UNIT – II

Single phase and three phase controlled rectifiers

Phase control technique – Single phase Line commutated converters

Unlike diode rectifiers, PCRs or phase controlled rectifiers has an advantage of regulating the output voltage. The diode rectifiers are termed as uncontrolled rectifiers. When these diodes are switched with Thyristors, then it becomes phase control rectifier. The o/p voltage can be regulated by changing the firing angle of the Thyristors. The main application of these rectifiers is involved in speed control of DC motor.

What is a Phase Controlled Rectifier?

The term PCR or Phase controlled rectifier is a one type of rectifier circuit in which the diodes are switched by Thyristors or SCRs (Silicon Controlled Rectifiers). Whereas the diodes offer no control over the o/p voltage, the Thyristors can be used to differ the output voltage by adjusting the firing angle or delay. A phase control Thyristor is activated by applying a short pulse to its gate terminal and it is deactivated due to line communication or natural. In case of heavy inductive load, it is deactivated by firing another Thyristor of the rectifier during the negative half cycle of i/p voltage.

Types of Phase Controlled Rectifier

The phase controlled rectifier is classified into two types based on the type of i/p power supply. And each kind includes a semi, full and dual converter.

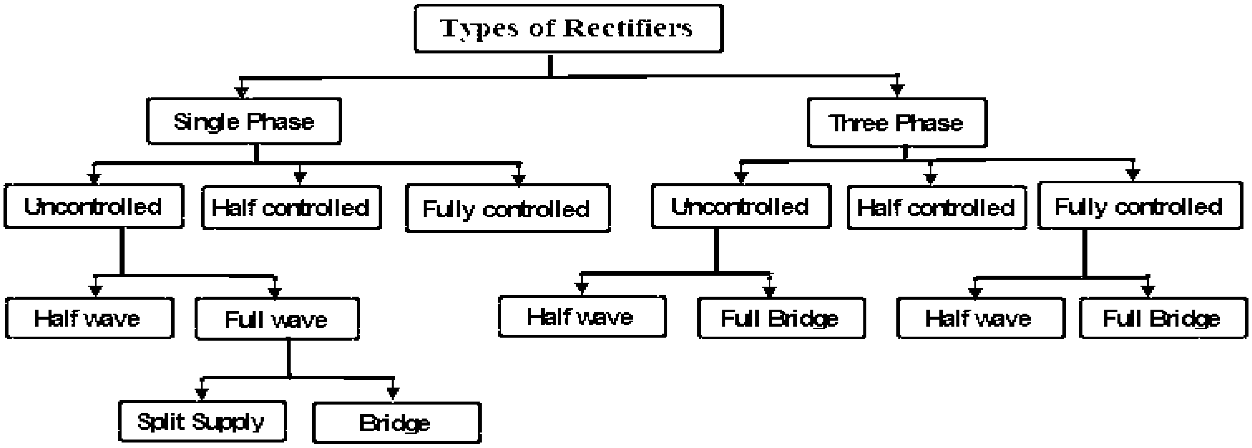

Figure: 2.1. Classification of rectifiers

Single-phase Controlled Rectifier

This type of rectifier which works from single phase AC i/p power supply

Single Phase Controlled Rectifiers are classified into different types

Half wave Controlled Rectifier: This type of rectifier uses a single Thyristor device to provide o/p control only in one half cycle of input AC supply, and it offers low DC output.

Full wave Controlled Rectifier: This type of rectifier provides higher DC output

- Full wave controlled rectifier with a center tapped transformer requires two Thyristors.
- Full wave bridge controlled rectifiers do not need a center tapped transformer

Three-phase Controlled Rectifier

This type of rectifier which works from three phase AC i/p power supply

- A semi converter is a one quadrant converter that has one polarity of o/p voltage and current.
- A full converter is a a two quadrants converter that has polarity of o/p voltage can be either +ve or –ve but, the current can have only one polarity that is either +ve or -ve.
- Dual converter works in four quadrants – both o/p voltage and o/p current can have both the polarities.

Operation of Phase Controlled Rectifier

The basic working principle of a PCR circuit is explained using a single phase half wave PCR circuit with a RL load resistive shown in the following circuit.

A single phase half wave Thyristor converter circuit is used to convert AC to DC power conversion. The i/p AC supply is attained from a transformer to offer the required AC supply voltage to the Thyristor converter based on the o/p DC voltage required. In the above circuit, the primary and secondary AC supply voltages are denoted with VP and VS.

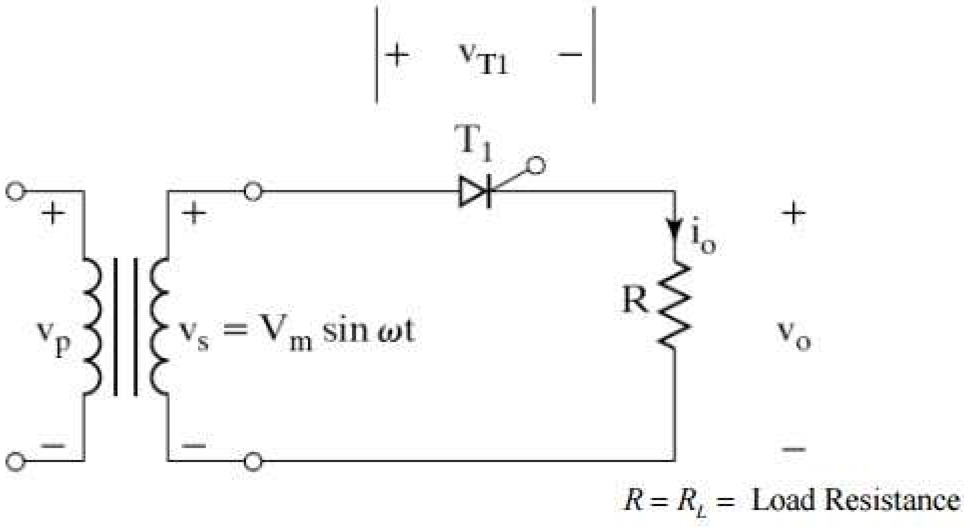

Figure: 2.2. Single phase half wave rectifier circuit

During the +ve half cycle of i/p supply when the upper end of the transformer secondary winding is at a + ve potential with respect to the lower end, the Thyristor is in a forward biased state.

The thyristor is activated at a delay angle of ωt =α, by applying an appropriate gate trigger pulse to the gate terminal of thyristor. When the thyristor is activated at a delay angle of ωt =α, the thyristor behaviors and assuming a perfect thyristor. The thyristor acts as a closed switch and the i/p supply voltage acts across the load when it conducts from ωt =α to π radians For a purely resistive load, the load current io that flows when the thyristor T1 is on, is given by the expression.

$$I_o = v_o / R_L, \text{ for } \alpha \leq \omega t \leq \pi$$

Applications of Phase Controlled Rectifier

Phase controlled rectifier applications include paper mills, textile mills using DC motor drives and DC motor control in steel mills.

- AC fed traction system using a DC traction motor.
- Electro-metallurgical and Electrochemical processes.
- Reactor controls.
- Magnet power supplies.
- Portable hand instrument drives.

- Flexible speed industrial drives.
- Battery charges.
- High voltage DC transmission.
- UPS (Uninterruptible power supply systems).

Operation of half converter with R and RL loads

Single Phase Half Wave Controlled Rectifier with 'R' load:

As shown in figure below primary of transformer is connected to ac mains supply with which SCR becomes forward bias in positive half cycle. T1 is triggered at an angle α, T1 conducts and voltage is applied across R.

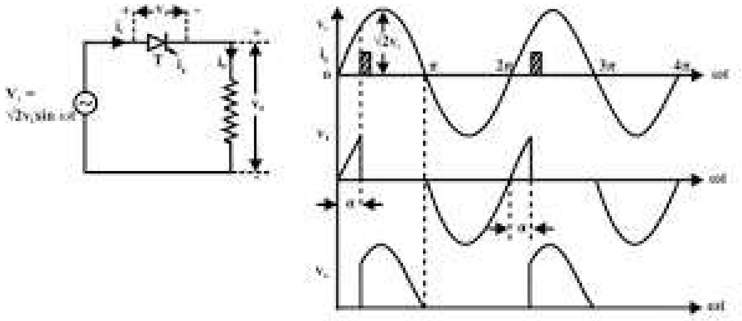

Figure: 2.3 Single phase half wave rectifier with R load with waveforms

The load current i_0 flows through 'R'
the waveforms for voltage & current are as shown above.
As load is resistive,
Output current is given as,

$$I_O = \frac{V_O}{R}$$

Hence shape of output current is same as output voltage
As T1 conducts only in positive half cycle as it is reversed bias in negative cycle, the ripple frequency of output voltage is-
fripple= 50 Hz (supply frequency)

Average output voltage is given as,

$$V_0(Avg) = \frac{1}{T}\int_o^T V_0(wt)\,dwt$$

i.e Area under one cycle.

Therefore T=2π&Vo(ωt) = Vm sinωt from α to π& for rest of the period Vo(ωt)=0

$$\therefore V_0(Avg) = \frac{1}{2\pi}\int_0^{2\pi} V_m \sin(wt)\,dwt$$

$$= \frac{V_m}{2\pi}\left[-\cos wt\right]_\alpha^\pi$$

$$= \frac{V_m}{2\pi}(1+\cos\alpha)$$

Power transferred to load,

$$P_0(Avg) = \frac{V_0^2(Avg)}{R}$$

Thus, power & voltage can be controlled by firing angle.

Single Phase Half Wave Controlled Rectifier with ‘RL’ load:

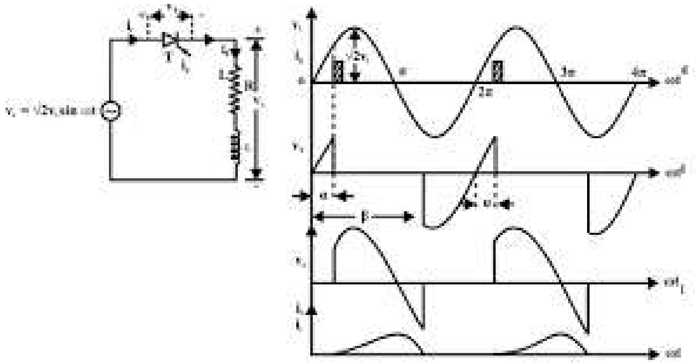

Figure: 2.4 Single phase half wave rectifier with RL load with waveforms

Figure above shows the single phase half wave rectifier with RL Load.

- Normally motors are inductive loads

L= armature of field coil inductance

R= Resistance of coil.

- In positive half cycle, SCR starts conduction at firing angle "α".
- Drop across SCR is small & neglected so output voltage is equal to supply voltage.
- Due to 'R_L' load, current through SCR increases slowly.
- At 'π', supply voltage is at zero where load current is at its max value.
- In positive half cycle, inductor stores energy & that generates the voltage.
- In negative half cycle, the voltage developed across inductor, forward biases SCR & maintains its conduction.
- Basically with the property of inductance it opposes change in current.
- Output current & supply current flows in same loop, so all the time $i_o=i_s$.
- After π the energy of inductor is given to mains & there is flow of 'io'. The energy reduces as if gets consumed by circuit so current also reduces.
- At 'β' energy stored in inductance is finished, hence 'i_o' becomes zero & 'T1' turns off.
- 'i_o' becomes zero from 'β' to '2π+α' hence it is discontinuous conduction.

The average output voltage $V_0 = \frac{1}{2\pi}\int_{\alpha}^{\beta} Vmsinwt\ d(wt) = \frac{\mathrm{Vm}}{2\pi}(\cos\alpha - \cos\beta)$

$$I_0 = \frac{\mathrm{Vm}}{2\pi \mathrm{R}}(\cos\alpha - \cos\beta)$$

RMS load voltage $V_{0r} = \left\{\frac{1}{2\pi}\int_{\alpha}^{\beta} Vm^2 sin^2 wt\ d(wt)\right\}^{1/2}$

$$= \frac{Vm}{2\sqrt{\pi}}\left[(\beta - \alpha) - \frac{1}{2}\{sin2\beta - sin2\alpha\}\right]^{1/2}$$

Single phase half controlled converter with RLE load

The diode D2 and D4 conducts for the positive and negative half cycle of the input voltage waveform respectively. On the other hand T1 starts conduction when it is fired in the positive half cycle of the input voltage waveform and continuous conduction till T3 is fired in the negative half cycle. Fig. shows the circuit diagram and the waveforms of a single phase half controlled converter supplying an R – L – E load.

