3. Magneto statics & Amphere's law

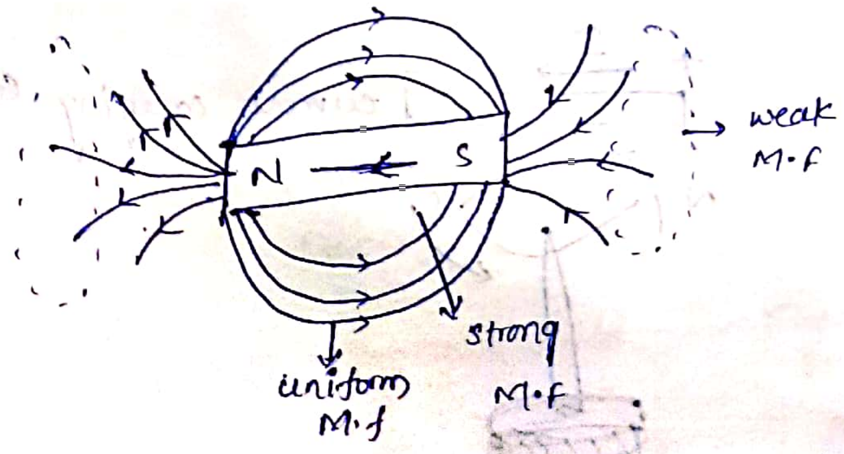
Introduction:-

In this chapter, the magnetic field is introduced the relation of stedy magnetic fields to its source is more complicated when compared to the relation b/n electro static field to its source.

The development of motors, transformers, high speed vehicles, Television etc. involve magnetic phenomena and play an important role in every day life.

There is a great similarity in the equations derived in electric fields and Magnetic fields.

* Magnetic field Intensity:-



- The region surrounding a Magnet in which the force of the Magnet can be experienced a field that is called as Magnetic field. it is indicated by H. AmpTurn/mt

Scanned by CamScanner

Note:-

- The lines flows from North- south (outside the magnet) and south - North (inside the Magnet)
- The lines shows both direction and strength.

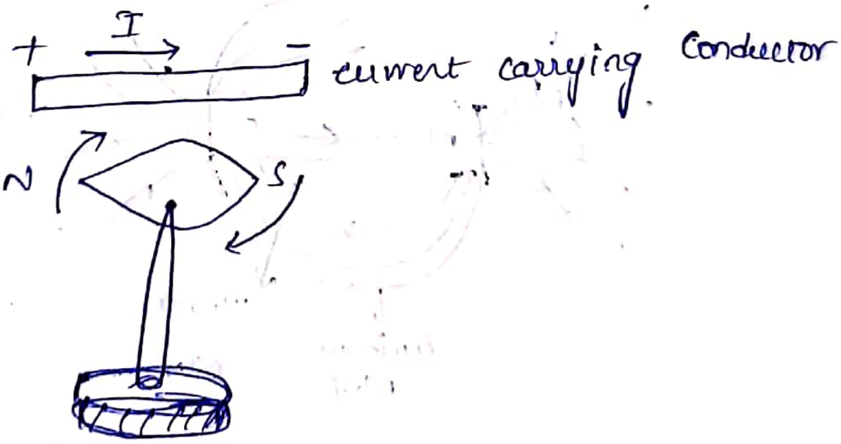
Magnetic flux density (B):-

It is defined as the magnetic flux per unit surface area. It is a vector quantity.

B units wb/m^2 or Tesla

Matthematically, $\bar{B} = \frac{\phi}{A}$ wb/m^2 (or) Tesla

*Oestered's Expement:-



- Oestered conducted an experiment in which a current carrying conductor was taken. A compass needle was kept under this conductor as shown in figure.

www.Jntufastupdates.com

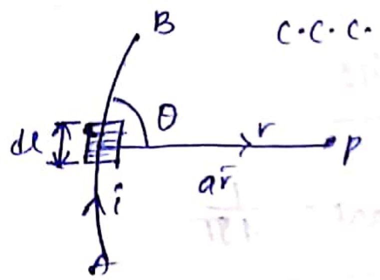
Scanned by CamScanner

- When there was no current through the conductor, then needle was pointing along North-south of the earth.

- But when the conductor carries current then the needle was attracted to the conductor or repulsion to the conductor, it moved and tended to stand at right angles to the conductor.

- From this experiment oestered shows that an electric current produces a Magnetic field.

* Biot- Savart's Law:-



- 1820's Biot and savarat' conducted a series experiment on steady magnetic field. finally they conclude that the required point "p" is obtained from the following relations [H or B]

i) The differential Magnetic field Intensity is directly proportional to current carrying through the conductor.

$$dH \propto I$$

ii) The differential MFI is directly proportional to differential length

$$dH \propto dL$$

www.Jntufastupdates.com

Scanned by CamScanner

iii) The differential MFI is directly proportional to sine angle

$$dH \propto \sin\theta$$

iv) The differential MFI is Inversly proportional to square of distance from differential length to point P.

$$dH \propto 1/r^2$$

$$\therefore dH \propto \frac{I dl \sin\theta}{r^2}$$

$$dH = \frac{k\, I dl \sin\theta}{r^2}$$

where k = constant = $\frac{1}{4\pi}$

$$\Rightarrow \boxed{dH = \frac{I dl \sin\theta}{4\pi r^2} \bar{a_r}}$$

If $\theta = 90^\circ$

$$dH = \frac{I dl}{4\pi r^2} \bar{a_r}$$

$$\boxed{H = \int \frac{I dl}{4\pi r^2} \sin\theta\, \bar{a_r}}$$

we know that $B = \mu H$

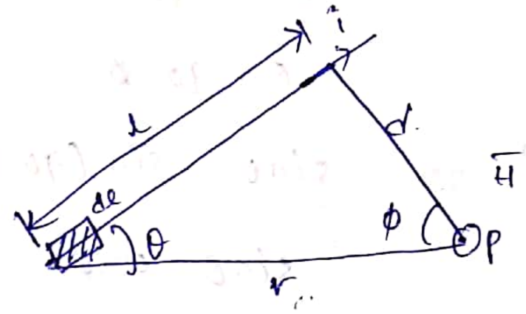
$$B = \mu \int \frac{I dl}{4\pi r^2} \sin\theta\, \bar{a_r}$$

www.Jntufastupdates.com

Scanned by CamScanner

$$B = \frac{\mu}{4\pi}\int \frac{I\,dl\,\sin\theta}{r^2}\,\bar{a_r}$$

* Magnetic field intensity due to current carrying conductor or wire:-



consider a straight wire of length 'l' carrying a steady current I. here we find the magnetic field intensity $\bar{H}$ at point 'P', with a distence of 'd' from the wire as shown in fig:

from the figure;

$$\sin\theta = d/r = \cos\phi$$

$$\cos\theta = l/r = \sin\phi$$

$$\tan\phi = \frac{l/r}{d/r} = l/d$$

$$l = d\tan\phi$$

diff. w.r.t. ϕ

$$\frac{dl}{d\phi} = d\sec^2\phi$$

$$\boxed{dl = d\sec^2\phi\; d\phi}$$

www.Jntufastupdates.com

Scanned by CamScanner

$r^2 = l^2 + d^2$

$r = \sqrt{l^2+d^2}$

$\phi + \theta = 90^\circ$ $\qquad$ $\phi + \theta = 90^\circ$

$\phi = 90 - \theta$ $\qquad$ $\theta = 90 - \phi$

$\sin\phi = \sin(90-\theta)$ $\qquad$ $\sin\theta = \sin(90-\phi)$

$\sin\phi = \cos\theta$ $\qquad$ $\sin\theta = \cos\phi$

W.K.T

The MFI at any point 'p' is,

$$d\bar{H} = \frac{I\,dl \sin\theta}{4\pi r^2}\,\bar{a_r}$$

$$d\bar{H} = \frac{I\,dl \cos\phi}{4\pi r^2}\,\bar{a_r}$$

$$\bar{H} = \int \frac{I\,dl \cos\phi}{4\pi r^2}\,\bar{a_r}$$

$$= \frac{I}{4\pi}\int \frac{dl\cos\phi}{r^2}\,\bar{a_r}$$

$$= \frac{I}{4\pi}\int \frac{d\sec^2\phi\, d\phi\, \cos\phi}{(l^2+d^2)}\,\bar{a_r}$$

$$= \frac{I}{4\pi}\int \frac{d\sec^2\phi\, d\phi\, (d/r)}{l^2+d^2}\,\bar{a_r}$$

$$= \frac{Id^2}{4\pi}\int \frac{\sec^2\phi\, d\phi}{(l^2+d^2)^{3/2}}\,\bar{a_r}$$

www.Jntufastupdates.com

Scanned by CamScanner

$$\bar{H} = \frac{I d^2}{4\pi} \int \frac{\sec^2\phi \, d\phi}{(l^2+d^2)^{3/2}} \bar{a_r}$$

