UNIT-1

Co-ordinate System:-

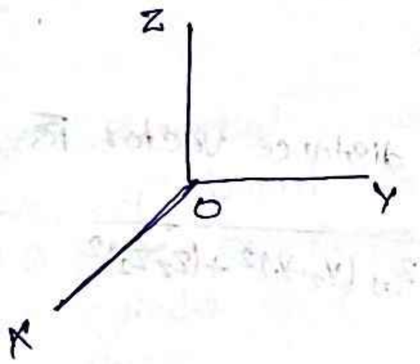
Describe Vector

vector → it's components

* Cartesian (or) Rectangular Co-ordinate System
* Cylindrical co-ordinate System
* Spherical Co-ordinate System

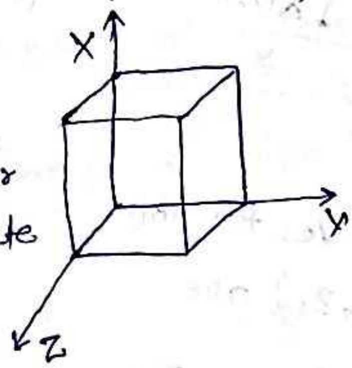
* Cartesian Co-ordinate System:-

* 3- Co-ordinate axix



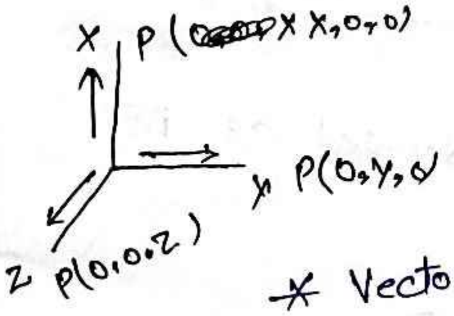
* Representation point in Cartesian Co-ordinate System
* A point has 3 Co-ordinates X, Y, Z

P (x_1, y_1, z_1)



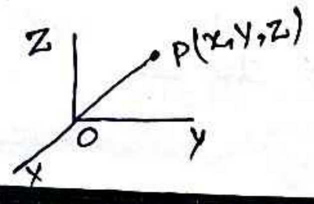
* Base Vector

* Base Vector is unit vector & directed along co-ordinate axix.



* Vector in its components
* Vector Component along X-axix $X\bar{a}_x$
* Vector component along Y-axix $Y\bar{a}_y$
* Vector component along Z-axix $Z\bar{a}_z$

* Position Vector:- positive vector of point 'p' is distance of point 'p' from origin.



It is denoted by $\bar{P}$ or $\overline{OP}$

$$\boxed{\bar{P} = x_1\bar{a}_x + y_1\bar{a}_y + z_1\bar{a}_z}$$

* Magnitude of position vector $\bar{p}$ is

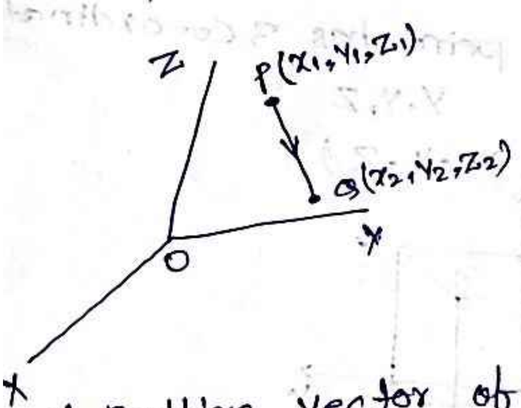
$$|\bar{P}| = |\overline{OP}| = \sqrt{x_1^2 + y_1^2 + z_1^2}$$

difference between point P & origin

* Unit vector along $\overline{OP}$ is given by

$$\boxed{\bar{a}_{op} = \frac{\overline{OP}}{|\overline{OP}|} = \frac{\bar{P}}{|\bar{P}|}}$$

* Distance vector



* Magnitude distance vector $\overline{PQ}$ is given by

$$|\overline{PQ}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

* Unit vector along $\overline{PQ}$ is given by

$$\boxed{\bar{a}_{PQ} = \frac{\overline{PQ}}{|\overline{PQ}|}}$$

* Position vector of $P(x_1, y_1, z_1)$ are

$$\bar{P} = \bar{x}_1\bar{a}_{x_1} + \bar{y}a_{y_1} + \bar{z}a_{z_1}$$

$$\bar{Q} = \bar{x}_2\bar{a}_x + y_2\bar{a}_y + z_2\bar{a}_z$$

Distance vector from P to Q is denoted as $\overline{PQ}$

$$\overline{PQ} = \bar{Q} - \bar{P}$$

$$\overline{PQ} = (x_2-x_1)\bar{a}_x + (y_2-y_1)\bar{a}_y + (z_2-z_1)\bar{a}_z$$

* Two points $A(2,3,1)$ $B(3,-4,2)$ are given in the cartesian system obtain the vector from A to B and unit vector directed from A to B.

Given points

* Position vector of $A(2,2,1)$ & $B(3,-4,2)$ are

$\bar{A} = 2\bar{a}_x + 2\bar{a}_y + 1\bar{a}_z$; $\bar{B} = 3\bar{a}_x - 4\bar{a}_y + 2\bar{a}_z$

* Distance vector from A to B is denoted as $\bar{AB}$

$$\bar{AB} = \bar{B} - \bar{A}$$

$$\bar{AB} = 1\bar{a}_x - 6\bar{a}_y + 1\bar{a}_z$$

* Magnitude distance vector $\bar{AB}$ is given by

$$|\bar{AB}| = \sqrt{(1)^2 + (-6)^2 + (1)^2} = 6.16$$

unit vector along $\bar{AB}$ is given by

$$\boxed{\bar{a}_{AB} = \frac{\bar{AB}}{|\bar{AB}|}} \qquad \bar{a}_{AB} = \frac{1\bar{a}_x - 6\bar{a}_x + 1\bar{a}_z}{6.16}$$

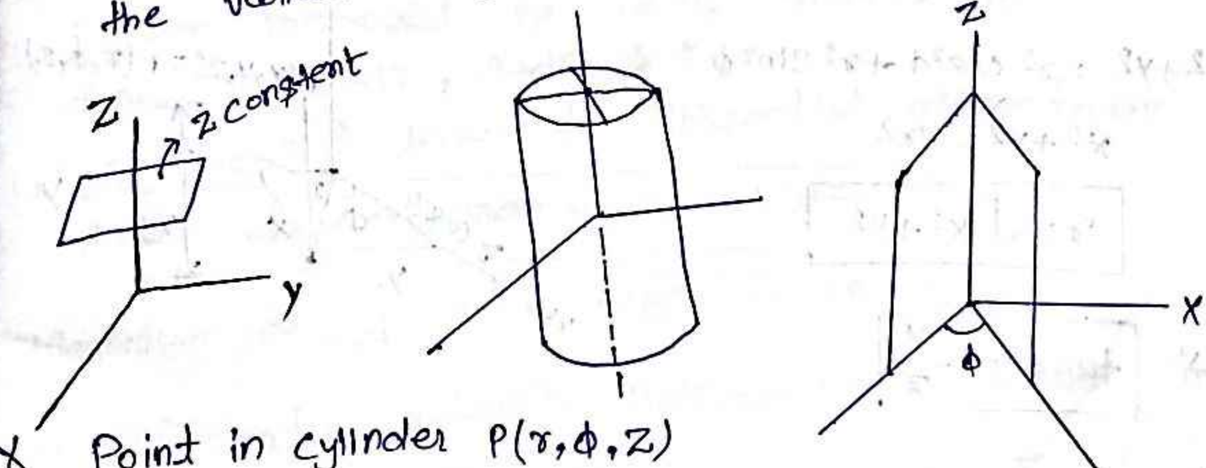
$$= 0.1622\bar{a}_x - 0.977\bar{a}_y + 0.1622\bar{a}_z$$

* Differential elements in cartesian co-ordinate system.

=> Cylindrical co-ordinate system formed by

* A plane of constant 'Z' which is parallel to x-y plane
* A cylinder of radius "r" with z-axix as the axix of cylinder.
* A half plane ⊥(er) to x-y plane at an angle (ϕ) with respect to x-y plane

The ranges of these variables $\boxed{\begin{array}{l} 0 \le r \le \infty \\ 0 \le \phi \le 2\pi \\ -\infty < z \le \infty \end{array}}$ these are the variables cylindrical co-ordinate.



Point in cylinder $P(r, \phi, z)$

$$\boxed{\bar{P} = r\bar{a}_r + \phi\bar{a}_\phi + \bar{z}\bar{a}_z}$$

* Base Vector

Unit vector $\bar{a}_r, \bar{a}_\phi, \bar{a}_z$ along r-direction, ϕ direction, Z-directions

* Position vector of point $P(r, \phi, z)$ in cylindrical co-ordinate system

$$\boxed{\bar{P} = r\bar{a}_r + \phi\bar{a}_\phi + z\bar{a}_z}$$

* Differential elements in cylindrical co-ordinate system :-

(1) Differential vector length dl is given by

$$\boxed{\bar{dl} = dr\bar{a}_r + r\,d\phi\,\bar{a}_\phi + dz\,\bar{a}_z}$$

$$|\bar{dl}| = \sqrt{(dr)^2 + (r\,d\phi)^2 + (dz)^2}$$

(2) Differential volume of differential element is

$$\boxed{dV = r\,dr\,d\phi\,dz}$$

(3) differential surface area of elements

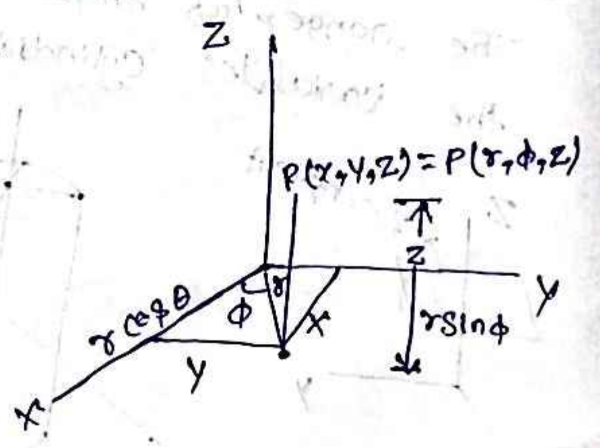
$d\bar{S}_r \Rightarrow r\,d\phi\,dz\,\bar{a}_r$

$d\bar{S}_\phi \Rightarrow dr \cdot dz\,\bar{a}_\phi$

$d\bar{S}_z \Rightarrow r\,dr\,d\phi \cdot \bar{a}_z$.

The relation between Cartesian and cylindrical system:-

* $x = r\cos\phi$, $y = r\sin\phi$, $z = z$



* $x^2 + y^2 = r^2\cos^2\phi + r^2\sin^2\phi$

$x^2 + y^2 = r^2$

$$\boxed{r = \sqrt{x^2 + y^2}}$$

* $\boxed{\tan\phi = \frac{y}{x}}$

* $\boxed{Z = z}$

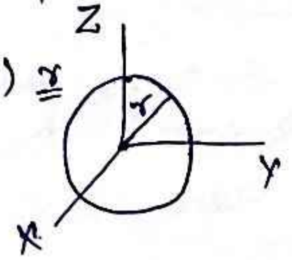
③ Spherical Co-ordinate System

* Point is formed by 3-points

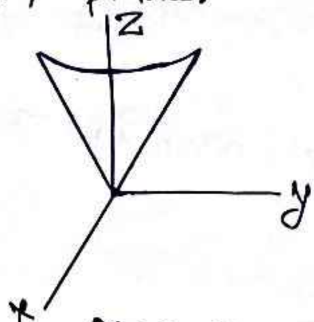
*(1) origin as the centre of sphere

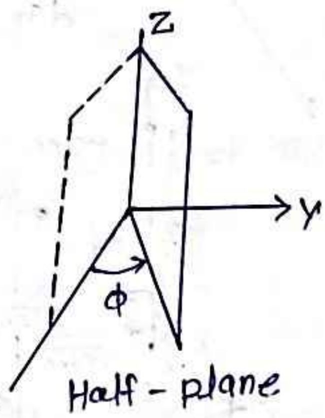
(2) A right circular cone with its APEX at the origin and its axix as Z-axix. its half angle is "θ" it's rotates about Z-axix. A half plate ⊥(er) to X-Y plane containing Z-axix making an angle "ϕ" with the X-Y plane.

(i) $\underline{\underline{r}}$



$\underline{\underline{\theta}}$





Ranges of r, θ, ϕ $\quad 0 \le r \le \infty \quad r < \theta < 2\pi \quad$ Half - plane

(1) A point in Spherical Co-ordinate System as $P(r, \theta, \phi)$

(2) Base Vector :- $\bar{a}_r, \bar{a}_\theta, \bar{a}_\phi$ are base Vectors (unit Vector) directed along 'r' 'θ' & ϕ direction.

(3)* Position Vector of point $P(r, \theta, \phi)$ as

$$\boxed{\bar{P} = r\bar{a}_r + \theta\bar{a}_\theta + \phi\bar{a}_\phi}$$

* Differential Elements in spherical Co-ordinate System increased by small amount in 'r' 'θ' & ϕ direction $r+dr$, $\theta+d\theta$, & $\phi+d\phi$ differential elements as

(1) Differential length of differential elements

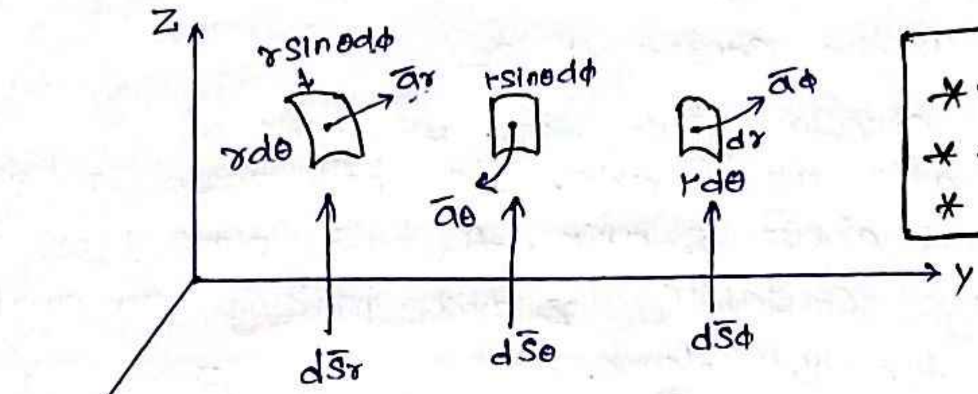
$$\boxed{\bar{dl} = dr\bar{a}_r + \cancel{rdrd\theta}\, rd\theta\bar{a}_\theta + r\sin\theta d\phi \bar{a}_\phi}$$

Magnitude $|\bar{dl}| = \sqrt{(dr)^2 + (rd\theta)^2 + (r\sin\theta d\phi)^2}$

(2) Differential Volume :- Differential elements

$$\boxed{d\bar{v} = dr \cdot rd\theta\, r\sin\theta d\phi}$$

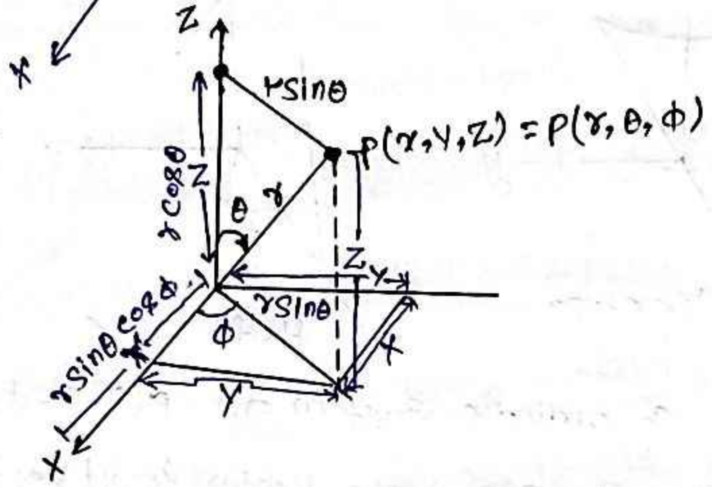
(3) Differential Surface Area:-



$* \ d\bar{S}_r = r^2 \sin\theta d\theta d\phi$

$* \ d\bar{S}_\theta = r\sin\theta dr d\phi$

$* \ d\bar{S}_\phi = r dr d\theta$



* Cartesian Co-ordinate in terms of Spherical co-ordinate:-

$x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

* Spherical co-ordinates in terms of cartesian co-ordinates.

$x^2 + y^2 + z^2 = (r\sin\theta\cos\phi)^2 + (r\sin\theta\sin\phi)^2 + (r\cos\theta)^2$

$r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$

$= r^2\sin^2\theta\,[\cos^2\phi + \sin^2\phi] + r^2\cos^2\theta$

$r^2\sin^2\theta + r^2\cos^2\theta = r^2(\cos^2\theta + \sin^2\theta)$

$= r^2(1) = r^2$

$x^2 + y^2 + z^2 = r^2$

$r = \sqrt{x^2 + y^2 + z^2}$

While $\tan\phi = \frac{y}{x}$ and $\cos\theta = \frac{z}{r}$

$\phi = \tan^{-1}\frac{y}{x}$; $\theta = \cos^{-1}\frac{z}{r}$

Substitute the value of r

$$\boxed{\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)} \quad ; \quad \boxed{\phi = \tan^{-1}\frac{y}{x}}$$

Vector Multiplication:-

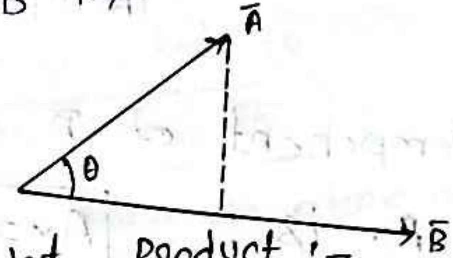
* Scaler or dot product.
* Vector or cross product.

* **Scaler or dot product:-**

Let $\bar{A}$ & $\bar{B}$ are two vectors its denoted as scaler dot product of $\bar{A}$ & $\bar{B}$

$\bar{A}\cdot\bar{B}$.

$$\boxed{\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta_{AB}}$$



* **Properties of Scaler or dot Product:-**

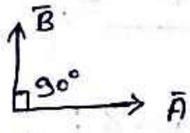
(1) If $\xrightarrow[\bar{A}]{}\xrightarrow[\bar{B}]{}$ (Parallel) two vectors are parallel to each other:

$\theta = 0°$ then $\cos\theta_{AB} = 1$

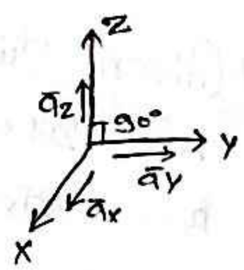
So, $\boxed{\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|}$ for parallel vectors.