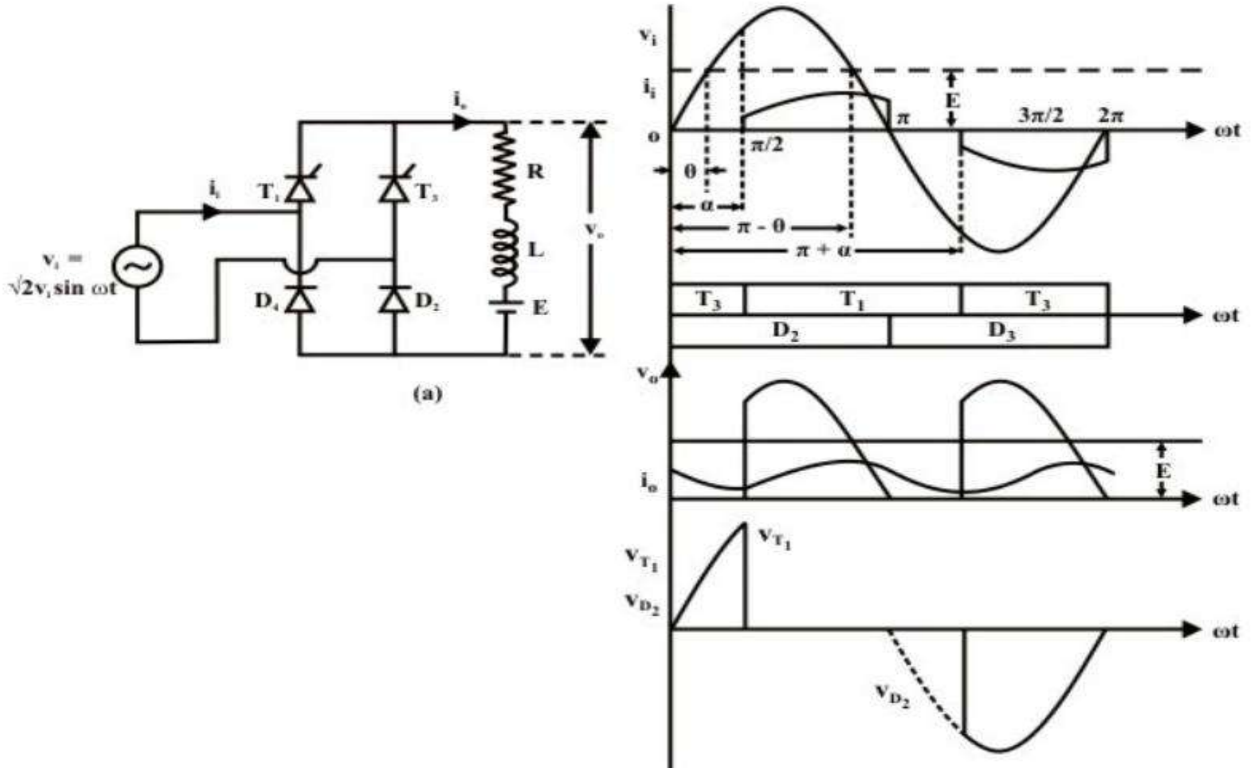

Figure: 2.5 single phase half controlled converter with RLE load

Referring to Fig T1 D2 starts conduction at ωt = α. Output voltage during this period becomes equal to vi. At ωt = π as vi tends to go negative D4 is forward biased and the load current commutates from D2 to D4 and freewheels through D4 and T1. The output voltage remains clamped to zero till T3 is fired at ωt = π + α. The T3 D4 conduction mode continues upto ωt = 2π. Where upon load current again free wheels through T3 and D2 while the load voltage is clamped to zero. From the discussion in the previous paragraph it can be concluded that the output voltage (hence the output current) is periodic over half the input cycle. Hence

$$V_{oav} = \frac{1}{\pi}\int_{o}^{\pi} v_o\, d\omega t = \frac{1}{\pi}\int_{\alpha}^{\pi} \sqrt{2}V_i \sin\omega t\, d\omega t = \frac{\sqrt{2}V_i}{\pi}(1+\cos\alpha)$$

$$I_{ov} = \frac{V_{oav} - E}{R} = \frac{\sqrt{2}V_i}{\pi R}(1+\cos\alpha - \pi\sin\theta)$$

Single phase half controlled converter with RLE load and freewheeling diode

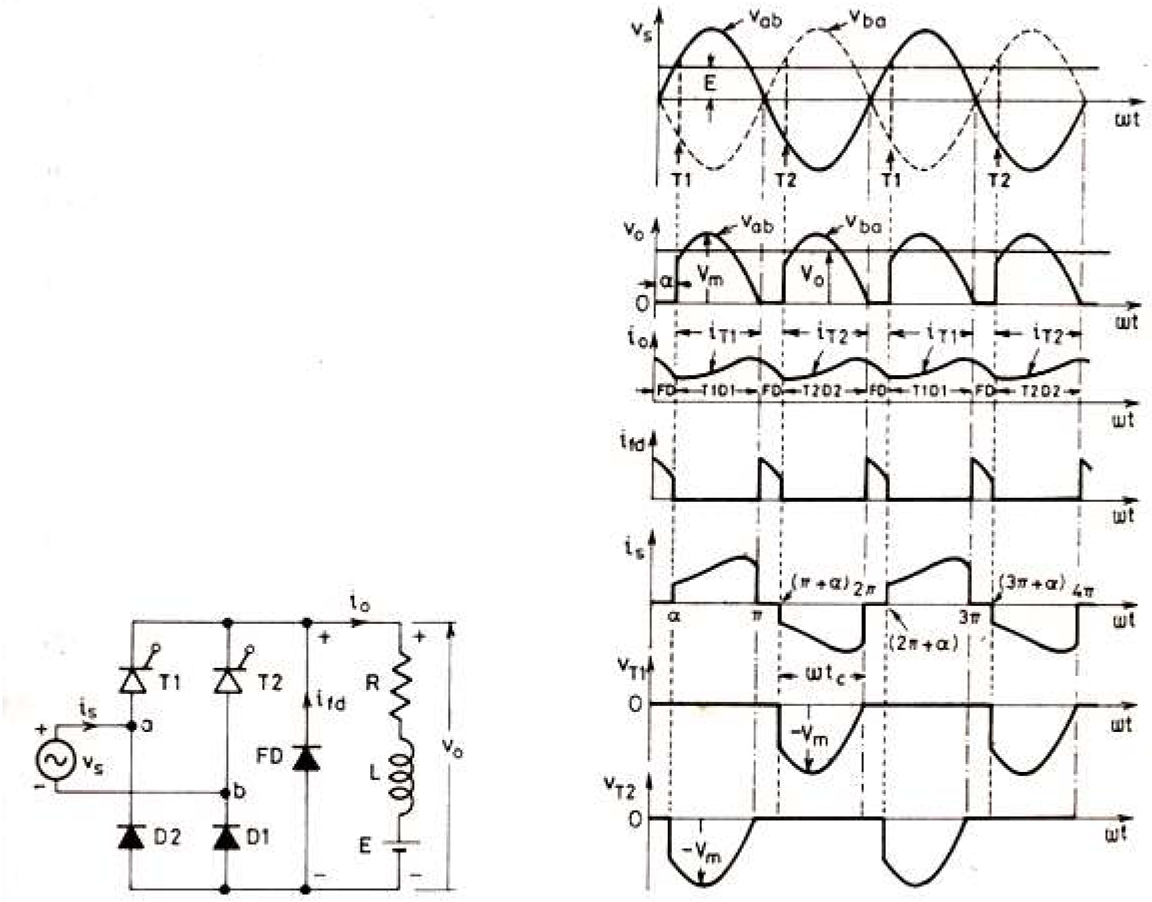

Figure: 2.6 single phase half controlled converter with RLE load and freewheeling diode

Numerical problems

1. A single phase 230V, 1 Kwheater is connected across 1 phase 230V, 50Hz supply through an SCR. For firing angle delay of 45^0 and 90^0, calculate the power absorbed in the heater element.

   Solution: Heater resistance R = $230^2/1000\ \Omega$

The rms value of voltage is $V_{or} = \frac{Vm}{2\sqrt{\pi}}\left[(\pi - \alpha) + \frac{1}{2}sin2\alpha\right]^{1/2}$

$$= \frac{\sqrt{2}\times 230}{2\sqrt{\pi}}\left[\left(\pi - \frac{\pi}{4}\right) + \frac{1}{2}sin90\right]^{1/2} = 155.071\text{V}$$

Power absorbed by the heater element for $\alpha = 45^0$ is

$$\frac{Vor^2}{R} = \left[\frac{155.071}{230}\right]^2 \times 1000 = 454.57\text{W}$$

for $\alpha = 90^0$ the rms voltage is

$$V_{or} = \frac{\sqrt{2}\times 230}{2\sqrt{\pi}}\left[\left(\pi - \frac{\pi}{2}\right) + \frac{1}{2}sin180\right]^{1/2} = 115\text{V}$$

Power absorbed by the heater element for $\alpha = 90^0$ is

$$\frac{Vor^2}{R} = \left[\frac{115}{230}\right]^2 \times 1000 = 250\text{W}$$

2. A resistive load of 10Ω is connected through a half-wave controlled rectifier circuit to 220V, 50 Hz, single phase source. Calculate the power delivered to the load for a firing angle of 60°. Find also the value of input power factor
3. A single phase semi converter delivers to RLE load with R=5Ω, L = 10mH and E = 80V. The source voltage is 230V, 50Hz. For continuous conduction, Find the average value of output current for firing angle = 50^o.

Single phase full wave controlled rectifier

Single Phase Full Wave Controlled Rectifier with ‘R’ load:

Figure below shows the Single phase Full Wave Controlled Rectifiers with R load

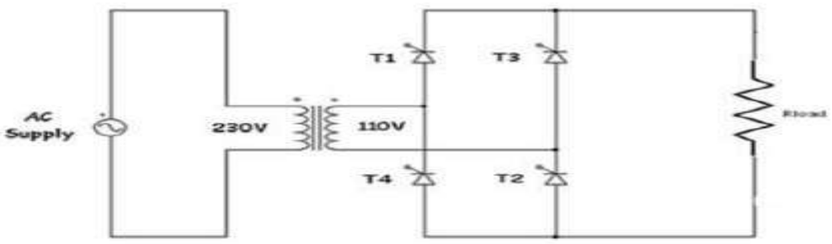

Figure: 2.7 single phase full converter circuit with R load

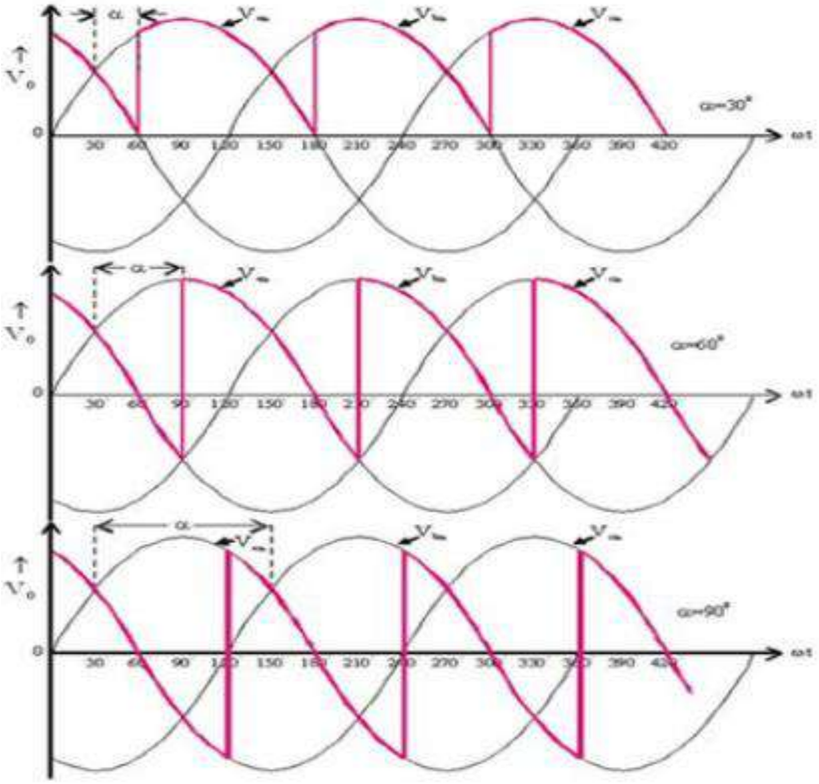

Figure: 2.8 single phase full converter circuit with R load input and output waveforms

- The single phase fully controlled rectifier allows conversion of single phase AC into DC. Normally this is used in various applications such as battery charging, speed control of DC motors and front end of UPS (Uninterruptible Power Supply) and SMPS (Switched Mode Power Supply).
- All four devices used are Thyristors. The turn-on instants of these devices are dependent on the firing signals that are given. Turn-off happens when the current through the device reaches zero and it is reverse biased at least for duration equal to the turn-off time of the device specified in the data sheet.
- In positive half cycle Thyristors T1 & T2 are fired at an angle α .
- When T1 & T2 conducts

Vo=Vs

IO=is=Vo/R=Vs/R

- In negative half cycle of input voltage, SCR's T3 &T4 are triggered at an angle of (π+α)
- Here output current & supply current are in opposite direction

∴ is=-io

T3 & T4 becomes off at 2π.

$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} Vmsinwt\ d(wt) = \frac{2Vm}{\pi}\ cos\alpha$

Single Phase Full Wave Controlled Rectifier with 'RL' load:

Figure below shows Single phase Full Wave Controlled Rectifiers with RL load.

