UNIT-5

Module

MECHANICAL DESIGN OF TRANSMISSION LINES

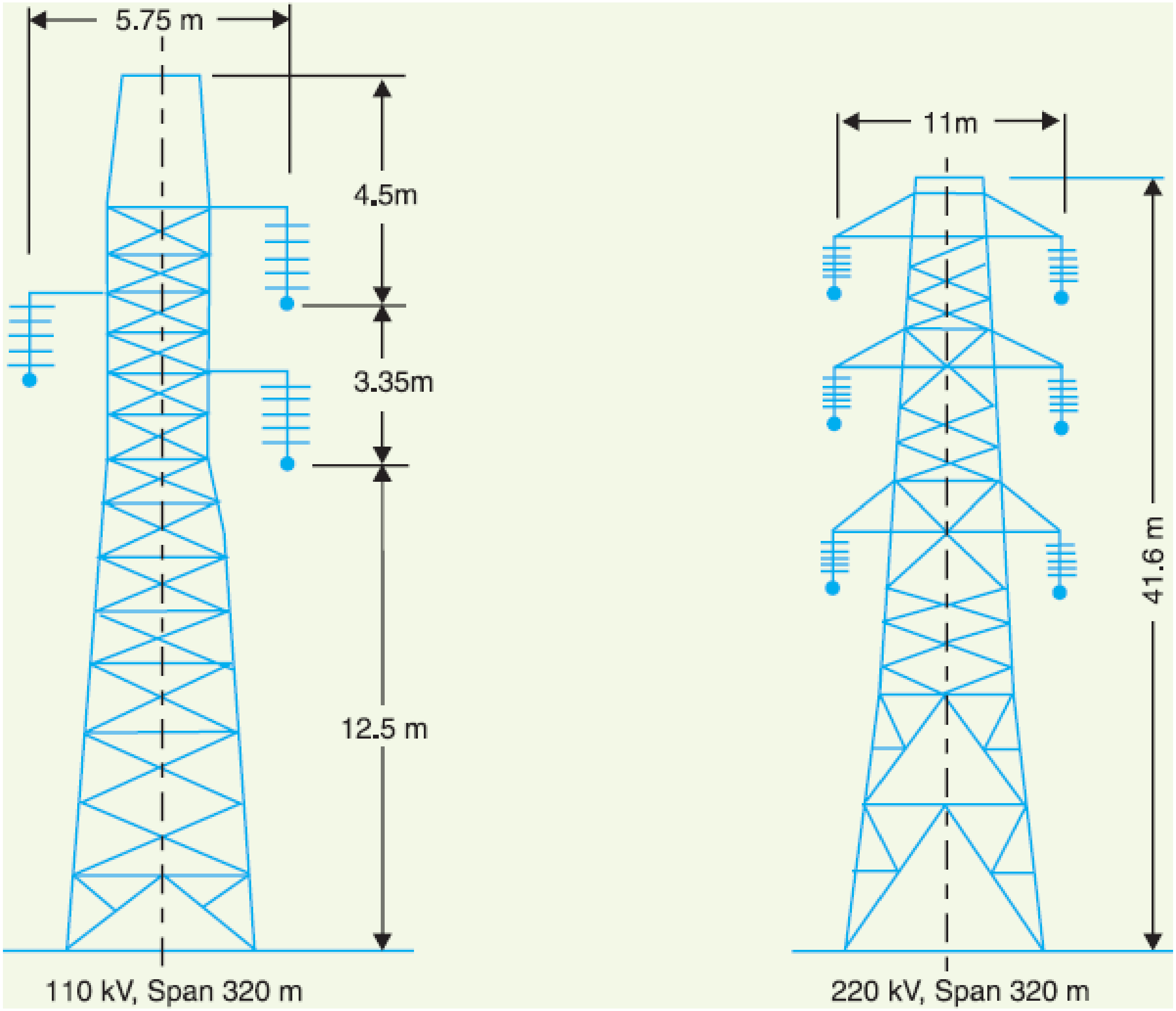
Module: MECHANICAL DESIGN OF TRANSMISSION LINES [12 Periods]

Overhead Line Insulators: Types of Insulators, String Efficiency and Methods for Improvement, Capacitance Grading and Static Shielding.
Corona: Corona Phenomenon, Factors Affecting Corona, Critical Voltages and Power Loss, Radio Interference.

Sag and Tension Calculations: Sag and Tension Calculations with Equal and Unequal Heights of Towers, Effect of Wind and Ice on Weight of Conductor, Stringing Chart and Sag Template and Its Applications, Numerical Problems.

Insulators

The overhead line conductors should be supported on the poles or towers in such a way that currents from conductors do not flow to earth through supports *i.e.*, line conductors must be properly insulated from supports. This is achieved by securing line conductors to supports with the help of *insulators*. The insulators provide necessary insulation between line conductors and supports and thus prevent any leakage current from conductors to earth. In general, the insulators should have the following desirable properties :



(i) High mechanical strength in order to withstand conductor load, wind load etc.

(ii) High electrical resistance of insulator material in order to avoid leakage currents to earth.

(iii) High relative permittivity of insulator material in order that dielectric strength is high.

(iv) The insulator material should be non-porous, free from impurities and cracks otherwise the permittivity will be lowered.

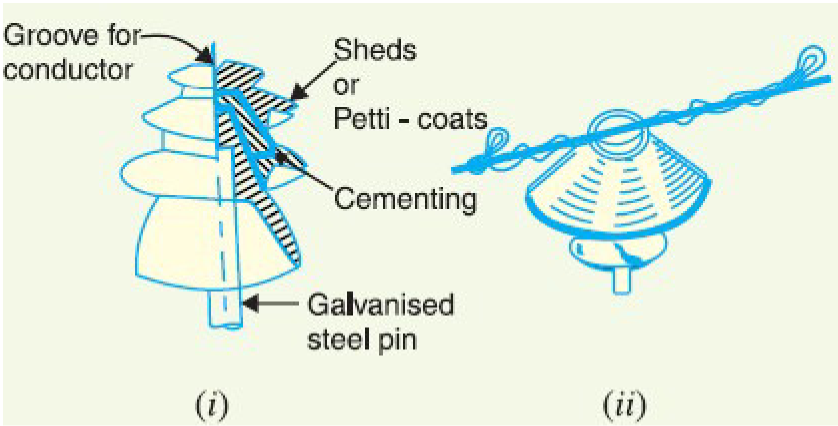
(v) High ratio of puncture strength to flashover.

The most commonly used material for insulators of overhead line is *porcelain* but glass, steatite and special composition materials are also used to a limited extent. Porcelain is produced by firing at a high temperature a mixture of kaolin, feldspar and quartz. It is stronger mechanically than glass, gives less trouble from leakage and is less effected by changes of temperature.

Types of Insulators

The successful operation of an overhead line depends to a considerable extent upon the proper selection of insulators. There are several types of insulators but the most commonly used are pin type, suspension type, strain insulator and shackle insulator.

1. Pin type insulators. The part section of a pin type insulator is shown in Fig. As the name suggests, the pin type insulator is secured to the cross-arm on the pole. There is a groove on the upper end of the insulator for housing the conductor. The conductor passes through this groove and is bound by the annealed wire of the same material as the conductor Pin type insulators are used for transmission and distribution of electric power at voltages upto 33 kV. Beyond operating voltage of 33 kV, the pin type insulators become too bulky and hence uneconomical.



Causes of insulator failure. Insulators are required to withstand both mechanical and electrical stresses. The latter type is pirmarily due to line voltage and may cause the breakdown of the insulator. The electrical breakdown of the insulator can occur either by *flash-over* or *puncture.* In flashover, an arc occurs between the line conductor and insulator pin (*i.e.*, earth) and the discharge jumps across the *air gaps, following shortest distance.

Fig. shows the arcing distance (*i.e.* $a + b + c$) for the insulator. In case of flash-over, the insulator will continue to act in its proper capacity unless extreme heat produced by the arc destroys the insulator. In case of puncture, the discharge occurs from conductor to pin through the body of the insulator. When such breakdown is involved, the insulator is permanently destroyed due to excessive heat. In practice, sufficient thickness of porcelain is provided in the insulator to avoid puncture by the line voltage. The ratio of puncture strength to flashover voltage is known as safety factor *i.e.,*

$$\text{Safety factor of insulator} = \frac{\text{Puncture strength}}{\text{Flash-over voltage}}$$

It is desirable that the value of safety factor is high so that flash-over takes place before the insulator gets punctured. For pin type insulators, the value of safety factor is about 10.