(2) If two vectors are perpendicular to each other:

$\theta_{AB} = 90°$ $\quad \cos 90° = 0$



(3) $\bar{a}_x, \bar{a}_y, \bar{a}_z$ are unit vectors are mutually ⊥r to each other & they are



$$\boxed{\bar{a}_x\cdot\bar{a}_y = \bar{a}_y\cdot\bar{a}_z = a_z a_x = 0}$$

(4) Dot product of unit vector with itself is unity.

$$\boxed{\bar{a}_x\cdot\bar{a}_x = \bar{a}_y\cdot\bar{a}_y = a_z\cdot a_z = 1}$$

(5) Dot product obey's Commutative law

$$\boxed{\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}}$$

(6) Dot product obey's distructive law.

$$\boxed{\bar{A}\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot\bar{B}+\bar{A}\cdot\bar{C}}$$

(7) Angle between two vectors $\bar{A}$ & $\bar{B}$

$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta_{AB}$$

$$\cos\theta_{AB} = \frac{\bar{A}\cdot\bar{B}}{|\bar{A}|\,|\bar{B}|} \; ; \; \boxed{\theta_{AB} = \cos^{-1}\left[\frac{\bar{A}\cdot\bar{B}}{|\bar{A}|\,|\bar{B}|}\right]}$$

(8) Component of $\bar{A}$ along $\bar{a}_x$ direction denoted as A_x is ; $\boxed{A_x = \bar{A}\cdot\bar{a}_x}$

(9) Component of $\bar{A}$ along $\bar{a}_y$ direction is denoted as A_y $\boxed{A_y = \bar{A}\cdot\bar{a}_y}$

(10) Component of $\bar{A}$ along $\bar{a}_z$ direction is denoted as $\boxed{A_z = \bar{A}\cdot\bar{a}_z}$

* Given two Vectors ;

$\bar{A} = 2\bar{a}_x - 5\bar{a}_y - 4\bar{a}_z$

$\bar{B} = 3\bar{a}_x + 5\bar{a}_y + 2\bar{a}_z$

Find the dot product and angle between the two vectors.

$\Rightarrow \bar{A}\cdot\bar{B} = (2\bar{a}_x - 5\bar{a}_y - 4\bar{a}_z)\cdot(3\bar{a}_x + 5\bar{a}_y + 2\bar{a}_z)$

$= 6(\bar{a}_x\cdot\bar{a}_x) + 10(\bar{a}_x\cdot\bar{a}_y) + 4(\bar{a}_x\cdot\bar{a}_z) - 15(\bar{a}_y\cdot\bar{a}_x) - 25(\bar{a}_y\cdot\bar{a}_y) - 10(\bar{a}_y\cdot\bar{a}_z) - 12(\bar{a}_z\cdot\bar{a}_x) - 20(\bar{a}_z\cdot\bar{a}_y) - 8(\bar{a}_z\cdot\bar{a}_z)$

$= 6(1) + 0 + 0 - 0 - 25(1) - 0 - 0 - 8(1)$

$= \bar{A}\cdot\bar{B} = -27$

∴ -ve gives angle $> 90^\circ$

(2) Find the angle between two vectors

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{|A||B|}\right)$$

$$|\bar{A}| = \sqrt{(2)^2+(-5)^2+(-4)^2} = \sqrt{45}$$

$$|\bar{B}| = \sqrt{(3)^2+(5)^2+(2)^2} = \sqrt{38}$$

$$\theta_{AB} = \cos^{-1}\left(\frac{\bar{A}\cdot\bar{B}}{|\bar{A}||\bar{B}|}\right) = 130.76$$

* Vector or Cross product:-

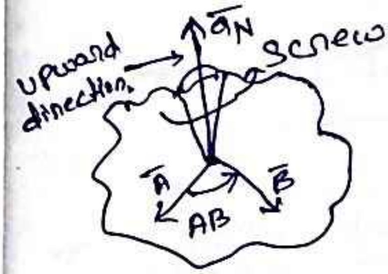
Let A & B are two vectors

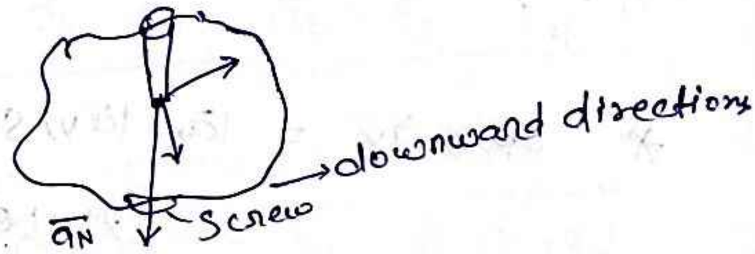
* Cross product of $\bar{A}$ & B is denoted as

$\bar{A}\times\bar{B}$ is = $\boxed{\bar{A}\times\bar{B} = |\bar{A}||\bar{B}|\sin\theta_{AB}\cdot\vec{a}_n}$

Where $\vec{a}_N$ is unit vector perpendicular to plane of A & B

* Direction of $\bar{a}_N$ is denoted Right hand screw rule.





Properties of Cross Product:-

* Cross product does not obey Commutative law.

$$\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$$

* But ✓ $\bar{B}\times\bar{A} = (\bar{A}\times\bar{B})$ Anti Commutative law.

(3) if $\bar{A}$ & $\bar{B}$ are parallel, $\theta = 0°$ $\sin 0° = 0$

$$\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \sin\theta_{AB} \cdot \bar{a}_N$$

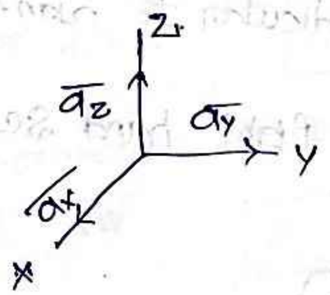
$$\boxed{\bar{A} \times \bar{B} = 0}$$

(4) if A & B are perpendicular, $\theta_{AB} = 90°$, $\sin 90° = 1$

$$\boxed{\bar{A} \times \bar{B} = |\bar{A}||\bar{B}| \cdot \bar{a}_N}$$

Cross product of unit vectors

(5) $\bar{a}_x$, $\bar{a}_y$, & $\bar{a}_z$ are unit vectors.



* $\bar{a}_x \times \bar{a}_y = |\bar{a}_x||\bar{a}_y| \sin\theta \cdot \bar{a}_N$

$$= 1 \times 1 \times 1 \, \bar{a}_N$$

$$\bar{a}_x \times \bar{a}_y = \bar{a}_N$$

$$= \bar{a}_N = \bar{a}_z$$

$$\boxed{\bar{a}_x \times \bar{a}_y = \bar{a}_z}$$

Similarly

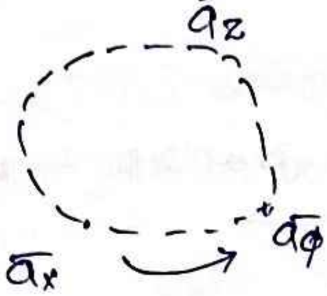
$\bar{a}_y \times \bar{a}_z = \bar{a}_x$, $\bar{a}_z \times \bar{a}_x = \bar{a}_y$

* if order of unit vector are changed

$\boxed{\bar{a}_y \times \bar{a}_x = -\bar{a}_z}$ $\boxed{\bar{a}_x \times \bar{a}_z = -\bar{a}_y}$,

$\boxed{\bar{a}_z \times \bar{a}_y = -\bar{a}_x}$

Cylindrical Co-ordinate System

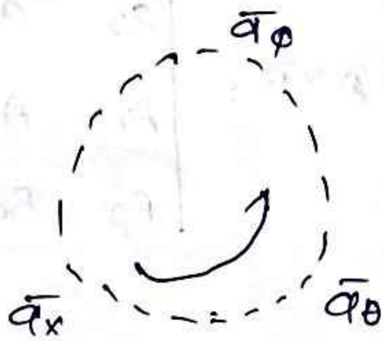


$\bar{a}_r \times \bar{a}_\phi = \bar{a}_z$

$\bar{a}_\phi \times \bar{a}_z = \bar{a}_r$

$\bar{a}_z \times \bar{a}_r = \bar{a}_\phi$

Spherical Co-ordinate System



$\bar{a}_r \times \bar{a}_\theta = \bar{a}_\phi$

$\bar{a}_\theta \times \bar{a}_\phi = \bar{a}_r$

$\bar{a}_\phi \times \bar{a}_r = \bar{a}_\theta$

* $\bar{a}_x \times \bar{a}_x = |\bar{a}_x||\bar{a}_x| \sin\theta_{xx} \cdot \bar{a}_N \quad (\theta_{xx} = 0)$

$\bar{a}_x \times \bar{a}_x = 0$

* $\therefore \boxed{\bar{a}_x \times \bar{a}_x = \bar{a}_y \times \bar{a}_y = \bar{a}_z \times \bar{a}_z = 0}$

* Cross product in Determinent form.
Cartesian System Co-ordinate are (x, y, z)

Let $\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$

$\bar{B} = B_x \bar{a}_x + B_y \bar{a}_y + B_z \bar{a}_z$

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - B_y A_z)\bar{a}_x - (A_x B_z - B_x A_z)\bar{a}_y + (A_x B_y - B_x A_y)\bar{a}_z$$

* If $\bar{A}$ & $\bar{B}$ are in cylindrical

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_r & \bar{a}_\phi & \bar{a}_z \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$$

* if $\bar{A}$ & $\bar{B}$ are in spherical

$$\bar{A} \times \bar{B} = \begin{vmatrix} \bar{a}_r & \bar{a}_\theta & \bar{a}_\phi \\ A_r & A_\theta & A_\phi \\ B_r & B_\theta & B_\phi \end{vmatrix}$$

Given the two coplaner vectors

$\bar{A} = 3\bar{a}_x + 4\bar{a}_y - 5\bar{a}_z$

$\bar{B} = -6\bar{a}_x + 2\bar{a}_y + 4\bar{a}_z$ oftened the unit vector normal to main countaining the vectors A and B.

Normal to = Cross product of two vectors.

$$\bar{a}_N = \frac{\bar{A} \times \bar{B}}{|\bar{A} \times \bar{B}|} =$$

* $\bar{A} \times \bar{B} = \begin{vmatrix} a_x & a_y & a_z \\ 3 & 4 & -5 \\ -6 & 2 & 4 \end{vmatrix} = (16+10)a_x - (12-30)\bar{a}_y + (6+24)\bar{a}_z$

$= 26\bar{a}_x + 18\bar{a}_y + 30\bar{a}_z$

$*\ |\bar{A}\times\bar{B}| = \sqrt{(26)^2+(18)^2+(30)^2} = 43.58$

$$\bar{a}_N = \frac{26\bar{a}_x + 18\bar{a}_y + 30\bar{a}_z}{43.58}$$

$$\boxed{\bar{a}_N = 0.596\bar{a}_x + 0.413\bar{a}_y + 0.688\bar{a}_z}$$

Another Method :-

$\bar{A} = 3\bar{a}_x + 4\bar{a}_y - 5\bar{a}_z$

$\bar{B} = -6\bar{a}_x + 2\bar{a}_y + 4\bar{a}_z$

$\bar{A}\times\bar{B} = (3\bar{a}_x + 4\bar{a}_y - 5\bar{a}_z)\times(-6\bar{a}_x + 2\bar{a}_y + 4\bar{a}_z)$

$= -18\underbrace{(\bar{a}_x\times\bar{a}_x)}_{0} + 6\underbrace{(\bar{a}_x\times\bar{a}_y)}_{a_z} + 12(\bar{a}_x\times\bar{a}_z)$

$- 24(\bar{a}_y\times\bar{a}_x) + 8(\bar{a}_y\times\bar{a}_y) + 16(\bar{a}_y\times\bar{a}_z)$

$+ 30(\bar{a}_z\times\bar{a}_x) - 10(\bar{a}_z\times\bar{a}_y) - 20(\bar{a}_z\times\bar{a}_z)$

Transformation of vector :-

(i) from Cartesian to cylindrical

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

(ii) Cylindrical to Cartesian

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix}$$

(iii) Cartesian to Spherical

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{bmatrix} \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

(iv) Spherical to Cartesian

$$\begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}$$

Transfer the vector field!-

$\bar{W} = 10\bar{a_x} - 8\bar{a_y} + 6\bar{a_z}$ to cylindrical co-ordinate system at a point (10, -8, 6)

$\Rightarrow$ $\bar{W} = 10\bar{a_x} - 8\bar{a_y} + 6\bar{a_z}$

$\bar{A} = A_x\bar{a_x} + A_y\bar{a_y} + A_z\bar{a_z}$

$\bar{A} = A_r\bar{a_r} + A_\phi\bar{a_\phi} + A_z\bar{a_z}$ (for cylindrical co-ordina

$$\begin{bmatrix} A_r \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ -8 \\ 6 \end{bmatrix}$$

* $A_r = 10\cos\phi - 8\sin\phi =$
* $A_\phi = -10\sin\phi - 8\cos\phi =$
* $A_z = 6$

Relation between cartession & cylindrical

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

$\phi = \tan^{-1}\left(\frac{-8}{10}\right) = -38.65$

Sub the val in A_r, A_ϕ, ~~A_z~~ :

So, $A_r = 10\cos(-38.65) - 8\sin(-38.65) = 12.8$

$A_\phi =$ ~~$-10\sin\phi - 8c$~~ $-10\sin(-38.65) - 8\cos(-38.65) = 0$

$\bar{A} = 12.8\bar{a_r} + 6\bar{a_z}$

* Given point $P(r=5, \phi=60^\circ, z=2)$ & $Q(r=2, \phi=110^\circ, z=-1)$ in cylindrical Co-ordinate System. find unit vector in cartesion quardent at P directed towards Q.

=> Cartesion quardent means Rectangle quardent Co-ordinated.

* P & Q are in cylindrical System
* Find Point P & Q in Cartessian System for point P

$x = r\cos\phi, \quad y = r\sin\phi, \quad z = z$

$x = 5\cos 60^\circ = 2.5, \quad y = 5\sin 60^\circ = 4.33, \quad z = 2$

$P(x, y, z) = P(2.5, 4.33, 2)$

for point Q

$x = r\cos\phi, \quad y = r\sin\phi, \quad z = z$

$x = 2\cos 110^\circ = -0.68, \quad y = 2\sin 110^\circ = 1.87, \quad z = -1$

$Q = (x, y, z) = Q(-0.68, 1.87, -1)$

* Find position of Vectors of 'P' & 'Q'

$\bar{P} = 2.5\bar{a_x} + 4.33\bar{a_y} + 2\bar{a_z}$

$\bar{Q} = -0.68\bar{a_x} + 1.87\bar{a_y} - 1\bar{a_z}$

* Find distance Vector from $\bar{P}$ to $\bar{Q}$

$\bar{PQ} = \bar{Q} - \bar{P}$

$$\boxed{\bar{PQ} = (-0.68 - 2.5)\bar{a_x} + (1.87 - 4.33)\bar{a_y} + (-1 - 2)\bar{a_z}}$$

* for unit vector

$$\bar{a}_{PQ} = \frac{\bar{PQ}}{|\bar{PQ}|}$$

$$|\overline{PQ}| = \sqrt{(-3.18)^2 + (-2.46)^2 + (-3)^2} = 5.01$$

$$\bar{a}_{PQ} = \frac{-3.18\,\bar{a}_x - 2.46\,\bar{a}_y - 3\bar{a}_z}{5.01}$$

$$\bar{a}_{PQ} = -0.6347\bar{a}_x - 0.4910\bar{a}_y - 0.5988\bar{a}_z$$

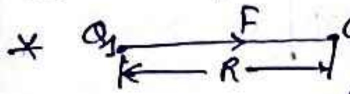
Unit-1 Electrostatics

Statics - Stationary

~~Coulomb~~ :-

* Point charges is in small dimension & having charge q.
* charge has +ve or -ve
* like charges repel each other.
* unlike charges attract each other.

Coulomb's Law :- Consider two point charges Q_1, Q_2 as shown seperated by a distance R. Charge Q_1 exerts a force on Q_2 while Q_2 also exerts a force on Q_1.



The force develope between the two point charges due to electric field and acted along the line joining of two point charges. is directly propotional to product of charges (Q_1, Q_2) of two point charges and inversily propotional to square of the distance between the two points charges that is

$$\boxed{F \propto \frac{Q_1 Q_2}{R^2}}$$

$$F = \frac{K Q_1 Q_2}{R^2}$$

Where Q_1 and Q_2 = Product of charges.

R = Distance between two points charges

K = Constant of proportionality.

$$\boxed{K = \frac{1}{4\pi\varepsilon}}$$

ε = Absolute permittivity of medium. (units is F/m).

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

ε_0 = Permittivity of free space or vacuum

$\varepsilon_0 = 8.854 \times 10^{-12}$ F/m

ε_r = Relative permittivity of medium w.r.t free space (or) dielectric constant

ε = Absolute permittivity

for the free space or vacuum, the relative permittivity $\varepsilon_r = 1$, hence

$$\varepsilon = \varepsilon_0$$

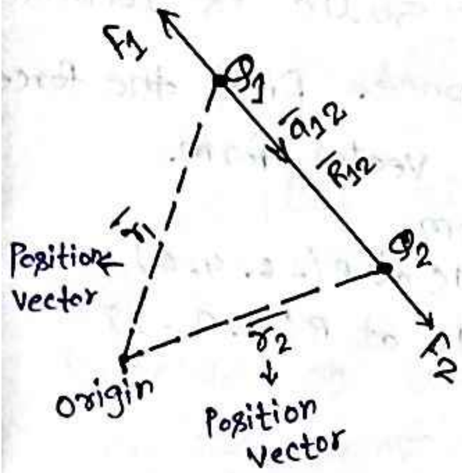
$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{R^2}}$$

The value of permittivity of free space ε_0 is,

$$\boxed{\varepsilon_0 = \frac{1}{36\pi} \times 10^{-9} = 8.854 \times 10^{-12} \text{ F/M}}$$

$$K = \frac{1}{4\pi\varepsilon_0} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 8.98 \times 10^{9} \simeq 9 \times 10^{9} \text{ m/F}$$

* Vector form of coulomb's law



* force $\overline{F_2}$ on Q_2 due to Q_1 is given as

$$\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \cdot \overline{a_{12}}$$

where $\overline{a_{12}}$ is unit vector along R_{12} direction.

$$\overline{a_{12}} = \frac{\overline{R_{12}}}{|R_{12}|}$$

* $\overline{R_{12}} = \overline{r_2} - \overline{r_1}$

$$\overline{a_{12}} = \frac{\overline{r_2}-\overline{r_1}}{|\overline{r_2}-\overline{r_1}|}$$

$$\boxed{\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{12}^2} \cdot \overline{a_{12}}} \quad \text{— (i)}$$

* force $\overline{F_1}$ on the Q_1 due to the Q_2 is given by

$$\boxed{\overline{F_1} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R_{21}^2} \overline{a_{21}}} \quad \text{— (i)}$$

$$\overline{a_{21}} = \frac{\overline{R_{21}}}{|R_{21}|} = \frac{\overline{r_1}-\overline{r_2}}{|\overline{r_1}-\overline{r_2}|}$$

But $\overline{r_1}-\overline{r_2} = -(r_2 - r_1)$

$\therefore \overline{a_{21}} = -\overline{a_{12}}$ $\quad (R_{12} = R_{21})$

$$\boxed{F_1 = \frac{q_1 q_2}{4\pi\varepsilon_0 R_{12}^2} \cdot (-a_{12})} \quad \text{--- (ii)}$$

from equ (i) and (ii)

$$\boxed{\overline{F_1} = -\overline{F_2}}$$

So, force acting on each charge is equal to magnitude and opposite in direction.

