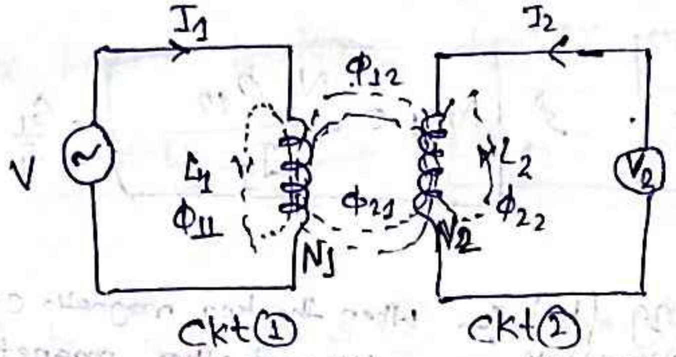
## Unit-5 Self Inductance & Mutual Inductance

Consider that two different circuits with Self Inductance and mutual inductance are kept close to each other.




flux linkage between two circuits

* Self Inductance

Flux linkage of coil ($\lambda$) $\propto$ Current (I)

$$\lambda = LI$$

$$N\phi = LI$$

L → Constant of proportionality = Inductance

$$\boxed{L = \frac{\lambda}{I} = \frac{N\phi}{I}}$$

* Self inductance coil (i) is given by,

$$\boxed{L_1 = \frac{N_1\phi_{11}}{I_1} = \frac{N_1\phi_{1}}{I_1}} \text{ H}$$

* Self inductance of coil (ii) is given by

$$\boxed{L_2 = \frac{N_2\phi_{22}}{I_2} = \frac{N_2\phi_2}{I_2}} \text{ H}$$

Mutual Inductance (M)

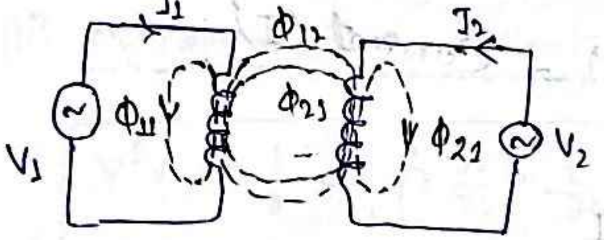
$\phi_{12}$ & $\phi_{21}$ are mutual fluxes (useful flux)

$M_{12}$ & $M_{21}$ are mutual inductance.

Mutual Inductance = $\dfrac{\text{flux linkage in one coil}}{\text{current in another coil.}}$

$$\boxed{M_{12} = \frac{N_1 \phi_{21}}{I_2}} \quad \& \quad \boxed{M_{21} = \frac{N_2 \phi_{12}}{I_1}} = \frac{\lambda_{21}}{I_1}$$

Cofficient of Coupling (K) :- When the two magnetic CKT kept closed to each other interacts with each other magnetically through the flux linkages in the CKT due to the current in other CKT the CKT are called magnetically coupled CKT.




* Self Inductances of Coils are

$$\boxed{L_1 = \frac{N_1 \phi_1}{I_1}} \text{ — (i)} \quad \& \quad \boxed{L_2 = \frac{N_2 \phi_2}{I_2}} \text{ — (ii)}$$

* ~~From linkage factor of~~ CKT (i)

$$K_1 = \frac{\phi_{21}}{\phi_1} \Rightarrow \phi_{21} = K_1 \phi_1 \text{ — (iii)}$$

* ~~From linkage factor of CKT (ii)~~

$$K_2 = \frac{\phi_{12}}{\phi_2} \Rightarrow \phi_{12} = K_2 \phi_2 \text{ —}$$

* Mutual Inductance between two Coils,

$$\boxed{M_{12} = \frac{N_1 \phi_{21}}{I_2}} \text{ — (iii)} \quad \& \quad \boxed{M_{21} = \frac{N_2 \phi_{12}}{I_1}} \text{ — (iv)}$$

* from leakage factor of CKT (i).

$$K_1 = \frac{\phi_{21}}{\phi_1} = \boxed{\phi_{21} = K_1\phi_1} \quad \text{--- (v)}$$

* From leakage factor of CKT (ii)

$$K_2 = \frac{\phi_{12}}{\phi_2} = \boxed{\phi_{12} = K_2\phi_2} \quad \text{--- (vi)}$$

