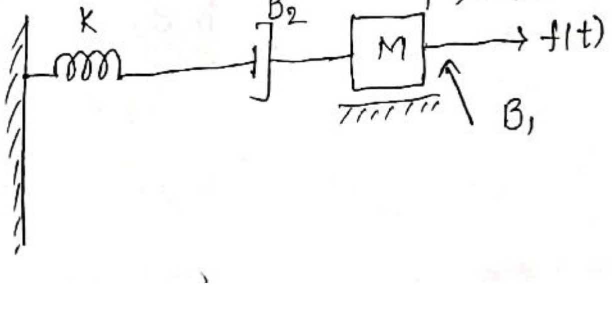
$$F(s) = Y_2(s)\left[\frac{(M_2 s^2 + K_2)(M_1 s^2 + K_1 + K_2 + Bs) - K_2^2}{K_2}\right]$$

$$\Rightarrow TF = \frac{Y_2(s)}{F(s)} = \frac{K_2}{(M_2 s^2 + K_2)(M_1 s^2 + K_1 + K_2 + Bs) - K_2^2}$$

3. Determine the transfer function $\frac{X_1(s)}{F(s)}$ and $\frac{X_2(s)}{F(s)}$ for the system shown below:
***



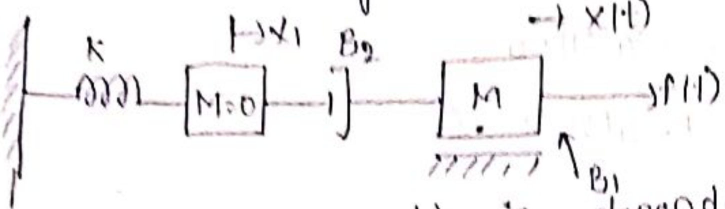
4. Write the equation of motion in s-domain for the system shown below. Determine the TF of the system.



Const

By observing the above diagram, when elements are connected in series consider a dummy node in b/w these elements.

$\because$ The above diagram can be equal to



No. of differential eqn's is dependent on no. of mass elements.

<u>At node M=0 :-</u>

$F_K \propto$ displacement

$= K x_1$

$F_M \propto$ acceleration

$= M \frac{d^2x_1}{dt^2}$ $(\because M=0)$

$F_M = 0$

$F_{B_2} \propto$ Velocity

$= B_2 (x_1 - x$

$= B_2 \frac{d}{dt} (x_1 - x)$

total force acting on Mass M=0 is equal to zero.

$$F_K + F_M + F_{B_2} = 0$$

$$Kx_1 + M\frac{d^2x_1}{dt^2} + B_2\frac{d(x_1 - x)}{dt} = 0$$

Apply L/T

$$Kx_1(s) + Ms^2x_1(s) + B_2[s\,x_1(s) - s\,x(s)] = 0$$

$$x_1(s)\,[K + Ms^2 + B_2 s] - B_2 s\,x(s) = 0$$

$$\Rightarrow x_1(s)\,[K + B_2 s] - B_2 s\,X(s) = 0 \quad \text{— (1)}$$

<u>At node M :-</u>

$F_M \propto$ acceleration

$= M\frac{d^2x}{dt^2}$

$F_{B_1} \propto$ Velocity

$= B_1 \frac{dx}{dt}$

$F_{B_2} \propto$ Velocity

$= B_2 \frac{d}{dt}(x - x_1)$

Applied force is equal to sum of opposition forces.

$$F_M + F_{B_1} + F_{B_2} = f(t)$$

$$M\frac{d^2x}{dt^2} + B_1\frac{dx}{dt} + B_2\frac{d}{dt}(x - x_1) = f(t)$$

Apply L.T on b.s

$$M s^2 x(s) + B_1 s\, x(s) + B_2[x(s) - x_1(s)]s = F(s)$$

$$x(s)\left[Ms^2 + B_1 s + B_2 s\right] - B_2 s\, x_1(s) = F(s) \quad -(2)$$

From eq (1)

$$x_1(s) = \frac{B_2 s\, x(s)}{k + B_2 s} \quad -(3)$$

eq (3) in eq (2)

$$x(s)\left(Ms^2 + B_1 s + B_2 s\right) - B_2 s\left(\frac{B_2 s\, x(s)}{k + B_2 s}\right) = F(s)$$

$$x(s)\left[\left(Ms^2 + B_1 s + B_2 s\right) - \frac{(B_2 s)^2}{k + B_2 s}\right] = F(s)$$

$$\Rightarrow \frac{x(s)}{F(s)} = \frac{\left[Ms^2 + (B_1 + B_2)s\right]\left[k + B_2 s\right] - (B_2 s)^2}{k + B_2 s} = F(s)$$

$$\Rightarrow TF = \frac{x(s)}{F(s)} = \frac{k + B_2 s}{(Ms^2 + B_1 s + B_2 s)(k + B_2 s) - (B_2 s)^2}$$

3) No. of differential equations is dependent on no. of mass elements.

At node M_1:

$F_{M_1} \propto$ acceleration

$$= M_1\frac{d^2x_1}{dt^2}$$

$F_{B_{12}} \propto$ Velocity

$$= B_{12}\frac{d(x_1 - x)}{dt}$$

$F_{K_1} \propto$ displacement

$$= K_1 x_1$$

$F_{B_1} \propto$ Velocity

$$= B_1\frac{dx_1}{dt}$$

Applied force is equal to sum of opposite forces.

$$F_{M_1} + F_{K_1} + F_{B_1} + F_{B_{12}} = f(t)$$

$\therefore f(t) = M_1 \frac{d^2x_1}{dt^2} + K_1x_1 + B_1\frac{dx_1}{dt} + B_{12}\frac{d}{dt}(x_1 - x_2)$

Apply LT on b.s

$F(s) = M_1 s^2 X_1(s) + K_1 x_1(s) + B_1 s x_1(s) + B_{12}[x_1(s) - x_2(s)]s$

$F(s) = X_1(s)\left[M_1s^2 + K_1 + B_1S + B_{12}S\right] - B_{12}S\,x_2(S)$ — (1)

At node M_2 :-

$F_{M_2} \propto$ acceleration

$= M_2\frac{d^2x_2}{dt^2}$

$F_{B_2} \propto$ Velocity

$= B_2\frac{dx_2}{dt}$

$F_{K_2} \propto$ displacement

$= K_2X_2$

$F_{B_{12}} \propto$ Velocity

$= B_{12}\frac{d}{dt}(x_2 - x_1)$

Total force acting on Mass M_2 is equal to Zero.

$F_{M_2} + F_{K_2} + F_{B_2} + F_{B_{12}} = 0$

$\Rightarrow M_2\frac{d^2x_2}{dt^2} + K_2x_2 + B_2\frac{dx_2}{dt} + B_{12}\frac{d}{dt}(x_2 - x_1) = 0.$

Apply LT on b.s

$\Rightarrow M_2 S^2 x_2(s) + K_2X_2(S) + B_2S\,X_2(s) + B_{12}[X_2(s) - X_1(s)]s = 0$

$\Rightarrow \left[M_2S^2 + K_2 + B_2S + B_{12}S\right]X_2(s) - B_{12}S\,X_1(s) = 0$ — (2)

From eq (2).

$\left[M_2S^2 + K_2 + B_2S + B_{12}S\right]X_2(S) = B_{12}SX_1(S)$

$X_1(S) = \frac{M_2S^2 + K_2 + B_2S + B_{12}S\,(x_2(s))}{B_{12}S}$ — (3)

$X_2(S) = \frac{B_{12}SX_1(S)}{M_2S^2 + K_2 + B_2S + B_{12}S}$ — (4)

Sub. eq (3) in eq (1)

$F(S) = \left(\frac{M_2S^2 + K_2 + B_2S + B_{12}S}{B_{12}S}\right)X_2(S)\left[M_1S^2 + K_1 + B_1S + B_{12}S\right] - B_{12}S\,x_2(S)$

$F(S) = X_2(S)\left[\frac{\left[M_2S^2 + K_2 + (B_2 + B_{12})S\right]\left[M_1S^2 + K_1 + B_1S + B_{12}S\right]}{B_{12}S} - B_{12}S\right]$

$$F(S) = X_2(S)\left\{\frac{\left[M_2S^2+K_2+(B_2+B_{12})S\right]\left[M_1S^2+K_1+B_1S+B_{12}S\right]-(B_{12}S)^2}{B_{12}S}\right\}$$

$$\Rightarrow \frac{X_2(S)}{F(S)} = \frac{B_{12}S}{(M_2S^2+K_2+B_2S+B_{12}S)(M_1S^2+K_1+B_1S+B_{12}S)-(B_{12}S)^2}$$

sub eq (4) in eq (2)

$$F(S) = X_1(S)(M_1S^2+K_1+B_1S+B_{12}S) - B_{12}S\left[\frac{B_{12}S\,X_1(S)}{M_2S^2+K_2+B_2S+B_{12}S}\right]$$

$$= X_1(S)\left[M_1S^2+K_1+B_1S+B_{12}S - \frac{(B_{12}S)^2}{M_2S^2+K_2+B_2S+B_{12}S}\right]$$

$$F(S) = X_1(S)\left[\frac{(M_1S^2+K_1+B_1S+B_{12}S)(M_2S^2+K_2+B_2S+B_{12}S)-(B_{12}S)^2}{M_2S^2+K_2+B_2S+B_{12}S}\right]$$

$$\frac{X_1(S)}{F(S)} = \frac{M_2S^2+K_2+B_2S+B_{12}S}{(M_1S^2+K_1+B_1S+B_{12}S)(M_2S^2+K_2+B_2S+B_{12}S)-(B_{12}S)^2}$$

Mathematical Model of Mechanical Transrotational system :-

We determine the mathematical model of a mechanical transrotational systems we require 3 elements. Those are Inertia of mass (J) ; torosional spring element (K) ; torosional damper / dashpot element (B)

I/p - Torque (T)

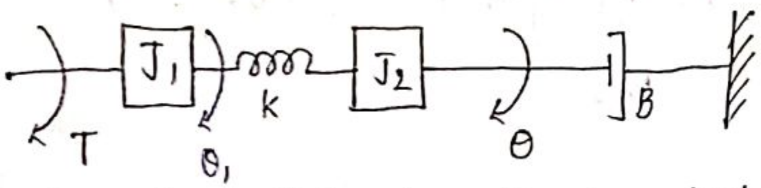
O/p - angular displacement (θ)

angular velocity (ω)

$$\omega = \frac{d\theta}{dt}$$

angular acceleration $a = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

1. Determine the transfer function for the given mechanical transrotational system.



No. of differential eq's is dependent on no. of inertia of mass elements i.e., 2.

