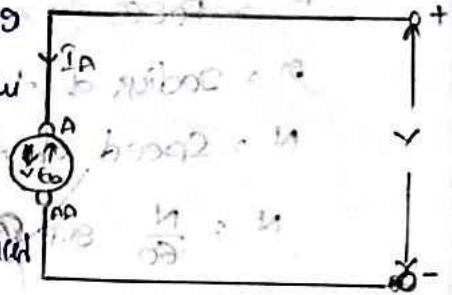


Torque & Back-emf equations of D.C. motor:Back-emf:

When the armature of a D.C. motor rotates, the conductors also rotate in the magnetic field. According to the Faraday's law of electro magnetic induction e.m.f is induced in the conductors whose direction as found by Fleming's right hand rule is in opposition to the applied voltage as shown in above fig. because of its opposition direction it's referred to as Back emf  $E_b$ . Oppositely  $v$  has to drive  $I_a$  against the opposing E.M.F  $E_b$ .



$$I_a = \frac{\text{Net voltage}}{\text{Resistance}} = \frac{v - E_b}{R_a}$$

Where

$R_a = \text{Armature Resistance}$

Induced emf (or) Back emf

$$\therefore E_b = \frac{\phi Z N}{60} \times \frac{P}{A} \text{ Volts}$$

Back emf depends upon armature speed. If speed is high  $E_b$  is high. Hence  $I_a$  is small. If speed is less, then  $E_b$  is less. Hence  $I_a$  is more current (or) high current flows to develop torque.

Torque:

Torque is defined as the twisting moment of a force about an axis.

Torque is the product of the force and the radius at which this force acts

$$\text{Torque } T = \text{Force} \times \text{radius} \quad \text{N.}$$

# Torque equation of D.C. motor

Let

$T_g$  = Armature (or) Gross Torque (N-m)

= Force  $\times$  radius

$F$  = Force in Newtons

$r$  = radius of the armature in m

$N$  = Speed of the armature in R.P.M.

$$N = \frac{N}{60} \text{ R.P.M.}$$

$$\text{Work done / revolution} = F \times 2\pi r \times N \text{ m}$$

$$\text{Work done / s} = F \times 2\pi r \times \frac{N}{60}$$

$$\text{W.D.} = \frac{2\pi N}{60} \times (F \times r) = \frac{2\pi N T_g}{60} \text{ N-m/s (or) J/s (or) Watts} \quad \text{--- (1)}$$

\* The voltage equation of D.C. motor is given by

$$V = E_b + I_a R_a$$

Multiplying both sides on  $I_a$

$$V I_a = E_b I_a + I_a^2 R_a$$

$$\left\{ \begin{matrix} \text{Electrical} \\ \text{I.P.} \end{matrix} \right\} = \left\{ \begin{matrix} \text{Electrical power (or)} \\ \text{mechanical power developed} \end{matrix} \right\} + \left\{ \begin{matrix} \text{Armature} \\ \text{Cu. losses} \end{matrix} \right\}$$

$$\text{Mechanical power developed} = E_b I_a \quad \text{--- (2)}$$

$\therefore$  Equating equation (1) & (2) we get

$$\frac{2\pi N T_g}{60} = E_b I_a + \frac{2\pi N}{60} \times \frac{P}{A} \times I_a$$

$$T_g = \frac{1}{2\pi} \phi Z I_a \times \frac{P}{A} \text{ N-m}$$

$$T_g = 0.159 \phi Z I_a \times \frac{P}{A} \text{ N-m}$$

$\phi$  = Flux per pole in wb  
 $P$  = No. of poles  
 $Z$  = No. of armature conductors  
 $A$  = No. of armature paths  
 $I_a$  = Armature current.

# Aerometer Reaction :-

→ The effect of aerometer stem on the distribution of pole flux in the air gap is known as aerometer reaction.

