Unit-II

Performance of DC Machines

DC Motors:-

Principle of operation:-

x - into the plane
• - out of the plane
Right hand thumb Rule.

As conductors are placed in the slots which are on the term perifery, the individual force experienced by the conductors acts as a twisting or turning force on the Armature which is called torque.

Torque is the product of force and the radius at which these force acts. So overall Armature experiences a torque and starts rotating.

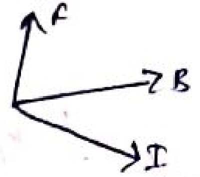
- The magnitude of the force experienced by the conductor in a motor is given by $f = BIL$ [flux cutting Law]

B - flux density
I = current passing through the conductor
L - length of the conductor

The direction of the rotation of the motor can be determined by flemings left hand Rule.

~~thus~~

Flemings left hand rule :-



thumb → gives direction of force experienced by conductor.

forefinger → gives direction of Magnetic field.

Middle finger → gives direction of current

* Significance of Back Emf :-

→ when a current carrying coil is placed in a magnetic field it experiences some force called Torque. This torque is to rotate the armature. The rotated armature cuts the magnetic flux and it induces some Emf.

⇒ According to Lenz's law the generated emf opposes the supply voltage. Here the generated emf called back emf.

$E_b = \frac{\phi Z N}{60} \left[\frac{P}{A}\right]$

$N \downarrow$

$E_g \propto N \Rightarrow E_g \downarrow$

$E_g < E_b$

$E_b \downarrow \propto N \downarrow \Rightarrow (V - E_b) \uparrow \cdot I_a \uparrow \; T \uparrow \; N \uparrow$

* Voltage Equation of a D.C. Motor:-

In a D.C. Motor, supply voltage (V) has to over come back emf E_b which is opposing the V and also various drops as armature resistance drop $I_a R_a$, brush drop etc..

$$V = E_b + I_a R_a + \text{brush drop}$$

$$I_a = \frac{V - E_b}{R_a}$$

* Power Equation of a DC Motor:-

$$V I_a = E_b I_a + I_a^2 R_a$$

$$\boxed{P_e = P_m + I_a^2 R_a}$$

where P_e is net electrical power input of the armature.

$I_a^2 R_a$ is the power loss due to the resistance of the armature.

P_m is the electrical equivalent of Gross mechanical power developed by the armature.

* Condition for Maximum power:-

$$E_b I_a = V I_a - I_a^2 R_a$$

To get max power. $\frac{dP_m}{dI_m} = 0$

$$P_m = P_e - I_a^2 R_a$$

$$I_a = \frac{V}{2R_a}$$

$I_a R_a = \frac{V}{2}$

substitute in voltage equation

$V = E_b + \frac{V}{2}$

$$\boxed{E_b = V/2}$$

* Torque equation of a DC Motor:-

- It is seen that the turning or twisting force about an axis in called Torque.

- Consider a wheel of radius R mt acted upon by a circumferential force f newtons, which is shown in the below figure.

- The wheel is rotating at a speed of N rpm.

- The angular speed of the wheel is $\omega = \frac{2\pi N}{60}$ rad/sec

- So work done in one revolution is

$W = F \times$ distance travelled in one revolution

$= F \times 2\pi R$ Jouls.

- power developed $= \frac{\text{work done}}{\text{time}}$

$= \frac{F \times 2\pi R}{\text{time for 1 sec}} = \frac{F \times 2\pi R}{60/N}$

$= \frac{2 F N \pi R}{60} = (F \times R)\left[\frac{2\pi N}{60}\right]$

$[P = T \times \omega]$

where T is Torque in Nmt.

ω is angular speed in rad/sec

- let T_a be the gross Torque developed by the armature of the motor.

- It is also called armature torque. The gross mechanical power developed in the armature is $E_b I_a$, and seen from the power Equation

- so If the speed of the Motor is N rpm then power in Armature is

$$\text{Power in Arm} = \text{Arma Torque} \times \omega$$

$$E_b I_a = T_a \times \frac{2\pi N}{60}$$

as we know

$$E_b = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$$

$$\frac{\phi Z N}{60} \cdot \frac{P}{A} \cdot I_a = T_a \times \frac{2\pi N}{60}$$

$$T_a = \frac{\phi I_a Z P}{2\pi A}$$

$$\boxed{T_a = 0.159\, \phi I_a \frac{PZ}{A} \text{ N-m}}$$

Different types of D.C. Motors:-

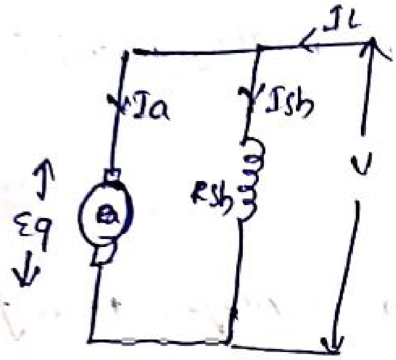
There are 3 types of D.C. Motors.