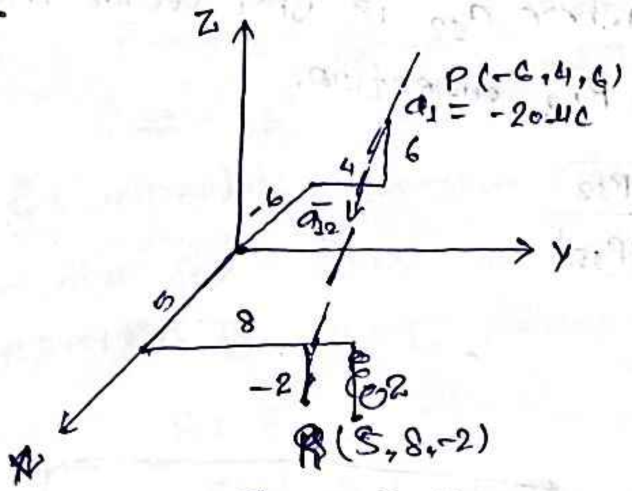
* A charge $q_1 = -20\mu C$ is located at point $P(-6,4,6)$ and a charge $q_2 = 50\mu C$ is located at $R(5,8,-2)$ in a free space. Find the force exicted on q_2 by q_1 in a vector form.

The distances are given in form.

$q_1 = -20\mu C$ at $P(-6,4,6)$

$q_2 = 50\mu C$ at $R(5,8,-2)$

=>



$q_2 = 50\mu C$

* Position $\overline{F_2}$ on q_2 due to q_1 is given by

$$\overline{F_2} = \frac{q_1 q_2}{4\pi\varepsilon_0 R^2_{PR}} \cdot \overline{a}_{PR}$$

$$\overline{a}_{PR} = \frac{\overline{PR}}{|\overline{PR}|} = \quad * \ \overline{PR} = \overline{R} - \overline{P}$$

$$\overline{PR} = 11\overline{a}_x + 4\overline{a}_y - 8\overline{a}_z$$

$$|\overline{PR}| = \sqrt{(11)^2 + (4)^2 + (8)^2} = 14.177$$

$$\overline{F_2} = \frac{-20\times10^{-6}\times50\times10^{-6}}{4\pi\times8.854\times10^{-12}(14.177)^2} \times \left(\frac{11\overline{a}_x + 4\overline{a}_y - 8\overline{a}_z}{14.177}\right)$$

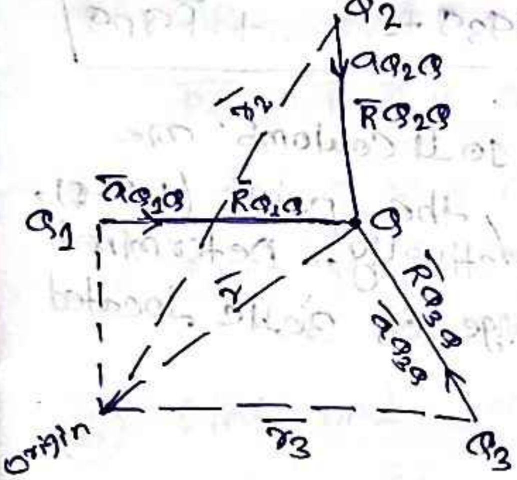
$$\overline{F_2} = -\frac{0.04471}{14.177}(11\overline{a}_x + 4\overline{a}_y - 8\overline{a}_z)$$

$$\overline{F_2} = -0.0346\,\bar{a}_x - 0.0126\,\bar{a}_y + 0.0252\,\bar{a}_z$$

$$|\overline{F_2}| = \sqrt{(-0.0346)^2 + (-0.0126)^2 + (0.0252)^2}$$

$$|\overline{F_2}| = 0.0446\ N = 44.6\ mN \quad \text{Ans}$$

Principle of Superposition:- If there are more then two point charges, then each will expert force on the other, then the net force on any other charge can be obtained by principle of superposition.



*Force

Consider a point charge q surrounded by three other point charges q_1, q_2 and q_3 as shown in fig.

* Force $\overline{F}_{q_1 q}$ on q due to q_1 only (neglected effect of q_2, q_3)

$$\overline{F}_{q_1 q} = \frac{q_1 q}{4\pi\varepsilon_0 R^2_{q_1 q}} \cdot \bar{a}_{q_1 q}$$

* Force $\overline{F}_{q_2 q}$ on q due to q_2 only (neglected effect of q_1 and q_3)

$$\overline{F}_{q_2 q} = \frac{q_2 q}{4\pi\varepsilon_0 R^2_{q_2 q}} \cdot \bar{a}_{q_2 q}$$

* Force $\overline{F}_{q_3 q}$ on q due to q_3 only (neglect effect of q_1 & q_2)

$$\overline{F}_{q_3 q} = \frac{q_3 q}{4\pi\varepsilon_0 R^2_{q_3 q}} \cdot \bar{a}_{q_3 q}$$

* According to principle of Superposition.

Total force $\bar{F}_q$ on q due to q_1, q_2 and q_3 is

$$\bar{F}_q = \bar{F}_{q_1 q} + \bar{F}_{q_2 q} + \bar{F}_{q_3 q}$$

* for 'n' charges, Total force $\bar{F}_q$ on q due to $q_1, q_2, q_3 \ldots q_n$

$$\boxed{\bar{F}_q = \bar{F}_{q_1 q} + \bar{F}_{q_2 q} + \bar{F}_{q_3 q} + \ldots\ldots + \bar{F}_{q_n q}}$$

Four point charges each of 10 μ Coulomb are placed in free space at the point (1,0,0), (-1,0,0), (0,1,0), (0,-1,0) meters respectively. Determined the force on a point charge of 30μC located at a point (0,0,1) m.

⇒ Given data

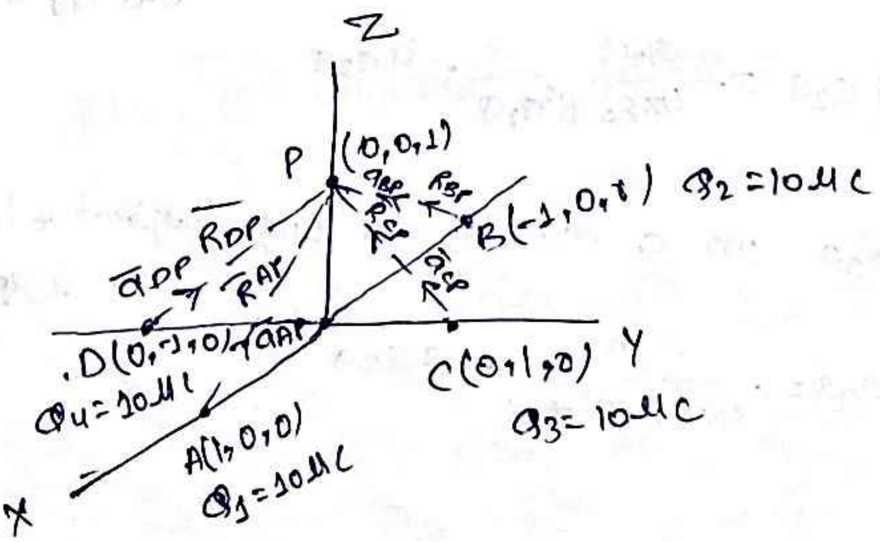
A=(1,0,1) q_1 = 10μC

B=(-1,0,0) q_2 = 10μC

C=(0,1,0) q_3 = 10μC

D=(0,-1,0) q_4 = 10μC

E=(0,0,1) q_5 = 30μC



* find the position vectors of points A, B, C, D & P

$\bar{A} = 1a_x$

$\bar{B} = -a_x$

$\bar{C} = \bar{a_y}$

$\bar{D} = -\bar{a_y}$

$\bar{E} = \bar{a_z}$

(3) find distance vectors.

$\overline{AP} = \bar{P} - \bar{A} = \bar{a_z} - \bar{a_x}$

$\overline{BP} = \bar{P} - \bar{B} = \bar{a_z} + \bar{a_x}$

$\overline{CP} = \bar{P} - \bar{C} = \bar{a_z} - \bar{a_y}$

$\overline{DP} = \bar{P} - \bar{D} = \bar{a_z} + \bar{a_y}$

(4) find unit vectors

$$\bar{a}_{AP} = \frac{\overline{AP}}{|\overline{AP}|} = \frac{\bar{a_z} - \bar{a_x}}{\sqrt{2}}$$

$$\bar{a}_{BP} = \frac{\overline{BP}}{|\overline{BP}|} = \frac{\bar{a_z} + \bar{a_x}}{\sqrt{2}}$$

$$\bar{a}_{CP} = \frac{\overline{CP}}{|\overline{CP}|} = \frac{\bar{a_z} - \bar{a_y}}{\sqrt{2}}$$

$$\bar{a}_{DP} = \frac{\overline{DP}}{|\overline{DP}|} = \frac{\bar{a_z} + \bar{a_y}}{\sqrt{2}}$$

(5) find Net force F_q on q due to q_1, q_2, q_3 & q_4 By principle of Superposition.

$$\boxed{\bar{F}_q = \bar{F}_{q_1 q} + \bar{F}_{q_2 q} + \bar{F}_{q_3 q} + \bar{F}_{q_4 q}}$$

$* \quad \bar{F}_{Q_1Q} = \frac{Q_1 Q}{4\pi\varepsilon_0 R^2_{AP}} \cdot \bar{a}_{AP}$

$$= \frac{10\times10^{-6} \times 30\times10^{-6}}{4\pi\times 8.854\times10^{-12}\times(\sqrt{2})^2} \cdot \frac{(\bar{a}_z - \bar{a}_x)}{\sqrt{2}} = 0.9542(\bar{a}_z - \bar{a}_x)$$

Similarly

$Q_1 = Q_2 = Q_3 = Q_4 = 10\mu C$

So magnitude are same only different is unit vector.

$* \quad \bar{F}_{Q_2Q} = 0.9542(\bar{a}_z + \bar{a}_x)$

$* \quad \bar{F}_{Q_3Q} = 0.9542(\bar{a}_z - \bar{a}_y)$

$* \quad \bar{F}_{Q_4Q} = 0.9542(\bar{a}_z + \bar{a}_y)$

$\therefore$ $\bar{F}_Q$ by principle of superposition.

$$\bar{F}_Q = 0.9542(\bar{a}_z - \bar{a}_x) + 0.9542(\bar{a}_z + \bar{a}_x) + 0.9542(\bar{a}_z - \bar{a}_y) + 0.9542(\bar{a}_z + \bar{a}_y)$$

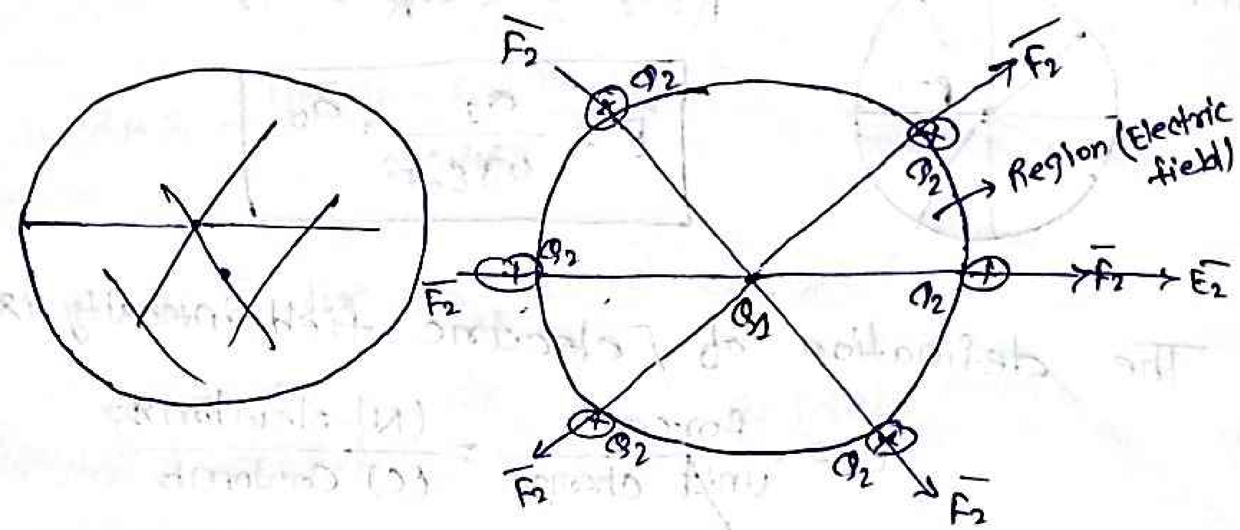
$$\bar{F}_Q = 0.9542(\bar{a}_z - \cancel{\bar{a}_x} + \bar{a}_z + \cancel{\bar{a}_x} + \bar{a}_z - \cancel{\bar{a}_y} + \bar{a}_z + \cancel{\bar{a}_y})$$

$$\bar{F}_Q = 0.9542(4\bar{a}_z)$$

$$\boxed{\bar{F}_Q = 3.8168\,\bar{a}_z}$$

Electric field intensity (E):- force per charge is called Electric field intensity = $\boxed{\overline{E} = \frac{F}{Q}}$

units = V/m (Volt/meter) & (newton/coulomb)



According to Coulomb's law

* force $\overline{F_2}$ on Q_2 due to Q_1 is given by Coulomb's law

$$\boxed{\overline{F_2} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2} \cdot \overline{a}_R}$$

The force per unit charge can be written as

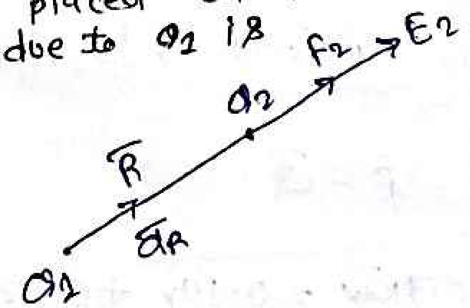
$$\text{*}\ \overline{E_2} = \boxed{\frac{\overline{F_2}}{Q_2} = \frac{Q_1}{4\pi\varepsilon_0 R^2} \cdot \overline{a_R}} \quad \text{--- (i)}$$

in General

$$\boxed{\overline{E} = \frac{Q_1}{4\pi\varepsilon_0 R_{1P}^2} \times a_{1P}} \text{ --(ii) due to } Q_1$$

Where P = position of any charge around Q_1

consider a charge Q_1 as shown in fig, and a other charge Q_2 is placed at a distance of R from Q_1. Then the force acting on Q_2 due to Q_1 is along the unit Vector $\overline{a_R}$



$$\boxed{\overline{E} = \frac{Q_1}{4\pi\varepsilon_0 R^2} \overline{a}_R} \quad \text{--- (iii)}$$

* In Spherical Co-ordinate System:- If a charge q_1 is located at the centre of the spherical coordinate system then unit vector $\bar{a}_R$ in the equation (3) become the radial unit vector $\bar{a}_r$. and the distance R is the radius of the sphere r.

$$R = r \ \& \ \bar{a}_R = \bar{a}_r$$

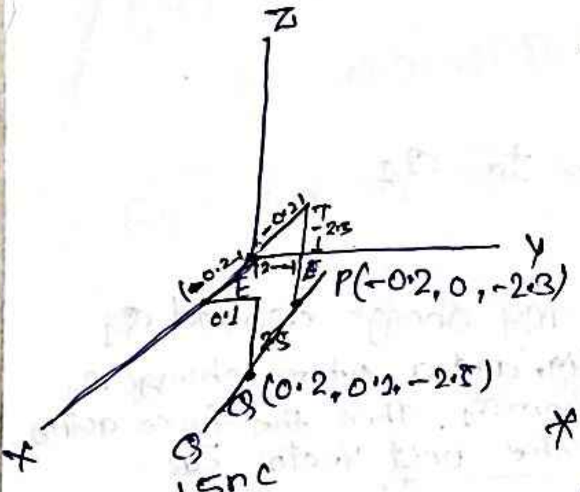
$$\boxed{\bar{E} = \frac{q_1}{4\pi\varepsilon_0 r^2} \cdot \bar{a}_r}$$

The defination of electric field intensity is,

$$\bar{E} = \frac{\text{force}}{\text{unit charge}} = \frac{(N) \text{ Newtons}}{(C) \text{ Coulomb}}$$

Hence unit of $\bar{E}$ is N/C & $\bar{E}$ is also measured in units V/m (Volte per metre).

* Determind Electric field intensity at P(-0.2, 0, -2.3) due to a point charge of 5nC at Q(0.2, 0.1, -2.5)m in air.



* Find Position Vector of point P & Q

$$\Rightarrow \bar{P} = -0.2\bar{a}_x - 2.3\bar{a}_z$$

$$\bar{Q} = 0.2\bar{a}_x + 0.1\bar{a}_y - 2.5\bar{a}_z$$

* Find Distance Vector of ~~Point~~ from ~~origin~~ Q to P

$$\overline{QP} = \bar{P} - \bar{Q}$$

$$\overline{QP} = -0.4\bar{a}_x - 0.1\bar{a}_y + 0.2\bar{z}$$

find unit vector

$$\bar{a}_R = \frac{\overline{QP}}{|\overline{QP}|} = \frac{-0.4\bar{a}_x - 0.1\bar{a}_y + 0.2\bar{a}_z}{\sqrt{(-0.4)^2 + (-0.1)^2 + (0.2)^2}}$$

$$\bar{a}_R = \frac{-0.4\bar{a}_x - 0.1\bar{a}_y + 0.2\bar{a}_z}{0.458}$$

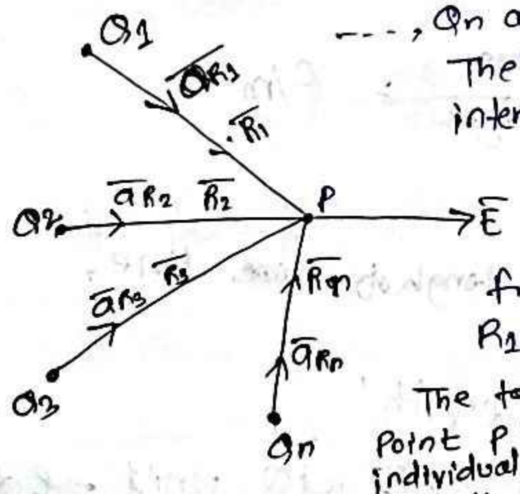
* find $\bar{E}$ due to point changes

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 R^2} \cdot \bar{a}_R \qquad R = |\overline{QP}| = 0.458$$

$$\bar{E} = \frac{5\times10^{-9}}{4\pi\times 8.854\times10^{-12}\times(0.458)^2} \cdot \frac{(-0.4\bar{a}_x - 0.1\bar{a}_y + 0.2\bar{a}_z)}{(0.458)}$$

$$\bar{E} = -187.09\,\bar{a}_x - 46.773\,\bar{a}_y + 93.546\,\bar{a}_z$$

* Electric field intensity due to discrete changes.