Sub equ (5) & (6) in equ (3) & (4)

$$M_{12} = \frac{N_1 K_1 \phi_1}{I_2}, \quad M_{21} = \frac{N_2 K_2 \phi_2}{I_1}.$$

$$M_{12} \times M_{21} = \left(\frac{N_1 K_1 \phi_1}{I_2}\right)\left(\frac{N_2 K_2 \phi_2}{I_1}\right)$$

Assumed medium arround the coils in linear

$$M_{12} = M_{21} = M \quad \text{[Both coils is linear/homogeneous]}$$

$$M^2 = K_1 K_2 \left(\frac{N_1 \phi_1}{\cancel{K} I_2}\right)\left(\frac{N_2 \phi_2}{I_1}\right)$$

$\boxed{\text{from (i) \& (ii)}}$

$$M^2 = K_1 K_2 L_1 L_2$$

$$K_1 = K_2 = K \quad \text{[Both coils is linear (homogeneous)]}$$

$$M^2 = K^2 L_1 L_2$$

$$\boxed{M = K\sqrt{L_1 L_2}}$$

$$\boxed{K = \frac{M}{\sqrt{L_1 L_2}}}$$

Where K is co-efficient of Coupling, between two coils.

Two magnetic circuit having self inductances $L_1$, $L_2$ and mutual inductance, are connected,

* Series aiding then their equivalent Inductance is $\boxed{L_{eq} = L_1 + L_2 + 2M}$

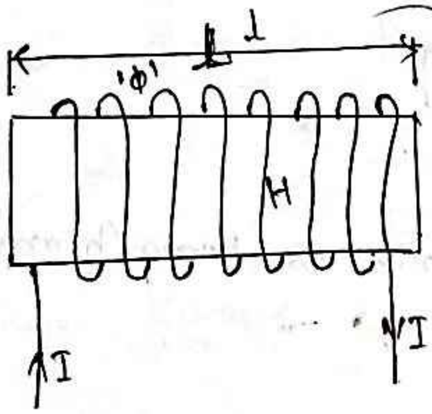
* Series opposing then their equivalent Inductance is $\boxed{L_{eq} = L_1 + L_2 - 2M}$

* Parallel aiding then their equivalent inductance is, $\boxed{L_{eq} = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}}$

* Parallel opposing then their equivalent inductance is

$$\boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}}$$

* Inductance of Solonoid :- Consider a Solenoid of N Turns as shown in fig.




Let the Current flowing through the Solenoid be I ampere. let the length of the Solenoid be l and the cross sectional area be A.

$$\boxed{\text{Inductance of Solonoid} = \frac{\text{Total flux linkage}}{\text{Total current}}}$$

* Total Flux linkage '$\lambda$' of Solonoid is

$$\boxed{\lambda = N\phi} \quad \text{--- (i)}$$

Where $\boxed{\phi = BA}$ --(ii) $\left(B = \dfrac{\phi}{A}\right)$

and

$\boxed{B = \mu H}$ --(iii)


~~magnetic field ...~~ 

* H is magnatic flux density inside Solenoid is $\boxed{H = \frac{NI}{l}}$ —(iv)

Sub the equ (iv) in (iii)

$\boxed{B = \frac{\mu N I}{l}}$ — (v)

we know that $\mu = \mu_r \mu_0$

Sub the equ (v) in equ (ii)

$$\boxed{\phi = \frac{\mu N I}{l} \cdot A} \quad \text{—(vi)}$$

Sub equ (vi) in equ (i)

Now total flux linkage ($\lambda$) =

$$\boxed{\lambda = \frac{N \mu N I A}{l} = \frac{\mu N^2 I A}{l}}$$

Thus the inductance of solenoid is given by.

* Inductance = $\frac{\text{Total flux linkage}}{\text{Total Current}}$

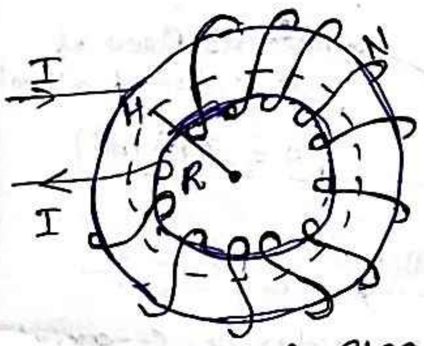
$$L = \frac{\lambda}{I}$$

$$L = \frac{N \mu N I A}{I l} = \frac{\mu N^2 A}{l}$$

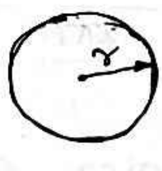
$$\boxed{L = \frac{\mu N^2 A}{l}} \text{ H.}$$

* Obtain an expression for the self-inductance of a toroid of a circular cross-section with N closely spaced turns.