i.e. Aerometer reaction takes place in air gap

Condition - 1

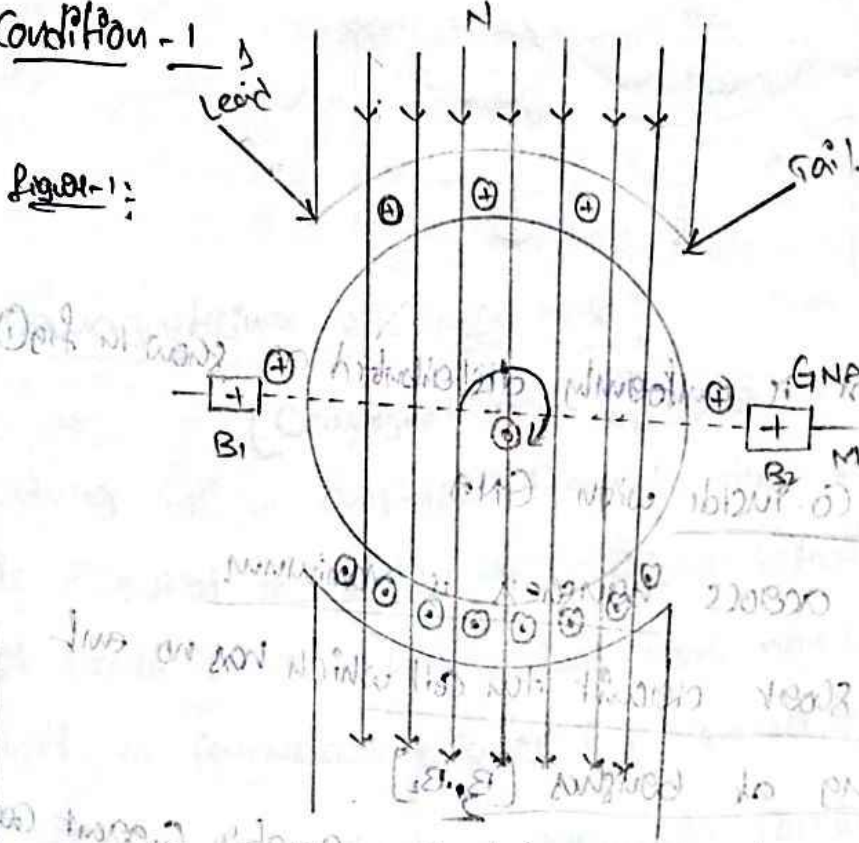


Figure-1:

→ MNA :- Magnetic neutral axis (MNA) plane

→ GNA :- Geometrical neutral axis (GNA) plane

→ Condition 2 :-

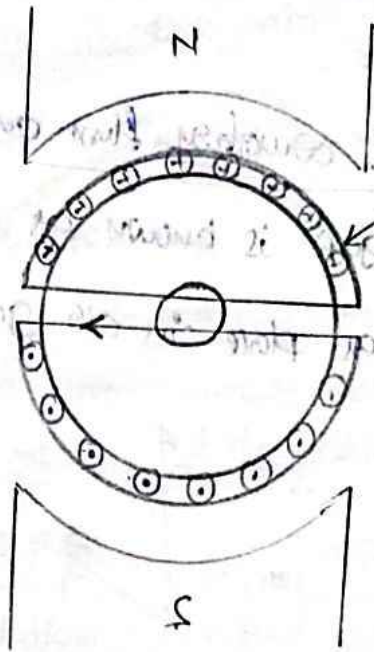
→ When the aerometer is not loaded no current passes

through the aerometer and no flux is produced by aerometer

i.e. there is no aerometer reaction

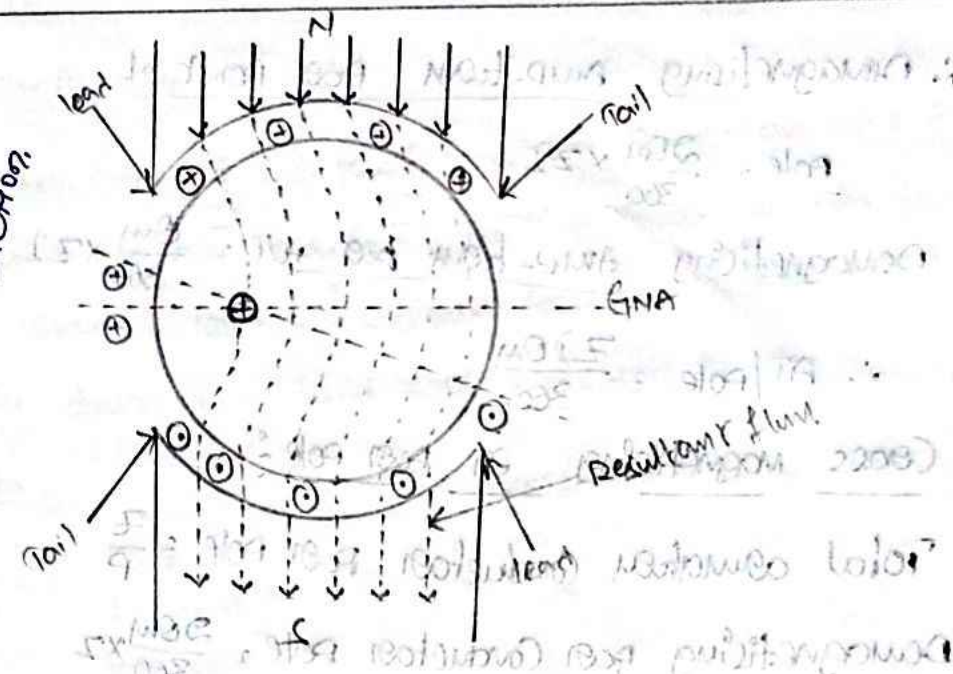
→ The condition under aerometer reaction is

Fig 01-2



1. The pole flux is uniformly distributed as shown in Fig (1)
2. The MNA coincide with GNA
3. The voltage across brushes is maximum
4. The brush short circuit the coil which has no emf
5. No sparking at brushes ( $B_1, B_2$ )
6. When the armature is loaded the armature current carries conductors and another flux is produced by armature flux
7. The armature flux is acting in the same direction with lagging pole flux at trailing pole tips and acting in opposite direction at leading pole tips
8. The Fig (3) shows the resultant flux direction action. The trailing pole is strengthened and leading poles are weakened

Figure 1  
 Fig shows Resultant flux structure reaction



Demagnetising AT. per pole

[Demagnetising AT per pole are neutralized by adding extra ampere turn to the main field winding] it is essential to calculate the number before proceeding further it should be remembered that the no. of turn are equal to half of conductors because two conductors constitute one turn.

