Conductor Materials:-

→ The materials to be used as Conductor in transmission and distribution must have the forward Characteristics

(i) Low specific Resistances which leads to high Conductivity

(ii) High tensil strength to withstand high mechanical stress.

(iii) Low specific gravity which leads to less weight

(iv) Low Cost in order to use longer diameter.

(v) flexibility for Easily handling.

→ Some of Conducting materials are,

1) Copper
2) Aluminium
3) Steal Cord Aluminium.

1) Copper:-

→ Copper is an ideal Material for over head lines with Electrical Conductivity, high tensile strength. But the Cost of the Copper is high, and availability is low.

2) Aluminium:-

→ Aluminium Conductors are low Cost, light weight as Compare to Copper for same Resistance. Aluminium Conductors have larger diameter Compare to Copper Conductors.

The availability of aluminium is high as compare to Copper Conductor Materials.

3) Steel Cold Aluminium:-

→ for transmitting high voltages Multi Conductors will be used, They are.

(i) Standard Copper Conductors.

(ii) Hallow Copper Conductors.

(iii) Aluminium Conductors Steel Reinforced (ACSR) are used.

→ An ACSR Conductor has Central Core of galvanised Core Covered With Successive Aluminium.

→ Which are Electrical parallel. Due to this the tension is increase. Will be Reduce, Reduces the span length. the tower is high and increase the span length.

Types of Conductors:-

→ Basically the Conductors are classified as

(i) Solid Conductors

(ii) Standard Conductors.

(iii) Hallow Conductors

(iv) Bundled Conductors.

(i) Solid Conductors:-

→ Solid Copper Conductors is Small Cross. section area used as Solid Conductors.

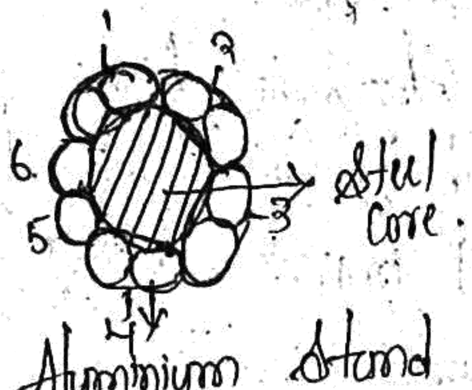
→ Conductors With large size Cannot be used because they are difficult to handle and transport

→ During the long spands they may be break a points are supporte.

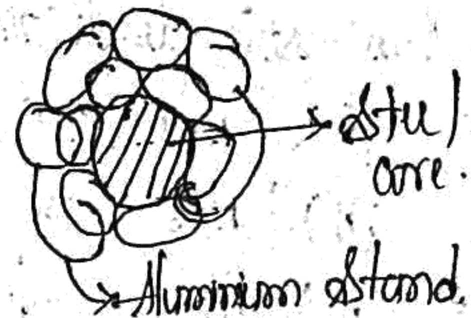
(ii) Standard Conductors:-

→ In, general, Standard Conductors are used in transmission lines for increasing the flexibility and over damped Material.

→ This Conductors Normally have a Central Wire around with the sucessfully layers of 6, 12, 18 are used.



6 - layers



12 - layers

→ The No - of Stands in a Standad Conductor is given by $\{N = 3n(n+1)+1\}$

Where,

n = no - of layers.

→ The diameter of Standard Conductors is given by $D = (2n+1)d$.

Where,

d = distance of Each Strand

n = No - of layer.

(iii) Hallow Conductors :-

→ Hallow Conductors have large diameter when Compared with Solid Conductors.

→ This Conductor have low inductance and low voltage gradient.

→ These type of Conductors are used for 400 kv line

(iv) Bundle Conductors :-

→ A Bundled Conductor Consists of two (or) more than Sub- Conductors in Each Space.

(i) These are used to transmit high

(ii) These are Supported by Constant distance.

(iii) These Conductor will improve voltage Regulation

* Measurement of line Parameters :-

(i) Resistances :-

→ Resistance is defined as voltage per unit Current at Constant temperature.

Resistance of a Conductor $R = V/I$ (or) $\frac{\rho l}{A}$.

Where,

ρ = Resistivity of Materials

l = length of Conductors.

A = Cross-Sectional area of the Conductor

Resistance of Standard Conductor was Slightly More than Solid Conductor with Equal Cross-sectional area.

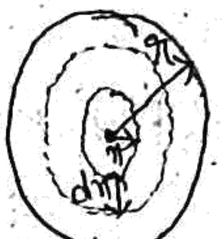
(ii) Inductance :-

→ The inductance Can be defined as the flux linkage per unit ampere of Current

$$L = \frac{\text{flux linkage}}{\text{Current}} \text{ (or) } \frac{\psi}{I}$$

→ In a transmission line the inductance is depends as the material is used and the diameter of the Conductor.

* Inductance due to internal flux linkage :-



Let, r = Radius of Conductor

I = Current passing through Conductor

I_x = Current Enclosed in a Circuit at a distance of "x" m.

H_x = Magnetic field intensity in a Circle at a distance of "x" m.

dx = thickness of a Circle at a distance of "x" m.

from Amper's law,

→ MMF in a Circle at a distance of "x" m is Equal to Current Enclosed in it.

$$\boxed{MMF = I_x} \rightarrow ①$$

→ from maxwell's law MMF in a Circle is Equal to line. line integral of magnetic field intensity

$$MMF = \oint H_x dx \rightarrow ②$$

from Eqn ① & ②

$$\oint H_x dx = I_x$$

$$2\pi x H_x = I_x.$$

Here, $$I_x = I \cdot \frac{\pi x^2}{\pi r^2}$$

$$2\pi x \cdot H_x = I \cdot \frac{\pi x^2}{\pi r^2}$$

$$H_x = I \cdot \frac{x}{2\pi r^2}$$

Magnetic flux density = $\mu \cdot H_x$

$$= \mu \cdot I \cdot x/2\pi r^2.$$

Magnetic flux = density x Area

$$d\phi = d\psi = \mu \cdot I \cdot \frac{x}{2\pi r^2} \times \frac{\pi x^2}{\pi r^2}$$

$$d\psi = \mu I \cdot \frac{x^3}{2\pi r^4}.$$

Extending these flux linkages upto radius "r" m.

$$\int d\psi = \int_0^r \frac{\mu I}{2\pi r^4} \cdot x^3 \, dx.$$

$$\psi = \frac{\mu I}{2\pi r^4} \int_0^r x^3 \, dx.$$

$$= \frac{\mu I}{2\pi r^4} \left[\frac{x^4}{4}\right]_0^r.$$

$$= \frac{\mu I}{2\pi r^4} \cdot \frac{r^4}{4}.$$

$$\psi = \mu I / 8\pi.$$

$$\text{Inductance} = \frac{\text{flux linkage}}{\text{Current}}$$

$$L = \psi/I = \frac{\mu I/8\pi}{I} = \mu/8\pi.$$

Where,

$\mu = \mu_0 \mu_r$.

μ_0 = absolute permiability = $4\pi \times 10^{-7}$

μ_r = Relative permiability = 1 for Air.

$$\text{Inductance } (L) = \frac{4\pi \times 10^{-7}}{8\pi} = \frac{1}{2} \times 10^{-7} \text{ H/m}.$$

$$\boxed{L = 0.5 \times 10^{-7} \text{ H/m}}$$

* Inductance due to External flux linkages :-

→ Let us Consider a With "r" m and two points 1 & 2 Where the flux linkages to be determine b/w them. of two Circles With Radius R_1, R_2 and also Consider an Elemental thickness "dx" at a distance of "x" m from the Center of Conductor.

→ The magnetic field intensity at any point on a Circle. is given by $H_x = \frac{I}{2\pi x}$ AT/m.

∴ Magnetic flux density $= \mu \cdot H_x$

$$= \mu I / 2\pi x.$$

→ Magnetic flux Enclosed in a Circle With an elemental thickness 'dx' at a distance of one meter is given by.

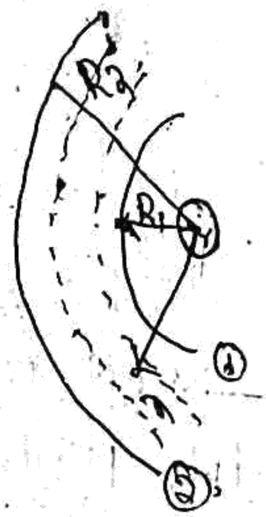
→ Magnetic flux = flux density x Area.

$$d\phi = \mu \cdot \frac{I}{2\pi x} dx \cdot 1$$

$$d\phi = \mu I \frac{dx}{2\pi x}$$

Flux linkage $= d\psi = d\phi = \mu I \cdot dx/2\pi x$.

→ The magnetic flux linkages b/w 1 and ② the distance R_1 and R_2 is given by.



$$\int d\psi = \int_{R_1}^{R_2} \mu I \cdot \frac{dx}{2\pi x}$$

$$= \frac{\mu I}{2\pi} \int_{R_1}^{R_2} \frac{1}{x} dx.$$

$$\psi = \frac{\mu I}{2\pi} \times \ln\left(R_2/R_1\right).$$

∴ Inductance due to External flux linkages.

$$L = \frac{\psi}{I} = \frac{\mu I}{I\, 2\pi} \times \ln\left(R_2/R_1\right).$$

$$L = \mu/2\pi \ln\left(R_2/R_1\right).$$

But $\mu = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 1 = 4\pi \times 10^{-7}$

$$\mu = 4\pi \times 10^{-7}$$

$$\text{Inductance } (L) = \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(R_2/R_1\right)$$

$$L = 2 \times 10^{-7} \ln\left(R_2/R_1\right) \text{ H/m.}$$

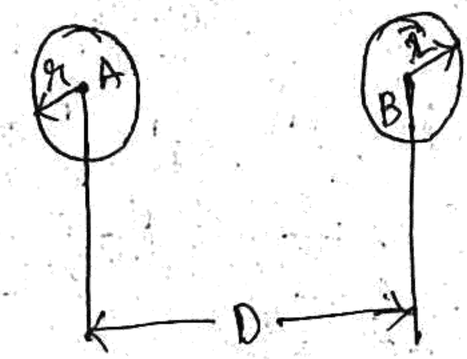
→ If a point will be consider at a distance of "D" m from the centre of conductor then $R_1 = r_1$, $R_2 = D - r$.

∴ Inductance due to external flux linkages is given by.

$$L = 2\times10^{-7} \ln\left(\frac{D-r}{r}\right) \quad \{D > r = r\}$$

$$L = 2\times10^{-7} \ln\left(\frac{D}{r}\right) \text{ H/m.}$$

* Inductance of Single Phase System :-



→ Let us consider a single phase overhead system of two circular conductors with a "r" m supported at a distance "d" m as shown in above figure.

