UNIT I

Principles of Electromechanical Energy Conversion

Topics to cover:

1) *Introduction*
2) *EMF in Electromechanical Systems*
3) *Force and Torque on a Conductor*
4) *Force and Torque Calculation from Energy and Co-energy*
5) *Model of Electromechanical Systems*

Introduction

For energy conversion between electrical and mechanical forms, electromechanical devices are developed. In general, electromechanical energy conversion devices can be divided into three categories:

(1) Transducers (for measurement and control)

These devices transform the signals of different forms. Examples are microphones, pickups, and speakers.

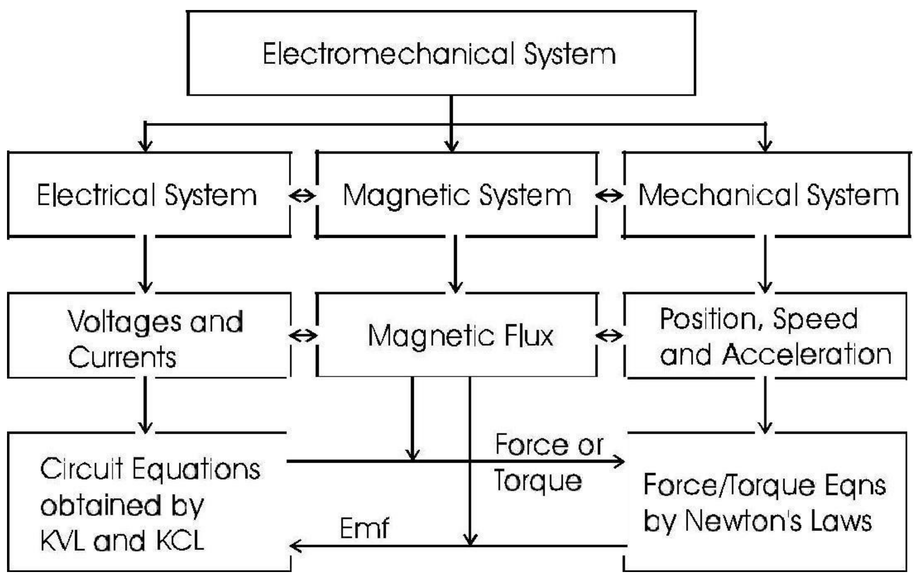
(2) Force producing devices (linear motion devices)

These type of devices produce forces mostly for linear motion drives, such as relays, solenoids (linear actuators), and electromagnets.

(3) Continuous energy conversion equipment

These devices operate in rotating mode. A device would be known as a generator if it convert mechanical energy into electrical energy, or as a motor if it does the other way around (from electrical to mechanical).

Since the permeability of ferromagnetic materials are much larger than the permittivity of dielectric materials, it is more advantageous to use electromagnetic field as the medium for electromechanical energy conversion. As illustrated in the following diagram, an electromechanical system consists of an electrical subsystem (electric circuits such as windings), a magnetic subsystem (magnetic field in the magnetic cores and airgaps), and a mechanical subsystem (mechanically movable parts such as a plunger in a linear actuator and a rotor in a rotating electrical machine). Voltages and currents are used to describe the state of the electrical subsystem and they are governed by the basic circuital laws: Ohm's law, KCL and KVL. The state of the mechanical subsystem can be described in terms of positions, velocities, and accelerations, and is governed by the Newton's laws. The magnetic subsystem or magnetic field fits between the electrical and mechanical subsystems and acting as a "ferry" in energy transform and conversion. The field quantities such as magnetic flux, flux density, and field strength, are governed by the Maxwell's equations. When coupled with an electric circuit, the magnetic flux interacting with the current in the circuit would produce a force or torque on a mechanically movable part. On the other hand, the movement of the moving part will could variation of the magnetic flux linking the electric circuit and induce an electromotive force (*emf*) in the circuit. The product of the torque and speed (the mechanical power) equals the active component of the product of the *emf* and current. Therefore, the electrical energy and the mechanical energy are inter-converted via the magnetic field.