1. Shunt (D.C) Motor:

2. DC Series Motor

3. DC Compound motor.

Shunt (D.C) Motor:-



$I_L = I_{sh} + I_a$

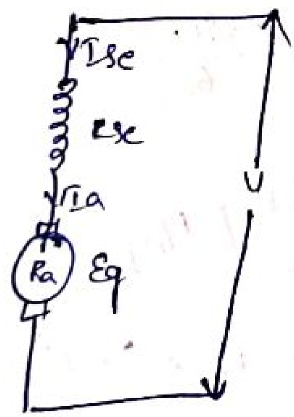
$V = E_q + I_a R_a$

$I_{sh} = \frac{V}{R_{sh}}$

$\phi \propto I_{sh}$

flux produced by the field winding is proportional to current passing through it.

DC series Motor:-



$I_L = I_{se} = I_a$

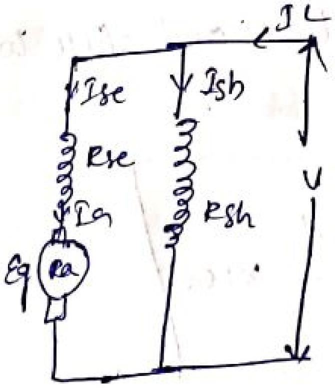
$V = E_q + I_a R_a + I_{se} R_{se}$

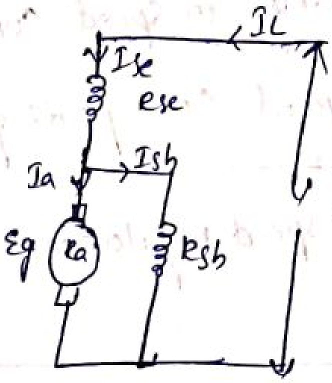
In series motor entire Armature current is passing through the series field winding. So flux produced is proportional to armature current

$\phi \propto I_{se} \propto I_a$

D.C. Compound Motor:-

Long shunt / short shunt





Long shunt:

$I_L = I_{sh} + I_{se}$

$I_{sh} = V/R_{sh}$

$V = E_g + I_a R_a + I_{se} R_{se}$

Short shunt:

$I_L = I_{se} + R_{sh}$

$I_{se} = I_a + I_{sh}$

$I_{sh} = \frac{V - I_{se}R_{se}}{R_{sh}}$

$V = E_g + I_a R_a + I_{se} R_{se}$

* Torque and Speed relations:-

$T = 0.159 \frac{PZ}{A} \phi I_a$

$T \propto \phi I_a$

In shunt field, $T \propto I_a$

In series

$T \propto \phi I_a$

$\phi \propto I_{se}$

$T \propto I_{se} I_a$

$I_{se} = I_a$

$T \propto I_a^2$

for series

$N \propto \frac{V - I_a R_a - I_{se} R_{se}}{\phi}$

$E_b = \frac{\phi Z N}{60} \left[P/A \right]$

$E_b = V - I_a R_a$

$E_b \propto \phi N$

$N \propto \frac{V - I_a R_a}{\phi}$

$N \propto \frac{E_b}{\phi}$

* Speed regulation:-

The Speed Regulation for a D.C. Motor is defined as the Ratio of change in speed corresponding to no load and full load condition to speed corresponding to full load.

$$\%\ \text{Speed regulation} = \frac{N_{noload} - N_{fullload}}{N_{full\ load}} \times 100$$

* Torque expression in terms of Eb.

$$E_b = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$$

$$\therefore \frac{\phi Z P}{A} = \frac{E_b \times 60}{N}$$

$$T = 0.159\left[\frac{PZ}{A}\right]\phi I_a$$

$$T = \left[\frac{PZ\phi}{A}\right] 0.159\, I_a$$

$$T = \frac{E_b \times 60}{N} \times 0.159\, I_a$$

$$T = 9.55\frac{E_b I_a}{N}$$

$$T \propto \frac{1}{N}$$

* Shaft Torque:-

The torque which is available at the motor shaft for doing useful work is known as shaft torque.