→ The inductance due to internal flux linkages with current (I) in the conductor "A" is

$$L_{internal} = 2\times10^{-7} \ln(D/r) \text{ H/m}$$

→ The inductance conductor A due to internal and external flux linkages is given by.

$$L_A = 0.5\times10^{-7} + 2\times10^{-7} \ln(D/r) \text{ H/m.}$$

$$= \frac{1}{2} \times 10^{-7} + 2\times 10^{-7} \ln (D/r).$$

$$= 2\times 10^{-7} \left[\cancel{2\times 10} \ \frac{1}{4} + \ln D/r \right].$$

$$= 2\times 10^{-7} \left[\ln D/re^{1/4} \right]$$

$$L_A = 2\times 10^{-7} \left[\ln D/r' \right]$$

Where,

r' = Effective radius of Conductor.

$= 0.7788 r$.

Similarly,

The inductance of Conductor "B"

$$L_B = 2\times 10^{-7} \ln D/r' \ H/m.$$

$\therefore$ Inductance of Single phase System.

$$L = L_A + L_B$$

$$L = 2\times 10^{-7} \ln (D/r') + 2\times 10^{-7} \ln (D/r')$$

$$L = 4\times 10^{-7} \ln (D/r') \ H/m.$$

Problem :-

1. Determine the inductive Reactance of 1-ϕ 50 Hz, 16 km transmission line, which consists of pair of Conductors of it's diameter 60 cm and spacing b/w Conductor is 1.2 m.

Sol :- Given data,

frequency = 50 HZ.

length = 16 Km.

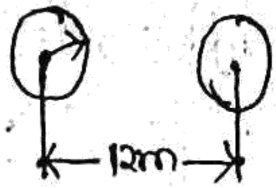
diameter = 60 cm, Radius = 30 cm = 30×10^{-2} m.

Spacing D = 1.2 m

Inductance Reactance $(X_L) = ?$

We know that $X_L = 2\pi f L$

We know that $L = 4\times10^{-7} \ln(D/r)$



$r' = 0.7788 \times r = 0.7788 \times 30 \times 10^{-2}$

$r' = 0.23364$

$L = 4\times10^{-7} \ln\left[\frac{1.2}{0.23364}\right]$

$= 6.54\times10^{-7}$ H/m.

$\therefore$ Inductances for 16 km of length.

$= 16\times10^{3}\times 6.54 \times10^{-7}$ H

$L = 0.01$ H.

$\therefore$ inductive Reactance $(X_L) = 2\pi f L$

$= 2\pi \times 50 \times 0.01$

$\boxed{X_L = 3.14\,\Omega}$

2. Calculate the inductance of 1-ϕ System with two parallel Conductors of 6mm diameter, distance b/w the Conductors are 1m apart. If material of a Conductor is (i) Copper (ii) Steel with Relative permiability of '50'.

Sol:- Given data,

diameter = 6 mm, Radius = 3

Spacing (D) = 1m

$= 3 \times 10^{-1}$ cm

$= 3\times10^{-3}$ m

Inductance = ?

for Copper $\mu_r = 1$, Steel $\mu_r = 50$

→ Inductance of Copper Conductor $= 4\times10^{-7}\times\mu_r\times\ln(D/r')$

$$r' = 0.7788\, r$$
$$= 0.7788\times3\times10^{-3}\, m$$
$$r' = 2.3364\times10^{-3}\, m.$$

$$\Rightarrow L = 4\times10^{-7}\ln\left(D/r'\right)$$
$$L = 4\times10^{-7}\ln\left[\frac{1}{2.3364\times10^{-3}}\right]$$
$$L = 2.42\times10^{-6}\, H/m$$

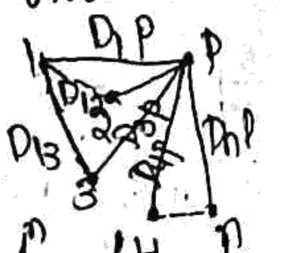
Inductance of Steel $L = 4\times10^{-7}\times\mu_r\times\ln\left(D/r'\right)$

$$L = 4\times10^{-7}\times50\times\ln\left[\frac{1}{2.334\times10^{-3}}\right]$$

$$\boxed{L = 0.248\times10^{-3}\, H/m}$$

* Flux linkages of a Conductor in a group of Conductors:-

→ Let us Consider a group of Conductors at a distance of $D_1P, D_2P, D_3P \ldots D_nP$ from point P. Now We Can Calculate the flux linkages of Conductor '1' due to it's own Current and due to other Conductors With $I_2, I_3 \ldots I_n$.



→ The flux linkages of Conductor '1' due to Current I_1 is $\psi_1 P = 2\times10^{-7} I_1 \ln\frac{D_1P}{r'}$.

→ The flux linkages of Conductor "2" due Current is

$$\psi_2 P = 2\times10^{-7} I_2 \ln\frac{D_2P}{D_{12}}.$$

→ The flux linkages of conductor '1' due to current I_3 is $\psi_{3P} = 2\times10^{-7} I_3 \ln \frac{D_{3P}}{D_{13}}$.

→ The flux linkages of conductor, due to current $I_1, I_2, I_3 \ldots\ldots I_n$ is given by.

$$\psi_1 = \psi_{11} + \psi_{12} + \psi_{13} + \cdots\cdots + \psi_n .$$

$$= 2\times10^{-7}\left[I_1 \ln \frac{D_1P}{r'} + I_2 \ln \frac{D_2P}{D_{12}} + I_3 \ln \frac{D_3P}{D_{13}} + \cdots + I_n \ln \frac{D_nP}{D_{1n}}\right]$$

$$= 2\times10^{-7}\left[I_1 \ln \frac{1}{r_1} + I_2 \ln \frac{1}{D_{12}} + I_3 \ln \frac{1}{D_{13}} + \cdots + I_n \ln \frac{1}{D_{1n}} + I_1 \ln D_1P + I_2 \ln D_2P + I_3 \ln D_3P + \cdots + I_n \ln D_nP\right]$$

In a closed circuit

$$I_1 + I_2 + I_3 + \cdots + I_n = 0$$

$$\Rightarrow I_n = -I_1 - I_2 - I_3 \cdots - I_{n-1}$$

$$\therefore \psi_1 = 2\times10^{-7}\left[I_1 \ln \frac{1}{r'} + I_2 \ln \frac{1}{D_{12}} + \cdots + I_n \ln \frac{1}{D_{1n}} + I_1 \ln D_1P + I_2 \ln D_2P + \cdots + (-I_1 - I_2 - I_3 \cdots - I_{n-1}) \ln D_nP\right]$$

$$\psi_1 = 2\times10^{-7}\left[I_1 \ln \frac{1}{r'} + I_2 \ln \frac{1}{D_{12}} + \cdots + I_n \ln \frac{1}{D_{1n}} + I_1 \ln \frac{D_1P}{D_nP} + I_2 \ln \frac{D_2P}{D_nP} + \cdots + I_n \ln \frac{D_nP}{D_nP}\right]$$

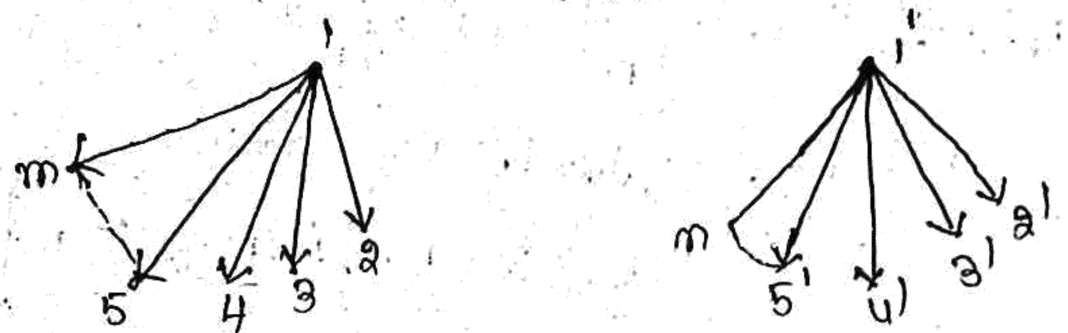
Here,

$D_{1P} \simeq D_{2P} \simeq D_{3P} \simeq \cdots \simeq D_{nP}$

flux linkages of conductor 1 due to $I_1, I_2, I_3 \ldots I_n$ is

$$\psi_1 = 2\times10^{-7}\left[I_1 \ln\frac{1}{r'} + I_2 \ln\frac{1}{D_{12}} + I_3 \ln\frac{1}{D_{13}} + \cdots + I_n \ln\frac{1}{D_{1n}}\right].$$

* Inductance of a single phase system of conductor A and B with m and N No-of strands:-



→ Let us consider conductor A and B with "m" and "n" no-of strands as shown in figure. The current pass-ing through conductor A and conductor B is I_A.

→ Let us assume that the current is equally divi-ded among "m" no-of strands and "n" no-of strands.

→ The current passing through each strand in cond-uctor A is equals to "I/m".

→ The current passing through each strand in conductor "B" is equals to "I/n".

→ The flux linkages of strand "1" in conductor "A" due to "m" no-of strands is given by.

$$\psi_{1m} = 2\times10^{-7}\left[\frac{I}{m}\ln\frac{1}{r'} + \frac{I}{m}\ln\frac{1}{D_{12}} + \frac{I}{m}\ln\frac{1}{D_{13}} + \cdots + \frac{I}{m}\ln\frac{1}{D_{1m}}\right]$$

→ Flux linkages of strands "1" in conductor "A" due to "n" No. of strands.

$$\psi_{1n} = 2\times10^{-7}\left(\frac{I}{n}\ln\frac{1}{D_{11}'} + \frac{I}{n}\ln\frac{1}{D_{12}} + \frac{I}{n}\ln\frac{1}{D_{13}'} + \cdots + \frac{I}{n}\ln\frac{1}{D_{1n}}\right)$$

→ The flux linkage of strands "1" due to "m" and "n" No. of strands.

$$\psi_1 = 2\times10^{-7}\frac{I}{m}\left[\ln\frac{1}{r'} + \ln\frac{1}{D_{12}} + \ln\frac{1}{D_{13}} + \cdots + \ln\frac{1}{D_{1m}}\right]$$

$$= -2\times10^{-7}\frac{I}{n}\left(\ln\frac{1}{D_{11}'} + \ln\frac{1}{D_{12}'} + \ln\frac{1}{D_{13}'} + \cdots + \ln\frac{1}{D_{1n}'}\right)$$

$$\psi_1 = 2\times10^{-7} I\left(\frac{1}{m}\ln\frac{1}{\sqrt[m]{r'\cdot D_{12}\cdot D_{13}\cdots D_{1m}}} - \frac{1}{n}\ln\frac{1}{D_{11}'\cdot D_{12}'\cdot D_{13}'\cdots D_{1n}'}\right)$$

$$\psi_1 = 2\times10^{-7} I\left[\ln\frac{1}{\sqrt[m]{r'\cdot D_{12}\cdot D_{13}\cdots D_{1m}}} - \ln\frac{1}{\sqrt[n]{D_{11}'\cdot D_{12}'\cdot D_{13}'\cdots D_{1n}}}\right]$$

$$\psi_1 = 2\times10^{-7} I\left[\ln\frac{\sqrt[n]{D_{11}'\cdot D_{12}'\cdot D_{13}'\cdots D_{1n}}}{\sqrt[m]{D_{11}\cdot D_{12}\cdot D_{13}\cdots D_{1m}}}\right]$$