Concept map of electromechanical system modeling

In this chapter, the methods for determining the induced *emf* in an electrical circuit and force/torque experienced by a movable part will be discussed. The general concept of electromechanical system modeling will also be illustrated by a singly excited rotating system.

Induced *emf* in Electromechanical Systems

The diagram below shows a conductor of length l placed in a uniform magnetic field of flux density **B**. When the conductor moves at a speed **v**, the induced *emf* in the conductor can be determined by

$$\mathbf{e} = l\mathbf{v} \times \mathbf{B}$$

The direction of the *emf* can be determined by the "right hand rule" for cross products. In a coil of N turns, the induced *emf* can be calculated by

$$e = -\frac{d\lambda}{dt}$$

where λ is the flux linkage of the coil and the minus sign indicates that the induced current opposes the variation of the field. It makes no difference whether the variation of the flux linkage is a result of the field variation or coil movement.

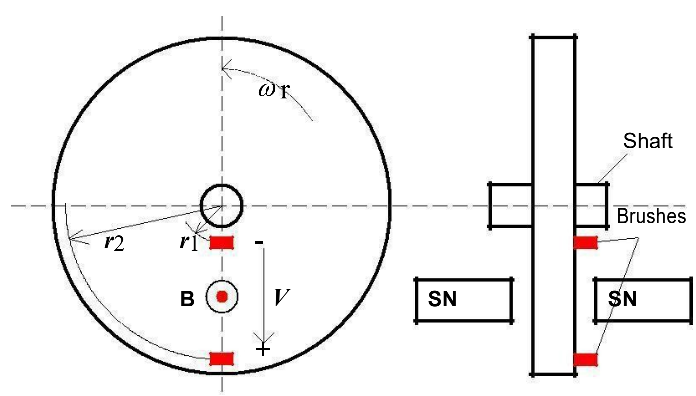
In practice, it would convenient if we treat the *emf* as a voltage. The above express can then be rewritten as

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

if the system is magnetically linear, i.e. the self inductance is independent of the current. It should be noted that the self ***inductance is a function of the displacement*** x since there is a moving part in the system.

Example:

Calculate the open circuit voltage between the brushes on a Faraday's disc as shown schematically in the diagram below.



Solution:

Choose a small line segment of length dr at position r ($r_1 \le r \le r_2$) from the center of the disc between the brushes. The induced emf in this elemental length is then

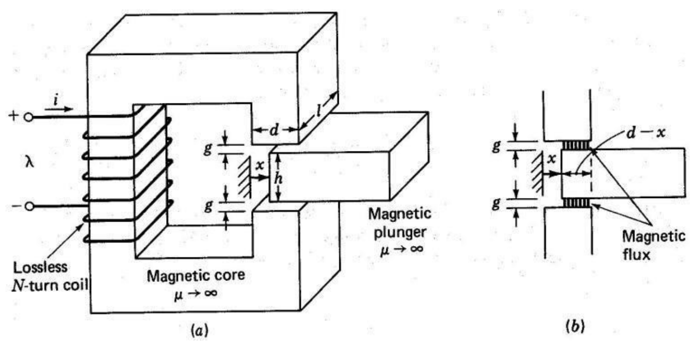
$$de = Bv\,dr = B\omega_r r\,dr$$

where $v = r\omega_r$. Therefore,

$$e = \int_{r_1}^{r_2} B\omega_r r\,dr = \omega_r B \left.\frac{r^2}{2}\right|_{r_1}^{r_2} = \omega_r B\,\frac{r_2^2 - r_1^2}{2}$$

Example:

Sketch L(x) and calculate the induced emf in the excitation coil for a linear actuator shown below.