At node J_1 :-

$T_{J_1} \propto$ angular acceleration $\qquad T_K \propto$ angular displacement

$$= J_1 \frac{d^2\theta_1}{dt^2} \qquad\qquad = K(\theta_1 - \theta)$$

Applied Torque is equal to sum of opposite torques.

$$T_{J_1} + T_K = T$$

$$J_1 \frac{d^2\theta_1}{dt^2} + K(\theta_1 - \theta) = T$$

Apply LT on b.s

$$J_1 s^2 \theta_1(s) + K[\theta_1(s) - \theta(s)] = T(s)$$

$$\theta_1(s)\left[J_1 s^2 + K\right] - K\theta(s) = T(s) \quad —(1)$$

At node J_2 :-

$T_{J_2} \propto$ angu. acceleration $\qquad T_B \propto$ ang. velocity

$$= J_2 \frac{d^2\theta}{dt^2} \qquad\qquad = B\frac{d\theta}{dt}$$

$T_K \propto$ angu. displacement

$$= K(\theta - \theta_1)$$

Total torque acting on element J_2 is zero.

$$T_{J_2} + T_B + T_K = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d\theta}{dt} + K(\theta - \theta_1) = 0$$

Apply LT on b.s

$$J_2 s^2 \theta(s) + B s\theta(s) + K[\theta(s) - \theta_1(s)] = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K\right] - K\theta_1(s) = 0 \quad —(2)$$

$$(2) \Rightarrow \quad \theta_1(s) = \frac{(J_2 s^2 + Bs + K)\,\theta(s)}{K} \quad —(3)$$

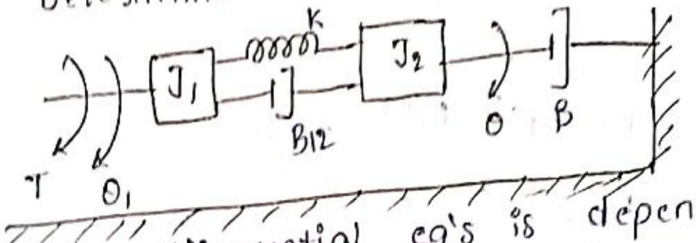
sub eq (3) in eq (1)

$$T(s) = \frac{(J_2s^2 + Bs + K)\,\theta(s)}{K}(J_1s^2 + K) - K\theta(s)$$

$$T(s) = \theta(s)\left[\frac{(J_2s^2 + Bs + K)(J_1s^2 + K) - K^2}{K}\right]$$

$$TF = \frac{\theta(s)}{T(s)} = \frac{K}{(J_2s^2 + Bs + K)(J_1s^2 + K) - K^2}$$

2. Determine the T.F for a given transrotational system.



No. of differential eq's is dependent on no. of inertia elements.

At node J_1:

$T_{J_1} \propto$ ang. acceleration

$$= J_1 \frac{d^2\theta_1}{dt^2}$$

$T_K \propto$ ang. displacement

$$= K(\theta_1 - \theta)$$

$T_{B_{12}} \propto$ ang. velocity

$$= B_{12}\frac{d}{dt}(\theta_1 - \theta)$$

Applied torque is equal to sum of opposite torques.

$$T = T_{J_1} + T_{B_{12}} + T_K$$

$$T = J_1\frac{d^2\theta_1}{dt^2} + B_{12}\frac{d}{dt}(\theta_1 - \theta) + K(\theta_1 - \theta)$$

Apply L.T on b.s

$$T(s) = J_1 s^2\theta_1(s) + B_{12}s\left[\theta_1(s) - \theta(s)\right] + K\left[\theta_1(s) - \theta(s)\right]$$

$$T(s) = \theta_1(s)\left[J_1s^2 + B_{12}s + K\right] - \left[B_{12}s + K\right]\theta(s) \quad (1)$$

At node J_2:

$T_{J_2} \propto$ ang. acceleration

$$= J_2\frac{d^2\theta}{dt^2}$$

$T_K \propto$ ang. displacement

$$= K(\theta - \theta_1)$$

$T_{B_{12}} \propto$ ang. velocity

$$= B_{12}\frac{d}{dt}(\theta - \theta_1)$$

$T_B \propto$ ang. velocity

$$= B\frac{d}{dt}(\theta\cancel{-\theta_1})$$

Scanned by CamScanner

Total torque acting on J_2 is zero.

$$T_{J_2} + T_B + T_K + T_{B_{12}} = 0$$

$$J_2 \frac{d^2\theta}{dt^2} + B\frac{d}{dt}(\theta - \theta_1) + K(\theta - \theta_1) + B_{12}\frac{d}{dt}(\theta - \theta_1) = 0$$

Apply L/T on b·s

$$\Rightarrow J_2 s^2 \theta(s) + B\, s[\theta(s) - \theta_1(s)] + K[\theta(s) - \theta_1(s)] + B_{12}\, s[\theta(s) - \theta_1(s)] = 0$$

$$\theta(s)\left[J_2 s^2 + Bs + K + B_{12}s\right] - \theta_1(s)\left[Bs + K + B_{12}s\right] = 0 \quad \text{—(2)}$$

From eq (2)

$$\Rightarrow \theta_1(s) = \frac{\left[J_2 s^2 + Bs + K + B_{12}s\right]}{Bs + K + B_{12}s}\,\theta(s) \quad \text{—(3)}$$

Substitute eq(3) in eq(1)

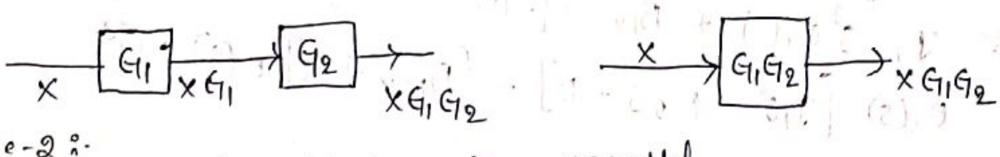
$$T(s) = \left[\frac{J_2 s^2 + Bs + K + B_{12}s}{Bs + K + B_{12}s}\right]\left[J_1 s^2 + B_{12}s + K\right]\theta(s) - \left[B_{12}s + K\right]\theta(s)$$

$$T(s) = \theta(s)\left\{\frac{(J_2 s^2 + Bs + K + B_{12}s)(J_1 s^2 + B_{12}s + K)}{Bs + K + B_{12}s} - \left[B_{12}s + K\right]\right\}$$

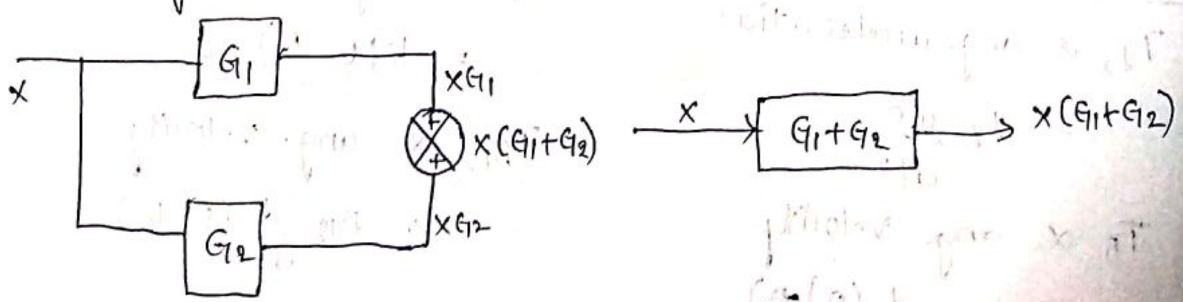
$$TF = \frac{\theta(s)}{T(s)} = \frac{Bs + K + B_{12}s}{(J_2 s^2 + Bs + K + B_{12}s)(J_1 s^2 + B_{12}s + K) - (B_{12}s + K)^2}$$

Mathematical Reduction Method :-

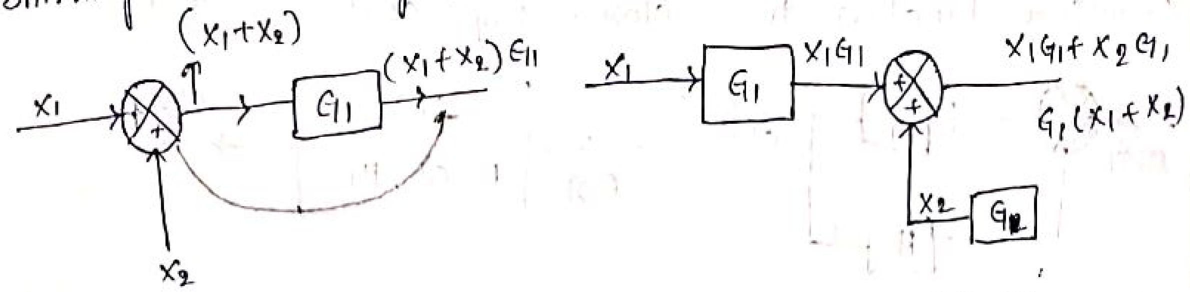
Rule-1 :-
Combining the blocks in series



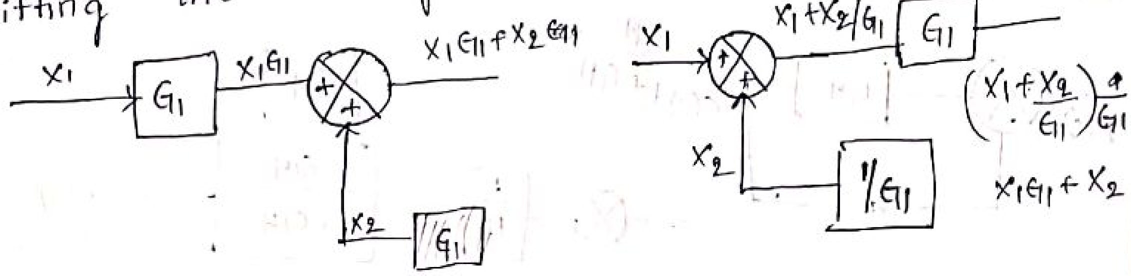
Rule-2 :-
Combining the blocks in parallel.



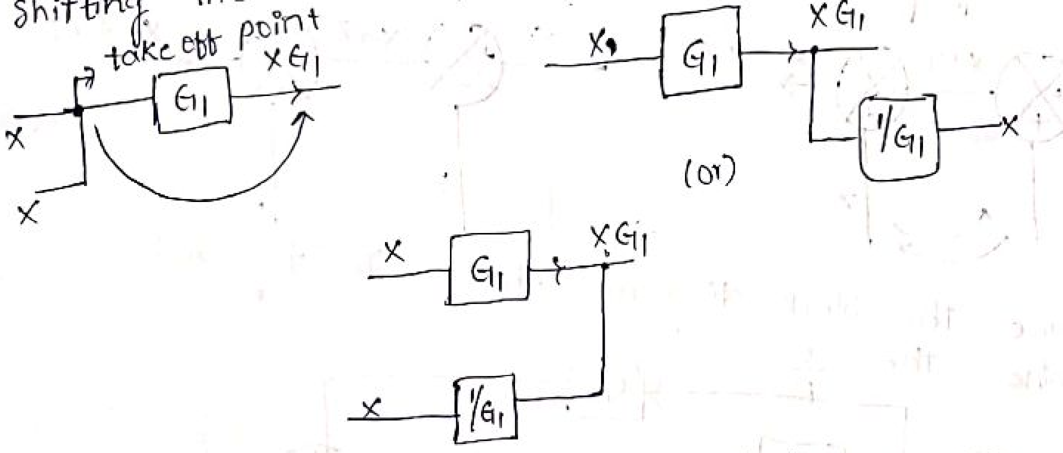
Rule-3 :-
Shifting the summing point after the block.



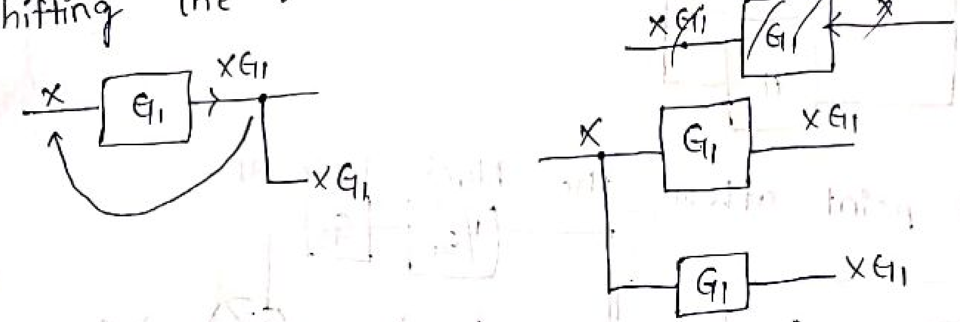
Rule-4 :-
Shifting the summing point before the block.



Rule-5 :-
Shifting the take off point after the ~~point~~ block.



Rule-6 :-
Shifting the take off before the block.