→ Inductance of strands "1" in conductor "A" is.

$$L_1 = \frac{\psi_1}{I/m} = \frac{2\times10^{-7}}{I/m} I\left[\ln\frac{\sqrt[n]{D_{11}'\cdot D_{12}'\cdot D_{13}'\cdots D_{1n}}}{\sqrt[m]{r'\cdot D_{11}\cdot D_{12}\cdot D_{13}\cdots D_{1m}}}\right].$$

$$L_1 = 2\times10^{-7} m\left[\ln\frac{\sqrt[n]{D_{11}'\cdot D_{12}'\cdot D_{13}'\cdots D_{1n}}}{\sqrt[m]{r'\cdot D_{11}\cdot D_{12}\cdot D_{13}\cdots D_{1m}}}\right]$$

→ Similarly,

Inductance of strand "2" in conductor "A" is.

$$L_2 = 2\times10^{-7}\, m \left[\ln \frac{\sqrt[n]{D_{21}' \cdot D_{22}' \cdot D_{23}' \cdots D_{2n}'}}{\sqrt[m]{D_{21} \cdot D_{22} \cdot D_{23} \cdots D_{2m}}}\right]$$

Inductance of Strand "m" in Conductor "A" is

$$L_m = 2\times10^{-7}\, m \left[\ln \frac{\sqrt[n]{D_{m1}' \cdot D_{m2}' \cdot D_{m3}' \cdots D_{mn}'}}{\sqrt[m]{D_{m1} \cdot D_{m2} \cdot D_{m3} \cdots D_{mm}}}\right]$$

$\therefore$ Average Inductance $= L_{avg} = \dfrac{L_1 + L_2 + L_3 + \cdots + L_m}{m}$

$\longrightarrow$ Inductance of Conductor A $= \dfrac{L_{avg}}{m} = \dfrac{L_{avg}}{m^2}$.

$\longrightarrow$ Inductance of Conductor A $= \dfrac{L_1 + L_2 + L_3 + \cdots + L_m}{m^2}$.

$$L_A = \frac{2\times10^{-7}}{m^2}\, m \left[\left(\ln \frac{\sqrt[n]{D_{11}' \cdot D_{12}' \cdot D_{13}' \cdots D_{1n}'}}{\sqrt[m]{D_{11}' \cdot D_{12} \cdot D_{13} \cdots D_{1m}}}\right) + \ln \frac{\sqrt[n]{D_{21}' \cdot D_{22}' \cdot D_{23}' \cdots D_{2n}'}}{\sqrt[m]{D_{21} \cdot D_{22} \cdot D_{23} \cdots D_{2m}}}\right.$$

$$\left. + \cdots + \ln \frac{\sqrt[n]{D_{m1}' \cdot D_{m2}' \cdot D_{m3}' \cdots D_{mn}}}{\sqrt[m]{D_{m1} \cdot D_{m2} \cdots D_{mm}}}\right]$$

$$L_A = 2\times10^{-7}\left[\frac{1}{m} \ln \frac{\sqrt[n]{(D_{11}' \cdot D_{12}' \cdots D_{1n})(D_{21}' \cdot D_{22}' \cdot D_{2n}) \cdots (D_{m1}' \cdot D_{m2}' \cdots D_{mn})}}{\sqrt[m]{(r' \cdot D_{12} \cdot D_{1m})(D_{21} \cdot D_{22} \cdot D_{2m}) \cdots (D_{m1} \cdot D_{m2} \cdots D_{mm})}}\right]$$

$$L_A = 2\times10^{-7}\left(\ln \frac{\sqrt[mn]{(D_{11}' \cdot D_{12}' \cdot D_{13}' \cdots D_{1n})(D_{21}' \cdot D_{22}' \cdots D_{23}') \cdots (D_{m1}' \cdot D_{m2}' \cdots D_{mn})}}{\sqrt[m^2]{(r' \cdot D_{12} \cdot D_{13} \cdots D_{1m})(D_{21} \cdot D_{22} \cdots D_{2m}) \cdots (D_{m1} \cdot D_{m2} \cdots D_{mm})}}\right)$$

$$L_A = 2\times10^{-7} \ln\left(\frac{D_m}{D_s}\right)$$

1/07/2019

Where,

D_m = GMD = Geometric Mean Distance.

DS = GMR = Geometric Mean Radius.

For Single phase system.

Inductance $L = L_A + L_B$

$$= 2\times10^{-7}\ln\left(\frac{D_m}{D_s}\right) + 2\times10^{-7}\ln\left(\frac{D_m}{D_s}\right)$$

$$\boxed{L = 4\times10^{-7}\ln\left(\frac{D_m}{DS}\right)\ H/m}$$

Problems

1. Calculate GMR of a Standard Conductor having "7" identical Strands Each of radius "r" m as shown in the fig. And also Calculate the ratio of GMR to overall Conductor Radius.

Sol:- We know that,

$$GMR = \sqrt[7]{DS_1 \cdot DS_2 \cdot DS_3 \cdot DS_4 \cdot DS_5 \cdot DS_6 \cdot DS_7}$$

Where, $DS_1 = \sqrt[7]{D_{11}\cdot D_{12}\cdot D_{13}\cdot D_{14}\cdot D_{15}\cdot D_{16}\cdot D_{17}}$

$DS_2 = \sqrt[7]{D_{21}\cdot D_{22}\cdot D_{23}\cdot D_{24}\cdot D_{25}\cdot D_{26}\cdot D_{27}}$

⋮

$DS_7 = \sqrt[7]{D_{71}\cdot D_{72}\cdot D_{73}\cdot D_{74}\cdot D_{75}\cdot D_{76}\cdot D_{77}}$.

Here,

$D_{11} = D_{22} = D_{33} = D_{44} = D_{55} = D_{66} = D_{77} = r' = 0.7788\,r$.

$DS_1 = \sqrt[7]{D_{11}\cdot D_{12}\cdot D_{13}\cdot D_{14}\cdot D_{15}\cdot D_{16}\cdot D_{17}}$

$\therefore D_{12} = D_{16} = D_{17} = 2r$.

$D_{14} = 4r$.

$D_{13} = D_{15} = \sqrt{(D_{14})^2 - (D_{43})^2} = \sqrt{(4r)^2 - (2r)^2}$

$D_{13} = D_{15} = \sqrt{16r^2 - 4r^2} = \sqrt{12r^2} = 2\sqrt{3}\,r$

$$D_{S_1} = \sqrt[7]{(0.7788\gamma)\times 2\gamma\times 2\sqrt{3}\gamma \times 4\gamma \times 2\sqrt{3}\gamma \times 2\gamma\times 2\gamma}$$

$$= \sqrt[7]{0.7788\gamma \times 12\gamma^2 \times 32\gamma^4}$$

$$D_{S_1} = \sqrt[7]{299.05\gamma^7} = 2.25\gamma.$$

$$\Rightarrow D_{S_2} = \sqrt[7]{D_{21}\cdot D_{22}\cdot D_{23}\cdot D_{24}\cdot D_{25}\cdot D_{26}\cdot D_{27}}$$

$D_{21} = D_{23} = D_{27} = 2\gamma$; $D_{22} = 0.7788\,\gamma.$

$D_{24} = D_{26} = D_{13} = D_{15} = 2\sqrt{3}.$

$D_{25} = 4\gamma.$

$$D_{S_2} = \sqrt[7]{2\gamma\times(0.7788\gamma)\times 2\gamma\times 2\sqrt{3}\gamma\times 4\gamma\times 2\sqrt{3}\gamma\times 2\gamma}$$

$$= \sqrt[7]{299.05\gamma^7} = 2.25\gamma!$$

$$\therefore D_{S_1} = D_{S_2} = D_{S_3} = D_{S_4} = D_{S_5} = D_{S_6} = 2.25\gamma$$

$$\Rightarrow D_{S_7} = \sqrt[7]{D_{71}\cdot D_{72}\cdot D_{73}\cdot D_{74}\cdot D_{75}\cdot D_{76}\cdot D_{77}}$$

$D_{71} = D_{72} = D_{73} = D_{74} = D_{75} = D_{76} = 2\gamma$

$D_{77} = 0.7788\,\gamma.$

$$D_{S_7} = \sqrt[7]{(2\gamma)^6\times 0.7788\,\gamma.}$$

$$\boxed{D_{S_7} = 1.72\,\gamma}$$

$$\therefore GMR = D_S = \sqrt[7]{D_{S_1}\cdot D_{S_2}\cdot D_{S_3}\cdot D_{S_4}\cdot D_{S_5}\cdot D_{S_6}\cdot D_{S_7}}$$

$$D_S = \sqrt[7]{(2.25\gamma)^6\times 1.77\,\gamma.}$$

$$\boxed{D_S = 2.16\,\gamma.}$$

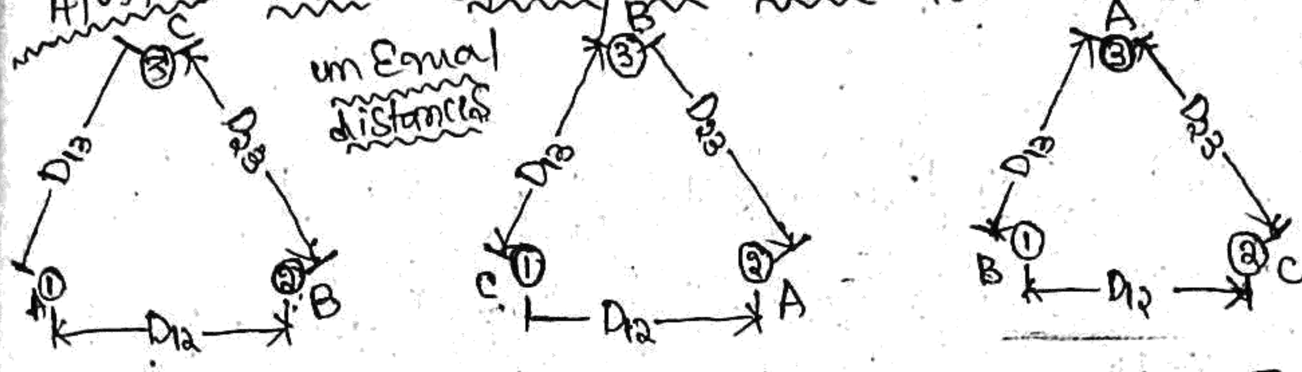
Ratio of = $\frac{GMR}{\text{overall Conductor Radius}}$

overall Radius = $\frac{\text{diameter}}{2} = \frac{2r+2r+2r}{2} = 3r$.

$$\text{Ratio} = \frac{2.16\cancel{r}}{3\cdot\cancel{r}} = 0.72.$$

2. Calculate the inductance of a single phase line consisting

Date 4/07/19 * Inductance of 3-ϕ System With unsymmetrical Spacing and With transposition:-



position - I position - II position - III

→ Let us Consider a 3-ϕ Circuit line, with Conductor A, B, C With unsymmetrical Spacing and transposition as Shown in the above fig.