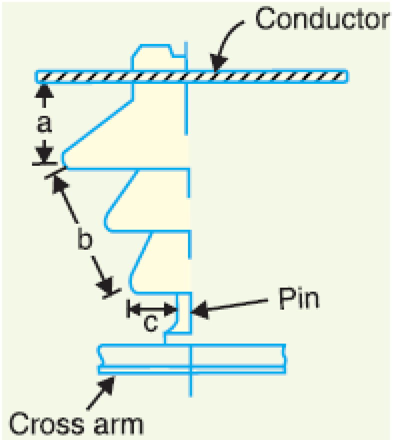
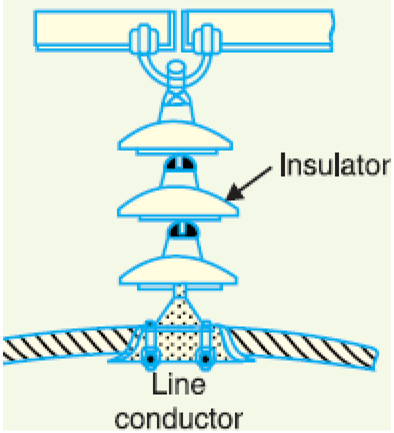


Fig. 8.6



2 Suspension type insulators. The cost of pin type insulator increases rapidly as the working voltage is increased. Therefore, this type of insulator is not economical beyond 33 kV. For high voltages (>33 kV), it is a usual practice to use suspension type insulators shown in Fig. . They consist of a number of porcelain discs connected in series by metal links in the form of a string. The conductor is suspended at the bottom end of this string while the other end of the string is secured to the cross-arm of the tower. Each unit or disc is designed for low voltage, say 11 kV. The number of discs in series would obviously depend upon the working voltage. For instance, if the working voltage is 66 kV, then six discs in series will be provided on the string.

Advantages

(i) Suspension type insulators are cheaper than pin type insulators for voltages beyond 33 kV.

(ii) Each unit or disc of suspension type insulator is designed for low voltage,usually 11 kV.

Depending upon the working voltage, the desired number of discs can be connected in series.

(iii) If any one disc is damaged, the whole string does not become useless because the damaged disc can be replaced by the sound one.

(iv) The suspension arrangement provides greater flexibility to the line. The connection at the cross arm is such that insulator string is free to swing in any direction and can take up the position where mechanical stresses are minimum.

(v) In case of increased demand on the transmission line, it is found more satisfactory to supply the greater demand by raising the line voltage than to provide another set of conductors. The additional insulation required for the raised voltage can be easily obtained in the suspension

arrangement by adding the desired number of discs.

(vi) The suspension type insulators are generally used with steel towers. As the conductors run below the earthed cross-arm of the tower, therefore, this arrangement provides partial protection from lightning.

3. Strain insulators. When there is a dead end of the line or there is corner or sharp curve, the line is subjected to greater tension. In order to relieve the line of excessive tension, strain insulators are used. For low voltage lines (< 11 kV), shackle insulators are used as strain insulators. However,

for high voltage transmission lines, strain insulator consists of an assembly of suspension insulators as shown in Fig. The discs of strain insulators are used in the vertical plane. When the tension in lines is exceedingly high, as at long river spans, two or more strings are used in parallel.

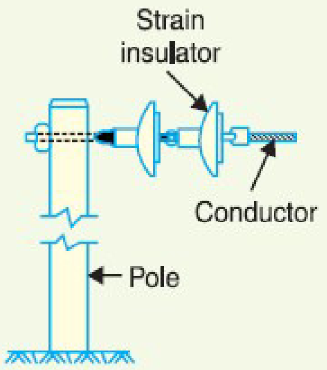


Fig. 8.8. Strain insulator.

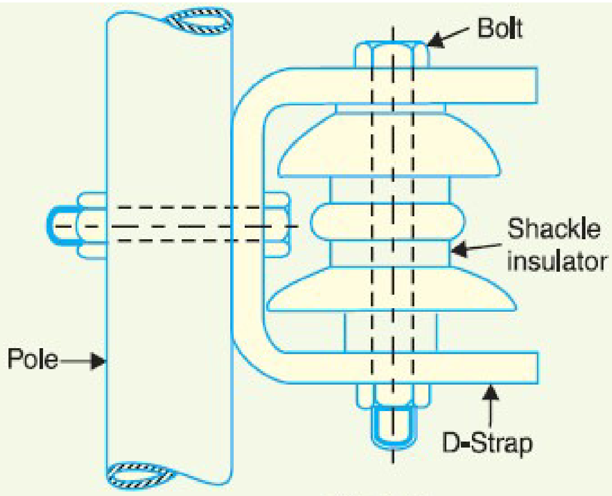


Fig. 8.9

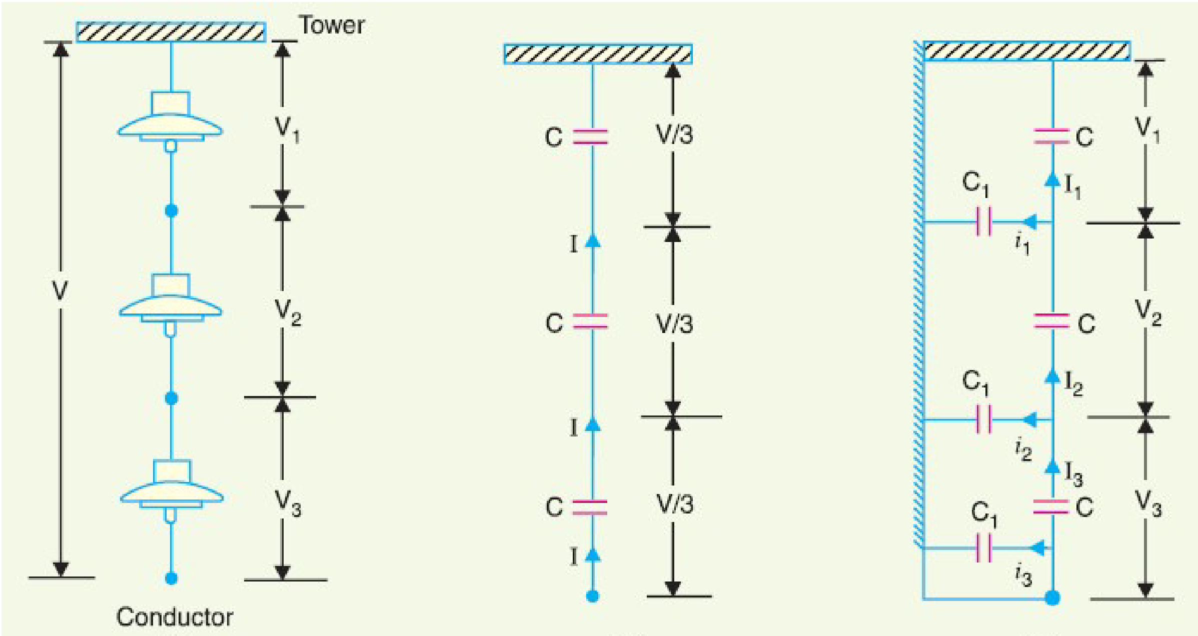
4. Shackle insulators. In early days, the shackle insulators were used as strain insulators. But now a days, they are frequently used for low voltage distribution lines. Such insulators can be used either in a horizontal position or in a vertical position. They can be directly fixed to the pole with a bolt or to the cross arm. Fig. shows a shackle insulator fixed to the pole. The conductor in the groove is fixed with a soft binding wire.

Potential Distribution over Suspension Insulator String

A string of suspension insulators consists of a number of porcelain discs connected in series through metallic links. Fig. (*i*) shows 3-disc string of suspension insulators. The porcelain portion of each disc is inbetween two metal links. Therefore, each disc forms acapacitor C as shown in Fig. (*ii*). This is known as *mutual capacitance* or *self-capacitance*.

If there were mutual capacitance alone, then charging current would have been the same through all the discs and consequently voltage across each unit would have been the same *i.e.,* $V/3$ as shown in Fig. (*ii*). However, in actual practice, capacitance also exists between metal fitting of each disc and tower or earth. This is known as *shunt capacitance* $C1$. Due to shunt capacitance, charging current is not the same through all the discs of the string [See Fig. (*iii*)].

Therefore, voltage across each disc will be different. Obviously, the disc nearest to the line conductor will have the maximum voltage. Thus referring to Fig.(*iii*), $V3$ will be much more than $V2$or$V1$.



(*i*) (*ii*) (*iii*)

The following points may be noted regarding the potential distribution over a string of suspension insulators :

(i) The voltage impressed on a string of suspension insulators does not distribute itself uniformly across the individual discs due to the presence of shunt capacitance.

(ii) The disc nearest to the conductor has maximum voltage across it. As we move towards the cross-arm, the voltage across each disc goes on decreasing.

(iii) The unit nearest to the conductor is under maximum electrical stress and is likely to be punctured. Therefore, means must be provided to equalise the potential across each unit. This is fully discussed in Art. 8.8.

(iv) If the voltage impressed across the string were d.c., then voltage across each unit would be the same. It is because insulator capacitances are ineffective for d.c.

String Efficiency

As stated above, the voltage applied across the string of suspension insulators is not uniformly distributed across various units or discs. The disc nearest to the conductor has much higher potential than the other discs. This unequal potential distribution is undesirable and is usually expressed in terms of string efficiency.

The ratio of voltage across the whole string to the product of number of discs and the voltage across the disc nearest to the conductor is known as **string efficiency** *i.e.,*

$$\text{String efficiency} = \frac{\text{Voltage across the string}}{n \times \text{Voltage across disc nearest to conductor}}$$

where n = number of discs in the string.

String efficiency is an important consideration since it decides the potential distribution along the string. The greater the string efficiency, the more uniform is the voltage distribution. Thus 100% string efficiency is an ideal case for which the volatge across each disc will be exactly the same. Although it is impossible to achieve 100% string efficiency, yet efforts should be made to improve it as close to this value as possible.