$(d^2)^{3/2}$ $(d^2)^{3/2} \cdot \left[\frac{l^2}{d^2}+1\right]$

$$= \frac{I d^2}{4\pi} \int \frac{\sec^2\phi \, d\phi}{d^3\left[1+\frac{l^2}{d^2}\right]^{3/2}} \bar{a_r}$$

$$= \frac{I}{4\pi d} \int \frac{\sec^2\phi \, d\phi}{(1+\tan^2\phi)^{3/2}} \bar{a_r}$$

$$= \frac{I}{4\pi d} \int \frac{\sec^2\phi \, d\phi}{(\sec^2\phi)^{3/2}} \bar{a_r}$$

$$= \frac{I}{4\pi d} \int \frac{1}{\sec\phi} d\phi \, \bar{a_r}$$

$$= \frac{I}{4\pi d} \int \cos\phi \, d\phi \, \bar{a_r}$$

$$\boxed{\bar{H} = \frac{I}{4\pi d} \sin\phi \, \bar{a_r}}$$

$$\boxed{\bar{H} = \frac{I\cos\theta}{4\pi d} \bar{a_r}}$$

Note:-

for infinite length i.e., $\frac{-\pi}{2}$ to $\frac{+\pi}{2}$ the limits of integration. ∴ the MFI due to ccc is (or) $\frac{I\cos\theta}{4\pi d}$

$$\bar{H} = \frac{I\sin\phi}{4\pi d} \bar{a_r} = \frac{I(\sin\phi_2 - \sin\phi_1)}{4\pi d} \bar{a_r}$$

$$= \frac{I}{4\pi d}\left[\sin\frac{\pi}{2} + \sin\frac{\pi}{2}\right] = I \qquad = \frac{I(\cos\theta_1 - \cos\theta_2)}{4\pi d}$$

$$\boxed{\bar{H} = \frac{I}{2\pi d}}$$

www.Jntufastupdates.com

Scanned by CamScanner

* Magnetic permiability :- (μ)

It is the ration flux density ($\bar{B}$) and produced in the Material or produced MFI.

$$\because B \propto H$$

$$B = \mu H$$

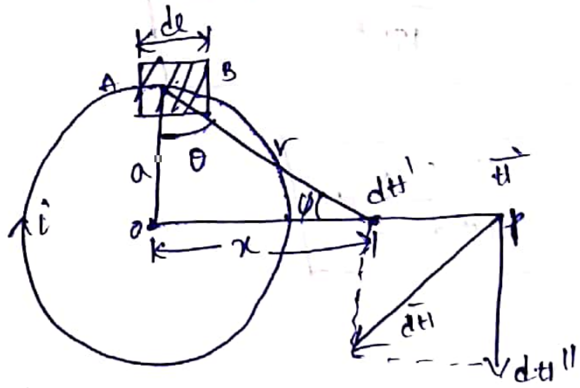
$$\boxed{\mu = B/H}$$

where $\mu = \mu_0 \mu_r$

μ_0 = Absolute permiability $= 4\pi \times 10^{-7}$ H/m

μ_r = relative permiability

* Magnetic F.I due to circular coil :-



consider a circular loop of a radius a carrying a current i amps we have to find the magnetic field intensity at point P on the axis of loop with a distance of x mt. from its centre O. Consider a segment AB whose elemental length is 'dl'. let r be the distance of the element from point P an θ be

www.Jntufastupdates.com

Scanned by CamScanner

the angle b/n the direction current with line joining the elementary length dl.

- from Biot - savart's law

The MFI at point p is $\bar{dH} = \frac{I dl \sin\theta}{4\pi r^2} \bar{a_r} \rightarrow (1)$

from the figure

$r^2 = x^2 + a^2$

$r = \sqrt{x^2 + a^2}$

$\sin\phi = \frac{a}{r}$

If $\theta = 90^\circ$ then the equation (1) is,

$$\bar{dH} = \frac{I dl}{4\pi r^2} \bar{a_r} \rightarrow (2)$$

from the figure, the resultant vector $\bar{dH}$

$$\bar{dH} = \bar{dH'} + \bar{dH''}$$

$\bar{dH''} = 0$ because it is vertical component.

$\therefore$ The resultant vector $\bar{dH} = \bar{dH'} \rightarrow (3)$

w.k.T $\bar{dH'} = \bar{dH} \sin\phi \rightarrow (4)$

substitute (4) in (3)

$$\bar{dH} = \bar{dH} \sin\phi.$$

$$\bar{dH} = \frac{I dl}{4\pi (x^2 + a^2)} \sin\phi \, \bar{a_r}$$

$$\bar{dH} = \frac{I dl}{4\pi (x^2 + a^2)} \cdot \frac{a}{r} \bar{a_r}$$

www.Jntufastupdates.com

Scanned by CamScanner

$$d\bar{H} = \frac{I\ a\ dl}{4\pi(x^2+a^2)(\sqrt{x^2+a^2})}\ \bar{a_r}$$

$$= \frac{Ia\ dl}{4\pi(x^2+a^2)^{3/2}}\ \bar{a_r}$$

$$\bar{H} = \int \frac{Ia\,dl}{4\pi(x^2+a^2)^{3/2}}\ \bar{a_r}$$

$$= \frac{I}{4\pi}\int \frac{dl \cdot a}{(x^2+a^2)^{3/2}}\ \bar{a_r}$$

$$= \frac{I \cdot a}{4\pi(x^2+a^2)^{3/2}} \cdot \int dl \cdot \bar{a_r}$$

$$= \frac{I \cdot a \cdot l}{4\pi(x^2+a^2)^{3/2}}\ \bar{a_r}$$

$$\bar{H} = \frac{I \cdot a \cdot (2\cancel{\pi}a)}{4\cancel{\pi}(x^2+a^2)^{3/2}}\ \bar{a_r}$$

$$\boxed{\bar{H} = \frac{Ia^2}{2(x^2+a^2)^{3/2}}\ \bar{a_r}}$$

If the required point is at centre:

$\therefore x = 0$

$$\bar{H} = \frac{Ia^2}{2a^3}\ \bar{a_r}$$

$$\boxed{\bar{H} = \frac{I}{2a}\ \bar{a_r}}$$

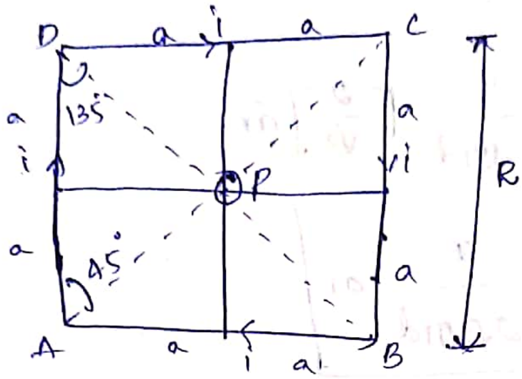
www.Jntufastupdates.com

Scanned by CamScanner

for N turn coils the magnetic field intensity is

$$\boxed{\bar{H} = \frac{IN}{2a}\, \bar{a_r}}$$

* Magnetic field Intensity at the centre of conductor or in the form of square:-



- Let the length of the each side by R mt and i denotes a steady direct current carried by loop. it is requires to determine the magnetic field intensity at point p at centre of square.