→ The Current passing through Conductor A, B & C are given by I_A, I_B & I_C

→ The flux linkages of Conductor A due to I_A, I_B & I_C in position "1" is given by

$$\psi_{A_1} = 2\times10^{-7}\left[I_A \ln\frac{1}{r'} + I_B \ln\frac{1}{D_{12}} + I_C \ln\frac{1}{D_{13}}\right].$$

→ The flux linkages of Conductor "A" due to I_A, I_B & I_C in position "2" is given by.

$$\psi_{A_2} = 2\times10^{-7}\left[I_A \ln\frac{1}{r'} + I_B \ln\frac{1}{D_{23}} + I_C \ln\frac{1}{D_{12}}\right].$$

→ The flux linkages of Conductor "A" due to I_A, I_B & I_C

$$\psi_{A} = 2\times10^{-7}\left[I_A \ln\frac{1}{r'} + I_B \ln\frac{1}{D_{13}} + I_C \ln\frac{1}{D_{23}}\right]$$

To make Each and Every phase of Inductance are Equal

→ Average flux linkage of Conductor 'A' due to I_A, I_B & I_C at position 3

$$\psi_A = \frac{\psi_{A1} + \psi_{A2} + \psi_{A3}}{3}$$

$$\psi_A = \frac{2\times10^{-7}}{3}\left[3 I_A \ln\frac{1}{r'} + I_B\left(\ln\frac{1}{D_{12}} + \ln\frac{1}{D_{23}} + \ln\frac{1}{D_{13}}\right) + I_C\left(\ln\frac{1}{D_{12}} + \ln\frac{1}{D_{13}} + \ln\frac{1}{D_{23}}\right)\right]$$

$$\psi_A = \frac{2\times10^{-7}}{3}\left[3 I_A \ln\frac{1}{r'} + I_B \ln\frac{1}{D_{12}\cdot D_{23}\cdot D_{13}} + I_C \ln\frac{1}{D_{12} D_{13} D_{23}}\right]$$

$$= \frac{2\times10^{-7}}{3}\left[3 I_A \ln\frac{1}{r'} + \ln\frac{1}{D_{12}\cdot D_{13}\cdot D_{23}}\left(I_B + I_C\right)\right]$$

$$I_A + I_B + I_C = 0$$

$$I_B + I_C = -I_A$$

$$\psi_A = \frac{2\times10^{-7}}{3}\left[3 I_A \ln\frac{1}{r'} - I_A \ln\frac{1}{D_{12}\cdot D_{13}\cdot D_{23}}\right]$$

$$= \frac{2\times10^{-7}}{3}\times 3\left[I_A\left(\ln\frac{1}{r'} - \frac{1}{3}\ln\frac{1}{D_{12}\cdot D_{13}\cdot D_{23}}\right)\right]$$

$$= \frac{2\times10^{-7}}{\cancel{3}}\left[I_A\left(\ln\frac{1}{r'} - \ln\frac{1}{\sqrt[3]{D_{12}\cdot D_{13}\cdot D_{23}}}\right)\right]$$

$$\psi_A = 2\times10^{-7}\left[I_A \ln\frac{\sqrt[3]{D_{12}\cdot D_{13}\cdot D_{23}}}{r'}\right]$$

∴ Inductance of Conductor "A" is

$$L_A = \frac{\psi_A}{I_A}$$

$$L_A = \frac{2\times10^{-7}\,\cancel{I_A}}{\cancel{I_A}}\ln\frac{\sqrt[3]{D_{12}\cdot D_{13}\cdot D_{23}}}{r'}$$

$$L_A = 2\times10^{-7}\ln\frac{\sqrt[3]{D_{12}\cdot D_{13}\cdot D_{23}}}{r'}$$

Similarly,

→ The flux linkages of Conductor "B" due to I_A, I_B & I_C is

$$\psi_{B_1} = 2\times10^{-7}\left[I_B \ln\frac{1}{r'} + I_A \ln\frac{1}{D_{12}} + I_C \ln\frac{1}{D_{23}}\right.$$

→ The flux linkage of Conductor "B" due to I_A, I_B & I_C is

$$\psi_{B_2} = 2\times10^{-7}\left[I_B \ln\frac{1}{r'} + I_A \ln\frac{1}{D_{23}} + I_C \ln\frac{1}{D_{13}}\right.$$

→ The flux linkage of Conductor "B" due to I_A, I_B & I_C.

$$\psi_{B_3} = 2\times10^{-7}\left[I_B \ln\frac{1}{r'} + I_A \ln\frac{1}{D_{13}} + I_C \ln\frac{1}{D_{12}}\right.$$

→ Average flux linkage of Conductor "B" due to I_A, I_B & I_C

$$\psi_B = \frac{\psi_{B_1} + \psi_{B_2} + \psi_{B_3}}{3}$$

$$\psi_B = \frac{2\times10^{-7}}{3}\left(3 I_B \ln\frac{1}{r'} + I_A\left(\ln\frac{1}{D_{12}} + \ln\frac{1}{D_{23}} + \ln\frac{1}{D_{13}}\right) + I_C\left(\ln\frac{1}{D_{23}} + \ln\frac{1}{D_{23}} \cdot \ln\frac{1}{D_{12}}\right)\right]$$

$$\psi_B = \frac{2\times10^{-7}}{3}\left[3 I_B \ln\frac{1}{r'} + I_A \ln\frac{1}{D_{12}\cdot D_{23}\cdot D_{13}} + I_C \ln\frac{1}{D_{12} D_{13} D_{23}}\right]$$

$$= \frac{2\times10^{-7}}{3}\left[3 I_B \ln\frac{1}{r'} + \ln\frac{1}{D_{12}\cdot D_{23}\cdot D_{13}}\left(I_A + I_C\right)\right]$$

$\therefore\ I_B + I_A + I_C = 0$

$I_A + I_C = -I_B$

$$\psi_B = \frac{2\times10^{-7}}{3}\left[3 I_B \ln\frac{1}{r'} - I_B \ln\frac{1}{D_{12}\cdot D_{23}\cdot D_{13}}\right]$$

$$= \frac{2\times10^{-7}}{3}\times 3\left[I_B\left(\ln\frac{1}{r'} - \frac{1}{3}\ln\frac{1}{D_{12}\cdot D_{23}\cdot D_{13}}\right)\right]$$

$$= 2\times10^{-7}\left[I_B\left(\ln\frac{1}{r'} - \ln\frac{1}{\sqrt[3]{D_{12}\cdot D_{23}\cdot D_{13}}}\right)\right]$$

$$\psi_B = 2\times10^{-7}\left[I_B \ln\frac{\sqrt[3]{D_{12}\cdot D_{23}\cdot D_{13}}}{r'}\right]$$

Inductance of Conductor,

$$L_B = \frac{\psi_B}{I_B}$$

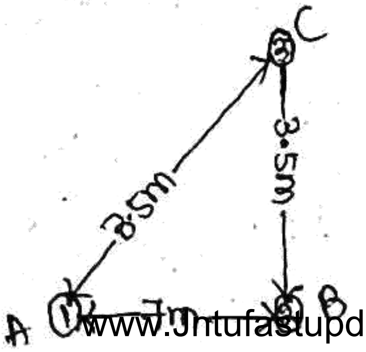
$$L_B = 2\times10^{-7}\ln\frac{\sqrt[3]{D_{12}\cdot D_{23}\cdot D_{13}}}{r'}\ \text{//}.$$

$||^{ly}$ $\boxed{L_C = 2\times10^{-7}\ln\frac{\sqrt[3]{D_{12}\cdot D_{23}\cdot D_{13}}}{r'}}$

→ With unsymmetrical spacing by doing trans-position we can get $L_A = L_B = L_C$

Problems

① Calculate the Inductance per phase of a 3-φ transmission line shown in fig. the radius of the conductor is 0.5cm, and the lines are untransposed



Sol:- Radius $(r) = 0.5\,cm$

$r = 0.5 \times 10^{-2}\, m.$

In unsymmetrical spacing without transposition.

Inductance / phase,

$$\rightarrow L_A = 2\times10^{-7}\left[\ln\frac{\sqrt{D_{12}\cdot D_{13}}}{r'} - j\frac{\sqrt{3}}{2}\ln\frac{D_{13}}{D_{12}}\right]$$

Here $r' = 0.7788\,r = 0.7788 \times 0.5\times10^{-2}$

$= 3.89 \times 10^{-3}$.

$D_{12} = 7m$, $D_{23} = 3.5\,m$, $D_{31} = 3.5\,m$.

$$\therefore L_A = 2\times10^{-7}\left[\ln\frac{\sqrt{7\times3.5}}{3.89\times10^{-3}} - j\frac{\sqrt{3}}{2}\ln\frac{3.5}{7}\right] H/m$$

$$= \cancel{2\times10^{-7}}\ 0.2\left[7.14 + j\,0.6\right] mH/Km$$

$L_A = 1.43 \angle 4.80\ mH/Km.$

$\rightarrow$ Inductance of Capacitor B,

$$L_B = 2\times10^{-7}\left[\ln\frac{\sqrt{D_{23}\cdot D_{12}}}{r'} - j\frac{\sqrt{3}}{2}\ln\frac{D_{12}}{D_{23}}\right].$$

$$L_B = 2\times10^{-7}\left[\ln\frac{\sqrt{3.5\times7}}{3.89\times10^{-3}} - j\frac{\sqrt{3}}{2}\ln\frac{7}{3.5}\right] H/m$$

$$= \cancel{2\times10^{-7}}\ 0.2\left[7.14 - j\,0.60\right] mH/Km.$$

$L_B = 1.43 \angle -4.80$

$\rightarrow$ Inductance of Capacitor C,

$$L_C = 2\times10^{-7}\left[\ln\frac{\sqrt{D_{13}\cdot D_{23}}}{r'} - j\frac{\sqrt{3}}{2}\ln\frac{D_{23}}{D_{13}}\right].$$

$$L_C = 2\times10^{-7}\left[\ln\frac{\sqrt{3.5\times3.5}}{3.89\times10^{-3}} - j\frac{\sqrt{3}}{2}\ln\left(\frac{3.5}{3.5}\right)\right] H/m$$

$$= \cancel{2\times10}\ 0.2\left[6.80 - 0\right] mH/m.$$

$$\boxed{L_C = 1.36}\ mH/m$$

$$\therefore \boxed{L_A \neq L_B \neq L_C}$$ //.