Let  $Z$  = Total no. of Armature conductors

$I$  = Current in each conductor =  $\frac{I_a}{Z}$  per simple wave winding

$I_a$  per simple lap winding

Total no. of conductors in angle AOC & BOD is

$$\frac{4\pi m}{360} \times Z$$

where  $\pi m$  = Perimeter leading mechanical: (deg)

Geometrical (deg) Angular degrees

As two conductors constitute one turn

Total no. of turns in the angle =  $\frac{2\pi m}{360} \times Z$

∴ Demagnetising Amp-turns per pole of

$$\text{pole} = \frac{20m}{360} \times Z_2$$

Demagnetising Amp-turns per pole =  $\frac{20m}{360} \times Z_2$

$$\therefore AT/\text{pole} = \frac{Z_2 20m}{360}$$

Cross magnetising AT per pole =

Total armature conductors per pole =  $\frac{Z}{P}$

Demagnetising per conductor pole =  $\frac{20m}{360} \times 2$

Cross magnetising conductors per pole =  $\frac{Z}{P} \times 2 \times \frac{20m}{360}$

$$= Z \left[ \frac{1}{P} - \frac{20m}{360} \right] \quad \left( Z \left[ \frac{1}{P} - \frac{20m}{360} \right] \right)$$

Cross-magnetising Amp conductors per pole

$$= Z \left[ \frac{1}{P} - \frac{20m}{360} \right]$$

Cross-magnetising Amp turns per pole

$$= Z \left[ \frac{1}{2P} - \frac{20m}{360} \right]$$

ATC per pole

$$= Z \left[ \frac{1}{2P} - \frac{20m}{360} \right]$$

NOTE =

For neutralising demagnetising effects of an armature on either side of armature there may be put on each pole

∴ No. of extra turns per pole

$$= \frac{AT_d}{I_a} \rightarrow \text{for shunt generator}$$

$$= \frac{AT_d}{I_a} \rightarrow \text{for series generator}$$

Here,  $I_a$  is armature current

$I_a$  is armature current

$d$  is demagnetising

Problem on Afd, Afc per pole:

Q1 A 8 pole lap connected D.C. shunt generator driven on old 2400 rpm. The armature has 1408 conductors & 160 commutator segments. If the brushes are given a lead of 4 segments from the no-load neutral axis estimate the demagnetising and cross magnetising Afd/pole.

Sol Given data:

$P = 8$   
 $T = 2400$   
 $V = 500$

$Z = 1408$   
 Commutator segments = 160  
 Brush lead = 4 segments

$I_a = \frac{V}{P} = \frac{500}{8} = 62.5$

$\theta_m = \frac{4 \times 360}{160} = 9^\circ$

$A_{fd}/\text{pole} = \frac{Z \theta_m}{360} = \frac{1408 \times 30 \times 9}{360} = 1056$

$A_{fc}/\text{pole} = Z \left[ \frac{1}{2P} - \frac{\theta_m}{360} \right] = 1408 \times 30 \left[ \frac{1}{2 \times 8} - \frac{9}{360} \right] = 1584$

Q2 A 22.38 kW, 440V, 4 pole wave wound D.C. shunt motor has 840 armature conductors and 140 commutator segments, its full load efficiency is 88% and the shunt field current is 8 A. If the brushes are shifted back used through 1/5 segments from the geometrical neutral axis. Find the demagnetising and distorting amp-turn per pole.

Sol Given data:

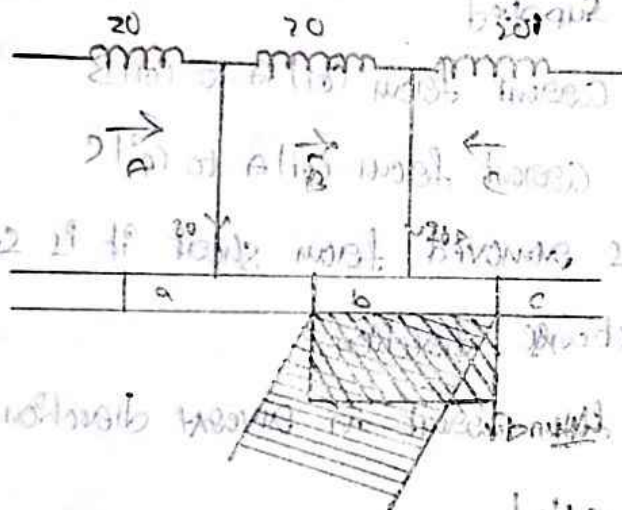
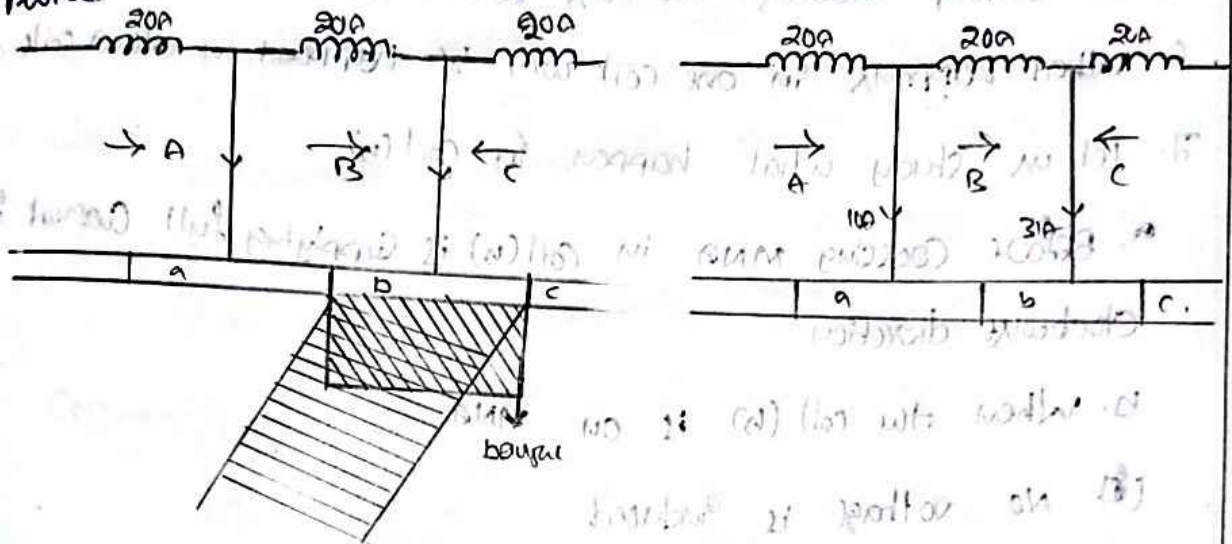
Motor output = 22.38 kW  
 Voltage (V) = 440V  
 $P = 4$   
 $Z = 840$   
 Commutator segments = 140  
 $I_{sh} = 8$  A





## Commutation:

Commutation means reverse of current thru current in the commutator coil reverse immediately after crossing MNA. The analysis gives the performance of commutator during commutation period.



1. During the commutation period the following conditions
2. All the coils side under each pole carry same current in the same direction
3. Each coil is connected across adjacent segment the segments are insulated from the each other the poles and brushes are in same position

4. The coils & segments rotate at a speed (this happens) does two things

- a. collect current from coils through segments
- b. It gives current to coils which are on MNA
- c. No voltage induces in coils which are on MNA
- d. what happens in one coil will be happen in other coil also

7. Let us study what happen in coil (b)

- a. Before crossing MNA in coil (b) is supplying full current in clockwise direction
- b. When the coil (b) is on MNA
  - (i) No voltage is induced
  - (ii) No current is supplied
  - (iii) The brush take current from coil A to coil B
  - (iv) The brush take current from coil A to coil C
- c. After the coil is removed from sheet it is supplying current in anticlockwise direction

8. The coil B gives full current in reverse direction after crossing MNA. It is called

- i. Smooth (or) Sparkless Commutation
- ii. Linear Commutation
- iii. Ideal (or) Swiftness Commutation

## Reactance voltage :-

$E = \text{Self Inductance} \times \text{rate of change of current}$  self-inductance  
 $= L$

Current in a coil  $= I - (I) = \Delta I$

$T_c = \text{Commutation period} = \frac{C_{ub-wm}}{V}$

where

$w_b = \text{width of brush in cm}$

$w_m = \text{width of mica}$

$V = \text{peripheral velocity of commutator}$

Segments in Cmls

$$E = \frac{L \Delta I}{T_c}$$

For Simulacal commutator  $= 1.11 \cdot E$

Problems :-

- ① Calculate reactance voltage for a machine having 55 conductors segments and speed - 900 rpm. If reactance inductance  $L = 1.53 \times 10^{-6}$  width of brushes is 1.74 cm. Current having 22A.

