Transients

when ever the supply voltage changes 0 to any voltage, the current drawn by the circuit is changes from one state to another state. to change from one state to another state it will takes some time period, this time is called a transient time.

Initial conditions:-

Resistor :- Input voltage is applied to a resistor, the property of Resistor is which opposes the flow of electrons: The current drawn by the resistive ~~elem~~ component is less in magnitude as compared with supply voltage. [If ~~step~~ we apply step voltage the current is altered]

Inductor:- the property of Inductor ~~of~~ is which oppose the sudden changes in the current at $T=0$ the switch is close. due to the property of the inductor, it does not allows the current. Hence inductor will acts as open circuit. At $T=0^+$ the switch is continuously on therefore the inductor is allow the current hence inductor acts as short circuit.

At $T=0$ inductor acts as open circuit.

At $T=0^+$ inductor acts as short circuit

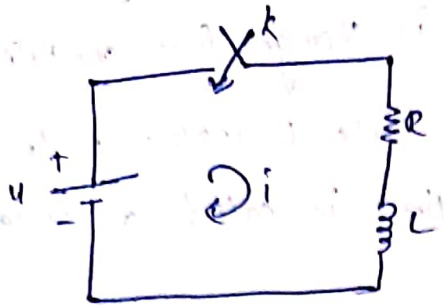
Capacitor:-

At $T=0$ the capacitor acts as short circuit.

at $T=0^+$ the capacitor acts as open circuit.

DC transients:-

i) RL-series circuit:-



- Initially the switch k is open, current passes through the circuit is zero. i.e., at $t=0^-$.

- at $t=0=0^+$ the switch k is closed.

apply KVL

$$V = Ri + L\frac{di}{dt}$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \longrightarrow (1)$$

[first order Non-Homogeneous equation]

$$\left(D + \frac{R}{L}\right) i = \frac{V}{L}$$

$$D + \frac{R}{L} = 0$$

$$\boxed{D = -R/L}$$

The solution of above equation consisting of complementary function and particular integral.

$\therefore$ complementary function $C.F = C_1 e^{-R/L\,t}$

$$P.I = e^{-Pt}\int Q.e^{Pt}\,dt$$

eqn (1) is compared with $\frac{di}{dt} + Pi = Q$,

$P = \frac{R}{L}$, $Q = \frac{V}{L}$

$\therefore$ The particular integral $PI = e^{-R/L\,t} \int \frac{V}{L} e^{R/L\,t} dt$

$$= e^{-R/L\,t} \cdot \frac{V}{L} \int e^{\frac{R}{L} t} dt$$

$$= \cancel{e^{-\frac{R}{L} t}} \cdot \frac{V}{\cancel{L}} \frac{\cancel{e^{\frac{R}{L} t}}}{\frac{R}{\cancel{L}}}$$

$$= \frac{V}{R}$$

$\therefore$ the general solution $i(t) = C.F + P.I$

$$i(t) = C_1 e^{-\frac{R}{L} t} + \frac{V}{R} \longrightarrow (2)$$

in order to find constant C_1, choose initial conditions.

i.e., at $t=0$

At $t=0$ inductor will acts as a open circuit hence the current passes through the circuit is zero.

i.e., at $t=0$ $i(t)=0$

substitute above condition's in eqn (2)

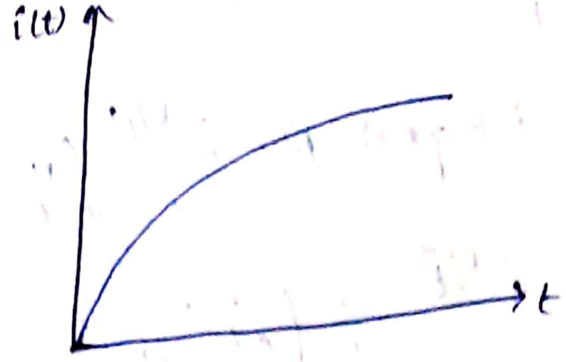
$$0 = C_1 e^{-\frac{R}{L}(0)} + \frac{V}{R}$$

$$C_1 = -\frac{V}{R}$$

$$i(t) = -\frac{V}{R} e^{-\frac{R}{L} t} + \frac{V}{R}$$

$$i(t) = \frac{V}{R}\left[1 - e^{-\frac{R}{L} t}\right]$$

From the above equation it is clear that the response of the current is exponentially increasing



The voltage across the resistor $v_R = i(t) R$

$$= \frac{V}{\cancel{R}}\left[1-e^{-\frac{R}{L}t}\right)\cancel{R}$$

$$\boxed{v_R = V\left[1-e^{-\frac{R}{L}t}\right]}$$

voltage across the inductor $v_L = L\frac{di(t)}{dt}$

$$= L\frac{d}{dt}\left[\frac{V}{R}\left[1-e^{-\frac{R}{L}t}\right]\right]$$

$$= \cancel{L}\cdot\frac{V}{\cancel{R}}\left[0-e^{-\frac{R}{L}t}\cdot -\frac{\cancel{R}}{\cancel{L}}\right]$$

$$\boxed{v_L = +Ve^{-\frac{R}{L}t}}$$

checking

$V = v_R + v_L$ [KVL]

$$= V - \cancel{Ve^{-\frac{R}{L}t}} + \cancel{Ve^{-\frac{R}{L}t}}$$

$V = V$

Power observed by the load Resistor $P_R = I^2 R$ or $\frac{V^2}{R}$

$$= \left[\frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right]^2 \cdot R$$

$$= \frac{V^2}{R^2}\left[1 - e^{-\frac{R}{L}t}\right]^2 \cdot R$$

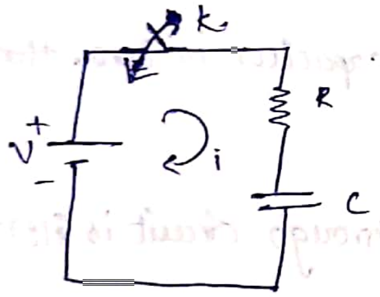
$$\boxed{P_R = \frac{V^2}{R}\left[1 - e^{-\frac{R}{L}t}\right]^2}$$

Power observed by the inductor $P_L = V_L\, i(t)$

$$= V e^{-\frac{R}{L}t} \cdot \frac{V}{R}\left[1 - e^{-\frac{R}{L}t}\right]$$

$$\boxed{P_L = \frac{V^2}{R} e^{-\frac{R}{L}t}\left[1 - e^{-\frac{R}{L}t}\right]}$$

R-C Series Circuit :-



Initially the switch k is open the current passes through the ~~current~~ ckt is zero at T=0 the switch is closed.

applying KVL in above circuit

$$V = iR + \frac{1}{C}\int i\,dt$$

Differentiate above eqn w.r.t 't'

$$\frac{dV}{dt} = R\frac{di}{dt} + \frac{1}{C} i$$

$$0 = R\frac{di}{dt} + \frac{1}{C} i$$

$$\frac{di}{dt} + \frac{1}{RC} i = 0$$

The above equation is homogeneous eqn

The above equation is first order differential equation.