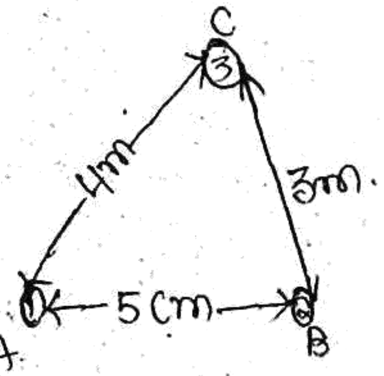
2. Calculate the inductance and inductive Reactance of Each phase of a "3-ϕ, 50 HZ" over head. high tension line which has Conductor diameter "2.5cm", the distance b/w the phases are 5m b/w A and B, "4m" b/w A & C. "3m" b/w B and B. Assume that the Conductors are transposed Regularly

Sol:-

$D_{ab} = 5m$

$D_{BC} = 3m$

$D_{CA} = 4m.$



$\Rightarrow r' = 0.7788\times r = 9.73\times10^{-3}$

Diameter = 2.5cm

radius = 1.25cm

$= 1.25\times0.2$

frequency = 50 HZ.

$$L_A = 2\times10^{-7}\ \ln\frac{\sqrt[3]{D_{12}\cdot D_{13}\cdot D_{23}}}{r'}$$

$$L_A = 2\times10^{-7}\ \ln\frac{\sqrt[3]{5\times4\times3}}{9.73\times10^{-3}}$$

$$L_A = 2\times10^{-7}\times 5.99\ H/m.$$

$$L_A = 0.2\times 5.99\ mH/km$$

$$\boxed{L_A = 1.19}$$

$\Rightarrow$ Inductive Reactive $(X_L) = \omega L$

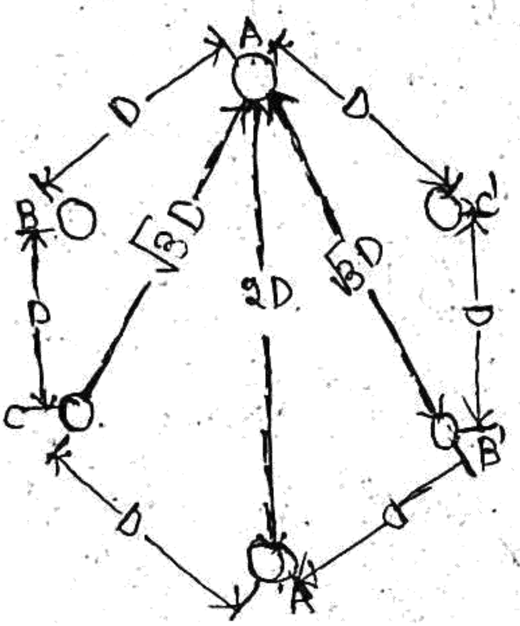
$$= 2\pi f \cdot L$$

$$= 2 \times 3.14 \times 50 \times 1.54$$

$$X_L = 376.8\ m\Omega$$

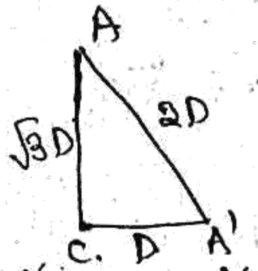
09/07/19

* Inductance of 3-ϕ double Circuit line With untransposed :-



$\rightarrow$ let us Consider a 3-ϕ double Circuit line of Conductors A, A', BB' & CC' with Currents of I_A, I_B & I_C and also I_A', I_B' & I_C'.

$\rightarrow$ The distance b/w Conductors A & C, A & B' is given by



$$\Rightarrow (AA')^2 = (AC)^2 + (A'C)^2$$

$$AC^2 = (AA')^2 - (A'C)^2$$

$$AC^2 = 4D^2 - D^2$$

$$\boxed{AC = \sqrt{3}D}$$

$\rightarrow$ The flux linkages of Conductor 'A' due to $I_A I_A'$, $I_B I_B'$ & $I_C I_C'$.

$$\psi_A = 2 \times 10^{-7} \left[\left(I_A \ln \frac{1}{r'} + I_A' \ln \frac{1}{2D} \right) + \left(I_B \ln \frac{1}{D} + I_B' \ln \frac{1}{\sqrt{3}D} \right) + \left(I_C \ln \frac{1}{D} + I_C' \ln \frac{1}{\sqrt{3}D} \right) \right]$$

Here, $I_A = I_A'$, $I_B = I_B'$, $I_C = I_C'$.

$$\psi_A = 2\times10^{-7}\left[\left(I_A \ln\frac{1}{r'} + I_A \ln\frac{1}{2D}\right) + \left(I_B \ln\frac{1}{D} + I_B \ln\frac{1}{\sqrt{3}D}\right) + \left(I_C \ln\frac{1}{D} + I_C \ln\frac{1}{\sqrt{3}D}\right)\right]$$

$$\psi_A = 2\times10^{-7}\left[I_A\left(\ln\frac{1}{r'} + \ln\frac{1}{2D}\right) + I_B\left(\ln\frac{1}{D} + \ln\frac{1}{\sqrt{3}D}\right) + I_C\left(\ln\frac{1}{D} + \ln\frac{1}{\sqrt{3}D}\right)\right]$$

$$\psi_A = 2\times10^{-7}\left[I_A \ln\frac{1}{2Dr'} + I_B \ln\frac{1}{\sqrt{3}D^2} + I_C \ln\frac{1}{\sqrt{3}D^2}\right]$$

$$\psi_A = 2\times10^{-7}\left[I_A \ln\frac{1}{2Dr'} + \ln\frac{1}{\sqrt{3}D^2}(I_B + I_C)\right]$$

But, $I_A + I_B + I_C = 0 \Rightarrow I_B + I_C = -I_A$.

$$\therefore \psi_A = 2\times10^{-7}\left[I_A \ln\frac{1}{2Dr'} - I_A \ln\frac{1}{\sqrt{3}D^2}\right]$$

$$= I_A\, 2\times10^{-7}\left[\ln\frac{\sqrt{3}D^2}{2Dr'}\right]$$

$$\psi_A = I_A\, 2\times10^{-7}\left[\ln \frac{\sqrt{3}}{2}\cdot\frac{D}{r'}\right]$$

→ Inductance of Conductor 'A' is

$$L_A = \psi_A / I_A = \cancel{I_A}/\cancel{I_A} \times 2\times10^{-7} \ln \frac{\sqrt{3}}{2}\cdot\frac{D}{r'}$$

$$\boxed{L_A = 2\times10^{-7} \ln \frac{\sqrt{3}}{2}\cdot\frac{D}{r'}}$$

→ The flux linkages of Conductor 'B' due to $I_A I_A'$, $I_B I_B'$, $I_C I_C'$.

$$\psi_B = 2\times10^{-7}\left[\left(I_B \ln\frac{1}{r'} + I_B' \ln\frac{1}{2D}\right) + \left(I_A \ln\frac{1}{D} + I_A' \ln\frac{1}{\sqrt{2}D}\right) + \left(I_C \ln\frac{1}{D} + I_C' \ln\frac{1}{\sqrt{2}D}\right)\right]$$

$$\psi_B = 2\times10^{-7}\left[\left(I_B \ln\frac{1}{r'} + I_B \ln\frac{1}{2D}\right) + \left(I_A \ln\frac{1}{D} + I_A \ln\frac{1}{\sqrt{2}D}\right) + \left(I_C \ln\frac{1}{D} + I_C \ln\frac{1}{\sqrt{2}D}\right)\right]$$

$$\psi_B = 2\times10^{-7}\left(I_B\left(\ln\frac{1}{r'} + \ln\frac{1}{2D}\right) + I_A\left(\ln\frac{1}{D} + \ln\frac{1}{\sqrt{2}D}\right) + I_C\left(\ln\frac{1}{D} + \ln\frac{1}{\sqrt{2}D}\right)\right)$$

$$\psi_B = 2\times10^{-7}\left[I_B \ln\frac{1}{2Dr'} + \ln\frac{1}{\sqrt{2}D^2}(I_A + I_C)\right]$$

But, $I_A + I_B + I_C = 0 \Rightarrow -I_B = I_A + I_C$

$$\psi_B = 2\times10^{-7}\left(I_B \ln\frac{1}{2Dr'} - I_B \ln\frac{1}{\sqrt{2}D^r}\right)$$

$$= I_B\, 2\times10^{-7}\left[\ln\frac{\sqrt{2}D^r}{2Dr'}\right]$$

$$\psi_B = I_B\, 2\times10^{-7}\left[\ln \sqrt{2}/2 \cdot D/r'\right]$$

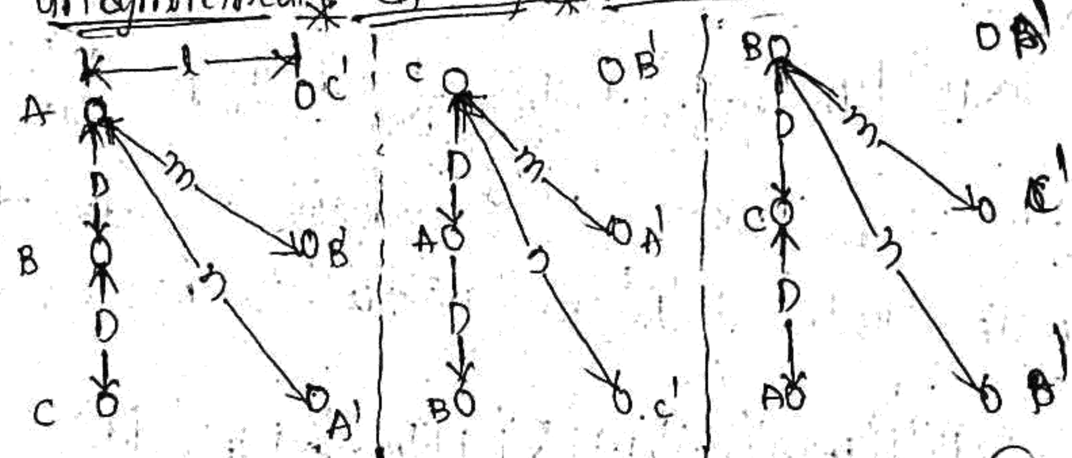
→ Inductance of Conductor 'B' is

$$L_B = \frac{\psi_B}{I_B} = \frac{\cancel{I_B}}{\cancel{I_B}}\times 2\times10^{-7}\left[\ln \sqrt{2}/2 \cdot D/r'\right]$$

$$L_B = 2\times10^{-7}\left[\sqrt{2}/2 \cdot D/r'\right].$$

$$\therefore \boxed{L_A \neq L_B}$$

* Inductance of a 3-ϕ double circuit line with unsymmetrical spacing and transposition:-



Position-I Position-II Position-III

→ Let us Consider a 3-ϕ double ckt line of Conductor A, B & C. With Currents $I_A I_A'$; $I_B I_B'$ & $I_C I_C'$ as shown in the figure.

