Module-II

Performance of Transmission Lines

SHORT TRANSMISSION LINES

The transmission lines are categorized as three types

1) Short transmission line – the line length is up to 80 km

2) Medium transmission line – the line length is between 80km to 160 km

3) Long transmission line – the line length is more than 160 km

Whatever may be the category of transmission line, the main aim is to transmit power from one end to another. Like other electrical system, the transmission network also will have some power loss and voltage drop during transmitting power from sending end to receiving end. Hence, performance of transmission line can be determined by its efficiency and voltage regulation.

$$\text{Efficiency of transmission line} = \frac{\text{power delivered at receiving end}}{\text{power sent from sending end}} \times 100\ \%$$

power sent from sending end – line losses = power delivered at receiving end

Voltage regulation of transmission line is measure of change of receiving end voltage from no-load to full load condition.

$$\%\ \text{regulation} = \frac{\text{no load receiving end voltage} - \text{full load receiving end voltage}}{\text{full load voltage}} \times 100\ \%$$

Every transmission line will have three basic electrical parameters. The conductors of the line will have resistance, inductance, and capacitance. As the transmission line is a set of conductors being run from one place to another supported by transmission towers, the parameters are distributed uniformly along the line.

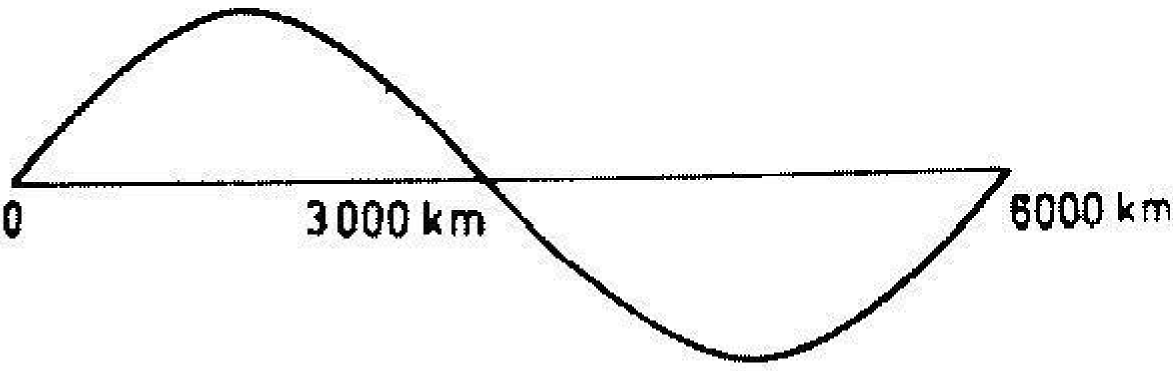
The electrical power is transmitted over a transmission line with a speed of light that is $3X10^8$ m ⁄ sec. Frequency of the power is 50Hz. The wave length of the voltage and current of the power can be determined by the equation given below,

$f.\lambda = v$ where f is power frequency, &labda is wave length and v is the speed of light.

$$\text{Therefore, } \lambda = \frac{v}{f}$$

$$\lambda = \frac{3X10^8}{50} = 6X10^6 \text{ meters} = 6000 \text{ km.}$$

Hence the wave length of the transmitting power is quite long compared to the generally used line length of transmission line.



Voltage distribution of 50 Hz supply

For this reason, the transmission line, with length less than 160 km, the parameters are assumed to be lumped and not distributed. Such lines are known as electrically short transmission line. This electrically short transmission lines are again categorized as short transmission line (length up to 80 km) and medium transmission line(length between 80 and 160 km). The capacitive parameter of short transmission line is ignored whereas in case of medium length line the capacitance is assumed to be lumped at the middle of the line or half of the capacitance may be considered to be lumped at each ends of the transmission line. Lines with length more than 160 km, the parameters are considered to be distributed over the line. This is called long transmission line.

ABCD PARAMETERS

A major section of power system engineering deals in the transmission of electrical power from one particular place (eg. Generating station) to another like substations or distribution units with maximum efficiency. So its of substantial importance for power system engineers to be thorough with its mathematical modeling. Thus the entire transmission system can be simplified to a **two port network** for the sake of easier calculations.

The circuit of a 2 port network is shown in the diagram below. As the name suggests, a 2 port network consists of an input port PQ and an output port RS. Each port has 2 terminals to connect itself to the external circuit. Thus it is essentially a 2 port or a 4 terminal circuit, having

Supply end voltage = VS

and Supply end current = IS

Given to the input port P Q.

And there is the Receiving end Voltage = VR

and Receiving end current = IR

Given to the output port R S.

As shown in the diagram below.

Now the **ABCD parameters** or the transmission line parameters provide the link between the supply and receiving end voltages and currents, considering the circuit elements to be linear in nature.