∴ The solution of the above equation consisting of complementating function only.

$$\left[D+\frac{1}{RC}\right]i=0$$

$$D=\frac{-1}{RC}$$

∴ The complementary function $= C_1 e^{-\frac{1}{RC}t}$

The general solution on the circuit $i(t)=C_1 e^{-\frac{1}{RC}t}$

In order to find out constant C_1 we use initial condition

Assume initially the charge at capacitor is zero. Hence the capacitors of as short circuit.

at $T=0$, the current passes through circuit is $i(t)=V/R$

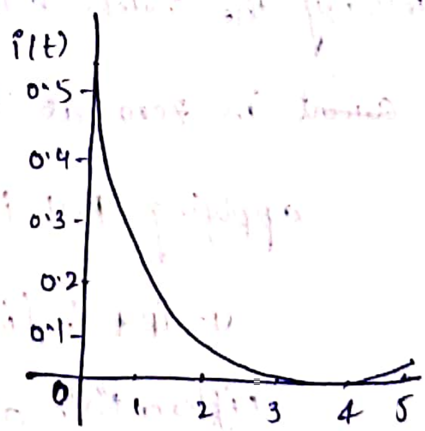
∴ The response $V/R = C_1 e^{-1/RC\, t(0)}$

$$V/R = C_1 e^0$$

$$V/R = C_1$$

$$\therefore i(t)=\frac{V}{R}e^{-\frac{1}{RC}t}$$

$$i(t)=\frac{V}{R}e^{-t/\tau}$$



τ is time constant $\Rightarrow \tau = RC$

voltage drop across the resistor $V_R = i(t)R$

$$=\frac{V}{\cancel{R}}e^{-\frac{1}{RC}t}\cdot\cancel{R}$$

$V_R = V e^{-\frac{1}{RC}t}$

Potential voltage drop across the capacitor $v_c = \frac{1}{C}\int i(t)dt$

$$= \frac{1}{C}\int \frac{V}{R} e^{-\frac{1}{RC}t} dt$$

$$= \frac{V}{RC} \frac{e^{-\frac{1}{RC}t}}{-\frac{1}{RC}}$$

$$= -V e^{-\frac{1}{RC}t}$$

power observed by the resistor $P_r = i^2(t) R$

$$= \left[\frac{V}{R} e^{-\frac{1}{RC}t}\right]^2 R$$

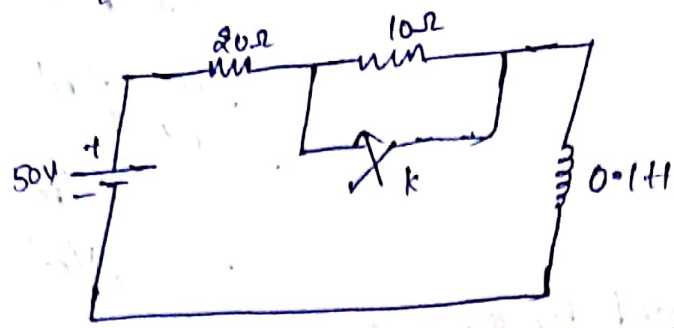
$$= \frac{V^2}{R} e^{-\frac{1}{RC}2t}$$

power observed by the capacitor $P_c = V_c\, i(t)$

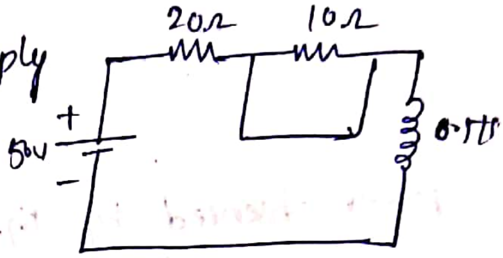
$$= -Ve^{-\frac{1}{RC}t} \cdot \frac{V}{R} e^{-\frac{1}{RC}t}$$

$$= \frac{-V^2}{R} e^{-\frac{2}{RC}t}$$

* A 50V is supplied to the circuit. at $t=0$ the switch is close find the current response of the circuit.



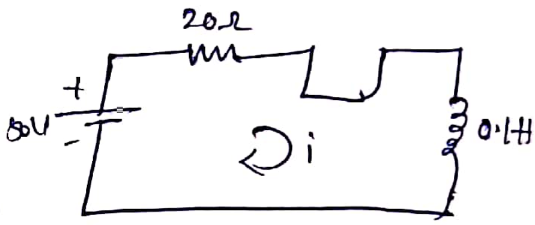
sol:- Initially the switch is open position. the initial current flowing through the circuit $i(0^-) = i(0^+) = \frac{50}{20+10} = 1.66$ Amp



At $t=0$ the switch is close Apply KVL for this circuit

$$50 = 20i + 0.1\frac{di}{dt}$$

$$\frac{di}{dt} + 200i = 500$$



The above 1st order D·equation is Non-Homogeneous

the general solution = C·F + P·I

$$(D+200)i = 0$$

$$D = -200$$

$$C\cdot F = C_1 e^{-200t}$$

$$P\cdot I = e^{-Pt}\int \cancel{50V}\,Q \cdot e^{Pt}\,dt$$

$$= e^{-200t}\int 500\, e^{200t}\,dt$$

$$= \cancel{e^{-200t}} \cdot \frac{500\,\cancel{e^{200t}}}{200}$$

$$= \frac{5}{2} = 2.5$$

$\therefore$ The General solution $i(t) = C.F + P.I$

$$= C_1 e^{-200t} + 2.5$$

To find constant C_1 use initial conditions at t

at $t=0$, $i(t) = 1.666$

$$\therefore 1.66 = C_1 e^{-200(0)} + 2.5$$

$$1.66 = C_1 + 2.5$$

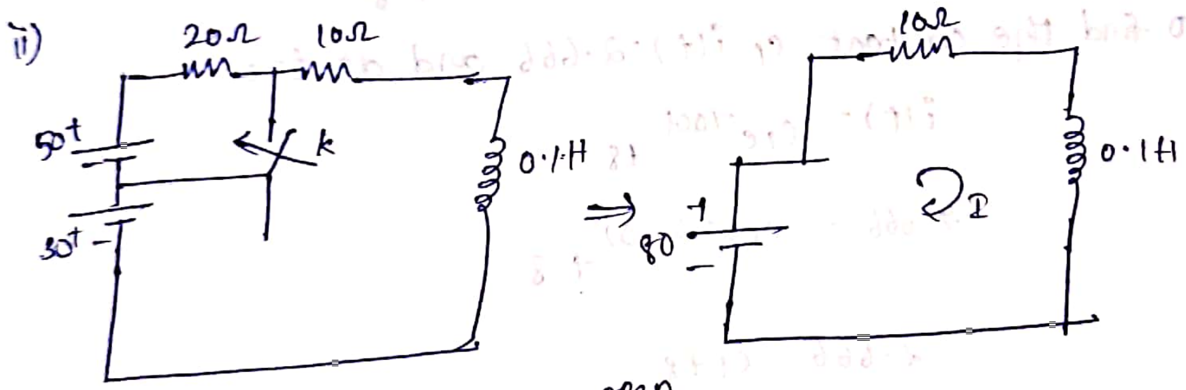
$$C_1 = 1.66 - 2.5$$

$$C_1 = -0.834$$

The response of the current

$$\therefore i(t) = -0.834\, e^{-200t} + 2.5$$

ii)



Initially the switch is ~~closed~~ open position. The initial flowing through the circuit $i(0) = i(0^+) = \dfrac{50+30}{20+10} = \dfrac{80}{30} = 2.666$ Amp

Now the switch is closed

Applying KVL for this circuit

$$80 = 10i + 0.1\frac{di}{dt}$$

$$\frac{di}{dt} + 100i = 800$$

The above first order D.E is Non-Homogeneous equation

$$(D+100)i = 0$$

$$D = -100$$

$$C.F = c_1 e^{-100t}$$

$$P.I = e^{-Pt}\int Q \cdot e^{Pt} dt$$

$$= e^{-100t}\int 800 \cdot e^{100t} dt$$

$$= \cancel{e^{-100t}}\, 8\cancel{00}\left[\frac{\cancel{e^{100t}}}{\cancel{100}}\right] = 8$$

The general solution = C.F + P.I

$$= c_1 e^{-100t} + 8$$

To find the constant c_1 $i(t) = 2.666$ and at $t=0$.

$$i(t) = c_1 e^{-100t} + 8$$

$$2.666 = c_1 e^{-100(0)} + 8$$

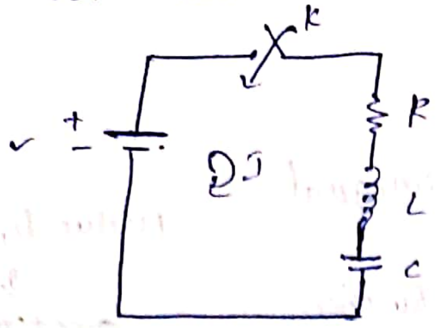
$$2.666 = c_1 + 8$$

$$c_1 = 2.666 - 8 = -5.334$$

$$c_1 = -5.334$$

The response of the current $i(t) = -5.334 e^{-100t} + 8$

* R-L-C Series circuit :-



Initially the switch is open, the current passes through the circuit is zero. at $t=0$ the switch is closed.