→ The flux linkages of Conductor A due to $I_A I_A'$, $I_B I_B'$, $I_C I_C'$ in position-I is given by

$$\psi_{A_1} = 2\times10^{-7}\left[\left(I_A \ln\frac{1}{r'} + I_A' \ln\frac{1}{h}\right) + \left(I_B \ln\left(\frac{1}{D} + I_B' \ln\frac{1}{m}\right)\right.\right.$$
$$\left.\left. + I_C \ln\frac{1}{2D} + I_C' \ln\frac{1}{l}\right)\right]$$

Here, $I_A = I_A'$, $I_B = I_B'$, $I_C = I_C'$

$$\psi_{A_1} = 2\times10^{-7}\left[\left(I_A \ln\frac{1}{r'} + I_A \ln\frac{1}{h}\right) + \left(I_B \ln\frac{1}{D} + I_B \ln\frac{1}{m}\right) + \left(I_C \ln\frac{1}{2D} + I_C \ln\frac{1}{l}\right)\right]$$

$$\psi_{A_1} = 2\times10^{-7}\left[I_A\left(\ln 1/r' + \ln 1/h\right) + I_B\left(\ln 1/D + \ln 1/m\right) + I_C\left(\ln 1/2D + \ln 1/l\right)\right]$$

$$\psi_{A_1} = 2\times10^{-7}\left[I_A\left(\ln\frac{1}{hr'}\right) + I_B \ln\frac{1}{Dm} + I_C \ln\frac{1}{2Dl}\right]$$

→ The flux linkages of conductor "A" due to $I_A I_A'$, $I_B I_B'$, $I_C I_C'$ in position-2 is given by,

$$\psi_{A2} = 2\times10^{-7}\left[\left(I_A \ln 1/r' + I_A' \ln\frac{1}{l}\right) + \left(I_B \ln 1/D + I_B' \ln(1/m)\right)\right.$$
$$\left. + \left(I_C \ln 1/D + I_C' \ln(1/m)\right)\right]$$

$$\psi_{A2} = 2\times10^{-7}\left[I_A \ln\frac{1}{r'l} + I_B \ln\frac{1}{Dm} + I_C \ln 1/Dm\right]$$

→ The flux linkages of conductor "A" due to $I_A I_A'$, $I_B I_B'$, $I_C I_C'$ in position-3 is given by

$$\psi_{A3} = 2\times10^{-7}\left[\left(I_A \ln 1/r' + I_A' \ln(1/h)\right) + \left(I_B \ln 1/2D + I_B' \ln\left(\frac{1}{l}\right)\right)\right.$$
$$\left. + \left(I_C \ln 1/D + I_C' \ln(1/m)\right)\right]$$

$$\psi_{A3} = 2\times10^{-7}\left[I_A \ln\frac{1}{hr'} + I_B \ln\frac{1}{2Dl} + I_C \ln\frac{1}{Dm}\right]$$

→ Average of flux linkages of conductor is,

$$\psi_A = \frac{\psi_{A1} + \psi_{A2} + \psi_{A3}}{3}$$

$$\psi_A = \frac{2\times10^{-7}}{3}\left[I_A \ln\left(\frac{1}{r'n}+\frac{1}{mr'}+\frac{1}{r'l}\right)+I_B \ln\left[\frac{1}{Dm}+\frac{1}{Dm}+\frac{1}{2Dl}\right]\right.$$
$$\left.+ I_C \ln\left(\frac{1}{2Dl}+\frac{1}{Dm}+\frac{1}{Dm}\right)\right]$$

$$\psi_A = \frac{2\times10^{-7}}{3}\left(I_A \ln\left(\frac{1}{r'n}+\frac{1}{mr'}+\frac{1}{r'l}\right)+\ln\left[\frac{1}{Dm}+\frac{1}{Dm}+\frac{1}{2Dl}\right](I_B+I_C)\right)$$

But, $I_A+I_B+I_C=0 \Rightarrow I_B+I_C=-I_A$.

$$\psi_A = \frac{2\times10^{-7}}{3}\left(I_A \ln\left[\frac{1}{n^2(r')^3 l}\right] - \ln\left(\frac{1}{2D^3m^2 l}\right)I_A\right)$$

$$\psi_A = I_A\frac{2\times10^{-7}}{3}\left[\ln\frac{2D^3m^2 l}{n^2(r')^3 l}\right]$$

$$\psi_A = I_A\frac{2\times10^{-7}}{3}\left[\ln\frac{2D^3m^2}{n^2(r')^3}\right].$$

→ Inductance of Conductor "A" is

$$L_A = \frac{\psi_A}{I_A} = \frac{\cancel{I_A}}{\cancel{I_A}}\frac{2\times10^{-7}}{3}\ln\frac{2D^3m^2}{n^2(r')^3}$$

$$\cancel{\boxed{L_A = \frac{2\times10^{-7}}{3}\ln\frac{2D^3m^2}{n^2(r')^3}}}$$

$$\Rightarrow \boxed{L_A = 2\times10^{-7}\ln\left(2\right)^{1/3}\left(D/r'\right)\left(m/n\right)^{2/3}} \text{ H/m}$$

→ The flux linkage of Conductor "B" is given in position is

$$\psi_{B_1} = 2\times10^{-7}\left(I_B\ln(1/r') + I_B'\ln 1/l\right) + \left(I_A\ln 1/D + I_A'\ln 1/m\right) + \left(I_C\ln(1/D)\right.$$
$$\left. + I_C'\ln(1/m)\right)$$

$$\psi_{B_1} = 2\times10^{-7}\left[I_B \ln \frac{1}{r'_1} + I_A \ln \frac{1}{D_m} + I_C \ln \frac{1}{D_m}\right]$$

→ The flux linkages of conductor "B" in position-2.

$$\psi_{B_2} = 2\times10^{-7}\left[\left(I_B \ln \frac{1}{r'} + I_B \ln \frac{1}{n}\right) + \left(I_A \ln \frac{1}{D} + I_A' \ln\left(\frac{1}{m}\right)\right) + \left(I_C \ln \frac{1}{2D} + I_C' \ln\left(\frac{1}{l}\right)\right)\right]$$

$$\psi_{B_2} = 2\times10^{-7}\left[I_B \ln \frac{1}{r'n} + I_A \ln \frac{1}{D_m} + I_C \ln \frac{1}{2Dl}\right]$$

→ The flux linkages of conductor 'B' in position-3.

$$\psi_{B_3} = 2\times10^{-7}\left[\left(I_B \ln \frac{1}{r'} + I_B' \ln\left(\frac{1}{n}\right)\right) + I_A \ln\left(\frac{1}{2D}\right) + I_A' \ln\left(\frac{1}{l}\right) + I_C \ln\left(\frac{1}{D}\right) + I_C'\left(\frac{1}{m}\right)\right]$$

$$\psi_{B_3} = 2\times10^{-7}\left[\left(I_B \ln\left(\frac{1}{nr'}\right)\right) + I_A \ln\left(\frac{1}{2Dl}\right) + I_C \ln\left(\frac{1}{D_m}\right)\right]$$

→ The flux linkage of conductor "B" is.

$$\psi_B = \frac{\psi_{B_1} + \psi_{B_2} + \psi_{B_3}}{3}$$

$$\psi_B = \frac{2\times10^{-7}}{3}\left[I_B \ln\left(\frac{1}{r'l} + \frac{1}{r'n} + \frac{1}{r'n}\right) + I_A \ln\left[\frac{1}{D_m} + \frac{1}{D_m} + \frac{1}{2Dl}\right] + I_C \ln\left[\frac{1}{D_m} + \frac{1}{2Dl} + \frac{1}{D_m}\right]\right].$$

$$\psi_B = 2\times10^{-7}\left[I_B \ln \frac{1}{(r')^3 (n)^2 l} \; \ln \frac{1}{2D^3 m^2 l}\,[I_A + I_C]\right]$$

$$\psi_B = 2\times10^{-7}\left(I_B \cdot \ln \frac{2D^3 m^2 l}{(r')^3 (n)^2 l}\right)$$

$$\psi_B = 2\times10^{-7} I_B \cdot \ln (2)^{1/3}\left(D/r'\right)\left(m/n\right)^{2/3}$$

→ Inductance of Conductor B is,

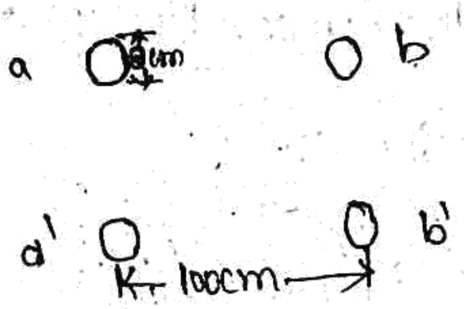
$L_B = \psi_B / I_B$

$$\boxed{L_B = 2\times10^{-7}\left[\ln (2)^{1/3} (D/r') (m/n)^{2/3}\right] H/m}$$

11/07/19

Problems

1. Calculate the inductance of 1-φ double ckt. line as shown in figure. The diameter of each conductor is 2cm



Sol:-

For double Circuit,

inductance /phase $= 2\times10^{-7} \ln \frac{Dm}{DS}$

$DS = \sqrt[4]{DS_1\, DS_2\, DS_3\, DS_4}$

$= \sqrt[4]{D_{aa}\, D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}}$

$D_{aa} = r' = 0.778\times 2/2 \times 10^{-2} = 0.7788\times10^{-2}$ m

$= 0.7788$ cm.

$D_{aa'} = D_{a'a} = 100$ cm

$D_{aa} = D_{a'a'} = \cancel{0.7788}\times10^{-2}$.

$DS = \sqrt[4]{(0.7788)\times100\times100\times0.778} = 8.825$ cm.

$$D_m = \sqrt[4]{D_{m1} D_{m2} D_{m3} D_{m4}}$$

$$= \sqrt[4]{D_{ab} \cdot D_{ab'} \cdot D_{a'b} \cdot D_{a'b'}}$$

$D_{ab'} = 100$ cm.

$$D_{a'b'} = \sqrt{(D_{ab})^2 + (D_{bb'})^2}$$

$$= \sqrt{(100)^2 + (100)^2}$$

$D_{ab'} = 141.42$ cm

$\Rightarrow D_{ab'} = D_{a'b} = 141.42$ cm

$$D_m = \sqrt[4]{100 \times 141.42 \times 141.42 \times 100}$$

$= 118.92$ cm.

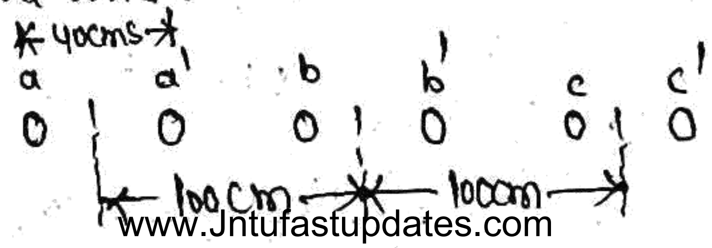
$\therefore$ Inductance $= 2 \times 10^{-7} \ln \frac{D_m}{D_S}$

$$= 2 \times 10^{-7} \ln \frac{118.92}{8.825}$$

$= 5.20 \times 10^{-7}$ H/cm

$$\boxed{L = 0.52 \text{ mH/km}}$$

Q. A three phase double ckt line with two sub conductors of phase has horizontal configuration as shown in the figure. Find the inductance per phase. If the radius of the conductor is 1.2 cms.



Sol: - Inductance / phase $= 2\times10^{-7} \ln \frac{Dm}{DS}$

Here, Radius = 1.2 cm.

Effective Radius = $D_{aa} = D_{a'a'} = r' = 0.7788 \times r$

$= 0.7788 \times 1.2$

$= 0.934$ cm.

$$D_4 = \sqrt[4]{D_{aa} \cdot D_{aa'} \cdot D_{a'a'} \cdot D_{a'a}}$$

$$= \sqrt[4]{0.934 \times 40 \times 0.934 \times 40}$$

$= 6.1$ cm.

$$Dm = \sqrt[8]{(D_{ab} \cdot D_{ab'} \cdot D_{ac} \cdot D_{ac'}) \cdot (D_{a'b} \cdot D_{a'b'} \cdot D_{a'c} \cdot D_{a'c'})}$$

$D_{ab} = 100$ cm, $D_{ab'} = 140$, $D_{ac} = 200$, $D_{ac'} = 240$.