A singly excited linear actuator

Solution:

$$L(x) = \frac{N^2}{R_g(x)}$$

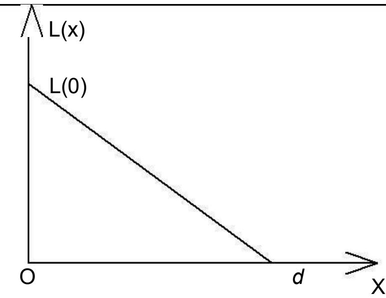
and

$$R_g(x) = \frac{2g}{\mu_o (d - x) l}$$

$$= \frac{\mu_o N^2 l (d - x)}{2g}$$

$$e = \frac{d\lambda}{dt} = L\frac{di}{dt} + i\frac{dL}{dx}\frac{dx}{dt}$$

$$= L(x)\frac{di}{dt} - i\frac{\mu_o N^2 l}{2g} v$$



Inductance vs. displacement

If i=Idc,

$$e = -I_{dc} \frac{\mu_o N^2 l}{2g} v$$

If i=Imsin ωt,

$$e = \frac{\mu_o N^2 l}{2g}(d-x)\omega I_m \cos\omega t - v I_m \sin\omega t \frac{\mu_o N^2 l}{2g}$$

$$= I_m \frac{\mu_o N^2 l}{2g}\left[(d-x)\omega\cos\omega t - v\sin\omega t\right]$$

$$= I_m \frac{\mu_o N^2 l}{2g}\sqrt{(d-x)^2\omega^2 + v^2}\left[\cos\left(\omega t + \arctan\frac{v}{(d-x)\omega}\right)\right]$$

Force and Torque on a Current Carrying Conductor

The force on a moving particle of electric charge q in a magnetic field is given by the Lorentz's force law:

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The force acting on a current carrying conductor can be directly derived from the equation as

$$\mathbf{F} = I\int_C d\boldsymbol{l} \times \mathbf{B}$$

where C is the contour of the conductor. For a homogeneous conductor of length $\boldsymbol{l}$ carrying current I in a uniform magnetic field, the above expression can be reduced to

$$\mathbf{F} = I(\boldsymbol{l} \times \mathbf{B})$$

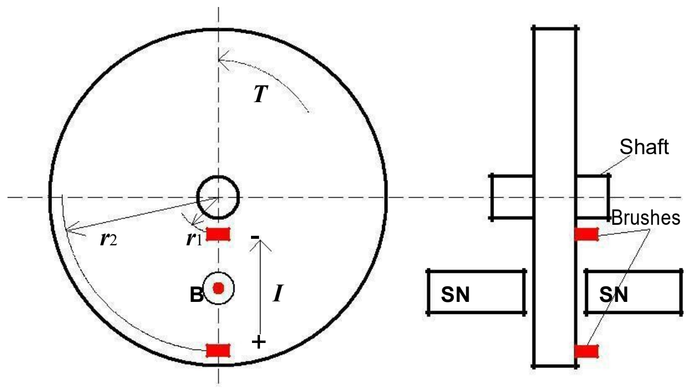
In a rotating system, the torque about an axis can be calculated by

$$\mathbf{T} = \mathbf{r} \times \mathbf{F}$$

where $\mathbf{r}$ is the radius vector from the axis towards the conductor.

Example:

Calculate the torque produced by the Faraday's disc if a dc current I_{dc} flows from the positive terminal to the negative terminal as shown below.



Solution:

Choose a small segment of length dr at position r ($r_1 \le r \le r_2$) between the brushes. The force generated by this segment is

$$d\mathbf{F} = (-Idr\hat{\mathbf{r}}) \times (B\mathbf{a}_z) = IBdr\mathbf{a}_\theta$$

where $\mathbf{a}_\theta$ is the unit vector in θ direction. The corresponding torque is

$$d\mathbf{T} = \mathbf{r} \times d\mathbf{F} = IBrdr\mathbf{a}_z$$

Therefore,

$$T = \int d\mathbf{T} = \int_{r_1}^{r_2} IBrdr\mathbf{a}_z = IB\frac{r_2^2 - r_1^2}{2}\mathbf{a}_z$$

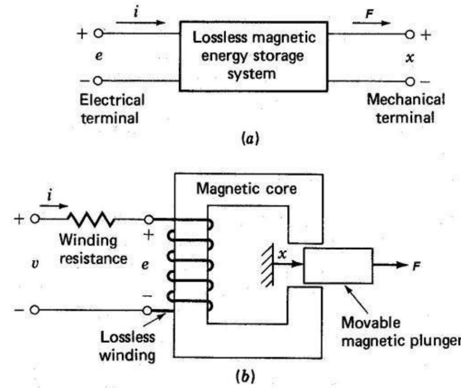
Force and Torque Calculation from Energy and Co-energy