~~Horse~~ Horsepower = 735.5

* Characteristics of a D.C. Motor:-

The characteristics of a DC Motor is divided into 3 types:

1. Torque vs Amature current
2. Speed vs Amature current,
3. Speed vs Torque.

DC Shunt Motor:-

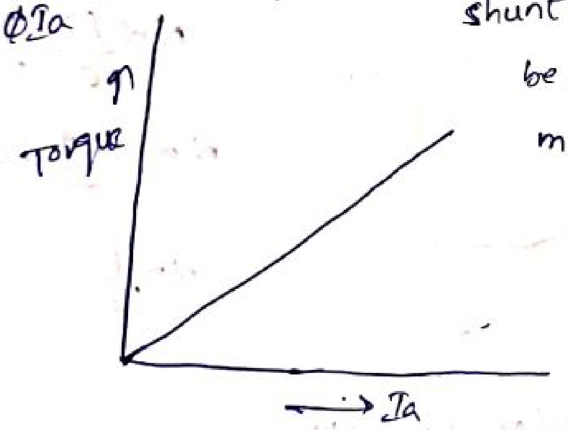
$T = 0.159 \frac{PZ}{A} \phi I_a$

ϕ is constant when we apply constant voltage.

$T \propto \phi I_a$

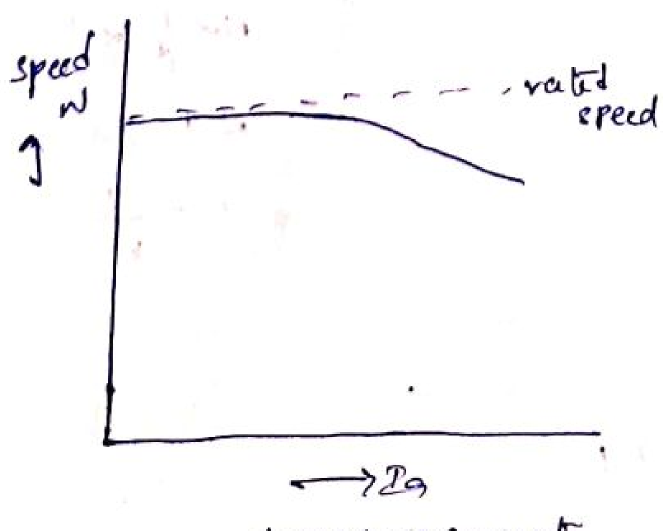
T vs Ia

Shunt motor can be assumed to be as a constant speed motor



Amature Current.

N vs Ia



$E_b = \frac{\phi Z N}{60} [P/A]$

$E_b = V - I_a R_a$

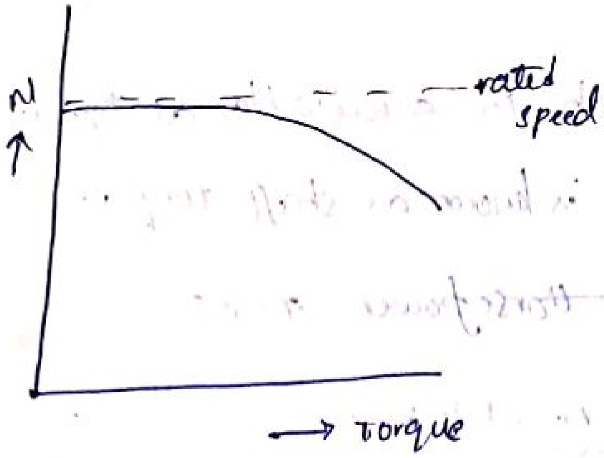
$N \propto V - I_a R_a$

Considered as

$V \approx E_b$ (Ra is very less)