- consider the side 'AD' of the loop the field intensity at a point distance are form a current carrying conductor of finite length is given by,

$$\bar{H} = \frac{Idl\cos\theta}{4\pi d}\, \bar{a_r} \quad \rightarrow (1)$$

$$= \frac{I}{4\pi d}\left[\cos\alpha_1 - \cos\alpha_2\right] \bar{a_r}$$

The side AD of the field intensity H1 at point P is given by

www.Jntufastupdates.com
Scanned by CamScanner

$$H_1 = \frac{I}{4\pi d}\left[\cos 45^\circ - \cos 135^\circ\right] \bar{a_r}$$

$\alpha_1 = 45^\circ$, $\alpha_2 = 135^\circ$

$$= \frac{I}{4\pi d}\left[\frac{1}{\sqrt{2}} - \left[\frac{-1}{\sqrt{2}}\right]\right] \bar{a_r}$$

$$= \frac{I}{4\pi d}\left[\frac{2}{\sqrt{2}}\right] \bar{a_r}$$

$$\boxed{\bar{H_1} = \frac{I}{2\sqrt{2}\pi d} \bar{a_r}}$$

it in clear that the field intensity H_1, H_2, H_3 and H_4 due to sides of AD, DC, CB, BA d BA (due to directions) are equal.

The total field intensity at point P due to square loop ABCD is

$$\bar{H} = H_1 + H_2 + H_3 + H_4$$

$$\Rightarrow \bar{H} = 4H_1$$

$$= 4\left[\frac{I}{2\sqrt{2}\pi d} \bar{a_r}\right]$$

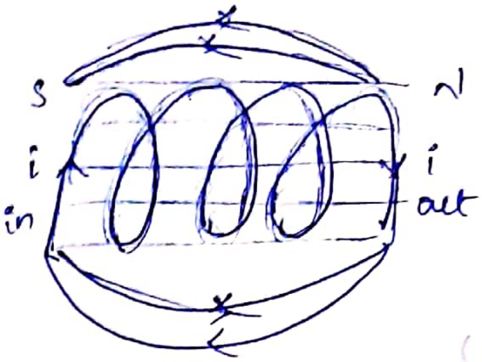
$$\boxed{\bar{H} = \frac{\sqrt{2}I}{\pi d} \bar{a_r}}$$

www.Jntufastupdates.com

Scanned by CamScanner

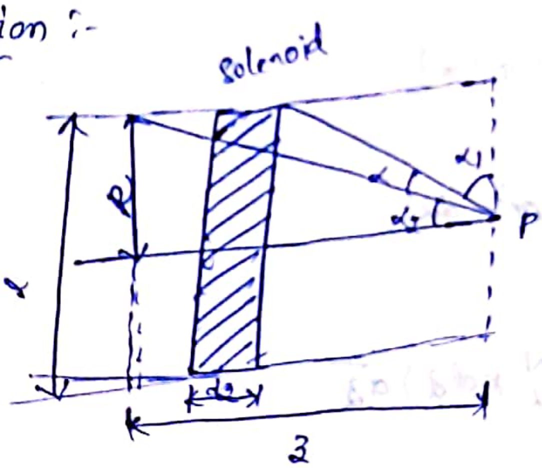
* MFI due to solenoid:-

Solenoid:-



- A solenoid consists of long conducting wire made up of many loops packed closely together. when the current passed through the solenoid then the magnetic field is produced.

Derivation:-



- consider a solenoid of n-turns of length l. let R be the radius of solenoid.
- let p be the point on the axis of the solenoid (z-axis)
- consider an element 'dz' at a distance 'z' from point P.
- No. of turns per unit length $= N/l$.

No. of turns in solenoid$(dz) = N/l \cdot dz \longrightarrow (1)$

www.Jntufastupdates.com
Scanned by CamScanner

Magnetic field intensity for one turn at point p is,

$$\bar{H} = \frac{Ia^2}{2(x^2+a^2)^{3/2}} \cdot \bar{a_r}$$

in solenoid $\bar{H} = \frac{IR^2}{2(z^2+R^2)^{3/2}} \cdot \bar{a_z}$

$$d\bar{H} = \frac{IR^2}{2(z^2+R^2)^{3/2}} \bar{a_z} \longrightarrow (2)$$

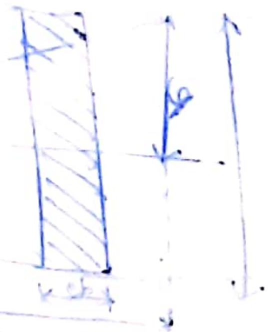
from the fig;

$$\cot\alpha = z/R$$

diff w.r.t. α

$$\frac{dz}{d\alpha} = R(-\operatorname{cosec}^2\alpha)$$

$$dz = -R\operatorname{cosec}^2\alpha \, d\alpha.$$



$$d\bar{H} = \frac{IR^2}{2(z^2+R^2)^{3/2}} \left(\frac{N}{l} \times dz\right) \bar{a_z}$$

$$= \frac{IR^2}{2(R^2+z^2)^{3/2}} \left(\frac{N}{l} \times (-R\operatorname{cosec}^2\alpha \, d\alpha)\right) \bar{a_z}$$

$$= \frac{-IR^3 N}{2l(R^2+z^2)^{3/2}} \operatorname{cosec}^2\alpha \, \bar{a_z}$$

$$= \frac{-IR^3 N}{2l} \int_{\alpha_1}^{\alpha_2} \frac{\operatorname{cosec}^2\alpha \, d\alpha}{(R^2+z^2)^{3/2}} \bar{a_z}$$

Scanned by CamScanner

$$\bar{H} = \frac{-IR^3N}{2l}\int_{\alpha_1}^{\alpha_2} \frac{\operatorname{cosec}^2\alpha \, d\alpha}{(R^2+R^2\cot^2\alpha)^{3/2}}\,\bar{a_3}$$

$$= \frac{-I\cancel{R^3}N}{2l\cancel{R^3}}\int_{\alpha_1}^{\alpha_2} \frac{\operatorname{cosec}^2\alpha}{(1+\cot^2\alpha)^{3/2}}\,d\alpha\,\bar{a_3}$$

$$= \frac{-IN}{2l}\int_{\alpha_1}^{\alpha_2} \frac{\operatorname{cosec}^2\alpha}{(\operatorname{cosec}^2\alpha)^{3/2}}\,d\alpha\,\bar{a_3}$$

$$= \frac{-IN}{2l}\int_{\alpha_1}^{\alpha_2} \frac{1}{\operatorname{cosec}\alpha}\,d\alpha\,\bar{a_3}$$

$$\bar{H} = \frac{-IN}{2l}\int_{\alpha_1}^{\alpha_2} \sin\alpha\,d\alpha\,\bar{a_3}$$

$$= \frac{-IN}{2l}(-\cos\alpha)_{\alpha_1}^{\alpha_2}\,\bar{a_3}$$

$$\boxed{\bar{H} = \frac{IN}{2l}(\cos\alpha_2 - \cos\alpha_1)\,\bar{a_3}}$$

case-I

$$\alpha_1+\alpha_2 = \pi$$

$$\alpha_1 = \pi-\alpha_2$$

$$\cos(\alpha_1) = \cos(\pi-\alpha_2)$$

$$= -\cos\alpha_2$$

$$\bar{H} = \frac{IN}{2l}(\cos\alpha_2 + \cos\alpha_2)\,\bar{a_3}$$

$$= \frac{IN}{2l}\,2\cos\alpha_2\,\bar{a_3}$$

$$\boxed{\bar{H} = \frac{IN}{l}\cos\alpha_2\,\bar{a_3}}$$

www.Jntufastupdates.com

Scanned by CamScanner

Case-2:-

when the length of the solenoid is

$$\alpha_2 = 0$$

$$\Rightarrow \cos\alpha_2 = \cos 0$$
$$= 1$$

$$\bar{H} = \frac{IN}{l}\cos\alpha_2\, \bar{a_z}$$

$$\boxed{\bar{H} = \frac{IN}{l}\bar{a_z}}$$

* Relation b/n M.F.D($\bar{B}$), M.F.I ($\bar{H}$) & Magnetic flux(ψ):-

$B \propto H$

$$\boxed{B = \mu H}$$

WKT

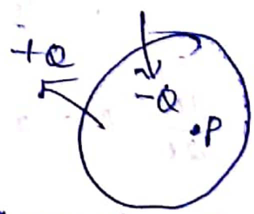
$$\bar{B} = \phi/A \text{ or } \phi/s \text{ or } \frac{\psi}{s}$$

$$\bar{B} = \frac{d\psi}{ds}$$

$$d\psi = \bar{B}ds$$

$$\boxed{\psi = \int_s \bar{B}\cdot d\bar{s}}$$

* Gauss Law in Magnetostatic's:-



consider a closed surface in a magnetic field for every flux line enters into a surface. there must always be flux lines emerging out of surface.

www.Jntufastupdates.com
Scanned by CamScanner

where total output flux = total input flux

- The outward flux is taken as +Q and inward as -Q

then the magnetic flux in entire closed surface is equal to 0.

