Unit 5: Three Phase Transformers

Introduction:

Almost all the major power generation and distribution systems in the world today are three-phase ac systems. Since three-phase systems play such an important role in modern life, it is necessary to understand how transformers are used in them.

Transformers for three-phase circuits can be constructed in two ways. One approach is simply to take three single-phase transformers and connect them in a three-phase bank. An alternative approach is to make a three-phase transformer consisting of three sets of windings wrapped on a common core.

These two possible types of transformer construction are shown in the figures below.

The construction of a single three-phase transformer is the preferred practice today, since it is lighter, smaller, cheaper, and slightly more efficient. The older construction approach was to use three separate transformers. That approach had the advantage that each unit in the bank could be replaced individually in the event of trouble, but that does not outweigh the advantages of a combined three phase unit for most applications. However, there are still a great many installations consisting of three single-phase units in service.

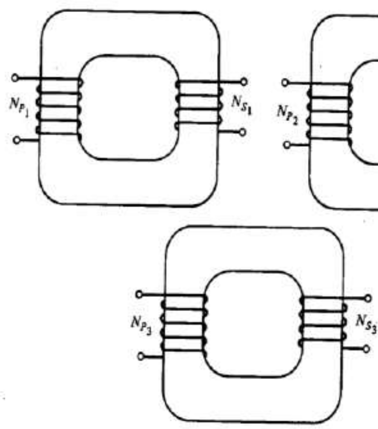


Fig: A three-phase transformer bank composed of independent transformers.

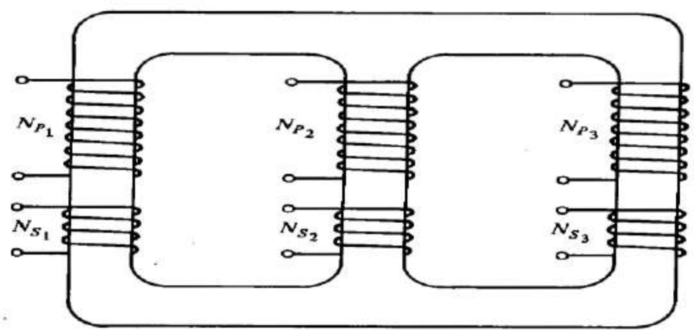


Fig: A three-phase transformer wound on a single three-legged core.

Three-Phase Transformer Connections:

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any three-phase transformer can be independently connected in either a Wye (Y) or a Delta (Δ). This gives a total of four possible connections for a three-phase transformer bank:

I. Wye(star)-Wye(star) (Y-Y)
2. Delta-Delta (Δ-Δ)
3. (star)Wye-Delta (Y -Δ)

4. Delta-Wye(star) (Δ-Y)

The key to analyzing any three-phase transformer bank is to look at a single transformer in the bank. *Any single transformer in the bank behaves exactly like the single-phase transformers already studied.* The impedance, voltage regulation, efficiency, and similar calculations for three-phase transformers are done on *a per-phase basis,* using exactly the same techniques already developed for single-phase transformers.

The advantages and disadvantages of each type of three-phase transformer connection are explained below along with the relevant connection diagrams.

WYE-WYE CONNECTION:

The Y-Y connection details of three-phase transformers are shown in the figure below.

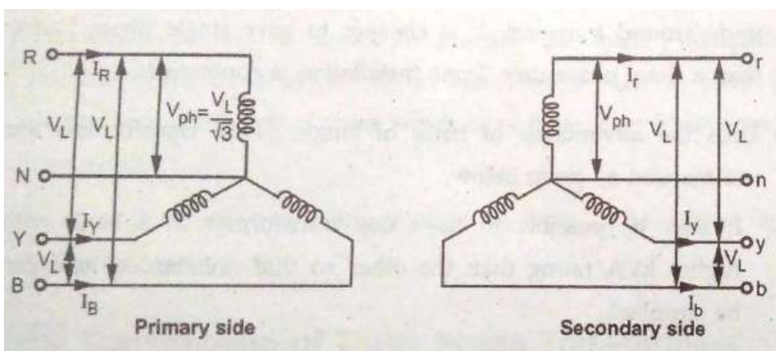


Figure (a): Star-Star(Y-Y) connection representation

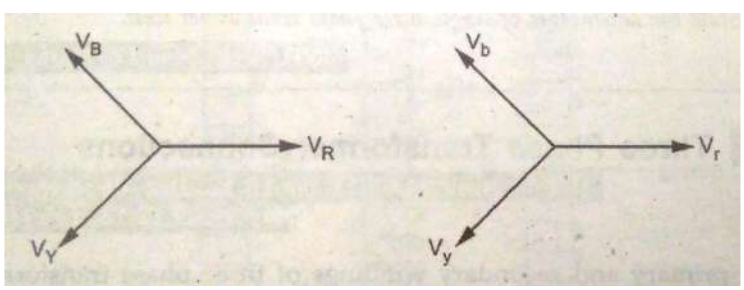


Figure (b): Star-Star(Y-Y) connection Phasor diagram

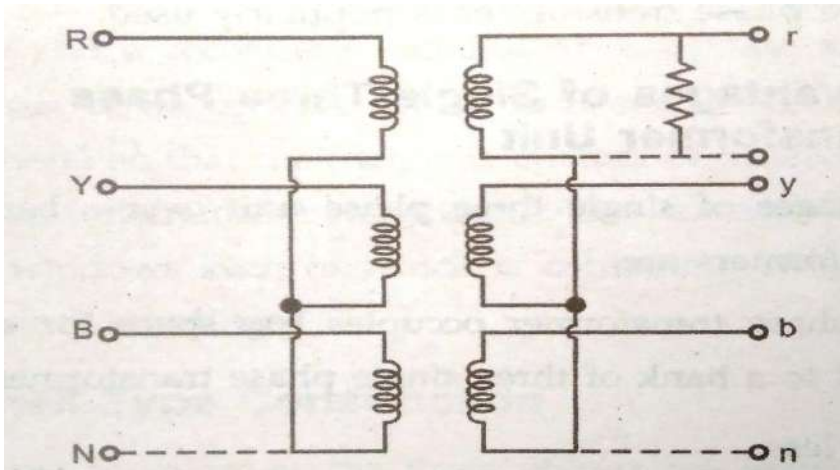


Figure (c): Star-Star(Y-Y) connection wiring diagram

In a Y-Y connection, the primary voltage on each phase of the transformer is given by $\mathbf{V_{\phi P} = V_{LP} / \sqrt{3}}$. The primary-phase voltage is related to the secondary-phase voltage by the turns ratio of the transformer. The phase voltage on the secondary is then related to the line voltage on the secondary by $\mathbf{V_{LS} = \sqrt{3} V_{\phi S}}$. Therefore, overall the voltage ratio **'a'** of the transformer is then given by:

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}V_{\phi P}}{\sqrt{3}V_{\phi S}} = a$$

Advantage/Application: This is useful and economical for low power high voltage transformers because the phase voltage is $1/\sqrt{3}$ times the line voltage. Hence the number of turns per phase and the strength of insulation required would be less.

Disadvantages:

1. If loads on the transformer circuit are unbalanced, then the voltages on the phases of the transformer can become severely unbalanced.

2. Third-harmonic voltages can be large.

When a three-phase set of voltages is applied to a Y - Y transformer, the voltages in any phase will be 120^0 apart from the voltages in any other phase. However, *the third-harmonic components of each of the three phases will be in phase with each other,* since there are three cycles in the third harmonic for each cycle of the fundamental frequency. There will always be some third-harmonic components in a transformer because of the nonlinearity of the core, and these components add up.

The result is a very large third-harmonic component of voltage on top of the 50 or 6O-Hz fundamental voltage. This third-harmonic voltage can be larger than the fundamental voltage itself.