Apply KVL $\quad v = RI + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$

differentiate above eqn with r.s.t t

$$\frac{dv}{dt} = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i \qquad \text{[constant DC V]}$$

$$0 = R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

The above equation is second order homogeneous differential equation. the solution of above equation is only complementary function.

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right)i = 0$$

$$D^2 + \frac{R}{L}D + \frac{1}{LC} = 0$$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - 4\frac{1}{LC}\cdot 1}}{2\cdot 1}$$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

Case-I:

If $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal

L value α_1, C value
⇓
overdamped condition
otherwise
underdamped condition

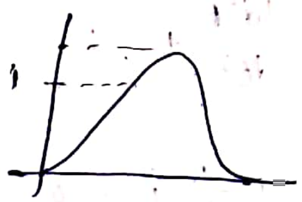
∴ The complementary function

$$C.F = c_1 e^{\alpha_1 t} + c_2 e^{\alpha_2 t}$$

here $\alpha_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$$\alpha_2 = \frac{-R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

This is a over damped condition



Case-II

If $\left[\frac{R}{2L}\right]^2 < \frac{1}{LC}$

the roots are complex conjugates and unequal.

∴ The complementary function

$$C.F = C_1 \cos(\alpha+\beta)t + C_2 \sin(\alpha-\beta)t$$

The system is under damped condition.

case - III.

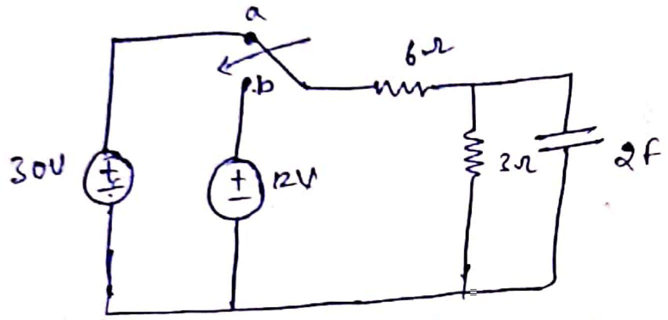
If $\left[\frac{R}{2L}\right]^2 = \frac{1}{LC}$

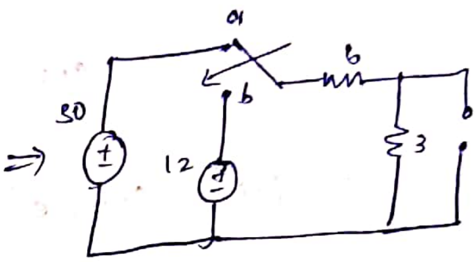
The roots are real and equal

The complementary function $c.f = (C_1 + x C_2) e^{\alpha t}$

The system is critically damped.

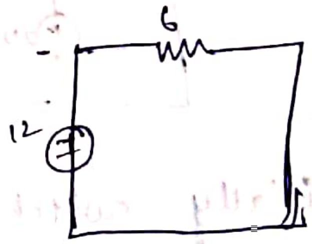
* The switch has been in position A for along time. At t=0 it moves to position B. Calculate $i(t)$ for all $t>0$.





sol: Initially the switch is at position A. at $t=0^+$ the capacitor will acts as open circuit. The initial current $i(0^-) = i(0^+) = \frac{30}{6+3} = 3.33$ Amps

at $t=0$ the switch is moved to b



apply KVL: $12 = 6i + \frac{1}{C}\int i\,dt$

$12 = 6i + \frac{1}{2}\int i\,dt$

differentiate it

$0 = 6\frac{di}{dt} + \frac{1}{2}i$

$$\frac{di}{dt} + \frac{1}{12} i = 0$$

$$\left(D + \frac{1}{12}\right) i = 0$$

$$D = -1/12$$

The solution complementary function $C_1 e^{-1/12 t}$

$$i(t) = C_1 e^{-0.08t}$$

in order to find constant C_1 using initial conditions.

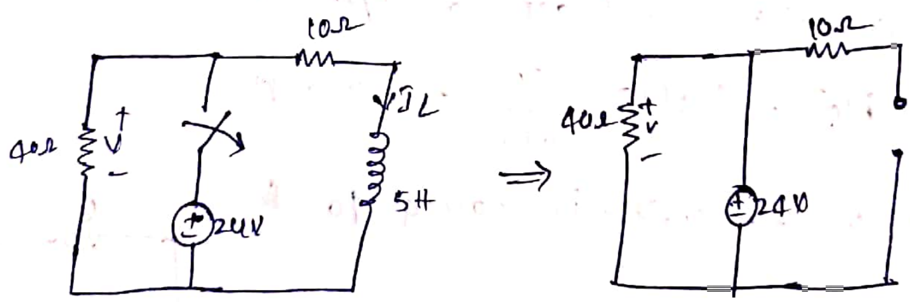
at $t=0$, $3.33 = C_1 e^{-0.08(0)}$

$$C_1 = 3.33$$

$$\therefore i(t) = 3.33 e^{-0.08t}$$

* for the circuit shown find the voltage labelled v at t=200 milli sec.

Initial condition



sol:- Initially switch is closed position. the current passes through the circuit is $I = \frac{24}{40} = 0.6$ Amp

at $t=0$, the switch is opened, apply KVL

$$0 = 40I + 10I + 5\frac{di}{dt}$$

$$5\frac{di}{dt}+50i=0$$

$$\frac{di}{dt}+10i=0$$

$$(D+10)i=0$$

$$D=-10$$

$\therefore$ $C.F = c_1 e^{-10t}$

$$i(t)= c_1 e^{-10t}$$

in order to find constant c_1 using initial conditions

at $t=0^+$ $\quad 0.6 = c_1 e^{-10(0)}$

$$c_1 = 0.6$$

$$i(t) = 0.6\, e^{-10t}$$

* at t = 200 millisec

$$t = 0.2\,sec$$

$$i(0.2) = 0.6\, e^{-10(0.2)}$$

$$= 0.6 e^{-2}$$

$$= 0.08\,Amps$$

* voltage $= 40 \times 0.6 e^{-10t}$

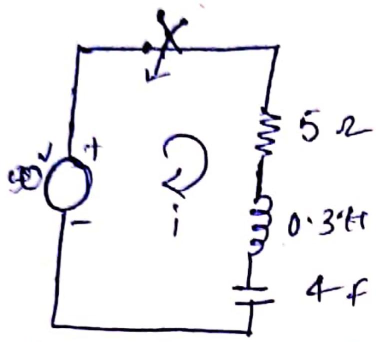
$$v = 24 e^{-10t}$$

$$= 24 e^{-10(0.2)}$$

$$= 3.24\,V$$

+ In a series RLC circuit L = 0.3H and C = 4μF. a DC voltage of 50 V is applied at t = 0 obtain an expression for current i(t) in the circuit, when