$D_{a'b} = 60$ cm, $D_{a'b'} = 100$ cm, $D_{a'c} = 160$, $D_{a'c'} = 200$.

$$Dm = \sqrt[8]{100 \times 200 \times 140 \times 240 \times 60 \times 100 \times 160 \times 200}$$

$$\boxed{Dm = 137.6 \text{ cm}}$$

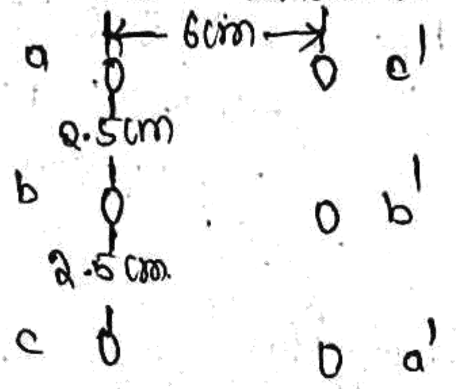
Induction / phase $L_A = 2\times10^{-7} \ln \frac{Dm}{DS}$

$= 2\times10^{-7} \ln \frac{137.6}{6.1}$

$= 6.23\times10^{-7}$ H/m.

$$\boxed{L_A = 0.623 \text{ mH/km}}$$

3. Calculate the inductance / phase of a 3-φ double Circuit line which is shown in the fig. the 'dia' of Each Conductor is 1.5 cms



Sol :- Inductance / phase $L_A = 2\times10^{-7} \frac{Dm}{DS}$

$$R = \frac{1.5}{2} = 0.75\,cm.$$

Effective Radius = $D_{aa} = r' = 0.7788\,r$

$= 0.7788 \times 0.75$

$= 0.5841$ c.m.

DSs =

$$L_A = 2\times10^{-7} \ln 2^{1/3} \left(D/r'\right)\left(m/n\right)^{2/3}$$

$r' = 0.7788 \times r$

$= 0.7788 \times 0.75$

$= 0.58$

$$M = \sqrt{(6\times10^{2})^2 + (2.5)^2} = 600\ cm.$$

$$n = \sqrt{(6\times10^{2})^2 + (2.5+2.5)^2} \simeq 600\ cm.$$

$$L_A = 2\times10^{-7} \ln 2^{1/3} \left(\frac{2.5}{0.58}\right)\left(\frac{600}{600}\right)^{2/3}$$

$= 3.38\times10^{-7}$ H/m $= 0.338 \times$ mH/km.

* The three Conductors of a 3-ϕ transmission line are arrange in a horizontal plane and are 5m apart the diameter of Each Conductor is 30mm. determine the inductance per km. of Each Conductor assume Balanced load RYB phase sequence Determine also the average inductance per phase for regularly transposed the.

* Calculate the inductance per phase of a three phase double Ckt line. If the Conductors are spaced at the vertices of Hexagonal spacing of side 2M Each the diameter of Each Conductor is 2cm.

* Calculate the inductance per phase. of a 3-ϕ double Ckt line With the Conductors are spaced at the vertices of Hexagonal. spacing of side 3.2m Each the diameter of the Conductor is 1.5cm.

* Determine the inductance of Each phase a 3-ϕ, 50HZ

15/07/19

* Capacitance :-

→ Capacitance is the ability (or) Capacity to store the Electric Charges in Conductors.

→ The Capacitance of a Capacitor is defined as the Charge per unit potential difference

$$\therefore \boxed{C = Q/V}$$

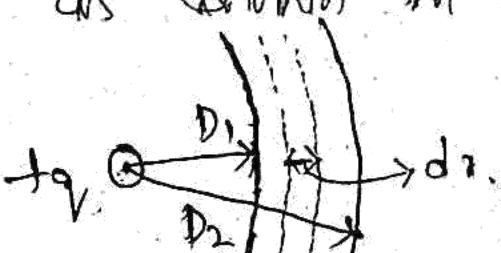
→ Potential difference B/W two points ~~has~~ due to a Charge.

→ The potential difference B/W two points is Equal to the Work done in J/col Necessary to move a Columb of Charge b/w two Points.

→ According to the Gauss theorem, the Electrical field intensity at an ~~distance~~ distance "x" m is given by

$$\boxed{E = \frac{Q}{2\pi x \varepsilon}} \ V/m .$$

→ Let us Consider a Straight Conductor Carried a Positive Charge "Q" as shown in the figure.



→ The points ① & ② are located at a distance of "D_1, D_2" met, from the Centre of a Conductor then the potential difference b/w points ① & ② is obtained by integrating the Electric field intensity over a Radial path.

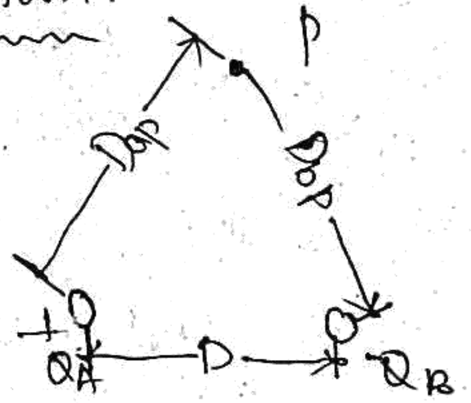
$$V_{12} = \int_{D_1}^{D_2} E_x \, dx$$

$$V_{12} = \int_{D_1}^{D_2} \frac{Q}{2\pi \varepsilon x} dx = \frac{Q}{2\pi\varepsilon} \int_{D_1}^{D_2} \frac{1}{x} dx$$

$$\boxed{V_{12} = Q/_{2\pi\varepsilon} \ln\left(\frac{D_2}{D_1}\right)}$$

* **Capacitance of 1-ϕ System:-**

→ let us Consider a 1-ϕ System with Conductors A & B and distance of "D" m. With a Radius of "R".



→ The Charges of Conductors A & B is Q_A & Q_B.

→ Let us Assume a point "P" at a far distance from Charge Q_A and Q_B.

Then the voltage b/w Conductors "A" & point "P" is given by "B_{AP}".

$$V_{ap} = \frac{Q}{2\pi\varepsilon} \ln \frac{D_{ap}}{r}.$$

Where.

D_{ap} = distance b/w Conductor "a" and "p"

r = Radius of Conductor

iiiy, the voltage (V_{ap}) due to charge on Co-
-nductor 'B' is given by.

$$V_{ap} = \frac{Q_B}{2\pi\varepsilon} \ln \frac{D_{bp}}{D}.$$

∴ The potential difference b/w Conductor 'A' and point "p" due to charge Q_A & Q_B is given by.

$$V_a = \frac{Q_A}{2\pi\varepsilon} \ln \frac{D_{ap}}{r} + \frac{Q_B}{2\pi\varepsilon} \ln \frac{D_{bp}}{D}.$$

Here $Q_A = -Q_B$.

$$V_a = \frac{Q_A}{2\pi\varepsilon} \ln \frac{D_{ap}}{r} - \frac{Q_A}{2\pi\varepsilon_0} \ln \frac{D_{bp}}{D}.$$

$$V_a = \frac{Q_A}{2\pi\varepsilon} \left[\ln \frac{D_{ap}}{r} - \ln \frac{D_{bp}}{d}\right]$$

$$V_a = \frac{Q_A}{2\pi\varepsilon} \left[\ln \frac{D_{ap}}{r} \times \frac{D}{D_{bp}}\right]$$

Here $\boxed{D_{ap} \simeq D_{bp}}$

$$V_a = \frac{Q_A}{2\pi\varepsilon} \left[\ln \frac{D_{ap}}{D_{ap}} \times D/r\right]$$

$$\boxed{V_a = \frac{Q_A}{2\pi\epsilon}\left[\ln D/r\right]}$$

$\therefore$ Capacitance of Conductor "A" is.

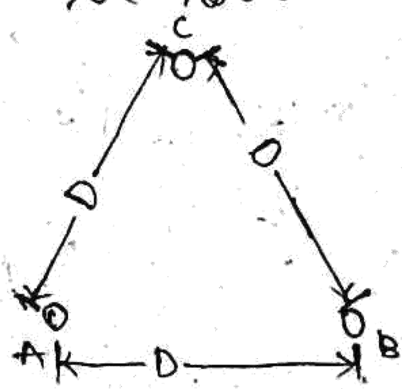
$$C_A = \frac{Q_A}{V_A} = \frac{Q_A}{\frac{Q_A}{2\pi\epsilon}\ln D/r}$$

$$\boxed{C_A = \frac{2\pi\epsilon}{\ln D/r}}$$

$$\Rightarrow C_A = \qquad \boxed{C_A = \frac{10^{-9}}{18 \ln D/r}} \text{ F/m}$$

16/07/19

* Capacitance of 3-ϕ System With Symmetrical Spacing



→ Let us Consider a 3-ϕ CKt line of Conductor A, B & C With Charges of Q_A, Q_B, Q_C at a distance of "D" meters as Shown in the figure.

→ let us Consider point 'p' at a far distance then the potential difference b/w Conductor "A" and point 'p" due to Charging Q_A, ~~Q_B~~ ~~Q_C~~ is given by, $\boxed{V_{AP} = \frac{Q_A}{2\pi\epsilon} \ln 1/r}$

The potential difference b/w Conductor "A" and point "p" due to Charge "Q_B".

$$\boxed{V_{APQ_B} = \frac{Q_B}{2\pi\varepsilon} \ln \frac{1}{D}}.$$

The potential difference b/w Conductor "A" and point "p" due to Charge "Q_C"

$$\boxed{V_{APQ_C} = \frac{Q_C}{2\pi\varepsilon} \ln \frac{1}{D}}$$

∴ The potential difference b/w Conductor "A" and point "p" due to Charge Q_A, Q_B & Q_C is given by.

$$V_a = V_{APQ_A} + V_{APQ_B} + V_{APQ_C}.$$

$$V_a = \frac{Q_A}{2\pi\varepsilon_0} \ln \frac{1}{r} + \frac{Q_B}{2\pi\varepsilon} \ln \frac{1}{D} + \frac{Q_C}{2\pi\varepsilon} \ln \frac{1}{D}.$$

$$V_a = \frac{1}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{r} + Q_B \ln \frac{1}{D} + Q_C \ln \frac{1}{D}\right]$$

$$V_a = \frac{1}{2\pi\varepsilon}\left[Q_A \ln\left(\frac{1}{r}\right) + \ln \frac{1}{D}\left(Q_B + Q_C\right)\right]$$

$$\Rightarrow \quad Q_A + Q_B + Q_C = 0$$

$$Q_B + Q_C = -Q_A$$

$$V_a = \frac{1}{2\pi\varepsilon}\left[Q_A \ln\left(\frac{1}{r}\right) - Q_A \ln \frac{1}{D}\right]$$

$$V_a = \frac{1}{2\pi\varepsilon_0} Q_A \ln \frac{D}{r}$$

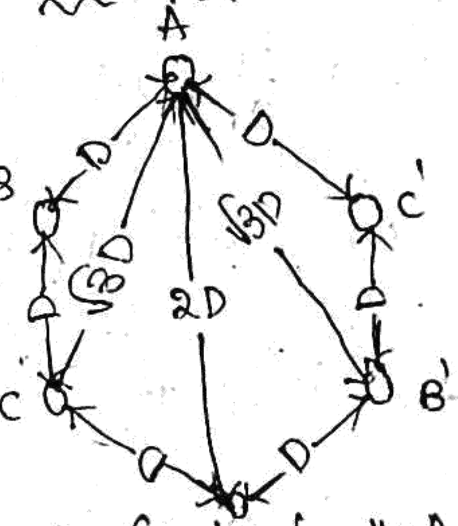
→ The Capacitance of Conductor "A"

$$C_A = \frac{Q_A}{V_A} = \frac{\cancel{Q_A}}{\frac{1}{2\pi\epsilon_0}\cancel{Q_A}\ln(D/r)}$$

$$= \frac{2\pi\epsilon}{\ln(D/r)} = \frac{10^{-9}}{18\ln(D/r)}$$

$$\therefore \boxed{C_A = \frac{10^{-9}}{18\ln(D/r)}} \text{ F/m.}$$

* Capacitance of 3-ϕ double ckt line ~~system~~ With Hexagonal Spacing:-



→ Let us Consider a 3-ϕ double ckt line of Conductors AA', BB' & CC' With Charges ~~Currents~~ Q_A, Q_B & Q_C and also Q_A', Q_B' & Q_C'.