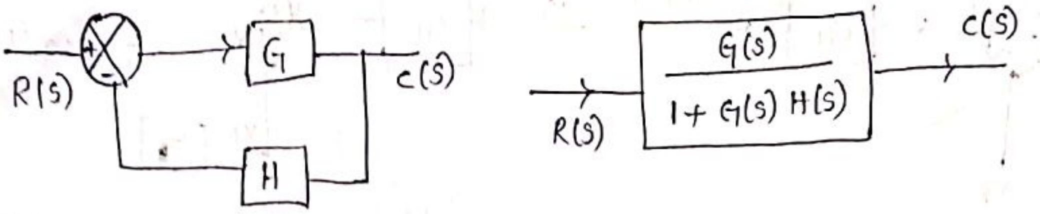


~~Rule~~ Note :-

* Summing point after the block G
* Summing point before the block $1/G$
* Take off point before the block : G
* Take off point after the block $1/G$

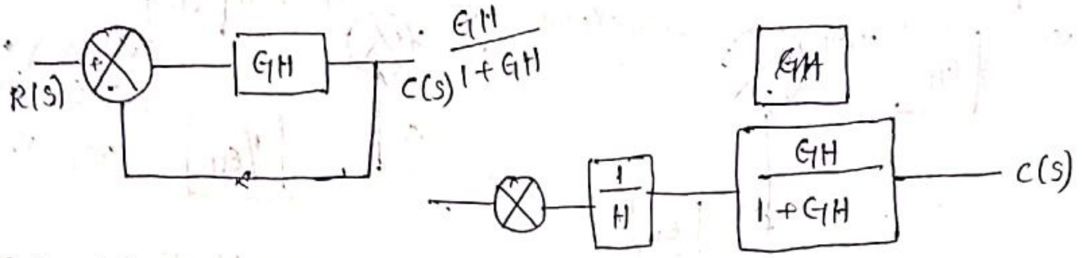
Rule - 7 :-

Transfer function for closed loop control system:



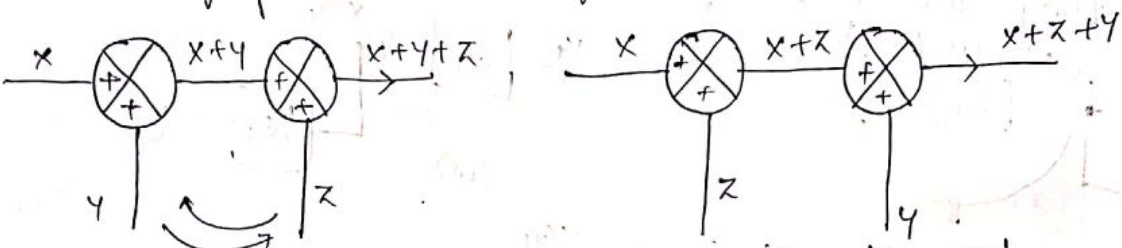
Rule - 8 :-

Block transformation | Interchanging the block.

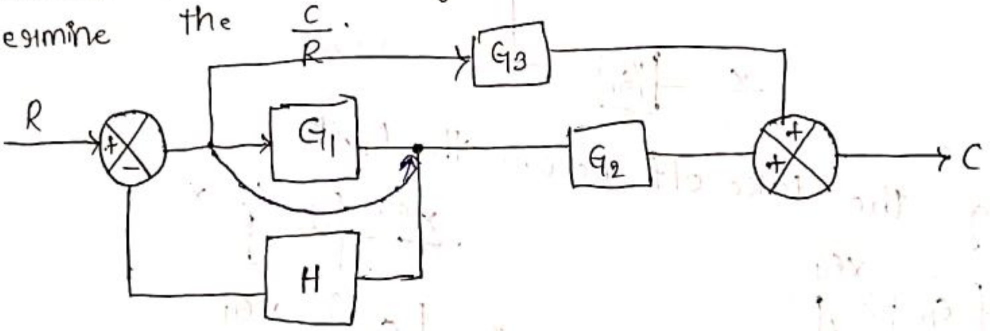


Rule - 9 :-

Interchanging the summing points :-

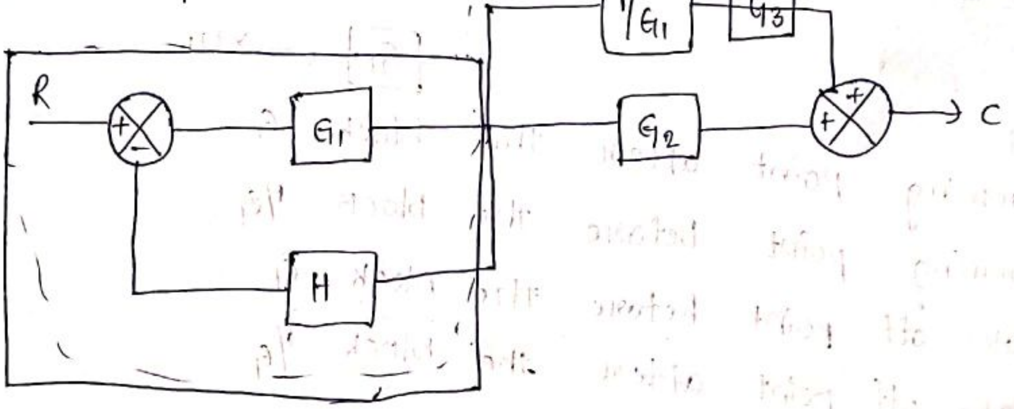


1. Reduce the block diagram shown in fig. and determine the $\frac{C}{R}$.

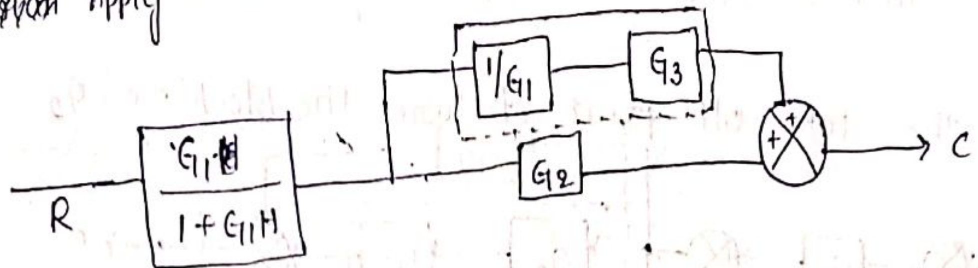


Step 1:

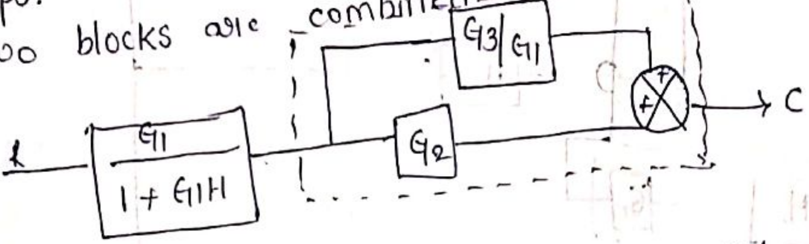
Take off point after the block i.e., G_1



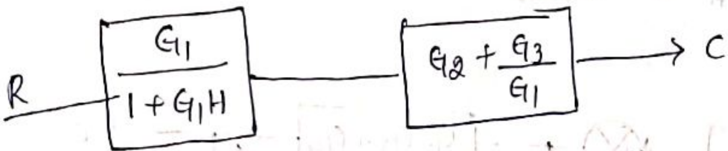
step 2:
Apply -Ve feedback for closed loop CS transfer function



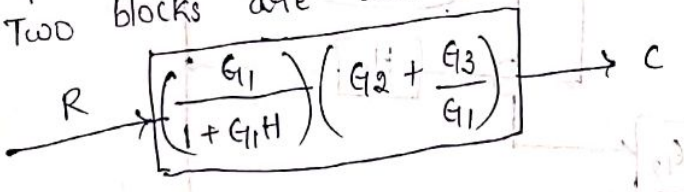
step 3:
Two blocks are combined in series.



step 4:
Two blocks are combined in parallel.



step 5:
Two blocks are connected in series.

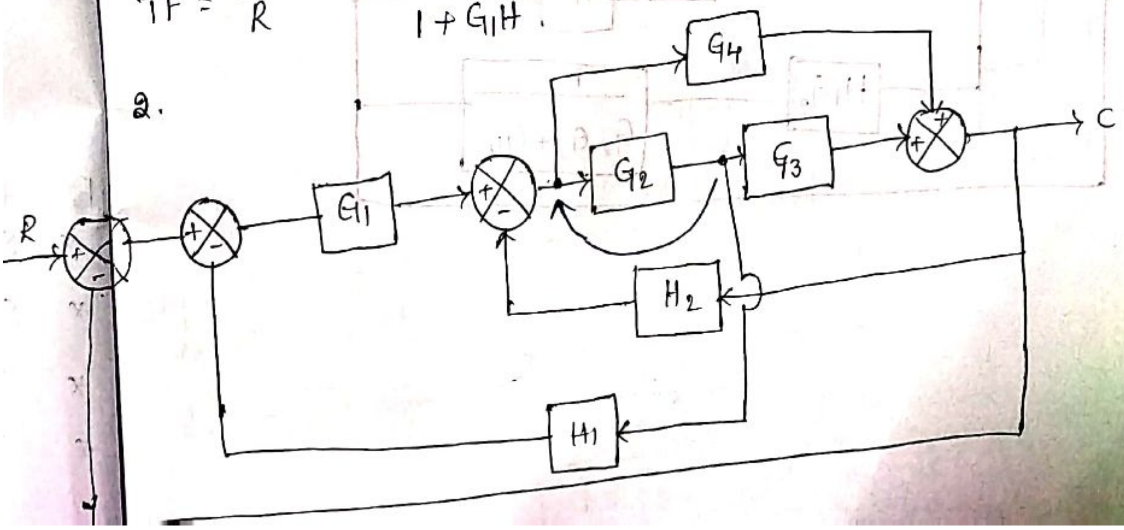


$$\left(\frac{G_1}{1+G_1H}\right)\left(G_2+\frac{G_3}{G_1}\right)=\frac{C}{R}$$

$$\left(\frac{\cancel{G_1}}{1+G_1H}\right)\left(\frac{G_1G_2+G_3}{\cancel{G_1}}\right)=\frac{C}{R}$$

$$TF=\frac{C}{R}=\frac{G_1G_2+G_3}{1+G_1H}$$

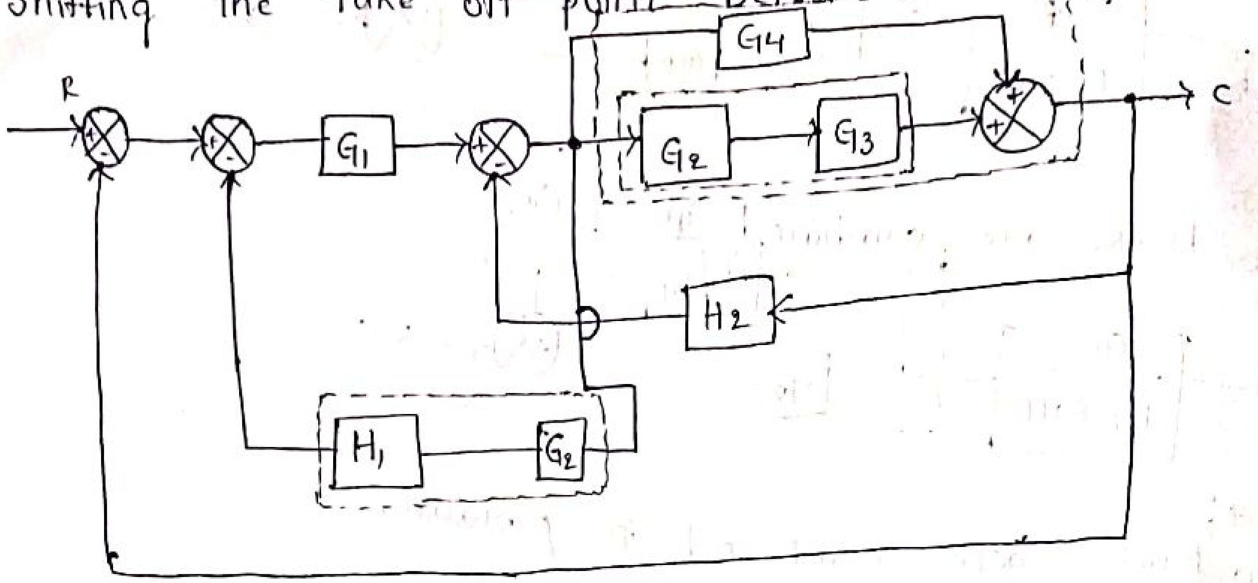
2.