i) R = 5Ω ii) R = 6Ω



Sol: when R = 5Ω

at t = 0 the switch is closed

Apply KVL

$$50 = 5i + 0.3\frac{di}{dt} + \frac{1}{4}\int i\,dt$$

$$0 = 5\frac{di}{dt} + 0.3\frac{d^2i}{dt^2} + \frac{1}{4}i\,dt$$

$$\frac{d^2i}{dt^2} + 16.66\frac{di}{dt} + 0.833\,i = 0$$

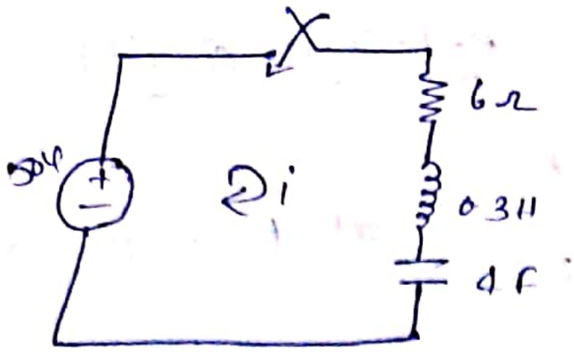
$$(D^2 + 16.66D + 0.833)\,i = 0$$

$$D^2 + 16.66D + 0.833 = 0$$

$$D = -0.05, \quad D = -16.60$$

$$C.F = C_1 e^{-0.05t} + C_2 e^{-16.60t}$$

when $R = 6\Omega$



$$50 = 6i + 0.3\frac{di}{dt} + \frac{1}{4}\int i\,dt$$

$$0 = 6\frac{di}{dt} + 0.3\frac{d^2i}{dt^2} + \frac{1}{4}i$$

$$\frac{d^2i}{dt^2} + 20\frac{di}{dt} + 0.833i = 0$$

$$(D^2 + 20D + 0.833)\,i = 0$$

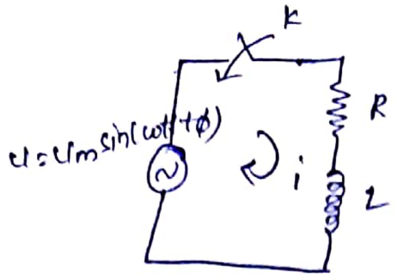
$$D^2 + 20D + 0.833 = 0$$

$$D_1 = -0.04, \quad D_2 = -19.95$$

$$C.F = C_1e^{-0.04t} + C_2e^{-19.95t}$$

Ac Transients:-

R-L series circuit



at $t=0$, the switch is closed

apply KVL

$$V_m \sin(\omega t+\phi) = iR + L\frac{di}{dt}$$

$$L\frac{di}{dt} + iR = V_m \sin(\omega t+\phi)$$

$$\frac{di}{dt} + \frac{R}{L} i = \frac{V_m}{L}\sin(\omega t+\phi) \longrightarrow (1)$$

Above equation is first order non-Homogeneous differential equation.

The solution consists complementary function and particular integral.

$$C.F = C_1 e^{-\frac{R}{L}t}$$

Assume $i_p = i = A\cos(\omega t+\phi) + B\sin(\omega t+\phi)$

$$\frac{di_p}{dt} = \frac{di}{dt} = -A\omega \sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)$$

Substitute $\frac{di_p}{dt}$, i_p in (1)

$$-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) + \frac{R}{L}\left(A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right) = \frac{V_m}{L}\sin(\omega t+\phi)$$

$$\left[-A\omega + \frac{R}{L}B\right]\sin(\omega t+\phi) + \left[B\omega + \frac{R}{L}A\right]\cos(\omega t+\phi) = \frac{V_m}{L}\sin(\omega t+\phi)$$

equating like terms on both sides

$$\frac{V_m}{L} = -A\omega + \frac{R}{L}B \rightarrow (2) \qquad B\omega + \frac{R}{L}A = 0 \longrightarrow (3)$$

$$B\omega = -\frac{R}{L}A$$

$$B = \frac{-R}{\omega L}A$$

$$\frac{V_m}{L} = -A\omega + \frac{R}{L}\left[\frac{-R}{\omega L}\right]A$$

$$\frac{V_m}{L} = -A\left[\omega + \frac{R^2}{\omega L^2}\right]$$

substitute (B) in (2)

$$\frac{V_m}{L} = -A\omega\left[1 + \frac{R^2}{\omega^2 L^2}\right]$$

$$B = \frac{-R}{\omega L}\left[\frac{-V_m\omega L}{\omega^2 L^2 + R^2}\right]$$

$$A = \frac{-V_m}{L\omega\left[1 + \frac{R^2}{\omega^2 L^2}\right]} \qquad = \frac{V_m R}{\omega^2 L^2 + R^2}$$

$$A = \frac{-V_m\omega L}{\omega^2 L^2 + R^2}$$

substitue A in B

substitute (A) & (B) in i_p

$$i_p = i = \frac{-V_m\omega L}{\omega^2 L^2 + R^2}\cos(\omega t + \phi) + \frac{V_m R}{\omega^2 L^2 + R^2}\sin(\omega t + \phi)$$

$$= \frac{V_m}{\omega^2 L^2 + R^2}\left[-\omega L\cos(\omega t + \phi) + R\sin(\omega t + \phi)\right]$$

Assume $R = \cos\theta$, $\omega L = \sin\theta$

$$= \frac{V_m}{\omega^2 L^2 + R^2}\left[\cos\theta\sin(\omega t + \phi) - \sin\theta\cos(\omega t + \phi)\right]$$

$$i_p = \frac{V_m}{\omega^2 L^2 + R^2}\sin\left[\omega t + \phi - \theta\right]$$

here $\theta = \tan^{-1}\left[\frac{\omega L}{R}\right]$

$$i_p = \frac{V_m}{R^2+L^2\omega^2}\left[\sin(\omega t+\phi - \tan^{-1}\left[\frac{\omega L}{R}\right]\right]$$

General solution is $i(t) = C.F + P.I$

$$i(t) = C_1 e^{-\frac{R}{L}t} + \frac{V_m}{R^2+\omega^2 L^2}\left[\sin(\omega t+\phi - \tan^{-1}\left[\frac{\omega L}{R}\right]\right]$$

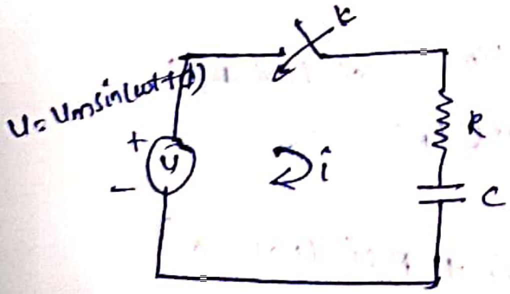
at $t=0$, $i(t)=0$

$$0 = C_1 e^{-\frac{R}{L}(0)} + \frac{V_m}{R^2+\omega^2 L^2}\left[\sin(0+\phi-\theta\right]$$

$$C_1 = \frac{-V_m}{R^2+\omega^2 L^2}\left[\sin(\phi-\theta)\right]$$

$$i(t) = \frac{-V_m}{R^2+\omega^2 L^2}\left[\sin(\phi-\theta)\right] e^{-\frac{R}{L}t} + \frac{V_m}{R^2+\omega^2 L^2}\left[\sin(\omega t+\phi-\theta)\right]$$

R-C series circuit:-



at $t=0$, the switch is in closed position

Apply KVL:

$$V_m \sin(\omega t+\phi) = iR + \frac{1}{C}\int i\,dt$$

$$V_m \omega \cos(\omega t+\phi) = R\frac{di}{dt} + \frac{1}{C} i$$

$$\frac{di}{dt} + \frac{1}{RC} i = \frac{V_m}{R}\omega\cos(\omega t+\phi) \rightarrow (1)$$

The above equation is first order differential equation

The solution consists C.F and P.I

$$C.F = C_1 e^{-\frac{1}{RC}t}$$

Assume $i_p = i = A\cos(\omega t+\phi) + B\sin(\omega t+\phi)$

$$\frac{di_p}{dt} = \frac{di}{dt} = -A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)$$

substitute $\frac{di_p}{dt}$, i_p in eq(1)

$$-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi) + \frac{1}{RC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{V_m}{R}\omega\cos(\omega t+\phi)$$

$$\left[-A\omega + \frac{B}{RC}\right]\sin(\omega t+\phi) + \left(B\omega + \frac{1}{RC}A\right]\cos(\omega t+\phi) = \frac{V_m}{R}\omega\cos(\omega t+\phi)$$

equating like terms on both sides

$$-A\omega + \frac{B}{RC} = 0 \qquad B\omega + \frac{A}{RC} = \frac{V_m\omega}{R}$$

$$A\omega = \frac{B}{RC} \qquad \omega(A\omega RC) + \frac{A}{RC} = \frac{V_m\omega}{R}$$