Consider a changes $Q_1, Q_2, \ldots, Q_n$ as shown in fig. The combined electric field intensity is to be obtained by at point P. The distance of point P from $Q_1, Q_2, \ldots, Q_n$ are $R_1, R_2, \ldots, R_n$ respectively. The total electric field intensity at point P is the vector sum of the individual field intensities produced by the various charges at the point P. $\boxed{E = E_1 + E_2 + E_3 + \ldots + \bar{E}_n}$

By principle of superposition

$$\boxed{E_P = \bar{E}Q_1 + \bar{E}Q_2 + \bar{E}Q_3 + \ldots\ldots + \bar{E}Q_n}$$

$$E = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i^2}\bar{a}_{R_i}$$

$$\boxed{E = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\cdot\bar{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2^2}\cdot\bar{a}_{R_2} + \ldots + \frac{Q_n}{4\pi\varepsilon_0 R_n^2}\cdot\bar{a}_{R_n}}$$

* Types of change distribution :-

(1) Point change

(2) Line change

(3) Surface change

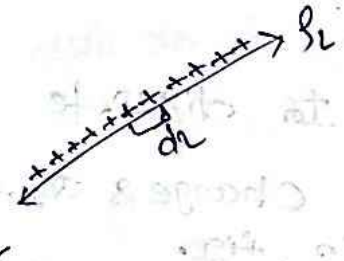
(4) Volume change

(1) * Point change :- (·)

* Point change have position but not dimension.

* Point change can be +V or -V.

(2) Line changes :-



ρ_L (line charge density)

* Line has finite or infinite charge

$$\rho_L = \frac{\text{Total charge}}{\text{Total length}} = C/m$$

ρ_L = Constant over lenght of ~~the~~ line.

* dl = differential length

charge ~~dQ unit~~ $\boxed{dQ \text{ unit} = \rho_L dl}$

$$\rho_L = \frac{dQ}{dl}$$

Total charge in line * ~~Total charge in~~ line

$Q = \int_L dQ = \int_L \rho_L dL$ ~~Q = ∫~~ = $\int \rho_L dl$

Types of Charge distribution.

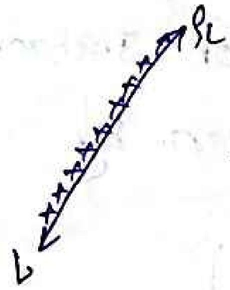
(i) Point charge (ii) Line charge

(iii) Surface charge (iv) Volume charge

(1) * Point charge:-

* Point charge have position but not dimension.
* Point charge can be +ve or -ve.

(2) Line Charge! -



ρ_L (Line charge density)

* Line has finite or infinite charge.

$$\rho_L = \frac{\text{Total charge}}{\text{Total length}} = C/m$$

ρ_L = Constent over length of line.

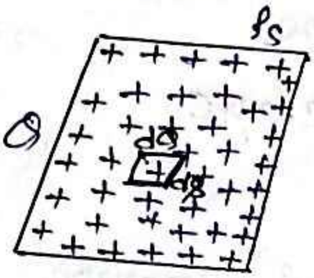
* dl = differential length

charge dq unit $= \rho_L dl$

$$\rho_L = \frac{dq}{dl}$$

$$q = \int_L dq = \int_L \rho_L dL$$

(3) Surface charge :- If the charge is distributed uniformly over a two dimensional surface then it is called a surface charge (or) a sheet of charge.



* ρ_s = Surface charge density

* $$\boxed{\rho_s = \frac{\text{Total charge}}{\text{Total Area}}}\quad C/m^2$$

Sheet of charge (Surface charge)

* ds → Differential surface Area

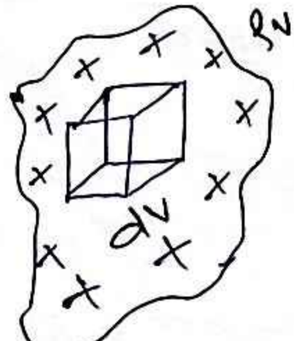
* dQ → dQ is differential charge on surface and Area is given by

$$\boxed{dQ = \rho_s d\cdot s} \quad \left(\rho_s = \frac{dQ}{dS}\right)$$

* Total charge on surface is

$$\boxed{Q = \int_S dQ = \int_S \rho_s dS}$$

(4) Volume charge :- If the charge distributed uniformly in a volume then it is called volume charge.



The volume charge is as shown in fig.

* ρ_v = Volume charge density

* $\boxed{\rho_v = \dfrac{\text{Total charge}}{\text{Total volume}} = \dfrac{C}{m^3}}$

* dv = differential volume

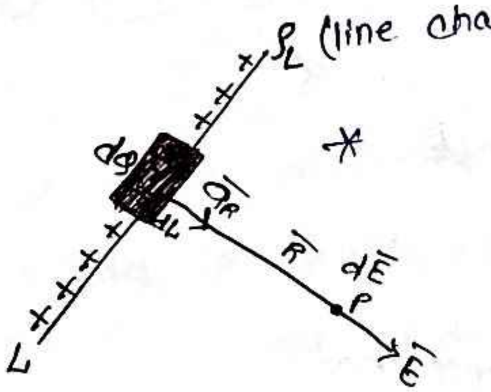
* dq = differential charge

$$dq = \rho_v dv \quad \left(\rho_v = \frac{dq}{dv}\right)$$

* $\boxed{q = \int_v dq = \int_v \rho_v dv}$

* $\overline{E}$ due to various charge distribution

* $\overline{E}$ due to Line charge



Consider a line charge distribution having a charge density ρ_L as shown in fig.

dl is differential length.

The charge dq on the differential ~~volume~~ length dl is,

* dl is differential length

* ~~dqe is~~ dq is charge on differential length is given by

$$\boxed{dq = \rho_L dl} \quad \text{—— (i)}$$

* $\overline{dE}$ is differential electric field intensity at point 'p' due to dq is

$$d\bar{E} = \frac{dq}{4\pi\varepsilon_0 R^2}\cdot \bar{a}_R \quad --(ii)$$

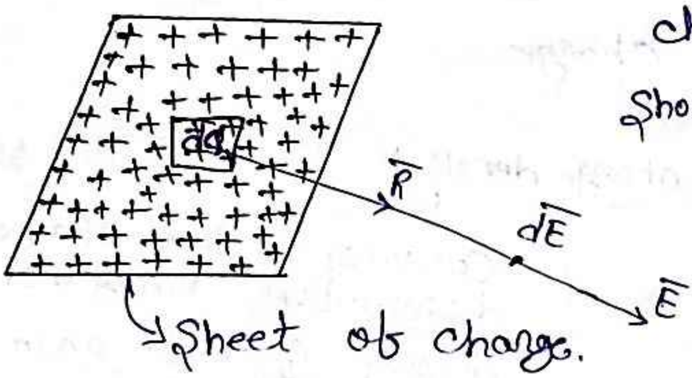
$$d\bar{E} = \frac{\rho_L dL}{4\pi\varepsilon_0 R^2}\cdot \bar{a}_R$$

* $\bar{E}$ at point p can be obtained

$$\bar{E} = \int_L d\bar{E}$$

$$\boxed{\bar{E} = \int_L \frac{\rho_L dL}{4\pi\varepsilon_0 R^2}\cdot \bar{a}_R}$$

* $\bar{E}$ due to Surface charge :- Consider a surface charge distribution having charge density ρ_S as shown in fig.



* ds is differential surface area
* dq is charge on differential surface area is given by

$$\boxed{dq = \rho_S \cdot ds} \quad --(i)$$

* $d\bar{E}$ is differential electric field intensity at point 'p' due to dq is

$$\boxed{d\overline{E} = \frac{dq}{4\pi\varepsilon_0 R^2}\cdot\overline{a_R}} \quad --(ii)$$

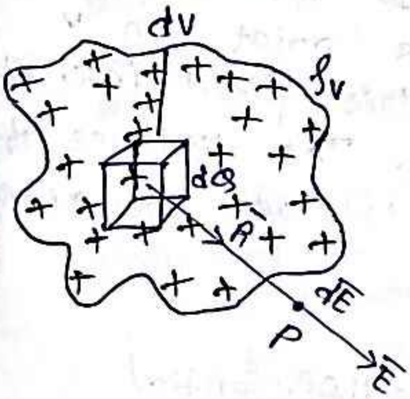
$$d\overline{E} = \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\cdot\overline{a_R}$$

* $\overline{E}$ at point P can be obtained

$$\overline{E} = \int_L d\overline{E}$$

$$\boxed{\overline{E} = \int_L \frac{\rho_s\, ds}{4\pi\varepsilon_0 R^2}\cdot\overline{a_R}}$$

* E due to volume charge :- Consider a volume charge distribution having a charge density ρ_v as shown in fig.



* dv is differential volume.
* dq is charge on differential volume is given by

$$dq = \rho_v dv \quad --(i)$$

* $d\overline{E}$ is differential Electric field intensity at point 'P' due to dq is

$$d\overline{E} = \frac{dq}{4\pi\varepsilon_0 R^2}\cdot\overline{a_R} \quad --(ii)$$

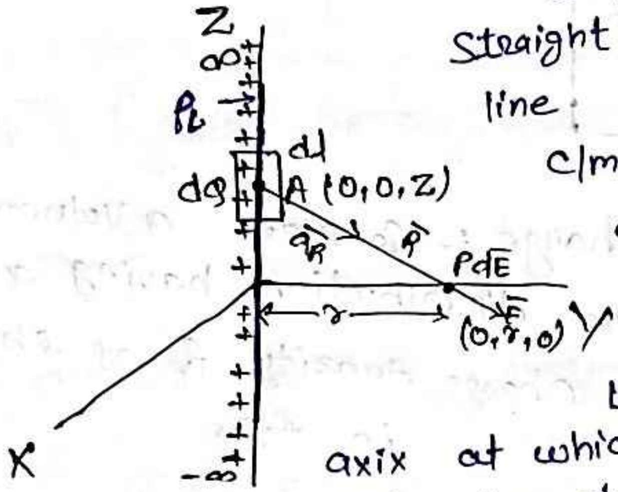
$$d\overline{E} = \frac{\rho_v\, dv}{4\pi\varepsilon_0 R^2}\cdot\overline{a_R}$$

* $\overline{E}$ at point 'p' can be obtained

$$E = \int d\overline{E}$$

$$\boxed{\overline{E} = \int_V \frac{\rho_v \, dv}{4\pi\varepsilon_0 R^2} \cdot \overline{a_R}}$$

$\overline{E}$ due to Infinite line charge :-



Consider an infinitely long straight line carrying uniform line charge having density ρ_L c/m. Let this line lies along Z-axis from $-\infty$ to ∞ and hence called infinite line charge. Let P is point on Y axis at which electric field intensity is to be determined. The distance of point P from the origin is 'r' as shown in fig.

* dl is differential length

dq is differential charge on differential length is given

$dq = \rho_L dl \quad , dl = dz$

$dq = \rho_L dz$ --- (i)

* dE is differential Electric field Intensity at point 'p' due to dq is

$$d\overline{E} = \frac{dq}{4\pi\varepsilon_0 R^2_{AP}} \cdot \overline{a_R}$$

from equ (i) Sub the value of dq

$$d\overline{E} = \frac{\rho_L dz}{4\pi\varepsilon_0 R^2_{AP}} \cdot \overline{a}_R$$

* $\overline{R_{AP}} = \overline{AP} = \overline{P} - \overline{A} = r\bar{a}_y - z\bar{a}_z$

$$|R_{AP}| = |\overline{AP}| = \sqrt{r^2+z^2}$$

$$d\overline{E} = \frac{\rho_L dz}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2} \cdot \frac{(r\bar{a}_y - z\bar{a}_z)}{\sqrt{r^2+z^2}}$$

$$\boxed{d\overline{E} = \frac{\rho_L dz \cdot (r\bar{a}_y - z\bar{a}_z)}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}} \quad \text{--- (ii)}$$

Note :- For a every charge on the positive z axix there is equal charge present on negative z axix. Hence the z component of electric field intensities produced by such charge at point P will cancel each other. Hence there will not be any z component of $\overline{E}$ at P.

Hence the equation of $d\overline{E}$ can be written by eliminating $\bar{a}_z$ component.
from (ii)

$$d\overline{E} = \frac{\rho_L dz}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \, r\bar{a}_y$$

E of point P due to dQ

$$\overline{E} = \int d\overline{E}$$

$$\boxed{\overline{E} = \int_L \frac{\rho_L dz}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot r\bar{a}_y} \quad \text{--- (iii)}$$

For such an intregation, use the substitution

$$\tan\theta = \frac{z}{R}, \quad r = \frac{z}{\tan\theta}$$

$$z = r\tan\theta \qquad (z^2 = r^2\tan^2\theta)$$

$$dz = r\sec^2\theta \, d\theta$$

Substitute the value of z^2 in equ (iii)

$$\bar{E} = \int_L \frac{\rho_L dz}{4\pi\varepsilon_0 (r^2 + r^2\tan^2\theta)^{3/2}} \cdot r \cdot \bar{a}_y$$

$$= \int \frac{\rho_L r \sec^2\theta d\theta \cdot r \cdot \bar{a}_y}{4\pi\varepsilon_0 \left(r^2(1+\tan^2\theta)\right)^{3/2}}$$ (Sub the Value of dz)

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\bar{E} = \frac{\rho_L \bar{a}_y}{4\pi\varepsilon_0} \int_L \frac{r^2 \sec^2\theta d\theta}{(r^2 \sec^2\theta)^{3/2}}$$

$$\bar{E} = \frac{\rho_L \bar{a}_y}{4\pi\varepsilon_0} \int_L \frac{r^2 \sec^2\theta d\theta}{r^3 \sec^3\theta}$$

$$\boxed{\frac{1}{\sec\theta} = \cos\theta}$$

$$\bar{E} = \frac{\rho_L \bar{a}_y}{4\pi\varepsilon_0 r} \int_L \cos\theta d\theta$$

$$\boxed{\begin{array}{l} Z = -\infty, \ \theta = \tan^{-1}\left(-\frac{\infty}{r}\right) = -\frac{\pi}{2} = -90^\circ \\ Z = +\infty, \ \theta = \tan^{-1}\left(+\frac{\infty}{r}\right) = \frac{\pi}{2} = 90^\circ \end{array}}$$

Changing the limits.

$$\bar{E} = \frac{\rho_L \bar{a}_y}{4\pi\varepsilon_0 r} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta$$

$$\bar{E} = \frac{\rho_L \bar{a}_y}{4\pi\varepsilon_0 r} \left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$\boxed{\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right] = 2}$$

$$\bar{E} = \frac{\rho_L \bar{a}_y}{{}_2 4\pi\varepsilon_0 r} \cdot \cancel{2}$$

$$\boxed{\bar{E} = \frac{\rho_L \bar{a}_y}{2\pi\varepsilon_0 r}} \quad \text{--(iv)}$$ But in generally $\bar{a}_y = \bar{a}_r$

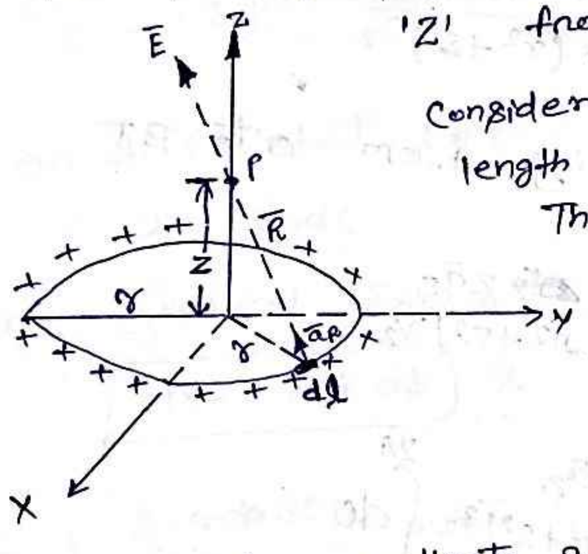
Substitute the value of $\bar{a}_r$ in equ (iv)

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r} \cdot \bar{a}_r \ V/m}$$

ELECTRIC FIELD DUE TO CHARGED CIRCULAR RING

* Consider a charged circular ring of radius r placed in xy plane with centre at origin, carrying a charge uniformly along its circumference. The charged density is ρ_L C/m.

The point P is at a perpendicular distance 'Z' from as shown in fig. Consider a small differential length (dl) on the ring. The charge on it is dQ.



In cylindrical co-ordinate system

$$dQ = \rho_L dl$$

$$dl = r \cdot d\theta$$

$$dQ = \rho_L r \cdot d\theta$$

* $\bar{E}$ at 'p' due to dQ

$$\bar{E} = \int_L \frac{dQ}{4\pi\varepsilon_0 R^2} \cdot \bar{a}_R$$

$$\bar{E} = \int_L \frac{\rho_L r \cdot d\theta}{4\pi\varepsilon_0 (\sqrt{r^2+z^2})^2} \cdot \frac{(-r\bar{a}_r + z\bar{a}_z)}{\sqrt{r^2+z^2}}$$

$$\boxed{\bar{E} = \int \frac{\rho_L r d\theta}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} (-r\bar{a}_r - z\bar{a}_z)}$$

Note :- The radial components of $\bar{E}$ at a point 'p' will be symmetrical placed in a plane parallel to x-y plane and are going to cancel each plane. Hence neglect radial component $\bar{E}$.

$$\bar{E} = \int \frac{\rho_L r \cdot d\theta \; z \cdot \bar{a}_z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

θ in varying from 0 to 2π

$$\bar{E} = \int_0^{2\pi} \frac{\rho_L r d\theta \cdot \cancel{z\bar{a}_z}\, z\bar{a}_z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}$$

$$\bar{E} = \frac{\rho_L r \cdot z\bar{a}_z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \int_0^{2\pi} d\theta$$

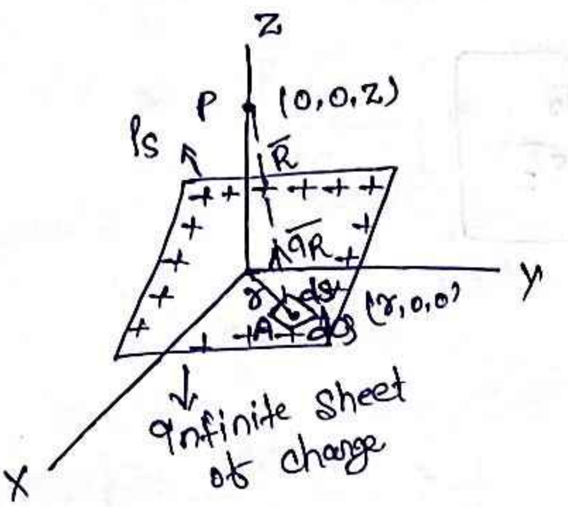
$$E = \frac{\rho_L r \cdot z \cdot \bar{a}_z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} [\theta]_0^{2\pi}$$

$$\boxed{\bar{E} = \frac{\rho_L r z \, \bar{a}_z}{2\varepsilon_0 (r^2+z^2)^{3/2}} \; V/m}$$

Where r = radius of the ring,

z = perpendicular distance of point P from the ring along the axis of the ring.