sol

Given data :-

Inductance  $L = 1.53 \times 10^{-6}$

Current  $I = 22 \text{ A}$

Brush width  $w_b = 1.74$

$v = \frac{900}{60} \times 55 = 825$

$w_m = 0, w_b = 1.74$

$\tau_c = \frac{w_b - w_m}{v} = \frac{1.74 - 0}{825}$

$\tau_c = 2.1 \times 10^{-3} \text{ s}$

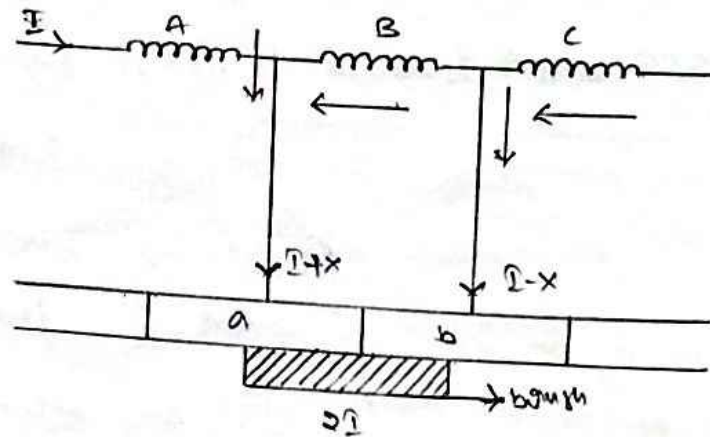
$E = L \times \frac{2I}{\tau_c}$

$= 1.5 \times 10^{-6} \times \frac{2 \times 220}{2.1 \times 10^{-3}}$

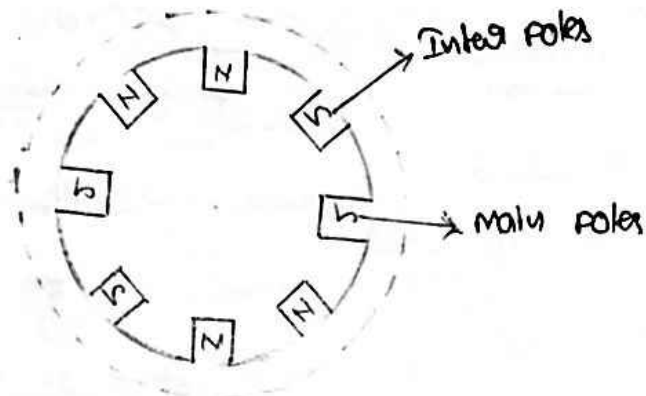
$E = 0.039 \text{ V}$

## Method of Improving Commutation :

### (i) Resistance Commutation Method :



### (ii) Emf Commutation using Interpoles :



- Interpoles are connected to the brush
- Interpoles polarity is same as main pole
- So they produce some emf which neutralizes the reaction voltage
- Its another function is to neutralize the commutation inductance and coast when compared to the interpoles. So interpoles used.

## Characteristics of D.C. motor :-

The performance of a D.C. motor can be judged from its characteristics curve known as motor characteristics

\* Torque and Armature Current characteristics ( $T_a/I_a$ )

\* Speed and Armature Current characteristics ( $N/I_a$ )

\* Speed and Torque characteristic ( $N/T_a$ ). It is also known as mechanical characteristic.

\* Electrical characteristics ( $T_a/I_a$ ) :-

$$I_a = I_L - I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

It will be seen that its field current  $I_{sh}$  remain almost constant so flux  $\phi$  which is proportional to  $I_{sh}$  will also remain constant

$$\text{We know that } T_a = \frac{1}{2\pi} \phi Z_a \times \frac{P}{A}$$

$$T_a \propto I_a$$

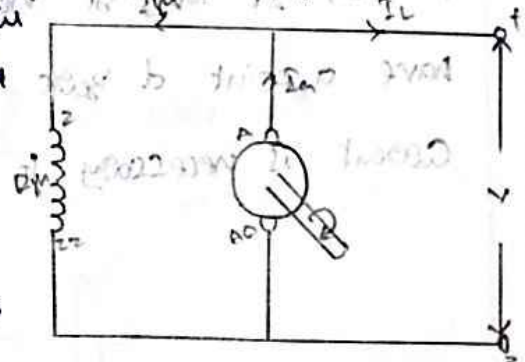
$$I_a \propto I_a$$

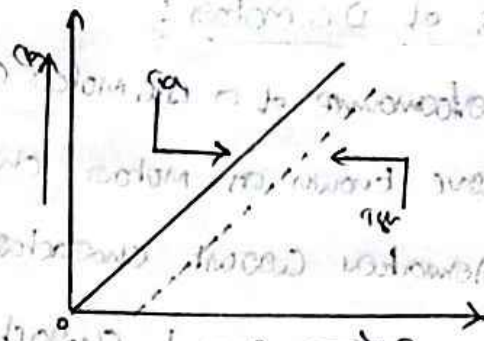
( $\phi$  is constant)

⇒ The electrical characteristic i.e. the curve  $T_a$  is proportional to the armature current  $I_a$  is practically a straight line from origin as shown in given fig.

The shaft torque ( $T_{sh}$ ) versus  $I_a$  is also shown dotted. It is clear that at large current  $I_a$  is required to start heavy load.

∴ Shunt motor should never be started with heavy load.

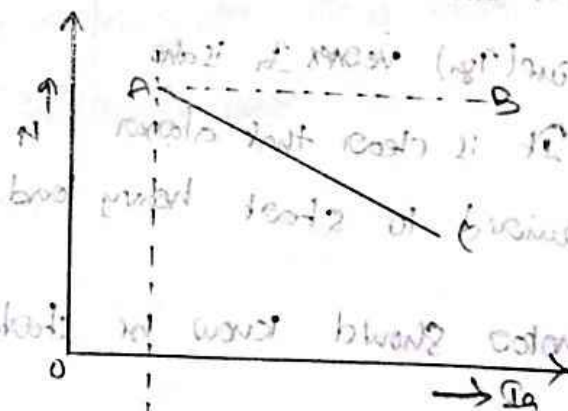




→ This operation Horse power (HP) of the motor is dependent on the shaft torque as shown in above fig.