Mathematical expression. Fig. 8.11 shows the equivalent circuit for a 3-disc string. Let us suppose that self capacitance of each disc is C. Let us further assume that shunt capacitance C_1 is some fraction K of self-capacitance *i.e.*, $C_1 = KC$. Starting from the cross-arm or tower, the voltage across each unit is V_1, V_2 and V_3 respectively as shown.

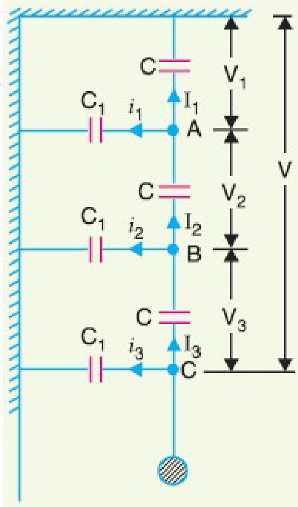


Fig. 8.11

Applying Kirchhoff's current law to node A, we get,

$$I_2 = I_1 + i_1$$

or $$V_2 \omega C^* = V_1 \omega C + V_1 \omega C_1$$

or $$V_2 \omega C = V_1 \omega C + V_1 \omega K C$$

∴ $$V_2 = V_1 (1 + K) \quad ...(i)$$

Applying Kirchhoff's current law to node B, we get,

$$I_3 = I_2 + i_2$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\, \omega C_1 ^\dagger$$

or $$V_3 \omega C = V_2 \omega C + (V_1 + V_2)\, \omega K C$$

or $$V_3 = V_2 + (V_1 + V_2)K$$
$$= KV_1 + V_2 (1 + K)$$
$$= KV_1 + V_1 (1 + K)^2 \quad [\because V_2 = V_1 (1 + K)]$$
$$= V_1 [K + (1 + K)^2]$$

∴ $$V_3 = V_1 [1 + 3K + K^2] \quad ...(ii)$$

Voltage between conductor and earth (*i.e.*, tower) is

$$V = V_1 + V_2 + V_3$$
$$= V_1 + V_1 (1 + K) + V_1 (1 + 3K + K^2)$$
$$= V_1 (3 + 4K + K^2)$$

∴ $$V = V_1 (1 + K)(3 + K) \quad ...(iii)$$

From expressions (*i*), (*ii*) and (*iii*), we get,

$$\frac{V_1}{1} = \frac{V_2}{1+K} = \frac{V_3}{1 + 3K + K^2} = \frac{V}{(1+K)(3+K)} \quad ...(iv)$$

∴ Voltage across top unit, $$V_1 = \frac{V}{(1+K)(3+K)}$$

Voltage across second unit from top, $$V_2 = V_1 (1 + K)$$

Voltage across third unit from top, $$V_3 = V_1 (1 + 3K + K^2)$$

$$\text{\%age String efficiency} = \frac{\text{Voltage across string}}{n \times \text{Voltage across disc nearest to conductor}} \times 100$$
$$= \frac{V}{3 \times V_3} \times 100$$

The following points may be noted from the above mathematical analysis :

(i) If $K = 0{\cdot}2$ (Say), then from exp. (*iv*), we get, $V2 = 1{\cdot}2\ V1$ and $V3 = 1{\cdot}64\ V1$. This clearly shows that disc nearest to the conductor has maximum voltage across it; the voltage across other discs decreasing progressively as the cross-arm in approached.

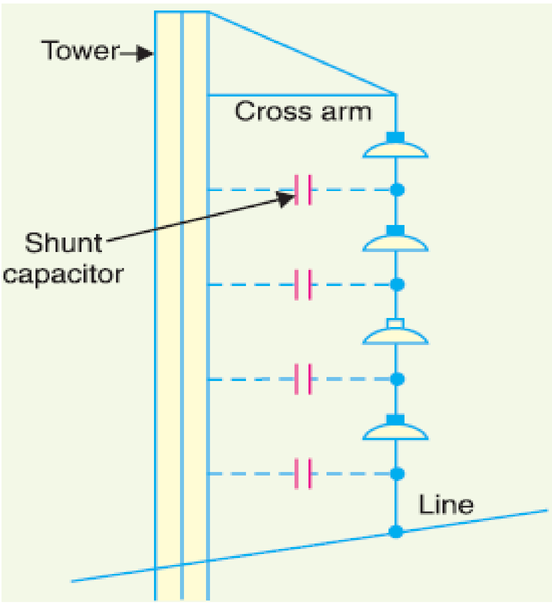
(ii) The greater the value of K (= $C1/C$), the more non-uniform is the potential across the discs and lesser is the string efficiency.

(iii) The inequality in voltage distribution increases with the increase of number of discs in the string. Therefore, shorter string has more efficiency than the larger one.

Methods of Improving String Efficiency

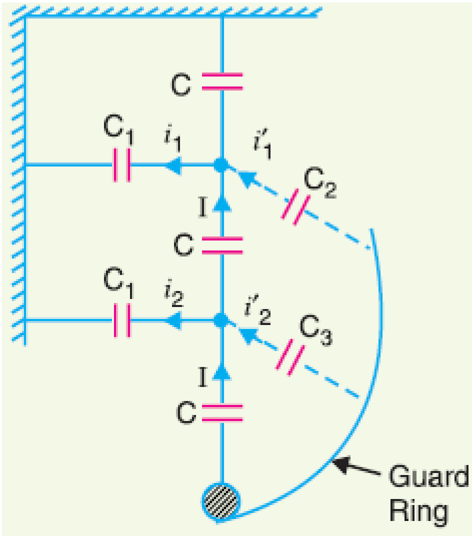
It has been seen above that potential distribution in a string of suspension insulators is not uniform. The maximum voltage appears across the insulator nearest to the line conductor and decreases progressively as the cross arm is approached. If the insulation of the highest stressed insulator (*i.e.* nearest to conductor) breaks down or flash over takes place, the breakdown of other units will take place in succession. This necessitates to equalise the potential across the various units of the string *i.e.* to improve the string efficiency. The various methods for this purpose are :

(i) *By using longer cross-arms*. The value of string efficiency depends upon the value of K *i.e.*, ratio of shunt capacitance to mutual capacitance. The lesser the value of K, the greater is the string efficiency and more uniform is the voltage distribution. The value of K can be decreased by reducing the shunt capacitance. In order to reduce shunt capacitance, the distance of conductor from tower must be increased *i.e.,* longer cross-arms should be used. However, limitations of cost and strength of tower do not allow the use of very long cross-arms. In practice, $K = 0{\cdot}1$ is the limit that can be achieved by this method.



(ii) *By grading the insulators*. In this method, insulators of different dimensions are so chosen that each has a different capacitance. The insulators are capacitance graded *i.e.* they are assembled in the string in such a way that the top unit has the minimum capacitance, increasing progressively as the bottom unit (*i.e.,* nearest to conductor) is reached. Since voltage is inversely proportional to capacitance, this method tends to equalise the potential distribution across the units in the string. This method has the disadvantage that a large number of different-sized insulators are required. However, good results can be obtained by using standard insulators for most of the string and larger units for that near to the line conductor.

(iii) *By using a guard ring*. The potential across each unit in a string can be equalised by using a guard ring which is a metal ring electrically connected to the conductor and surrounding the bottom insulator as shown in the Fig. The guard ring introduces capacitance between metal fittings and the line conductor. The guard ring is contoured in such a way that shunt capacitance currents $i1$, $i2$ etc. are equal to metal fitting line capacitance currents $i21$, $i22$ etc. The result is that same charging current I flows through each unit of string. Consequently, there will be uniform potential distribution across the units.



Important Points

While solving problems relating to string efficiency, the following points must be kept in mind:

(i) The maximum voltage appears across the disc nearest to the conductror (*i.e.,* line conductor).

(ii) The voltage across the string is equal to phase voltage *i.e.,* Voltage across string = Voltage between line and earth = Phase Voltage

(iii) Line Voltage = 3 √ Voltage across string

CORONA

Electric-power transmission practically deals in the bulk transfer of electrical energy, from generating stations situated many kilometers away from the main consumption centers or the cities. For this reason the long distance transmission cables are of utmost necessity for effective power transfer, which in-evidently results in huge losses across the system. Minimizing those has been a major challenge for power engineers of late and to do that one should have a clear understanding of the type and nature of losses. One of them being the **corona effect in power system**, which has a predominant role in reducing the efficiency of EHV(extra high voltage lines) which we are going to concentrate on, in this article.

What is corona effect in power system and why it occurs?

For corona effect to occur effectively, two factors here are of prime importance as mentioned below:-

1) Alternating potential difference must be supplied across the line.

2) The spacing of the conductors, must be large enough compared to the line diameter.

Corona Effect in Transmission Line

When an alternating current is made to flow across two conductors of the transmission line whose spacing is large compared to their diameters, then air surrounding the conductors (composed of ions) is subjected to di-electric stress. At low values of supply end voltage, nothing really occurs as the stress is too less to ionize the air outside. But when the potential difference is made to increase beyond some threshold value of around 30 kV known as the critical disruptive voltage, then the field strength increases and then the air surrounding it experiences stress high enough to be dissociated into ions making the atmosphere conducting. This results in electric discharge around the conductors due to the flow of these ions, giving rise to a faint luminescent glow, along with the hissing sound accompanied by the liberation of ozone, which is readily identified due to its characteristic odor. This phenomena of electrical discharge occurring in transmission line for high values of voltage is known as the **corona effect in power system**. If the voltage across the lines is still increased the glow becomes more and more intense along with hissing noise, inducing very high power loss into the system which must be accounted for.