Using Block diagram reduction technique, find closed loop transfer function for the given system.

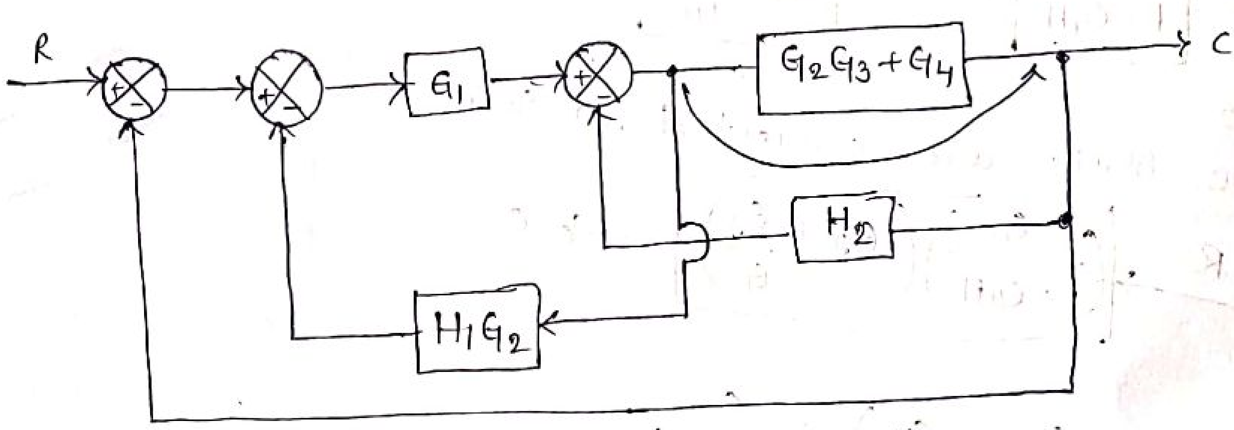
Step 1:

Shifting the take off point before the block i.e., G_2



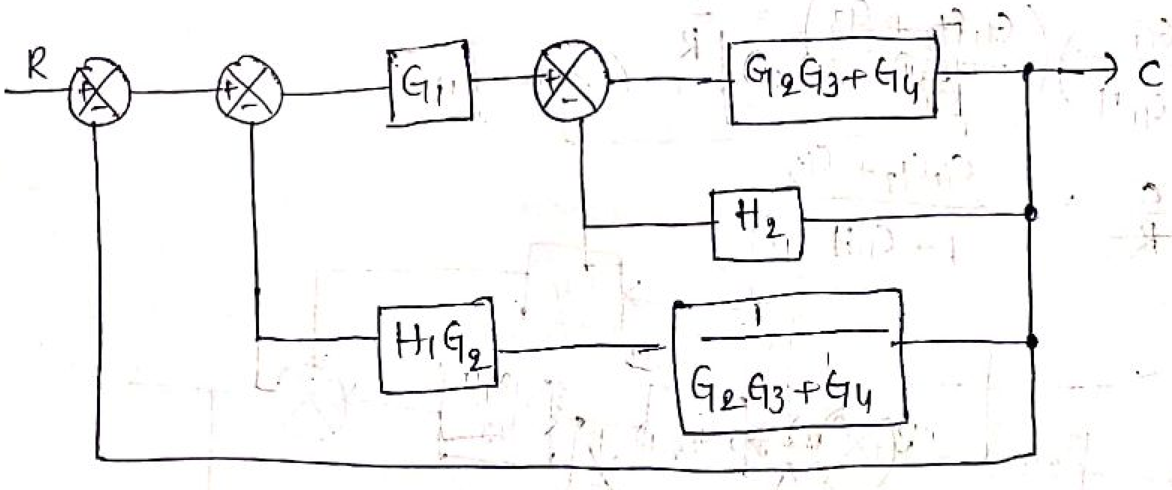
Step 2:

Combining the block in Series.

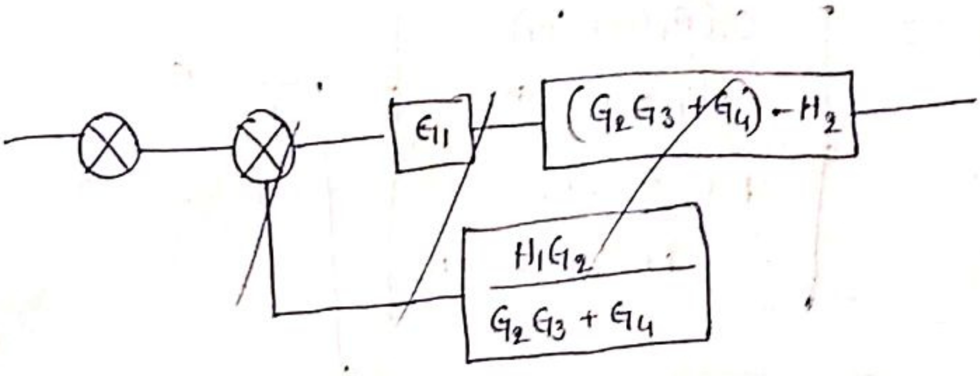


Step 3:

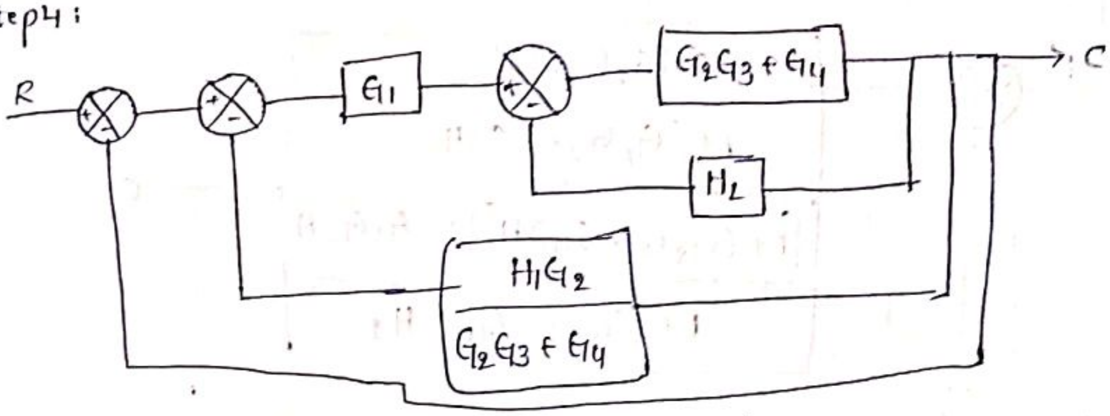
Shift the take off point after the block.



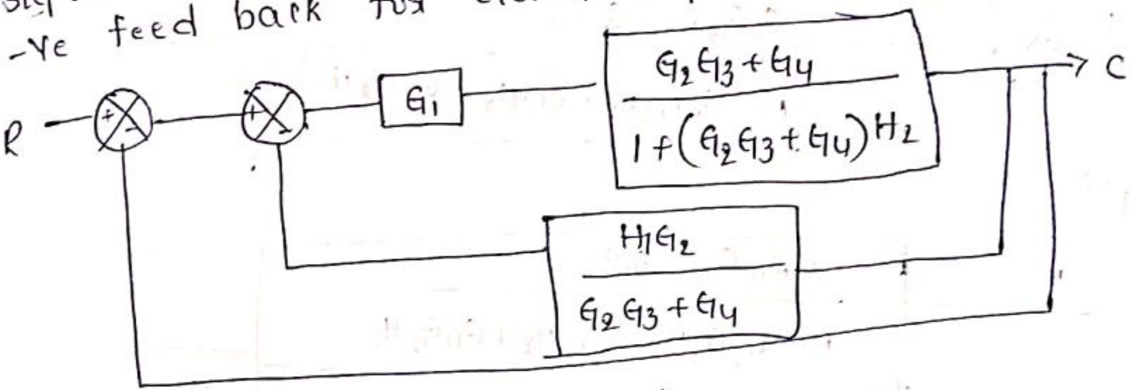
step 4:



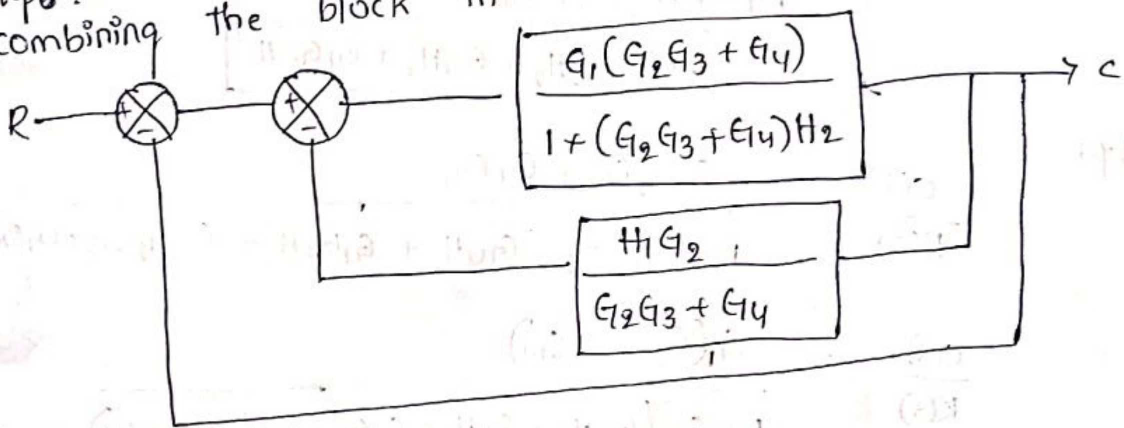
step 4:



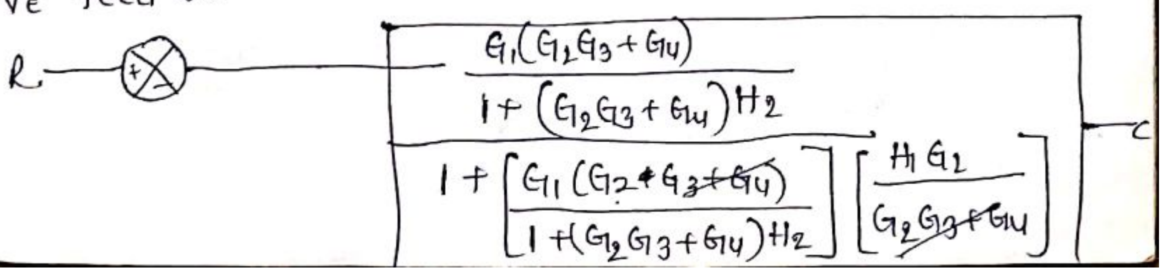
step 5:
-ve feed back for closed loop CS.



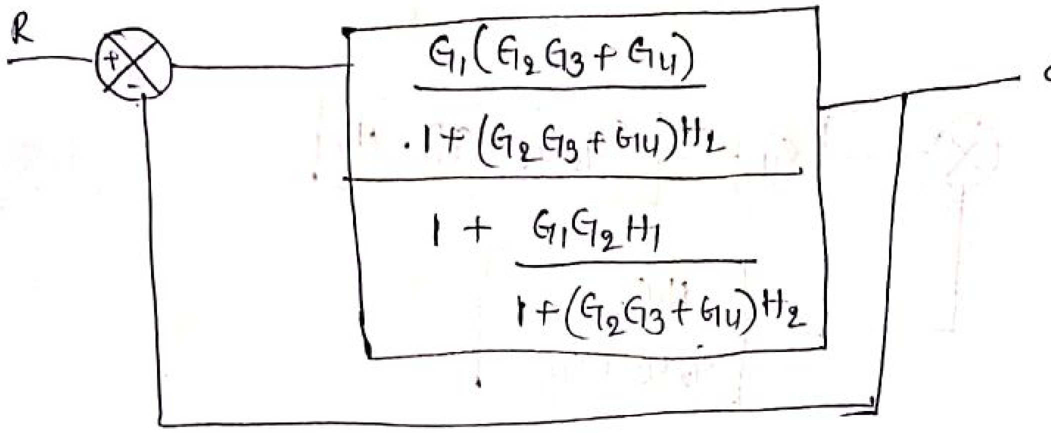
step 6:
combining the block in series.