$\therefore \Rightarrow \psi = 0$

$$\boxed{\int_S \bar{B} \cdot ds = 0}$$

* Maxwell's second Equation ($\nabla \cdot \bar{B} = 0$) :-

we know that the magnetic flux flowing through a surface is given by

$$\int \bar{B} \cdot ds = \psi = \phi \longrightarrow (1)$$

Acc. to gauss law the Magnetic flux in entire closed surface is equal to zero.

$$\psi = 0 \longrightarrow (2)$$

substitute (2) in (1)

$$\int_S \bar{B} \cdot ds = 0 \longrightarrow (3)$$

Apply the divergence theorem in (3)

i.e, $\int_V \nabla \bar{B} \cdot dv = 0$

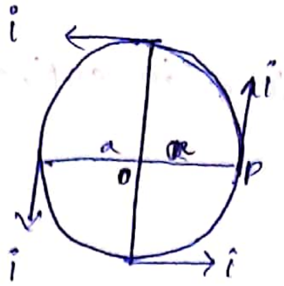
$\nabla \cdot \bar{B} \, dv = 0$

$\boxed{\nabla \cdot \bar{B} = 0}$ (or) $\boxed{(div \bar{B}) = 0}$

www.Jntufastupdates.com

Scanned by CamScanner

Ampere's Circuit Law :-



statement:- The line integral of Magnetic field intensity ($\bar{H}$) at any closed path [solenoid, square, conducting wire or circular coil] is equal to the current enclosed by that path.

i.e., $$\boxed{\int_l \bar{H} \cdot dl = I} \longrightarrow (1)$$

WKT current density $J = I/s$

$$J = dI/ds$$

$$dI = J ds$$

$$I = \int_s J \cdot ds \longrightarrow (2)$$

substitute (2) in (1)

$$\boxed{\int_l \bar{H} \cdot d\bar{l} = \int_s J \cdot ds = I}$$

Scanned by CamScanner

* calculate the magnetic flux density at the centre of a current carrying loop when the radius of loop is 2cm, loop current is 1mAmp and loop is placed in air

sol:- Given

$I = 1 \times 10^{-3}$ Amp

radius of loop $a = 2cm = 2 \times 10^{-2} m$

we know that $\bar{H} = \frac{I}{2a}$

$= \frac{10^{-3}}{2 \times 2 \times 10^{-2}} = \frac{1}{40}$

$= 0.025$ A-T/mt

$\bar{B} = \mu \bar{H}$ [air]

$= 4\pi \times 10^{-7} \times 0.025$

$= 0.314 \times 10^{-7}$

$= 3.14 \times 10^{-8}$

$=$

square

$= \frac{\sqrt{2} I}{\pi d}$

$= \frac{\sqrt{2} \times 10^{-3}}{\pi \times 2 \times 10^{-2}}$

$= \frac{\sqrt{2}}{20 \times \pi}$

$= 0.022$ At/m

* the magnitude of $\bar{H}$ at a radius of 1mt from a long linear conductor is 1 AmT/m. find current in wire.

sol:- magnetic field intensity $\bar{H} = \frac{I}{2a}$

$\bar{H} = 1$ A-T/mt, $a = 1$ mt

Ampher's circuit law

$$\oint_s \bar{H} \cdot dl = I$$

$H \cdot \oint dl = I \Rightarrow H(2\pi r) = I$

$I = 6.28$ Amp

www.Jntufastupdates.com

Scanned by CamScanner

* The magnitude of H at a radius of 4 mt from a long linear conductor is 2A-T/mt. find the current.

sol:-

$r = 4\,mt$

$\bar{H} = 2$

$\oint \bar{H} \cdot \bar{dl} = I$

$I = H \oint_S dl$

$I = \cancel{4} \times 2\pi r \times 2$

$= 2 \times \pi \times 4 \times 2$

$= 50.26$ Amp

* A circular coil of radius 1.5 cm carries a current 1.5 Amps if the coil has 25 turns find the field at centre.

sol:-

$\bar{H} = \frac{NI}{2a}$

$= \frac{25 \times 1.5}{2 \times 1.5 \times 10^{-2}}$

$\bar{B} = \mu H = \frac{\overset{2}{\cancel{4}}\pi \times 10^{-7} \times 25 \times 1.\cancel{5}}{\cancel{2} \times 1.\cancel{5} \times 10^{-2}}$

$= 157.07 \times 10^{-5}\ wb/m^2$

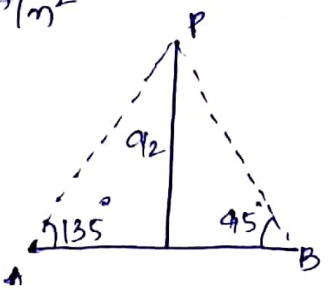
www.Jntufastupdates.com

Scanned by CamScanner

* using Amphere's circuit law find $\bar{H}$ and $\bar{B}$ inside a long straight magnetic conductor of radius r mm

* a) A steady current of I amp flow in a conductor bent in the form of square loop of amt. find magnetic field intensity at the centre of loop.

b) using Amphere's circuit law find $\bar{H}$ and $\bar{B}$ inside a long straight magnetic conductor of radius 8mm carrying a current density of 50 kA/m^2



sol:- i) $$\bar{H} = \frac{I}{4\pi d} \cos\theta \, \bar{a_r}$$

$$= \frac{I}{4\pi d} (\cos\alpha_1 - \cos\alpha_2)$$

$$= \frac{I}{4\pi d} [\cos 45 - \cos 135°]$$

$$= \frac{I}{4\pi d} \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$$

$$= \frac{I}{4\pi d} \cdot \frac{2}{\sqrt{2}} = \frac{I}{2\sqrt{2}\pi d}$$

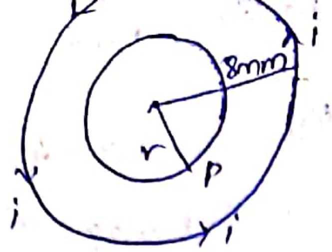
$$= \frac{I}{2\sqrt{2}\pi a}$$

ii) $$\bar{H} = 4\bar{H} = 4 \cdot \frac{I}{2\sqrt{2}\pi a}$$

$$= \frac{\sqrt{2} I}{\pi a} = \cancel{\sqrt{2}}X$$

Scanned by CamScanner

ii)



$\oint \bar{H} \cdot dl = I$

$\oint \bar{H}\, dl = J \cdot A$

$H \cdot \oint dl = J \cdot A$

$H \cdot (2\cancel{\pi r}) = J \cdot \cancel{\pi} r r$

$\bar{H} \cdot 2 = J \cdot r$

$= \dfrac{50 \times 10^{+3} \times \overset{4}{\cancel{8}} \times 10^{-3}}{\cancel{2}}$