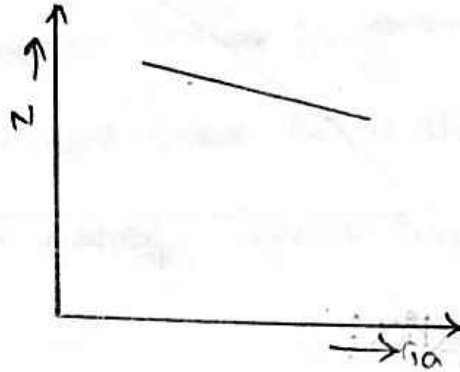
### N/Ia Characteristics:

- \* The speed  $N$  of a motor is given by  $N \propto \frac{E_b}{\phi}$
- \* The flux  $\phi$  and back emf  $E_b$  in a shunt motor are almost constant. Under normal condition, therefore speed of a shunt motor will remain constant as the armature current varies slightly. Specifying, when the load is increased ( $E_b = V - I_a R_a$ ) and  $\phi$  decreases due to the armature resistance drop and armature reaction respectively. However  $E_b$  decreases slightly more than  $\phi$  so that the speed of the motor decreases slightly with load (linear). It may be noted that the characteristic does not have a point of zero armature current because a small current is necessary to maintain rotating of the motor at no-load.



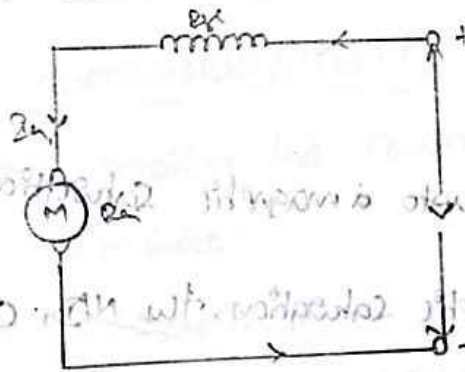
### 3. N vs Ia characteristics :

This curve is obtained by plotting the values of  $N$  vs  $I_a$  for various armature currents as shown fig. It may be seen that speed falls same what as the load torque increases



### Characteristics of D.C. series motor :

The d.c. series motor is the current passing through the field winding is the same as that in the armature. If the mechanical load on the motor increases the armature current also increases. The flux in a series motor increases with the increase in armature current and vice-versa.



### 1. Ta vs Ia characteristics :

We know that

$$T_a \propto I_a$$

upto magnetic saturation of  $I_a$

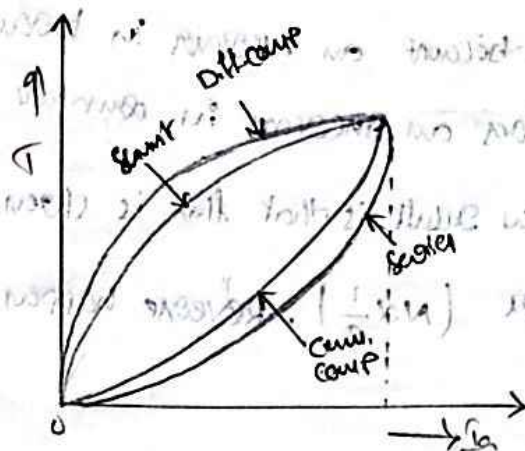
$$\text{so that } T_a \propto I_a^2$$

After magnetic saturation  $\phi$  is constant, so that  $T_a \propto I_a$

Thus upto magnetic saturation the armature torque is directly



to the motor. The result is hence there is a decrease in load is  
 rate at which the motor become incant. with load. Another draw  
 back is weakening of flux with increase in load, speed is instability



2. Speed & Armature current characteristics (N/Ia) :

long shunt compound motor

$$V = E_b + I_a (R_a + R_c)$$

$$E_b = k \phi N$$

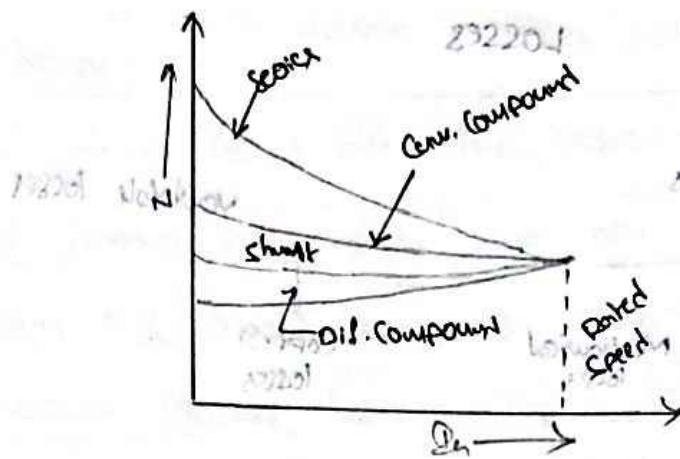
$$= k (\phi_{sh} + \phi_{sc}) N$$

$\phi_{sc}$ ,  $\phi_{sc}$  are flux due to  $I_{sc}$  &  $I_a$

$$N = \frac{E_b}{k(\phi_{sh} + \phi_{sc})} = \frac{V - I_a(R_a + R_c)}{k(\phi_{sh} + \phi_{sc})}$$