* $\bar{E}$ due to Infinite sheet of charge (Surface charge)



Consider a differential Surface Area (A) which the distance as r from origin point. and Charge of this Surface is dq.

* $\bar{E}$ at point 'P' due to dq is given by.

$$\bar{E} = \int_S \frac{dq}{4\pi\varepsilon_0 R^2} \cdot \bar{a}_R$$

* dq on differential Surface area.

$$dq = \rho_s \cdot ds$$

in Cylindrical System

$$\boxed{ds = r\,dr\,d\phi} \;?$$

* $$\bar{E} = \int_S \frac{\rho_s \cdot r\,dr\,d\phi}{4\pi\varepsilon_0 R^2} \cdot \bar{a}_R$$

* $$\bar{R} = \overline{AP} = \bar{P} - \bar{A} = -r\bar{a}_r + Z\bar{a}_Z \;\; ?$$

* $$\bar{E} = \int_S \frac{\rho_s\, r \cdot dr\,d\phi}{4\pi\varepsilon_0 \left(\sqrt{r^2+Z^2}\right)^2} \times \frac{(-r\bar{a}_Z + Z\bar{a}_Z)}{\sqrt{r^2+Z^2}}$$

$$\boxed{\bar{E} = \int_S \frac{\rho_s\, r \cdot dr\,d\phi \cdot (-r\bar{a}_Z + Z\bar{a}_Z)}{4\pi\varepsilon_0 \,(r^2+Z^2)^{3/2}}}$$

Radial Component of E at P

get Cancel each other,

$$\boxed{\bar{E} = \int_S \frac{\rho_s r \cdot dr\, d\phi \cdot z\bar{a}_r}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}}}$$

Put $r^2+z^2 = u^2$

$2r\,dr = 2u\,du$

$r\,dr = u\,du$

$$\bar{E} = \int_S \frac{\rho_s \cdot u\,du\, d\phi}{4\pi\varepsilon_0 u^{3}} \cdot z\bar{a}_z$$

$$\bar{E} = \int_S \frac{\rho_s du\, d\phi \cdot z \cdot \bar{a}_z}{4\pi\varepsilon_0 u^2}$$

Infinite sheet of charge placed in XY plane (Parallel to XY plane.)

$\phi = 0$ to 2π

$r = 0$ to ∞

$r = 0, \quad u = z,$

$r = \infty, \quad u = \infty$

$$\bar{E} = \int_{\phi=0}^{2\pi} \int_{u=z}^{u=\infty} \frac{\rho_s \cdot d\phi\, du}{4\pi\varepsilon_0 \cdot u^2}$$

$$\bar{E} = \frac{\rho_s \cdot z\bar{a_z}}{4\pi\varepsilon_0} \int_{\phi=0}^{2\pi} d\phi \int_{u=z}^{u=\infty} \frac{1}{u^2}\, du$$

$$\bar{E} = \frac{\rho_s z\bar{a_z}}{4\pi\varepsilon_0} \left[\phi\right]_0^{2\pi} \left[\frac{-1}{u}\right]_z^{\infty}$$

$$\bar{E} = \frac{\rho_s z\bar{a_z}}{\cancel{2}\cancel{4}\pi\varepsilon_0 \atop 2} (\cancel{2}\pi) \left(-\frac{1}{\infty} + \frac{1}{z}\right)$$

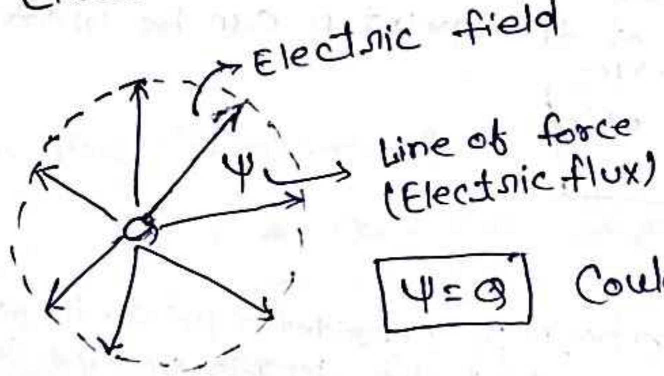
$$\bar{E} = \frac{\rho_s z \cdot \bar{a_z}}{2\cancel{\pi}\varepsilon_0} \times \frac{1}{z}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \times \bar{a_z}}$$

$\bar{a_z}$ is normal ds

$$\boxed{\bar{a_z} = \bar{a_n}}$$

* Electric flux:-

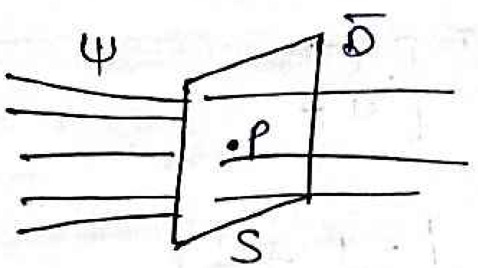


$\boxed{\psi = Q}$ Coulomb

* Scaler field

* Displacement flux.

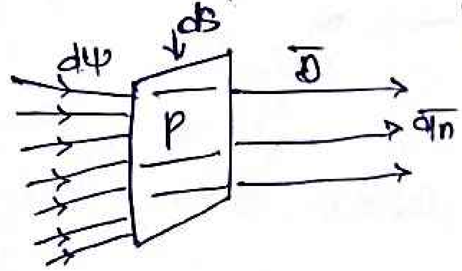
* ELECTRIC FIELD DENSITY ($\bar{D}$)



$$\boxed{\bar{D} = \frac{\Psi}{S}} \quad C/m^2$$

Vector field

* Vector field of Electric flux density.



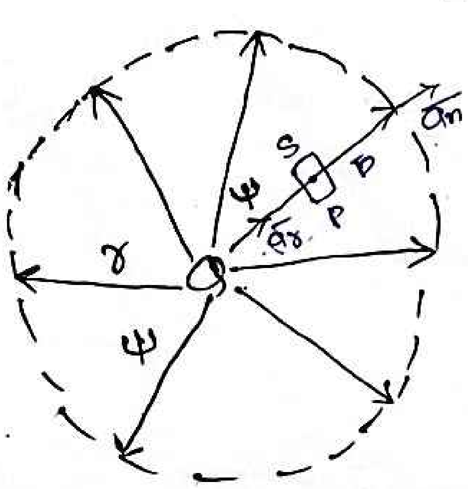
Consider the differential surface area ds at point P. The flux crossing through this differential area is dΨ. The direction of $\bar{D}$ is same as that of direction of flux lines at that point.

* $\bar{a_n}$ → Unit vector is in normal to ds

Hence the flux density $\bar{D}$ at the point P can be written as in the vector form

$$\boxed{\bar{D} = \frac{d\Psi}{dS} \cdot \bar{a_n}}$$

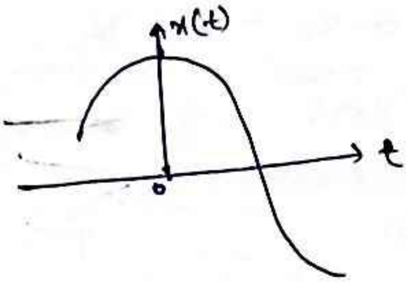
X $\bar{D}$ due to point charge:- Consider a point charge to placed at the centre of the imaginary sphere of radius r.



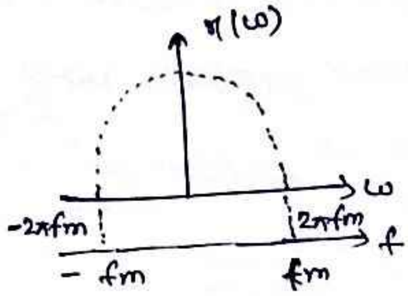
$\bar{a_n} = \bar{a_r}$

Sampling theorem : Changing Analog to Discreat Signal.

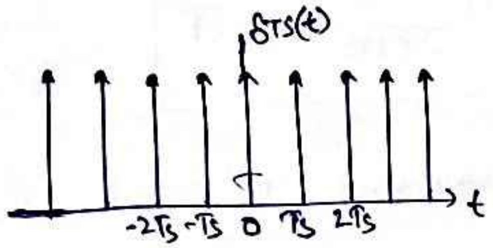
(a) Continious time signal



(b) Spectrum of Continious Time Signal

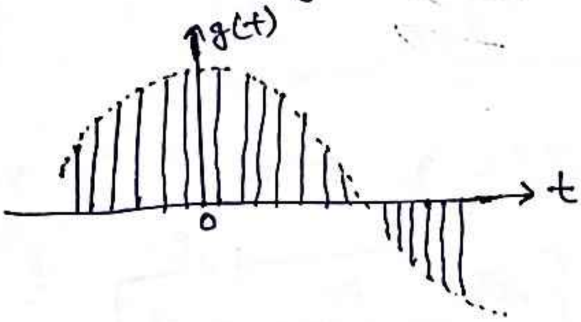


(c) Impulse Train

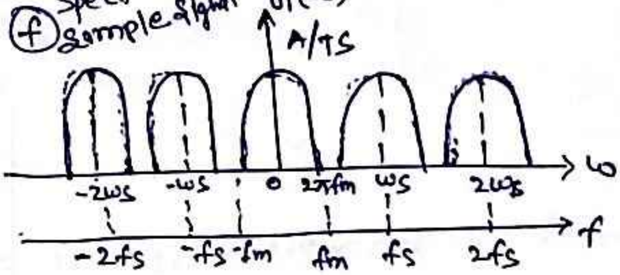


(d) multiplier

(e) Sampled Signal



(f) Spectrum of sample signal



↔ Spectrum of Sample Signal

Sampling theorem :

Statement : A Continue Signal may be Completly represented in it Sample & recoverd back if the Sampling freq. is $f_s \geq 2 f_m$

here

f_s = Sampling freq.

f_m = maximum freq. present in Signal.

Quantization : Discreat to Digital.

* $\bar{D} = \frac{\psi}{S} \cdot \bar{a_n}$ $\frac{\text{Total flux } (\psi)}{\text{Total area } (S)}$

* $\psi = q =$ Total flux

* $S = 4\pi r^2$ (Sphere) Total surface area.

$$\bar{D} = \frac{q}{4\pi r^2} \cdot \bar{a_n} \quad (\bar{a_n} = \bar{a_r})$$

$$\boxed{\bar{D} = \frac{q}{4\pi r^2} \cdot \bar{a_r}}$$

* Relation Between $\bar{D}$ & $\bar{E}$ ($\bar{D}$ and $\bar{E}$ due to a point charge +q we get)

* $\bar{D}$ at point P which at distance (r) from q.

$$\boxed{\bar{D} = \frac{q}{4\pi r^2} \cdot \bar{a_r}}$$

* $\bar{E}$ at point P which is at distance 'r' from q.

$$\boxed{\bar{E} = \frac{q}{4\pi \varepsilon_0 r^2} \cdot \bar{a_r}}$$

$$\frac{\bar{D}}{\bar{E}} = \frac{\frac{\cancel{q}}{\cancel{4\pi r^2}} \cdot \cancel{\bar{a_r}}}{\frac{\cancel{q}}{4\pi \varepsilon_0 r^2} \cdot \cancel{\bar{a_r}}}$$

$$\frac{\bar{D}}{\bar{E}} = \varepsilon_0$$

$$\boxed{\bar{D} = \varepsilon_0 \bar{E}} \quad \text{for free space}$$

* for any medium.

$$\boxed{\bar{D} = \varepsilon \bar{E}} \quad \text{or} \quad \boxed{\bar{D} = \varepsilon_0 \varepsilon_r \bar{E}}.$$

$\boxed{\bar{D} = \text{Electric flux Density}}$

$\bar{D}$ due to Various change distribution :-

(1) * Line change

* Fine line change :-

$$\bar{D} = \frac{Q}{4\pi r^2} \cdot \bar{a_r}$$

$$Q = \int_L \rho_L \, dl$$

$$\bar{D} = \frac{\int_L \rho_L \, dL}{4\pi r^2} \cdot \bar{a_r}$$

* Infinite line change

$$* \; \bar{E} = \frac{\rho_L}{2\pi \varepsilon_0 r} \cdot \bar{a_r}$$

$$* \; \bar{D} = \varepsilon_0 \bar{E}$$

$$\bar{D} = \frac{\varepsilon_0 \times \rho_L}{2\pi \varepsilon_0 r} \cdot \bar{a_r}$$

$$\boxed{\bar{D} = \frac{\rho_L}{2\pi r} \cdot \bar{a_r}}$$

(2) * Surface change

* For finite surface change

$$\bar{D} = \frac{Q}{4\pi r^2} \cdot \bar{a_r}$$

$$Q = \int_S \rho_S \, dS$$

$$\boxed{\bar{D} = \frac{\int_s \rho_s \, ds}{4\pi r^2} \cdot a_r}$$

* For Infinite Surface charge:-

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a_r}}$$

$$\bar{D} = \varepsilon_0 \bar{E}$$

$$\bar{D} = \varepsilon_0 \times \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a_r}$$

$$\boxed{\bar{D} = \frac{\rho_s}{2} \cdot \bar{a_r}}$$

(3) Volume charge

$$Q = \int_v \rho_v \, dv$$

$$* \ \bar{D} = \frac{Q}{4\pi r^2} \cdot \bar{a_r}$$

$$\bar{D} = \frac{\int_v \rho_v \, dv}{4\pi r^2} \bar{a_r}$$

Gauss's Law:-

Gauss's Law statement :- The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

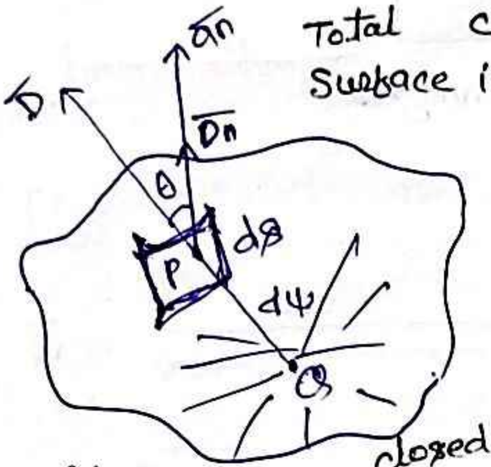
$$\boxed{\psi = Q}$$

ψ => Net flux coming out of the surface

Q = Charge enclosed by the surface

* Mathematically Representation of Gauss's Law.

Consider any object of irregular shape. Total charge enclosed by that closed surface is Q coulombs.



The total flux that has to pass through the closed surface is Q. Consider a small differential surface ds at a point P.

flux through closed irregular surface:-

* Because of irregular surface $|\bar{D}|$ & Directional of $\bar{D}$ get changes from point to point.

* $\boxed{d\bar{s} = ds \cdot \bar{a}_n}$ — (i)

* '$d\psi$' passing through ds is given

$$\boxed{d\psi = D_n \cdot ds} \text{ — (ii)}$$

D_n = Component of $\bar{D}$ along $\bar{a}_n$

$$\boxed{D_n = \bar{D} \cos\theta} \text{ — (iii)}$$

sub (iii) in (ii) $\boxed{d\psi = \bar{D} \cdot \cos\theta \cdot ds}$ — (iv)

$$d\psi = \bar{D} \cdot ds \cdot \cos\theta$$

from Scaller Vectors (dot product)

$$\boxed{\bar{A} \cdot \bar{B} = |\bar{A}||\bar{B}| \cos\theta_{AB}}$$

So, we can write equ (iv),

$$\boxed{d\psi = \bar{D} \cdot \overline{ds}}$$

According to Gauss's law,

Total flux passing through surface is given by integrating

$$\boxed{\Psi = \oint_S d\Psi}$$

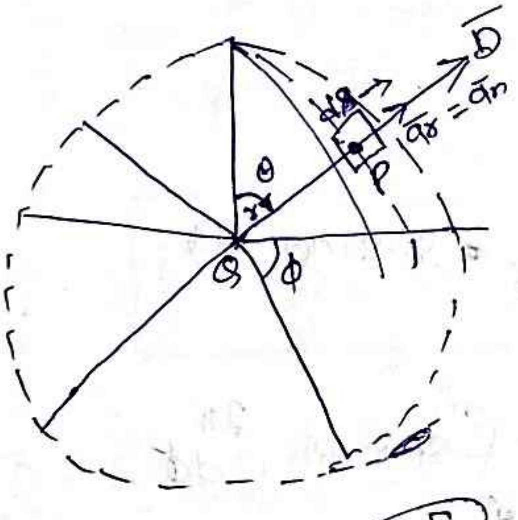
Closed surface integral $\oint$ because closed irregular surface is their

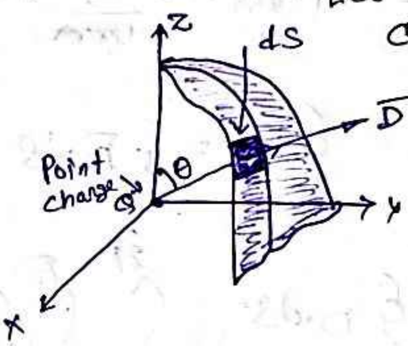
$$\boxed{\Psi = \oint_S \bar{D}\cdot d\bar{S}}$$

from the gauss law statement,

$$\boxed{\Psi = \oint_S \bar{D}\cdot d\bar{S} = Q}$$

* Proof of Gauss's Law :- (Due to point charge Q)





Let a point charge Q located at origin. Consider a differential surface area dS as shown

$$\cancel{\Psi = \oint_S \bar{D}\cdot ds = \frac{Q}{4\pi}\left[\cos\theta\right]_0^{4}} \left[\phi\right]_0^{2\pi}$$

$$\Psi = \oint_S \bar{D}\cdot ds = \frac{Q}{\cancel{4\pi}_{2}}\left[-\cos(\pi) + \cos 0^\circ\right] 2\pi$$

* $\bar{D}$ due to 'Q' is directed along $\bar{a_r}$ directional

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a_r}} \quad \text{—} \; (1)$$

* 'ds' differential surface is normal to $\bar{a_n}$

$$\boxed{d\bar{s} = ds \cdot \bar{a_n}} \quad \text{— (ii)}$$

* In spherical coordinate system

$$\boxed{ds = r^2 \sin\theta \cdot d\theta \cdot d\phi} \quad \text{— (iii)}$$

Sub (iii) in (ii),

$$\boxed{d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \cdot \bar{a_r}} \quad \text{— (iv)} \quad (\bar{a_n} = \bar{a_r})$$

$$\boxed{D \cdot ds = \frac{Q}{4\pi r^2} \cdot \bar{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \cdot \bar{a_r}}$$ from (i) & (iv),

$$\bar{D} \cdot ds = \frac{Q}{4\pi} \sin\theta \, d\theta \cdot d\phi \; (\bar{a_r} \cdot \bar{a_r})$$

$$d\psi = \bar{D} \cdot ds = \frac{Q}{4\pi} \sin\theta \, d\theta \cdot d\phi \quad \text{— (iii)}$$

$$\psi = \oint_S d\psi = \oint_S \bar{D} \cdot ds$$

$$\psi = \oint_S D \cdot ds = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \frac{Q}{4\pi} \sin\theta \, d\theta \, d\phi$$

$$\psi = \oint_S \bar{D} \cdot ds = \frac{Q}{4\pi} \int_0^{\pi} \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

By Gauss law, $\psi = \oint_S \bar{D} \cdot ds = \frac{Q}{4\pi} \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$

$$\psi = \oint_S \bar{D} \cdot ds = \frac{Q}{4\pi} \left[-\cos(\pi) - \cos(0)\right] [2\pi]$$

$$\therefore \psi = \oint_S \bar{D} \cdot ds = \frac{Q}{4\pi} [2 \times 2\pi]$$

$$\boxed{\psi = \oint_S \bar{D} \cdot ds = Q}$$

$\bar{D}$ & E is obtained by from Gauss's Law:-

* from Gauss's Law

$$\oint_S \bar{D}\cdot ds = Q \quad --(1)$$

* In Spherical Co-ordinate System for Sphere of radius 'r'.