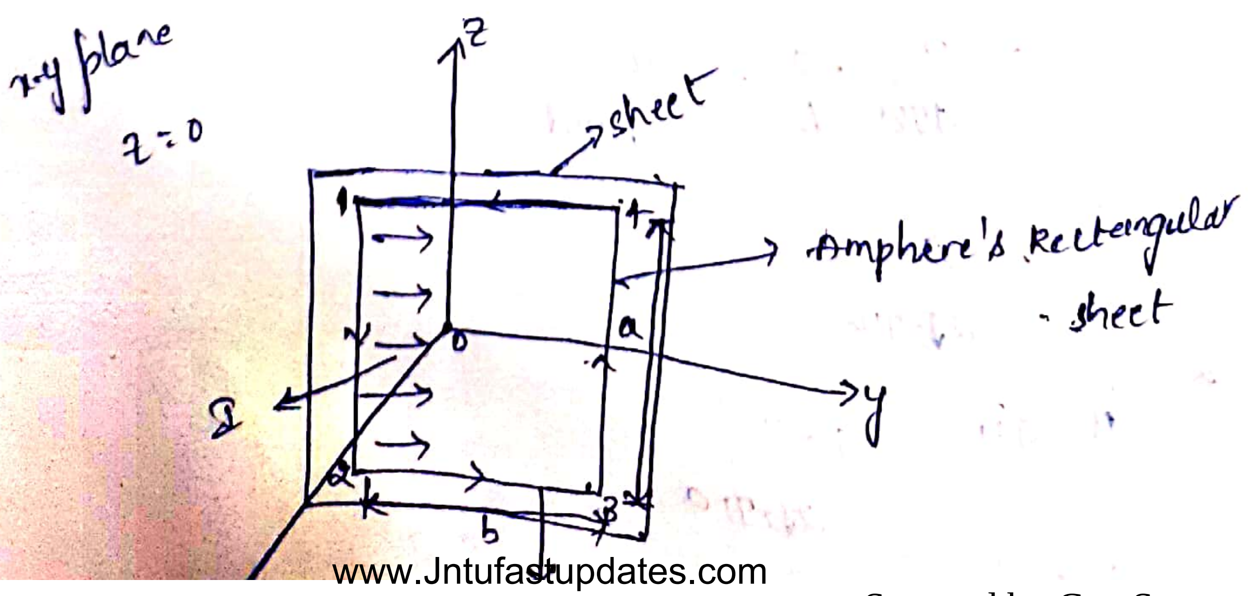
$\bar{H} = 200$ AT/m

$\bar{B} = \mu \cdot \bar{H}$

$= 4\pi \times 10^{-7} \times 200$

$= 2.51 \times 10^{-4}$ Tesla

MFI due to infinite sheet of charge :-



www.Jntufastupdates.com

Scanned by CamScanner

- Here we consider the infinite sheet in z=0 plane nothing but x-y plane sheet.

- Take the Amphere's Surface. if the sheet place that at the top and bottom.

- Amphere's surface sheet or 1234 sheet i.e., in rectangular form. by using this sheet we are cutting that infinite sheet (x-y -plane sheet). that time the current direction will be shown in figure. i.e., nothing but current density.

- we know that the Amphere's Circuit Law

$$\oint H \cdot dl = I \longrightarrow (1)$$

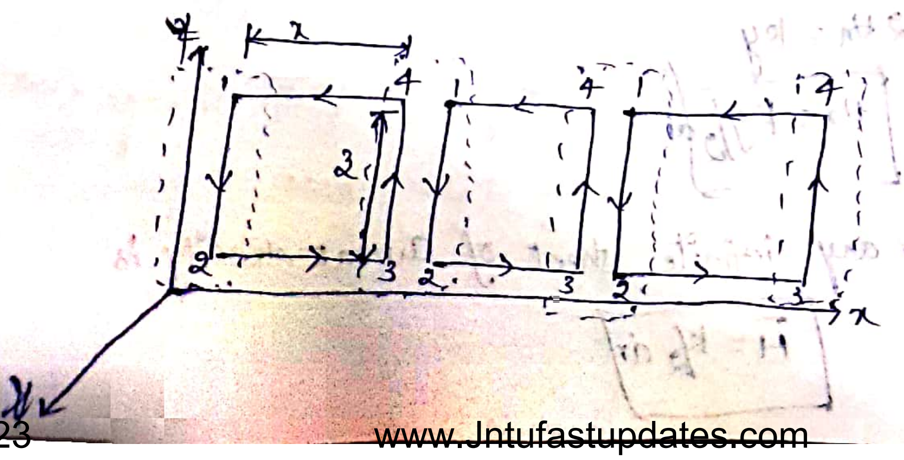
Applying Amphere's circuit law to the rectangular sheet is given by.

$$\oint H \cdot dl = k_y \cdot b = I \longrightarrow (2)$$

where k_y = Surface Current density

b = length

1234 - Infinite sheets



www.Jntufastupdates.com
Scanned by CamScanner

→ 3-4 sides in Amphere's surface is in up direction of vector and 1-2 Amphere's surface sheet is in down coard direction then both are in opposite in direction.

$$\int H \cdot dl = 0$$

∴ The total Magnetic field intensity in rectangular sheet (1234) is,

$$\underbrace{\int_1^2 \bar{H}\cdot dl}_{0} + \int_2^3 \bar{H}\cdot dl + \underbrace{\int_3^4 \bar{H}\cdot dl}_{0} + \int_4^1 \bar{H}\cdot dl = k_y \cdot b$$

$$\int_2^3 \bar{H}\cdot dl + \int_4^1 \bar{H}\cdot dl = k_y \cdot b$$

$$\int_2^3 H_x\, dx + \int_4^1 H_x \cdot dx = k_y \cdot b$$

$$H_x \int_2^3 dx + H_x \int_4^1 dx = k_y \cdot b$$

the integration of dx is b in figure. $[\int dx = b]$

∴ $H_x b + H_x b = k_y \cdot b$

$$2H_x \cancel{b} = k_y \cdot \cancel{b}$$

$$2H_x = k_y$$

$$\boxed{H_x = k_y/2 \; a_r}$$

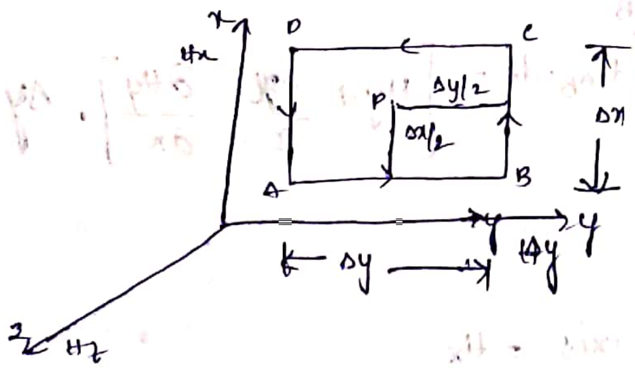
In general for any infinite sheet of current density is

$$\boxed{\bar{H} = k/2 \; \bar{a_r}}$$

www.Jntufastupdates.com

Scanned by CamScanner

Point-form of Amphere's Circuit law:-



$+, +$
$-, -$
$-, +$
$+, -$

- consider a point P the magnetic field intensity be H and its components be Hx, Hy and Hz.
- The values of both H and its components change with distance as ~~moving~~ move in away direction.
- consider a rectangular loop ABCD parallel to xy-plane. such that point-P is located exactly its centre, so inorder to evaluate the closed line integral of 'H' over the path in the direction of ABCD, it can be divided into 4 parts namely AB, BC, CD & DA segments/sides.

AB segment:-

with the help of Tylor's series theorem.