→ The potential difference of Conductor "A" due to $Q_A Q_A'$, $Q_B Q_B'$, $Q_C Q_C'$ is given by.

$$V_A = \frac{1}{2\pi\epsilon}\left[\left(Q_A\ln\frac{1}{r'} + Q_A'\ln\frac{1}{2D}\right) + \left(Q_B\ln\frac{1}{D} + Q_B'\ln\frac{1}{\sqrt{3}D}\right) + \left(Q_C\ln\frac{1}{D} + Q_C'\ln\frac{1}{\sqrt{3}D}\right)\right].$$

Here, $Q_A = Q_A'$, $Q_B = Q_B'$, $Q_C = Q_C'$.

$$V_A = \frac{1}{2\pi\epsilon}\left[\left(Q_A\ln 1/r' + Q_A\ln\frac{1}{2D}\right) + \left(Q_B\ln 1/D + Q_B\ln\frac{1}{\sqrt{3}D}\right) + \left(Q_C\ln 1/D + Q_C\ln\frac{1}{\sqrt{3}D}\right)\right]$$

$$V_A = \frac{1}{2\pi\varepsilon}\left[Q_A\left(\ln \frac{1}{r} + \ln \frac{1}{2D}\right) + Q_B\left(\ln \frac{1}{D} + \ln \frac{1}{\sqrt{3}D}\right) + Q_C\left(\ln \frac{1}{D} + \ln \frac{1}{\sqrt{3}D}\right)\right]$$

$$V_A = \frac{1}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{2Dr} + Q_B \ln \frac{1}{\sqrt{3}D^2} + Q_C \ln \frac{1}{\sqrt{3}D^2}\right]$$

$$V_A = \frac{1}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{2Dr} + \ln \frac{1}{\sqrt{3}D^2}\left(Q_B + Q_C\right)\right]$$

Here, $Q_A + Q_B + Q_C = 0$; $Q_B + Q_C = -Q_A$.

$$V_A = \frac{1}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{2Dr} - Q_A \ln \frac{1}{\sqrt{3}D^2}\right]$$

$$V_A = \frac{1}{2\pi\varepsilon}\left[Q_A \ln \frac{\sqrt{3}D^{2}}{2\cancel{D}r}\right]$$

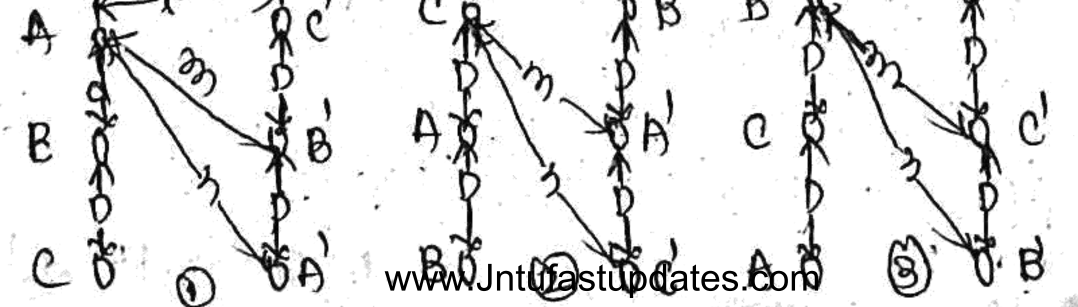
$$V_A = \frac{1}{2\pi\varepsilon} Q_A \ln \frac{\sqrt{3}}{2}\cdot \frac{D}{r}$$

→ Capacitance of Conductor "A" is.

$$C_A = \frac{Q_A}{V_A} = \frac{\cancel{Q_A}}{\frac{1}{2\pi\varepsilon}\cancel{Q_A} \ln \frac{\sqrt{3}}{2}\cdot\frac{D}{r}}$$

$$C_A = \frac{2\pi\varepsilon}{\ln \frac{\sqrt{3}}{2}\cdot \frac{D}{r}} = \frac{10^{-9}}{18 \ln \frac{\sqrt{3}}{2}\cdot\frac{D}{r}}$$

* Capacitance of 3-φ double ckt line with unsymmetrical spacing and transposition.



→ let us Consider a three phase double ckt line of Conductor A, B & C with Charges $Q_A Q_A'$, $Q_B Q_B'$ & $Q_C Q_C'$ as shown in figure.

→ The potential difference of Conductor "A" due to $Q_A Q_A'$, $Q_B Q_B'$, $Q_C Q_C'$ is given by

$$V_{A1} = \frac{Q}{2\pi\varepsilon}\left[\left(Q_A \ln \frac{1}{r} + Q_A' \ln \frac{1}{n}\right) + \left(Q_B \ln \frac{1}{D} + Q_B' \ln \frac{1}{m}\right) + \left(Q_C \ln \frac{1}{2D} + Q_C' \ln \frac{1}{l}\right)\right]$$

Here, $Q_A = Q_A'$, $Q_B = Q_B'$, $Q_C = Q_C'$

$$V_{A1} = \frac{Q}{2\pi\varepsilon}\left[\left(Q_A \ln \frac{1}{r} + Q_A \ln \frac{1}{n}\right) + \left(Q_B \ln \frac{1}{D} + Q_B' \ln \frac{1}{m}\right) + \left(Q_C \ln \frac{1}{2D} + Q_C' \ln \frac{1}{l}\right)\right]$$

$$V_{A1} = \frac{Q}{2\pi\varepsilon}\left[Q_A\left(\ln \frac{1}{r} + \ln \frac{1}{n}\right) + Q_B\left(\ln \frac{1}{D} + \ln \frac{1}{m}\right) + Q_C\left(\ln \frac{1}{2D} + \ln \frac{1}{l}\right)\right]$$

$$V_{A1} = \frac{Q}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{rn} + Q_B \ln \frac{1}{Dm} + Q_C \ln \frac{1}{2Dl}\right]$$

→ The potential difference of Conductor "A" due to $Q_A Q_A'$, $Q_B Q_B'$, $Q_C Q_C'$.

$$V_{A2} = \frac{Q}{2\pi\varepsilon}\left[\left(Q_A \ln \frac{1}{r} + Q_A' \ln \frac{1}{l}\right) + \left(Q_B \ln \frac{1}{D} + Q_B' \ln\left(\frac{1}{m}\right)\right) + \left(Q_C \ln \frac{1}{D} + Q_C' \ln \frac{1}{m}\right)\right].$$

Here $Q_A = Q_A'$, $Q_B = Q_B'$, $Q_C = Q_C'$

$$V_{A2} = \frac{Q}{2\pi\varepsilon}\left[Q_A \ln \frac{1}{rl} + Q_B \ln \frac{1}{Dm} + Q_C \ln \frac{1}{Dm}\right].$$

→ The potential difference of Conductor "A" due to $Q_A Q_A'$, $Q_B Q_B'$, $Q_C Q_C'$

$$V_{A3} = \frac{Q}{2\pi\varepsilon}\left[\left(Q_A \ln \frac{1}{r} + Q_A' \ln \frac{1}{n}\right) + \left(Q_B \ln \frac{1}{2D} + Q_B' \ln \frac{1}{l}\right) + \left(Q_C \ln \frac{1}{D} + Q_C' \ln \frac{1}{m}\right)\right.$$

Here $Q_A = Q_A'$, $Q_B = Q_B'$, $Q_C = Q_C'$

$$V_{A3} = \frac{1}{2\pi\varepsilon}\left[Q_A \ln\frac{1}{nr'} + Q_B \ln\frac{1}{2D\lambda} + Q_C \ln\frac{1}{Dm}\right],$$

$\longrightarrow$ Average of potential of Conductor is,

$$V_A = \frac{V_{A1} + V_{A2} + V_{A3}}{3}$$

$$V_A = \frac{1}{\frac{2\pi\varepsilon_0}{3}}\left[Q_A \ln\left(\frac{1}{r'n} + \frac{1}{n'r'} + \frac{1}{r'_1}\right) + Q_B \ln\left(\frac{1}{Dm} + \frac{1}{Dm} + \frac{1}{2D\lambda}\right) + Q_C \ln\left(\frac{1}{2D\lambda} + \frac{1}{Dm} + \frac{1}{Dm}\right)\right]$$

$$V_A = \frac{1}{\frac{2\pi\varepsilon}{3}}\left[Q_A \ln\left[\frac{1}{n^2 (r')^3 \cdot \lambda}\right] + \ln\left[\frac{1}{2(D^3)m^2 \cdot \lambda}\right](Q_B + Q_C)\right]$$

Here, $Q_A + Q_B + Q_C = 0 \Rightarrow Q_B + Q_C = -Q_A$.

$$V_A = \frac{1}{\frac{2\pi\varepsilon}{3}}\left[Q_A \ln\frac{1}{n^2 (r')^3 \cdot \lambda} - Q_A \ln\frac{1}{2D^3 m^2 \cdot \lambda}\right]$$

$$V_A = \frac{Q_A}{\frac{2\pi\varepsilon}{3}}\left[\ln\frac{2D^3 m^2 \cancel{\lambda}}{n^2 (r')^3 \cdot \cancel{\lambda}}\right].$$

$\longrightarrow$ Capacitance of Conductor 'A' is

$$C_A = \frac{Q_A}{V_A} = \frac{\cancel{Q_A}}{\frac{\cancel{Q_A}}{\frac{2\pi\varepsilon}{3}}\left[\ln\left[\frac{2D^3 m^2}{n^2 (r')^3}\right]\right]}$$

$$\boxed{C_A = \frac{10^{-9}}{18 \ln (2)^{1/3} (D/r') \cdot (m/n)^{2/3}}}$$ //