Factors Affecting Corona

The phenomenon of corona is affected by the physical state of the atmosphere as well as by the conditions of the line. The following are the factors upon which corona depends :

(i) ***Atmosphere***. As corona is formed due to ionsiation of air surrounding the conductors, therefore, it is affected by the physical state of atmosphere. In the stormy weather, the number of ions is more than normal and as such corona occurs at much less voltage as compared with fair weather.

(ii) ***Conductor size.*** The corona effect depends upon the shape and conditions of the conductors. The rough and irregular surface will give rise to more corona because unevenness ofthe surface decreases the value of breakdown voltage. Thus a stranded conductor has irregular surface and hence gives rise to more corona that a solid conductor.

(iii) ***Spacing between conductors.*** If the spacing between the conductors is made very large as compared to their diameters, there may not be any corona effect. It is because larger distance between conductors reduces the electro-static stresses at the conductor surface, thus avoiding corona formation.

(iv) ***Line voltage***. The line voltage greatly affects corona. If it is low, there is no change in the condition of air surrounding the conductors and hence no corona is formed. However, if the line voltage has such a value that electrostatic stresses developed at the conductor surface make the air around the conductor conducting, then corona is formed.

Important Terms

The phenomenon of corona plays an important role in the design of an overhead transmission line. Therefore, it is profitable to consider the following terms much used in the analysis of corona effects:

(*i*) Critical disruptive voltage. *It is the minimum phase-neutral voltage at which corona occurs.* Consider two conductors of radii *r* cm and spaced *d* cm apart. If *V* is the phase-neutral potential, then potential gradient at the conductor surface is given by:

$$g = \frac{V}{r \log_e \frac{d}{r}} \text{ volts / cm}$$

In order that corona is formed, the value of *g* must be made equal to the breakdown strength of air. The breakdown strength of air at 76 cm pressure and temperature of 25ºC is 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*) and is denoted by *go*. If *Vc* is the phase-neutral potential required under these conditions, then,

$$g_o = \frac{V_c}{r \log_e \frac{d}{r}}$$

where g_o = breakdown strength of air at 76 cm of mercury and 25°C = 30 kV/cm (*max*) or 21·2 kV/cm (*r.m.s.*)

$$\therefore \text{ Critical disruptive voltage, } V_c = g_o\, r \log_e \frac{d}{r}$$

The above expression for disruptive voltage is under standard conditions *i.e.,* at 76 cm of Hg and 25ºC. However, if these conditions vary, the air density also changes, thus altering the value of *go*. The value of *go* is directly proportional to air density. Thus the breakdown strength of air at a barometric pressure of *b* cm of mercury and temperature of *t*ºC becomes *go* where

$$\delta = \text{air density factor} = \frac{3 \cdot 92b}{273 + t}$$

Under standard conditions, the value of $\delta = 1$.

$\therefore$ Critical disruptive voltage, $V_c = g_o \delta r \log_e \frac{d}{r}$

Correction must also be made for the surface condition of the conductor. This is accounted for by multiplying the above expression by irregularity factor m_o.

$\therefore$ Critical disruptive voltage, $V_c = m_o g_o \delta r \log_e \frac{d}{r}$ kV/phase

where m_o = 1 for polished conductors
= 0·98 to 0·92 for dirty conductors
= 0·87 to 0·8 for stranded conductors

(*ii*) Visual critical voltage. *It is the minimum phase-neutral voltage at which corona glow appears all along the line conductors.*

It has been seen that in case of parallel conductors, the corona glow does not begin at the disruptive voltage V_c but at a higher voltage V_v, called *visual critical voltage.* The phase-neutral effective value of visual critical voltage is given by the following empirical formula :

$$V_v = m_v g_o \delta r \left(1 + \frac{0 \cdot 3}{\sqrt{\delta r}}\right) \log_e \frac{d}{r} \text{ kV/phase}$$

where m_v is another irregularity factor having a value of 1·0 for polished conductors and 0·72 to 0·82 for rough conductors.

(*iii*) Power loss due to corona. Formation of corona is always accompanied by energy loss which is dissipated in the form of light, heat, sound and chemical action. When disruptive voltage is exceeded, the power loss due to corona is given by :

$$P = 242 \cdot 2 \left(\frac{f + 25}{\delta}\right) \sqrt{\frac{r}{d}} (V - V_c)^2 \times 10^{-5} \text{ kW / km / phase}$$

where f = supply frequency in Hz
V = phase-neutral voltage (*r.m.s.*)
V_c = disruptive voltage (*r.m.s.*) per phase

Advantages and Disadvantages of Corona

Corona has many advantages and disadvantages. In the correct design of a high voltage overhead line, a balance should be struck between the advantages and disadvantages.

Advantages

(i) Due to corona formation, the air surrounding the conductor becomes conducting and hence virtual diameter of the conductor is increased. The increased diameter reduces the electrostatic stresses between the conductors.

(ii) Corona reduces the effects of transients produced by surges.

Disadvantages

(i) Corona is accompanied by a loss of energy. This affects the transmission efficiency of the line.

(ii) Ozone is produced by corona and may cause corrosion of the conductor due to chemical action.

(iii) The current drawn by the line due to corona is non-sinusoidal and hence non-sinusoidal voltage drop occurs in the line. This may cause inductive interference with neighbouring communication lines.

Methods of Reducing Corona Effect

It has been seen that intense corona effects are observed at a working voltage of 33 kV or above. Therefore, careful design should be made to avoid corona on the sub-stations or bus-bars rated for 33 kV and higher voltages otherwise highly ionised air may cause flash-over in the insulators or between the phases, causing considerable damage to the equipment. The corona effects can be reduced by the following methods :

(i) *By increasing conductor size.* By increasing conductor size, the voltage at which corona occurs is raised and hence corona effects are considerably reduced. This is one of the reasons that *ACSR* conductors which have a larger cross-sectional area are used in transmission lines.

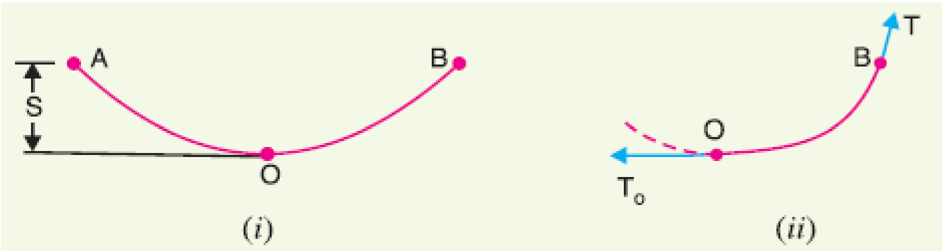
(ii) *By increasing conductor spacing.* By increasing the spacing between conductors, the voltage at which corona occurs is raised and hence corona effects can be eliminated. However, spacing cannot be increased too much otherwise the cost of supporting structure (*e.g.,* bigger cross arms and supports) may increase to a considerable extent.

Sag in Overhead Lines:

While erecting an overhead line, it is very important that conductors are under safe tension. If the conductors are too much stretched between supports in a bid to save conductor material, the stress in the conductor may reach unsafe value and in certain cases the conductor may break due to excessive tension. In order to permit safe tension in the conductors, they are not fully stretched but are allowed to have a dip or sag.

The difference in level between points of supports and the lowest point on the conductor is *called* **sag.**

Fig. shows a conductor suspended between two equilevel supports *A* and *B*. The conductoris not fully stretched but is allowed to have a dip. The lowest point on the conductor is *O* and the sag is *S*. The following points may be noted :



(i) When the conductor is suspended between two supports at the same level, it takes the shape of catenary. However, if the sag is very small compared with the span, then sag-span curve is like a parabola.

(ii) The tension at any point on the conductor acts tangentially. Thus tension *TO* at the lowest point *O* acts horizontally as shown in Fig.(*ii*).

(iii) The horizontal component of tension is constant throughout the length of the wire.

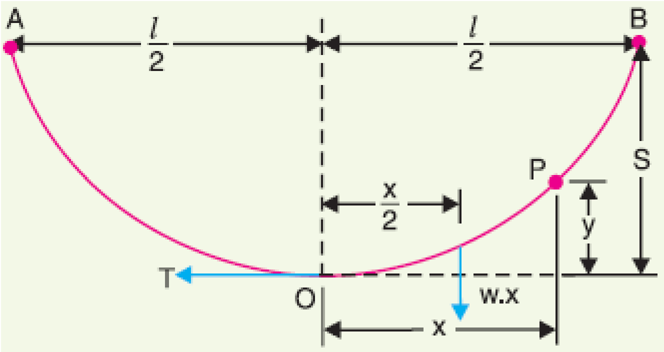
(iv) The tension at supports is approximately equal to the horizontal tension acting at any point on the wire. Thus if *T* is the tension at the support *B*, then *T* = *TO*.