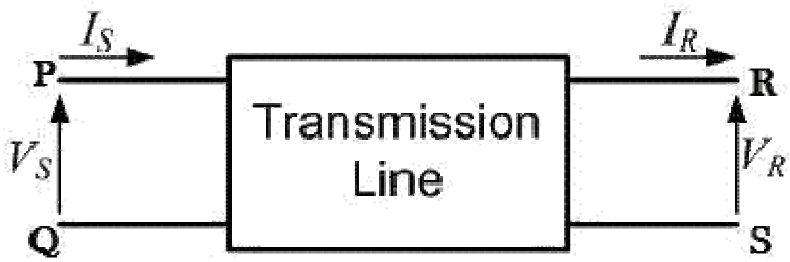
Thus the relation between the sending and receiving end specifications are given using **ABCD parameters** by the equations below.

VS = A VR + B IR ———————-(1)

IS = C VR + D IR ———————-(2)

Now in order to determine the ABCD parameters of transmission line let us impose the required circuit conditions in different cases.

ABCD parameters, when receiving end is open circuited



The receiving end is open circuited meaning receiving end current IR = 0.

Applying this condition to equation (1) we get.

$$V_S = A\,V_R + B\,0 \Rightarrow V_S = A\,V_R + 0$$

$$A = \left.\frac{V_S}{V_R}\right|_{I_R = 0}$$

Thus its implies that on applying open circuit condition to ABCD parameters, we get parameter A as the ratio of sending end voltage to the open circuit receiving end voltage. Since dimension wise A is a ratio of voltage to voltage, A is a dimension less parameter.

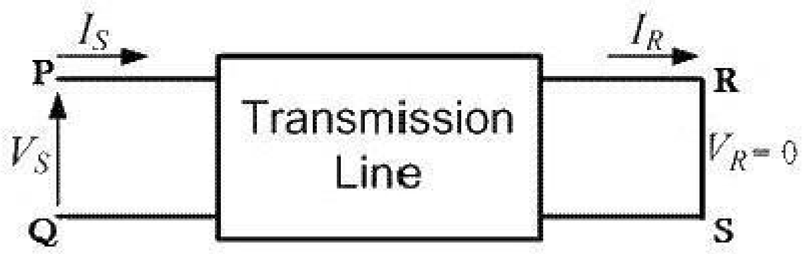
Applying the same open circuit condition i.e IR = 0 to equation (2)

$$I_S = C\,V_R + D\,0 \Rightarrow I_S = C\,V_R + 0$$
$$C = \left.\frac{I_S}{V_R}\right|_{I_R = 0}$$

Thus its implies that on applying open circuit condition to ABCD parameters of transmission line, we get parameter C as the ratio of sending end current to the open circuit receiving end voltage. Since dimension wise C is a ratio of current to voltage, its unit is mho.

Thus C is the open circuit conductance and is given by C = IS ⁄ VR mho.

ABCD parameters when receiving end is short circuited



Receiving end is short circuited meaning receiving end voltage VR = 0

Applying this condition to equation (1) we get

$$V_S = A\,0 + B\,I_R \Rightarrow V_S = 0 + B\,I_R$$
$$B = \left.\frac{V_S}{I_R}\right|_{V_R = 0}$$

Thus its implies that on applying short circuit condition to ABCD parameters, we get parameter B as the ratio of sending end voltage to the short circuit receiving end current. Since dimension wise B is a ratio of voltage to current, its unit is Ω. Thus B is the short circuit resistance and is

given by

B = VS ⁄ IR Ω.

Applying the same short circuit condition i.e VR = 0 to equation (2) we get

$$I_S = C\,0 + D\,I_R \Rightarrow I_S = 0 + D\,I_R$$
$$D = \left.\frac{I_S}{I_R}\right|_{V_R = 0}$$

Thus its implies that on applying short circuit condition to ABCD parameters, we get parameter D as the ratio of sending end current to the short circuit receiving end current. Since dimension wise D is a ratio of current to current, it's a dimension less parameter. ∴ the ABCD parameters of transmission line can be tabulated as:-