* **Inductance of Toroid:-** Consider a toroidal ring with N turns and carrying current I. Let the radius of the toroide be R as shown in fig.




Toroid of Ring




r: radius of cross-section of a ring.

* Total Flux linkage '$\lambda$' of Toroid is

$$\boxed{\lambda = N\phi} \quad \text{- - - (i)}$$

But $\boxed{\phi = BA}$ --(ii) $\left( B = \frac{\phi}{A} \right)$

Where
A = Area of cross-section of a toroidal ring.

& $\boxed{B = \mu H}$ -- (iii)

H is magnatic field intensity in Toroid

$$\boxed{H = \frac{NI}{2\pi R}} \text{ -- (iv)}$$

Sub (iv) in (iii)

$$\boxed{B = \frac{\mu \cdot NI}{2\pi R}} \text{ -- (v)}$$

Sub (v) in (ii)

$$* \boxed{\phi = \frac{\mu NI \cdot A}{2\pi R}} \text{ --- (vi)}$$

Sub (vi) in (i)

$$* \boxed{\lambda = \frac{N\mu NIA}{2\pi R}}$$

Now

$$* \text{Inductance (L)} = \frac{\text{Flux linkage } (\lambda)}{\text{Current (I)}}$$

$$L = \frac{\lambda}{I}$$

$$L = \frac{\mu N^2 \cancel{I} A}{\cancel{I}\, 2\pi R}$$

$$\boxed{L = \frac{\mu N^2 A}{2\pi R}} \text{ H}$$

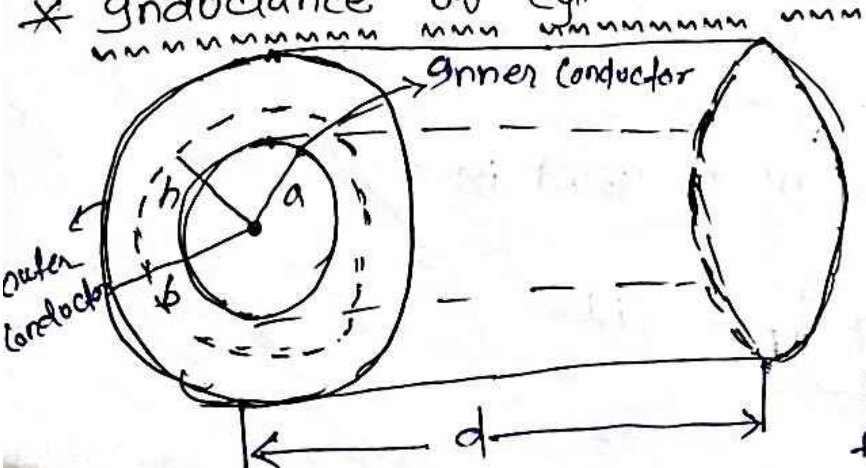
Where A = Area of cross-section of a toroidal ring.

$\boxed{A = \pi r^2 \ m^2}$

Now for a toroid with N number of turns h as the height of toroid with $r_1$ as the inner radius and $r_2$ as outer radius, the inductance is given by.

$$\boxed{L = \frac{\mu N^2 h}{2\pi} \ln\left(\frac{r_2}{r_1}\right)} \text{ H}$$

## * Inductance of cylindrical co-axial cable :-




Consider a Co-axial Cable with Inner Conductor radius a and outer Conductor radius b shown in fig.

Let the Current through the Coaxial Cable is I.

* Flux linkage '$\lambda$' of cable in

$$\boxed{\lambda = N\phi} \quad \text{---(i)}$$

where
$$\boxed{\phi = \oint_S \overline{B}\cdot ds} \quad \text{---(ii)} \qquad \boxed{\Psi = Q = \oint_S \overline{D}\cdot ds}$$

~~sub (ii)~~

where
$$\boxed{\overline{B} = B\phi \cdot \overline{a}\phi} \quad \text{---(iii)}$$

$$\boxed{B\phi = \mu H} \quad \text{---(iv)}$$

* H is magnatic field intensity in cylindrical co-axial cable $(a<r<b)$

$$\boxed{H = \frac{I}{2\pi r}} \quad \text{---(v)}$$

Sub (v) in (iv)

$$\boxed{B\phi = \frac{\mu I}{2\pi r}} \quad \text{---(vi)}$$

Sub (vi) in (iii)

$$\boxed{* \overline{B} = \frac{\mu I}{2\pi r} \cdot \overline{a}\phi} \quad \text{---(vii)}$$