Conductor sag and tension. This is an important consideration in the mechanical design of overhead lines. The conductor sag should be kept to a minimum in order to reduce the conductor material required and to avoid extra pole height for sufficient clearance above ground level. It is also desirable that tension in the conductor should be low to avoid the mechanical failure of conductor and to permit the use of less strong supports. However, low conductor tension and minimum sag are not possible. It is because low sag means a tight wire and high tension, whereas a low tension means a loose wire and increased sag. Therefore, in actual practice, a compromise in made between the two.

8.16 Calculation of Sag

In an overhead line, the sag should be so adjusted that tension in the conductors is within safe limits. The tension is governed by conductor weight, effects of wind, ice loading and temperature variations. It is a standard practice to keep conductor tension less than 50% of its ultimate tensile strength *i.e.,* minimum factor of safety in respect of conductor tension should be 2. We shall now calculate sag and tension of a conductor when (*i*) supports are at equal levels and (*ii*) supports are at unequal levels.

(i) When supports are at equal levels. Consider a conductor between two equilevel supports *A* and *B* with *O* as the lowest point as shown in Fig. It can be proved that lowest point will be at the mid-span.



Let

l = Length of span

w = Weight per unit length of conductor

T = Tension in the conductor.

Consider a point P on the conductor. Taking the lowest point O as the origin, let the co-ordinates of point P be x and y. Assuming that the curvature is so small that curved length is equal to its horizontal projection (*i.e., OP* = *x*), the two forces acting on the portion *OP* of the conductor are :

(a) The weight wx of conductor acting at a distance $x/2$ from o

(b) The tension T acting at O.

Equating the moments of above two forces about point O, we get,

$$T\,y = w\,x \times \frac{x}{2}$$

or $$y = \frac{w\,x^2}{2\,T}$$

The maximum dip (sag) is represented by the value of y at either of the supports A and B.

At support A, $x = l/2$ and $y = S$

$$\therefore \quad \text{Sag,} \quad S = \frac{w(l/2)^2}{2T} = \frac{w\,l^2}{8\,T}$$

(ii) When supports are at unequal levels. In hilly areas, we generally come across conductors suspended between supports at unequal levels. Fig. shows a conductor suspended between two supports A and B which are at different levels. The lowest point on the conductor is O.

Let

l = Span length

h = Difference in levels between two supports

$x1$ = Distance of support at lower level (*i.e., A*) from O

$x2$ = Distance of support at higher level (*i.e. B*) from O

T = Tension in the conductor

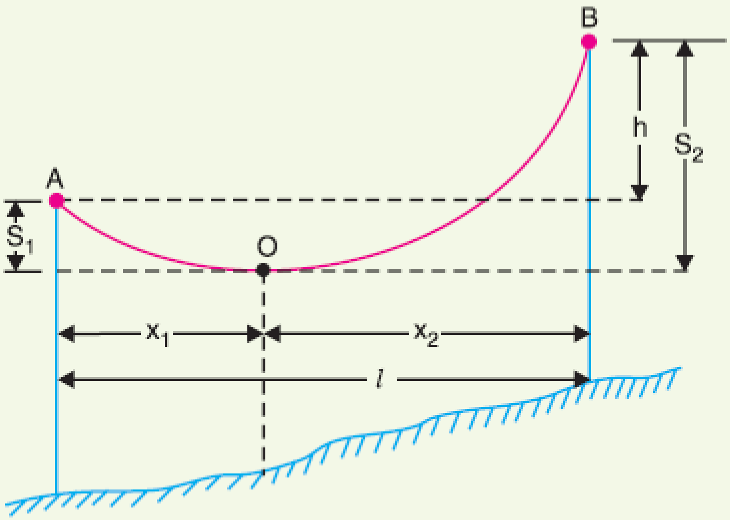


Fig. 8.25

If w is the weight per unit length of the conductor, then,

$$\text{Sag } S_1 = \frac{w\, x_1^{2*}}{2T}$$

and $$\text{Sag } S_2 = \frac{w\, x_2^{2}}{2T}$$

Also $$x_1 + x_2 = l$$

Now $$S_2 - S_1 = \frac{w}{2T}[x_2^2 - x_1^2] = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

∴ $$S_2 - S_1 = \frac{w\,l}{2T}(x_2 - x_1) \qquad [\because x_1 + x_2 = l]$$

But $$S_2 - S_1 = h$$

∴ $$h = \frac{w\,l}{2T}(x_2 - x_1)$$

or $$x_2 - x_1 = \frac{2\,Th}{w\,l} \qquad ...(ii)$$

Solving exps. (*i*) and (*ii*), we get,

$$x_1 = \frac{l}{2} - \frac{T\,h}{w\,l}$$

$$x_2 = \frac{l}{2} + \frac{T\,h}{w\,l}$$

Having found $x1$ and $x2$, values of $S1$ and $S2$ can be easily calculated.

Effect of wind and ice loading. The above formulae for sag are true only in still air and at normal temperature when the conductor is acted by its weight only. However, in actual practice, a conductor may have ice coating and simultaneously subjected to wind pressure. The weight of ice acts vertically downwards *i.e.*, in the same direction as the weight of conductor. The force due to the wind is assumed to act horizontally *i.e.*, at right angle to the projected surface of the conductor. Hence, the total force on the conductor is the vector sum of horizontal and vertical forces as shown in Fig. (*iii*).

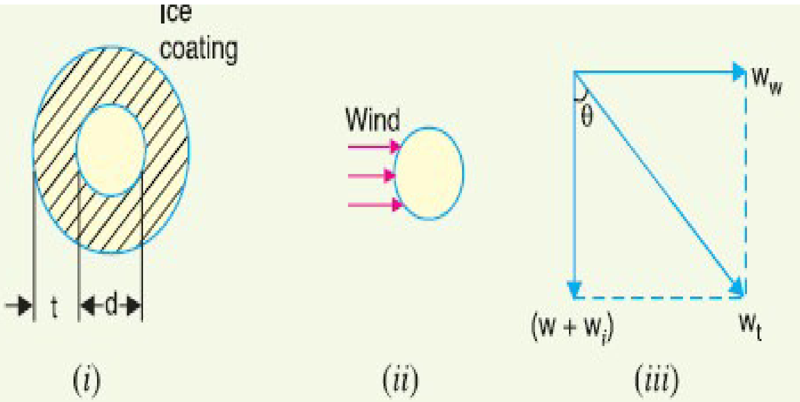


Fig. 8.26

Total weight of conductor per unit length is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

where w = weight of conductor per unit length

= conductor material density × volume per unit length

w_i = weight of ice per unit length

= density of ice × volume of ice per unit length

$= \text{density of ice} \times \frac{\pi}{4}[(d + 2t)^2 - d^2] \times 1$

$= \text{density of ice} \times \pi\, t\, (d + t)$*

w_w = wind force per unit length

= wind pressure per unit area × projected area per unit length

$= \text{wind pressure} \times [(d + 2t) \times 1]$

When the conductor has wind and ice loading also, the following points may be noted :

(*i*) The conductor sets itself in a plane at an angle θ to the vertical where

$$\tan\theta = \frac{w_w}{w + w_i}$$

(*ii*) The sag in the conductor is given by :

$$S = \frac{w_t\, l^2}{2T}$$

Hence S represents the slant sag in a direction making an angle θ to the vertical. *If no specific mention is made in the problem, then slant slag is calculated by using the above formula.*

(*iii*) The vertical sag $= S\cos\theta$

Example 8.18. *A transmission line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm². The tension in the conductor is 2000 kg. If the specific gravity of the conductor material is 9·9 gm/cm³ and wind pressure is 1·5 kg/m length, calculate the sag. What is the vertical sag?*

Solution.

Span length, l = 150 m; Working tension, T = 2000 kg

Wind force/m length of conductor, w_w = 1·5 kg

Wt. of conductor/m length, w = Sp. Gravity × Volume of 1 m conductor

$= 9{\cdot}9 \times 2 \times 100 = 1980 \text{ gm} = 1{\cdot}98 \text{ kg}$

Total weight of 1 m length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1{\cdot}98)^2 + (1{\cdot}5)^2} = 2{\cdot}48 \text{ kg}$$

$$\therefore \quad \text{Sag, } S = \frac{w_t l^2}{8T} = \frac{2{\cdot}48 \times (150)^2}{8 \times 2000} = \mathbf{3{\cdot}48 \text{ m}}$$

This is the value of slant sag in a direction making an angle θ with the vertical. Referring to Fig. 8.27, the value of θ is given by ;

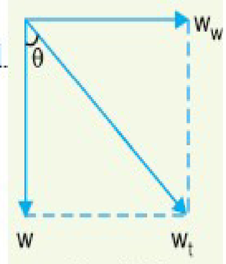


Fig. 8.27

$$\tan\theta = w_w/w = 1{\cdot}5/1{\cdot}98 = 0{\cdot}76$$

$$\therefore \quad \theta = \tan^{-1} 0{\cdot}76 = 37{\cdot}23°$$

$$\therefore \quad \text{Vertical sag} = S\cos\theta$$

$$= 3{\cdot}48 \times \cos 37{\cdot}23° = \mathbf{2{\cdot}77 \text{ m}}$$

Example 8.19. *A transmission line has a span of 200 metres between level supports. The conductor has a cross-sectional area of 1·29 cm², weighs 1170 kg/km and has a breaking stress of 4218 kg/cm². Calculate the sag for a safety factor of 5, allowing a wind pressure of 122 kg per square metre of projected area. What is the vertical sag?*

Solution.