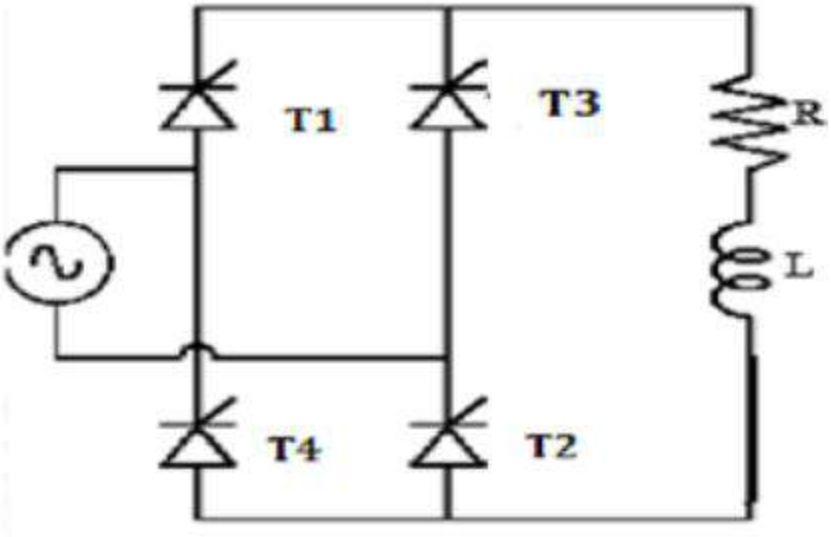

Figure: 2.9 single phase full converter circuit with RL load

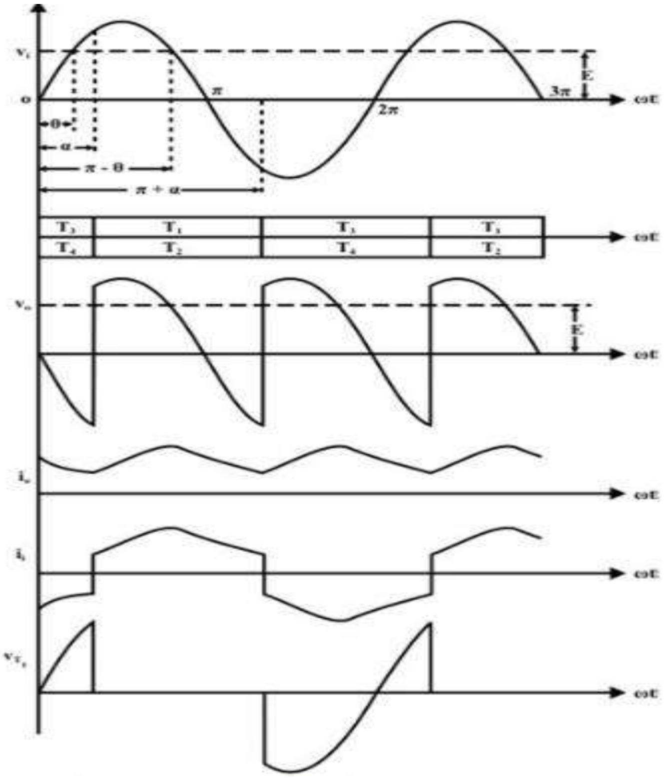

Figure: 2.10 single phase full converter circuit with RL load input and output waveforms

Operation of this mode can be divided between four modes

Mode 1 (α toπ)

- In positive half cycle of applied ac signal, SCR’s T1 & T2 are forward bias & can be turned on at an angle α.
- Load voltage is equal to positive instantaneous ac supply voltage. The load current is positive, ripple free, constant and equal to Io.
- Due to positive polarity of load voltage & load current, load inductance will store energy.

Mode 2 (π toπ+α)

- At wt=π, input supply is equal to zero & after π it becomes negative. But inductance opposes any change through it.
- In order to maintain a constant load current & also in same direction. A self inducedemf appears across ‘L’ as shown.
- Due to this induced voltage, SCR’s T1 & T2 are forward bais in spite the negative supply voltage.
- The load voltage is negative & equal to instantaneous ac supply voltage whereas load current is positive.
- Thus, load acts as source & stored energy in inductance is returned back to the ac supply.

Mode 3 (π+α to 2π)

- At wt=π+α SCR’s T3 & T4 are turned on & T1, T2 are reversed bias.
- Thus , process of conduction is transferred from T1,T2 to T3,T4.
- Load voltage again becomes positive & energy is stored in inductor
- T3, T4 conduct in negative half cycle from (π+α) to 2π
- With positive load voltage & load current energy gets stored

Mode 4 (2π to 2π+α)

- At wt=2π, input voltage passes through zero.
- Inductive load will try to oppose any change in current if in order to maintain load current constant & in the same direction.
- Induced emf is positive & maintains conducting SCR’s T3 & T4 with reverse polarity also.

- Thus VL is negative & equal to instantaneous ac supply voltage. Whereas load current continues to be positive.
- Thus load acts as source & stored energy in inductance is returned back to ac supply
- At wt=α or 2π+α, T3 & T4 are commutated and T1,T2 are turned on.

$$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} Vmsinwt\ d(wt) = \frac{2Vm}{\pi}\ cos\alpha$$

Single phase fully controlled converters with RLE load

The circuit diagram of a full wave bridge rectifier using thyristors in shown in figure below. It consists of four SCRs which are connected between single phase AC supply and a load.

This rectifier produces controllable DC by varying conduction of all SCRs.

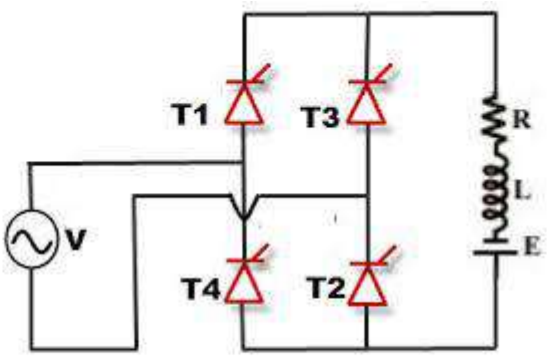

Figure: 2.11 single phase full converter circuit with RLE load

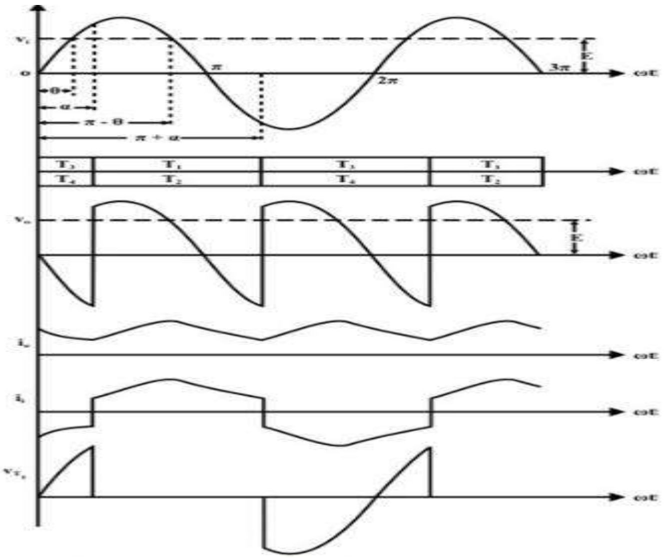

Figure: 2.12 single phase full converter circuit with RLE load input and output waveforms

In positive half-cycle of the input, Thyristors T1 and T2 are forward biased while T3 and T4 are reverse biased. Thyristors T1 and T2 are triggered simultaneously at some firing angle in the positive half cycle, and T3 and T4 are triggered in the negative half cycle.

The load current starts flowing through them when they are in conduction state. The load for this converter can be RL or RLE depending on the application.

By varying the conduction of each thyristor in the bridge, the average output of this converter gets controlled. The average value of the output voltage is twice that of half-wave rectifier.

The average output voltage is

$V_0 = \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} Vmsinwt\ d(wt) = \frac{2Vm}{\pi}\ cos\alpha$

Line commutated converters

For single phase half wave converter

1. Average DC load voltage: (V_{oavg})

$$V_{oavg} = V_0 = \frac{1}{T}\int_{0}^{T} Vmsinwt\ d(wt)\ \text{ where T is time period}$$

$$V_{oavg} = \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} Vmsinwt\ d(wt) + \int_{\pi}^{2\pi+\alpha} 0\ d(wt)\right]$$

$$= \frac{1}{2\pi}\left[\int_{\alpha}^{\pi} Vmsinwt\ d(wt)\right]$$

$$= \frac{Vm}{2\pi}[-coswt]_{\alpha}^{\pi}$$

$$= \frac{Vm}{2\pi} - [cos\pi - cos\alpha]$$

$$= \frac{Vm}{2\pi}[1 + cos\alpha]$$

If $\alpha = 0$ $V_{oavg\ max} = \frac{Vm}{\pi}$

If $\alpha = 180$ $V_{oavg} = 0$

2. Average DC load current is given as

$$I_{oavg} = \frac{V0avg}{R}$$

$$I_{oavg} = \frac{Vm}{2\pi R}[1 + cos\alpha]$$

3. RMS load voltage

$$V_{rms} = \left\{\frac{1}{T}\int_0^T Vm^2 sin^2 wt\ d(wt)\right\}^{1/2}$$

$$V_{rms} = \left\{\frac{1}{2\pi}\int_\alpha^\Pi Vm^2 sin^2 wt\ d(wt)\right\}^{1/2}$$

$$V_{rms} = \frac{Vm}{2\sqrt{\pi}}\left[(\pi - \alpha) + \frac{1}{2}sin2\alpha\right]^{1/2}$$

If $\alpha = 0$ $V_{rms} = \frac{Vm}{2}$

If $\alpha = 180$ $V_{rms} = 0$

The RMS voltage may be varied from 0 to $\frac{Vm}{2}$ by varying α from 180 to 0

4. Power delivered to the resistive load is given

$$P_L = \text{(RMS load voltage)(RMS load current)}$$
$$= V_{rms} X\ I_{rms}$$
$$= \frac{Vrms^2}{R} = Irms^2 XR$$

5. Input volt amperes = (RMS source voltage)(RMS line current)

$$= V_s\ I_{rms}$$
$$= V_s \frac{\sqrt{2}Vs}{R2\sqrt{\pi}}\left[(\pi - \alpha) + \frac{1}{2}sin2\alpha\right]^{1/2}$$
$$= \frac{Vs^2}{\sqrt{2\pi}XR}\left[(\pi - \alpha) + \frac{1}{2}sin2\alpha\right]^{1/2}$$

6. Input power factor: It is defined as the ratio of total mean input power to the total rms input volt amperes

$$\text{Input power factor} = \frac{\frac{\sqrt{2}Vs}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{1}{2}sin2\alpha\right]^{1/2}}{Vs}$$

$$= \frac{1}{\sqrt{2\pi}}\left[(\pi-\alpha)+\frac{1}{2}sin2\alpha\right]^{1/2}$$

7. Form factor: Form factor is defined as the ratio of RMS voltage to the average DC voltage

$$\text{Form Factor} = \frac{\text{Vrms}}{\text{Vavg}}$$

8. Effective value of the AC component of the output voltage

$$V_{ac} = [\text{Vrms}^2 - \text{Vavg}^2\ ]^{1/2}$$

9. Ripple factor (R_f)

   It is defined as the ratio of AC component to the DC. Where ripple is the amount of AC component present in DC component

$$R_f = \frac{\text{Vac}}{\text{Vavg}} = \frac{[\text{Vrms}^2-\text{Vavg}^2\ ]^{1/2}}{\text{Vavg}} = \left[\left(\frac{\text{Vrms}}{\text{Vavg}}\right)^2 - 1\right]^{1/2} = \sqrt{FF^2 - 1}$$

10. Transformer Utilization Factor (TUF):

    It is defined as the ratio of output DC power to the volt ampere rating of the transformer

$$\text{TUF} = \frac{\text{Pdc}}{\text{VA rating of secondary winding of the transformer}}$$

11. Rectifier efficiency:

    It is defined as the ratio of output DC power to the input ac power

$$\eta = \frac{\text{VavgIavg}}{\text{VrmsIrms}}$$

12. Peak inverse voltage (PIV):

    It is defined as the maximum voltage that an SCR can be subjected to in the reverse biased condition

    In the case of Half wave rectifier it is V_m

Effect of source inductance in single phase rectifier

Fig. below shows a single phase fully controlled converter with source inductance. For simplicity it has been assumed that the converter operates in the continuous conduction mode. Further, it has been assumed that the load current ripple is negligible and the load can be replaced by a dc current source the magnitude of which equals the average load current. Fig. shows the corresponding waveforms

It is assumed that the Thyristors T3 and T4 were conducting at t = 0. T1 and T2 are fired at ωt = α. If there were no source inductance T3 and T4 would have commutated as soon as T1 and T2 are turned ON.