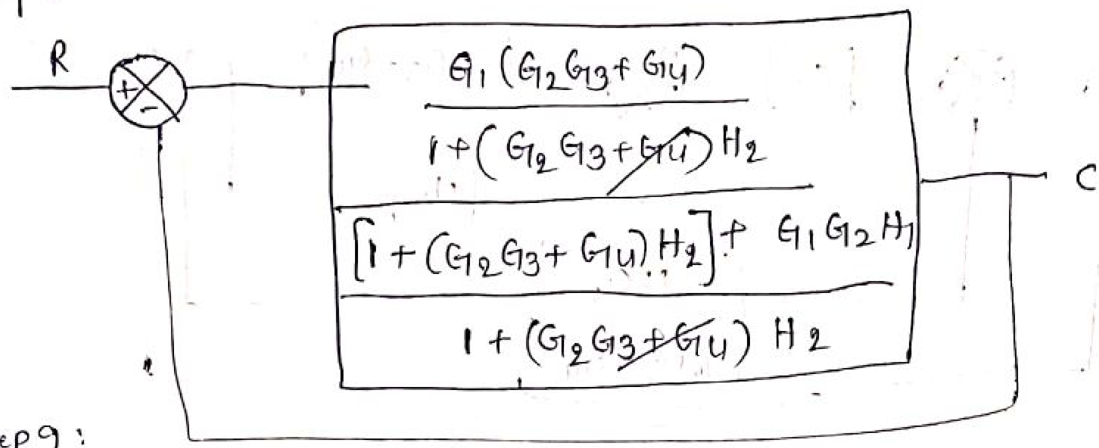
step 7:
-ve feed back CS



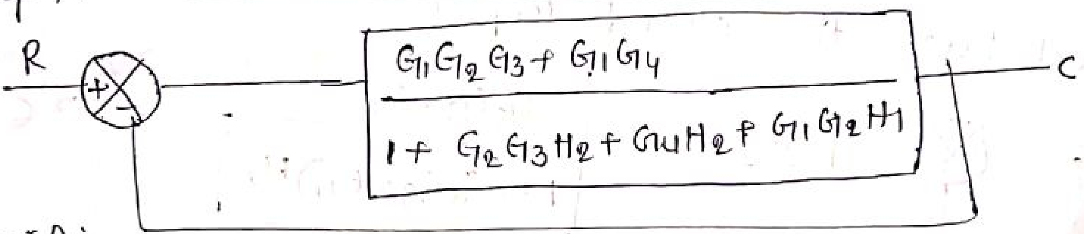
step7:



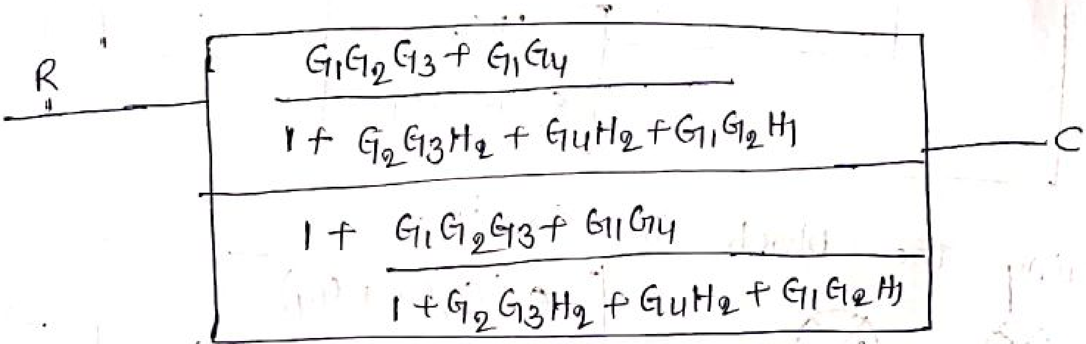
step8:



step9:



step10:



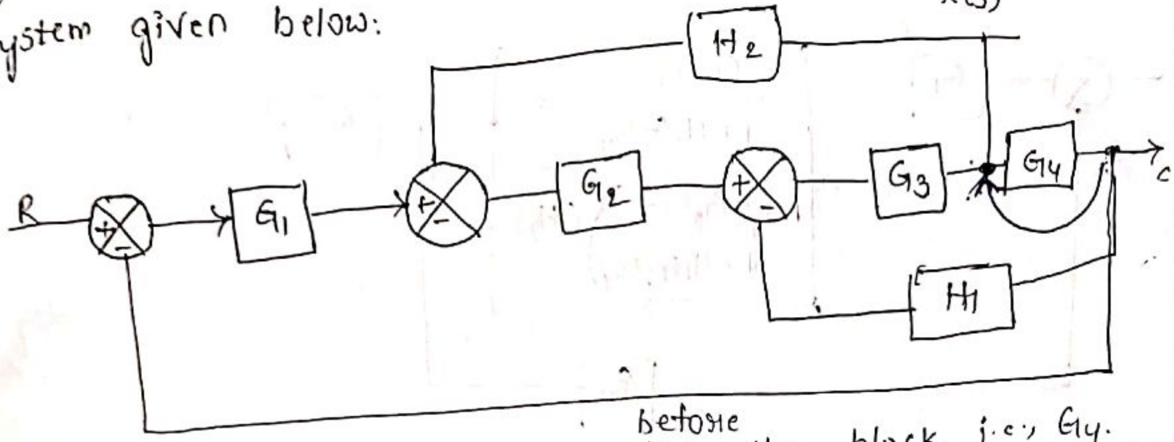
step1:

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1 + G_2G_3H_2 + G_4H_2 + G_1G_2H_1 + G_1G_2G_3 + G_1G_4}$$

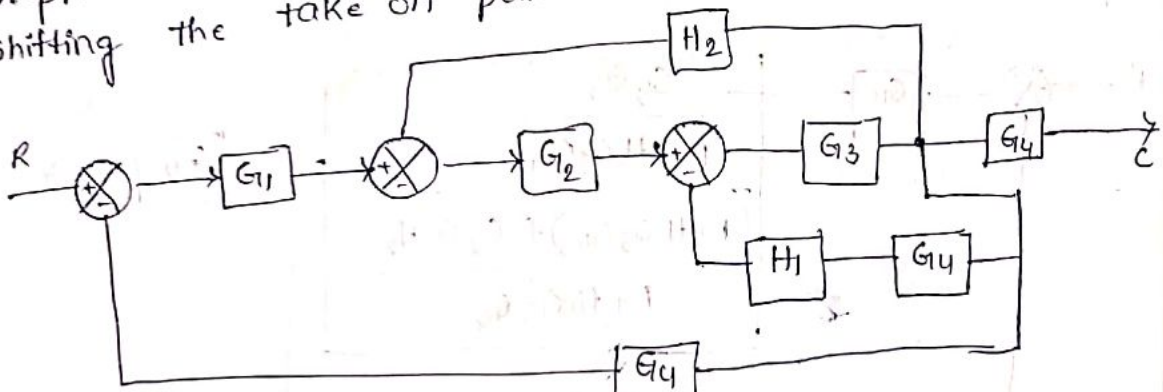
$$\frac{C(s)}{R(s)} = \frac{G_1(G_2G_3 + G_4)}{1 + G_2(G_3H_2 + G_1H_1 + G_1G_3) + G_4(H_2 + G_1)}$$

Scanned by CamScanner

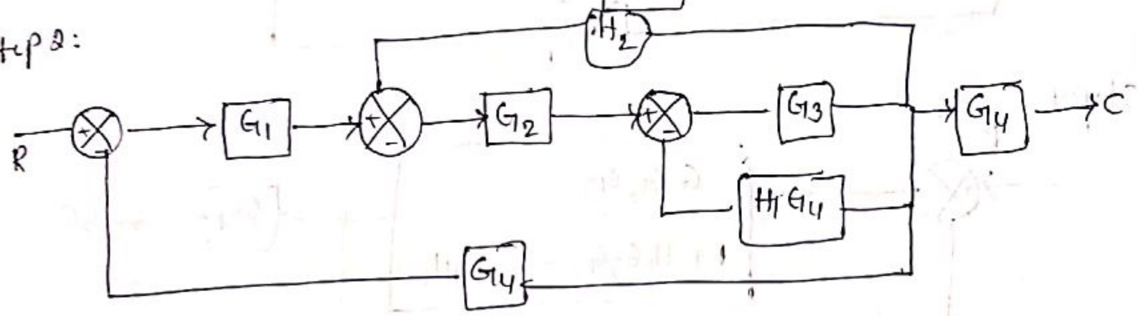
3) Determine the overall transfer function $\frac{C(S)}{R(S)}$ for the system given below:



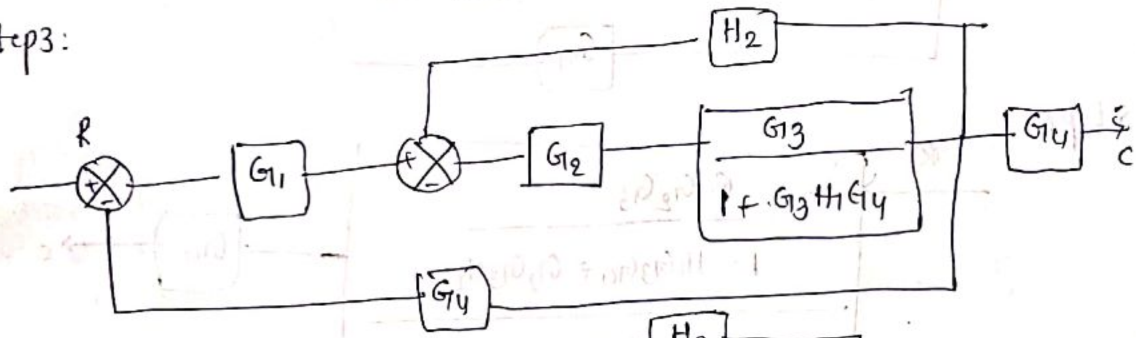
step1:
shifting the take off point ~~after~~ before the block. i.e., G4.