AB ∥ to y-axis = Hy

at distance = Δy

at point P = Δx/2

The magnetic field intensity AB segment,

$$H_{AB} = H_y + \frac{\Delta x}{2} \frac{\partial H_y}{\partial x}$$

www.Jntufastupdates.com

Scanned by CamScanner

the Line integral of AB is,

$$\int_A^B H_{AB}\cdot dl = \left[H_y + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\cdot \Delta y \longrightarrow (2)$$

BC Segment:-

BC || to x-axis $= H_x$

Distance $= \Delta x$

at point $p = \frac{\Delta y}{2}$

with By using tylor's Series theorem

$$H_{BC} = H_x + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$\int_B^C H_{BC}\cdot dl = -\left[H_x + \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\Delta x \longrightarrow (2)$$

CD segment:-

CD || y-axis $= H_y$

Distance $= \Delta y$

at point $p = \Delta x/2$

The magnetic field intensity of CD segment is,

$$H_{CD} = -H_y + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}$$

$$\int_C^D H_{CD}\cdot dl = \left[-H_y + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\right]\cdot \Delta y \longrightarrow (3)$$

www.Jntufastupdates.com

Scanned by CamScanner

DA segment :-

DA ∥ to x-axis = Hx

Distance = Δx

at point p = $-\Delta y/2$

By using tylor's series theorem

The magnetic field intensity of DA segment is

$$H_{DA} = H_x - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}$$

$$\int_D^A H_{DA}\cdot dl = \left[H_x - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\right]\cdot\Delta x \longrightarrow (4)$$

the line integral of ~~ABCD~~ rectan MFI ($\vec{H}$). of ABCD rectangle loop is

$$\int H_{ABCD}\cdot dl = \int_A^B H_{AB}\cdot dl + \int_B^C H_{BC}\cdot dl + \int_C^D H_{CD}\cdot dl + \int_D^A H_{DA}\cdot dl..$$

$$= \cancel{H_y \Delta y} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\cdot\Delta y - \cancel{H_x\Delta x} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\Delta x - \cancel{H_y\Delta y} + \frac{\Delta x}{2}\frac{\partial H_y}{\partial x}\Delta y + \cancel{H_x\Delta x} - \frac{\Delta y}{2}\frac{\partial H_x}{\partial y}\Delta x$$

$$= \Delta x\Delta y\frac{\partial H_y}{\partial x} - \Delta x\Delta y\frac{\partial H_x}{\partial y}$$

$$H_{ABCD}\cdot dl = \Delta x\Delta y\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

$$\boxed{I = \Delta x\Delta y\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]}$$

www.Jntufastupdates.com

Scanned by CamScanner

current density $J = I/A$

$I = J \cdot A$

$$J \cdot A = \Delta x \cdot \Delta y \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]$$

let $A = \Delta x \cdot \Delta y$

$$J \cdot [\cancel{\Delta x \cdot \Delta y}] = \cancel{\Delta x \cdot \Delta y} \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right]$$

$$J = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \longrightarrow (5)$$

$$\boxed{J = \cancel{\ } \nabla \times \bar{H}}$$

∂H

Now applying the unit vector for each component $[a_x, a_y, a_z]$

$$\Rightarrow \left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right] \bar{a}_z + \left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right] \bar{a}_x + \left[\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right] \bar{a}_y = J$$

It can be expressed in matrix form as

$$\begin{vmatrix} a_x & a_y & a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix} = J$$

$$\therefore \boxed{\nabla \times \bar{H} = J}$$

Scanned by CamScanner

The above equation is also called as Maxwell's third Equation.

Maxwell's Third Equation [$\nabla \times \bar{H} = \bar{J}$] :-

We know that

According to Ampher's circuit Law

$$\int_l \bar{H} \cdot dl = I \longrightarrow (1)$$

WKT current density $J = I/s$

$$I = J \cdot s$$

$$dI = J \cdot ds$$

$$I = \int_s J \cdot ds \longrightarrow (2)$$

Substitute (2) in (1)

$$\int_l \bar{H} \cdot dl = \int_s J \cdot ds$$

Now applying strokes theorem

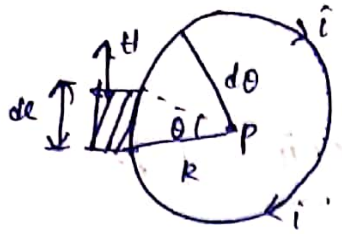
$$\iint_s \nabla \times \bar{H} \cdot \cancel{ds} = \iint_s \bar{J} \cdot \cancel{ds}$$

$$\boxed{\nabla \times \bar{H} = J}$$

www.Jntufastupdates.com

Scanned by CamScanner

* Magnetic field due to Circular loop:-



from the figure dl=

$$dl = R\,d\theta$$

from Biot-Savart's law

$$d\bar{H} = \frac{I\,dl\sin\theta}{4\pi R^2}\,\bar{a_r}$$

$$= \frac{I\,R\,d\theta\sin\theta}{4\pi R^2}\,\bar{a_r}$$

If $\theta = 90^\circ$

$\sin\theta = 1$

$$d\bar{H} = \frac{I\,d\theta}{4\pi R}\,\bar{a_r}$$

$$H = \int_0^{2\pi} \frac{I\,d\theta}{4\pi R}\,\bar{a_r}$$

$$= \frac{I}{4\pi R}\,\bar{a_r}\int_0^{2\pi} d\theta$$

$$= \frac{I\,\bar{a_r}}{4\pi R}\left[2\pi - 0\right]$$

$$\boxed{\bar{H} = \frac{I}{2R}\,\bar{a_r}}\ \text{Amp/mt}$$

www.Jntufastupdates.com

Scanned by CamScanner

Magnetic flux density

$$\bar{B} = \mu H$$

$$\boxed{\bar{B} = \frac{\mu I}{2R} \bar{a_r}} \text{ Tesla}$$

www.Jntufastupdates.com

Scanned by CamScanner

Force in Magnetic fields

Introduction :-

Magnetic field :-

The study of interaction between the ~~moving~~ charges in moving position is called as Magnetic field.

The Electric charges at rest produce electric field, the charges at moving produce magnetic field.

However, magnetic field produce a moving charges only when charged particles having charge Q travells with velocity v in a magnetic field $\bar{B}$ it experience a force that force is called as Magnetic force [~~F~~ F_m].

$$\therefore \boxed{F_m = Q(\bar{v} \times \bar{B})}$$

The above equation is called as Magnetic force equation. if $\bar{v}$ Parallel to $\bar{B}$ then force is 0.

$$\bar{v} \parallel \bar{B} \Rightarrow \boxed{F_m = 0}$$

By Taking The Magnitudes in equation the force is proportional to $\bar{v}$ and $\bar{B}$.

Similary The force is proportional to $\sin\theta$, because we take the magnitudes.

$$\boxed{F_m = BvQ\sin\theta\,\bar{a}_r}$$

www.Jntufastupdates.com

Scanned by CamScanner

Force on a Moving charge / Lorentz force Equation:-

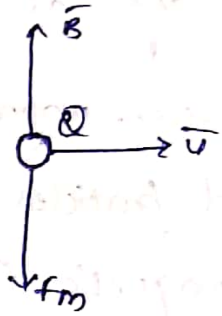
In electrostatic field ($\bar{E}$) The force on a charged particle is,

$$F_e = QE \longrightarrow (1)$$

A charged particle in motion in a magnetic field of flux density $\bar{B}$ to experience a force is given by,

$$F_m = Q(\bar{U} \times \bar{B}) \longrightarrow (2)$$

There is a fundamental difference between electric field and Magnetic field.



$\therefore$ The force is given by,

$$F = F_e + F_m$$

$$= Q\bar{E} + Q(\bar{U} \times \bar{B})$$

$$\boxed{F = Q[\bar{E} + (\bar{U} \times \bar{B})]}$$

where U = velocity of charged particles

$\bar{E}$ = Electric field intensity

$\bar{B}$ = Magnetic flux density

Q = Charge

$\therefore$ The above equation is called as force on a Moving charge / Lorentz force Equation.

www.Jntufastupdates.com

Scanned by CamScanner

* **Force on a current element in Magnetic field:-**

We know that force on a Magnetic field.

$$F_m = Q(\bar{U} \times \bar{B})$$

Now we consider linecharge, surface charge and volume charge.