Amature Current

N vs Ia

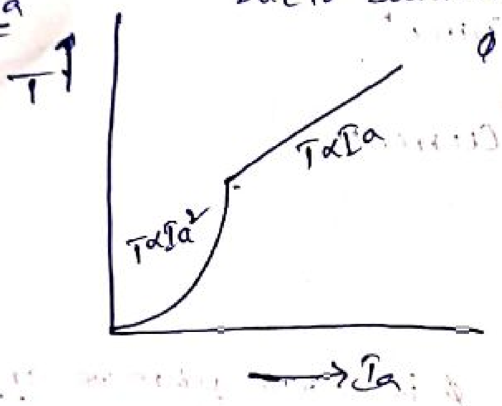


$T \propto I_a$

$T \propto \frac{1}{N}$

* Series Motor :-

T vs Ia



Due to saturation ϕ will be const. $T \propto \phi . I_a$

$E_b = \frac{\phi Z N}{60} \left(\frac{P}{A}\right)$

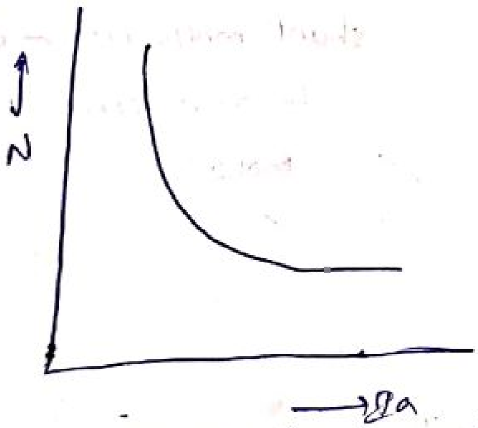
$E_b \propto \phi . N$

bcoz ϕ is varied we are connected in series with f.w. armature

$T \propto I_a I_a$

$T \propto I_a^2$

N vs Ia



rectangular hyperbola

$N \propto \frac{E_b}{\phi}$, $\phi \propto I_a$

$N \propto \frac{V - I_a R_a - I_{se} R_{se}}{I_a}$

$N \propto \frac{1}{I_a}$

N vs T

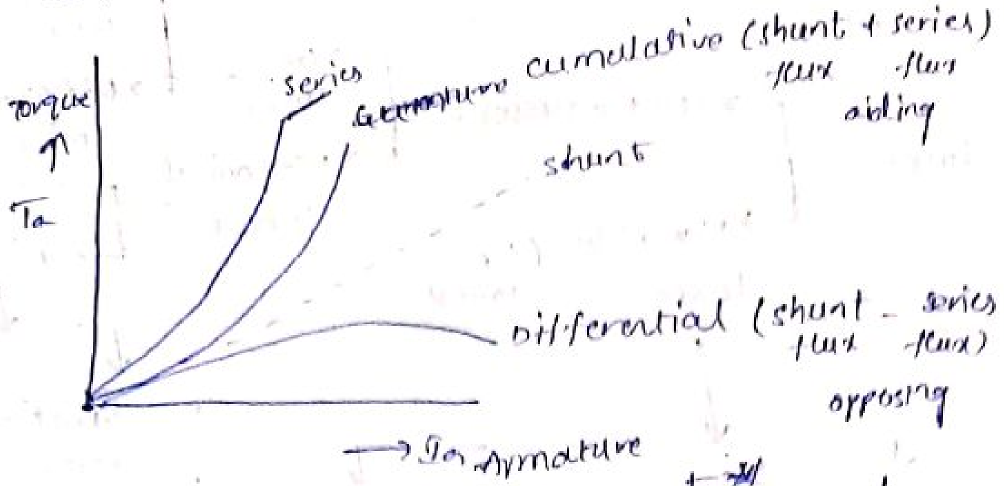
$N \propto \frac{1}{\phi} \propto \frac{1}{I_a}$