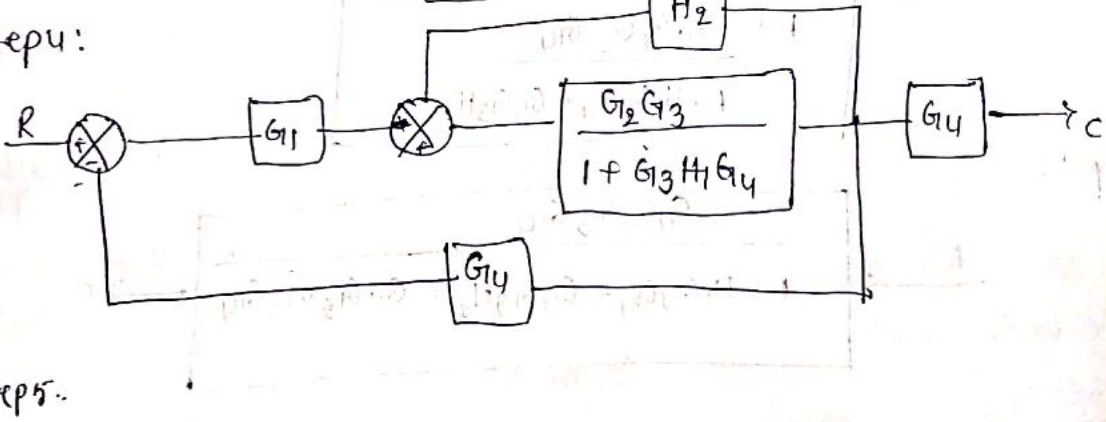
step2:



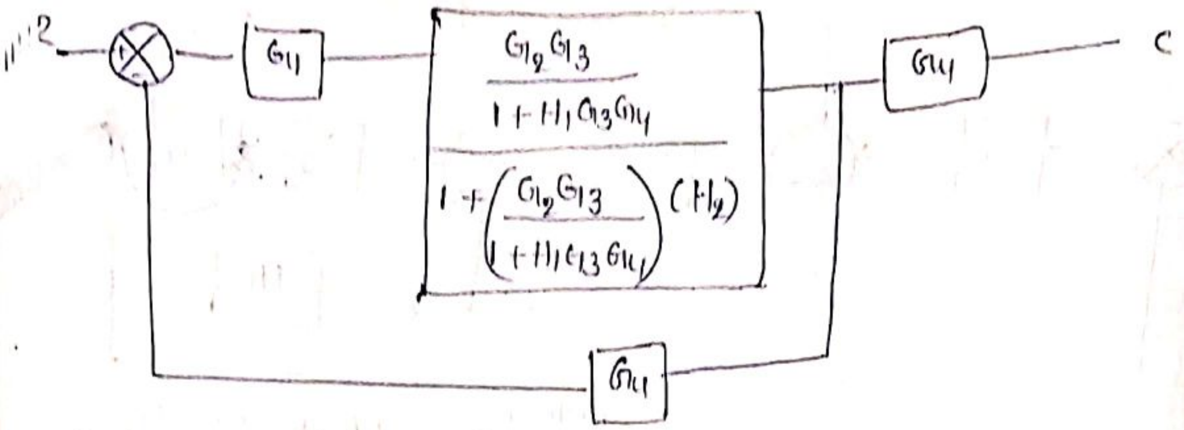
step3:



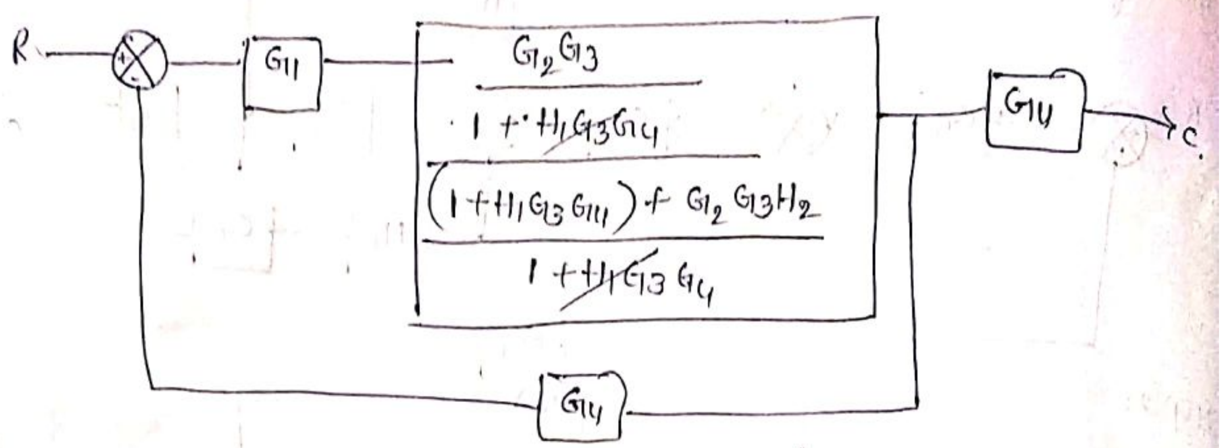
step4:



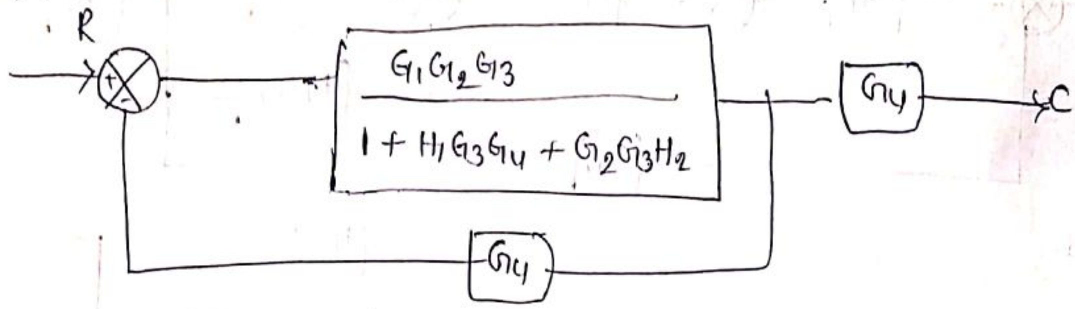
step5:



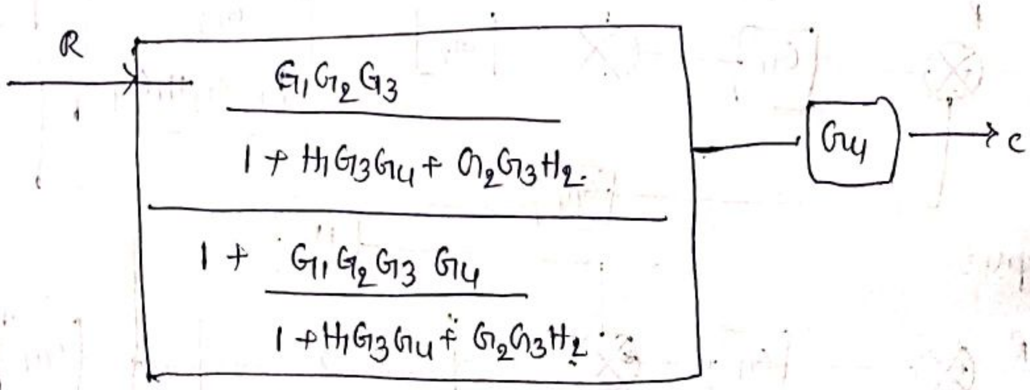
Step 6:



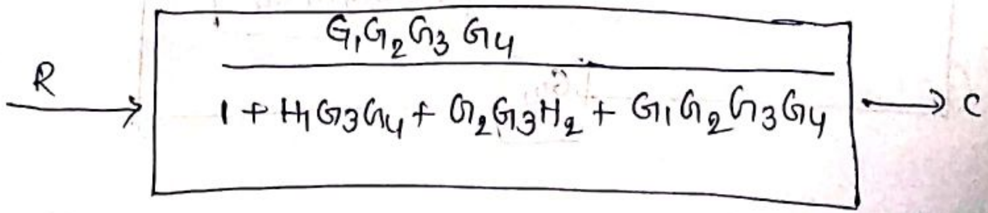
Step 7:



Step 8:



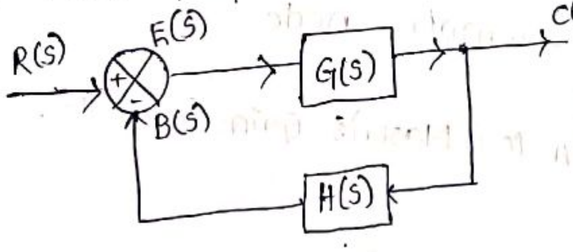
Step 9:



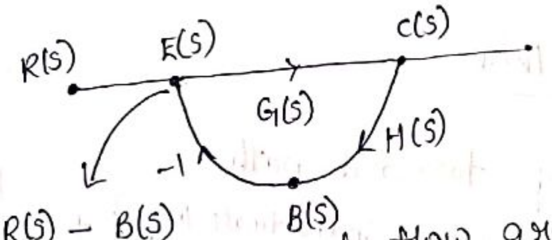
step10:

$$\frac{C(S)}{R(S)} = \frac{G_1G_2G_3G_4}{1+G_3\left[H_1G_4+G_2H_2+G_1G_2G_4\right]}$$

Signal Flow Graph Method :- the transfer function of the
It is used to determine the complexity existing in
system. Inorder to avoid the technique with the help of
the block diagram reduction representation
signal flow graph technique. It is graphical represents
of control System in which it is node branches.
system variable and are connected direct
Signal Flow Graph for closed loop Control System :-



Closed loop CS in Canonical form



technique :-
Terminology for Signal flow graph and it is
Node :- It represents system variable incoming
having which is equal to the sum of all
signals at it.
Source Node/ Input Node :-
It is the node which is having out going branches.
Output Node /Sink Node :-
It is the node which is having all incoming
branches.
Mixed Node :-
It is a node, which is having both incoming
and outgoing branches.
Path :-
It is the transversal of connecting branches
in the direction of branch arrow. such that
node is traverse more than once.

Forward Path :-
It is the path from i/p node to o/p node.

Loop :-
It is the path which originates and terminates at the same node.

Note :-
1. Self loop on i/p nodes are not valid loops and should not be considered when writing the transfer function.
2. Loops (or) self loops on o/p nodes are valid loops while writing the transfer function.

Non touching loops :-
Two (or) more loops are said to be non touching loops, which is having no common node.

*Mason's Gain formula :-
Overall transfer function (or) Mason's Gain formula =

$$= \frac{\Sigma P_K \Delta_K}{\Delta}$$

$$= \frac{P_1\Delta_1 + P_2\Delta_2 + \cdots + P_K\Delta_K}{\Delta}$$

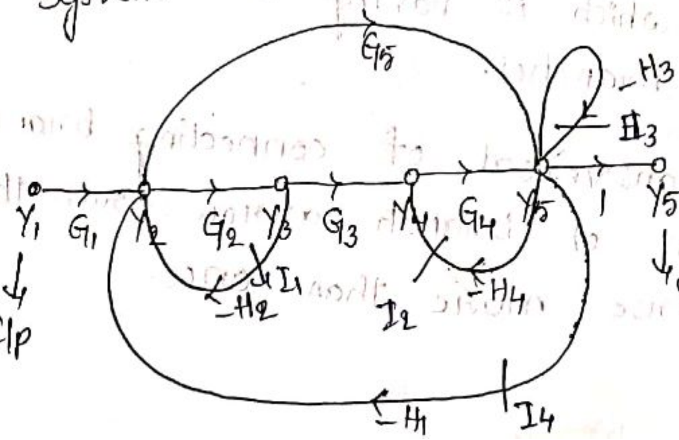
where P_K is path gain of forward path.

Δ is $1 -$ {Sum of gain of individuals loops} + {Sum of gain products of two nontouching loops} − {Sum of gain products of three nontouching loops}

Δ_K is the value of 'Δ' obtained by removing all the loops touching the K^{th} forward path.