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\cdot \bar{a_r}} \quad --$$

$$\boxed{D_r = \frac{Q}{4\pi r^2}}$$

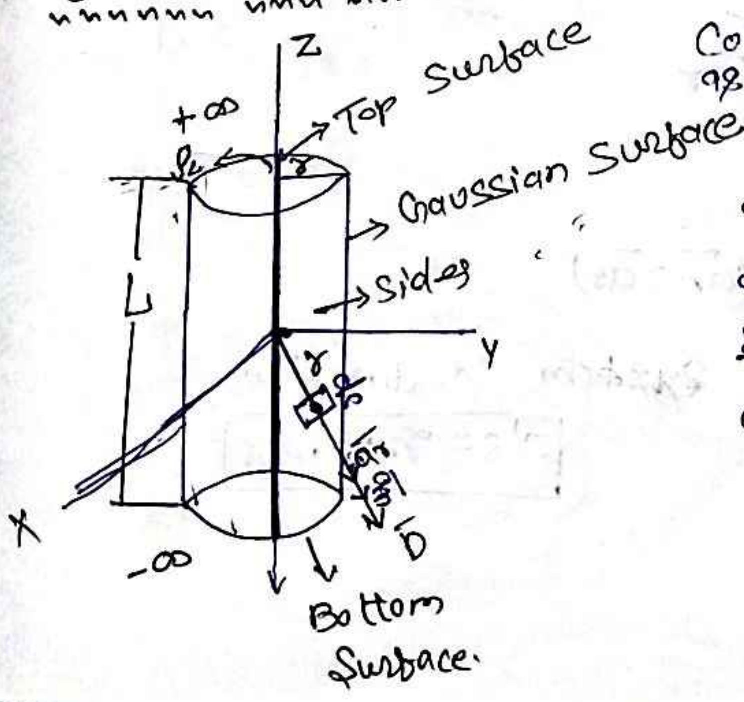
$$\bar{D} = D_r\cdot\bar{a_r}$$

$$* \ \bar{D} = \varepsilon_0 \bar{E}$$

$$\bar{E} = \frac{\bar{D}}{\varepsilon_0}$$

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2 \cdot \bar{a_r}}}$$

* $\bar{D}$ & $\bar{E}$ due to Infinite charge:-



Consider the Gaussian Surface, as the right circular cylinder with z-axix as its axix and radius r as shown in fig. & the length of the cylinder is L.

* From Gauss law.

$$\phi = \oint_S \bar{D} \cdot \bar{ds}$$

$$\phi = \oint_{Side} \bar{D} \cdot \bar{ds} + \oint_{Top} \bar{D} \cdot ds + \oint_{Bottom\ Surface} \bar{D} \cdot ds$$

* As line charged placed on z-axis & $\bar{D}$ at point p has only radial component is ($\bar{D}$ has not z-components)

$$\boxed{\oint_{Top} \bar{D} \cdot ds = \oint_{Bottom} \bar{D} \cdot ds = 0}$$

$$\phi = \oint_{Side} \bar{D} \cdot ds$$

* D at point on surface is directed along radial or direction in give by

$$\boxed{\bar{D} = D_r \cdot \bar{a_r}} \quad (\bar{a_n} = \bar{a_r})$$

* $\bar{ds}$ is normal to $\bar{a_r}$

* $\bar{ds} = ds \cdot \bar{a_n}$

$$\boxed{\bar{ds} = ds \cdot \bar{a_r}} \quad (\bar{a_n} = \bar{a_r})$$

in cylindrical ~~system~~ cordination

$$\boxed{ds = r d\phi \cdot dz}$$

$$\boxed{\bar{ds} = r \cdot d\phi\, dz \cdot \bar{a_r}}$$

$$\phi = \oint D_r \cdot r \cdot d\phi \cdot dz \; (\overline{a_r} \cdot \overline{a_r})$$

$$\phi = \int_{\phi=0}^{2\pi} \int_{z=0}^{L} D_r r \, d\phi \, dz.$$

$$\phi = D_r r \int_{z=0}^{L} dz \int_{\phi=0}^{2\pi} d\phi$$

$$\phi = D_r r \left[z\right]_0^L \left[\phi\right]_0^{2\pi}$$

$$\boxed{\phi = 2\pi D_r r \cdot L}$$

$$D_r = \frac{\phi}{2\pi r L}$$

$$\rho_L = \frac{\phi}{L}$$

$$D_r = \frac{\rho_L}{2\pi r}$$

$$\overline{D} = D_r \cdot \overline{a_r}$$

$$\overline{D} = \frac{\rho_L}{2\pi r} \cdot \overline{a_r}$$

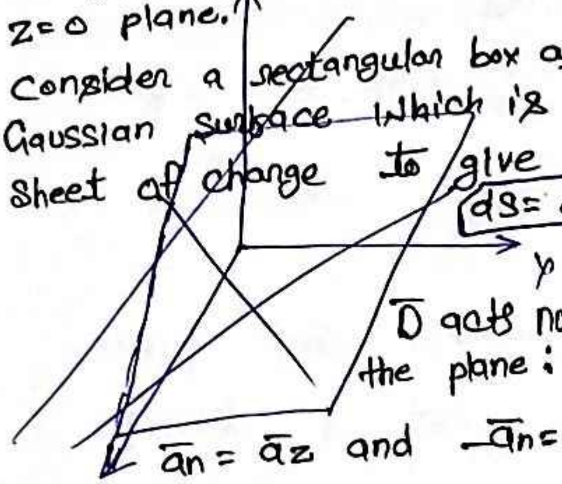
$$\overline{E} = \frac{\overline{D}}{\varepsilon_0}$$

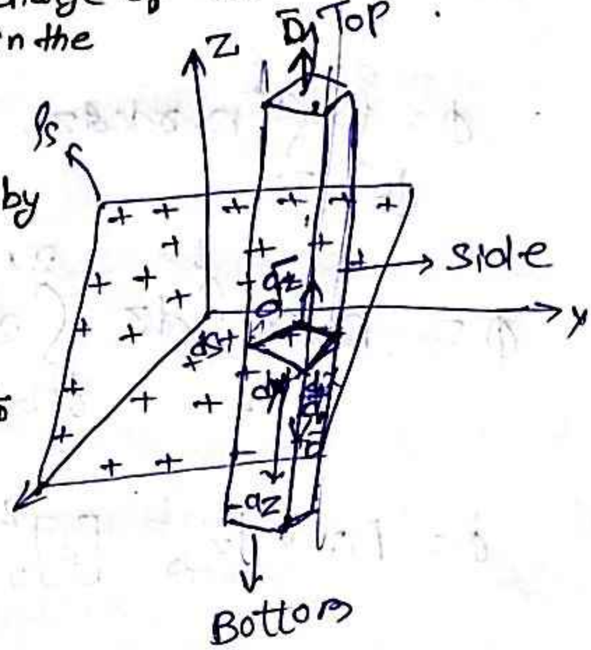
$$\boxed{\overline{E} = \frac{\rho_L}{2\pi r \varepsilon_0} \cdot \overline{a_r}} \quad V/m$$

* $\overline{D}$ & E due to Infinite Sheet of charge:-

Consider the infinite sheet of charge of uniform charge density ρ_s C/m², laying in the $z=0$ plane.

Consider a rectangular box as a Gaussian surface which is cut by sheet of charge to give $\boxed{ds = dx\,dy}$





$\overline{D}$ acts normal to the plane; i.e.

$\overline{a_n} = \overline{a_z}$ and $-\overline{a_n} = -\overline{a_z}$ direction.

Hence $\overline{D} = 0$ in x and y direction.

* From Gauss's law charge enclosed can be written as,

$$\boxed{Q = \oint_S \overline{D}\cdot ds}$$

$$Q = \oint_{Side} \overline{D}\cdot dS + \oint_{Top} \overline{D}\cdot dS + \oint_{bottom} \overline{D}\cdot dS$$

* D has only z- components & not having component along x- & y- direction

$$\oint_{Side} \overline{D}\cdot dS = 0$$

$$Q = \oint_{Top} \overline{D}\cdot dS + \oint_{bottom} \overline{D}\cdot dS$$

For top Surface

For top Surface

$\bar{D}$ is directed along $\bar{a_z}$ direction & is given by

$\boxed{\bar{D} = D_r(\bar{a_z})}$ $(\bar{a_z} = \bar{a_n})$

$\boxed{* d\bar{s} \text{ is normal to } \bar{a_z}}$

$\boxed{d\bar{s} = ds(\bar{a_z})}$

In Rectangular Co-ordinate system

$$ds = dx\,dy$$

$$d\bar{s} = dx\,dy\,(\bar{a_z})$$

ten

$$\oint \bar{D} \cdot d\bar{s} = \oint_S D_r(\bar{a_z})\,dx\,dy\,(\bar{a_z})$$

for Top $\boxed{\oint \bar{D}\cdot d\bar{s} = \oint_S D_r\,dx\,dy}$ (a)

For bottom Surface

$\bar{D}$ is directed along $-\bar{a_z}$ direction & is given by

$$\bar{D} = D_r(-\bar{a_z}) \quad (\bar{a_z} = \bar{a_n})$$

$*$ ds is normal to $-\bar{a_z}$

$$d\bar{s} = ds(-\bar{a_z})$$

In rectangular Co-ordinate system,

$$\boxed{ds = dx\,dy}$$

$$d\bar{s} = dx\,dy\,(-\bar{a_z})$$

$$\underset{\text{Bottom}}{\oint \bar{D}\cdot ds} = \oint_S D_r(-\bar{a_z})\;dx\,dy\,(-\bar{a_z})$$

$$\oint \bar{D}\cdot d\bar{s} = \oint_S D_r\,dx\,dy$$

<u>Now</u>

$$Q = \underset{\text{Top}}{\oint_S \bar{D}\cdot ds} + \underset{\text{Bottom}}{\oint_S \bar{D}\cdot ds}$$

$$Q = \underset{\text{Top}}{\oint D_r\,dx\,dy} + \underset{\text{Bottom}}{\oint D_r\,dx\,dy} = A = \text{Surface Area}$$

$$Q = 2D_r A \quad \text{——} \quad (1)$$

From Surface charge density ρ_s is

$$\boxed{\rho_s = \frac{Q}{A}}$$

$$Q = \rho_s A \quad \text{——} \quad (2)$$

$$(1) = (2)$$

$$2D_r\cancel{A} = \rho_s\cancel{A}$$

$$D_r = \frac{\rho_s}{2}$$

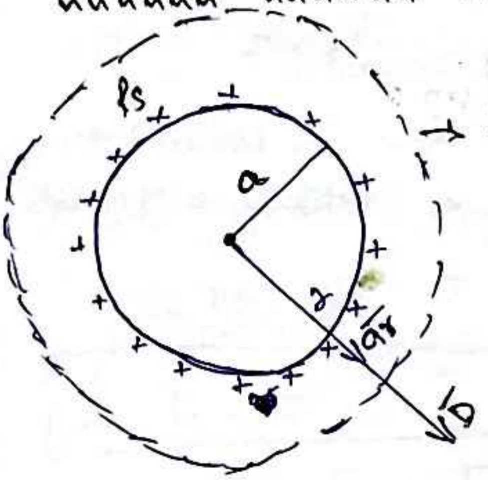
$$\bar{D} = D_r\cdot\bar{a_z}$$

$$\boxed{\bar{D} = \frac{\rho_s}{2}\cdot\bar{a_z}}$$

$$* \; \bar{E} = \frac{\bar{D}}{\varepsilon_0}$$

$$\boxed{\bar{E} = \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a_z}}$$

* $\bar{D}$ & $\bar{E}$ due to Spherical Shell of charge :-



9 Three Conditions are aplicable,

* Point P at outside the Shell ($r>a$)
* " " on the Shell ($r=a$)
* " " on the inside the Shell ($r<a$)

(1)st Case :- Point 'P' outside the Shell ($r>a$)

* According to Gauss's Law

$$\boxed{Q = \oint_S \bar{D} \cdot ds} \quad \text{--- (i)}$$

* D is directed along radial (or) directions & is given by

$$\boxed{\bar{D} = D_r \cdot \bar{a_r}} \quad \text{--- (ii)}$$

* $d\bar{s}$ is normal to $\bar{a_r}$ in Spherical Co-ordinate system $\boxed{d\bar{s} = r^2 \sin\theta \, d\theta \, d\phi \cdot \bar{a_r}}$ --- (iii)

Sub (ii) & (iii) in (i)

$Q = \oint \bar{D} \cdot ds$; $\oint_S D_r \cdot \bar{a_r} \cdot r^2 \sin\theta \, d\theta \, d\phi \cdot \bar{a_r}$

$$Q = \oint_S D_r \cdot r^2 \sin\theta \, d\theta \, d\phi \qquad (\bar{a_r} \cdot \bar{a_r} = 1)$$

$$Q = D_r \, r^2 \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$Q = D_r \cdot r^2 \left[-(-1) + \cos(0°)\right] \times 2\pi$$

$$Q = 4\pi r^2 \cdot D_r$$

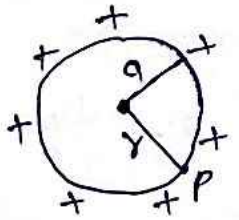
$$D_r = \frac{Q}{4\pi r^2}$$

$$\boxed{\bar{D} = D_r \cdot \bar{a_r} = \frac{Q}{4\pi r^2} \cdot \bar{a_r}} \quad \text{—— (iv)}$$

* $\bar{E} = \dfrac{\bar{D}}{\varepsilon_0}$

$$\boxed{\bar{E} = \frac{Q}{4\pi \varepsilon_0 r^2} \cdot \bar{a_r}}$$

(IInd) Case:- Point P on the Shell $(r = a)$



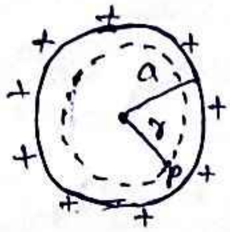
$\boxed{r = a}$

Sub the value of r in equ (iv)

$$\boxed{\bar{D} = \frac{Q}{4\pi a^2} \cdot \bar{a_r}}$$

and $\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 a^2} \cdot \bar{a_r}}$

(III) rd Case :- Point 'p' inside the Shell ($r<a$)

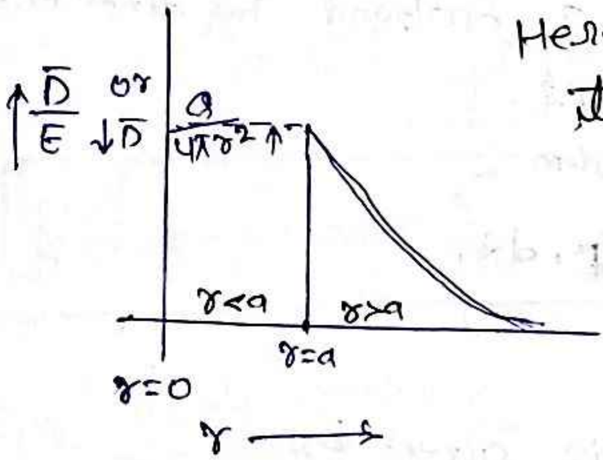


Total charge enclosed surface inside Spherical shell is zero

i.e $Q = 0$

$$\boxed{\therefore \bar{D} = 0 \;\&\; \bar{E} = 0}$$

Variation of D & E w.r.t r



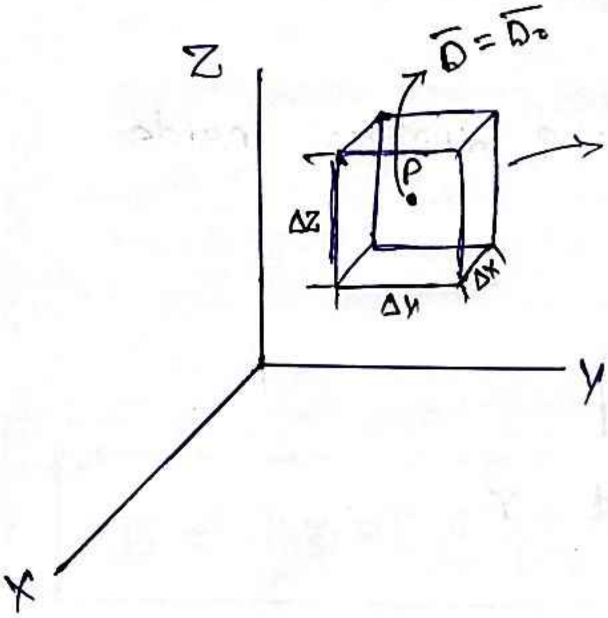
Here D is directly propotional to R.