$T \propto I_a^2$

$N \propto \frac{1}{\sqrt{T}}$

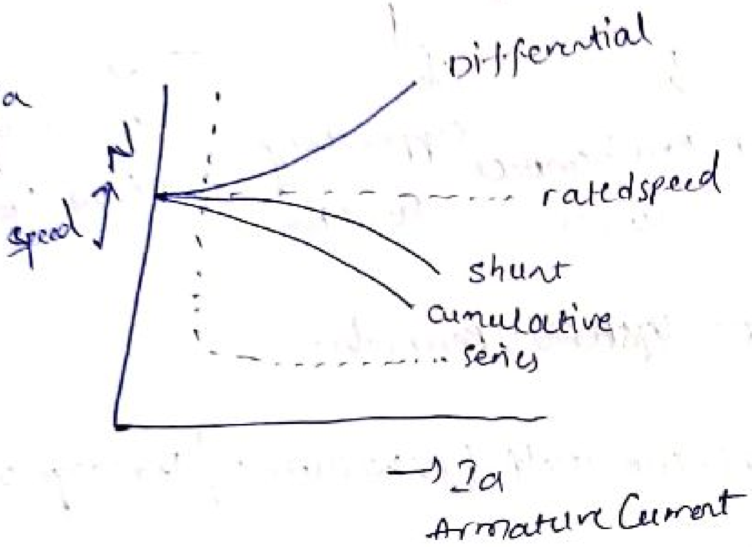
→ Compound Motor:-

T vs Ia

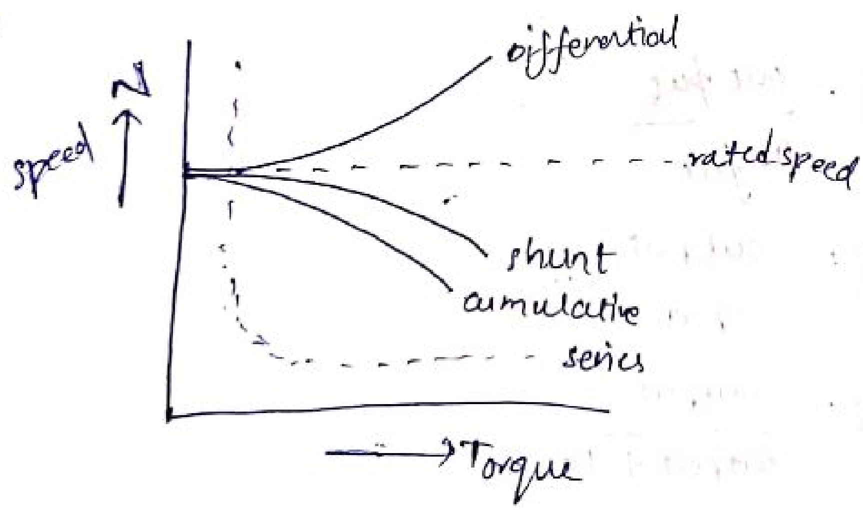


[* at no load runs series motor runs with with high speed]

N vs Ia



N vs Ta



* losses:-

- loses
  - constant losses
    - core losses
      - Hysterisis loses
      - Eddy current losses
    - Mechanical losses
      - windage losses
      - friction losses
        - Brush friction losses
        - Bearing friction losses
  - variable losses (I^2R losses)
    - copper losses
      - stator copper losses
      - Rotor copper losses
      - Brush contact losses
    - stray load losses
      - copper stray load losses
      - core stray load losses

Hysterisis:- Magnetic ~~hysterisis~~ Reversals.

Eddy currents: Appear in core itself (extra currents / leackage currents)

Mechanical :- friction due to Bolts

* Efficiency (η) :-

$$\eta = \frac{\text{output}}{\text{Input}}$$

$$\%\eta = \frac{\text{output}}{\text{Input}} \times 100$$

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\eta = \frac{\text{Input} - \text{losses}}{\text{Input}}$$

* A 220V shunt generator supplies a load current of 120A at a rated speed of 800 rpm. The armature and shunt-field resistance are 0.02Ω and 110Ω respectively. If the overall efficiency of the Machine at this load is 88%. determine

Armature and shunt-field copper loss.

2- stray losses.

Sol: $I_L = 120A$

$N = 800 rpm$

Ar cu. loss = $I_a^2 R_a$

$I_a = I_L + I_{sh}$

$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{110} = 2 Amp$

$I_a = 120 + 2 = 122 Amp$

Aratur. current loss = $(122)^2 \cdot 0.02$

$= 297.7 W$

shunt field copper loss $= V \times I_{sh}$

$= 220 \times 2$

$= 440 W$

total copper loss = Armature + shunt field

$= 297.7 + 440$

$= 737.7 W$

out put of generator = $VI = 220 \times 120$

$= 26400 W$

Mechanical Input = $\frac{\text{output}}{\text{Efficiency}}$

$= \frac{26400}{0.88} = 30,000 W$

Total losses = ~~output~~ Input - output

$= 30,000 - 26400$

$= 3600 W$

stray load losses = total losses - copper loss

$= 3600 - 737.7$

$= 2862.32 W$

* The following data pertain to a short shunt compound generator

Armature resistance = 0.04 Ω

Series field resistance = 0.025 Ω

shunt field resistance = 45 Ω.