Both the unbalance problem and the third-harmonic problem can be solved using one of the two following techniques:

1.*Solidly ground the neutrals of the transformers,* especially the primary winding's neutral. This connection permits the additive third-harmonic components to cause a current flow in the neutral instead of building up large voltages. The neutral also provides a return path for any current imbalances in the load.

2. *Add a third (tertiary) winding connected in* Δ to the transformer bank. If a third Δ connected winding is added to the transformer, then the third-harmonic components of voltage in the Δ will add up, causing a circulating current flow within the winding. This suppresses the third-harmonic components of voltage in the same manner as grounding the transformer neutrals.

The Δ connected tertiary windings need not even be brought out of the transformer case, but they often are used to supply lights and auxiliary power within the substation where it is located. The tertiary windings must be large enough to handle the circulating currents, so they are usually made about one-third the power rating of the two main windings.

One or the other of these correction techniques *must* be used any time a Y-Y transformer is installed. In practice, very few Y-Y transformers are used, since the same jobs can be done by one of the other types of three-phase transformers.

DELTA-DELTA CONNECTION:

The Δ- Δ connection details are shown in the figure below.

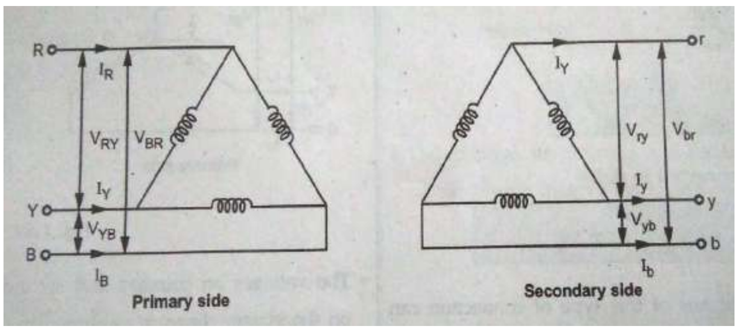


Figure (a): Delta-Delta (Δ-Δ) connection representation

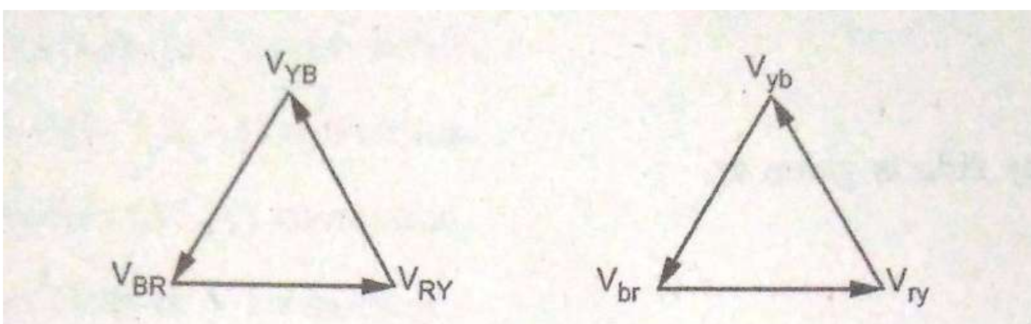


Figure (b): Delta-Delta (Δ-Δ) connection Phasor diagram

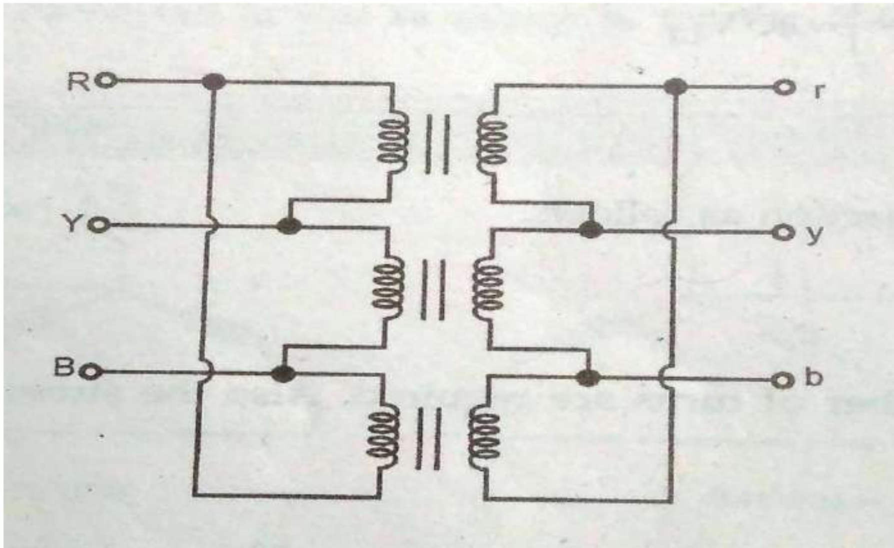


Figure (c): Delta-Delta (Δ-Δ) connection wiring diagram

In a Δ- Δ connection, $\mathbf{V_{LP}} = \mathbf{V_{\phi P}}$ and $\mathbf{V_{LS}} = \mathbf{V_{\phi S}}$, so the relationship between primary and secondary line voltages is given by:

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad \Delta\text{-}\Delta$$

Advantages/Application:

- Transformers with this configuration are economical for high power low voltage application since the number of turns required for a given line voltage are more (since line voltage is same as phase voltage)
- This transformer has no phase shift associated with it and no problems with unbalanced loads or harmonics.
- For the secondary voltage to be perfect sinusoidal the magnetizing currents must contain third harmonic components. The Delta configuration provides a closed path for the circulation of third harmonic components of current. Hence the flux remains sinusoidal thus resulting in better sinusoidal voltages.

- The phase current is lesser than the Line current (by 1/**√3**). Hence the conductor cross sectional area can be smaller thus resulting in saving of conductor material.

Disadvantages:

- Due to the nonavailability of the Neutral point this configuration is not suitable for three phase four wire systems

WYE-DELTA CONNECTION:

The Y - Δ connection details of three-phase transformers is shown in the figures below. In this connection, the primary line voltage is related to the primary phase voltage by $\mathbf{V_{LP} = \sqrt{3}V_{\phi P}}$ while the secondary line voltage is equal to the secondary phase voltage $\mathbf{V_{LS} = V_{\phi S}}$. The voltage ratio of each phase is:

$$\mathbf{V_{\phi P} / V_{\phi S} = a}$$

so the overall relationship between the line voltage on the primary side of the bank and the line voltage on the secondary side of the bank is:

$$\mathbf{V_{LP} / V_{LS} = \sqrt{3}V_{\phi P} / V_{\phi S} = \sqrt{3}\ a}$$

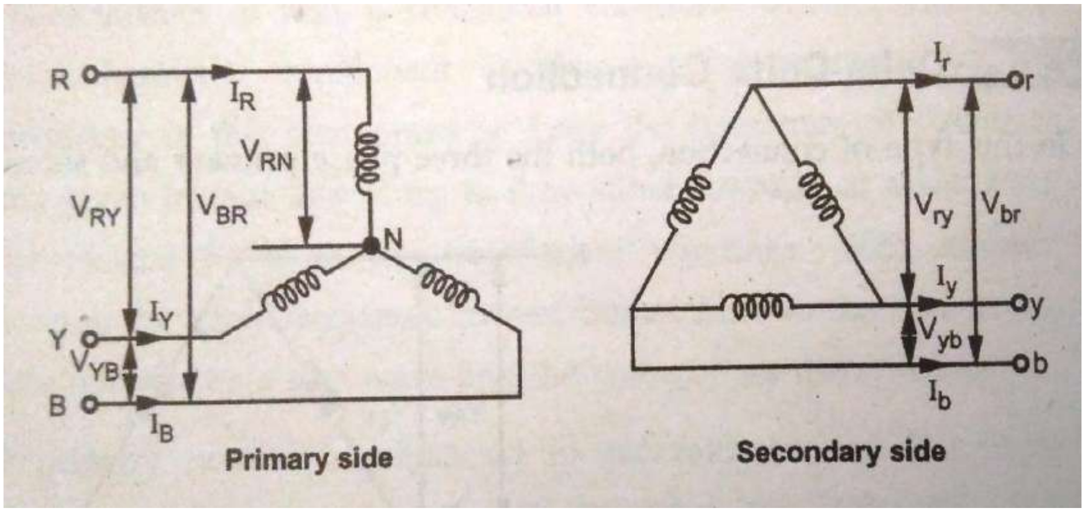


Figure (a): Wye-Delta (Y-Δ) connection representation

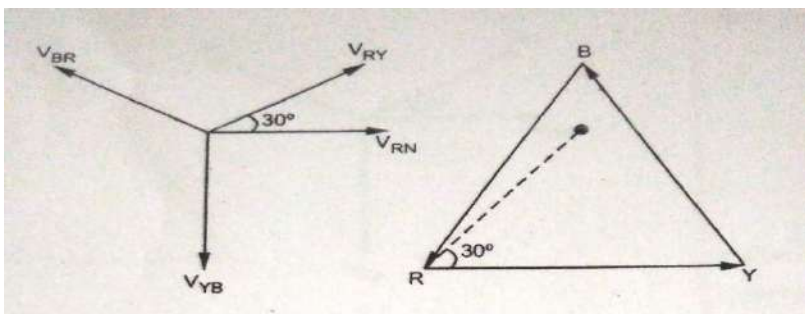


Figure (b): Wye -Delta (Y-Δ) connection Phasor diagram

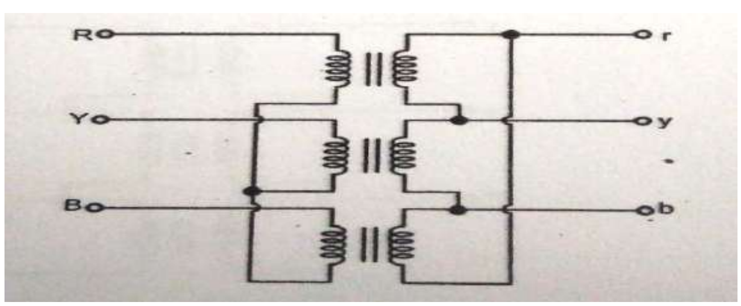


Figure (c): Wye -Delta (Y-Δ) connection wiring diagram

Advantages/Application:

- This connection is advantageous/economical for high power high voltage step ***down*** power transformers. Primary in star configuration can be used for higher voltage since line voltage is $\sqrt{3}$ times the phase voltage and thus the number of turns required per phase will be lesser for a higher line voltage. The delta side with lower line voltage (line voltage being equal to phase voltage) can be used as secondary.
- The neutral available in primary can be earthed to avoid distortion
- Hence transformers with this type of connection are used in the main receiving end of a transmission line where a step ***down*** transformer is required.
- The Y - Δ connection has no problem with third-harmonic components in its voltages, since they are consumed in a circulating current on the Δ side.
- This connection is also more stable with respect to unbalanced loads, since the Δ partially redistributes any imbalance that occurs. i.e. Load side (secondary) large unbalanced loads can be handled satisfactorily.

Disadvantages:

- This arrangement has one problem. Because of this type of connection, the secondary voltage is shifted 30^0 relative to the primary voltage of the transformer which can be further positive shift or negative shift. The fact that a phase shift has occurred can cause problems in paralleling the secondaries of two transformer banks together. The phase angles of transformer secondaries must be equal if they are to be paralleled, which means that attention must be paid to the direction of the 30^0 phase shift occurring in each transformer bank to be paralleled together.

Star/Delta (Y/D) Connection (Alternate explanation for Phase grouping):

Star connection is formed on primary side by connecting together 1 suffixed terminals with 2 suffixed terminals connected to appropriate lines. The delta is *formed by connecting c1a2, a1b2 and b1c2 with the lines connected to these junctions being labeled as a, b and c respectively as shown in Fig. (a). The phasor diagram is drawn in Fig. (b). It is seen from the phasor diagram on the delta side that the sum of voltages around delta is zero. This is a must as otherwise closed delta would mean a short circuit. It is also observed from the phasor diagram that phase a to neutral voltage (equivalent star basis) on the delta side lags by – 30° to the phase-to-neutral voltage on the star side. This is also the phase relationship between the respective line to- line voltages. This connection, therefore, is known as – 30°-connection. Or YD1 representing 1^{O} clock position. With this notation secondary Delta lags the primary star by 30^{0}.*

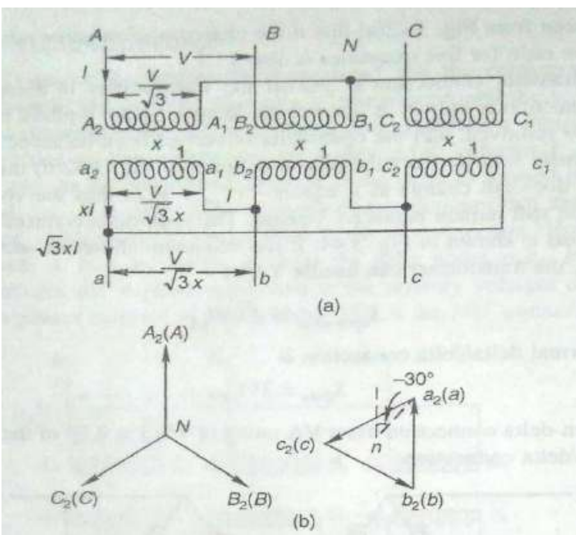


Fig: -30^{0} connection

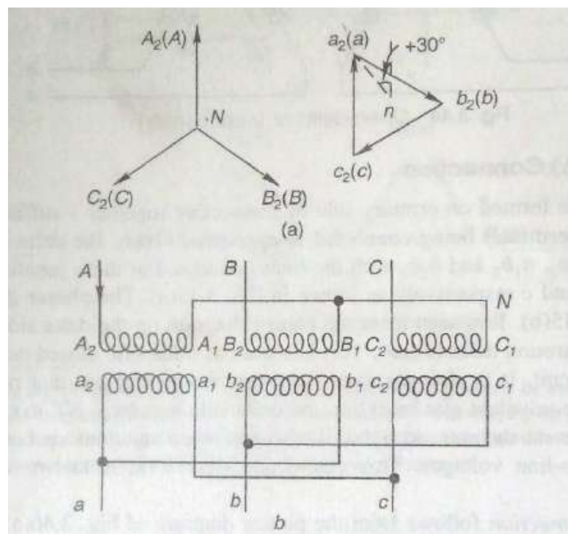


Fig: $+30^{0}$ connection

The + *30°-connection* follows from the phasor diagram of Fig. (a) above with the corresponding connection diagram shown in Fig. (b).