Gauss Law is applied for Differential Volume Element :-

* For Symmetrical Surfaces ⟶ Gauss's Law
* for unsymmetrical Surfaces → (Not can aplied directly) X

By Consediring close differential Gaussian Surface making

$$\boxed{\Delta V = 0}$$



Rectangular box as closed differential Gaussian Surface

* Total Changed Q enclosed by differential Volume element.

$$Q = \left[\int_{Front}^{Back} \int_{Left}^{Right} \int_{Top}^{Bottom} \bar{D} \cdot ds \right]$$

* $\bar{D}$ at point 'p' is given by

$$\boxed{\bar{D} = D_0 = D_{X0}\bar{a}_X + D_{Y0}\bar{a}_Y + D_{Z0}\bar{a}_Z} \quad --- (1)$$

D_{X0}, D_{Y0}, D_{Z0} are of Components of D & changes with distances in X, Y, Z direction front.

* For Front Surface

$$\oint_{\text{Front}} \bar{D}\cdot d\bar{s} \cong \bar{D}_{\text{front}}\cdot ds$$

$\bar{D}_{\text{front}} = D_{x\,\text{front}} \cdot \bar{a}_x$

$d\bar{s} = \Delta y \cdot \Delta z \cdot \bar{a}_x$

$\bar{D}_{\text{front}} \cdot d\bar{s} = D_{x\,\text{front}} \cdot \Delta y\, \Delta z \quad (\bar{a}_x \cdot \bar{a}_x = 1)$

Where

$D_{x\,\text{Front}} = D_{x0} + \left[\begin{array}{c}\text{Rate of change } D_x \\ \text{w.r.t } x\end{array}\right] \left[\begin{array}{c}\text{Distance of} \\ \text{surface from} \\ \text{point P}\end{array}\right]$

$$D_{x\,\text{front}} = + \left[D_{x0} + \frac{\partial D_x}{\partial x} \cdot \frac{\Delta x}{2}\right]$$

$$\boxed{\oint \bar{D}\cdot d\bar{s} = \left[D_{x0} + \frac{\partial D_x}{\partial x} \cdot \frac{\Delta x}{2}\right] \Delta y \Delta z} \quad \text{--- (ii)}$$

* For Back Surface

$$\oint_{\text{Back}} \bar{D}\cdot ds \cong \bar{D}_{\text{Back}}\cdot ds$$

$\bar{D}_{\text{Back}} = D_{x\,\text{Back}} \cdot (-\bar{a}_x)$

$d\bar{s} = \Delta y\, \Delta z \cdot (-\bar{a}_x)$

$$\boxed{\bar{D}_{\text{Back}} \cdot d\bar{s} = D_{x\,\text{Back}} \cdot \Delta y \Delta z} \quad (\bar{a}_x \cdot \bar{a}_x = 1) \text{ --- (iii)}$$

Where

$$D_x \text{ Back} = -\left[D_{x0} - \begin{bmatrix}\text{Rate of change} \\ D_x \text{ w.r.t } x\end{bmatrix}\begin{bmatrix}\text{Distance of surface} \\ \text{from point } P\end{bmatrix}\right]$$

$$D_x \text{ Back} = -\left[D_{x0} - \frac{\partial D_x}{\partial x}\cdot\frac{\Delta x}{2}\right]$$

$$\boxed{\oint_{Back} \bar{D}\cdot ds = -\left[D_{x0} - \frac{\partial D_x}{\partial x}\cdot\frac{\Delta x}{2}\cdot \Delta y\,\Delta z\right]} \leftarrow \text{(iv)}$$

<u>Now</u>

<u>Adding of front Back:-</u>

$$\oint_{front} \bar{D}\cdot ds + \oint_{Back} \bar{D}\cdot ds = \cancel{D_{x0}} + \frac{\partial D_x}{\partial x}\,\Delta y\Delta z\,\frac{\Delta x}{2} - \cancel{D_{x0}} + \frac{\partial D_x}{x}\,\Delta y\Delta z\cdot\frac{\Delta x}{2}$$

$$= 2\left(\frac{\partial D_x}{\partial x}\,\Delta y\Delta z\cdot\frac{\Delta x}{\cancel{2}}\right)$$

$$\boxed{\oint_{front}\bar{D}\cdot ds + \oint_{Back}\bar{D}\cdot ds = \frac{\partial D_x}{\partial x}\cdot\Delta x\cdot\Delta y\cdot\Delta z}$$

~~rechanges with di~~

|||'ly

$$\boxed{\oint_{Left}\bar{D}\cdot\bar{ds} + \oint_{Right}\bar{D}\cdot\bar{ds} = \frac{\partial D_y}{\partial y}\,\Delta x\cdot\Delta y\cdot\Delta z}$$

$$\boxed{\oint_{Top}\bar{D}\cdot\bar{ds} + \oint_{Bottom}\bar{D}\cdot\bar{ds} = \frac{\partial D_z}{\partial z}\,\Delta x\cdot\Delta y\cdot\Delta z}$$

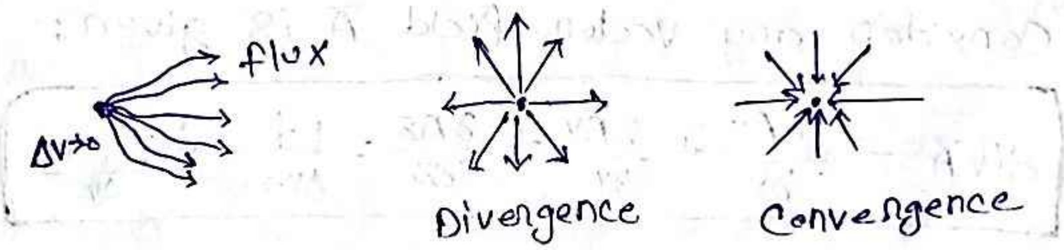
$$Q = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x\cdot\Delta y\cdot\Delta z$$

$$\boxed{\text{But } \Delta x \cdot \Delta y \cdot \Delta z = \Delta V}$$

$$\boxed{Q = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta V}$$

* Divergence :- The divergence of flux lies means how much flux lines are leaving from small valume(or) per unit Volume. or there is no direction associated with divergence.

The divergence of the flux density is the out flow of flux from a small closed surface per unit volume as the volume reduces to 0.



↑↑↑

↓↓↓

Zero divergence

Divergence of flux density $\bar{D}$

* Total charge enclosed by differential volume element is given by

$$Q = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta V \quad \text{--- (1)}$$

* From Gauss's Law

$$Q = \oint_S \bar{D} \cdot ds \quad \text{—— (ii)}$$

$$(1) = (2)$$

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \Delta V = \oint \bar{D} \cdot d\bar{s}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \frac{\oint \bar{D} \cdot d\bar{s}}{\Delta V}$$

* Divergence of $\bar{D}$ is given by:

$$\boxed{\text{div } \bar{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \underset{\Delta V \to 0}{Lt} \oint \frac{\bar{D} \cdot d\bar{s}}{\Delta V}}$$

Eg:- Consider any vector field $\bar{A}$ is given:

$$\boxed{\text{div} \bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = \underset{\Delta V \to 0}{Lt} \oint \frac{\bar{D} \cdot ds}{\Delta V}}$$

Operator (or) Vector operator (∇)

$$\boxed{\nabla \to \text{Del}}$$

* Operation of ∇ is given by,

$$\boxed{\nabla = \frac{\partial}{\partial x} \cdot \bar{a}_x + \frac{\partial}{\partial y} \cdot \bar{a}_y + \frac{\partial}{\partial z} \cdot \bar{a}_z} \quad \text{—— (i)}$$

* Any Vector $\bar{A}$ is given by

$$\boxed{\bar{A} = A_x \cdot \bar{a}_x + A_y \cdot \bar{a}_y + A_z \cdot \bar{a}_z} \quad \text{—— (ii)}$$

$\text{div}\,\bar{A} = \nabla\cdot\bar{A}$

So, from (I) & (II);

$$\text{div}\,\bar{A} = \nabla\cdot\bar{A} = \left(\frac{\partial}{\partial x}\cdot\bar{a_x} + \frac{\partial}{\partial y}\cdot\bar{a_y} + \frac{\partial}{\partial z}\cdot\bar{a_z}\right)\cdot\left(A_x\cdot\bar{a_x} + A_y\cdot\bar{a_y} + A_z\cdot\bar{a_z}\right)$$

$$\nabla\cdot\bar{A} = \frac{\partial}{\partial x}A_x(\bar{a_x}\cdot\bar{a_x}) + \frac{\partial}{\partial x}\cdot A_y(\bar{a_x}\cdot\bar{a_y}) + \frac{\partial}{\partial x}\cdot A_z(\bar{a_x}\cdot\bar{a_z}) +$$

$$\frac{\partial}{\partial y}A_x(\bar{a_y}\cdot\bar{a_x}) + \frac{\partial}{\partial y}A_y(\bar{a_y}\cdot\bar{a_y}) + \frac{\partial}{\partial y}A_z(\bar{a_y}\cdot\bar{a_z}) +$$

$$\frac{\partial}{\partial z}A_x(\bar{a_z}\cdot\bar{a_x}) + \frac{\partial}{\partial z}A_y(\bar{a_z}\cdot\bar{a_y}) + \frac{\partial}{\partial z}A_z(\bar{a_z}\cdot\bar{a_z})$$

Now dot vector of similar unit vector Ex:- $a_x\cdot a_x = 1$ & Dot vector of disimilar unit vector Ex:- $a_x\cdot a_y = 0$

$$\nabla\cdot\bar{A} = \frac{\partial}{\partial x}A_x + 0 + 0 + 0 + \frac{\partial}{\partial y}A_y + 0 + 0 + 0 + \frac{\partial}{\partial z}A_z$$

Scaler quantity, $\Rightarrow$ $\boxed{\text{div}\,\bar{A} = \nabla\cdot\bar{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z}$

Divergence in different Co-ordinate System.

[1] Cartesian System :-

* $\nabla\cdot\bar{A} = \text{div}\,\bar{A} = \frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z$

* $dv = dx\,dy\,dz$

[2] Cylindrical System :-

* $\nabla\cdot\bar{A} = \text{div}\,\bar{A} = \frac{1}{r}\frac{\partial}{\partial r}(rA_r) + \frac{1}{r}\frac{\partial}{\partial\phi}A_\phi + \frac{\partial}{\partial z}A_z$

* $dv = r\cdot dr\,d\phi\,dz$

[3] Spherical Co-ordinate System :-

$$* \; \text{div}\,\bar{A} = \nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\cdot A_\theta) +$$

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\phi}A_\phi$$

$$* \; dv = r^2\sin\theta\, dr\, d\phi\, d\theta$$

* Maxwell's Equation :-

Maxwell's 1st Equation :-

Divergence of (Flux density) $\bar{D}$ is given by

$$\boxed{\text{div}\,\bar{D} = \nabla\cdot\bar{D} = \underset{\Delta V\to 0}{Lt}\oint\frac{\bar{D}\cdot d\bar{s}}{\Delta V} = \underset{\Delta V\to 0}{Lt}\oint\frac{Q}{\Delta V}}$$

$$\underset{\Delta V\to 0}{Lt}\oint\frac{Q}{\Delta V}$$

* Total change Q in differential volume dv is given by

$$\boxed{Q = \int_V \rho_v\, dv} \; --(ii)$$

ρ_v = Volume change density.

$$\boxed{\text{div}\,\bar{D} = \nabla\cdot\bar{D} = \rho_v}$$ Maxwell's 1st equation

Maxwell's first equ (or) point form or differential form of Gauss law.

* Divergence Theorem :-

* From Gauss's Law

$$\boxed{Q = \oint_S \bar{D} \cdot ds} \quad \text{--- [i]}$$

* Charge 'Q' in differential Volume dv is given by

$$\boxed{Q = \int_V \rho_v \, dv} \quad \text{--- [ii]}$$

From Maxwell's equation

$$\boxed{\rho_v = \nabla \cdot \bar{D}} \quad \text{--- (iii)}$$

Substitute equ(3) in equ(2)

$$\boxed{Q = \int_V \nabla \cdot \bar{D} \, dv} \quad \text{--- (iv)}$$

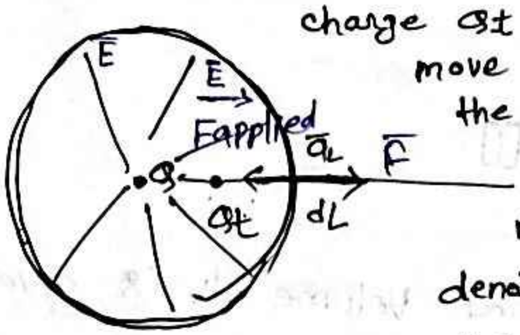
from 1 & 4

$$\boxed{\oint_S \bar{D} \cdot ds = \int_V \nabla \cdot \bar{D} \, dv} \quad \text{(Divergence theorem);}$$

Note :- With help of Divergence theorem, a closed surface integral can be converted into volume integral.

Electric Work and potential :- The ammout of work done (or) Energy required in moving a unit positive charge from one point to another point against electric field entensity is called (or) is known as Work done.

* Work done :- Consider a point charge Q and its electric field intensity E. If a positive test charge Q_t is placed in this field, it will move due to force of repulsion. Let the movement of charge Q_t is dL. The direction in which the movement has taken place is denoted by unit vector $\bar{a}_L$, in the direction of dL.



* $Q_t \rightarrow$ unit positive charge

* According to Coulomb's Law

force $\bar{F}$ exerted on Q_t in ~~the~~ $\bar{E}$ is give by

$$\bar{F} = Q_t \bar{E} \quad \text{--- (i)} \qquad \left[\bar{E} = \frac{\bar{F}}{Q}\right]$$

* F_L in components of $\bar{F}$ along $\bar{a}_L$ direction:-

$$\boxed{F_L = \bar{F} \cdot \bar{a}_L} \text{ --- (ii)}$$

Sub (i) in (ii),

$$\boxed{F_L = Q_t \bar{E} \cdot \bar{a}_L} \text{ --- (iii)}$$

* For charge equillibrium:-

$$F_{applied} = -\bar{F}_L$$

from (iii),

$$\boxed{F_{applied} = -Q_t \bar{E} \cdot \bar{a}_L} \text{ --- (iv)}$$

* 'dW' differential workdone in moving charge Q_t through length dL is given by.

$$dW = \bar{F}_{applied} \, \bar{dL}$$

sub (iv),

$$\boxed{dW = -Q_t \bar{E} \bar{a}_L \cdot \bar{dL}} \text{ --- (v)}$$

$\boxed{\overline{dL} = dL \cdot \bar{a}_L}$ = differential length vector

sub $\bar{a}_L$ value in (v)

$$\boxed{dw = -Q_t \bar{E} \cdot \overline{dL}}$$

v* $W = \int_{initial}^{final} dw$

$$W = \int_{initial}^{final} -Q\bar{E} \cdot d\bar{L}$$

$$W = -Q_t \int_{initial}^{final} \bar{E} \cdot dL$$

$$\boxed{W = -Q_t \int_B^A \bar{E} \cdot dl}$$

~~Note~~ :

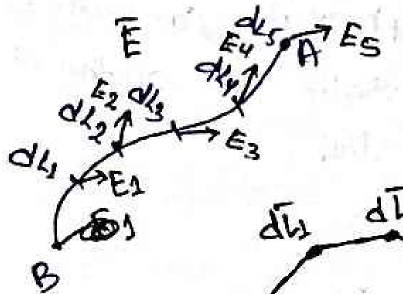
<u>Note :-</u> Work done is independent of the path selected from B to A. But it depends such has end point B and A.

* Workdone is dependent on the length initial and final point but not depends at path selected.

* $W = -Q \int_B^A \bar{E} \cdot dl$

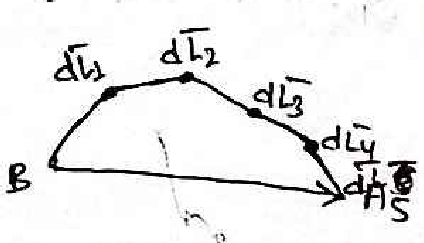
Where $E \cdot dl$ is component of $\bar{E}$ along $\bar{a}_L$ direction

$$W = -Q\left[E_1 d\bar{L}_1 + E_2 d\bar{L}_2 + E_3 d\bar{L}_3 + E_4 d\bar{L}_4 + E_5 dL_5\right]$$



for uniform electric field,

$$\bar{E}_1 = \bar{E}_2 = \bar{E}_3 = \bar{E}_4 = \bar{E}_5 = \bar{E}$$



$$W = -QE\left[\overline{dL_1} + \overline{dL_2} + \overline{dL_3} + \overline{dL_4} + \overline{dL_5}\right]$$

$$dL_1 + dL_2 + dL_3 + dL_4 + dL_5 = L_{BA}$$

$$\boxed{W = -QEL_{BA}}$$

∴ Workdone is independent of path selected but depends on end point B & A.

* Potential difference:-

* Work done in moving charge Q from point B to A against E is given by

$$\boxed{W = -Q\int_B^A E \cdot \overline{dL}}$$

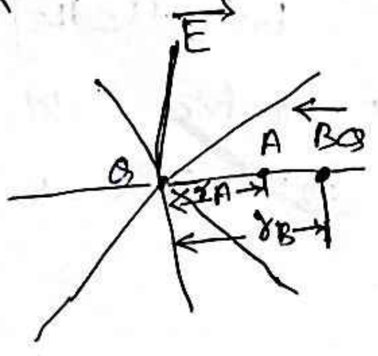
* Work done per unit charge in moving a charge Q from B to A in E is called p.d (Potential difference),

$$V = \frac{W}{Q}$$; units is Voltage (or) joule/coulomb

$$\boxed{V_{AB} = -\int_B^A \overline{E} \cdot \overline{dL}} \; V$$

* Potential due to point charge:-

consider a point charge, located at the origin of a spherical co-ordinate system, producing E radially in all the directions as shown in fig.