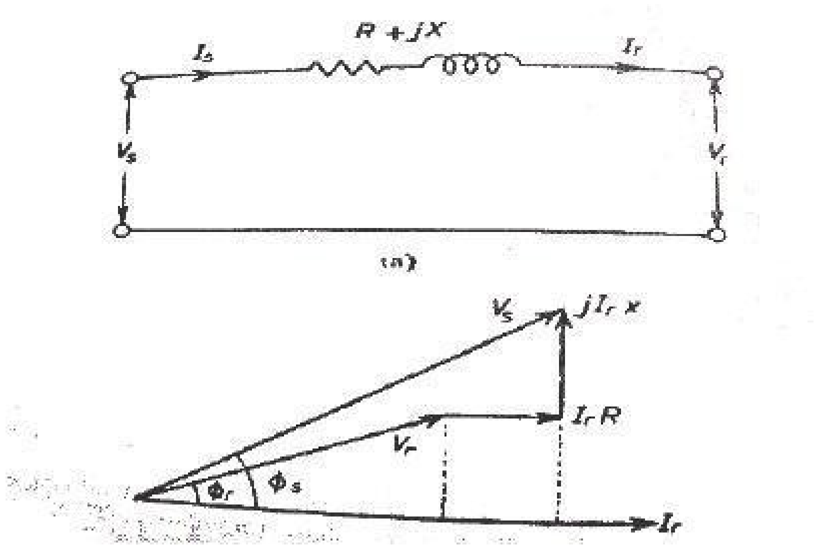
Parameter	Specification	Unit
A = VS / VR	Voltage ratio	Unit less
B = VS / IR	Short circuit resistance	Ω
C = IS / VR	Open circuit conductance	mho
D = IS / IR	Current ratio	Unit less

SHORT TRANSMISSION LINE

The transmission lines which have length less than 80 km are generally referred as **short transmission lines**.

For short length, the shunt capacitance of this type of line is neglected and other parameters like resistance and inductance of these short lines are lumped, hence the equivalent circuit is represented as given below,

Let's draw the vector diagram for this equivalent circuit, taking receiving end current Ir as reference. The sending end and receiving end voltages make angle with that reference receiving end current, of φs and φr, respectively.



As the shunt capacitance of the line is neglected, hence sending end current and receiving end current is same, i.e.

Is = Ir.

Now if we observe the vector diagram carefully, we will get, Vs is approximately equal to

Vr + Ir.R.cosφr + Ir.X.sinφr

That means,

Vs ≅ Vr + Ir.R.cosφr + Ir.X.sinφr as the it is assumed that φs ≅ φr

As there is no capacitance, during no load condition the current through the line is considered as zero, hence at no load condition, receiving end voltage is the same as sending end voltage

As per dentition of voltage regulation,

$$\% \text{ regulation} = \frac{V_s - V_r}{V_r} X\ 100\ \%$$

$$= \frac{I_r.R.\cos\varphi_r + I_r.X.\sin\varphi_r}{V_r} X\ 100\ \%$$

$$\text{per unit regulation} = \frac{I_r.R}{V_r}\cos\varphi_r + \frac{I_r.X}{V_r}\sin\varphi_r = v_r\cos\varphi_r + v_x\sin\varphi_r$$

$$A = \frac{V_s}{V_r}\bigg|_{I_r = 0}$$

Here, vr and vx are the per unit resistance and reactance of the short transmission line.

Any electrical network generally has two input terminals and two output terminals. If we consider any complex electrical network in a black box, it will have two input terminals and output terminals. This network is called two – port network. Two port model of a network simplifies the network solving technique. Mathematically a two port network can be solved by 2 by 2 matrixes.

A transmission as it is also an electrical network; line can be represented as two port network. Hence two port network of transmission line can be represented as 2 by 2 matrixes. Here the concept of ABCD parameters comes. Voltage and currents of the network can represented as ,

Vs= AVr + BIr… ....(1)

Is= CVr + DIr…........... (2)

Where A, B, C and D are different constant of the network.

If we put Ir = 0 at equation (1), we get

Hence, A is the voltage impressed at the sending end per volt at the receiving end when receiving end is open. It is dimension less.

If we put Vr = 0 at equation (1), we get

$$B = \frac{V_s}{I_r}\bigg|_{V_r = 0}$$

That indicates it is impedance of the transmission line when the receiving terminals are short circuited. This parameter is referred as transfer impedance.

$$C = \frac{I_s}{V_r}\bigg|_{I_r = 0}$$

C is the current in amperes into the sending end per volt on open circuited receiving end. It has the dimension of admittance.

$$D = \frac{I_s}{I_r}\bigg|_{V_r = 0}$$

D is the current in amperes into the sending end per amp on short circuited receiving end. It is dimensionless.

Now from equivalent circuit, it is found that,

$$V_s = V_r + I_r Z \text{ and } I_s = I_r$$

Comparing these equations with equation 1 and 2 we get,

A = 1, B = Z, C = 0 and D = 1. As we know that the constant A, B, C and D are related for passive network as

Here, A = 1, B = Z, C = 0 and D = 1

AD − BC = 1.

⇒ 1.1 − Z.0 = 1

So the values calculated are correct for short transmission line.