The input current polarity would have changed instantaneously. However, if a source inductance is present the commutation and change of input current polarity cannot be instantaneous. s. Therefore, when T1 and T2 are turned ON T3 T4 does not commutate immediately. Instead, for some interval all four Thyristors continue to conduct as shown in Fig. 2.14. This interval is called “overlap” interval.

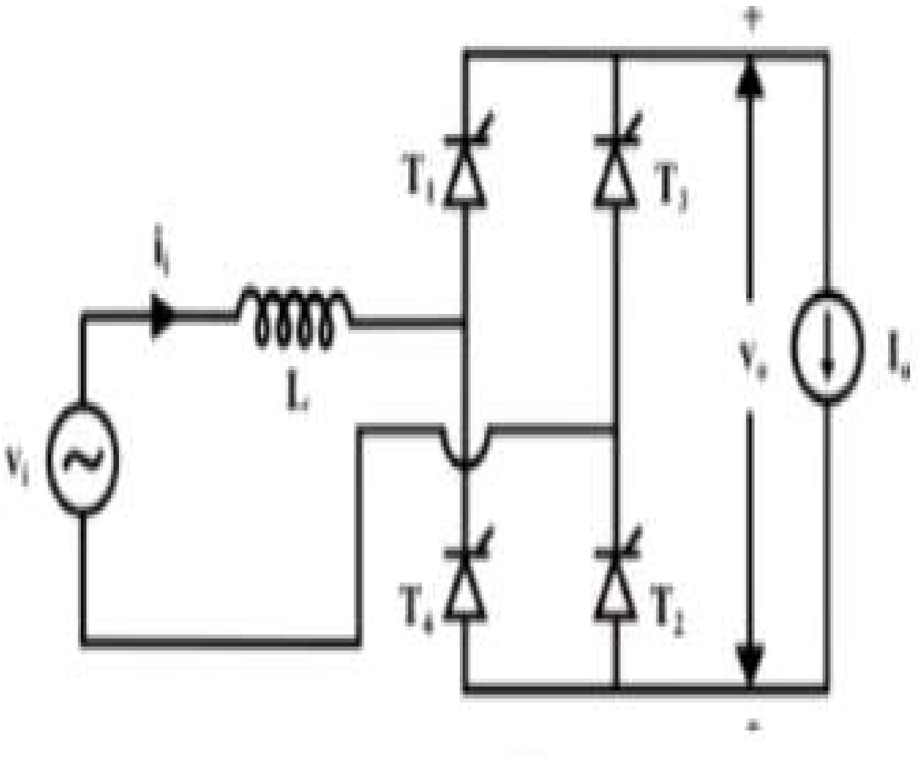

Figure: 2.13 single phase full converter circuit with source inductance

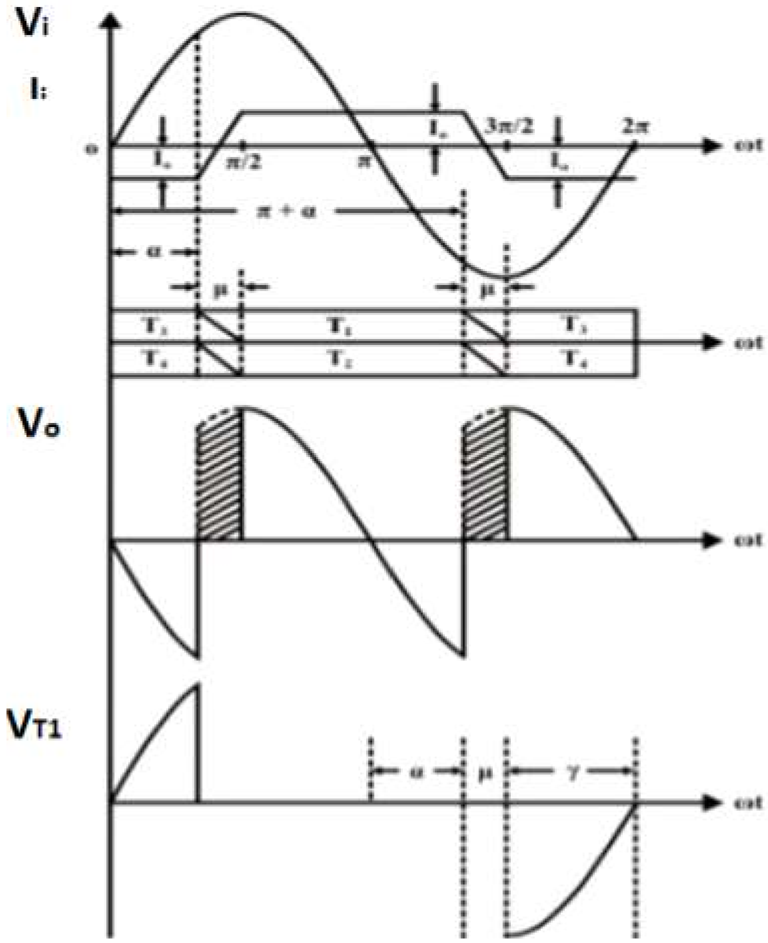

Figure: 2.14 single phase full converter output waveforms with source inductance

1. During overlap interval the load current freewheels through the thyristors and the output voltage is clamped to zero. On the other hand, the input current starts changing polarity as the current through T1 and T2 increases and T3 T4 current decreases. At the end of the overlap interval the current through T3 and T4 becomes zero and they commutate, T1 and T2 starts conducting the full load current
2. The same process repeats during commutation from T1 T2 to T3T4 at ωt = π + α . From Fig. 2.14 it is clear that, commutation overlap not only reduces average output dc voltage but also reduces the extinction angle γ which may cause commutation failure in the inverting mode of operation if α is very close to 180°.
3. In the following analysis an expression of the overlap angle "μ" will be determined. From the equivalent circuit of the converter during overlap period.

$$L\frac{di_i}{dt} = v_i \;\; for \;\; \alpha \le \omega t + \mu$$

$$i_i(\omega t = \alpha) = -I_0$$

$$i_i = I - \frac{\sqrt{2}V_i}{\omega L} cos\omega t$$

$$\therefore i_i|_{\omega t - \alpha} = I - \frac{\sqrt{2}V_i}{\omega L} cos\alpha = -I_0$$

$$I = \frac{\sqrt{2}V_i}{\omega L}\cos\alpha - I_0$$

$$\therefore \qquad i_i = \frac{\sqrt{2}V_i}{\omega L}(\cos\alpha - \cos\omega t) - I_0$$

$$\text{at} \quad \omega t = \alpha + \mu \quad i_i = I_0$$

$$I_0 = \frac{\sqrt{2}V_i}{\omega L}(\cos\alpha - \cos(\alpha+\mu)) - I_0$$

$$\therefore \qquad \cos\alpha - \cos(\alpha+\mu) = \frac{\sqrt{2}\omega L}{V_0} I_0$$

$$V_0 = \frac{I}{\pi}\int_{\alpha}^{\alpha+\pi} V_i d\,\omega t$$

or

$$V_0 = \frac{I}{\pi}\int_{\alpha+\mu}^{\alpha+\pi} \sqrt{2}v_i \sin\,\omega t d\,\omega t$$

$$= \frac{\sqrt{2}v_i}{\pi}[\cos(\alpha+\mu) - \cos(\pi+\alpha)$$

$$= \frac{\sqrt{2}v_i}{\pi}[\cos\alpha + \cos(\alpha+\mu)]$$

$$\therefore V_0 = 2\sqrt{2}\frac{v_i}{\pi}[cos\alpha - cos(\alpha+\mu)]$$

$$\therefore V_0 = \frac{2\sqrt{2}}{\pi}v_i cos\alpha - \frac{2}{\pi}\omega L I_0$$

The Equation can be represented by the following equivalent circuit

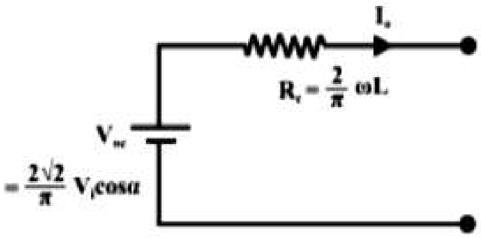

Figure: 2.15 Equivalent circuit of the given equation

Equivalent circuit representation of the single phase fully controlled rectifier with source inductance

The simple equivalent circuit of Fig. 2.15 represents the single phase fully controlled converter with source inductance as a practical dc source as far as its average behavior is concerned. The open circuit voltage of this practical source equals the average dc output voltage of an ideal converter (without source inductance) operating at a firing angle of α. The voltage drop across the internal resistance "RC" represents the voltage lost due to overlap shown in Fig. 2.14 by the hatched portion of the Vo waveform. Therefore, this is called the "Commutation resistance". Although this resistance accounts for the voltage drop correctly there is no power loss associated with this resistance since the physical process of overlap does not involve any power loss. Therefore this resistance should be used carefully where power calculation is involved.

Numerical problems

1. For the single phase fully controlled bridge is connected to RLE load. The source voltage is 230 V, 50 Hz. The average load current of 10A continuous over the working range. For R= 0.4 Ω and L = 2mH, Compute (a) firing angle for E = 120V (b) firing angle for E = -120V (c) in case output current is constant find the input power factors for both parts a and b

Solution:

a) For E = 120 the full converter is operating as a controlled rectifier

$$\frac{2Vm}{\pi} \cos\alpha = E + I_0R$$

$$\frac{2\sqrt{2}.230}{\pi} \cos\alpha = 120 + 10X0.4 = 124V$$

$$\alpha = 53.21^0$$

For $\alpha = 53.21^0$ power flows from ac source to DC load.

b) For E = -120 the full converter is operating as a controlled rectifier

$$\frac{2Vm}{\pi}\ cos\alpha = E+I_0R$$

$$\frac{2\sqrt{2}.230}{\pi}\ cos\alpha = -120+ 10X0.4 = -116V$$

$$\alpha = 124.1^0$$

For $\alpha = 124.1^0$ power flows from DC source to ac load.

c) For constant load current, rms value of load current is

$$I_{or} = I_o = 10A$$

$$V_s\, I_{or} \cos\Phi = EI_o+I_{or}^2R$$

For $\alpha = 53.21^0$ $\cos\Phi = \frac{120\times10+10^2\times0.4}{230\times10} = 0.5391$ lag

For $\alpha = 124.1^0$ $\cos\Phi = \frac{120\times10-10^2\times0.4}{230\times10} = 0.5043$ lag

2. A single phase two pulse converter feeds power to RLE load with R= 6Ω, L= 6mH, E= 60V, AC source voltage is 230V, 50Hz for continuous condition. Find the average value of load current for a firing angle of 50°. In case one of the 4 SCRs gets open circuited. Find the new value of average load current assuming the output current as continuous.
3. For the single phase fully controlled bridge converter having load of 'R', determine the average output voltage, rms output voltage and input power factor if the supply is 230V, 50 Hz, single phase AC and the firing angle is 60 degrees