Span length, l = 200 m

Wt. of conductor/m length, w = 1170/1000 = 1·17 kg

Working tension, *T = 4218 × 1·29/5 = 1088 kg

$$\text{Diameter of conductor, } d = \sqrt{\frac{4 \times \text{area}}{\pi}} = \sqrt{\frac{4 \times 1{\cdot}29}{\pi}} = 1{\cdot}28 \text{ cm}$$

Wind force/m length, w_w = Pressure × projected area in m²

$= (122) \times (1{\cdot}28 \times 10^{-2} \times 1) = 1{\cdot}56 \text{ kg}$

Total weight of conductor per metre length is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1{\cdot}17)^2 + (1{\cdot}56)^2} = 1{\cdot}95 \text{ kg}$$

$$\therefore \quad \text{Slant sag,} \quad S = \frac{w_t\, l^2}{8T} = \frac{1 \cdot 95 \times (200)^2}{8 \times 1088} = \mathbf{8 \cdot 96\ m}$$

The slant sag makes an angle θ with the vertical where value of θ is given by :

$$\theta = \tan^{-1}(w_w/w) = \tan^{-1}(1 \cdot 56/1 \cdot 17) = 53 \cdot 13°$$

$$\therefore \quad \text{Vertical sag} = S \cos\theta = 8 \cdot 96 \times \cos 53 \cdot 13° = \mathbf{5 \cdot 37\ m}$$

Example 8.20. *A transmission line has a span of 275 m between level supports. The conductor has an effective diameter of 1·96 cm and weighs 0·865 kg/m. Its ultimate strength is 8060 kg. If the conductor has ice coating of radial thickness 1·27 cm and is subjected to a wind pressure of 3·9 gm/cm² of projected area, calculate sag for a safety factor of 2. Weight of 1 c.c. of ice is 0·91 gm.*

Solution.

Span length, l = 275 m ; Wt. of conductor/m length, w = 0·865 kg

Conductor diameter, d = 1·96 cm ; Ice coating thickness, t = 1·27 cm

Working tension, T = 8060/2 = 4030 kg

Volume of ice per metre (*i.e.,* 100 cm) length of conductor

$$= \pi\, t\,(d + t) \times 100 \text{ cm}^3$$

$$= \pi \times 1 \cdot 27 \times (1 \cdot 96 + 1 \cdot 27) \times 100 = 1288 \text{ cm}^3$$

Weight of ice per metre length of conductor is

$$w_i = 0 \cdot 91 \times 1288 = 1172 \text{ gm} = 1 \cdot 172 \text{ kg}$$

Wind force/m length of conductor is

$$w_w = [\text{Pressure}] \times [(d + 2t) \times 100]$$

$$= [3 \cdot 9] \times (1 \cdot 96 + 2 \times 1 \cdot 27) \times 100 \text{ gm} = 1755 \text{ gm} = 1 \cdot 755 \text{ kg}$$

Total weight of conductor per metre length of conductor is

$$w_t = \sqrt{(w + w_i)^2 + (w_w)^2}$$

$$= \sqrt{(0 \cdot 865 + 1 \cdot 172)^2 + (1 \cdot 755)^2} = 2 \cdot 688 \text{ kg}$$

$$\therefore \quad \text{Sag} = \frac{w_t\, l^2}{8T} = \frac{2 \cdot 688 \times (275)^2}{8 \times 4030} = \mathbf{6 \cdot 3\ m}$$

Example 8.21. *A transmission line has a span of 214 metres between level supports. The conductors have a cross-sectional area of 3·225 cm². Calculate the factor of safety under the following conditions :*

Vertical sag = 2·35 m ; Wind pressure = 1·5 kg/m run

Breaking stress = 2540 kg/cm² ; Wt. of conductor = 1·125 kg/m run

Solution.

Here, l = 214 m ; w = 1·125 kg ; w_w = 1·5 kg

Total weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1 \cdot 125)^2 + (1 \cdot 5)^2} = 1 \cdot 875 \text{ kg}$$

If f is the factor of safety, then,

$$\text{Working tension,} \quad T = \frac{\text{Breaking stress} \times \text{conductor area}}{\text{safety factor}} = 2540 \times 3 \cdot 225/f = 8191/f \text{ kg}$$

$$\text{Slant Sag,} \quad S = \frac{\text{Vertical sag}}{{}^*\cos\theta} = \frac{2 \cdot 35 \times 1 \cdot 875}{1 \cdot 125} = 3 \cdot 92 \text{ m}$$

Now $\quad S = \dfrac{w_t\, l^2}{8T}$

or $\quad T = \dfrac{w_t\, l^2}{8\,S}$

$\therefore \quad \dfrac{8191}{f} = \dfrac{1{\cdot}875 \times (214)^2}{8 \times 3{\cdot}92}$

or Safety factor, $\quad f = \dfrac{8191 \times 8 \times 3{\cdot}92}{1{\cdot}875 \times (214)^2} = \mathbf{3}$

Example 8.22. *An overhead line has a span of 150 m between level supports. The conductor has a cross-sectional area of 2 cm^2. The ultimate strength is 5000 kg/cm^2 and safety factor is 5. The specific gravity of the material is 8·9 gm/cc. The wind pressure is 1·5 kg/m. Calculate the height of the conductor above the ground level at which it should be supported if a minimum clearance of 7 m is to be left between the ground and the conductor.*

Solution.

Span length, $l = 150$ m ; $\quad$ Wind force/m run, $w_w = 1{\cdot}5$ kg

Wt. of conductor/m run, $\quad w$ = conductor area × 100 cm × sp. gravity

$= 2 \times 100 \times 8{\cdot}9 = 1780$ gm $= 1{\cdot}78$ kg

Working tension, $\quad T = 5000 \times 2/5 = 2000$ kg

Total weight of one metre length of conductor is

$$w_t = \sqrt{w^2 + w_w^2} = \sqrt{(1{\cdot}78)^2 + (1{\cdot}5)^2} = 2{\cdot}33 \text{ kg}$$

$$\text{Slant sag, } S = \frac{w_t\, l^2}{8T} = \frac{2{\cdot}33 \times (150)^2}{8 \times 2000} = 3{\cdot}28 \text{ m}$$

$$\text{Vertical sag} = S\cos\theta = 3{\cdot}28 \times w/w_t = 3{\cdot}28 \times 1{\cdot}78/2{\cdot}33 = 2{\cdot}5 \text{ m}$$

Conductor should be supported at a height of 7 + 2·5 = **9·5 m**

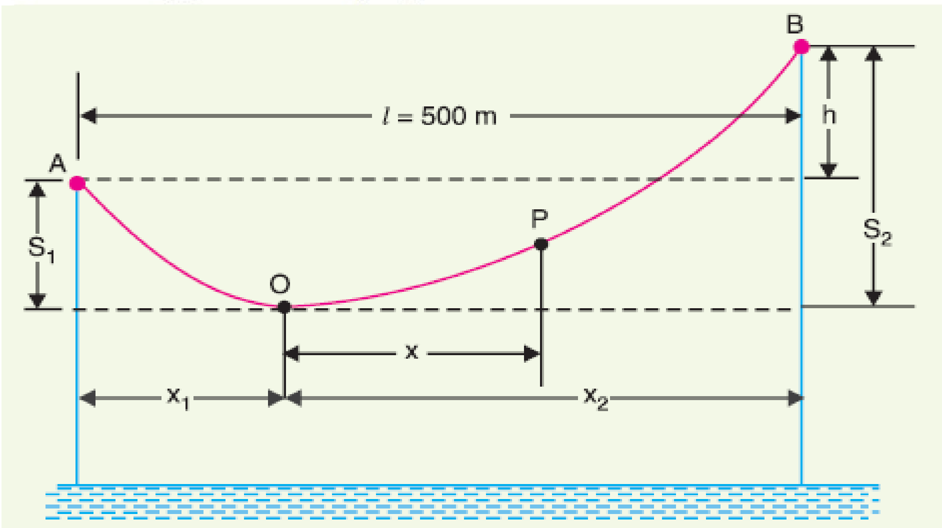
Example 8.23. *The towers of height 30 m and 90 m respectively support a transmission line conductor at water crossing. The horizontal distance betwen the towers is 500 m. If the tension in the conductor is 1600 kg, find the minimum clearance of the conductor and water and clearance mid-way between the supports. Weight of conductor is 1·5 kg/m. Bases of the towers can be considered to be at water level.*

Solution. Fig. 8.28 shows the conductor suspended between two supports *A* and *B* at different levels with *O* as the lowest point on the conductor.

Here, $l = 500$ m ; $\quad w = 1{\cdot}5$ kg ; $T = 1600$ kg.

Difference in levels between supports, $h = 90 - 30 = 60$ m. Let the lowest point *O* of the conductor be at a distance x_1 from the support at lower level (*i.e.*, support *A*) and at a distance x_2 from the support at higher level (*i.e.*, support *B*).