Sub (vii) in (ii)

$$\boxed{\phi = \int_S \frac{\mu I}{2\pi r} \overline{a}\phi \cdot ds} \quad \text{---(viii)}$$

* In cylindrical co-ordinate system,

$$\boxed{\overline{ds} = dr \cdot dz \, \overline{a}\phi} \quad \text{---(ix)}$$

~~[illegible]~~

Sub (ix) in (viii)

$$\phi = \int_S \frac{\mu I}{2\pi r} dr\cdot dz \qquad \text{[illegible]} \boxed{\overline{a}\phi \cdot \overline{a}\phi = 1}$$

$$\phi = \frac{\mu I}{2\pi} \int_a^b \frac{1}{r} \cdot \int_0^d dz$$

$$\phi = \frac{\mu I}{2\pi} \left[\ln r\right]_a^b \left[z\right]_0^d$$

$$\boxed{\phi = \frac{\mu I}{2\pi} \ln \frac{b}{a} \cdot d} \quad \text{---(x)}$$

Sub (x) in (i)

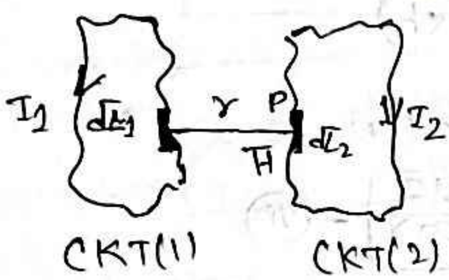
$$* \; \lambda = N\phi = \frac{\mu N I}{2\pi} \ln \frac{b}{a} \cdot d$$

$$\boxed{\text{Inductance} = \frac{\text{Flux linkage } (\lambda)}{(I) \text{ total current}}}$$

$$L = \frac{\lambda}{I} = \frac{\frac{\mu N I}{2\pi} \ln \frac{b}{a} \cdot d}{I}$$

$$\boxed{L = \frac{\mu N \ln\left(\frac{b}{a}\right) d}{2\pi}} \; H$$

* Necuman's formulla:- Consider two closed CKT(1) & CKT(2) in any random shap shape as shown in fig. Let $I_1$ and $I_2$ be the current flowing through closed paths or CKT(1) & CKT(2). Let $r$ be the distance of seperation between $c_1$ and $c_2$.




Fluxe linkage $\lambda_{21}$ with CKT 2 due to CKT (1), Current $I_1$ is given by,

$$\boxed{\lambda_{21} = N_2 \phi_{12}}$$

* assuming both CKT having single turn

$$\boxed{N_1 = N_2 = 0}$$

* $\lambda_{21} = \phi_{12} = \oint_S \bar{B}_1 \cdot d\bar{S}_2$ --- (i)

* ($B_1$ in magnatic flux density) interms of vector magnitude. Potential ($A_1$ in)

~~substitute equ (ii)~~ in (i)

$\bar{B}_1 = \nabla \times \bar{A}_1$ --- (ii)

Sub equ (iii) in (i)

$$\lambda_{21} = \phi_{12} = \oint_S \nabla \times \bar{A}_1 \cdot d\bar{S}_2 \text{ - - - (iii)}$$

* form Strokes theorem,

Surface integral is converted into line integral.

$$\oint_C \nabla \times A_1 d\bar{S}_2 = \oint_{C_2} \bar{A}_1 \cdot d\bar{L}_2$$

$$\lambda_{21} = \phi_{12} = \oint \bar{A}_1 d\bar{L}_2 \text{ - - (iv)}$$

Where $\bar{A}_1$ is vector magnitude potential & is givenby.