Then force on a current element in a M.F is,

$$J = \rho_v \cdot \bar{U} \longrightarrow (1)$$

$$J \cdot dv = K \cdot ds = I \cdot dl \longrightarrow (2)$$

J = current density

K = surface charge density [ρ_s]

I.dl = current element

let

$$J \cdot dv = I \cdot dl \longrightarrow (3)$$

substitute (1) in (3)

$$\rho_v \cdot \bar{U} dv = I \cdot dl$$

$$\rho_v dv \cdot \bar{U} = I \cdot dl$$

$$\boxed{dQ \cdot \bar{U} = I \cdot dl} \longrightarrow (4)$$

$\rho_v = Q/v = \frac{dQ}{dv}$

$dQ = \rho_v \cdot dv.$

WKT: $F_m = Q \cdot \bar{U} \times \bar{B}$ —

$$dF_m = dQ \cdot \bar{U} \times \bar{B} \longrightarrow (5)$$

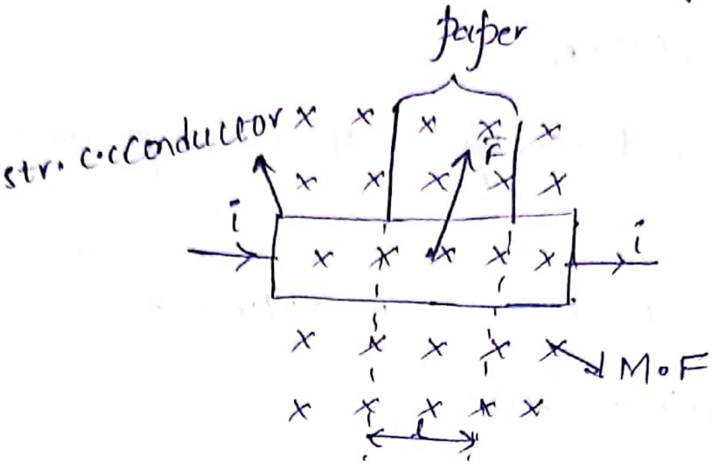
substitute (4) in (5)

$dF_m = I \cdot dl \times \bar{B}$ or $dF_m = K ds \times \bar{B}$ or $dF_m = J \cdot dv \times \bar{B}$

$F_m = \int I \cdot dl \times \bar{B}$ $F_m = \int K \cdot ds \times \bar{B}$ $F_m = \int J \cdot dv \times \bar{B}$

www.Jntufastupdates.com
Scanned by CamScanner

* force on a straight and long current carrying conductor in a Magnetic field (or) filamentary current.



WKT force on M.F is

$$F_m = Q(\bar{u} \times \bar{B}) \longrightarrow (1)$$

WKT $\bar{u} = \frac{dl}{dt} \longrightarrow (2)$

substitute (2) in (1)

$$F_m = Q\frac{dl}{dt} \times \bar{B}$$

$$dF_m = dQ\,\frac{dl}{dt} \times \bar{B}$$

$$dF_m = \frac{dQ}{dt}\,dl \times \bar{B}$$

$$dF_m = I\,dl \times \bar{B}$$

$$\boxed{F_m = BIL\sin\theta}$$

where θ represents the angle b/w vectors.

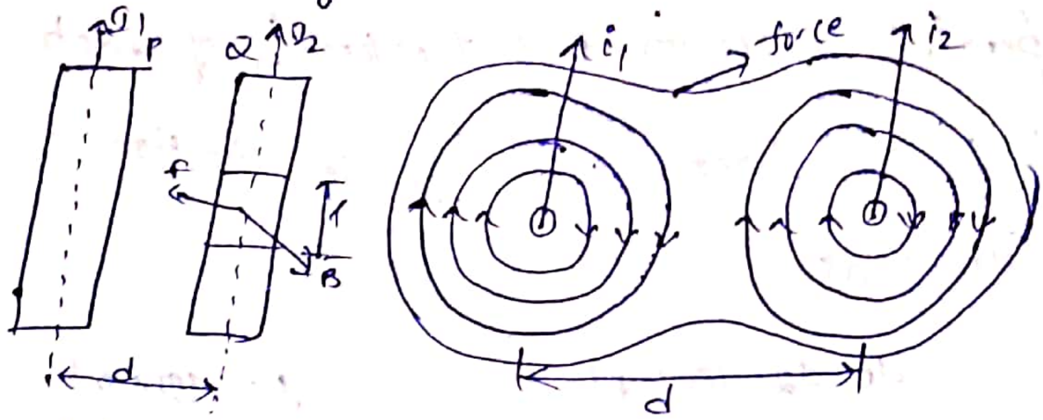
i.e., direction of current flow and direction of $\bar{B}$ flux.

∴ The above equation is called as filamentary / familiar equation.

 www.Jntufastupdates.com

Scanned by CamScanner

-* force on a 2-straight and parallel current carrying conductors due to Magnetic field.



straigh c.c.c

WKT Magnetic field intensity due straight c.c.c is,

$$\bar{H} = \frac{I_1}{2\pi d} \quad \text{[for P-conductor]}$$

WKT $\quad \bar{B} = \mu \bar{H}$

$$\bar{B} = \frac{\mu I_1}{2\pi d} \longrightarrow (1)$$

force on conductor Q, due to P is

$$f = B L I_2 \longrightarrow (2)$$

$$f = \frac{\mu I_1 L I_2}{2\pi d}$$

$$\boxed{\frac{f}{l} = \frac{\mu I_1 I_2}{2\pi d}} \ N/m$$

Scanned by CamScanner

* 2 long parallel conduction carrying 100 amps. If the conductors are seperated by 200mm find the force/m of each conductor if the current flow direction is in opposite direction.

Sol: Given that

$I_1 = I_2 = 100$ Amps $\qquad \mu = 4\pi \times 10^{-7}$

$d = 200mm = 200 \times 10^{-3}$ mt

$\frac{F}{l} = ?$

$$\frac{F}{l} = \frac{\mu I_1 I_2}{2\pi d} = \frac{\cancel{4}^{2}\cancel{\pi} \times 10^{-7} \times 100 \times 100}{\cancel{2}\cancel{\pi} \times \cancel{0.2}_{0.1}}$$

$= 0.001$ N/m

* A Galvanometer has a rectangular coil suspended in a radial magnetic field so that the magnetic field always act across the plane of coil of 10mm by 10mm side and has 1000 turns if the magnetic provides a magnetic flux density of 0.3 T. find the torque exerted on the coil for a current of 10mA.

Sol: Given that

$A = 10 \times 10$ mm $\qquad N = 1000$

$= 100 \times 10^{-6}$ $\qquad T = ?$

$I = 10$mA

$B = 0.3$T

Torque = $\underline{B \cdot M}$ M = magnetic dipolemom

$M = I \cdot A = NIA = 1000 \times 100 \times 10^{-6} \times 10 \times 10^{-3}$

$= 10^{-3}$

www.Jntufastupdates.com
Scanned by CamScanner

$T = 0.3 \times 10^{-3}$

$= .3 \times 10^{-4}$ Ntw.

* A distributional line consist of 2-straight parallel conductors supported on the cross arms of wooden poles spaced 100m. the normal spacing b/w the two conductors in 20cm suppose a current of 10,000 Amp. determine the force/~~unit~~.

sol: Given that $I_1 = I_2 = 10{,}000$

$d = 20cm = 20 \times 10^{-2}$ mt

$l = 100m$

$$f = \frac{\mu I_1 I_2}{2\pi d} l = \frac{4\pi \times 10^{-7} \times 10{,}000 \times 10{,}000 \times 100}{2\pi \times 20 \times 10^{-2}}$$

$= 10^4$ N.

* The force b/w 2-long parallel conductors is 15N/m. The conductor spacing of 10cm. if one conductor carries twice the current of other: calculate the current in each conductor.

sol: $f/l = 15 N/m$

$d = 10cm = 10 \times 10^{-2}$ mt

$I_2 = 2I_1$

$$\frac{f}{l} = \frac{\mu \cdot I_1 \cdot I_2}{2\pi d}$$

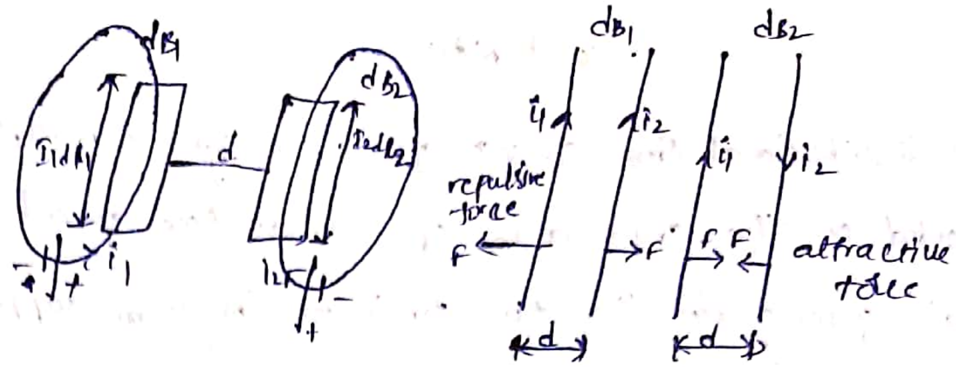
$$15 = \frac{4\pi \times 10^{-7} \times 2I^2}{2\pi \times 10^{-1}} = 40 \times 10^{-7} \times I^2$$

$$I^2 = \frac{15}{40 \times 10^{-7}} = \frac{10^6 \times \cancel{15}^{3.75}}{\cancel{4}}$$

$I_1 = 1936$ $I_2 = 3872$

Scanned by CamScanner

* Force b/n Two differential Current Elements:-



→ let us consider two current elements $I_1 d\bar{l}_1$, $I_2 d\bar{l}_2$ as shown in figure.