$B = A\omega RC \rightarrow (a)$

substitute B in

$$ARC\left[\omega^2 + \frac{1}{R^2C^2}\right] = \frac{V_m\omega}{R}$$

$$ARC\left[\frac{\omega^2R^2C^2+1}{R^2C^2}\right] = \frac{V_m\omega}{R}$$

$$A\left[\frac{1+\omega^2R^2C^2}{RC}\right] = \frac{V_m\omega}{R}$$

$$A = \frac{V_m\omega RC}{R\left[1+\omega^2R^2C^2\right]}$$

$$A = \frac{V_m\omega C}{1+\omega^2R^2C^2}$$

$$B = \frac{V_m\omega C}{1+\omega^2R^2C^2}\cdot\omega RC$$

$$= \frac{RV_m\omega^2C^2}{\frac{1+\omega^2R^2C^2}{\omega^2C^2}}$$

substitute A and B in ip

$$i_p = i = \frac{V_m \omega C}{1+\omega^2C^2R^2}\cos(\omega t+\phi) + \frac{\frac{RV_m \omega^2C^2}{1+\omega^2C^2R^2}}{\omega C} \sin(\omega t+\phi)$$

$$= \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\frac{1}{\omega}\cos(\omega t+\phi) + R\omega C \sin(\omega t+\phi)\right]$$

$$= \frac{V_m \omega C}{1+\omega^2 C^2 R^2} \quad \text{let } \frac{1}{\omega} = \sin\theta$$

$$R\omega C = \cos\theta$$

$$= \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\sin\theta \cos(\omega t+\phi) + \cos\theta \sin(\omega t+\phi)\right]$$

$$= \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\sin(\omega t+\phi+\theta)\right]$$

$$\tan\theta = \frac{1}{\omega R\omega C}$$

$$\theta = \tan^{-1}\left[\frac{\frac{1}{\omega}}{R\omega C}\right]$$

$$i_p = i = \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\sin\left(\omega t+\phi+\tan^{-1}\left[\frac{\frac{1}{\omega}}{R\omega C}\right]\right)\right]$$

General solution is CF + P.I

$$i(t) = C_1 e^{-\frac{1}{RC}t} + \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\sin(\omega t+\phi+\theta)\right]$$

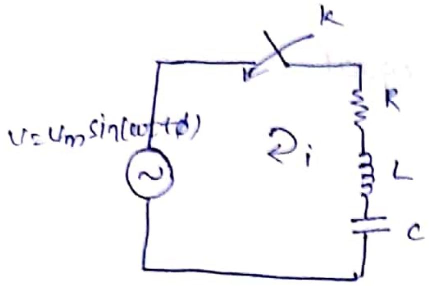
at $t=0$ $i(t)=0$

$$0 = C_1 e^{\frac{-1}{RC}(0)} + \frac{V_m \omega^2 C}{1+\omega^2C^2R^2}\left[\sin(\phi+\theta)\right]$$

$$C_1 = \frac{-V_m \omega C}{1+\omega^2C^2R^2}\left[\sin(\phi+\theta)\right]$$

$$i(t) = \frac{-V_m \omega C}{1+\omega^2 C^2 R^2} \sin(\phi+\theta)\, e^{\frac{-1}{RC}t} + \frac{V_m \omega^2 C}{1+\omega^2 C^2 R^2}\left[\sin(\omega t+\phi+\theta)\right]$$

R-L-C Series Circuit :-



at $t=0$ the switch is closed. Apply KVL

$$V_m \sin(\omega t+\phi) = RI + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

$$V_m \omega \cos(\omega t+\phi) = R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C} i$$

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} i = \frac{V_m \omega}{L} \cdot \cos(\omega t+\phi) \longrightarrow (1)$$

Non ~~Homogeneous~~ - second order equation.

the solution consisting of C.F and P.I

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) i = 0$$

$$D^2 + \frac{R}{L}D + \frac{1}{CL} = 0$$

$$D = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4 \cdot \frac{1}{LC}}}{2}$$

$$= \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$= \alpha \pm \beta$$

$\alpha = \frac{-R}{2L}$, $\beta = \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

case(i) if $\alpha \ne \beta$ $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

The roots are real and unequal

$\therefore C.F = C_1 e^{(\alpha+\beta)t} + C_2 e^{(\alpha-\beta)t}$

If case-2 $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

The roots are complex conjugates

$C.F = C_1 \cos(\alpha+\beta)t + C_2 \sin(\alpha-\beta)t$

case-3:- if $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

The roots are equal and Real

$C.F = (C_1 + x C_2)\, e^{\left(\frac{-R}{2L}\right)t}$

Assume $i_p = A\cos(\omega t+\phi) + B\sin(\omega t+\phi)$

$\frac{di_p}{dt} = -A\omega \sin(\omega t+\phi) + B\omega \cos(\omega t+\phi)$

$\frac{d^2 i_p}{dt} = -A\omega^2 \cos(\omega t+\phi) - B\omega^2 \sin(\omega t+\phi)$

substitute Above equation's in equation (1)

$$\left[-A\omega^2\cos(\omega t+\phi) - B\omega^2\sin(\omega t+\phi)\right] + \frac{R}{L}\left(-A\omega\sin(\omega t+\phi) + B\omega\cos(\omega t+\phi)\right] + \frac{1}{LC}\left[A\cos(\omega t+\phi) + B\sin(\omega t+\phi)\right] = \frac{V_m\,\omega\,\cancel{\sin}\cos(\omega t+\phi)}{L}$$

$$-A\omega^2 + B\omega\frac{R}{L} + \frac{A}{LC} = \frac{V_m\omega}{L} \rightarrow (2)$$

$$-B\omega^2 - \frac{A\omega R}{L} + \frac{B}{LC} = 0 \rightarrow (3)$$

$$B\left[\frac{1}{LC} - \omega^2\right] = \frac{A\omega R}{L}$$

$$B = \frac{A\omega R}{L\left[\frac{1}{LC} - \omega^2\right]} = \frac{A\omega R}{L\left[\frac{1}{LC} - \omega^2\right]}$$

$$B = \frac{A\omega R C}{1 - LC\omega^2} = \frac{A\omega R}{\frac{1}{C} - L\omega^2}$$

substitute (B) in (A)

$$-A\omega^2 + \left[\frac{A\omega RC}{1 - LC\omega^2}\right]\frac{\omega R}{L} + \frac{A}{LC} = \frac{V_m\omega}{L}$$

$$-A\omega^2 + \frac{A\omega^2R^2C}{1 - L^2C\omega^2} + \frac{A}{LC} = \frac{V_m\omega}{L}$$

$$A\left[\frac{1}{LC} + \frac{\omega^2R^2C}{1 - L^2C\omega^2} - \omega^2\right] = \frac{V_m\omega}{L}$$

$$A = \frac{V_m\,\omega}{L\left[\frac{1}{LC} + \frac{\omega^2R^2C}{1 - L^2C\omega^2} - \omega^2\right]}$$

$$A = \frac{V_m\,\omega}{L\left[\frac{1}{LC} + \frac{\omega^2R^2C}{1-L^2C\omega^2} - \omega^2\right]}$$

$$= \frac{V_m\ \omega}{\frac{1}{C} + \frac{L\omega^2R^2C}{1-L^2C\omega^2} - L\omega^2}$$

$$= \frac{V_m\ \omega \times C\cdot(1-LC\omega^2)}{1-L^2C\omega^2 + L\omega^2R^2C^2 - LC\omega^2}$$

$$= \frac{V_m\omega\cdot L\left[\frac{C}{L} - \cancel{C}\,C^2\omega^2\right]}{L^3\left[\frac{\omega^2R^2C^2}{L^2} - \frac{C\omega^2}{L^2} - \frac{C\omega^2}{L} + \frac{1}{L^3}\right]}$$

$$= \frac{V_m\omega\, LC^2\left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2R^2C^2}{L^2}\right] - \frac{L^3C\omega^2}{L^2}\left[1+\frac{1}{L}\right] + \frac{1}{L^3}\cdot L^3}$$