Obviously, $\quad x_1 + x_2 = 500 \text{ m} \qquad \ldots(i)$



Now $\qquad$ Sag $S_1 = \dfrac{w\,x_1^2}{2T}$ and Sag $S_2 = \dfrac{w\,x_2^2}{2T}$

$\therefore \qquad h = S_2 - S_1 = \dfrac{w\,x_2^2}{2T} - \dfrac{w\,x_1^2}{2T}$

or $\qquad 60 = \dfrac{w}{2T}(x_2 + x_1)(x_2 - x_1)$

$\therefore \qquad x_2 - x_1 = \dfrac{60 \times 2 \times 1600}{1 \cdot 5 \times 500} = 256 \text{ m}$ ...(*ii*)

Solving exps. (*i*) and (*ii*), we get, $x_1 = 122$ m; $x_2 = 378$ m

Now, $\qquad S_1 = \dfrac{w\,x_1^2}{2T} = \dfrac{1 \cdot 5 \times (122)^2}{2 \times 1600} = 7 \text{ m}$

Clearance of the lowest point O from water level

$= 30 - 7 =$ **23 m**

Let the mid-point P be at a distance x from the lowest point O.

Clearly, $\qquad x = 250 - x_1 = 250 - 122 = 128$ m

Sag at mid-point P, $\qquad S_{mid} = \dfrac{w\,x^2}{2T} = \dfrac{1 \cdot 5 \times (128)^2}{2 \times 1600} = 7 \cdot 68 \text{ m}$

Clearance of mid-point P from water level

$= 23 + 7{\cdot}68 =$ **30·68 m**

Example 8.24. *An overhead transmission line conductor having a parabolic configuration weighs 1·925 kg per metre of length. The area of X-section of the conductor is 2·2 cm² and the ultimate strength is 8000 kg/cm². The supports are 600 m apart having 15 m difference of levels. Calculate the sag from the taller of the two supports which must be allowed so that the factor of safety shall be 5. Assume that ice load is 1 kg per metre run and there is no wind pressure.*

Solution. Fig. 8.29. shows the conductor suspended between two supports at A and B at different levels with O as the lowest point on the conductor.

Here, $\qquad l = 600$ m ; $w_i = 1$ kg ; $h = 15$m

$w = 1{\cdot}925$ kg ; $T = 8000 \times 2{\cdot}2/5 = 3520$ kg

Total weight of 1 m length of conductor is

$$w_t = w + w_i = 1{\cdot}925 + 1 = 2{\cdot}925 \text{ kg}$$

Let the lowest point O of the conductor be at a distance x_1 from the support at lower level (*i.e.*, A) and at a distance x_2 from the support at higher level (*i.e.*, B).

Clearly, $\qquad x_1 + x_2 = 600$ m ...(*i*)

Now, $\qquad h = S_2 - S_1 = \dfrac{w_t\,x_2^2}{2T} - \dfrac{w_t\,x_1^2}{2T}$

or $\qquad 15 = \dfrac{w_t}{2T}(x_2 + x_1)(x_2 - x_1)$

$\therefore \qquad x_2 - x_1 = \dfrac{2 \times 15 \times 3520}{2 \cdot 925 \times 600} = 60 \text{ m}$ ...(*ii*)

Solving exps. (*i*) and (*ii*), we have, $x_1 = 270$ m and $x_2 = 330$ m

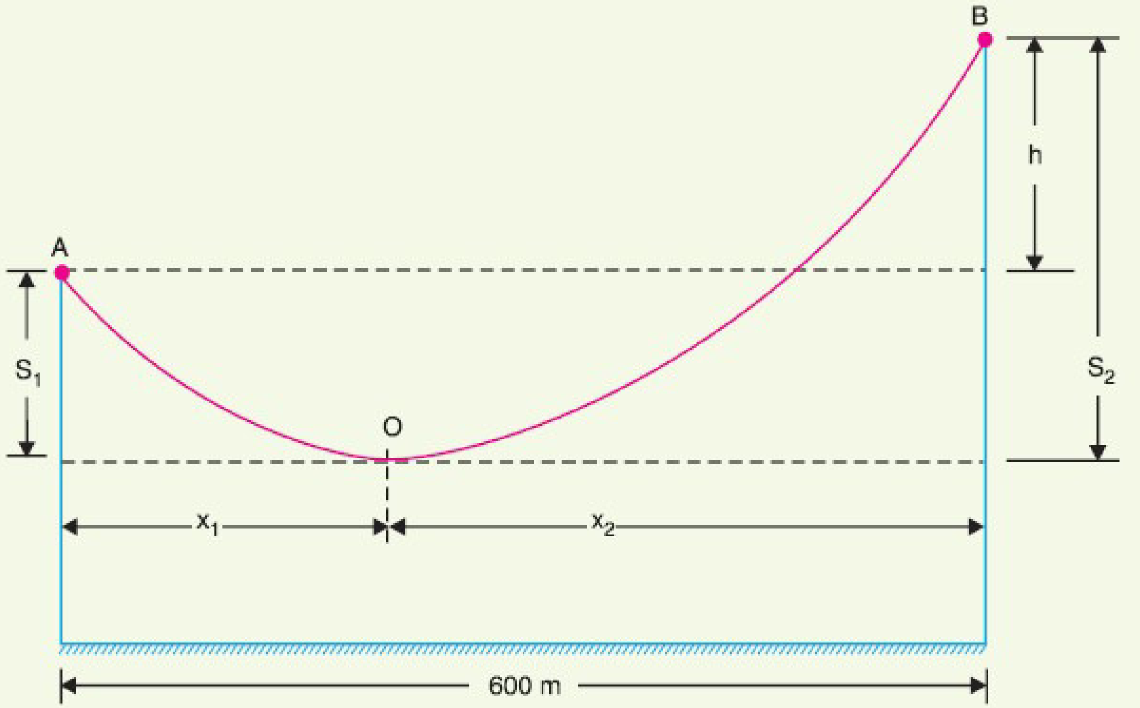


Fig. 8.29

Sag from the taller of the two towers is

$$S_2 = \frac{w_t x_2^2}{2T} = \frac{2 \cdot 925 \times (330)^2}{2 \times 3520} = \mathbf{45 \cdot 24\ m}$$

Example 8.25. *An overhead transmission line at a river crossing is supported from two towers at heights of 40 m and 90 m above water level, the horizontal distance between the towers being 400 m. If the maximum allowable tension is 2000 kg, find the clearance between the conductor and water at a point mid-way between the towers. Weight of conductor is 1 kg/m.*

Solution. Fig. 8.30 shows the whole arrangement.

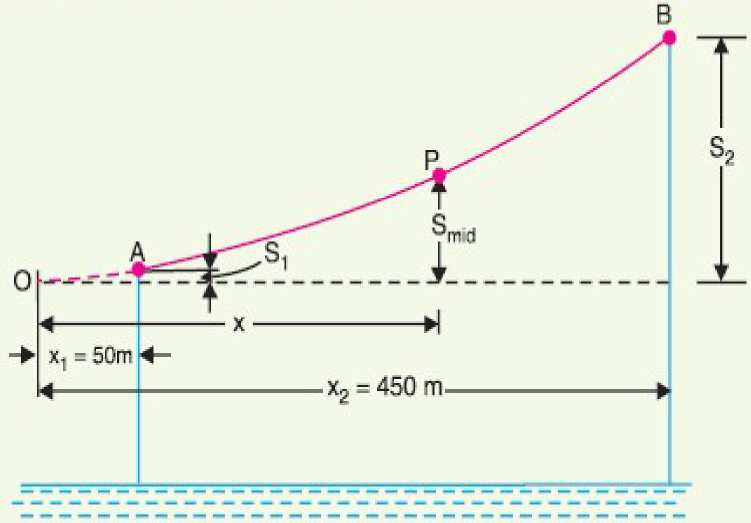


Fig. 8.30

Here, $h = 90 - 40 = 50$ m; $l = 400$ m

$T = 2000$ kg; $w = 1$ kg/m

Obviously, $x_1 + x_2 = 400$ m ...(*i*)

Now $$h = S_2 - S_1 = \frac{wx_2^2}{2T} - \frac{wx_1^2}{2T}$$

or $$50 = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

∴ $$x_2 - x_1 = \frac{50 \times 2 \times 2000}{400} = 500 \text{ m} \quad ...(ii)$$

Solving exps. (*i*) and (*ii*), we get, $x_2 = 450$ m and $x_1 = -50$ m

Now x_2 is the distance of higher support *B* from the lowest point *O* on the conductor, whereas x_1 is that of lower support *A*. As the span is 400 m, therefore, point *A* lies on the same side of *O* as *B* (see Fig. 8.30).

Horizontal distance of mid-point *P* from lowest point *O* is

$$x = \text{Distance of } A \text{ from } O + 400/2 = 50 + 200 = 250 \text{ m}$$

∴ Sag at point *P*, $$S_{mid} = \frac{w\,x^2}{2T} = \frac{1 \times (250)^2}{2 \times 2000} = 15{\cdot}6 \text{ m}$$

Now Sag $$S_2 = \frac{w\,x_2^2}{2T} = \frac{1 \times (450)^2}{2 \times 2000} = 50{\cdot}6 \text{ m}$$

Height of point *B* above mid-point *P*

$$= S_2 - S_{mid} = 50{\cdot}6 - 15{\cdot}6 = 35 \text{ m}$$

∴ Clearance of mid-point *P* above water level

$$= 90 - 35 = \mathbf{55 \text{ m}}$$

Example 8.26. *A transmission line over a hillside where the gradient is 1 : 20, is supported by two 22 m high towers with a distance of 300 m between them. The lowest conductor is fixed 2 m below the top of each tower. Find the clearance of the conductor from the ground. Given that conductor weighs 1 kg/m and the allowable tension is 1500 kg.*

Solution. The conductors are supported between towers *AD* and *BE* over a hillside having gradient of 1 : 20 as shown in Fig. 8.31. The lowest point on the conductor is *O* and sin $\theta = 1/20$.