From above equation (1),

$$V_s = AV_r + BI_r$$

When $I_r = 0$ that means receiving end terminals is open circuited and then from the equation 1, we get receiving end voltage at no load

$$V_r' = \frac{V_s}{A}$$

and as per definition of voltage regulation,

$$\%\ \text{voltage regulation} = \frac{V_s / A - V_r}{V_r} X100\ \%$$

Efficiency of Short Transmission Line

The efficiency of short line as simple as efficiency equation of any other electrical equipment, that means

$$\% \text{ efficiency } (\mu) = \frac{\text{Power received at receiving end}}{\text{Power delivered at sending end}} X\ 100\ \%$$

$$\% \mu = \frac{\text{Power received at receiving end}}{\text{Power received at receiving end} + 3I_r^2.R} X\ 100\ \%$$

MEDIUM TRANSMISSION LINE

The transmission line having its effective length more than 80 km but less than 250 km, is generally referred to as a **medium transmission line**. Due to the line length being considerably high, admittance Y of the network does play a role in calculating the effective circuit parameters, unlike in the case of short transmission lines. For this reason the modelling of a **medium length transmission line** is done using lumped shunt admittance along with the lumped impedance in series to the circuit.

These lumped parameters of a medium length transmission line can be represented using two different models, namely.

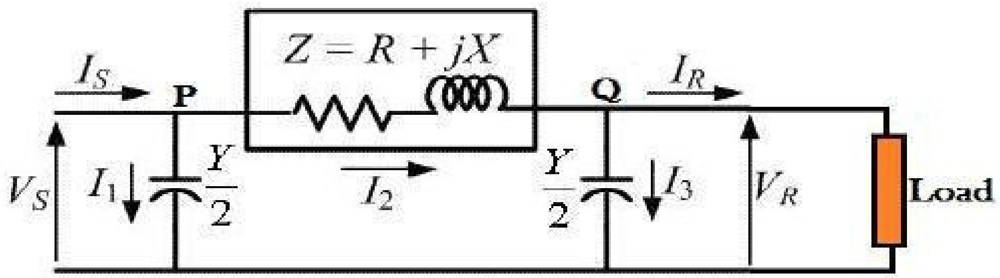
1) Nominal **Π** representation.

2) Nominal **T** representation.

Let's now go into the detailed discussion of these above mentioned models.

Nominal Π representation of a medium transmission line

In case of a nominal **Π** representation, the lumped series impedance is placed at the middle of the circuit where as the shunt admittances are at the ends. As we can see from the diagram of the Π network below, the total lumped shunt admittance is divided into 2 equal halves, and each half with value $Y/2$ is placed at both the sending and the receiving end while the entire circuit impedance is between the two. The shape of the circuit so formed resembles that of a symbol **Π**, and for this reason it is known as the nominal Π representation of a medium transmission line. It is mainly used for determining the general circuit parameters and performing load flow analysis.



Nominal π network of medium transmission line.

As we can see here, VS and VR is the supply and receiving end voltages respectively, and Is is the current flowing through the supply end.

IR is the current flowing through the receiving end of the circuit.

I1 and I3 are the values of currents flowing through the admittances.

And I2 is the current through the impedance Z.

Now applying KCL, at node P, we

get. IS = I1 + I2 ————(1)

Similarly applying KCL, to node

Q. I2 = I3 + IR ————(2)

Now substituting equation (2) to equation

(1) IS = I1 + I3 + IR

$$= \frac{Y}{2}V_S + \frac{Y}{2}V_R + I_R \text{---------(3)}$$

Now by applying KVL to the circuit,

VS = VR + Z I2

$$= V_R + Z(V_R\frac{Y}{2} + I_R)$$

$$= (Z\frac{Y}{2} + 1)\, V_R + ZI_R \text{---------(4)}$$

Now substituting equation (4) to equation (3), we get.

$$I_s = \frac{Y}{2}[(\frac{Y}{2}Z + 1)V_R + ZI_R] + \frac{Y}{2}V_R + I_R$$

$$= Y(\frac{Y}{4}Z + 1)V_R + (\frac{Y}{2}Z + 1)I_R \text{-----------(5)}$$

Comparing equation (4) and (5) with the standard ABCD parameter equations

VS = A VR + B IR

IS = C VR + D IR

We derive the parameters of a medium transmission line as:

$$A = (\frac{Y}{2}Z + 1)$$

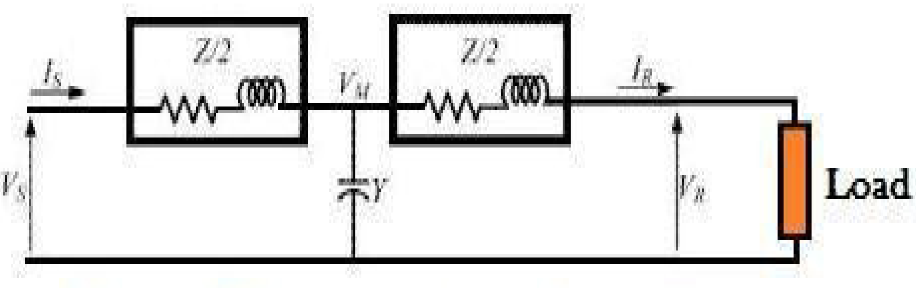
$$B = Z\ \Omega$$

$$C = Y(\frac{Y}{4}Z + 1)$$

$$D = (\frac{Y}{2}Z + 1)$$

Nominal T representation of a medium transmission line

In the **nominal T** model of a medium transmission line the lumped shunt admittance is placed in the middle, while the net series impedance is divided into two equal halves and and placed on either side of the shunt admittance. The circuit so formed resembles the symbol of a capital **T**, and hence is known as the nominal T network of a medium length transmission line and is shown in the diagram below.



Nominal T representation of a medium transmission line.

Here also Vs and Vr is the supply and receiving end voltages respectively, and

Is is the current flowing through the supply end. Ir is the current flowing through the receiving end of the circuit. Let M be a node at the midpoint of the circuit, and the drop at M, be given by

Vm. Applying KVL to the above network we get

$$\frac{V_S - V_M}{Z/2} = Y V_M + \frac{V_m - V_R}{Z/2}$$

$$\text{Or } V_M = \frac{2(V_S + V_R)}{YZ + 4} \text{ ----------------(6)}$$

And the receiving end current

$$\text{Or } I_R = \frac{2(V_M - V_R)}{Z/2} \text{ ----------------(7)}$$

Now substituting V_M from equation (6) to (7) we get,

$$\text{Or } I_R = \frac{[(2V_S + V_R)/YZ+4] - V_R}{Z/2}$$

Rearranging the above equation:

$$V_S = (\frac{Y}{2}Z + 1)V_R + Z(\frac{Y}{4}Z + 1)I_R \text{ ----------------(8)}$$

Now the sending end current is

Is = Y VM + IR ——————(9)

Substituting the value of VM to equation (9) we get,

$$\text{Or } I_S = Y V_R + (\frac{Y}{2}Z + 1)I_R \text{ ----------------(10)}$$

Again comparing Comparing equation (8) and (10) with the standard ABCD parameter equations

VS = A VR + B IR

IS = C VR + D IR

The parameters of the **T** network of a medium transmission line are

$$A = (\frac{Y}{2}Z + 1)$$

$$B = Z(\frac{Y}{4}Z + 1)\ \Omega$$

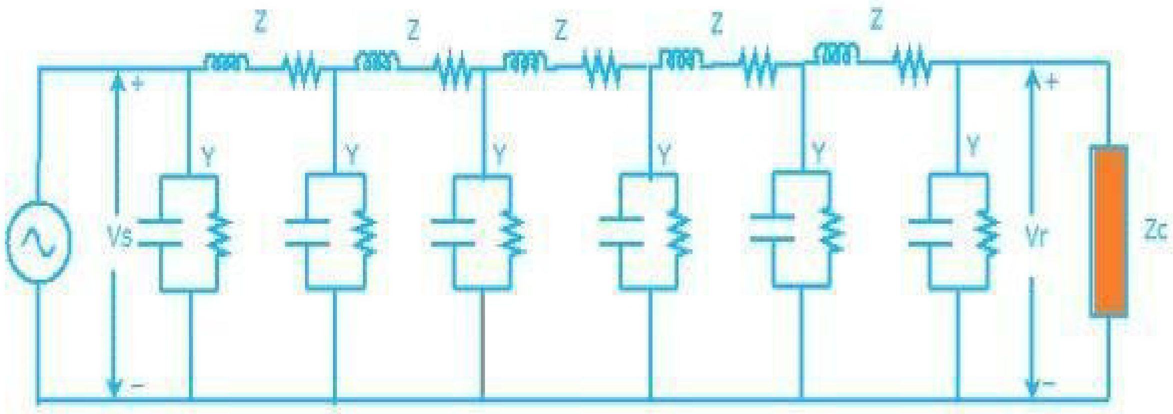
$$C = Y \text{ mho}$$

$$D = (\frac{Y}{2}Z + 1)$$

Performance of Long Transmission Lines

LONG TRANSMISSION LINE

A power transmission line with its effective length of around 250 Kms or above is referred to as a **long transmission line**. Calculations related to circuit parameters (ABCD parameters) of such a power transmission is not that simple, as was the case for a short or medium transmission line. The reason being that, the effective circuit length in this case is much higher than what it was for the former models(long and medium line) and, thus ruling out the approximations considered there like.