Delta/Star (D/Y) Connection (Alternate explanation for Phase grouping):

This connection is simply the interchange of primary and secondary roles in the star/delta connection. One just interchanges capital and small letter suffixing in *the above figures .But what was the – 30°-connection will now be the + 30°-connection and vice versa.*

DELTA-WYE CONNECTION:

Δ-Y connection details of three-phase transformers are shown in the figures below. In a Δ-Y connection, the primary line voltage is equal to the primary-phase voltage $\mathbf{V_{LP} = V_{\phi P}}$, while the secondary voltages are related by $\mathbf{V_{LS} = \sqrt{3}V_{\phi S}}$ **. T**herefore, the line-to-line voltage ratio of this transformer connection is given by :

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}V_{\phi S}}$$

$$\boxed{\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}} \qquad \Delta\text{-Y}}$$

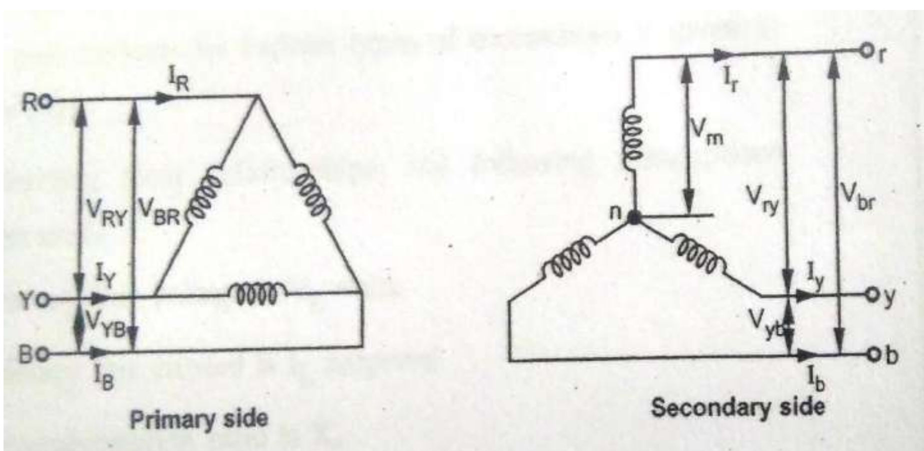


Figure (a): Delta - Wye (Δ-Y) connection representation

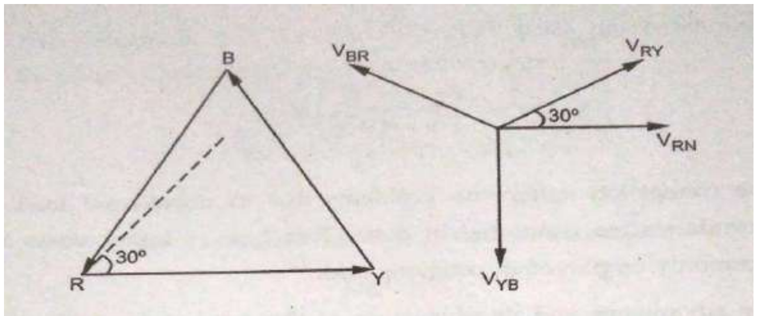


Figure (b): Delta - Wye (Δ-Y) connection Phasor diagram

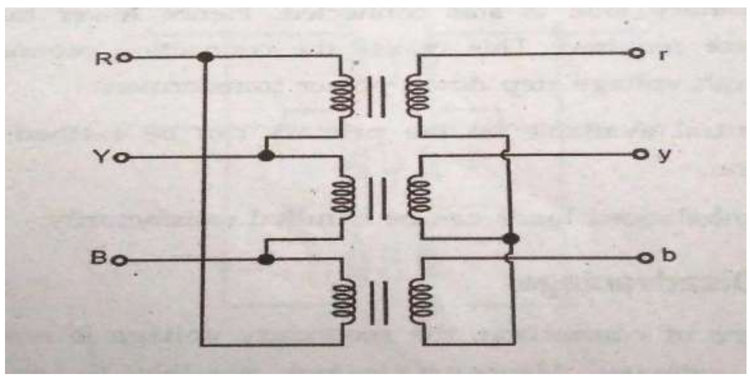


Figure (c): Delta - Wye (Δ-Y) connection wiring diagram

Advantages/Application:

- This connection is advantageous/economical for high power high voltage *step* ***up*** *power transformers.* Primary in Delta configuration can be used for lower voltage (line voltage being equal to phase voltage) And secondary in Wye (star) configuration can be used for higher voltage since line voltage is **√3** times the phase voltage and thus the number of turns required per phase will be lesser for a higher line voltage.
- Hence transformers with this type of connection are used at the starting (Generating station) end of a transmission line where a *step* ***up*** *transformer* is required.

Disadvantages:

This connection has the same disadvantages and the same phase shift as the Y - Δ transformer. The connection shown in the figure above makes the secondary voltage differ the primary voltage by 30° as in **Y -Δ.**

The Open Δ (or V-V) Connection:

In some situations a full transformer bank may not be used to accomplish three phase transformation. For example, suppose that a Δ - Δ transformer bank consisting of three separate transformers has a damaged phase which has to be removed for repair.

The resulting configuration is known as open Δ (or V-V) Connection and is shown in the figure below.

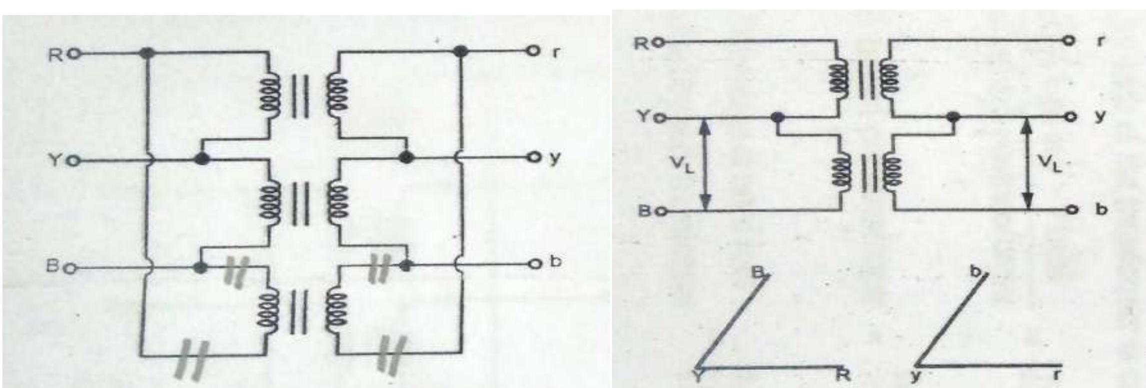


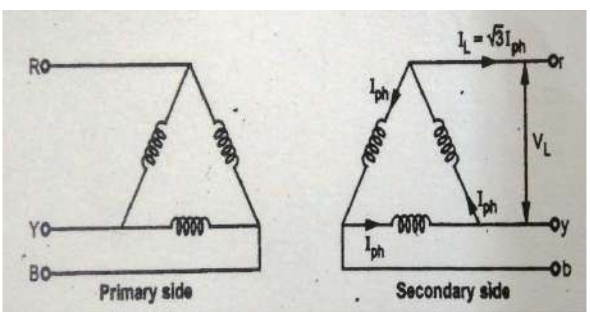
Figure: Third transformer (V_{BR}) removed from the Three transformer Bank and the corresponding Phasor diagram

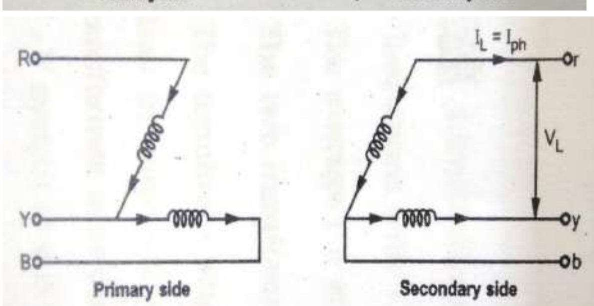
If the open Delta primary is now excited from a balanced three phase supply, then the voltage across the gap where the third transformer used to be would exactly be the same voltage that would be present if the third transformer were still there .

Thus, the open-delta connection lets a transformer bank work as a three phase transformer with only two transformers, allowing some *reduced* power flow to continue even with a damaged phase removed.