Effective height of each tower (*AD* or *BE*)

$$= 22 - 2 = 20 \text{ m}$$

Vertical distance between towers is

$$h = EC = DE \sin\theta = 300 \times 1/20 = 15 \text{ m}$$

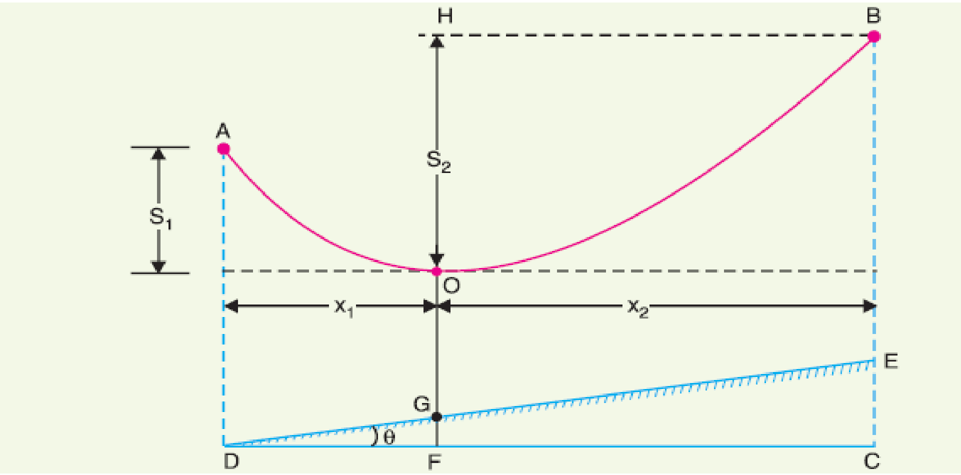
Horizontal distance between two towers is

$$DC = \sqrt{DE^2 - EC^2} = \sqrt{(300)^2 - (15)^2} \simeq 300 \text{ m}$$

or $$x_1 + x_2 = 300 \text{ m} \quad ...(i)$$

Now $$h = \frac{w\,x_2^2}{2T} - \frac{w\,x_1^2}{2T} = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

or $$x_2 - x_1 = \frac{2T\,h}{w\,(x_2 + x_1)} = \frac{2 \times 1500 \times 15}{1 \times 300} = 150 \text{ m} \quad ...(ii)$$



Solving exps. (*i*) and (*ii*), we have, $x_1 = 75$ m and $x_2 = 225$ m

$$\text{Sag } S_2 = \frac{w\,x_2^2}{2T} = \frac{1 \times (225)^2}{2 \times 1500} = 16{\cdot}87 \text{ m}$$

Now $\quad BC = BE + EC = 20 + 15 = 35$ m

Clearance of the lowest point O from the ground is

$$OG = HF - S_2 - GF$$

$$= BC - S_2 - GF \qquad (\because BC = HF)$$

$$[\text{Now } GF = x_1 \tan\theta = 75 \times 0{\cdot}05 = 3{\cdot}75 \text{ m}]$$

$$= 35 - 16{\cdot}87 - 3{\cdot}75 = \mathbf{14{\cdot}38 \text{ m}}$$

Example 8.27. *A transmission tower on a level ground gives a minimum clearance of 8 metres for its lowest conductor with a sag of 10 m for a span of 300 m. If the same tower is to be used over a slope of 1 in 15, find the minimum ground clearance obtained for the same span, same conductor and same weather conditions.*

Solution. On level ground

$$\text{Sag, } S = \frac{w\,l^2}{8T}$$

$$\therefore \quad \frac{w}{T} = \frac{8S}{l^2} = \frac{8 \times 10}{(300)^2} = \frac{8}{9 \times 10^3}$$

Height of tower = Sag + Clearance = 10 + 8 = 18 m

On sloping ground. The conductors are supported between towers *AD* and *BE* over a sloping ground having a gradient 1 in 15 as shown in Fig. 8.32. The height of each tower (*AD* or *BE*) is 18 m.

Vertical distance between the two towers is

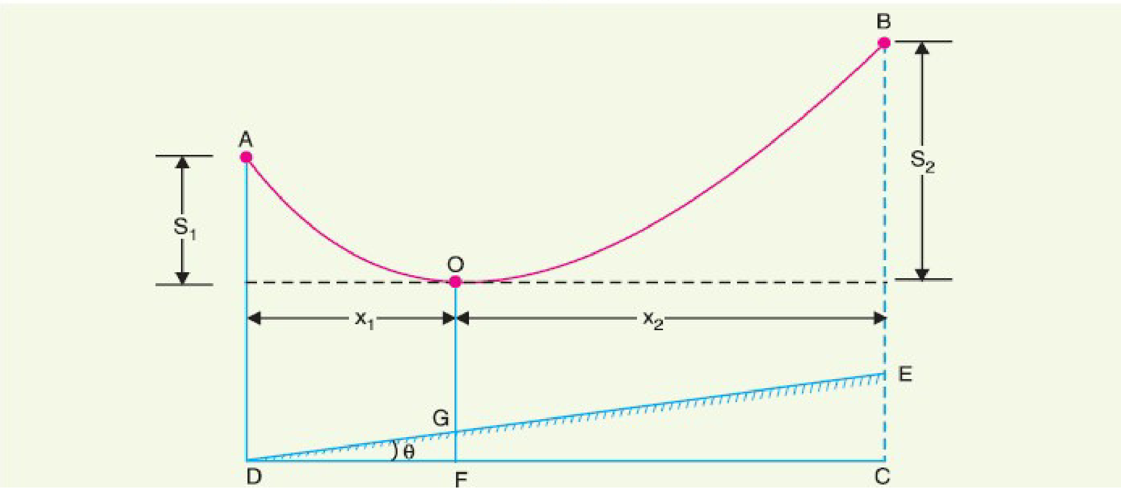
$$h = EC = {}^*DE \sin\theta = 300 \times 1/15 = 20 \text{ m}$$

Now $\quad x_1 + x_2 = 300$ m $\qquad$ ...(*i*)

Also

$$h = \frac{w\,x_2^2}{2T} - \frac{w\,x_1^2}{2T} = \frac{w}{2T}(x_2 + x_1)(x_2 - x_1)$$

$$\therefore \quad x_2 - x_1 = \frac{2\,T\,h}{w(x_2 + x_1)} = \frac{2 \times 9 \times 10^3 \times 20}{8 \times 300} = 150 \text{ m} \qquad \text{...}(ii)$$



Solving exps. (*i*) and (*ii*), we have, $x_1 = 75$ m and $x_2 = 225$ m

Now $$S_1 = \frac{w x_1^2}{2T} = \frac{8 \times (75)^2}{2 \times 9 \times 10^3} = 2{\cdot}5 \text{ m}$$

$$S_2 = \frac{w x_2^2}{2T} = \frac{8 \times (225)^2}{2 \times 9 \times 10^3} = 22{\cdot}5 \text{ m}$$

Clearance of point *O* from the ground is

$$OG = BC - S_2 - GF = 38 - 22{\cdot}5 - 5 = 10{\cdot}5 \text{ m}$$

$$[\because GF = x_1 \tan\theta = 75 \times 1/15 = 5\text{m}]$$

Since *O* is the origin, the equation of slope of ground is given by :

$$y = m x + A$$

Here $$m = 1/15 \text{ and } A = OG = -10{\cdot}5 \text{ m}$$

$$\therefore \quad y = \frac{x}{15} - 10{\cdot}5$$

$\therefore$ Clearance *C* from the ground at any point *x* is

$$C = \text{Equation of conductor curve} - y = \left(\frac{w x^2}{2T}\right) - \left(\frac{x}{15} - 10 \cdot 5\right)$$

$$= \frac{8x^2}{2 \times 9 \times 10^3} - \left(\frac{x}{15} - 10 \cdot 5\right)$$

$$\therefore \quad C = \frac{x^2}{2250} - \frac{x}{15} + 10 \cdot 5$$

Clearance will be minimum when $dC/dx = 0$ *i.e.,*

$$\frac{d}{dx}\left[\frac{x^2}{2250} - \frac{x}{15} + 10 \cdot 5\right] = 0$$

or $$\frac{2x}{2250} - \frac{1}{15} = 0$$

or $$x = \frac{1}{15} \times \frac{2250}{2} = \mathbf{75 \text{ m}}$$

i.e., minimum clearance will be at a point 75 m from *O*.

$$\text{Minimum clearance} = \frac{x^2}{2250} - \frac{x}{15} + 10 \cdot 5 = (75)^2/2250 - 75/15 + 10{\cdot}5$$

$$= 2{\cdot}5 - 5 + 10{\cdot}5 = \mathbf{8 \text{ m}}$$