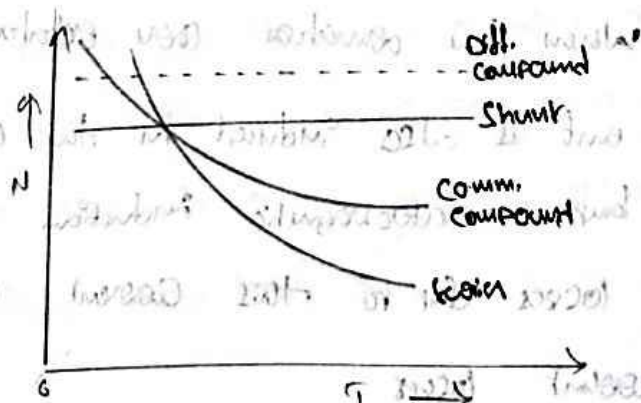
with increase in  $I_a$ ,  $\phi_{sc}$  increases and  $V - I_a(R_a + R_c)$   
 decreases. Thus with the increase in  $I_a$  the speed drops at  
 faster rate in cumulative compound motor than in shunt motor  
 if the shunt field is stronger than series field the curve  
 tends to shunt motor curve and if series field is stronger  
 than shunt field it tends to series field curve.

The Composition of Characteristics

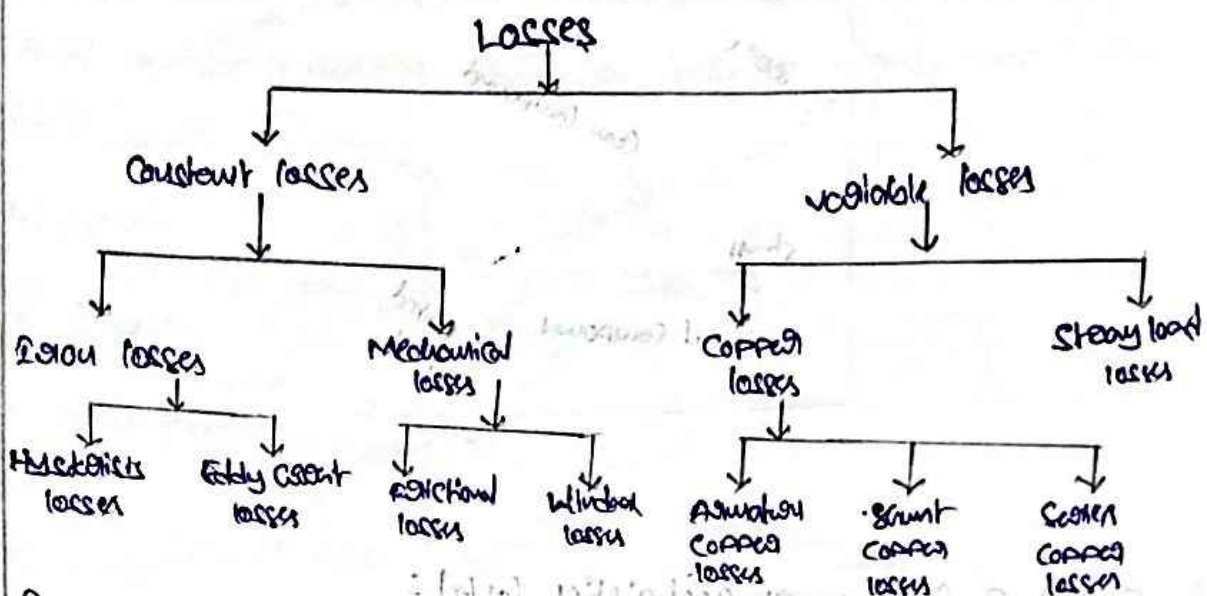


### 3. Speed & Torque Characteristics (N/T) :

Speed characteristic is also known as mechanical characteristic and is derived from speed-current & torque-current characteristics. It is seen that an increase in torque increases the armature current and also the flux, at the same time decreasing the speed rapidly in case of compound motor. The decrease is more predominant than compound with shunt motor. Therefore the speed torque characteristic approaches the shunt motor characteristic if the shunt field is stronger. If the series flux is stronger than shunt flux speed torque characteristic approaches the series motor characteristic. Depending upon the relative strength of shunt and series fields the speed torque can occupy any position.



# Losses in D.C. motor :



## Iron losses :

As the commutator core is made up of iron it rotates in a magnetic field, a small current gets induced in the core itself. Due to this current, eddy current loss, hysteresis loss occurs in the core.

### Hysteresis loss

It is due to the reversal of magnetisation of commutator core. The loss depends on the volume and grade of iron, frequency of magnetic reversal and value of flux density. Hysteresis loss is given by

$$W_h = N B_m f v$$

### Eddy current loss :

When the commutator core rotates in the magnetic field, an EMF is also induced in the core. According to Faraday's law of electromagnetic induction

→ Power losses due to this current is known as

eddy current loss

Eddy Current losses :  $k D^2 E^2 V B_m^2$  where:

Copper losses:

The copper losses are the losses taking place due to the current flowing in windings. These are basically armature and field winding. The copper losses are proportional to the square of the current flowing through this winding.