A Singly Excited Linear Actuator

Consider a singly excited linear actuator as shown below. The winding resistance is R. At a certain time instant *t*, we record that the terminal voltage applied to the excitation winding is *v*, the excitation winding current *i*, the position of the movable plunger *x*, and the force acting on the plunger **F** with the reference direction chosen in the positive direction of the *x* axis, as shown in the diagram. After a time interval *dt*, we notice that the plunger has moved for a distance *dx* under the action of the force **F**. The mechanical done by the force acting on the plunger during this time interval is thus

$$dW_m = Fdx$$

The amount of electrical energy that has been transferred into the magnetic field and converted into the mechanical work during this time interval can be calculated by subtracting the power loss dissipated in the



A singly excited linear actuator

winding resistance from the total power fed into the excitation winding as

$$dW_e = dW_f + dW_m = vidt - Ri^2 dt$$

Because $$e = \frac{d\lambda}{dt} = v - Ri$$

we can write $$dW_f = dW_e - dW_m = eidt - Fdx$$

$$= id\lambda - Fdx$$

From the above equation, we know that the energy stored in the magnetic field is a function of the flux linkage of the excitation winding and the position of the plunger. Mathematically, we can also write

$$dW_f(\lambda, x) = \frac{\partial W_f(\lambda, x)}{\partial \lambda} d\lambda + \frac{\partial W_f(\lambda, x)}{\partial x} dx$$

Therefore, by comparing the above two equations, we conclude

$$i = \frac{\partial W_f(\lambda, x)}{\partial \lambda} \quad \text{and} \quad F = -\frac{\partial W_f(\lambda, x)}{\partial x}$$

From the knowledge of electromagnetics, the energy stored in a magnetic field can be expressed as

$$W_f(\lambda, x) = \int_0^{\lambda} i(\lambda, x) d\lambda$$

For a magnetically linear (with a constant permeability or a straight line magnetization curve such that the inductance of the coil is independent of the excitation current) system, the above expression becomes

$$Wf(\lambda, x) = \frac{1}{2}\frac{\lambda^2}{L(x)}$$

and the force acting on the plunger is then

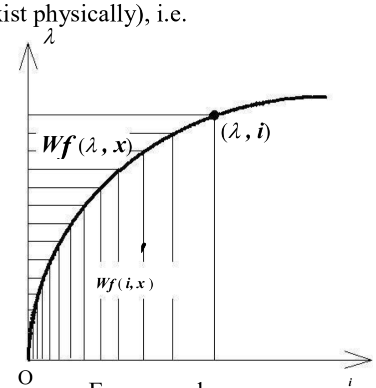
$$F = -\frac{\partial Wf(\lambda, x)}{\partial x} = \frac{1}{2}\left[\frac{\lambda}{L(x)}\right]^2 \frac{dL(x)}{dx} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

In the diagram below, it is shown that the magnetic energy is equivalent to the area above the magnetization or λ-i curve. Mathematically, if we define the area underneath the magnetization curve as the ***co-energy*** (which does not exist physically), i.e.

$$Wf'(i, x) = i\lambda - Wf(\lambda, x)$$

we can obtain

$$\begin{aligned} dWf'(i, x) &= \lambda di + i d\lambda - dWf(\lambda, x) \\ &= \lambda di + F dx \\ &= \frac{\partial W_f'(i, x)}{\partial i} di + \frac{\partial W_f'(i,x)}{\partial x} dx \end{aligned}$$