Operation of three phase half wave rectifier with R and RL loads

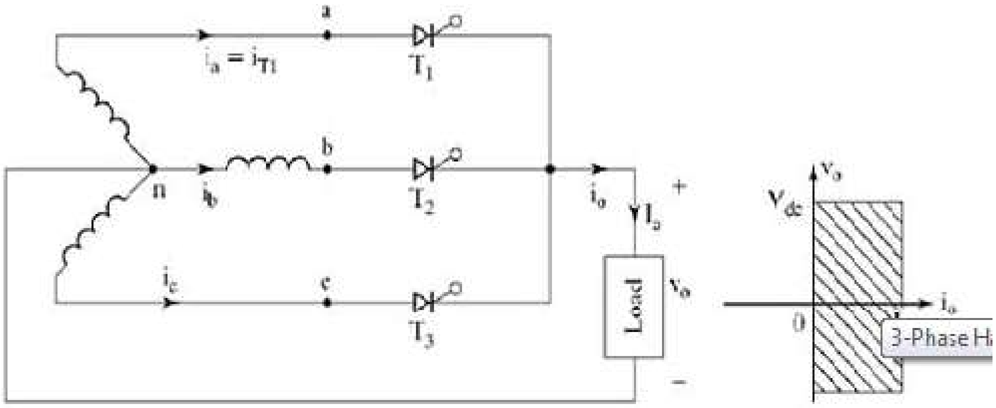

Figure: 2.16 circuit diagram three phase half wave rectifier

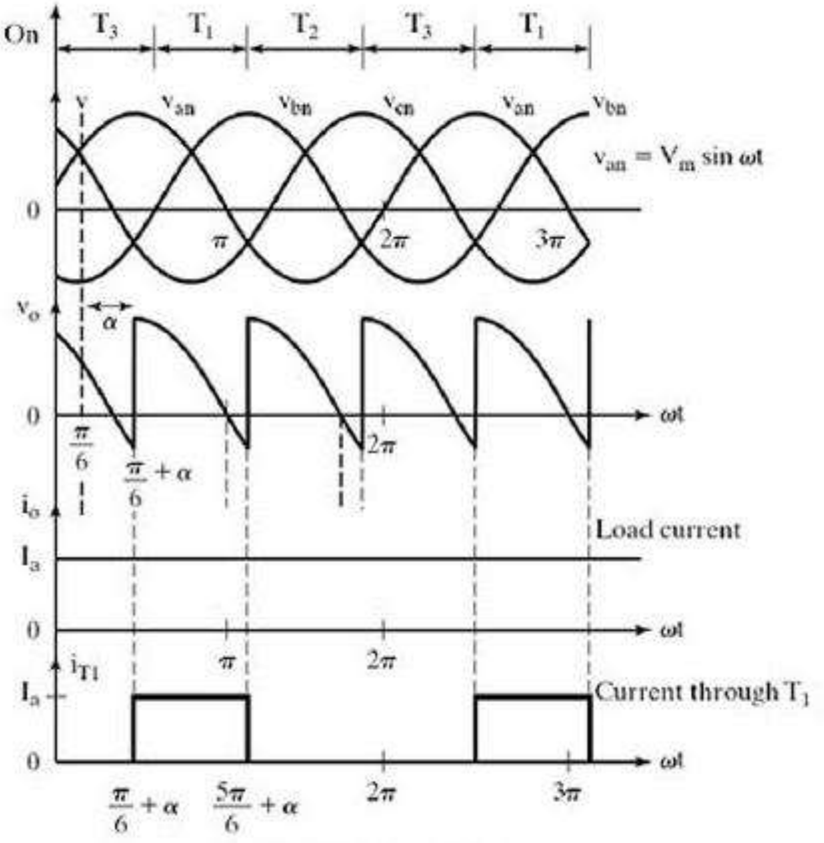

Figure: 2.17 input and output waveforms of three phase half wave rectifier

Three phase supply voltage equations

We define three line neutral voltages (3 phase voltages) as follows

$V_{RN} = V_{an} = V_m$ sinwt where V_m is the maximum voltage

$V_{YN} = V_{bn} = V_m \sin(wt-\frac{2\pi}{3})$

$V_{BN} = V_{cn} = V_m \sin(wt-\frac{4\pi}{3})$

The **3-phase half wave converter** combines three **single phase half wave controlled rectifiers in one** single circuit feeding a common load. The thyristor T_1 in series with one of the supply phase windings *'a-n'* acts as one half wave controlled rectifier The second thyristor T_2 in series with the supply phase winding *'b-n'* acts as the second half wave controlled rectifier. The third thyristor T_3in series with the supply phase winding acts as the third half wave controlled rectifier.

The 3-phase input supply is applied through the star connected supply transformer as shown in the figure. The common neutral point of the supply is connected to one end of the load while the other end of the load connected to the common cathode point.

When the thyristor T_1 is triggered at $\omega t=(\prod/6 + \alpha)=(30° + \alpha)$, the phase voltage V_{an} appears across the load when T_1 conducts. The load current flows through the supply phase winding *'a-n'* and through thyristor T_1 as long as T_1 conducts.

When thyristor T_2 is triggered at $\omega t=(5\prod/6\alpha)$, T_1 becomes reverse biased and turns-off. The load current flows through the thyristor and through the supply phase winding *'b-n'* . When T_2 conducts the phase voltage v_{bn} appears across the load until the thyristor T_3 is triggered .

When the thyristor T_3 is triggered at $\omega t=(3\prod/2 + \alpha)=(270°+\alpha)$, T_2 is reversed biased and hence T_2turns-off. The phase voltage V_{an} appears across the load when T_3 conducts.

When T_1 is triggered again at the beginning of the next input cycle the thyristor T_3 turns off as it is reverse biased naturally as soon as T_1 is triggered. The figure shows the 3-phase input supply voltages, the output voltage which appears across the load, and the load current assuming a constant and ripple free load current for a highly inductive load and the current through the thyristor T_1.

For a purely resistive load where the load inductance 'L = 0' and the trigger angle $\alpha > (\prod/6)$, the load current appears as discontinuous load current and each thyristor is naturally commutated when the polarity of the corresponding phase supply voltage reverses. The frequency of output ripple frequency for a **3-phase half wave converter** is f_s, where f_s is the input supply frequency. 3

The **3-phase half wave converter** is not normally used in practical converter systems because of the disadvantage that the supply current waveforms contain dc components (i.e., the supply current waveforms have an average or dc value).

To derive an expression for the average output voltage of a 3-phase half wave converter for continuous load current

The reference phase voltage is $v_{RN}=v_{an}=V_m sin\omega t$. The trigger angle is measured from the cross over points of the 3-phase supply voltage waveforms. When the phase supply voltage V_{an} begins its positive half cycle at $\omega t=0$, the first cross over point appears at $\omega t=(\prod/6) radians$ 30°.
The trigger angle α for the thyristor T_1 is measured from the cross over point at . The thyristor T_1 is forward biased during the period ωt=30° to 150° , when the phase supply voltage v_{an} has higher amplitude than the other phase supply voltages. Hence T_1 can be triggered between 30° to 150°. When the thyristor T_1 is triggered at a trigger angle α, the average or dc output voltage for continuous load current is calculated using the equation

$$V_{avg} = \frac{3}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha} \mathrm{Vmsinwt\,d(wt)}$$

$$= \frac{3Vm}{2\pi}\left[(-cos\alpha)_{\frac{\pi}{6}+\alpha}^{\frac{5\pi}{6}+\alpha}\right]$$

$$= \frac{3\sqrt{3}Vm}{2\pi}\ cos\alpha$$

$$= \frac{3Vml}{2\pi}\ cos\alpha$$

Operation of three phase half controlled rectifier with R and RL loads

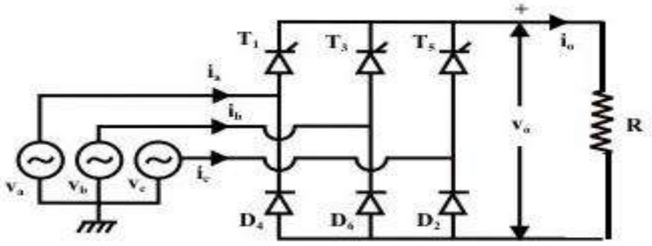

Figure: 2.18 circuit diagram three phase half controlled rectifier

Three phase half wave controlled rectifier output voltage waveforms for different trigger angles with R load

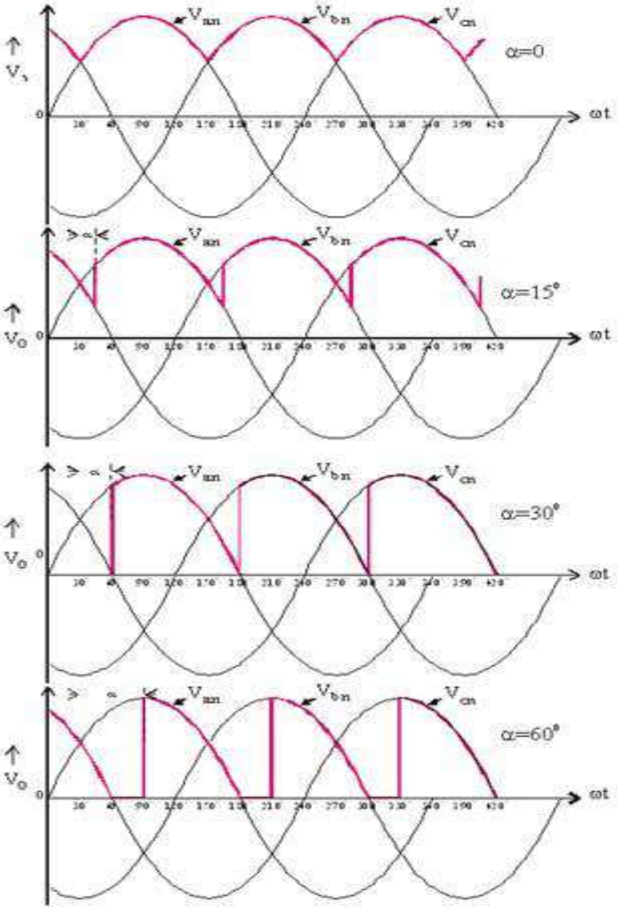

Figure: 2.19 input and output waveforms of three phase half controlled rectifier with R load

Three single phase half wave converters can be connected to form a three phase half wave converter. Similarly three phase semi converter uses 3 SCRs T1, T3 & T5 and 3 diodes D2, D4&D6 In the circuit shown above when any device conducts, line voltage is applied across load. so line voltage are necessary to draw Phase shift between two line voltages is 60 degree & between two phase voltages it is 120 degree Each phase & line voltage is sine wave with the frequency of 50 Hz. R,Y,B are phase voltages with respect to 'N'.

In the case of a **three-phase half wave controlled** rectifier with resistive load, the thyristor T_1 is triggered at $\omega t=(30°+\alpha)$and T_1 conducts up to $\omega t=180°=$&pron; radians. When the phase supply voltage decreases to zero at , the load current falls to zero and the thyristor T_1 turns off. Thus T_1 conducts from $\omega t=(30° + \alpha)$ *to* $(180°)$.

Hence the average dc output voltage for a 3-pulse converter (3-phase half wave controlled rectifier) is calculated by using the equation

The average output voltage $V_{avg} = \frac{3}{2\pi}\int_{\frac{\pi}{3}+\alpha}^{\frac{2\pi}{3}} \mathrm{Vmlsinwt}\, d(wt) + \int_{\frac{\pi}{3}}^{\frac{2\pi}{3}+\alpha} \mathrm{Vmlsinwt}\, d(wt)$

$$= \frac{3Vml}{2\pi}\,(1 + cos\alpha)$$

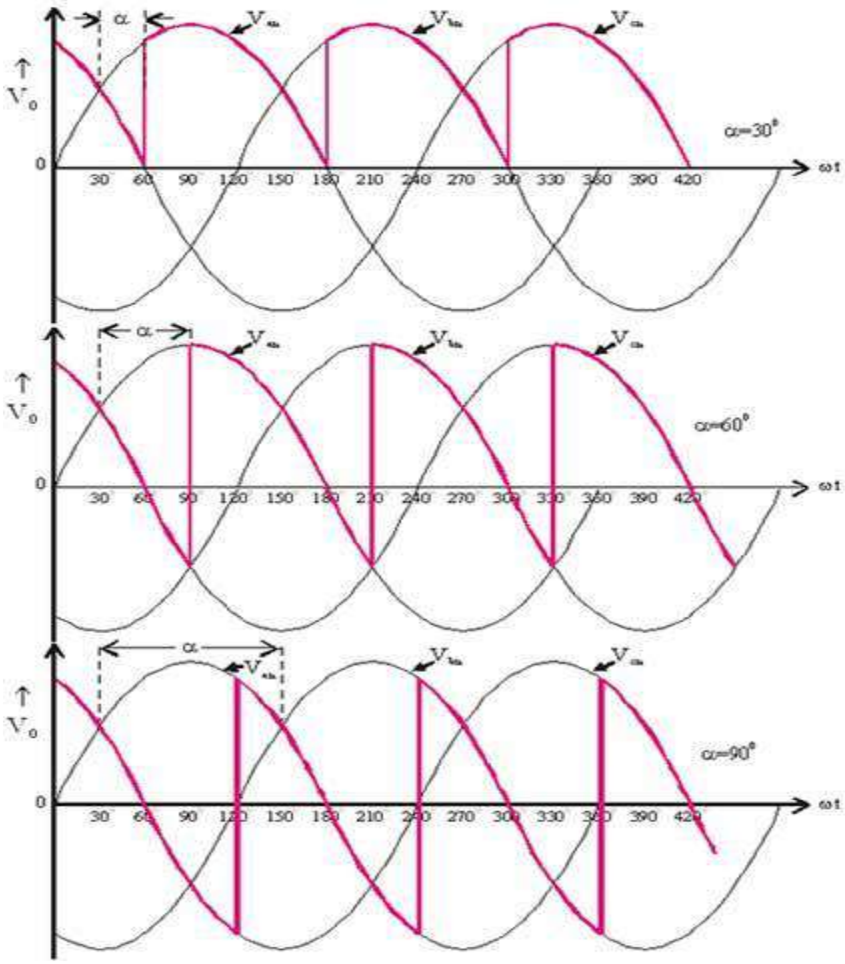

Figure: 2.19 Input and output waveforms of three phase half controlled rectifier with RL load

Numerical Problems on three phase rectifiers:

1. A three phase semi converter feeds power to a resistive load of 10Ω. For a firing angle delay of 30^0 the load takes 5 Kw. Find the magnitude of per phase input supply voltage.