$K =$ no. of forward paths

1. Determine the transfer function by using signal flow graph method (or) Mason's Gain formula for the given system



No. of forward paths (K) = 2

Mason's Gain formula = $\frac{\Sigma P_K \Delta_K}{\Delta}$

$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$P_1 = G_1G_2G_3G_4(1) = G_1G_2G_3G_4$

$P_2 = G_1G_5(1) = G_1G_5$

No. of individuals loops : 4

$I_1 = -G_2H_2$

$I_2 = -G_4H_4$

$I_3 : -H_3$

$I_4 : -G_2G_3G_4H_1$

Non-touching loops (L) :

$L_1 = I_1I_2 = (-G_2H_2)(-G_4H_4) = G_2G_4H_2H_4$

$L_2 : I_1I_3 = (-G_2H_2)(-H_3) = G_2H_2H_3$

$\Delta = 1 - [-G_2H_2 - G_4H_4 - H_3 - G_2G_3G_4H_1] + [G_2G_4H_2H_4 + G_2H_2H_3 + 0]$

$\Delta = 1 + G_2H_2 + G_4H_4 + H_3 + G_2G_3G_4H_1 + G_2G_4H_2H_4 + G_2H_2H_3$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$\Delta_1 = \Delta_2 = 1$

Mason's Gain formula = $\frac{\Sigma P_K \Delta_K}{\Delta}$

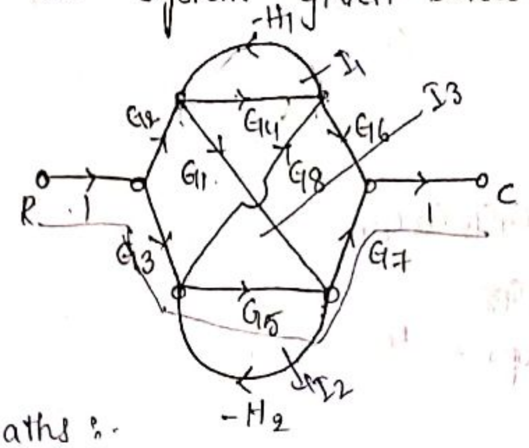
$$= \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

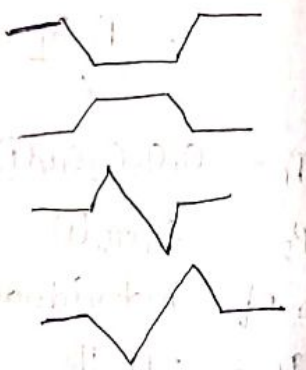
$$= \frac{G_1G_2G_3G_4(1) + G_1G_5(1)}{1 + G_2H_2 + G_4H_4 + H_3 + G_2G_3G_4H_1 + G_2G_4H_2H_4 + G_2H_2H_3}$$

$$= \frac{G_1(G_2G_3G_4 + G_5)}{1 + G_2H_2(1 + G_4H_4 + H_3) + G_4H_4 + H_3 + G_2G_3G_4H_1}$$

2) Det

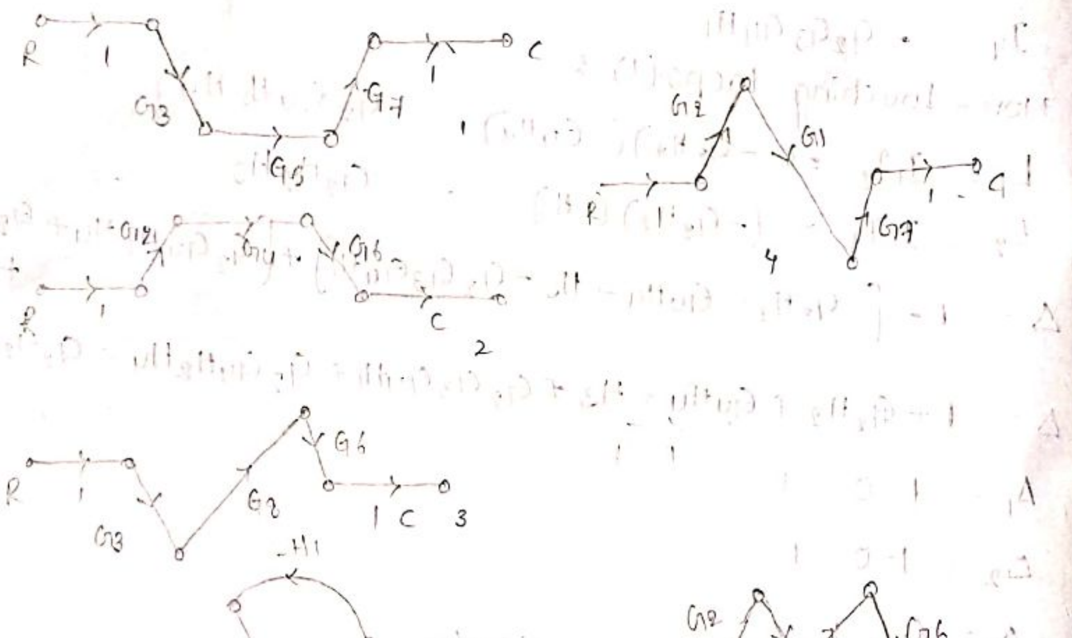
2) Determine the transfer function by using signal flow graph method for the system given below:

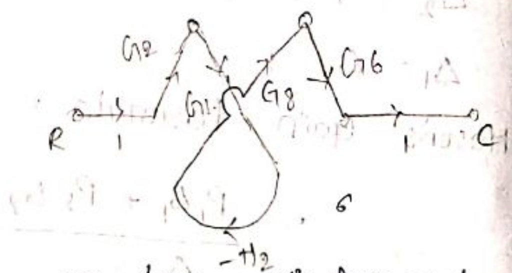




Forward paths :-

No. of forward paths (K) = 6





By observing the given system, we have six forward paths i.e., K = 6.

P_1 = (1) $G_3 G_5 G_7$ (1) : $G_3 G_5 G_7$

P_2 : (1) $G_2 G_4 G_6$ (1) = $G_2 G_4 G_6$

P_3 : (1) $G_3 G_8 G_6$ (1) = $G_3 G_8 G_6$

P_4 = (1) $G_2 G_1 G_7$ (1) : $G_2 G_1 G_7$

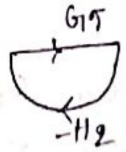
P_5 = (1) $G_3 G_8 (-H_1) G_1 G_7$ (1) = $- G_3 G_8 G_1 G_7 H_1$

P_6 = (1) $G_2 G_1 (-H_2) G_8 G_6$ (1) = $- G_2 G_1 H_2 G_8 G_6$

Individual loops (I)

$I_1 = G_4(-H_1) = -G_4H_1$

$I_2 = -G_5H_2$



$I_3 = G_1(-H_2)G_8(-H_1) = G_1G_8H_1H_2$

Non touching loops (L) :

$L_1 = I_1I_2 = (-G_4H_1)(-G_5H_2) = G_4G_5H_1H_2$

$\Delta_1 = 1-\{I_1\}+0 = 1+G_4H_1$

$\Delta_2 = 1-I_2 = 1+G_5H_2$

$\Delta_3 = 1-0 = 1$

$\Delta_4 = 1-0 = 1$

$\Delta_5 = 1-0 = 1$

$\Delta_6 = 1-0 = 1$

$\therefore \Delta_3 = \Delta_4 = \Delta_5 = \Delta_6 = 1$

$\Delta = 1-(I_1+I_2+I_3)+(L_1)$

$\Delta = 1-(-G_4H_1-G_5H_2+G_1G_8H_1H_2)+G_4G_5H_1H_2$

$\Delta = 1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2$

WKT the Mason's Gain formula $= \dfrac{\sum_{K=1}^{6} P_K\Delta_K}{\Delta}$

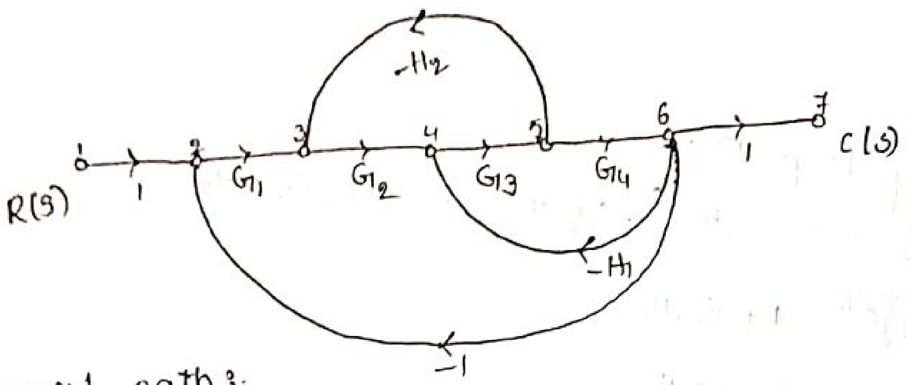
$$= \frac{P_1\Delta_1+P_2\Delta_2+P_3\Delta_3+P_4\Delta_4+P_5\Delta_5+P_6\Delta_6}{\Delta}$$

$$= \frac{(G_3G_5G_7)(1+G_4H_1)+(G_2G_4G_6)(1+G_5H_2)+G_3G_8G_6(1)+G_2G_1G_7(1)+(-G_3G_8G_1G_7H_1)-G_2G_1H_2G_8G_6}{1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2}$$

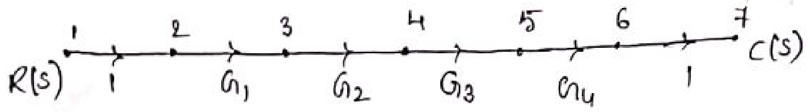
$$= \frac{G_3G_5G_7+G_3G_4G_5G_7H_1+G_2G_4G_6+G_2G_4G_5G_6H_2+G_3G_6G_8+G_1G_2G_7-G_1G_3G_7G_8H_1-G_1G_2G_6G_8H_2}{1+G_4H_1+G_5H_2-G_1G_8H_1H_2+G_4G_5H_1H_2}$$

$$\frac{G_3G_{15}G_{17}(1+G_{14}H_1)+G_{12}G_{14}G_{16}(1+G_{15}H_2)+G_{13}G_{18}(G_{16}-G_1G_{17}H_1)+G_{11}G_{12}(G_{17}-G_{16}G_{18}H_2)}{1+G_{14}H_1+G_{15}H_2-H_1H_2(G_{11}G_{18}-G_{14}G_{15})}$$

3) Find the overall gain $\frac{C(S)}{R(S)}$ for the given system



Forward path :-

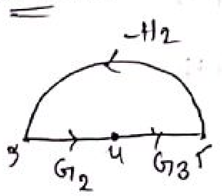


No. of forward paths = 1

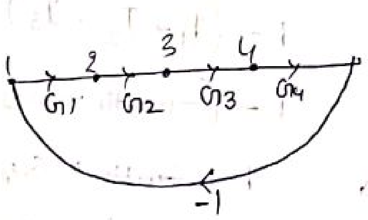
$K = 1$

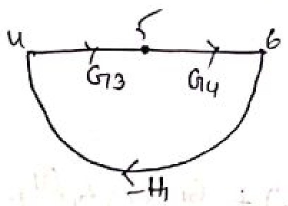
$P = (1)\, G_1 G_2 G_3 G_4 (1) = G_1 G_2 G_3 G_4$

Individual Loops :-

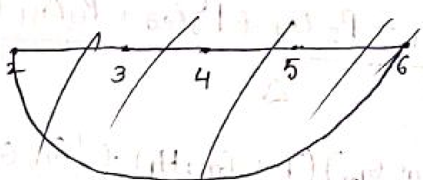


$I_1 = -G_2 G_3 H_2$





$I_2 = -G_3 G_4 H_1$ $\quad I_3 = -G_1 G_2 G_3 G_4$



There are no non-touching loops.