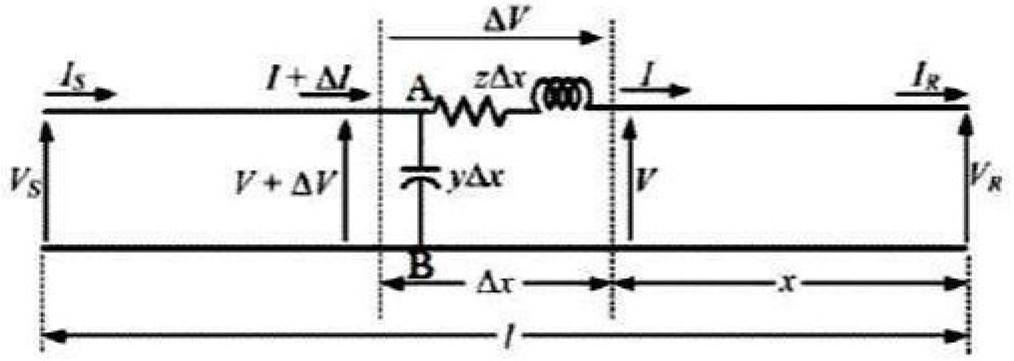
Long Transmission Line model

a) Ignoring the shunt admittance of the network, like in a small transmission line model.

b) Considering the circuit impedance and admittance to be lumped and concentrated at a point as was the case for the medium line model.

Rather, for all practical reasons we should consider the circuit impedance and admittance to be distributed over the entire circuit length as shown in the figure below.

The calculations of circuit parameters for this reason is going to be slightly more rigorous as we will see here. For accurate modeling to determine circuit parameters let us consider the circuit of the **long transmission line** as shown in the diagram below.



Long Transmission Line.

Here a line of length l > 250km is supplied with a sending end voltage and current of VS and IS respectively, where as the VR and IR are the values of voltage and current obtained from the receiving end. Lets us now consider an element of infinitely small length Δx at a distance x from the receiving end as shown in the figure where.

V = value of voltage just before entering the element Δx.

I = value of current just before entering the element Δx.

V+ΔV = voltage leaving the element Δx.

I+ΔI = current leaving the element Δx.

ΔV = voltage drop across element Δx.

zΔx = series impedence of element Δx

yΔx = shunt admittance of element Δx

Where Z = z l and Y = y l are the values of total impedance and admittance of the long transmission line.

∴ the voltage drop across the infinitely small element Δx is given by

$\Delta V = I z \Delta x$

Or $I z = \Delta V / \Delta x$

Or $I z = dV / dx$ ——————————(1)

Now to determine the current ΔI, we apply KCL to node A.

$\Delta I = (V+\Delta V)y\Delta x = V y\Delta x + \Delta V y\Delta x$

Since the term ΔV yΔx is the product of 2 infinitely small values, we can ignore it for the sake of easier calculation.

∴ we can write $dI/dx = V y$ ————(2)

Now derevating both sides of eq (1) w.r.t x,

$d^2 V / d x^2 = z\, dI / dx$

Now substituting $dI/dx = V\,y$ from equation (2)

$d^2 V / d x^2 = zyV$

or $d^2 V / d x^2 - zyV = 0$ ————(3)

The solution of the above second order differential equation is given by.

$V = A1\, e^{x\sqrt{yz}} + A2\, e^{-x\sqrt{yz}}$ —————(4)

Derivating equation (4) w.r.to x.

$dV/dx = \sqrt{(yz)}\, A1\, e^{x\sqrt{yz}} - \sqrt{(yz)} A2\, e^{-x\sqrt{yz}}$ ————

(5) Now comparing equation (1) with equation (5)

$$I = \frac{dV}{dx} = \frac{zA_1 e^{x\sqrt{(yz)}}}{\sqrt{(z/y)}} - \frac{zA_1 e^{-x\sqrt{(yz)}}}{\sqrt{(z/y)}} \text{-----------------(6)}$$

Now to go further let us define the characteristic impedance Zc and propagation constant δ of a long transmission line as

$Zc = \sqrt{(z/y)}$

$\Omega\ \delta = \sqrt{(yz)}$

Then the voltage and current equation can be expressed in terms of characteristic impedance and propagation constant as

$V = A1\, e^{\delta x} + A2\, e^{-\delta x}$ ————(7)

$I = A1/ Zc\, e^{\delta x} + A2 / Zc\, e^{-\delta x}$ —————(8)

Now at x=0, V= VR and I= Ir. Substituting these conditions to equation (7) and (8) respectively. VR = A1 + A2 —————(9)

IR = A1/ Zc + A2 / Zc —————(10)

Solving equation (9) and (10), We get values of A1 and A2 as,

A1 = (VR + ZCIR) / 2

And A1 = (VR − ZCIR)

Now applying another extreme condition at x=l, we have V = VS and I = IS.