Contact drop per brush = 1 V

Magnetic losses = 2.5 kW

Mechanical losses = 1 kW

find the Efficiency of the Generator when it delivers 400A at 440V

Sol:- ~~Armature~~ $R_a = 0.04 \Omega$

$R_{se} = 0.025$

$R_{sh} = 45 \Omega$

Ma

voltage across shunt field winding = $V + I R_{se}$

$= 440 + 400 \times 0.025$

$= 450\ V$

$I_{sh} = \frac{V}{R_{sh}} = \frac{450}{45}$

$= 10$ Amp

$I_a = I_L + I_{sh}$

$= 450 + 10$

$= 460$ Amp

Amature copper loss $= I_a^2 R_a = (410)^2 \times 0.04$

$= 6,724\ W$

series field copper loss $= I^2 R_{se} = (400)^2 \times 0.025$

$= 4000\ W$

shunt field copper loss $= V \cdot I_{sh} = 450 \times 10 = 4500\ W$

brush contact loss $= 2 \times I_a$

$= 2 \times 410$

$= 820\ W$

total losses = Amature + seriesfield + shuntfield + brush + Mech + Mag

$= 6724 + 4000 + 820 + 4500 + 2500 + 1000$

$= 19544\ W$

output $= V \times I$

$= 440 \times 400$

$= 176000\ W$

% Efficiency $\eta = \frac{output}{output + loss}$

$= \frac{176000}{176000 + 19544}$

$= \frac{176000}{195544}$

$= 0.9 \times 100$

$= 90\%$

* A 440V shunt motor has armature resistance of 0.8Ω and field resistance of 200Ω. Determine the backemf when giving an output of 7.46 kW at efficiency 85%.

Solⁿ:-

$V = 440V$

$R_a = 0.8\Omega$

Output $= 7.46$ kW

$\eta = 85\%$

$R_{sh} = 200\Omega$

$E_b = ?$

Power input = $\frac{output}{\eta}$

= $\frac{7.46\times10^3}{0.85}$ = 8776.47 ~~[illegible]~~ kW

we know that

$E_b = V - I_a R_a$

$I_a = I_L + I_{sh}$

$I_L = \frac{1}{\frac{V}{P_i}}$ $\qquad I_{sh} = \frac{V}{R_{sh}}$

$= \frac{1}{\frac{440}{8776.47}}$ $\qquad = \frac{440}{200}$

= ~~[illegible]~~ A $\qquad = 2.2\,A$

= ~~[illegible]~~ = 19.95A

I_a = ~~[illegible]~~

19.95 − 2.2 = 17.75A

$E_b = 440 - 17.752\times0.8$

= 440 − ~~[illegible]~~ 14.2

= 425.8 V

* Find the useful flux per pole on load of 250V, 6 pole shunt motor having a wave connected Armature winding with 110 turns. The Armature resistance is 0.2Ω. The armature Current is 13.3 A at the no-load speed of 908 rpm.

sol^n $\phi = ?$

$N = 908$ rpm $Z = 110 \times 2 = 220$

$A = 2$

$P = 6$ $V = 250$

$R_a = 0.2$

$I_a = 13.3$

$E_b = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$

$E_b = V - I_a R_a$

$= 250 - (13.3 \times 0.2)$

$= 247.34V$

$247.34 = \frac{\phi \times 220 \times 908}{60}\left[\frac{6}{2}\right]$

$= \phi \times 9988$

$\phi = 24.7$ mW

* A 250V, shunt motor on-no-load runs at 1000rpm and takes 5 Amp. the total Armature and shunt field resistances are 0.2Ω and 250Ω respectively. calculate the speed when loaded and Taking current of 50 Amp if Armature reaction weakens the field by 3%.

sol:- $V = 250V$ $R_{sh} = 250$

$N = 1000$

$I_L = 5$ $I_L = 50$

$R_a = 0.2\Omega$ Armature reaction: 3%

at NoLoad condition $I_L = 5$

$I_a = I_L - I_{sh}$

$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1$

$I_a = 5 - 1 = 4 Amp$

$E_{b1} = 250 - 4 \times 0.2$

$= 249.2V$

at loaded condition

$I_L = 50$

$I_a = I_L - I_{sh} = 50 - 1$
$= 49A$

$E_{b2} = V - I_a R_a$

$= 250 - 49 \times 0.2$

$= 240.2V$

$$\frac{E_{b2}}{E_{b1}} = \frac{\phi_2 N_2}{\phi_1 N_1}$$

$$N_2 = \frac{E_{b2}}{E_{b1}} \times \frac{\phi_1 N_1}{\phi_2}$$

$$= \frac{240.2 \times 1 \times 1000}{249.2 \times 0.97}$$

$= 994$ rpm

* A 6-pole DC Motor has a wave connected Armature with 87 - slots, each slot containing 6 conductors. The flux per pole is 30mW and the armature has a resistance of 0.10 Ω. calculate the Speed when the motor is connected to a 250V supply, and taking an Armature current of 80Amp. calculate also the torque in N-m. developed by The Armature.