$$= \frac{V_m\omega\, LC^2\left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2R^2C^2}{L^2}\right] - \frac{L^3C\omega^2(L+1)}{L^3} + 1}$$

$$= \frac{\omega V_m LC^2\left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2R^2C^2}{L^2}\right] - \frac{L^4C\omega^2}{L^3} + \frac{L^3C\omega^2}{L^3} + 1}$$

$$= \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2 R^2}{L^2}\right] - \frac{L^4\omega^2}{CL^3} + \frac{C^3\omega^2}{CL^3} + 1}$$

$$= \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2 R^2}{L^2}\right] - \frac{L^4\omega^2 + L^3\omega^2}{CL^3} + 1}$$

$$= \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right]}{L^3\left[\frac{\omega^2 R^2}{L^2}\right] + L^2\left[\frac{-L^2\omega^2 + L\omega^2}{CL^3}\right] + 1}$$

$$= \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right]}{\frac{\omega^2 R^2}{L^2} + \left[-\omega^2 + \frac{1}{LC}\right]^2 L^2}$$

$$A = \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right]}{\frac{\omega^2 R^2}{L^2} + \left[\frac{1}{LC} - \omega^2\right]^2 L^2}$$

$$B = \frac{V_m \omega L \left[\frac{1}{LC} - \omega^2\right] \omega R}{L\left(\frac{1}{LC} - \omega^2\right)\left[\frac{\omega^2 R^2}{L^2} + \left(\frac{1}{LC} - \omega^2\right)^2 L^2\right]}$$

$$B = \frac{V_m \omega^2 R}{\frac{\omega^2 R^2}{L^2} + L^2\left[\frac{1}{LC} - \omega^2\right]^2}$$

$$i_p = \frac{V_m \omega L\left[\frac{1}{LC}-\omega^2\right]}{\frac{\omega^2R^2}{L^2}+L^2\left[\frac{1}{LC}-\omega^2\right]^2}\cos(\omega t+\phi) + \frac{V_m \omega^2 R}{\frac{\omega^2R^2}{L^2}+L^2\left[\frac{1}{LC}-\omega^2\right]^2}\sin(\omega t+\phi)$$

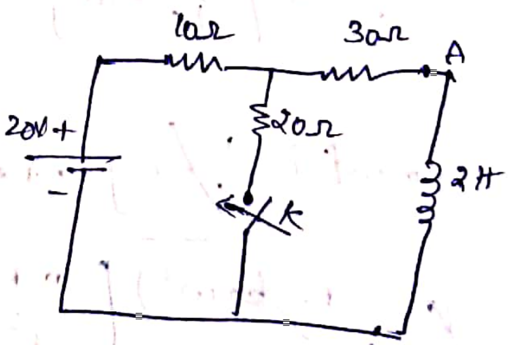
$$= \frac{1}{\frac{\omega^2R^2}{L^2}+L^2\left[\frac{1}{LC}-\omega^2\right]^2}\left[V_m \omega L\left[\frac{1}{LC}-\omega^2\right]\cos(\omega t+\phi) + V_m\omega^2 R\sin(\omega t+\phi)\right]$$

∴ General solution case:-

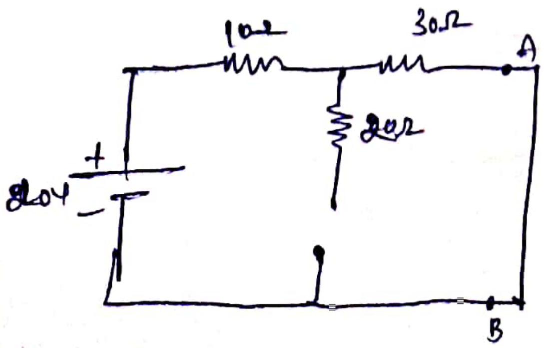
$$i(t) = C_1 e^{s_1 t}$$

* Find the i(t) for the given circuit at t=0 the switch is closed and reaches to steady state

Sol^n:- Inductance L=2H



initially the switch is open. under steady state condition the inductor will acts as short circuit

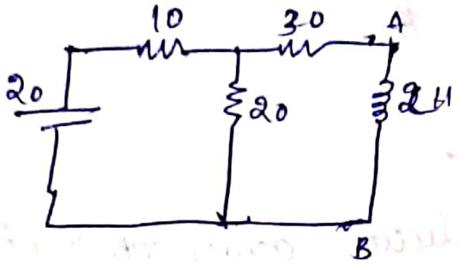


$\therefore$ Initial current $i(0^-) = i(0^+) = \frac{V}{R_{eq}}$

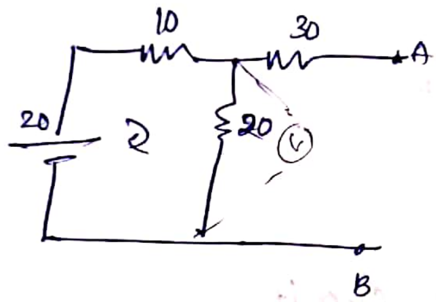
$$= \frac{20}{10+30}$$

$$= \frac{20}{40} = 0.5\,Amp$$

At $t=0$ the switch is closed



find V_{oc}:-

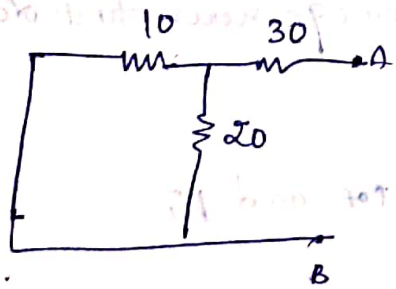


$$I = \frac{V}{R_{eq}} = \frac{20}{10+20} = 0.666\,Amp$$

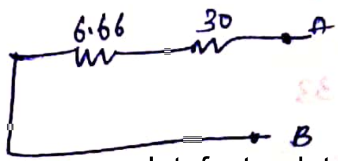
$$V_{oc} = i \times 20 = 0.666 \times 20$$

$$= 13.33\,V$$

R_{th}



$$10 \| 20 \Rightarrow \frac{10 \times 20}{10+20} = 6.66$$

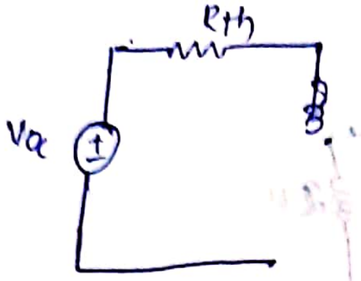


30 is series 6.66

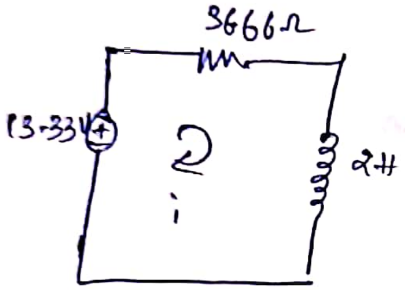
$R_{th} = 30 + 6.66$

$= 36.66\,\Omega$

Thevinins equivalent circuit



Connect the inductor across AB Terminals



Apply KVL for this circuit

$$13.33 = i(36.66) + 2.\frac{di}{dt}$$

$$\frac{di}{dt} + 18.33\,i = 6.66$$

Above equation is Non-Homogeneous first order differential equation.