$$\bar{A}_1 = \mu \int_{C_1} H_1 \cdot d\bar{L}_1$$

$$\bar{A}_1 = \mu \int_{C_1} \frac{I_1 \cdot d\bar{L}}{2\pi r}$$

Sub the value of $A_1$ in equ (5)

$$\lambda_{21} = \phi_{12} = \oint_{C_2} \oint_{C_1} \frac{\mu I_1 \cdot d\bar{L}_1}{2\pi r} \cdot d\bar{L}_2 \text{ - - (vi)}$$

for linear medium,

$$M_{12} = M_{21} = M$$

$$\lambda_{12} = \lambda_{21} \quad \& \quad \phi_{12} = \phi_{21}$$

$$* \; M = \frac{\lambda_{21}}{I_1}$$

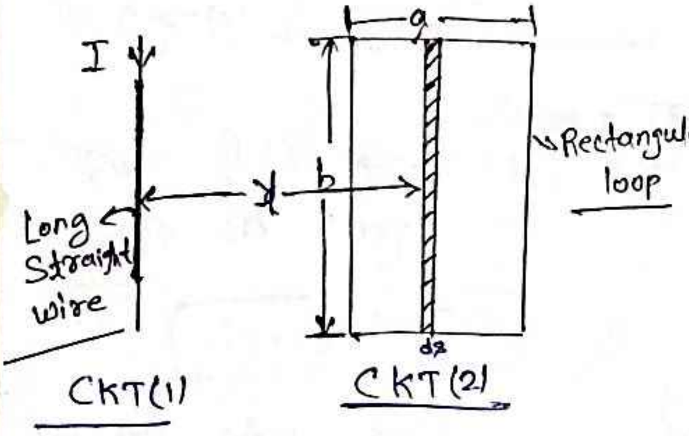
Sub the value of $\lambda_{21}$ from equ (vi).

$$\boxed{M = \oint_{c_1}\oint_{c_2} \frac{\mu}{2\pi r}\, d\bar{L}_1\, d\bar{L}_2}$$ Neuman's formula.

* Mutual Inductance between a long straight wire and square loop/Rectangular loop lying in same plane.

⇒ Let us consider the a long straight line which carrying a current I. and a Rectangular closed loop having dimension a and b which is placed in a same plane.

Consider long straight wire is CKT (1) and Rectangular loop is circuit (2).




Let us consider a differential surface ds at a distance of x from CKT(i).

* Flux linkage $\lambda_{21}$ with Rectangular loop due to CKT current $I_1$ is.

$$\lambda_{21} = N_2 \phi_{12}$$

Both CKT having single Turn

$$N_1 = N_2 = 1$$

$$\boxed{\lambda_{21} = \phi_{12} = \oint \bar{B}_1 \cdot \bar{ds}}$$

* $B_1$ in directed along $\bar{a}_\phi$ direction

$$\bar{B}_1 = B_{1\phi} \cdot \bar{a}_\phi$$

$$B_{1\phi} = \mu H_{1\phi} = \frac{\mu I_1}{2\pi l}$$

$$\boxed{\overline{B_1} = \frac{\mu I_1}{2\pi l} \cdot \overline{a}_\phi}$$

* $d\overline{S_2} = ds \cdot \overline{a}_\phi$

$dS_2 = b \cdot dl$

$dS_2 = b \cdot dl \cdot \overline{a}_\phi$

* $\boxed{\lambda_{21} = \phi_{12} = \oint_S \frac{\mu I_1}{2\pi l} \cdot b \cdot dl}$ $(\overline{a}_\phi \cdot \overline{a}_\phi = 1)$

* for linear midium.

$\boxed{M_{12} = M_{21} = M}$

$\boxed{\lambda_{21} = \lambda_{12}}$

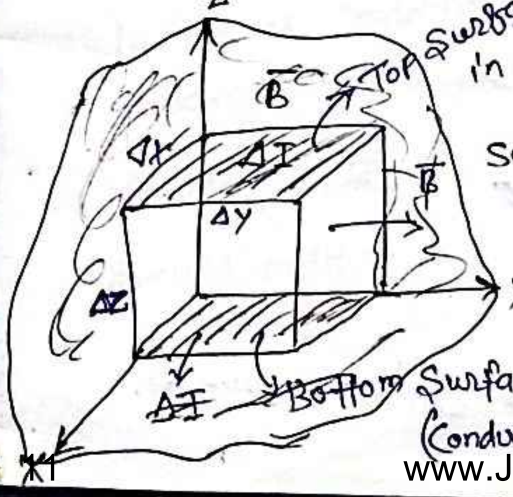
* $M = \frac{\lambda_{21}}{I_1} = \frac{\mu b}{2\pi} \int_0^a \frac{1}{l} dl = \frac{\mu b}{2\pi} \left[ \ln l \right]_0^a =$

$$\boxed{\frac{\mu b}{2\pi} \ln a}$$

* Energy Stored in ~~Magnetic field~~:- Inductance of differential Volume

Consider a differential volume in magnetic field $\overline{B}$ as shown in fig.