* E at point due to charge Q, which at a radial 'r' distance from origin.

$$\boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \cdot \bar{a_r}}$$

* $\bar{dL}$ in differential along $\bar{a_r}$ direction is given by in Spherical Co-ordinate system.

$$\bar{dL} = dr\,\bar{a_r} + r\,d\theta\,\bar{a_\theta} + r\sin\theta\,d\phi\,\bar{a_\phi}$$

* potential difference between two points A & B is given by

$$\boxed{V_{AB} = \int_B^A \bar{E} \cdot \bar{dL}}$$

$$V_{AB} = -\int_B^A \frac{Q}{4\pi r^2 \varepsilon_0} \cdot \bar{a_r} \cdot (dr\,\bar{a_r} + r\,d\theta\,\bar{a_\theta} + r\sin\theta\,d\phi\,\bar{a_\phi})$$

$$V_{AB} = -\int_B^A \frac{Q}{4\pi r^2 \varepsilon_0} \cdot dr$$

$B = r_B$ & $A = r_A$

$$V_{AB} = \frac{-Q}{4\pi\varepsilon_0} \int_{r_B}^{r_A} \frac{1}{r^2}\,dr$$

$$V_{AB} = \frac{-Q}{4\pi\varepsilon_0} \left[-\frac{1}{r}\right]_{r_B}^{r_A}$$

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]}$$

~~VAB~~ V_{AB} is +ve

when $\frac{1}{r_A} > \frac{1}{r_B}$ ie; $\boxed{r_B > r_A}$

Absolute potential:- due to charge :-

Pottential difference between points A & B is given by.

$$\boxed{V_{AB} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]}$$

the point B is treated as reference point Which is at infinite.

$$\frac{1}{r_B} = \frac{1}{\infty} = 0$$

* Potential of point 'A' is given by

$$\boxed{V_A = \frac{Q}{4\pi\varepsilon_0 r_A}}$$

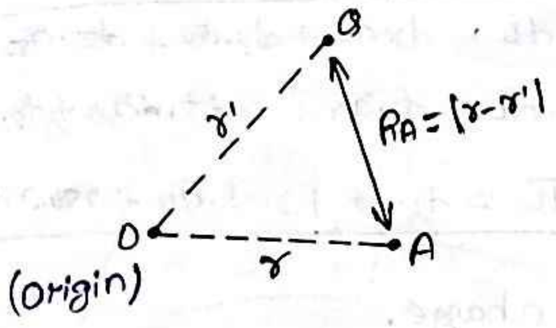
* lll'y potential of point B is given by

$$\boxed{V_B = \frac{Q}{4\pi\varepsilon_0 r_B}}$$

$$\boxed{V_{AB} = V_A - V_B}$$

In terms of Absolute potential.

* Potential Due to point charge Not an origin:-



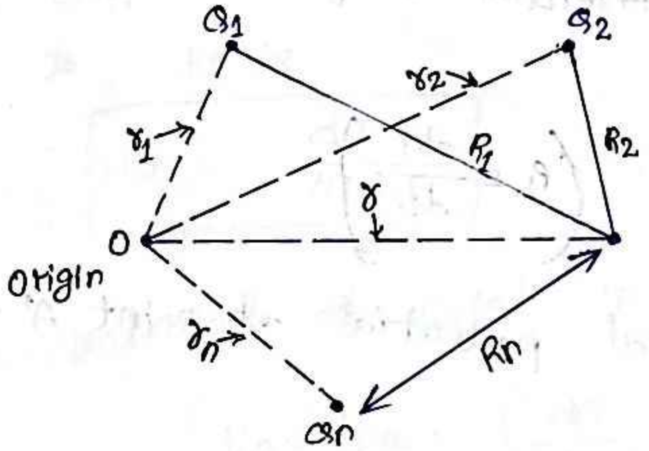
If the point charge Q is not located at the origin of a spherical system then obtain the positive vector r' of the point where Q is located.

Potential Voltage point 'A' is given by,

$$\boxed{V_A = \frac{Q}{4\pi\varepsilon_0 R}} \ V$$

Potential at A due to Q.

* Potential at point due to no. of charge.



* According to Supperposition principle,

Net total potential at point A =

$$\boxed{V_1 + V_2 + V_3 + V_4 + \ldots + V_n}$$

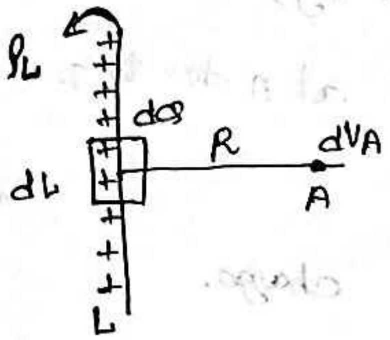
$$\boxed{V_A = \frac{Q_1}{4\pi\varepsilon_0 R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2} + \ldots + \frac{Q_n}{4\pi\varepsilon_0 R_n}}$$

Algebric sum of individual due to $Q_1, Q_2, Q_3, \ldots$ & Q_n.

* Selection of $\bar{dL}$:-

* In Cartesian System — $\bar{dL} = dx\,\bar{a_x} + dy\,\bar{a_y} + dz\,\bar{a_z}$
* In Cylindrical System — $\bar{dL} = dr\,\bar{a_r} + r\,d\phi\,\bar{a_\phi} + dz\,\bar{a_z}$
* In Spherical System — $\bar{dL} = dr\,\bar{a_r} + r\,d\theta\,\bar{a_\theta} + r\sin\theta\,d\phi\,\bar{a_\phi}$

* Potential due to line charge.



* dQ is diffrential charge on different length dL is given by

$$\boxed{dQ = \rho_L\,dL} \qquad \left(\rho_L = \frac{dQ}{dL}\right)$$

* dV_A is diffrential potential at point 'A' due to dQ is

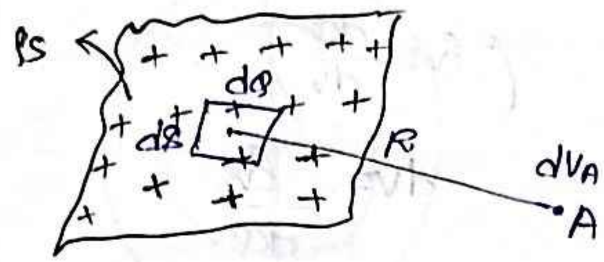
$$\boxed{dV_A = \frac{dQ}{4\pi\varepsilon_0 R}}$$

* Total potential at point 'A' due to 'Q' is.

$$V_A = \int dV_A = \oint \frac{dQ}{4\pi\varepsilon_0 R}$$

$$\boxed{V_A = \int \frac{\rho_L\,dL}{4\pi\varepsilon_0 R}}$$

* Potential due to Surface charge.



* dQ is differential charge on differential Surface 'ds' is given by.

$$\boxed{dQ = \rho_s ds}$$ $$\boxed{\left(\rho_s = \frac{dQ}{ds}\right)}$$

* dV_A in differential pottential at point 'A' due to dQ is

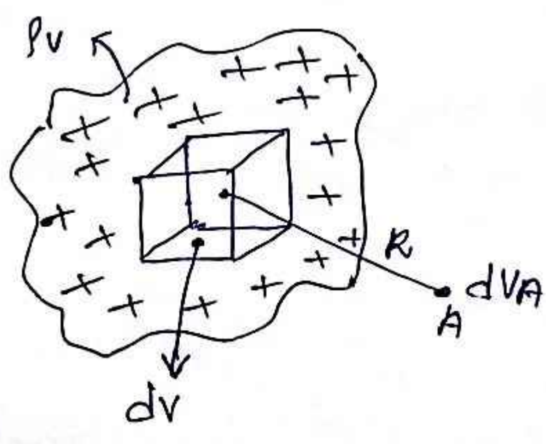
$$\boxed{dV_A = \frac{dQ}{4\pi\varepsilon_0 R}}$$

* Total potential at point 'A' due to 'Q' is

$$V_A = \int dV_A = \int \frac{dQ}{4\pi\varepsilon_0 R}$$

$$\boxed{V_A = \int \frac{\rho_s ds}{4\pi\varepsilon_0 R}}$$

* Potential due to volume charge.



* dq is differential Volume on differental ~~charge~~. Volume 'ds' is given by

$$dq = \rho_v dv \qquad \left(\rho_v = \frac{dq}{dv}\right)$$

$$dv = \frac{\rho_v}{dq}$$

* dV_A in differntgral potential at point 'A' due to dq is

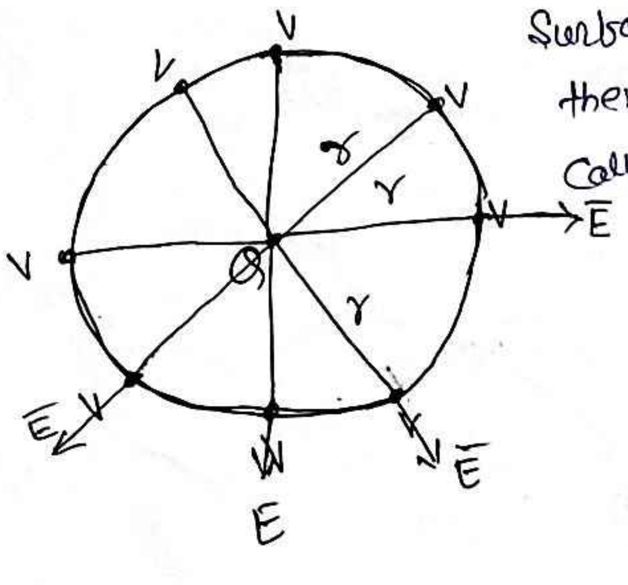
$$dV_A = \frac{dq}{4\pi\varepsilon_0 R}$$

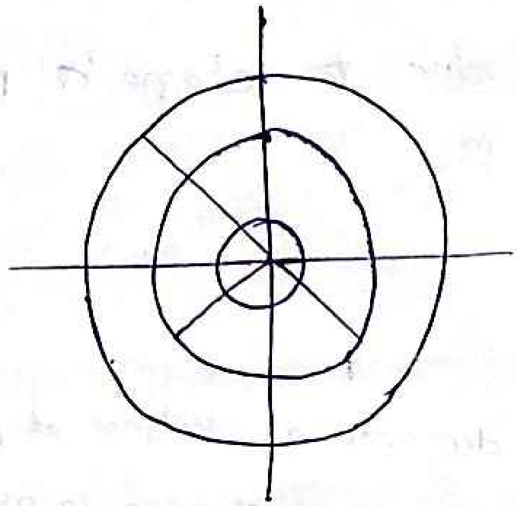
* Total potential at point 'A' due to 'q' is

$$V_A = \int dV_A = \int \frac{dq}{4\pi\varepsilon_0 R}$$

$$V_A = \frac{\rho_v dv}{4\pi\varepsilon_0 R}$$

* Equipotential surface:- ~~surface~~ At each and every point on the surface having same potential then that surface is called equipotential surface.

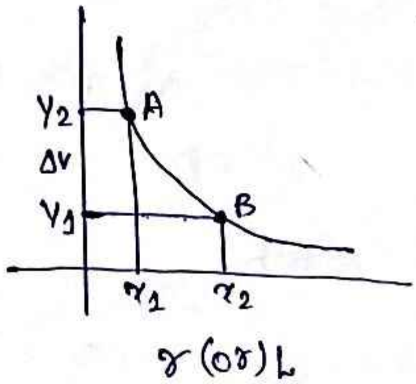




* Potential Gradient :-

* Potential at a point due to charge 'q' placed at centre of sphere is

$$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}}$$



The potential decreases as distance of point from the charge increases. This is shown the fig.

then $\text{Slope} = \dfrac{V_2 - V_1}{x_2 - x_1} = \dfrac{\Delta V}{\Delta L}$

Where $\Delta V \rightarrow$ incremental potential

$\Delta L \rightarrow$ incremental length (distance)

* To find potential at small point, take $\underset{\Delta L \to 0}{Lt}$

$$\boxed{\frac{dL}{dV} = \underset{\Delta L \to 0}{Lt} \frac{\Delta V}{\Delta L} = \text{Potential gradient}}$$

The rate of change of potential with respect to length is known as potential gradient.

* Relation between $\bar{E}$ & V :-

* For small incremental length 'ΔL'
The incremental potential is

$$\Delta V = -\bar{E} \cdot \Delta L \quad \left[V = -\int \bar{E} \cdot d\bar{L}\right]$$

$$\bar{E}\cdot\bar{\Delta L} = E\,\Delta L \cos\theta$$

$$\Delta V = -E\,\Delta V \cos\theta$$

* To find ΔV at a point, take $\lim \Delta L \to 0$

$$\lim_{\Delta L \to 0} \frac{\Delta V}{\Delta L} = -E\cos\theta$$

* ~~To~~ find a potential at small point, take $\underset{\Delta L \to 0}{Lt}$

But

$$\underset{\Delta L\to 0}{Lt} \frac{\Delta V}{\Delta L} = \frac{dv}{d\Delta} = \text{Potential gradient.}$$

$$\frac{dv}{dL} = -E\cos\theta$$

* To find potential at small point, take $\underset{\Delta L\to 0}{Lt}$

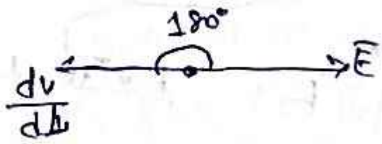
$$\underset{\Delta L\to 0}{Lt} \frac{\Delta V}{\Delta L} = \frac{dv}{dL}$$

$$\frac{dv}{dL} = -E\cos\theta$$

$\frac{dv}{dL}$ attains max value when

$\cos\theta = -1$, or $\theta = 180°$ This indicates that ΔL must be in the direction opposite to E.

$$\left(\frac{dv}{dL}\right)_{max} = E$$



But $\bar{E}$ & $\left(\frac{dv}{dL}\right)_{max}$ are in opposition.

* $$\left(\frac{dv}{dL}\right)_{max} = -\bar{E}$$

$\bar{E}$ can be expressed as,

$$\bar{E} = -\left(\frac{dv}{dL}\right)_{max}$$

$$\left(\frac{dv}{dL}\right)_{max} = \text{Gradient of potential.}$$

The maximum Value of the rate of change of potential w.r.t length is known as gradient of potential

$$\left(\frac{dv}{dL}\right)_{max} = \text{grad } V = \nabla.V$$

$$E = -(\text{grad } V)$$

$$\boxed{\bar{E} = -\nabla.V} \qquad \boxed{\bar{E} = -\nabla.V = -(\text{grad} v)}$$

∇ - operator :-

* Potential 'V' is unique function of x, y, z Co-ordinates & denoted V(x,y,z) is given by

$$dv = \frac{\partial v}{\partial x}.dx + \frac{\partial v}{\partial y}.dy + \frac{\partial v}{\partial z}.dz \quad \text{--- (i)}$$

* Total differential potential dv is given by

$$dv = -\bar{E}.d\bar{L} \quad \text{-- (ii)}$$

Where

$$\bar{E} = E_x\bar{a_x} + E_y.\bar{a_y} + E_z.\bar{a_z}$$

$$d\bar{L} = dx.\bar{a_x} + dy.\bar{a_y} + dz.\bar{a_z}$$

Sub the value of E & $d\bar{L}$ is equ (2)

$$dv = -\left[(E_x\bar{a_x} + E_y\bar{a_y} + E_z\bar{a_z}).(dx.\bar{a_x} + dy.\bar{a_y} + dz.\bar{a_z})\right]$$

$$dv = -\left[E_x.dx + E_y dy + E_z dz\right] \quad \text{-- (iii)}$$

Comparision between (i) & (iii)

$$-E_x dx - E_y dy - E_z dz = \frac{\partial v}{\partial x} \cdot dx + \frac{\partial v}{\partial y} \cdot dy + \frac{\partial v}{\partial z} \cdot dz$$

$$E_x = -\frac{\partial v}{\partial x}, \quad E_y = -\frac{\partial v}{\partial y}, \quad E_z = -\frac{\partial v}{\partial z}$$

Sub these in equ $\bar{E}$

$$\bar{E} = -\frac{\partial v}{\partial x} \cdot \bar{a_x} - \frac{\partial v}{\partial y} \cdot \bar{a_y} - \frac{\partial v}{\partial z} \cdot \bar{a_z}$$

$$\bar{E} = -v \left(\frac{\partial}{\partial x} \cdot \bar{a_x} + \frac{\partial}{\partial y} \cdot \bar{a_y} + \frac{\partial}{\partial z} \cdot \bar{a_z} \right)$$

$$\bar{E} = \frac{\partial}{\partial x} \bar{a_x} + \frac{\partial}{\partial y} \bar{a_y} + \frac{\partial}{\partial z} \bar{a_z} = \nabla$$

$$\boxed{E = -\nabla V}$$

* Poison's and Laplace's Equation:-

Poison's equation:- From the Gauss law in the point form, Poison's equation can be derived

$$\boxed{\nabla \cdot \bar{D} = \rho_v} \text{ --(i)}$$

Where, $\bar{D}$ = flux density and ρ_v = Volume charge density.

For a homogeneous, isotropic and linear medium,

$$\boxed{\bar{D} = \varepsilon \bar{E}} \text{ --(ii)}$$

Sub (ii) in (i) →

$$\boxed{\nabla \cdot \varepsilon \bar{E} = \rho_v} \text{ --(iii)}$$

from the gradient relationship

$$\boxed{E = -\nabla V} \text{ --(iv)}$$

Sub (IV) in (iii)

$$\boxed{\nabla \cdot \varepsilon(-\nabla V) = \rho_v} \text{ --(v)}$$

Taking $-\varepsilon$ outside the as constant.

$$-\varepsilon [\nabla \cdot \nabla V] = \rho_v$$

$$\boxed{\nabla \cdot \nabla V = -\frac{\rho_v}{\varepsilon}} \text{ --(vi)}$$

Now $\nabla \cdot \nabla$ operation called 'del squared' operation and denoted as ∇^2.

$$\boxed{\nabla^2 V = \frac{-\rho_v}{\varepsilon}} \text{ --(vii)}$$

equ (vii) is called as poison's equation.

* Laplace Equation:- if $\rho_v = 0$ even through here is presence of point line surface charge.

$$\boxed{\nabla^2 v = 0}$$ Lablace equation.

* ∇^2 operation in various co-ordinate system.

[i] Cartesian System:-

$$\nabla^2 v = \nabla \cdot \nabla v = \left(\frac{\partial}{\partial x}\cdot \bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z\right)\cdot\left[\frac{\partial v}{\partial x}\bar{a}_x + \frac{\partial v}{\partial y}\bar{a}_y + \frac{\partial v}{\partial z}\bar{a}_z\right]$$

$$\boxed{\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}}$$

$$\boxed{\nabla^2 v = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = 0}$$ Laplace equation in Cartesian system

[ii] Cylindrical Co-ordinate System:-

$$\boxed{\nabla^2 v = \frac{1}{r}\cdot\frac{\partial}{\partial r}\left[r\frac{\partial v}{\partial r}\right] + \frac{1}{r^2}\left(\frac{\partial^2 v}{\partial \phi^2}\right) + \frac{\partial^2 v}{\partial z^2} = 0}$$ Laplace equation. co-ordinate system

[iii] Spherical co-ordinate System:-

$$\boxed{\nabla^2 v = \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left[r^2\frac{\partial v}{\partial r}\right] + \frac{1}{r^2 \sin\theta}\cdot\frac{\partial}{\partial \theta}\left[\sin\theta\frac{\partial v}{\partial \theta}\right] + \frac{1}{r^2\sin^2\theta}\cdot\frac{\partial^2 v}{\partial \theta^2} = 0}$$

Lablace equation, in Spherical co-ordinate system.