The solution consists cof and PI

$$\text{cof} = (D + 18.33)\,i = 0$$

$$D + 18.33 = 0$$

$$D = -18.33$$

$$C.F = C_1 e^{-18.33t}$$

$$P.I = e^{-18.33t} \int 6.66 \cdot e^{18.33t} \, dt$$

$$= \cancel{e^{-18.33t}} \cdot 6.66 \, \frac{\cancel{e^{18.33t}}}{18.33}$$

$$= 0.363$$

General solution: $C.F + P.I$

$$i(t) = C_1 e^{-18.33t} + 0.363$$

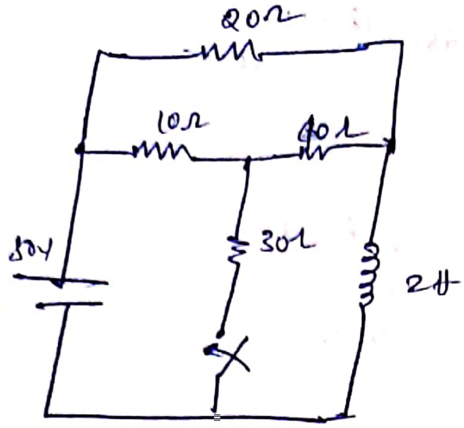
at $t=0$ the current $i = 0.5$

$$0.5 = C_1 e^{-18.33(0)} + 0.363$$

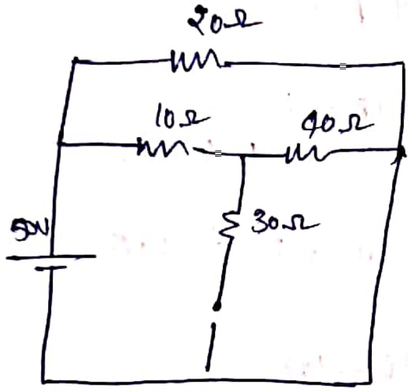
$$C_1 = -0.363 + 0.5 = 0.137$$

$$\therefore i(t) = +0.137 \, e^{-18.33t} + 0.363$$

* For the circuit find i(t). initially the switch is opened, at t=0 the switch is closed and reaches to steady state.



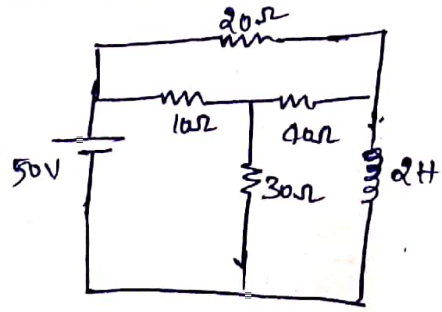
Initialy the switch is open the inductor act as short circuit



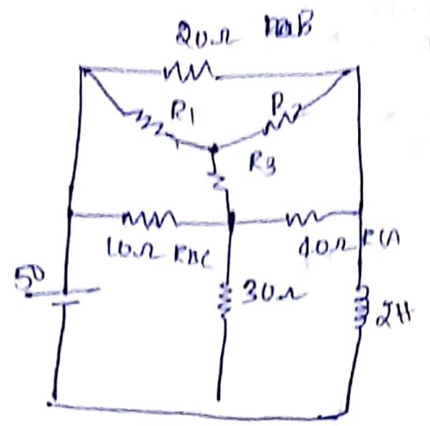
$$I(0^-) = \frac{50}{(10+40)\,||\,20} = \frac{50}{50\,||\,20}$$

$$= \frac{50}{14.285} = 3.5$$

Now, the switch is closed and at t=0



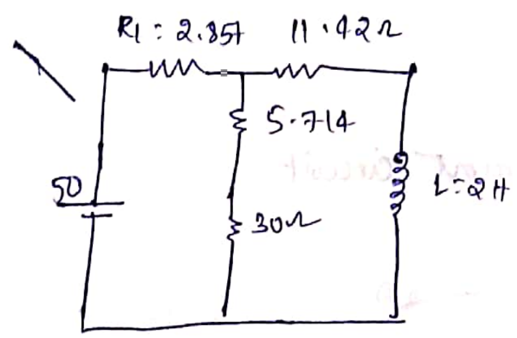
Convert the Delta connection into star connection.



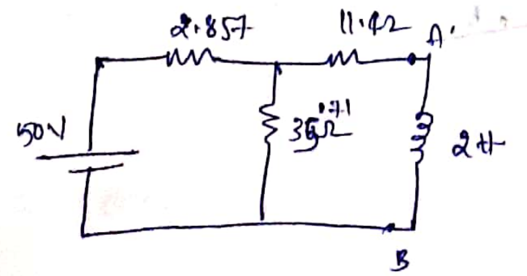
$$R_1 = \frac{R_{AB} \times R_{BC}}{R_{AB} + R_{BC} + R_{CA}} = \frac{20 \times 10}{20+10+40} = 2.857\,\Omega$$

$$R_2 = \frac{R_{AB} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{20 \times 40}{20+10+40} = 11.42\,\Omega$$

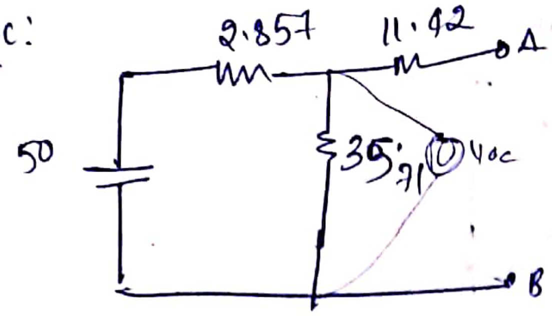
$$R_3 = \frac{R_{BC} \times R_{CA}}{R_{AB} + R_{BC} + R_{CA}} = \frac{10 \times 40}{20+10+40} = 5.714\,\Omega$$



$30 + 5.714 = 35.714$



Voc:

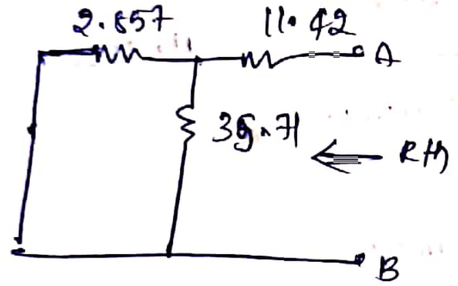


$$I = \frac{50}{35.71 + 2.857} = 1.297 \text{ Amp}$$

$$V_{OC} = I \times 30 = 1.29 \times 35.71$$

$$= 46.06\,V$$

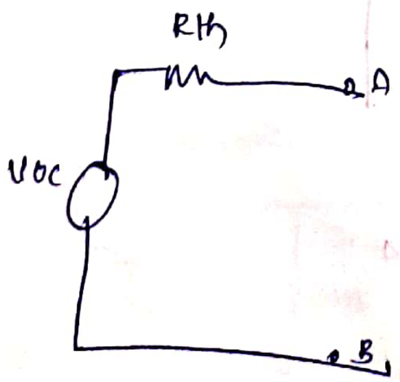
Rth:



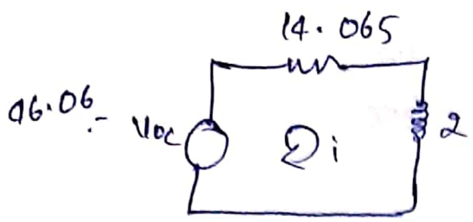
$$R_{th} = (35.71 \,||\, 2.857) + 11.42$$

$$= 14.065\,\Omega$$

Thevinin's equivalent circuit



Connect inductor across AB Terminals



Apply KVL

$$14.065i + 2\frac{di}{dt} = 46.06$$

$$\frac{di}{dt} + 7.03i = 23.03$$

This is first order non-Homogeneous equation

It consists C.F + P.I

$$(D + 7.03)I = 0$$

$$D + 7.03 = 0$$

$$D = -7.03$$

$$C.F = C_1 e^{-7.03t}$$

$$P.I = e^{-7.03t}\int 23.03\, e^{7.03t}\, dt$$

$$= \cancel{e^{-7.03}} \cdot \frac{23.03 \cdot \cancel{e^{7.03}}}{7.03}$$

$$= 3.27$$

General solution = C.F + P.I

$$i(t) = C_1 e^{-7.03t} + \frac{3.27}{\cancel{6.05}}$$

at $t=0$ the current $i = 3.5$

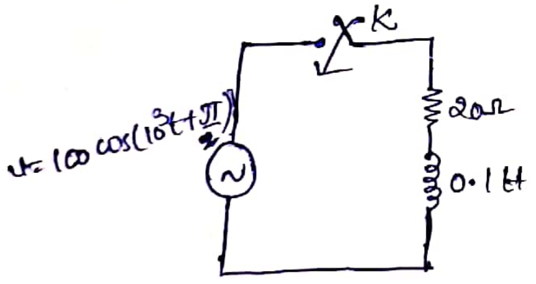
$$3.5 = C_1 e^{-7.03 i} + \frac{3.27}{\cancel{6.05}}$$

$$C_1 = 3.5 - \frac{3.27}{\cancel{6.05}}$$

$$= \cancel{-2.05}\ 0.23$$

$$i(t) = \frac{0.23}{\cancel{6.05}} e^{-7.03 i} + 6.55$$

* for the given circuit. determine complete solution $i(t)$. at $t=0$ the switch is closed. $R = 20\Omega$, $L = 0.1H$