Sol:-

$P = 6$, $A = 2$

$Z = 87 \times 6 = 522$ $N = ?$

$V = 250V$

$\phi = 30mW = 0.003$

$I_a = 80$

$R_a = 0.1 \Omega$.

$E_b = V - I_a R_a$

$= 250 - 80 \times 0.1$

$= 250 - 8$

$= 242 V$

$$E_b = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$$

$$N = \frac{E_b \times 60 \times A}{\phi \times Z \times P}$$

$$= \frac{242 \times 60 \times 2}{0.03 \times 522 \times 6}$$

$= 310$ rpm

Torque $T_a = 0.159 \times \phi \times I_a \times \frac{PZ}{A}$ (or) $9.55 \times \frac{E_b I_a}{N}$

$$= 0.159 \times 0.03 \times 80 \times \frac{6 \times 522}{2}$$

$= 597.58$ N-m

* A four pole 250 V series motor has a wave connected Armature with 1254 conductors. The flux per pole is 22mW when the motor is taking 50 A : Iron and friction losses amount to 1.0kW and armature resistance is 0.2Ω and series field resistance is 0.2Ω, calculate BHP (Break Horse power), speed, shaft torque/useful torque.

Sol:- $P = 4$, $A = 2$ $I_L = 50$ $V = 250$

$Z = 1254$, $\phi = 22 \times 10^{-3}$ $R_a = 0.2\Omega$

friction losses = 1×1000 W, $R_{se} = 0.2\Omega$

$I_L = I_{se} = I_a$

$$E_b = \frac{\phi Z N}{60}\left[P/A\right]$$

$$E_b = V - I_a R_a - I_{se} R_{se}$$

$$= 250 - 50 \times 0.2 - 50 \times 0.2$$

$$= 240\,V - 10 = 230\,V$$

$$230 = \frac{22 \times 10^{-3} \times 1254 \times N}{60}\left[4/2\right]$$

$$230 = 0.919 \times N$$

$$N = 250\ rpm$$

) total resistance = $R_a + R_{se}$

$= 0.2 + 0.2$

$= 0.4$

Mechanical power developed in the armature

$= V I_a$.

Mech power developed $= VI_a - I_a^2(R_a + R_{se})$

$= 250 \times 50 - (50)^2 (0.4)$

$P_m = 11500 W$

useful power $= P_m$ - Iron and friction losses

$= 11500 - 1000$

$= 10,500 W$

useful power $P_m = \frac{2\pi N T_{shaft}}{60}$

$10500 = \frac{2 \times \pi \times 250 \times T}{60}$

$10500 = 26.18 \times T$

$T_i = 384.179$

$T_{sh} = 401 N\text{-}m$

$BHP = \frac{10500}{746} = 14.075 HP$

& A 250V 4-pole shunt motor has two circuit armature winding with 500 conductors. the armature circuit resistence is 0.25Ω field resistence in 125Ω and flux per pole = 0.02Wb neglect armature reaction. find the speed and torque developed if the motor draws 14A from the mains.

sol: $V = 250$, $A = 2$, (wave connected = two circuit armature)

$P = 4$, $Z = 500$, $R_a = 0.25$, $R_{sh} = 125$, $\phi = 0.02$

$I_L = 14$

$I_a = I_L - I_{sh}$

$I_a = 14 - 2 = 12A$

$E_b = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$

$E_b = V - I_a R_a$

$= 247V$

$N = \frac{E_b \times 60 \times A}{\phi \times Z \times P} = \frac{E_b \times 60 \times 2}{0.02 \times 500 \times 4}$

$= 741 rpm$

$E_b I_a = \frac{2\pi N T}{60}$

$247 \times 12 = \frac{2\pi \times 741 \times T}{60}$

$T = 38.19 N\text{-}m$

* Determine the torque developed when a current of 30A passes through Armature of a motor with the following particulars lapwinding, 310 conductors, 4-poles; pole shoes 16.2 cm long subtending an angle 60° at the centre, Bore radius 16.2cm, flux density in the airgap b = 0.7 Tesla.