Solution:

$$V_{or} = \left[\frac{3}{2\pi}\left[\int_{-\left(\frac{\pi}{6}-\alpha\right)}^{\frac{\pi}{6}} Vml^2\sin^2 wt\, d(wt) + \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}+\alpha} Vml^2\sin^2 wt\, d(wt)\right]\right]^{1/2}$$

$$V_{or}{}^2 = \frac{3Vml^2}{4\pi}\left[\left|wt + \frac{sin2wt}{2}\right|\frac{-\frac{\pi}{6}}{-\left(\frac{\pi}{6}-\alpha\right)} + \left|wt + \frac{sin2wt}{2}\right|\frac{\frac{\pi}{6}+\alpha}{-\left(\frac{\pi}{6}\right)}\right]$$

$$V_{or} = \frac{Vml}{2}\sqrt{\frac{3}{\pi}}\left[\frac{2\pi}{3} + \frac{\sqrt{3}}{2}(1 + cos2\alpha\right]^{1/2}$$

For $\alpha = 30^0$

$$P = V^2/R$$

$$5000x10 = \frac{2Vs^2}{4}\frac{3}{\pi}\left[\frac{2\pi}{3} + \frac{\sqrt{3}}{2}(1 + cos60\right]$$

$V_s = 175.67V$ and $V_{ph} = 101.43V$

2. A three-phase half-wave controlled rectifier has a supply of 200V/phase. Determine the average load voltage for firing angle of 0^o, $30^o$and 60^o assuming a thyristor volt drop of 1.5V and continuous load current
3. A three phase half wave converter is supplying a load with a continuous constant current of 50A over a firing angle from 0^o to 60^0. What will be the power dissipated by the load at these limiting values of firing angle. The supply voltage is 415V (line).

Operation of three phase fully controlled rectifier with R and RL loads

Three phase full converter is a fully controlled bridge controlled rectifier using six thyristors connected in the form of a full wave bridge configuration. All the six thyristors are controlled switches which are turned on at a appropriate times by applying suitable gate trigger signals.

The **three phase full converter** is extensively used in industrial power applications upto about 120kW output power level, where two quadrant operations is required. The figure shows a **three phase full converter** with highly inductive load. This circuit is also known as three phase full wave bridge or as a six pulse converter.

The thyristors are triggered at an interval of (∏/3) radians (i.e. at an interval of 30°). The frequency of output ripple voltage is $6f_s$ and the filtering requirement is less than that of **three phase semi and half wave converters**.

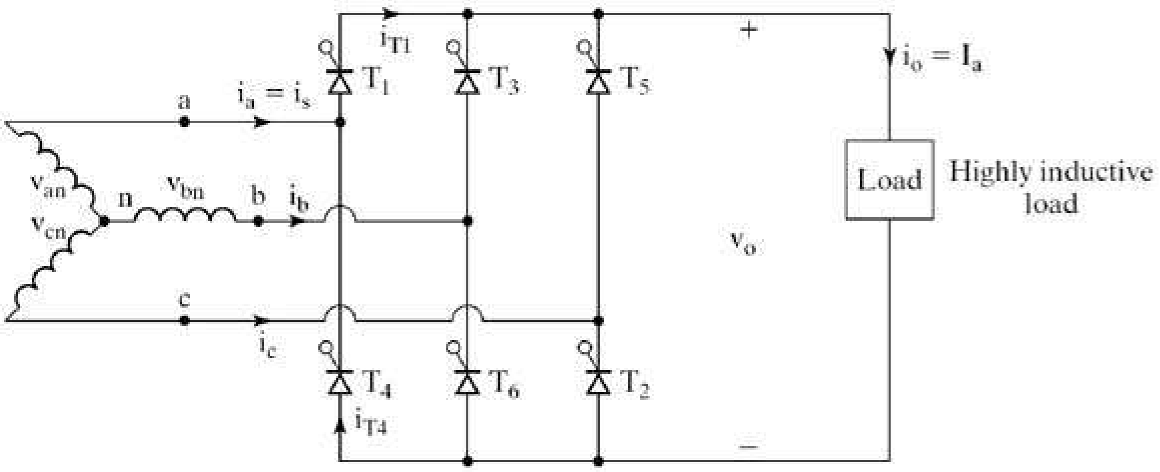

Figure: 2.20 circuit diagram three phase fully controlled rectifier with R and RL load

At $\omega t=(\prod/6 +\alpha)$, thyristor is already conducting when the thyristor is turned on by applying the gating signal to the gate of . During the time period $\omega t=(\prod/6 +\alpha)$ to $(\prod/2 +\alpha)$, thyristors and conduct together and the line to line supply voltage appears across the load.

At $\omega t=(\prod/2 +\alpha)$, the thyristor T_2 is triggered and T_6 is reverse biased immediately and T_6 turns off due to natural commutation. During the time period $\omega t=(\prod/ +\alpha)$ *to* $(5\prod/6 +\alpha)$, thyristor T_1 and T_2 conduct together and the line to line supply voltage appears across the load.

The thyristors are numbered in the circuit diagram corresponding to the order in which they are triggered. The trigger sequence (firing sequence) of the thyristors is 12, 23, 34, 45, 56, 61, 12, 23, and so on. The figure shows the waveforms of three phase input supply voltages, output voltage, the thyristor current through T_1 and T_4, the supply current through the line ‘a’.

We define three line neutral voltages (3 phase voltages) as follows

$V_{RN} = V_{an} = V_m$ sinwt where V_m is the maximum voltage

$V_{YN} = V_{bn} = V_m \sin(wt - \frac{2\pi}{3})$

$V_{BN} = V_{cn} = V_m \sin(wt - \frac{4\pi}{3})$

The corresponding line to line voltages are

$V_{RY} = V_{ab} = V_{an} - V_{bn} = \sqrt{3}\,\mathrm{Vm}\sin\left(\mathrm{wt} + \frac{\pi}{6}\right)$

$V_{YB} = V_{bc} = V_{bn} - V_{cn} = \sqrt{3}\,\mathrm{Vm}\sin\left(\mathrm{wt} - \frac{\pi}{2}\right)$

$V_{BR} = V_{ca} = V_{cn} - V_{an} = \sqrt{3}\,\mathrm{Vm}\sin\left(\mathrm{wt} + \frac{\pi}{2}\right)$

To derive an expression for the average output voltage of **three phase full converter** with highly inductive load assuming continuous and constant load current

The output load voltage consists of 6 voltage pulses over a period of 2∏ radians, hence the average output voltage is calculated as

$$V_{avg} = \frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \mathrm{Vod(wt)}$$

$$V_o = V_{ab} = \sqrt{3}\,\mathrm{Vm}\sin\left(\mathrm{wt} + \frac{\pi}{6}\right)$$

$$V_{avg} = \frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} \sqrt{3}\,\mathrm{Vm}\sin\left(\mathrm{wt} + \frac{\pi}{6}\right)\mathrm{d(wt)}$$

$$= \frac{3\sqrt{3}Vm}{\pi}\ cos\alpha$$

$$= \frac{3Vml}{\pi}\ cos\alpha$$

The RMS value of the output voltage is found from

$$V_{orms} = \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} Vo^2 d(wt)\right]^{1/2}$$

$$= \left[\frac{6}{2\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} Vab^2 d(wt)\right]^{1/2}$$

$$= \left[\frac{3}{\pi}\int_{\frac{\pi}{6}+\alpha}^{\frac{\pi}{2}+\alpha} 3\ Vm^2 \sin^2\left(wt + \frac{\pi}{6}\right) d(wt)\right]^{1/2}$$

$$= \sqrt{3}Vm\left(\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}cos2\alpha\right)^{1/2}$$

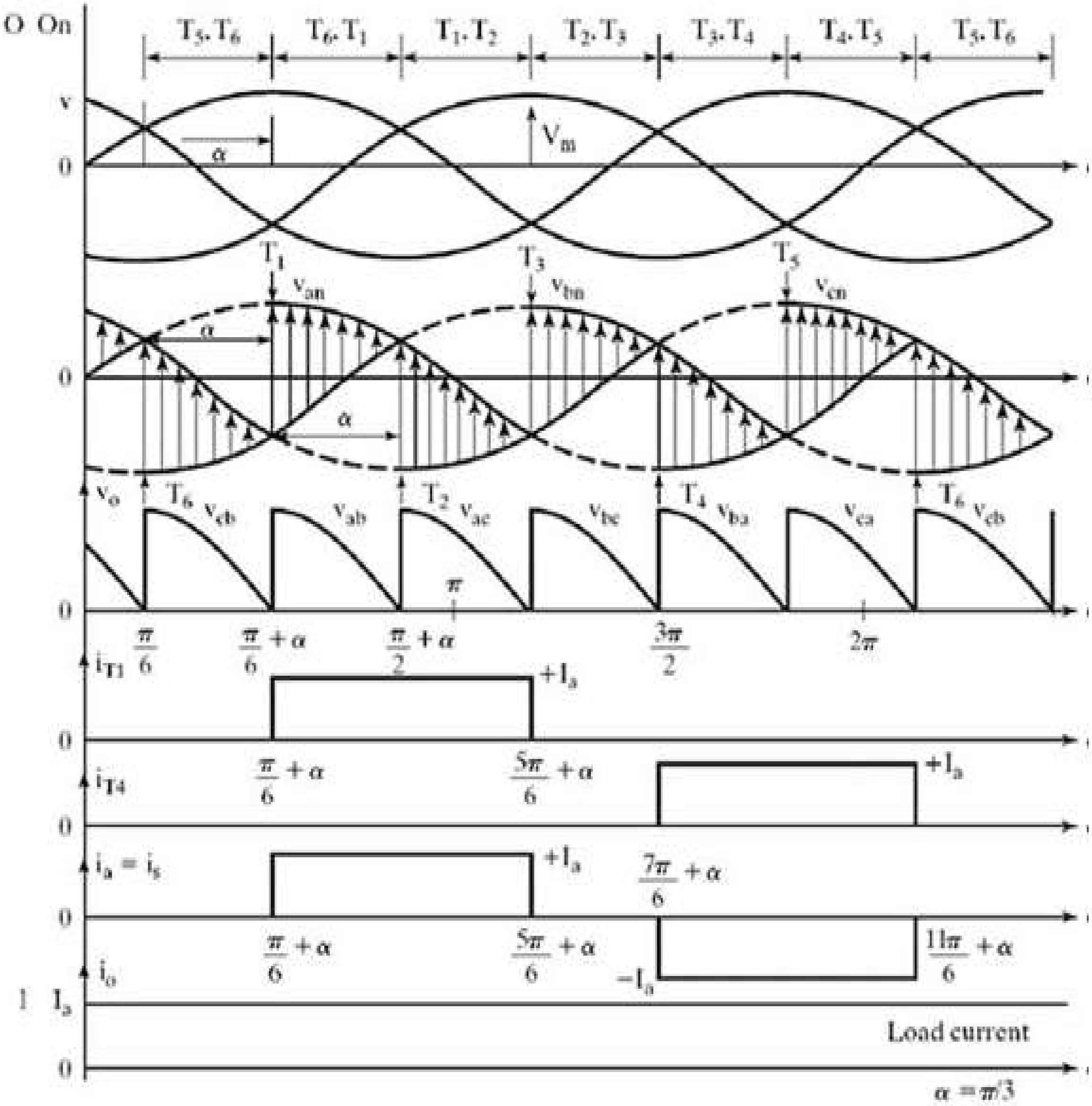

Figure: 2.21 Input and output waveforms of three phase fully controlled rectifier

Operation of three phase half wave rectifier with RLE loads

A three phase fully controlled converter is obtained by replacing all the six diodes of an uncontrolled converter by six thyristors as shown in Figure

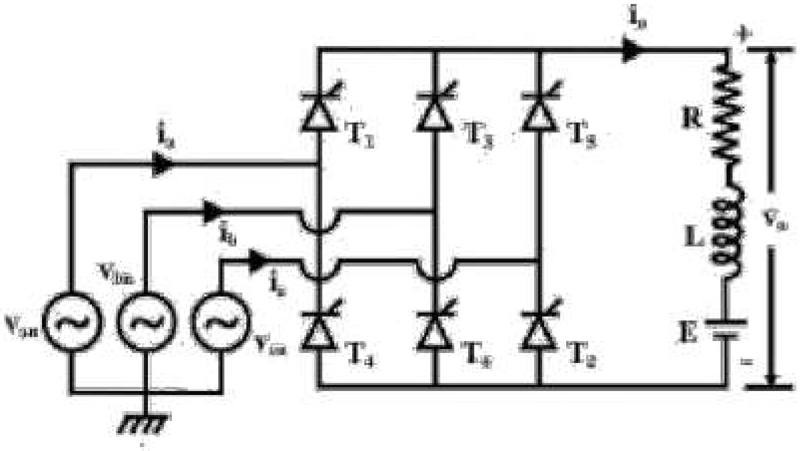

Figure: 2.22 circuit diagram of three phase fully controlled rectifier with RLE load

For any current to flow in the load at least one device from the top group (T1, T3, T5) and one from the bottom group (T2, T4, T6) must conduct. It can be argued as in the case of an uncontrolled converter only one device from these two groups will conduct.