- Note that the direction's of I_1 & I_2 are same. the current elements produced magnetic fields ($d\bar{B}_1$ & $d\bar{B}_2$)

- As the currents are flowing in same direction through the elements the force will be developed. ($d(dF_1)$) on element I_1dl_1 due to magnetic field $d\bar{B}_2$.

- As the currents are flowing in opposite direction through the elements the force will be developed ($d(dF_2)$) on element $I_2 dl_2$ due to magnetic field $d\bar{B}_1$.

from the equation of the force on a differential current element is $d(dF_1) = I_1 dl_1 \times d\bar{B}_2$ → (a)

Acc. to Biot-Savart's law

$$dH_2 = \frac{I_2 dl_2}{4\pi R_2^2} \cdot \bar{a}_{r_{21}} \rightarrow (1)$$

Scanned by CamScanner

W.K.T.

$$\bar{B} = \mu \bar{H}$$

$$d\bar{B_2} = \mu d\bar{H_2} \longrightarrow (2)$$

substitute (1) in (2)

$$d\bar{B_2} = \frac{\mu I_2 dl_2}{4\pi R_2^2} \cdot \bar{a_{r21}} \cdot \longrightarrow (3)$$

substitute (3) in (a)

$$d(d\bar{f_1}) = \frac{I_1 dl_1 \times \mu I_2 dl_2}{4\pi R_2^2} \bar{a_{r21}}$$

$$df_1 = \int_{l_1} \frac{\mu I_1 I_2 \bar{dl_1} \bar{dl_2}}{4\pi R_2^2} \bar{a_{r21}}$$

$$f_1 = \int_{l_1}\int_{l_2} \frac{\mu I_1 I_2 \cdot \bar{dl_1} \bar{dl_2}}{4\pi R_2^2} \bar{a_{r21}}$$

Exactly, following the same steps we can conclude that,

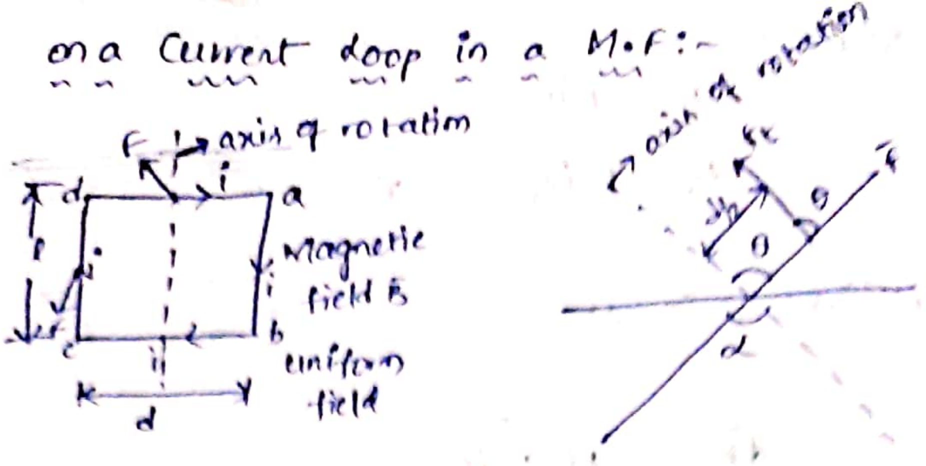
$$d(df_2) = I_2 \bar{dl_2} \times d\bar{B_1}$$

$$f_2 = -f_1$$

Thus the above equation indicates the both the forces $f_1 \& f_2$ obeys Newton's 3rd law.

Scanned by CamScanner

* Torque on a Current loop in a M.F:-



W.K.T
The force on a current element in a Magnetic field

$\bar{dF} = I(\bar{dl} \times \bar{B})$

$\bar{F} = BIL\sin\theta$

consider a rectangular loop ABCD with a sides of L & d mt carrying current of i amps is placed in a uniform Magnetic field $(\bar{B})$. i.e., shown in figure.

The force on each element of a loop side 'AB' of the length 'L' mt

i.e., $\bar{dF} = I(\bar{dl} \times \bar{B})$

$\bar{F} = BIL\sin\theta$

$\theta = 90^\circ$

$\bar{F} = BIL \longrightarrow (1)$

* If the plane of the loop is made an angle θ, i.e., shown in figure. The tangential force is w.r.t. $\bar{B}$

$F_t = F \times \cos\theta$

$F_t = BIL\cos\theta \longrightarrow (2)$

Scanned by CamScanner

the total torque on the loop is

$\tau = 2 \times (\text{torque on each side})$

$= 2 \times (F_t \times d/2)$

$= F_t \times d$

$= BIl\cos\theta \times d$

$= BI\cos\theta\,(l \times d)$

$= BIA\cos\theta$ $\bar{m}$ = dipole movement.

$= B(IA)\cos\theta$

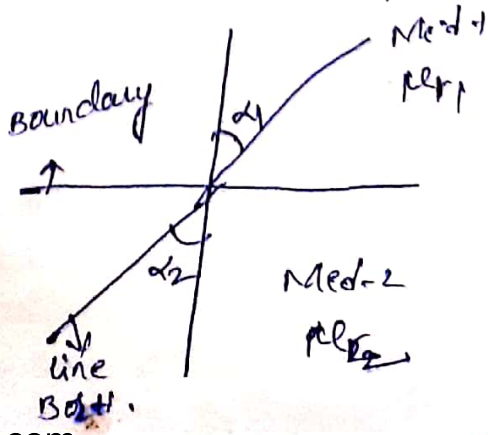
$= Bm\cos\theta$

$\boxed{\tau = Bm\cos\theta}$

If $\theta = 0$

$\boxed{\tau = Bm}$

* Derive the boundary conditions at the ~~Magnitude~~ Magnetic interfaces and show that $\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\mu_{r1}}{\mu_{r2}}$. The figure shows a magnetic field line passing through the interface b/w 2 magnetic faces of permeabilities μ_1 & μ_2. inorder to develop the boundary at an angle α_1 in media-1, α_2 in media-2. let us assume that both medials are having H and B are in same direction and $\bar{B} = \mu_0\mu_r H$ is valid



www.Jntufastupdates.com
Scanned by CamScanner

Sol: Now the relation's for magnetic boundary,

$$B_1 \cos\alpha_1 = B_2 \cos\alpha_2 \rightarrow (1)$$

$$H_1 \sin\alpha_1 = H_2 \sin\alpha_2 \rightarrow (2)$$

$$(1) \Rightarrow B_1 \cos\alpha_1 = B_2 \cos\alpha_2$$

$$\mu_0 \mu_{r_1} H_1 \cos\alpha_1 = \mu_0 \mu_{r_2} H_2 \cos\alpha_2 \rightarrow (3)$$

$$(2)/(3) \Rightarrow \frac{\cancel{H_1} \sin\alpha_1}{\mu_0 \mu_{r_1} \cancel{H_1} \cos\alpha_1} = \frac{\cancel{H_2} \sin\alpha_2}{\cancel{\mu_0} \mu_{r_2} \cancel{H_2} \cos\alpha_2}$$

$$\frac{\tan\alpha_1}{\mu_{r_1}} = \frac{\tan\alpha_2}{\mu_{r_2}} \qquad \left[\cancel{\mu_{r_1} = \mu_1} \quad \cancel{\mu_{r_2} = \mu_2} \right]$$

$$\boxed{\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{\mu_{r_1}}{\mu_{r_2}}}$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_{r_1}}{\mu_{r_2}}}$$

Scanned by CamScanner