Power Delivered in Open Delta configuration:

How much apparent power can the bank supply with one of its three transformers removed? At first, it seems that it could supply two-thirds of its rated apparent power, since two-thirds of the transformers are still present. Things are not that simple. To understand what happens when a transformer is removed, let us see the figures (a) and (b) below. Figure (a) shows **Δ - Δ** connection and figure (b) shows **V-V** connection.





Fig(a) : Δ - Δ connection **Fig(a) : V- V connection**

We know that the power output from a three phase system is $\sqrt{3}\, V_L I_L \cos\phi$ where $\cos\phi$ is the power factor. Hence in figure (a) **Δ - Δ** capacity = $\sqrt{3}\, V_L I_L \cos\phi = \sqrt{3}\, V_L \sqrt{3} I_{Ph} \cos\phi$ (since $I_L = \sqrt{3} I_{Ph}$)

$$= \mathbf{3 V_L I_{Ph} \cos\phi}$$

But in figure (b) **V-V** capacity **=** $\sqrt{3}\, V_L I_L \cos\phi = \mathbf{\sqrt{3}\, V_L I_{Ph} \cos\phi}$ (since $I_L = I_{Ph}$)

*Therefore **V-V** capacity **/ Δ - Δ** capacity* **=** $\sqrt{3}\, V_L I_{Ph} \cos\varphi \,/\, 3 V_L I_{Ph} \cos\varphi = 1/\sqrt{3}$ ***= 0.577=57.7%***

Summary conclusion:

The total load carried by an open **Δ (V- V)** connection is 57.7 % of the total capacity of the **Δ- Δ** and not $2/3^{rd}$ (66.6 %) of the total capacity of the **Δ- Δ** as might be expected. Thus for example, in a **Δ- Δ** bank of three transformers each of 100kVA if one transformer is removed, then the total capacity of the resulting **V-V** bank becomes 57.7 % of 300kVA i.e. 173.2kVA and not 200kVA.

In other words in V-V configuration the resultant capacity becomes 86.6% of the rated capacity of the two transformers put together. i.e. 0.866 x 200kVA = 173.2 kVA .

The factor 0.866 is called ***utility factor***
= (operating capacity of the transformers in **V-V**) / (Available capacity of the transformers in **V-V)**

V – V Configuration is intentionally employed in the following applications:

- When the three phase load is too small to warrant the installation of full three phase transformer bank.
- When one of the three transformers in a **Δ - Δ** bank is disabled service is at a reduced capacity is adequate , till the faulty transformer is repaired and restored.
- When it is expected that in the future , the total load will increase necessitating the closing of the open **Δ.**

Illustrative examples:

Example1: The following test results were obtained for a 20 kVA,2400/240 V distribution transformer : O.C test(l.v side)240 V,1.066 A,126.6 WS.C test (h.v side) 57.5V, 8.34 A, 284 W Calculate a) Equivalent circuit parameters when referred to h.v side and draw it. b) Efficiency of the transformer at half full-load with 0.8 power factor lagging. C) Regulation at full load 0.8 power factor lagging.

(JNTU May-15)

Solution : From O.C test , V_0 = 240 V, I_0 = 1.066 A, W_0 = 126.6 W,
Measurements are on LV, side i.e. secondary hence results will give parameters referred to secondary.

∴ $\cos\emptyset_0 = \frac{W_0}{V_0 I_0} = \frac{126.6}{240 \times 1.066} = 0.4948, \Phi_0 = 60.34^o$

i) $I_C = I_0 \cos\Phi_0 = 1.066 \times 0.4948 = 0.5275A$

$$I_m = I_0 \sin\Phi_0 = 1.066 \times 0.8689 = 0.9263A$$

$$R_0' = \frac{V_0}{I_0} = \frac{240}{0.5275} = 454.97\Omega$$

$$X_0' = \frac{V_0}{I_m} = 259.095\Omega$$

$$K = \frac{V_1}{V_2} = \frac{2400}{240} = 10$$

$$R_0 = R_0' \times K^2 = 45.497k\Omega$$

$$X_0 = X_0' x K^2 = 25.91 k\Omega$$ ....Referred to h.v side

From S.C. test, V_{sc} = 57.5 V, I_{sc} = 8.34 A, W_{sc} = 284 W

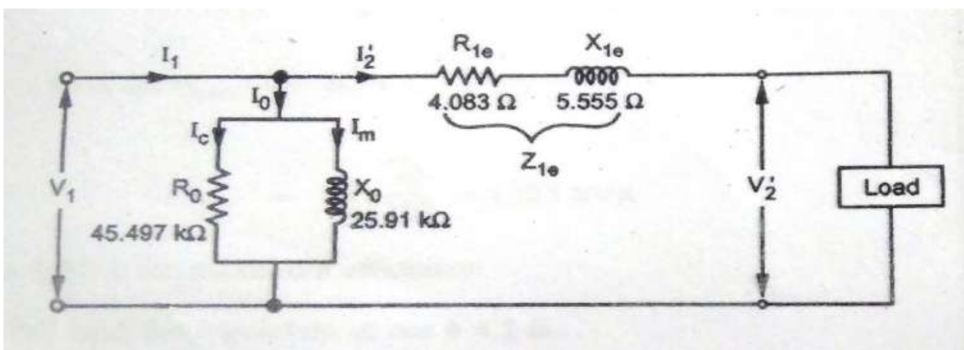
The meters are on h.v. side hence we get parameters referred to h.v. side.

$$R_{1e} = \frac{W_{sc}}{I_{le}^2} = \frac{284}{(8.34)^2} = 4.083 k\Omega$$

$$\therefore \quad Z_{1e} = \frac{V_{sc}}{I_{sc}} = 6.8944\Omega$$

$$X_{1e} = \sqrt{Z_{1e}^2 - R_{1e}^2} = 5.555\Omega$$

The equivalent circuits referred to h.v. side is shown in the figure below.



ii) From O.C test,

P_i = iron loss = 126.6 W

From S.C test P_{cu} = 284 W for I_{sc} = 8.34 A

$$I_1(FL) = \frac{VA}{V_1} = \frac{20\times10^3}{2400} = 8.34A$$

As I_1(FL) = I_{sc}, W_{sc} = P_{cu}(FL) = 284 W

$$\therefore \quad \%\eta_{HL} = \frac{nVA\cos\Phi}{nVA\cos\Phi + P_i + N^2 P_{cu}(FL)} \times 100$$ .....n = 0.5 on Half load

$$= \frac{0.5\times 20\times 10^3 \times 0.8}{0.5\times 20\times 10^3 \times 0.8 + 126.6 + [0.5^2 \times 284]}\times 100$$

= **97.589 %**

iii) I_1(FL) = 8.34 A, V_1 = 2400 V, cos Ø = 0.8

∴ $$\%R = \frac{I_1(FL)[R_{1e}\cos\Phi + X_{1e}\sin\Phi]}{V_1}\times 100$$

$$= \frac{8.34[4.083\times 0.8 + 5.55\times 0.6]}{2400}\times 100$$

= **2.293 %**

Example 2 : Two similar 200 kVA, single phase transformers gave the following results in Sumpner's test : Mains wattmeter W1 = 4 kW, Series wattmeter W2 = 6 kW at full load current. Find out individual transformer efficiencies at i) Full load at unity p.f.and ii) Half load at 0.8 p.f. lead. (JNTUK April-12)