Consider that at the top and bottom surfaces of a differential volume, Conducting sheets with current $\Delta I$ are present.



* Energy stored in Inductance of differential Volume is,

$$\boxed{\Delta W = \frac{1}{2} \Delta L (\Delta I)^2} \quad \text{-- (i)}$$

Where

* $$\boxed{\Delta L = \frac{\Delta \phi}{\Delta I}} \quad \text{— (ii)}$$

* $$\boxed{\Delta \phi = \bar{B} \cdot \Delta S} \quad \text{— (iii)}$$

& * $\boxed{\bar{B} = \mu H}$ — (iv)

$\Delta S$ is directed along y- direction

$$\boxed{\Delta S = \Delta x \cdot \Delta z} \quad \text{-- (v)}$$

~~Sub the value of ΔS in equ (iii)~~

Sub the Value of $\Delta S$ & $\bar{B}$ in equ (iii); $\boxed{\Delta \phi = \mu H \cdot \Delta x \cdot \Delta z}$ (6)

So, $$\boxed{\Delta L = \frac{\mu H \cdot \Delta x \cdot \Delta z}{\Delta I}} \text{(from equ (ii)) --- (vii)}$$

* Current $\Delta I$ passing through Conducting Sheet in y- direction.

$$\boxed{\Delta I = H \cdot \Delta y} \quad \text{-- (viii)}$$

So. $$\boxed{H = \frac{\Delta I}{\Delta y}}$$

Now in equ (i) & Sub the Value of $\Delta I$, & $\Delta L$ from (viii) & (vii)

$$\Delta W = \frac{1}{2} \left( \frac{\mu H \Delta x \Delta z}{H \cdot \Delta y} \right) (H \cdot \Delta y)^2$$

$$\Delta W = \frac{1}{2} \frac{\mu \Delta x \Delta z}{\Delta y} \cdot H^2 \Delta y^2$$

$$\Delta W = \frac{1}{2} \mu H^2 (\Delta x \Delta z \Delta y)$$

But the differential Volume can be written as,

$\boxed{\Delta x \cdot \Delta y \cdot \Delta z = \Delta v}$

$$\Delta W = \frac{1}{2}\mu H^2 \Delta V$$

$$\frac{\Delta W}{\Delta V} = \frac{1}{2}\mu H^2$$

The magnetostatic energy density function is defined as,

$$\boxed{W_m = \lim_{\Delta V \to 0} \frac{\Delta W}{\Delta V}} \rightarrow \text{Magnostatic Energy density}$$

<u>Therefore</u>

$$\boxed{W_m = \frac{1}{2}\mu H^2}$$

for linear medium total energy stored in inductance of differential volume

$$W = \int W_m dv$$

$$\boxed{W = \int \frac{1}{2}\mu H^2 dv = \int \frac{1}{2}\mu \cdot H \cdot H dv = \int \frac{1}{2}\bar{B}\cdot\bar{H} dv}$$

from (iv), $\bar{B} = \mu H$

or

$$\boxed{W = \int \frac{1}{2}\frac{B^2}{\mu} dv}$$

* If a coil of 800 μH is magnatically coupled to another coil of 200 μH. To Co-efficient of btw two coil is 0.05. Calculate inductance if two coils are connected in orbit (a) Series adding (b) Series oppositing (c) Parallel adding (d) Parallel oposing.

=> Given data

(i) Series adding,

$$l_{eq} = L_1 + L_2 + 2m$$

$$m = K\sqrt{L_1 L_2}$$

$= 0.05\sqrt{800 \times 200} = 20\mu H$

$L_{eq} = 800+200+20\times 2 = 1040\ \mu H$

(ii) Series opposing

$L_{eq} = L_1+L_2-2M$

$M = 20$

$= 800+200-2\times 20 = 960\ \mu H$

(iii) Parallel adding,

$$\frac{L_1 L_2 - M^2}{L_1+L_2-2M}$$

$$= \frac{800\times 200-(20)^2}{800+200-2(20)} = 166.25\ \mu H$$

(iv) Parallel opposing :

$$L_{eq} = \frac{L_1L_2-M^2}{L_1+L_2+2M} = \frac{800\times 200-(20)^2}{800+200+2(20)} =$$

$153.46\ \mu H$

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