\* Armature copper losses =  $I_a^2 R_a$

\* Shunt field copper losses =  $I_{sh} R_{sh}$

\* Series field copper losses =  $I_a^2 R_{se}$

In a compound D.C. machine with shunt & series field copper losses are present in addition to the copper losses. There exist branch contact resistance drop. This drop is usually included in the armature copper losses. The copper losses induced so-called full load losses.

Mechanical losses:

This losses consists of friction & windage losses. Some power is diverted required over some mechanical friction & windage resistance at this shaft.

Stray losses:

In addition to the losses stated above there may be small losses present which are called as stray losses.

This losses are to the design & modeling of the machine. Most of the times stray losses assumed to 1% of the full load losses.

20 Total losses =  $\text{Copper losses} + \text{Mechanical losses} + \text{Stray losses}$

Efficiency ( $\eta$ ) :-

It is the ratio of output & machine to input & machine

$$\% \eta = \frac{\text{Output}}{\text{Input}} \times 100$$

Efficiency when running as motor:

$$\text{Input} = P_{in} = VI$$

$$\text{Output} = P_{out} = I_a R_a$$

$$\text{Copper losses} = I_a^2 R_a$$

$$\text{Constant losses} = W_{cl}$$

$$\eta_m = \frac{O/P}{I/P} \times 100 = \frac{I_a R_a - \text{losses}}{I_a R_a} \times 100$$

$$\eta_m = \frac{I_a R_a - (I_a^2 R_a + W_{cl})}{I_a R_a} \times 100$$

$$\eta_m = \frac{I_a R_a - \{ (I_a - I_m)^2 R_a + W_{cl} \}}{I_a R_a} \times 100$$

Efficiency when running as generator:

$$I_a = I_L + I_m$$

$$O/P = I_a E_b$$

$$I/P = I_a V$$

$$\eta_g = \frac{I_a E_b}{I_a V} \times 100$$

$$\eta_g = \frac{I_a E_b}{I_a (E_b + I_a R_a + W_{cl})} \times 100$$

## Applications of D.C. Motor:

### 1. Shunt Motor:

\* For driving constant speed line shafting

\* Lathe machines

\* Centrifugal pumps

\* Blowers & fans

\* Reciprocating pumps

### 2. Series Motor:

\* For traction system

\* Electric locomotives

\* Trolley cars

\* cranes & hoists & conveyors

### 3. Cumulative compound:

\* Rolling mill, Press machines

\* Printing presses

\* Air compressors

\* For intermittent high-torque load

### 4. Differential compound:

These are not in common use. Limited use for experimental research & laboratory purposes.

## Key points:

### Kirchoff's Current law (KCL):

"The algebraic sum of all the current meeting at a point in an electrical network is zero."

$$\text{Sum of Incoming currents} = \text{Sum of outgoing currents}$$

### Kirchoff's Voltage law (KVL):

"In a closed path (loop) mesh, the algebraic sum of the products of currents and resistances in each the conductors plus the algebraic sum of the emfs in that path is equal to zero."

## Formulas:

### EMF Equation of D.C. Generator:

$$\text{Generated e.m.f } E_g = \frac{\phi Z N P}{60} \text{ volts}$$

### For lap winding: (P = A)

$$E_g = \frac{\phi Z N}{60} \text{ volts}$$

### For wave winding:

$$E_g = \frac{\phi Z N P}{120} \text{ volts}$$

### D.C. Series motor:

$$I_a = I_x = I_L = I$$

$$V = E_g + I_a (R_a + R_x)$$

$$P_g = E_g \cdot I_a$$

O.C. shunt motor:

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

voltage  $V = E_g - I_a R_a$

Generated emf  $E_g = V + I_a R_a + \text{Brush contact drop}$

Long shunt compound generator:

$$I_a = I_L + I_{sh}$$

$$I_{sh} = \frac{V}{R_{sh}}$$

$$V = E_g - I_a (R_a + R_{se})$$

$$E_g = V + I_a (R_a + R_{se})$$

$$P_g = E_g \cdot I_a$$

Short shunt compound generator:

$$I_L = I_a = I_a - I_{sh}$$

$$I_{sh} = \frac{V + I_{sh} R_{sh}}{R_{sh}}$$

$$V = E_g - I_a R_a + I_{sh} R_{sh}$$

$$E_g = V + I_a R_a + I_{sh} R_{sh}$$

Torque equation of O.C motor:

$$T_g = \frac{1}{2\pi} \phi Z I_a \times \frac{P}{A} \quad N-m$$

$$T_g = 0.159 \phi Z I_a \times \frac{P}{A} \quad N-m$$

Demagnetising amp-tum/pole =  $\frac{Z I_a \theta_m}{360}$

Cross-magnetising amp-tum/pole =  $2I_a \left[ \frac{1}{2\pi} - \frac{\theta_m}{360} \right]$