Now to determine VS and IS we substitute x by l and put the values of A1 and A2 in equation (7) and (8) we get

$VS = (VR + ZC\ IR)e^{\delta l} / 2 + (VR - ZC\ IR)e^{-\delta l}/2$ —————(11)

$IS = (VR / ZC + IR)e^{\delta l}/2 - (VR / ZC - IR)e^{-\delta l}/2$————— (12

By trigonometric and exponential operators we know

$\sinh \delta l = (e^{\delta l} - e^{-\delta l}) / 2$

And $\cosh \delta l = (e^{\delta l} + e^{-\delta l})/2$

∴ equation(11) and (12) can be re-written as

VS = VRcosh δl + ZC IR sinh δl

IS = (VR sinh δl)/ZC + IRcosh δl

Thus comparing with the general circuit parameters equation, we get the ABCD parameters of a long transmission line as,

C = sinh δl/ZC A = cosh δl D = cosh δl B = ZC sinh δl

Surge Impedance: The **characteristic impedance** or **surge impedance** (usually written Z_0) of a uniform transmission line is the ratio of the amplitudes of voltage and current of a single wave propagating along the line; that is, a wave travelling in one direction in the absence of reflections in the other direction. Characteristic impedance is determined by the geometry and materials of the transmission line and, for a uniform line, is not dependent on its length. The SI unit of characteristic impedance is the ohm.

The characteristic impedance of a lossless transmission line is purely real, with no reactive component. Energy supplied by a source at one end of such a line is transmitted through the line without being dissipated in the line itself. A transmission line of finite length (lossless or lossy) that is terminated at one end with an impedance equal to the characteristic impedance appears to the source like an infinitely long transmission line and produces no reflections.

The surge impedance loading:

The surge impedance loading (SIL) of a line is the power load at which the net reactive power is zero. So, if your transmission line wants to "absorb" reactive power, the SIL is the amount of reactive power you would have to produce to balance it out to zero. You can calculate it by dividing the square of the line-to-line voltage by the line's characteristic impedance. Transmission lines can be considered as, a small inductance in series and a small capacitance to earth, - a very large number of this combinations, in series. Whatever voltage drop occurs

due to inductance gets compensated by capacitance. If this compensation is exact, you have surge impedance loading and no voltage drop occurs for an infinite length or, a finite length terminated by impedance of this value (SIL load). (Loss-less line assumed!). Impedance of this line (Zs) can be proved to be sqrt (L/C). If capacitive compensation is more than required, which may happen on an unloaded EHV line, then you have voltage rise at the other end, the ferranti effect. Although given in many books, it continues to remain an interesting discussion always.

The capacitive reactive power associated with a transmission line increases directly as the square of the voltage and is proportional to line capacitance and length.

Capacitance has two effects:

1 Ferranti effect

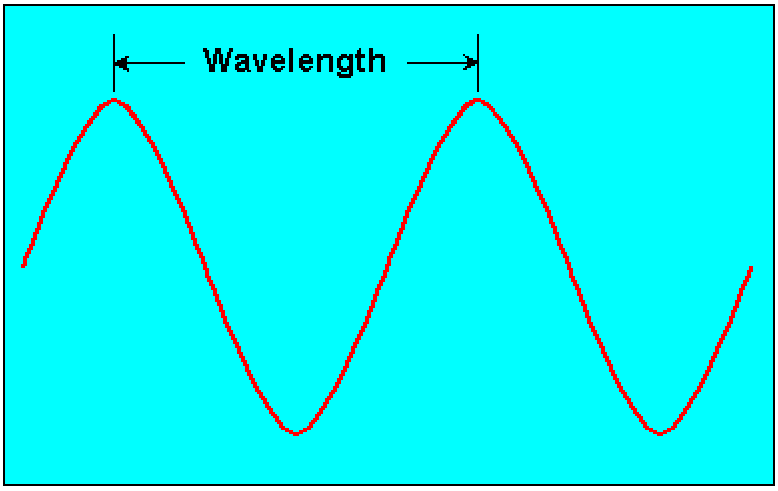
2 rise in the voltage resulting from capacitive current of the line flowing through the source impedances at the terminations of the line.

SIL is Surge Impedance Loading and is calculated as (KV x KV) / Zs their units are megawatts.

Where Zs is the surge impedance....be aware...one thing is the surge impedance and other very different is the surge impedance loading.

Wavelength:

Wavelength is the distance between identical points in the adjacent cycles of a waveform signal propagated in space or along a wire, as shown in the illustration. In wireless systems, this length is usually specified in meters, centimeters, or millimeters. Inthe case of infrared, visible light, ultraviolet, and gamma radiation, the wavelength ismore often specified in nanometers (units of 10^{-9}meter) or Angstrom units(units of 10^{-10} meter).



Wavelength is inversely related to frequency. The higher the frequency of the signal, the shorter the wavelength. If*f* is the frequency of the signal as measured in megahertz, and *w* isthe wavelength as measuredin meters, then

$w = 300/f$ and conversely

$f = 300/w$

Wavelength is sometimes represented by the Greek letter lambda.