Then from symmetry consideration it can be argued that each thyristor conducts for 120° of the input cycle. Now the thyristors are fired in the sequence T1 → T2 → T3 → T4 → T5 → T6 → T1 with 60° interval between each firing. Therefore thyristors on the same phase leg are fired at an interval of 180° and hence can not conduct simultaneously. This leaves only six possible conduction mode for the converter in the continuous conduction mode of operation. These are T1T2, T2T3, T3T4, T4T5, T5T6, T6T1. Each conduction mode is of 60° duration and appears in the sequence mentioned. Each of these line voltages can be associated with the firing of a thyristor with the help of the conduction table-1. For example the thyristor T1 is fired at the end

of T5 T6 conduction interval. During this period the voltage across T1 was vac. Therefore T1 is fired α angle after the positive going zero crossing of vac. similar observation can be made about other thyristors.

Fig. 2.23 shows the waveforms of different variables. To arrive at the waveforms it is necessary to draw the conduction diagram which shows the interval of conduction for each thyristor and can be drawn with the help of the phasor diagram of fig. 2.22. If the converter firing angle is α each thyristor is fired “α” angle after the positive going zero crossing of the line voltage with which it's firing is associated. Once the conduction diagram is drawn all other voltage waveforms can be drawn from the line voltage waveforms and from the conduction table of fig. 2.22. Similarly line currents can be drawn from the output current and the conduction diagram. It is clear from the waveforms that output voltage and current waveforms are periodic over one sixth of the input cycle. Therefore this converter is also called

the "six pulse" converter. The input current on the other hand contains only odds harmonics of the input frequency other than the triplex (3rd, 9th etc.) harmonics. The next section will analyze the operation of this converter in more details.

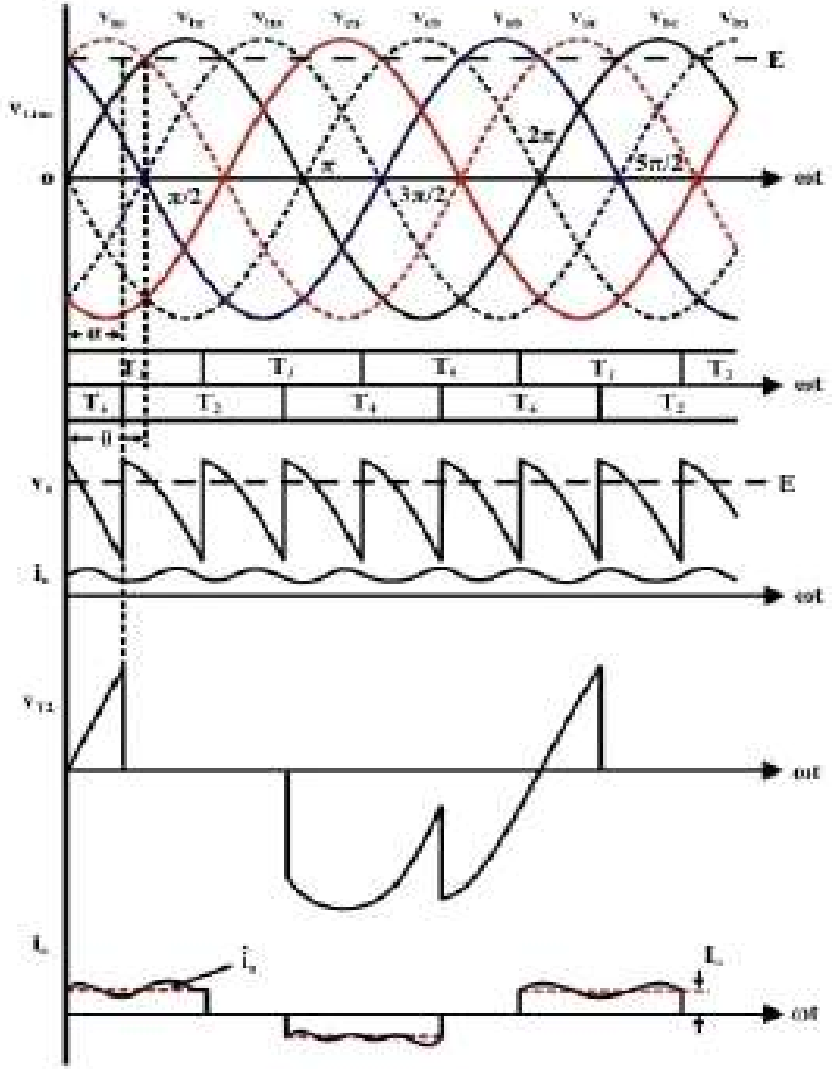

Figure: 2.23 Input and output waveforms of three phase fully controlled rectifier in rectifier mode

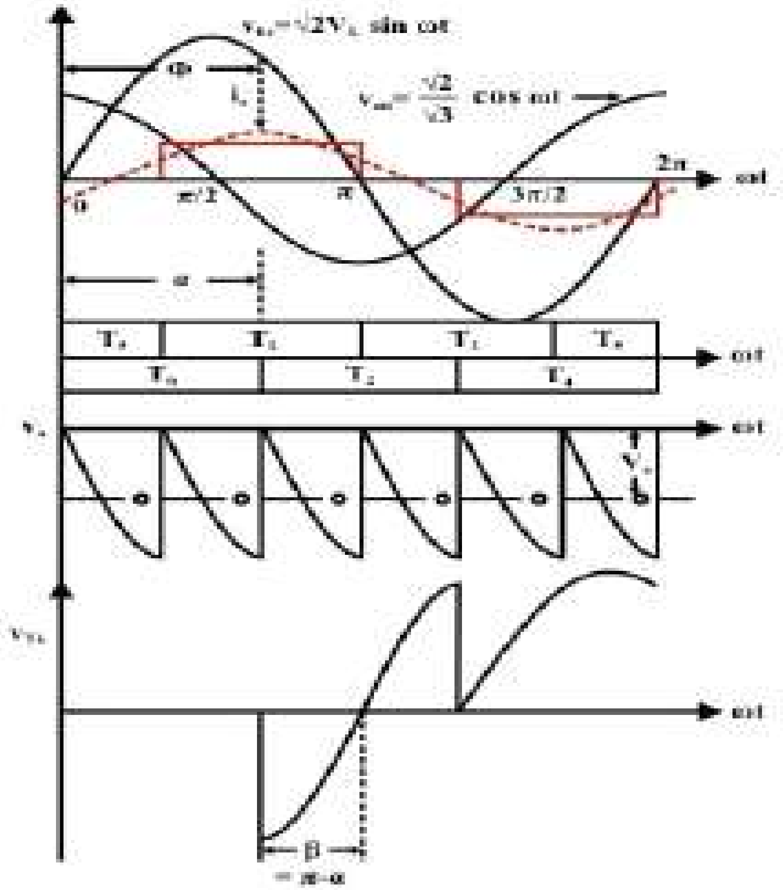

Figure: 2.24 Input and output waveforms of three phase fully controlled rectifier in inversion mode

Effect of source inductance in three phase rectifiers

The three phase fully controlled converter was analyzed with ideal source with no internal impedance. When the source inductance is taken into account, the qualitative effects on the performance of the converter is similar to that in the case of a single phase converter. Fig. 2.25 shows such a converter. As in the case of a single phase converter the load is assumed to be highly inductive such that the load can be replaced by a current source.

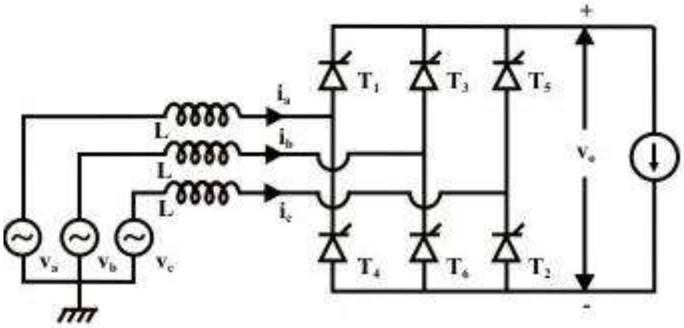

Figure: 2.25 circuit diagram for three phase rectifier with source inductance

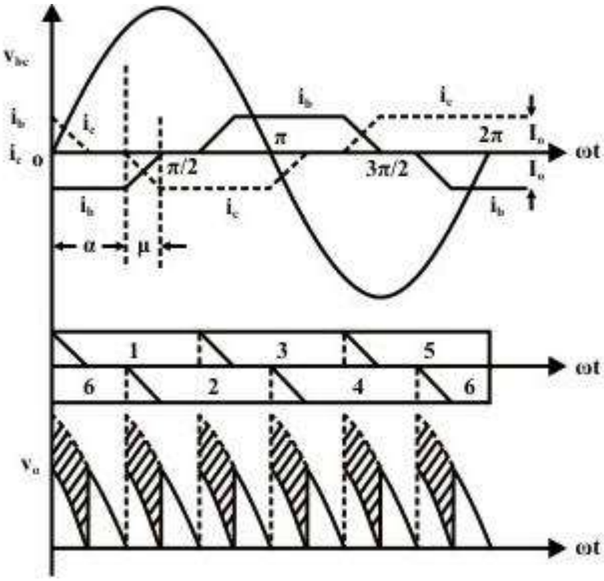

Figure: 2.26 waveforms for three phase rectifier with source inductance

As in the case of a single phase converter, commutations are not instantaneous due to the presence of source inductances. It takes place over an overlap period of “μ_1” instead. During the overlap period three thyristors instead of two conducts. Current in the outgoing thyristor gradually decreases to zero while the incoming thyristor current increases and equals the total load current at the end of the overlap period. If the duration of the overlap period is greater than 60° four thyristors may also conduct clamping the output voltage to zero for some time. However, this situation is not very common and will not be discussed any further in this lesson. Due to the conduction of two devices during commutation either from the top group or the bottom group the instantaneous output voltage during the overlap period drops (shown by the hatched portion of Fig. 2.26 resulting in reduced average voltage. The exact amount of this reduction can be calculated as follows.

In the time interval $\alpha < \omega t \leq \alpha + \mu$, T_6 and T_2 from the bottom group and T_1 from the top group conducts.

The equivalent circuit of the converter during this period is given by the circuit diagram of Fig. 2.27

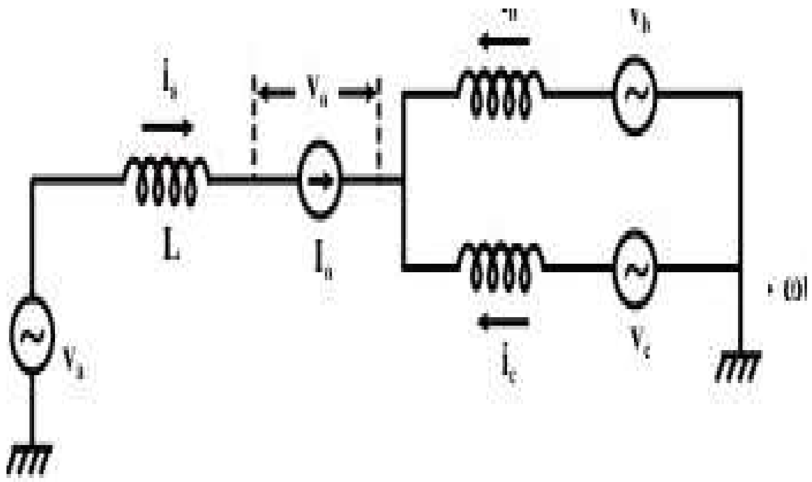

Figure: 2.27 Equivalent circuit of waveforms with source inductance

Therefore, in the interval $\alpha < \omega t \leq \alpha + \mu$

$$v_b = L\frac{di_b}{dt} - L\frac{di_c}{dt} + v_c$$

or, $$v_{bc} = L\frac{d}{dt}(i_b - i_c)$$

but $i_b + i_c + I_0 = 0$ $\quad\therefore\quad \frac{di_b}{dt} = -\frac{di_c}{dt}$

$$\therefore\quad 2L\frac{d}{dt}i_b = v_{bc} = \sqrt{2}V_L \sin\omega t$$

$$\therefore\quad i_b = C - \frac{\sqrt{2}V_L}{2\omega L}\cos\omega t$$

at $\omega t = \alpha$, $\quad i_b = -I_0 \quad \therefore\quad C = \frac{\sqrt{2}V_L}{2\omega L}\cos\alpha - I_0$

$$\therefore\quad i_b = \frac{\sqrt{2}V_L}{2\omega L}(\cos\alpha - \cos\omega t) - I_0$$

at $\omega t = \alpha + \mu$, $\quad i_b = 0$

$$\therefore\quad \frac{\sqrt{2}V_L}{2\omega L}(\cos\alpha - \cos(\alpha + \mu)) = I_0$$