Sol:- Initially the switch is open. At $t=0$ the switch is closed

initial current $I = 0$ $\quad i = \frac{V}{R} = \frac{100\cancel{\cos(10^3t + \pi/2)}}{20}$

Apply KVL $\quad = 5\cancel{\cos(10^3t + \pi/2)}$

$$100\cos(10^3 t + \frac{\pi}{2}) = 20i + 0.1\frac{di}{dt}$$

$$\frac{di}{dt} + 200i = 1000\cos(10^3 t + \frac{\pi}{2})$$

Above equation is Non-Homogeneous first order differential equation.

The solution consists C.F and P.I

$$(D+200)\, i = 0$$

$$D+200 = 0$$

$$D = -200$$

$\therefore$ The $C.F = C_1 e^{-200t}$

$$P.I = i_p = \frac{V_m}{\sqrt{R^2+(\omega L)^2}} \cos\left[\omega t+\phi - \tan^{-1}\left(\frac{\omega L}{R}\right)\right]$$

from circuit $V_m = 100$, $\phi = \pi/2$
$R = 20$
$\omega = 1000$
$L = 0.1$

$$= \frac{100}{\sqrt{(20)^2+(1000\times0.1)^2}} \cos\left[10^3 t + \frac{\pi}{2} - \tan^{-1}\left[\frac{1000\times0.1}{20}\right]\right]$$

$$= \frac{100}{101.98} \cos\left[10^3 t + \frac{\pi}{2} - \tan^{-1}\left[\frac{100}{20}\right]\right]$$

$$= 0.98 \cos\left[10^3 t + \frac{\pi}{2} - 78.69\right]$$

$$i_p = 0.98 \cos\left[10^3 t + 11.31\right]$$

General solution $i(t) = C.F + P.I$

$$i(t) = C_1 e^{-200t} + 0.98 \cos(1000t + 11.31)$$

at $t=0$, $i=0$

$$0 = C_1 e^{-200(0)} + 0.98 \cos(1000(0) + 11.31)$$

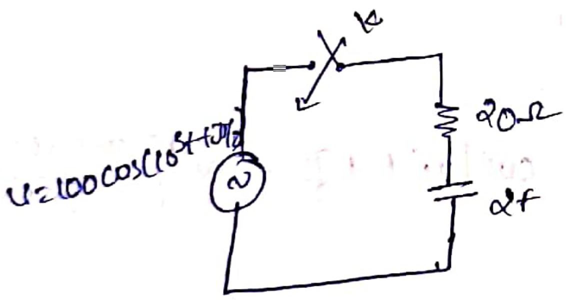
$$0 = C_1 + 0.9609$$

P

$$C_1 = -0.9609$$

$$\therefore i(t) = -0.9609\, e^{-200t} + 0.98 \cos(1000t + 11.31)$$

*



Sol: Initially the switch is open. at $t=0$, switch is closed

initial current $I_0 = \frac{V}{R} = \frac{100\cos(10^3 t + \pi/2)}{20}$

$= 5\cos(10^3 t + \pi/2)$

Apply KVL

$$100\cos(10^3 t + \pi/2) = 20i + \frac{1}{C}\int i\,dt$$

$$-100\sin(10^3 t + \pi/2) \times 1000 = 20\frac{di}{dt} + \frac{1}{2} i$$

$$\frac{di}{dt} + 0.025\, i = -5000 \sin(10^3 t + \pi/2)$$

$$(D + 0.025)\, i = 0$$

$$D + 0.025 = 0$$

$$D = -0.025$$

$$C.F = C_1 e^{-0.025 t}$$

(here taking magnitude)

$$P.I = i_p = \frac{V_m}{\sqrt{\frac{1+\omega^2 C^2 R^2}{\omega^2 C^2}}} \left[\cos(\omega t + \phi + \tan^{-1}\left[\frac{1}{R\omega C}\right]\right]$$

$$= \frac{100 \times (1000)^2 \times 2}{\sqrt{1 + (1000)^2 (2)^2 (20)^2}} \left[\cos(1000t + \frac{\pi}{2} + \tan^{-1}\left[\frac{1}{R \times 1000 \times 2}\right]\right]$$

$$= \frac{200000000}{\sqrt{16{,}000\,000\,00}} \left[\cos\left[1000t + \frac{\pi}{2} + 0.0284\right]\right]$$

$$= \frac{200000000}{40{,}000} \left[\cos(1000t + 90.001\right]$$

$$= 5000 \left[\cos(1000t + 90.001)\right]$$

General solution: $i(t) = C.F + P.I$

$$i(t) = C_1 e^{-0.025t} + 5000 \left[\cos(1000t + 90.001)\right]$$

at $t = 0$ $i = 5\cos(10^3 t + \pi/2)$

$$5\cos(10^3 t + \pi/2) = C_1 e^{-0.025(0)} + 5000 \left[\cos(1000(0)) + 90.001\right]$$

nearly (0)

$$C_1 = \frac{5\cos(10^3 t + \pi/2)}{0.0087}$$

$\therefore i(t) = \cancel{0.0084}\, e^{-0.025t} + 5000\cos(1000t + 90.001)$

$5\cos(10^3 t + \pi/2)$

$$* \int_0^\infty e^{-st}\, \cancel{(\ldots)} dt = \int$$

$f(t) = 1$

$$= \int_0^\infty e^{-st} dt = \left[\frac{e^{-st}}{-s}\right]_0^\infty$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{-0}}{-s} = 0 + \frac{1}{s} = \frac{1}{s}$$

$$= \frac{1}{\cancel{+}s} + \frac{1}{\cancel{+}s} 0 = +\frac{2}{s}$$

$$* \int_0^\infty e^{-st} \sin\omega t\, dt$$

$$= \int_0^\infty e^{-st}\left[\frac{e^{j} + e^{-j}}{2}\right] dt$$

$$= \cancel{\frac{1}{2}\int_0^\infty \frac{e^{(1-s)t}}{2}} \quad * \quad = \frac{1}{2}\left[\int_0^\infty \frac{e^{-st}\cdot e^{j\omega t}}{\cancel{2}} dt + \int_0^\infty \frac{e^{-st}\, e^{-j\omega t}}{\cancel{2}} dt\right]$$

$$= \cancel{\frac{1}{2}[} \qquad = \frac{1}{2}\left[\int_0^\infty e^{(-s+j\omega)t} dt + \int_0^\infty e^{(-s-j\omega)t} dt\right]$$

$$= \frac{1}{2}\left[\left.\frac{e^{(-s+j\omega)t}}{-s+j\omega}\right|_0^\infty + \left.\frac{e^{(-s-j\omega)t}}{-s-j\omega}\right|_0^\infty\right]$$

$$= \frac{1}{2}\left[\left[\frac{e^{\infty}}{-s+j\omega} - \frac{e^{0}}{-s+j\omega}\right] + \left[\frac{e^{-\infty}}{-s-j\omega} - \frac{e^{0}}{-s-j\omega}\right]\right]$$

$$= \frac{1}{2}\left[\frac{-1}{-s+j\omega} - \frac{1}{-s-j\omega}\right] = \frac{1}{2}\left[\frac{1}{s-j\omega} + \frac{1}{s+j\omega}\right]$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}(s-j\omega+s+j\omega)}{s^2-j^2\omega^2}\right] = \frac{s}{s^2+\omega^2}$$