$\Delta = 1 - (I_1 + I_2 + I_3)$

$\Delta = 1 - [-G_2 G_3 H_2 - G_3 G_4 H_1 - G_1 G_2 G_3 G_4]$

$\Delta = 1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4$

$\Delta_1 = 1 - 0 = 1$

By Mason's Gain formula, $\dfrac{\sum_{k=1} P_K \Delta_K}{\Delta}$

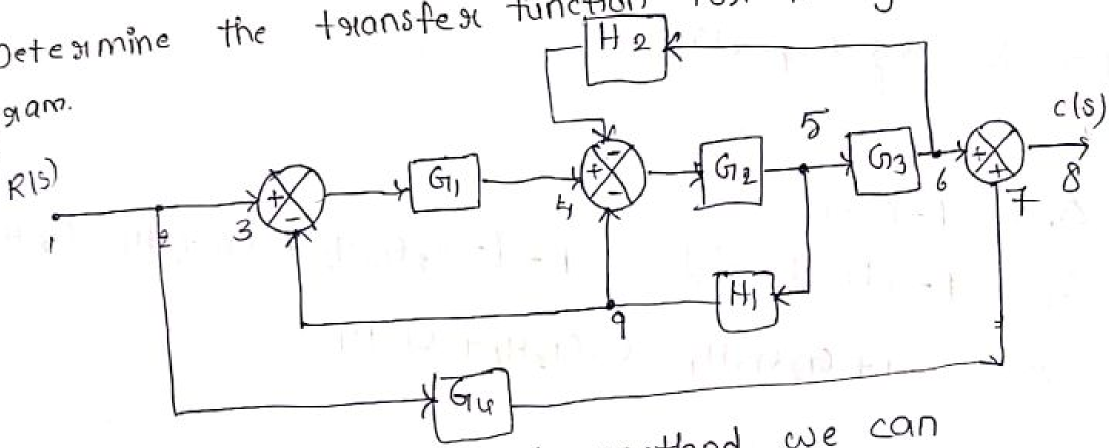
$$= \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{G_1 G_2 G_3 G_4 (1)}{1 + G_2 G_3 H_2 + G_3 G_4 H_1 + G_1 G_2 G_3 G_4}$$

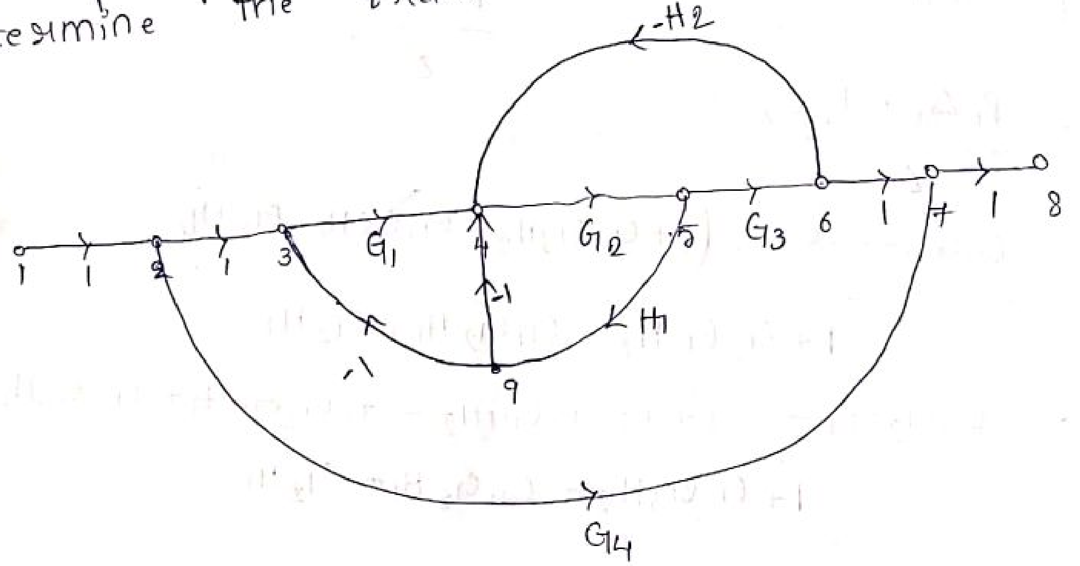
$$= \frac{G_1 G_2 G_3 G_4}{1 + G_2 (G_3 H_2 + G_1 G_3 G_4) + G_3 G_4 H_1}$$

Feed back characteristics and its advantages, linearizing effect of feed back: Nagrath & Gopal

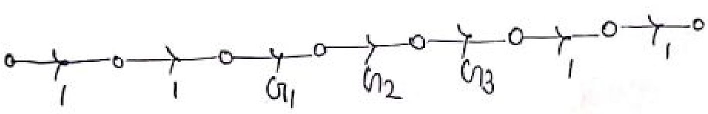
* Determine the transfer function for the given block diagram.



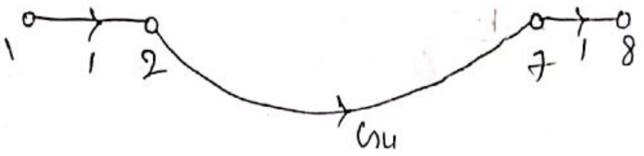
By using signal flow graph method, we can determine the transfer function.



Forward paths :-



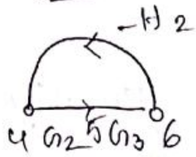
$P_1 = G_1 G_2 G_3$

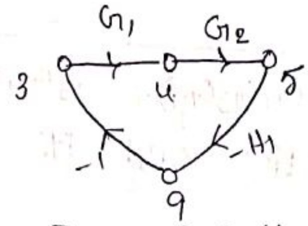


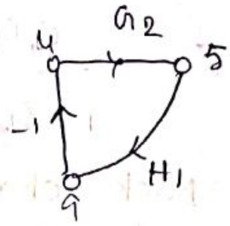
$P_2 = G_4$

No. of forward paths (K) = 2

Individual loops (I) :-







$I_1 = -G_2 G_3 H_2$ $\quad I_2 = G_1 G_2 H_1$ $\quad I_3 = -G_2 H_1$

Non touching loops (L) :-

$L = 0.$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - (I_1 + I_2 + I_3) = 1 - (-G_2 G_3 H_2 + G_1 G_2 H_1 - G_2 H_1)$

$\Delta_2 = 1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_2 H_1$

$\Delta = \cancel{1}\, \Delta_2$

By Masons gain formula, $\dfrac{\sum_{K=1}^{2} P_K \Delta_K}{\Delta}$

$$= \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 + G_4 (1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_2 H_1)}{1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_2 H_1}$$

$$= \frac{G_1 G_2 G_3 + G_4 + G_2 G_3 G_4 H_2 - G_1 G_2 G_4 H_1 + G_2 G_4 H_1}{1 + G_2 G_3 H_2 - G_1 G_2 H_1 + G_2 H_1}$$

Linearizing effect of feed back:-

Yet another property of feed back is its linearizing effect which is illustrated by means of the simple-single-loop static system of fig (a). In a static system, various gains are independent of time. we shall assume that the forward block function is nonlinear expresses as

$$c = f(e) = e^2, \text{ square low function}$$

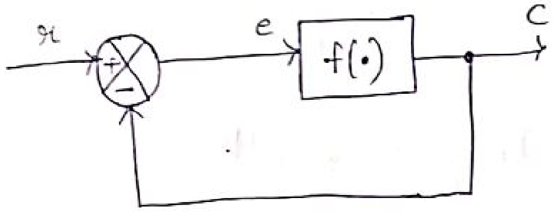
when the feed back is open,

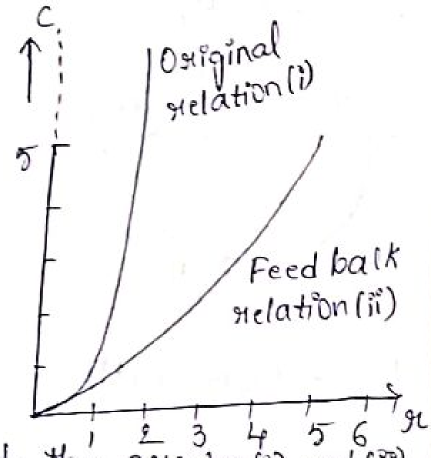
$$e = r \Rightarrow c = r^2$$

which is plotted in fig (b). On the other hand when the loop is closed, we have

$$e = r - c$$

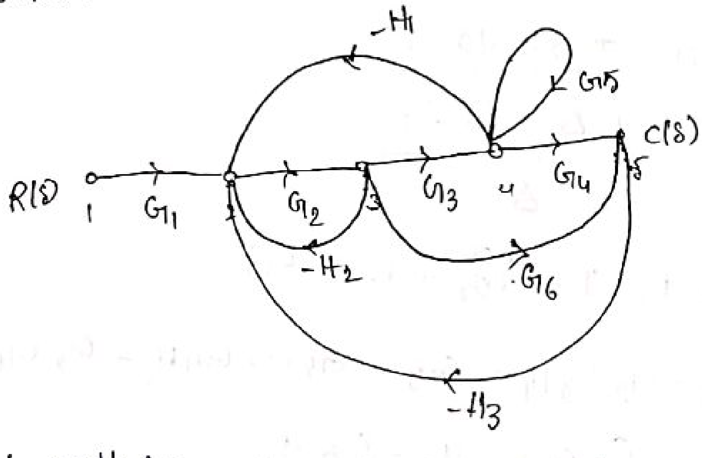
and so $c = f(e) = (r-c)^2$.



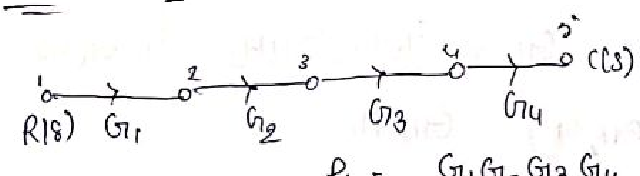


It is easily seen by comparision of the graphs (i) and (ii) that the input-output relation [c(r)] is approximately linear over a much wider range for the closed-loop system compared to its open-loop behaviour

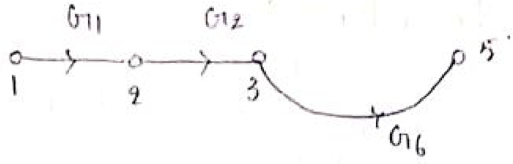
Find the overall gain $\frac{C(S)}{R(S)}$ for the signal flow graph given below:



Forward path:-



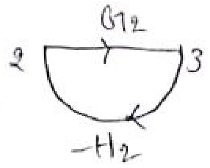
$$P_1 = G_1 G_2 G_3 G_4$$



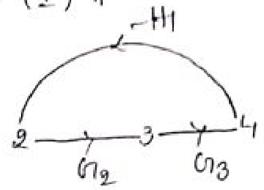
$P_2 = G_1 G_2 G_6$

No. of forward paths (K) = 2

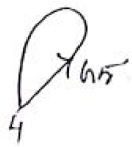
Individual loops :- (I) :-



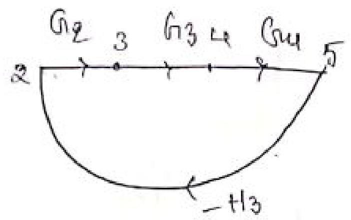
$I_1 = -G_2 H_2$



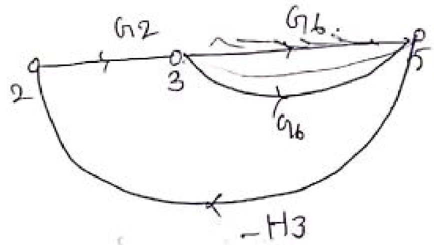
$I_2 = -G_2 G_3 H_1$



$I_3 = G_5$



$I_4 = -G_2 G_3 G_4 H_3$



$I_5 = -G_2 G_6 H_3$

Non touching loops (L) :-

$L_1 = I_1 I_3 = -G_2 G_5 H_2$

$L_2 = I_3 I_5 = -G_5 H_3$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - G_5$

By Mason's Gain formula,

$$\frac{\Sigma P_K \Delta_K}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$\Delta = 1 - (I_1 + I_2 + I_3 + I_4 + I_5) + (L_1 + L_2)$$

$$= 1 - (-G_2 H_2 - G_2 G_3 H_1 + G_5 - G_2 G_3 G_4 H_3 - G_2 G_6 H_3) + (-G_2 G_5 H_2 - G_5 H_3)$$

$$= 1 + G_2 H_2 + G_2 G_3 H_1 - G_5 + G_2 G_3 G_4 H_3 + G_2 G_6 H_3 - G_2 G_5 H_2 - G_5 H_3$$

$$\frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$\Rightarrow \frac{G_1G_2G_3G_4 + G_1G_2G_6(1-G_5)}{1+G_2H_2+G_2G_3H_1 - G_5 + G_2G_3G_4H_3 + G_2G_6H_3 - G_2G_5H_2 - G_5H_3}$$

$$\Rightarrow \frac{G_1G_2(G_3G_4+G_6-G_5G_6)}{1+G_2H_2(1-G_5)+G_2G_3(H_1+G_4H_3) - G_5 + G_2G_6H_3 - G_5H_3}$$