sol:-

$T = 0.159\, \phi I_a \left[\frac{Z}{A} P\right]$

$I_a = 30 \quad A = 4$

$Z = 310$

$P = 4$

$D = 2R = 2 \times 16.2$

$= 32.4\,cm$

$B = 0.7\,Tes$

$\text{pole arc} = \left[\frac{60}{360}\right] \pi d$

$= \frac{\pi D}{6} = \frac{\pi \times 32.4}{6} = 16.9\,cm$

Flux per pole = flux density × pole arc × core length

$= 0.7 \times 16.9 \times 16.2\ cm$

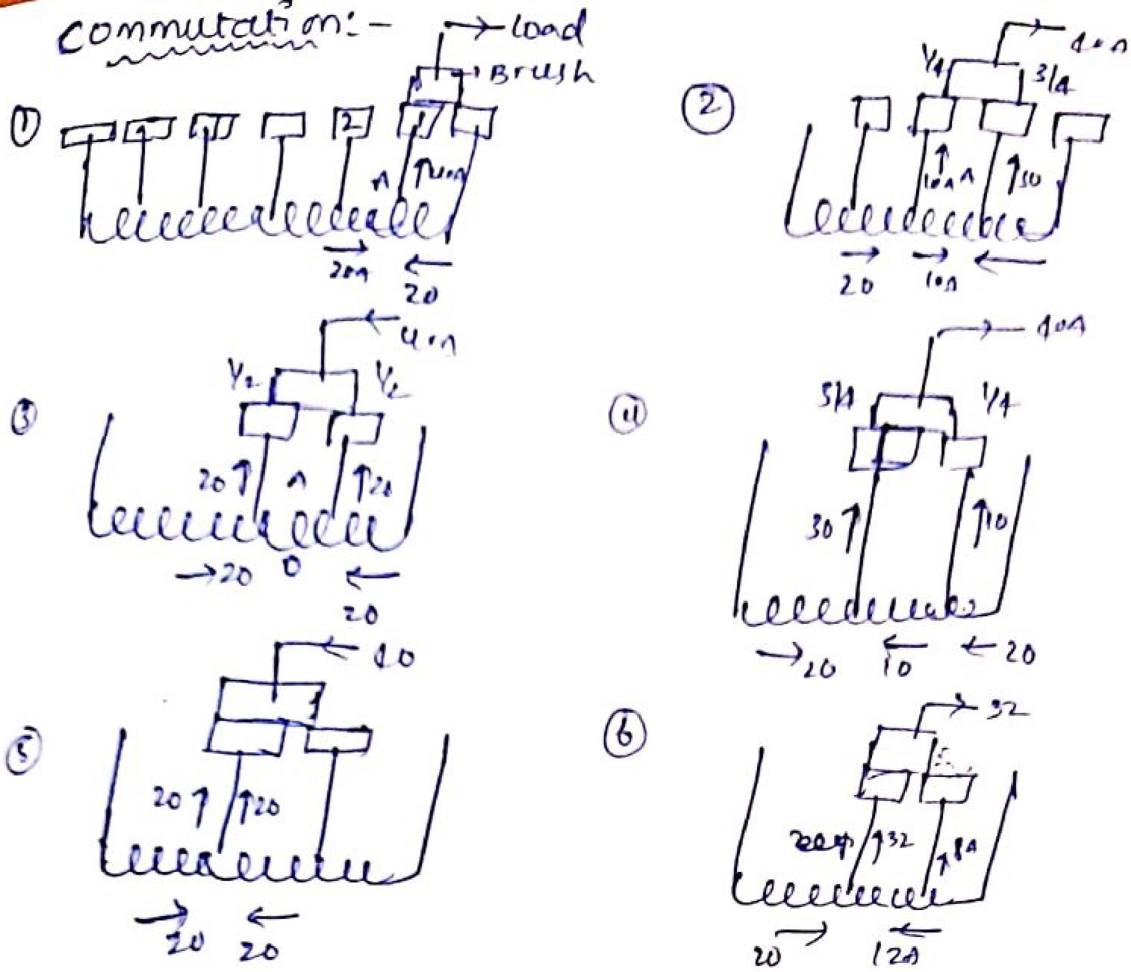
$= 0.7 \times 0.169 \times 0.162\ m$

$= 0.0193\,wb$

$$T = \frac{0.159 \times 0.0193 \times 30 \times 310 \times 4}{4}$$

$= 28.5$

Commutation:-



Commutation period $= \dfrac{w_b - w_m}{v}$

Reactance voltage $= L \times \dfrac{2I}{T_c}$

w_b = brush width

w_m = insulation width (mica)