Solution : The given values are: Rating = 200 kVA, W_1 = 4 kW, W_2 = 6 kW

W_1 = iron loss of both the transformers = 4 kW

∴ Pi = Iron loss for each transformer = $\frac{W_1}{2} = \frac{4}{2}$ = 2 kW

W_2 = Full load copper loss for both the transformers = 6 kW

∴ (P_{cu})F.L. = Full load copper loss for each transformer = $\frac{W_2}{2} = \frac{6}{2}$ = 3 kW

i) At full load,

$$\%\ \eta = \frac{VArating\cos\Phi_2}{VArating\cos\Phi_2 + P_i + (p_{cu})F.L}\times 100$$ with cos Ø2 = 1

$$= \frac{200 \times 10^3 \times 1}{200 \times 10^3 \times 1 + 2 \times 10^3 + 3 \times 10^3} \times 100$$

= **97.56 %**

ii) At half load, cos $\emptyset_2$ = 0.8 and n = $\frac{1}{2}$ = 0.5

$$\therefore \quad \% \ \eta = \frac{n \times (VArating) \times \cos\Phi_2}{n \times (VArating) \times \cos\Phi_2 + P_i + n^2 \times (P_{cu}) F.L} \times 100$$

(P_{cu})H.L. = n2 ×(P_{cu}) F.L. where n = Fraction of full load

$$\therefore \quad \% \ \eta = \frac{0.5 \times 200 \times 10^3 \times 0.8}{0.5 \times 200 \times 10^3 \times 0.8 + 2 \times 10^3 + (0.5)^2 \times 3 \times 10^3} \times 100$$

= 96.67 %

Example 3: A 200 V, 60 Hz single phase transformer has hysteresis and eddy current losses of 250 watts and 90 watts respectively. If the transformer is now energised from 230 V, 50 Hz supply. Calculate its core losses. Assume Steinmentz's constant equal to 1.6 . (JNTUK April-12)

Solution : Let P_h = Hysteresis loss and P_e = Eddy current loss

Then they are given by $P_h = K_h f B_m^x$ where x= Steinmentz's constant =1.6

and $P_e = K_e f^2 B_m^2$

$E = 4.44 f N B_m A$ where $\emptyset_m = B_m A$

$\therefore$ $B_m \propto \frac{E}{f}$ i.e. $P_h = K_h f (E/f)^{1.6}$

and $P_e = K_e f^2 (E/F)^2$

For E = 200 V, f = 60 Hz, P_h = 250 W and P_e = 90 W

$$250 = K_h \times 60 \times \left(\frac{200}{60}\right)^{1.6} \quad \text{i.e.} \quad K_h = 0.607$$

$$90 = K_e \times 60^2 \times \left(\frac{200}{60}\right)^2 \quad \text{i.e.} \quad K_e = 0.00225$$

Now E = 230 V, f = 50 Hz

$$\therefore \quad P_h = 0.607 \times 50 \times \left(\frac{230}{50}\right)^{1.6} = 348.795 \text{ W}$$

$$P_e = 0.00225 \times 50^2 \times \left(\frac{230}{50}\right)^2 = 119.025 \text{ W}$$

∴ Core loss = $P_h + P_e$ = 467.82 W at 230 V, 50 Hz

Example 4: In a test for determination of the losses of a 440V,50 Hz transformer, the total iron losses were found to be 2500 W at normal voltage and frequency. When the applied voltage and frequency were 220 V and 25 Hz, the iron losses were found to be 850W. Calculate the eddy current loss at normal voltage and frequency. (JNTU Feb-10 ,May-15)

Solution : We have the data: V_1 = 440 V, f_1= 50 Hz, V_2 = 220 V, f_2 = 25 Hz

$$\therefore \quad \frac{V_1}{f_1} = 8.8 \text{ and } \frac{V_2}{f_2} = 8.8$$

Thus V/f is constant hence flux density B_m remains constant.

$$\therefore \quad P_h = A f \quad \text{and} \quad P_e = B f^2$$

$$\therefore \quad P_i = P_h + P_e = A f + B f^2$$

∴In test - 1: 2500 = A x 50 + B x 2500 ....(1)

And in test - 2 850 = A x 25 + B x 625 ....(2)

Solving we get A = 18 and B = 0.64

Thus eddy current loss a normal voltage and frequency is,

$$P_e = B f^2 = 0.64 \times (50)^2 = \mathbf{1600\ W}$$

Example 5: Two single phase transformer with equal turns have impendance of (0.5 +j3) ohm and (0.6 +j10) ohm with respect to the secondary.If they operate in parallel, determine how they will share a total load of 100 kW at p.f. 0.8 lagging ?

(JNTU April-04, Nov-04)

Solution : We have the data: $Z_1 = 0.5 + j3\ \Omega$, $Z_2 = 0.6 + j10\ \Omega$,

Total load = 100 kW

p.f. of load = 0.8 lag.

Cos Ø = 0.8, Ø = $\cos^{-1} 0.8 = 36.86^0$ lag

kVA of load = $\frac{100}{0.8}$ = 1245

Hence Q = $125\underline{/-36.86^0}$ kVA

Load shared by transformer 1 = $Q\left(\frac{Z_2}{Z_1 + Z_2}\right)$

$$= [125\ \underline{/-36.86^0}]\left[\frac{0.6 + j10}{(0.5 + j3) + (0.6 + j10)}\right]$$

$$= \frac{(125/\_-36.860)(10.017/\_86.56^0)}{1.1 + j13}$$

$$= \frac{1252.125/\_49.7^0}{13.046/\_85.16^0} = \mathbf{95.97\ \underline{/-35.46^0}\ kVA}$$

p.f. = cos 35.46^0 = 0.8145 lag

Load shared by transformer 2 = $Q\left(\frac{Z_2}{Z_1 + Z_2}\right)$

$$= [125 \underline{/-36.86^0}]\left[\frac{0.5+j3}{(0.5+j3)+(0.6+j10)}\right]$$

$$= \frac{(125/\_-36.86^0)(3.041/\_80.53^0)}{13.046<85.16^0}$$

$$= \frac{380.125/\_43.67^0}{13.046/\_85.16^0}$$ = **29.13 $\underline{/-41.49^0}$ kVA**

Example 6: Two single phase transformer A and B of equal voltage ratio are running in parrallel and supplying a load of 1000 A at 0.8 p.f. lag. The equivalent impedance of the two transformers are (2+j3)and (2.5+j5) ohms respectively. Calculate the current supplied by each transformer and the ratio of the kW output of the two transformer.

Solution : For transformer A, Z_A = 2+ j 3 Ω

For transformer B, Z_B = 2.5+ j 5 Ω

Current supplied by transformer A,

$$I_A = \frac{I.Z_B}{Z_A+Z_B} \quad ....(1)$$

Current supplied by transformer B,

$$I_B = \frac{I.Z_A}{Z_A+Z_B} \quad ....(2)$$

Taking ratio of equations (1) and (2) we have,

$$\frac{I_A}{I_B} = \frac{Z_B}{Z_A} = \frac{2.5+j5}{2+j3} = \frac{5.5901/\_63.43^0}{3060555/\_56.30^0}$$

$= 1.5504 \underline{/7.13^0}$

∴ $\frac{I_A}{I_B}$ = 1.54+j 0.1924 I_A = (1.54 + j 0.1924) I_B

Total current I = 1000 A and p.f. of total current = 0.8 lag

∴ $\cos\phi = 0.8,\ \phi = \cos^{-1} 0.8 = 36.86^0$

∴ $I = 1000 \underline{/-36.86^0}$ A = (800-j 600) A

We have, $I = I_A + I_B$

∴ 800 - J 600 = [1.54 + j 0.1924] $I_B + I_B$

= [2.54 + j 0.1924] I_B

∴ $I_B = \dfrac{800 - j600}{2.54 + j0.1924} = \dfrac{1000/\_-36.86^0}{2.5472/\_-4.33^0}$

= 392.58 $\underline{/-41.19^0}$ A

∴ **I_B = 392.58 $\underline{/-41.19^0}$ A = 295.42 -j258.53 A**

Now, I_A = (1.54+ j 0.1924) I_B

= [1.5519 <7.12^0][392.58<-41.19^0]

∴ **I_A = 609.24 $\underline{/-34.07^0}$ A = 504.66 - j 341.29 A**

The ratio of kW outputs is nothing but the ratio of inphase components of the two currents.

$$\frac{output of Transformer A}{output of Transformer B} = \frac{504.66}{295.42}$$ **= 1.7**

Example 7: Two transformers A and B are connected in parallel to a load of (2+ j1.5) Ω Their impedance in secondary terms are Z_A = (0.15+j0.5) Ω and Z_B = (0.1+j0.6) Ω.Their no load terminal voltages are $E_A = 207 \underline{/0^0}$ V, $E_B = 205 \underline{/0^0}$ volts. Find the power output and power factor of each transformer.