Or, $$\cos\alpha - \cos(\alpha + \mu) = \frac{\sqrt{2}\omega L}{V_L}I_0$$

for $\mu \le 60°$. It can be shown that for this condition to be satisfied

$$I_0 \le \frac{V_L}{\sqrt{2}\omega L} \cos\left(\alpha - \frac{\pi}{3}\right)$$

To calculate the dc voltage

For $\alpha \le \omega t \le \alpha + \mu$

$$v_0 = v_a - v_b + L\frac{di_b}{d_t} = \frac{3}{2}v_a$$

for $\alpha + \mu \le \omega t \le \alpha + \frac{\pi}{3}$ $\qquad v_0 = v_{ac}$

$$\therefore \qquad V_0 = \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\mu} \frac{3}{2} v_a \, d\omega t + \int_{\alpha+\mu}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\mu} \left(v_{ac} + \frac{3}{2}v_a - v_{ac}\right) + \int_{\alpha+\mu}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t\right]$$

$$= \frac{3}{\pi}\left[\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_{ac} \, d\omega t + \int_{\alpha}^{\alpha+\mu} \left(\frac{v_a}{2} + v_c\right) d\omega t\right]$$

$$= \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3}{2\pi}\int_{\alpha}^{\alpha+\mu} v_{bc} \, d\omega t$$

or $$V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3\sqrt{2}V_L}{2\pi}\int_{\alpha}^{\alpha+\mu} \sin\omega t \, d\omega t$$

$$= \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3\sqrt{2}V_L}{2\pi}\left[\cos\alpha - \cos(\alpha + \mu)\right]$$

$$V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha - \frac{3}{\pi}\omega L \, I_0$$

Introduction to dual converters

Dual converter, the name itself says two converters. It is really an electronic converter or circuit which comprises of two converters. One will perform as rectifier and the other will perform as inverter. Therefore, we can say that double processes will occur at a moment. Here, two full converters are arranged in anti-parallel pattern and linked to the same dc load. These converters can provide four quadrant operations. The basic block diagram is shown below

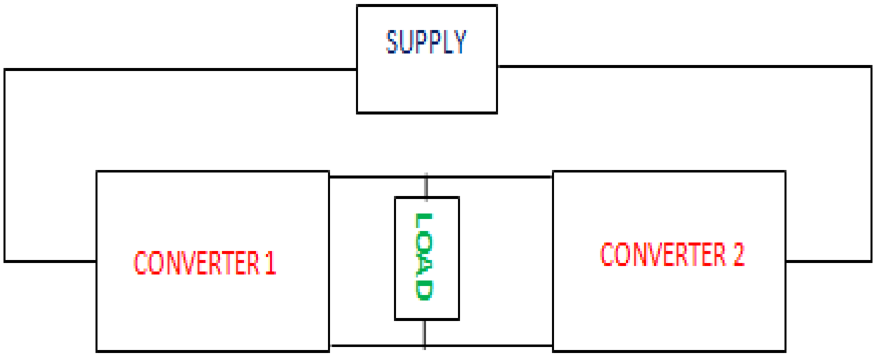

Figure: 2.28 Block diagram of dual converter

Modes of Operation of Dual Converter

There are two functional modes: Non-circulating current mode and circulating mode.

Non Circulating Current Mode

- One converter will perform at a time. So there is no circulating current between the converters.
- During the converter 1 operation, firing angle (α_1) will be $0<\alpha_1< 90^o$; V_{dc} and I_{dc} are positive.
- During the converter 2 operation, firing angle (α_2) will be $0<\alpha_2< 90^o$; V_{dc} and I_{dc} are negative.

Circulating Current Mode

- Two converters will be in the ON condition at the same time. So circulating current is present.
- The firing angles are adjusted such that firing angle of converter 1 (α_1) + firing angle of converter 2 (α_2) = 180^o.
- Converter 1 performs as a controlled rectifier when firing angle be $0<\alpha_1< 90^o$ and Converter 2 performs as an inverter when the firing angle be $90^o<\alpha_2< 180^o$. In this condition, V_{dc} and I_{dc} are positive.
  - Converter 1 performs as an inverter when firing angle be $90^o<\alpha_1< 180^o$ and Converter 2 performs as a controlled rectifier when the firing angle be $0<\alpha_2< 90^o$ In this condition, V_{dc} and I_{dc} are negative.
  - The four quadrant operation is shown below

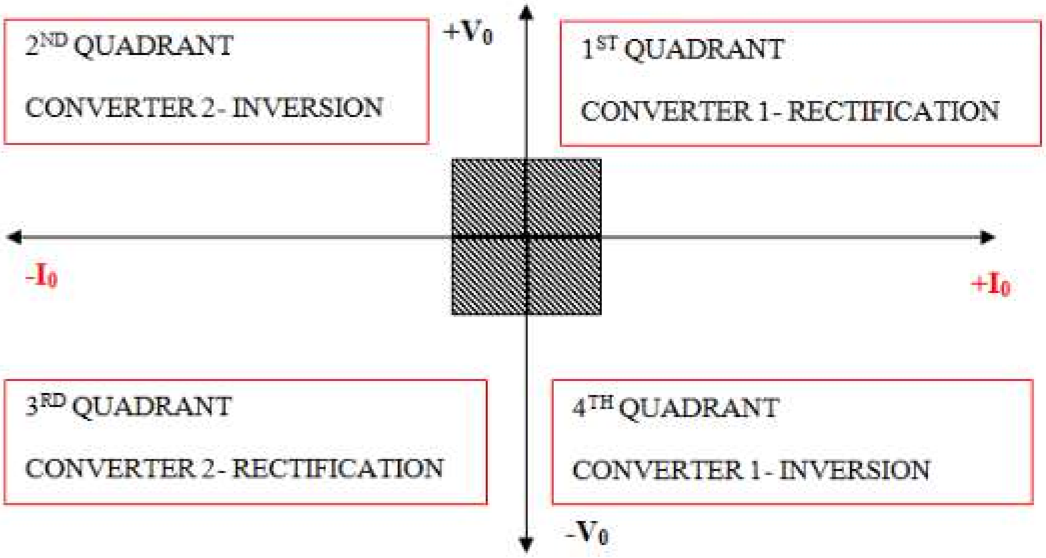

Figure: 2.29 Four quadrant operations of dual converter

Ideal Dual Converter

The term 'ideal' refers to the ripple free output voltage. For the purpose of unidirectional flow of DC current, two diodes (D_1 and D_2) are incorporated between the converters. However, the direction of current can be in any way. The average output voltage of the converter 1 is V_{01} and converter 2 is V_{02}. To make the output voltage of the two converters in same polarity and magnitude, the firing angles of the Thyristors have to be controlled.

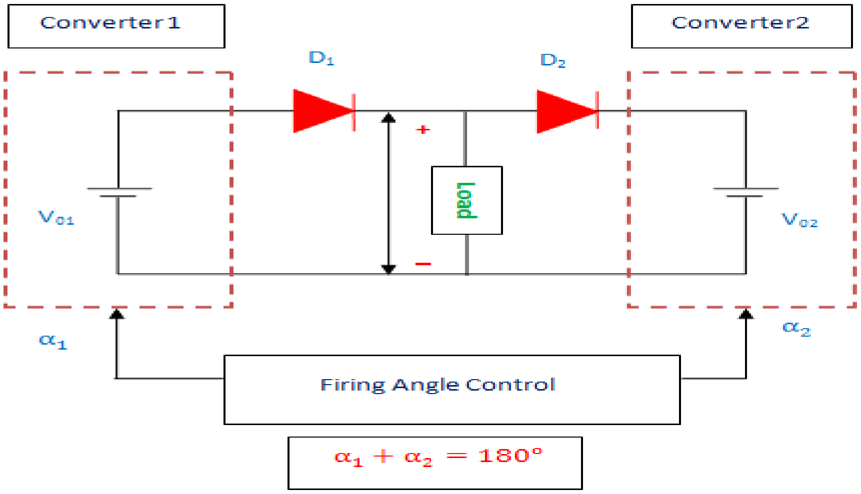

Figure: 2.30 Ideal dual converter

Single Phase Dual Converter

The source of this type of converter will be single-phase supply. Consider, the converter is in non-circulating mode of operation. The input is given to the converter 1 which converts the AC to DC by the method of rectification. It is then given to the load after filtering. Then, this DC is provided to the converter 2 as input. This converter performs as inverter and converts this DC to AC. Thus, we get AC as output. The circuit diagram is shown below.

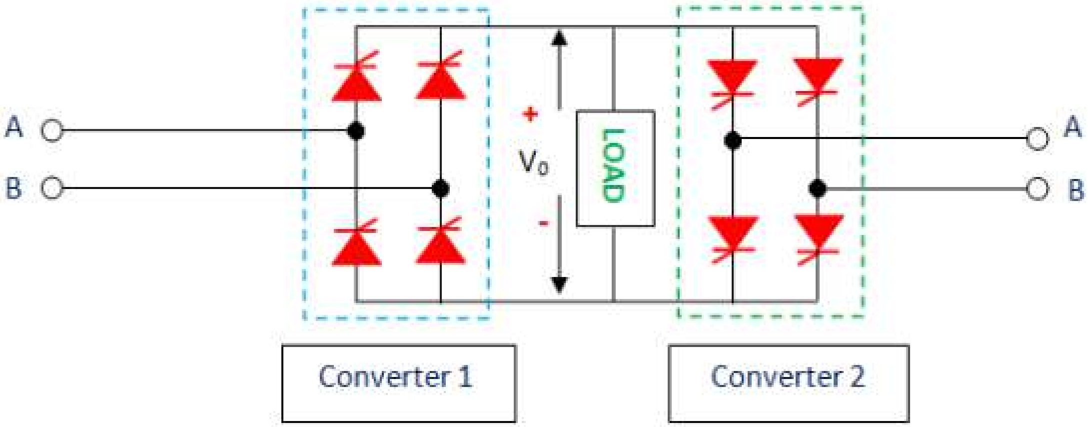

Figure: 2.31 Single phase Dual converter

Average output voltage of Single-phase converter= $\frac{2V_m Cos\ \alpha}{\pi}$

Average output voltage of Three-phase converter = $\frac{3V_{ml} Cos\ \alpha}{\pi}$

For converter 1, the average output voltage, $V_{01} = V_{max} Cos\alpha_1$

For converter 2, the average output voltage, $V_{02} = V_{max} Cos\alpha_2$

$$\begin{aligned} &V_0 = V_{01} = -V_{02} \\ &V_{max} Cos\alpha_1 = -V_{max} Cos\alpha_2 \\ &Cos\alpha_1 = Cos(180^o - \alpha_2)\ or\ Cos\alpha_2 = Cos(180^o + \alpha_2) \end{aligned}$$

Output voltage, $\alpha_1 + \alpha_2 = 180^o\ And\ \alpha_1 - \alpha_2 = 180^o$

The firing angle can never be greater than 180°. So, $\alpha_1 + \alpha_2 = 180^o$

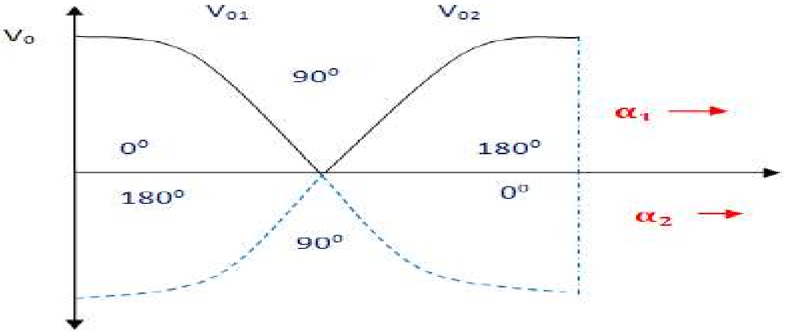

Figure: 2.32 output voltage variation with firing angle

Three Phase Dual Converter

Here, three-phase rectifier and three-phase inverter are used. The processes are similar to single-phase dual converter. The three-phase rectifier will do the conversion of the three-phase AC supply to the DC. This DC is filtered and given to the input of the second converter. It will do the DC to AC conversion and the output that we get is the three-phase AC. Applications where the output is up to 2 megawatts. The circuit is shown below.

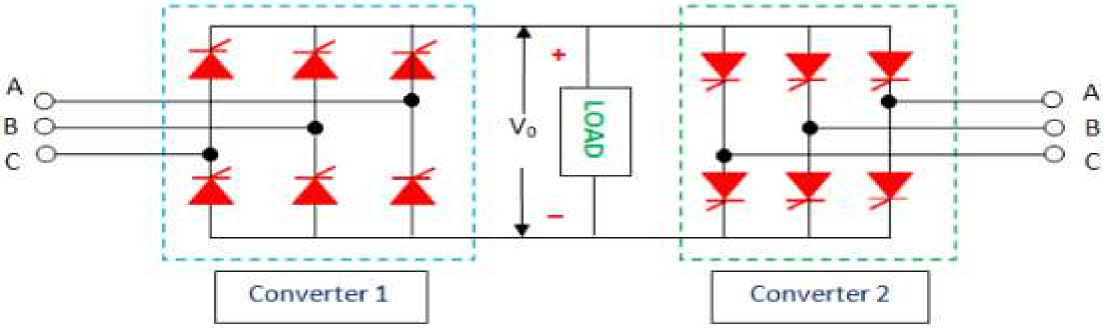

Figure: 2.33 Three phase dual converter

Application of Dual Converter

- Direction and Speed control of DC motors.
- Applicable wherever the reversible DC is required.
- Industrial variable speed DC drives.