Velocity of Propagation:

Velocity of propagation is a measure of how fast a signal travels over time, or the speed of the transmitted signal as compared to the speed of light.
In computer technology, the velocity of propagation of an electrical or electromagnetic signal is the speed of transmission through a physical medium such as a coaxial cable or optical fiber.

There is also a direct relation between velocity of propagation and wavelength. Velocity of propagation is often stated either as a percentage of the speed of light or as time-to distance.

FERRANTI EFFECT

In general practice we know, that for all electrical systems current flows from the region of higher potential to the region of lower potential, to compensate for the potential difference that exists in the system. In all practical cases the sending end voltage is higher than the receiving end, so current flows from the source or the supply end to the load. But Sir S.Z. Ferranti, in the year 1890, came up with an astonishing theory about medium or long distance transmission lines suggesting that in case of light loading or no load operation of transmission system, the receiving end voltage often increases beyond the sending end voltage, leading to a phenomena known as **Ferranti effect in power system**.

Why Ferranti effect occurs in a transmission line?

A long transmission line can be considered to composed a considerably high amount of capacitance and inductance distributed across the entire length of the line. Ferranti Effect occurs when current drawn by the distributed capacitance of the line itself is greater than the current associated with the load at the receiving end of the line(during light or no load). This capacitor charging current leads to voltage drop across the line inductance of the transmission system which is in phase with the sending end voltages. This voltage drop keeps on increasing additively as we move towards the load end of the line and subsequently the receiving end voltage tends to get larger than applied voltage leading to the phenomena called Ferranti effect in power system. It is illustrated with the help of a phasor diagram below.

Thus both the capacitance and inductance effect of transmission line are equally responsible for this particular phenomena to occur, and hence Ferranti effect is negligible in case of a short transmission lines as the inductance of such a line is practically considered to be nearing zero. In general for a 300 Km line operating at a frequency of 50 Hz, the no load receiving end voltage has been found to be 5% higher than the sending end voltage.

Now for analysis of Ferranti effect let us consider the phasor diagrame shown above.

Here Vr is considered to be the reference phasor, represented by OA.

Thus $V_r = V_r (1 + j0)$

Capacitance current, $I_c = j\omega C V_r$

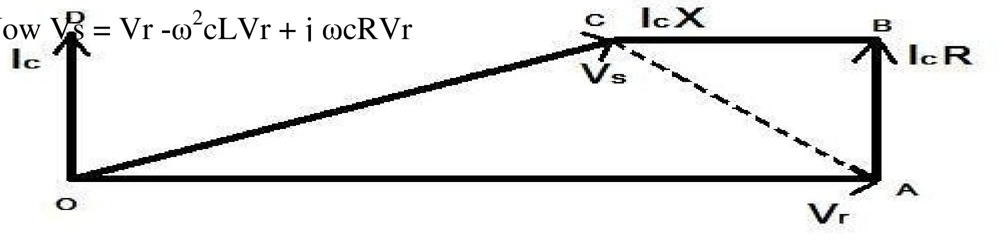
Now sending end voltage Vs = Vr + resistive drop + reactive drop.

$= V_r + I_cR + jI_cX$

$= V_r + I_c(R + jX)$

$= V_r + j\omega cV_r(R + j\omega L)$ [since $X = \omega L$]

Now $V_s = V_r - \omega^2 cLV_r + j\,\omega cRV_r$



Ferranti effect in transmission lines.

This is represented by the phasor OC.

Now in case of a long transmission line, it has been practically observed that the line resistance is negligibly small compared to the line reactance, hence we can assume the length of the phasor Ic R = 0, we can consider the rise in the voltage is only due to OA – OC = reactive drop in the line.

Now if we consider c0 and L0 are the values of capacitance and inductance per km of the transmission line, where l is the length of the line.

Thus capacitive reactance $X_c = 1/(\omega\, l\, c_0)$

Since, in case of a long transmission line the capacitance is distributed throughout its length, the average current flowing is,

$I_c = \frac{1}{2} V_r/X_c = \frac{1}{2} V_r \omega\, l\, c_0$

Now the inductive reactance of the line $= \omega\, L_0\, l$

Thus the rise in voltage due to line inductance is given by,

$I_cX = \frac{1}{2} V_r \omega\, l\, c_0 \times \omega\, L_0\, l$

Voltage rise $= \frac{1}{2} V_r \omega^2 l^2 c_0 L_0$

From the above equation it is absolutely evident, that the rise in voltage at the receiving end is directly proportional to the square of the line length, and hence in case of a long transmission line it keeps increasing with length and even goes beyond the applied sending end voltage at times, leading to the phenomena called Ferranti effect in power system.