Solution : We have the data : Z_L = (2 + j1.5)Ω , Z_A = (0.15 + j0.5)Ω , Z_B = (0.1 + j0.6) Ω , $E_A = 207 \underline{/0^0}$ volts and $E_B = 205 \underline{/0^0}$ volts. Using the formulae for I_A and I_B

$$I_A = \frac{E_A Z_B + Z_L(E_A - E_B)}{Z_A Z_B + Z_L(Z_A - Z_B)}$$

And $$I_B = \frac{E_B Z_A - Z_L(E_A - E_B)}{Z_A Z_B + Z_L(Z_A - Z_B)}$$

$$I_A = \frac{[207\angle 0^0][0.1 + j0.6] + (2 + j1.5)[207\angle 0^0 - 205\angle 0^0]}{(0.15 + j0.5)(0.1 + j0.6) + (2 + j1.5)[(0.15 + j0.5) + (0.1 + j0.6)]}$$

Simplifying we get $I_A = (42.196 \angle -38.84^0)$ A =(32.866-j26.463) A

and similarly we get $I_B = \dfrac{E_B Z_A - Z_L(E_A - E_B)}{Z_A Z_B + Z_L(Z_A - Z_B)}$

$$= \frac{[205\angle 0^0][0.15 + j0.5] + (2 + j1.5)[207\angle 0^0 - 205\angle 0^0]}{(0.15 + j0.5)(0.1 + j0.6) + (2 + j1.5)[(0.15 + j0.5) + (0.1 + j0.6)]}$$

Solving, $I_B = (33.5534 \angle -42.89^0)$ A

= (24.5832 - j 22.8362) A

Now total current is given by,

$I_L = I_A + I_B$

= (32.866 - j 26.463) + (24.5832 - j 22.8362)

= (57.4492 - j 49.2992) A

= $75.70 \angle -40.63^0$ A

The load voltage, $V_L = I_L Z_L = (75.70 \angle -40.63^0)(2+j\,1.5)$

= $(75.70 \angle -40.63^0)(2.5 \angle 36.86^0)$

= $189.25 \angle -3.770$ volts

The angle between V_L and I_A can be calculated as,

$\phi_A = (-38.84^0) - (-3.77^0) = -35.07^0$

∴ p.f. = cos ϕ_A = cos (35.07)

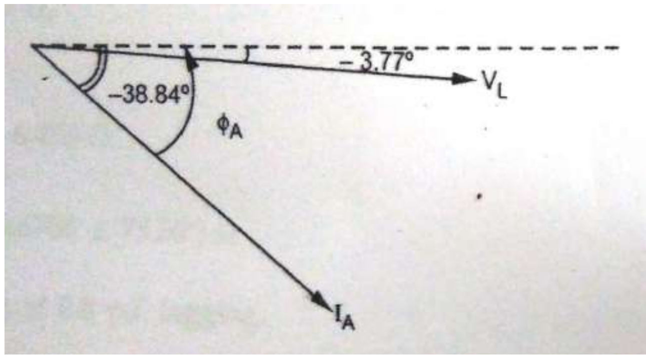
= 0.8184 (lagging)

The angle between V_L and I_B can be Calculated as,

$\phi_B = (-42.89^0) - (-3.77^0) = -39.12^0$

p.f. = cos ϕ_B = cos (39.12)

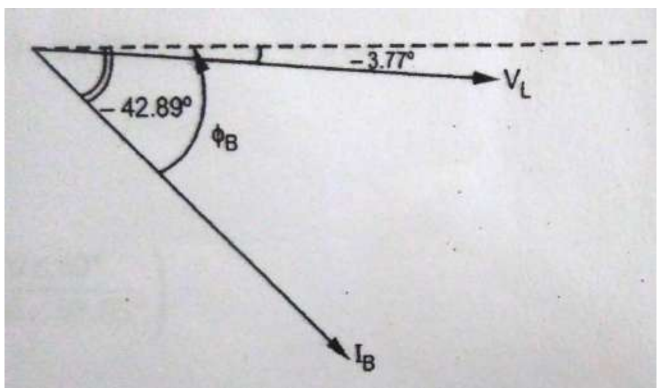
= 0.7758 (lagging



Power output of transformer A = V_L I_A cos $Ø_A$

= 189.25×42.196×0.8184

= 6535.40 W = 6.5354 Kw



Power output of transformer B = V_L I_B cos $Ø_B$

= (189.25) (33.5534) (0.7758)

= 4926.31 W = 49263 kW

Illustrative Examples on three phase transformers:

Example 1: An ideal 3-Ø step down transformer connected in delta/star delivers power to a balanced 3 - Ø load of 120 Kva at 0.8 pf. The input line voltage is 11 Kv and the turn's ratio of transformer (Phase to Phase) is 10. Determine the line voltage, line currents, and phase voltages, phase currents on both primary and secondary sides.

Solution:

∴ $K = \frac{N_1}{N_2} = 10 = \frac{V_{ph1}}{V_{ph2}}$ and $\mathbf{V_{ph1} = 11\ Kv}$

∴ $V_{ph2} = \frac{V_{ph1}}{10} = \frac{11 \times 10^3}{10}$ = **1100 V**

∴ $V_{L2} = \sqrt{3}\ V_{ph2}$ = **1.9052 kV**

Load $VA = \sqrt{3}\ V_{L2}\ I_{L2}$ and is given as 120kVA

i.e. $I_{L2} = \frac{120 \times 10^3}{\sqrt{3} \times 1.9052 \times 10^3}$

∴ I_{L2} = **36.36 A**

∴ $I_{ph2} = I_{L2}$ = **36.36 A**

$K = \frac{I_{ph2}}{I_{ph1}} = 10$ i.e. $I_{ph1} = \frac{36.36}{10}$ = **3.636 A**

∴ $I_{L1} = \sqrt{3}\ I_{ph1} = \sqrt{3} \times 3.636$ = **6.298 A**

Example 2: A bank of three single phase transformers has its h.v. terminals connected to 3 wire, 3-phase, 11kV system. It's l.v. terminals are connected to a 3 wire, 3-phase load rated at 1500 kVA , 2200 V. specify the voltage, current and kVA ratings of each transformer for both h.v

and l.v windings for the following connections.

i) Y -Δ ii) Δ - Y iii) Y - Y

Solution:

The load is 1500 kVA hence the rating of each transformer is (1500/3) = ***500 kVA*** *and is same for all configurations. Since input is 11kV and output is 2200V, in all configurations the line voltages are also same on both primary side and secondary side. . i.e. V_{L1}=11kV and V_{L2} = 2,200 V. The voltage and current ratings are specified on Phase basis only and hence lets us find out $\mathbf{V_{Ph}}$ and $\mathbf{I_{Ph}}$ on both HV side and LV side for the given four configurations.*

i) 11 kV Y and 2200 V Δ

$$\mathbf{V_{ph1}} = \frac{11\times10^3}{\sqrt{3}} = \mathbf{6350.8529\ V}$$

$V_{ph1}\ I_{ph1} = 500 \times 10^3$ i.e. $\mathbf{I_{ph1} = 78.729\ A}$

V_{ph2} = **2200 V**

$V_{ph2}\ I_{ph2} = 500 \times 10^3$ i.e. $\mathbf{I_{ph2} = 227.2727\ A}$

ii) 11 kV Δ and 2200 V Y

$$\mathbf{V_{ph1}} = 11\text{ kV} \text{ and } \mathbf{V_{ph2}} = \frac{2200}{\sqrt{3}} = \mathbf{1270.1705\ V}$$

$$\therefore \quad \mathbf{I_{ph1}} = \frac{500\times10^3}{11\times10^3} = \mathbf{45.45\ A}$$

$$\therefore \quad \mathbf{I_{ph2}} = \frac{500\times10^3}{1270.1705} = \mathbf{393.6479\ A}$$

iii) 11 kV Y and 2200 V Y

$$\mathbf{V_{ph1}} = \frac{11\times10^3}{\sqrt{3}} = \mathbf{6350.8529\ V},\ \mathbf{V_{ph2}} = \frac{2200}{\sqrt{3}} = \mathbf{1270.1705\ V}$$

$$\therefore \quad \mathbf{I_{ph1}} = \frac{500\times10^3}{6350.8529} = \mathbf{78.729\ A}$$

$$\therefore \quad \mathbf{I_{ph2}} = \frac{1500\times10^3}{1270.1705} = \mathbf{393.6479\ A}$$

Example 3 : A 3-Ø, 1200 kVA , 6.6/1.1kV transformer has Delta/Star connection. The per phase resistance is 2 Ω and 0.03 Ω on primary and secondary respectively. Calculate the efficiency on full load at 0.9 p.f. lagging, if iron losses are 20 kW.

Solution : V_{L1} = 6.6 kV, V_{L2} = 1.1 kV, $V_{ph1} = V_{L1}$, $V_{ph2} = V_{L2}/\sqrt{3}$

$$I_1(FL) = \frac{1200\times10^3}{\sqrt{3}VL1} = \frac{1200\times10^3}{\sqrt{3}\times6.6\times10^3} = 104.9727\text{ A (line value)}$$

$$\therefore \quad I_1\text{ (Ph)} = \frac{I_1(FL)}{\sqrt{3}} = 60.606\text{ A}$$

$$K = (V_{Ph1}/V_{Ph2}) = (6.6 \times 10^3)/(1.1 \times 10^3/\sqrt{3}) = 10.393$$

∴ $R_{1e} = R_1 + R'_2 = R_1 + K^2R_2 = 2 + (10.393)^2(0.03)$ x = 5.24 Ω

∴ $P_{cu}(FL) = 3\times [I_1(ph)]^2 \times R_{1e} = 3 \times (60.606)^2 \times 5.24 = 57740.9313$ W

∴ $$\%\eta_{FL} = \frac{VA\cos\Phi}{VA\cos\Phi + P_i + P_{cu}(FL)} \times 100$$

$$= \frac{1200\times 10^3 \times 0.9}{1200\times 10^3 \times 0.9 + 20\times 10^3 + 57740.9313}\times 100 = \mathbf{93.285\%}$$

Example 4: A 5000 kVA, 3 phase transformer 6.6 /33 kV, Δ/ Y has a no load loss of 15 kW and full load of 50 kW. The impedance drop at full load is 7%. Calculate the primary voltage when a load of 3200 kW at 0.8 p.f. is delivered at 33 kV.

Solution: Secondary is star connected with V_{L2} = 33 kV.

∴ $$I_{L2} = \frac{VA}{\sqrt{3}V_{L2}} = \frac{5000\times 10^3}{\sqrt{3}\times 33\times 10^3} = 87.4773 \text{ A} = I_{ph2}$$

Impedance drop per phase = 7 % of $V_{Ph2} = \frac{7}{100} \times \frac{33\times 10^3}{\sqrt{3}}$ = 1333.6791 V

∴ $$Z_{2e} = \frac{1333.6791}{I_{ph2}} = \frac{1333.6791}{87.4773} = 15.2459\ \Omega/\text{ph}$$

$P_{cu}(FL)$ = Total loss - No load loss = 50 -15 = 35 kW

But, $P_{cu}(FL) = 3 \times I_{2ph}^2 R_{2e}$ i.e. $R_{2e} = \frac{35\times 10^3}{3\times (87.4773)^2}$ = 1.524 Ω

∴ $$X_{2e} = \sqrt{Z_{2e}^2 - R_{2e}^2} = 15.169\ \Omega/\text{ph}$$

When load is P_2 = 3200 kW, cos Ø = 0.8, V_{L2} = 33 kV

I_{L2} = $\frac{P_2}{\sqrt{3}V_{L2}\cos\Phi} = \frac{3200\times10^3}{\sqrt{3}\times33\times10^3\times0.8}$ = 69.98 A = I_{2ph}

∴ % R = $\frac{I_{2ph}[R_{2e}\cos\Phi + X_{2e}\sin\Phi]}{V_{2ph}}$ × 100

= $\frac{69.98[1.524\times0.8+15.169\times0.6]}{(33\times10^3/\sqrt{3}}$ × 100 = 3.79 %

Thus primary voltage must be increased by 3.79 % to maintain 33 kV at the secondary.

∴ V_1 = 6.6+3.79 % of 6.6 = **6.8501 kV**

Example 5 : A 500 kVA, 3-phase, 50 Hz transformer has a voltage ratio (line voltage) of 33/11 kV and is delta/star connected. The resistances per phase are : high voltage 35 Ω, low voltage 0.876 Ω and the iron loss is 3050 W. Calculate the value of efficiency at full load and one-half of full load with 0.8 lagging power factor.

Solution : Since primary is Delta **:** V_{L1} = V_{ph1} = 33 kV

Since Secondary is Star: V_{L2} = 11 kV, V_{ph2} = $\frac{11}{\sqrt{3}}$ = 6.35 kV

∴ K = $\frac{V_{ph1}}{V_{ph2}} = \frac{33}{6.35}$ = 5.1975

R_1 = 35 Ω, R_2 = 0.876 Ω

∴ R_{1e} = R_1 + R'_2 = R_1 + K^2R_2 = 58.6643 Ω (R'₂ = R₂ referred to primary)

I_1(FL) = $\frac{VA}{\sqrt{3}V_{L1}} = \frac{500\times10^3}{\sqrt{3}\times33\times10^3}$ = 8.7477 A Therefore : I_{1ph}(FL)= $\frac{I_1(FL)}{\sqrt{3}}$

∴ $P_{cu}(FL) = 3\times I_{1ph}^{2}(FL) \times R_{1e}$ and $I_{1ph} = \frac{I_1(FL)}{\sqrt{3}}$

$$= 3 \times \left(\frac{8.7477}{\sqrt{3}}\right)^2 \times 58.6643 = 4489.1245\ W$$

∴ $$\%\eta_{FL} = \frac{VA\cos\Phi}{VA\cos\Phi + P_i + P_{cu}(FL)} \times 100$$

$$= \frac{500\times10^3\times0.8}{500\times10^3\times0.8+3050+4489.12} \times 100$$

= **98.15 %**

$$\%\eta_{HL} = \frac{0.5\times VA\cos\Phi}{0.5\times VA\cos\Phi + P_i + [(0.5)^2\times P_{cu}(FL)]} \times 100$$

$$= \frac{0.5\times VA\cos\Phi}{0.5\times VA\cos\Phi + 3050 + [(0.5)^2\times 4489.12]} \times 100$$ = **97.956 %**