Energy and co-energy

Therefore,

$$\lambda = \frac{\partial Wf'(i, x)}{\partial i}$$

and

$$F = \frac{\partial Wf'(i, x)}{\partial x}$$

From the above diagram, the co-energy or the area underneath the magnetization curve can be calculated by

$$Wf'(i, x) = \int_0^i \lambda(i, x) di$$

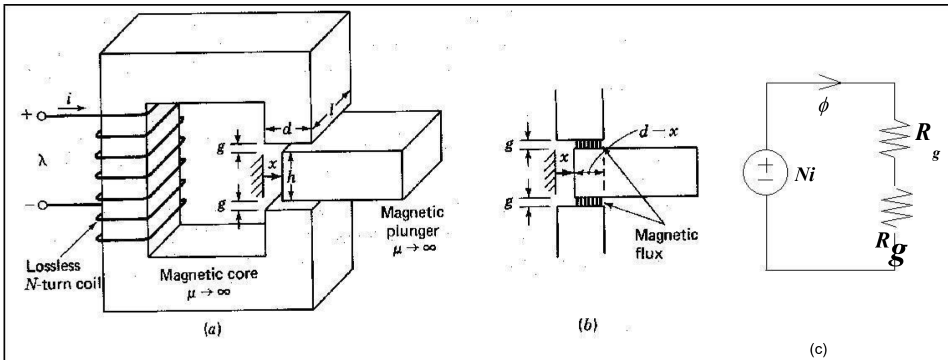
For a magnetically linear system, the above expression becomes

$$Wf'(i, x) = \frac{1}{2} i^2 L(x)$$

and the force acting on the plunger is then

$$F = \frac{\partial Wf'(i, x)}{\partial x} = \frac{1}{2} i^2 \frac{dL(x)}{dx}$$

Example: *Calculate the force acting on the plunger of a linear actuator discussed in this section.*



A singly excited linear actuator

Solution:

Assume the permeability of the magnetic core of the actuator is infinite, and hence the system can be treated as magnetically linear. From the equivalent magnetic circuit of the actuator shown in figure (c) above, one can readily find the self inductance of the excitation winding as

$$L(x) = \frac{N^2}{2R_g} = \frac{\mu_0 N^2 l(d-x)}{2g}$$

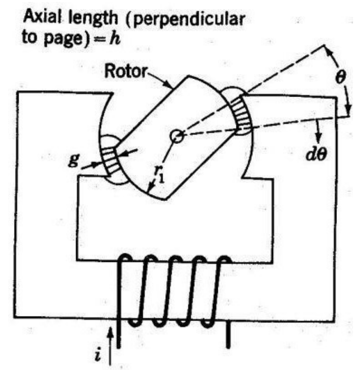
Therefore, the force acting on the plunger is

$$F = \frac{1}{2} i^2 \frac{dL(x)}{dx} = -\frac{\mu_0 l}{4g}(Ni)^2$$

The minus sign of the force indicates that the direction of the force is to reduce the displacement so as to reduce the reluctance of the air gaps. Since this force is caused by the variation of magnetic reluctance of the magnetic circuit, it is known as the ***reluctance force****.*

Singly Excited Rotating Actuator

The singly excited linear actuator mentioned above becomes a singly excited rotating actuator if the linearly movable plunger is replaced by a rotor, as illustrated in the diagram below. Through a derivation similar to that for a singly excited linear actuator, one can readily obtain that the torque acting on the rotor can be expressed as the negative partial derivative of the energy stored in the magnetic field against the angular displacement or as the positive partial derivative of the co-energy against the angular displacement, as summarized in the following table.



A singly excited rotating actuator

Table: Torque in a singly excited rotating actuator

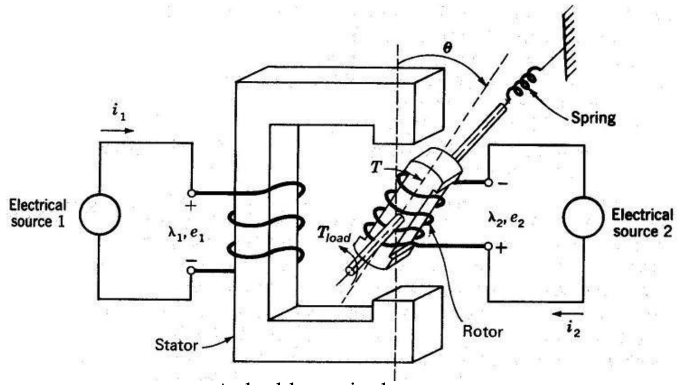
	Energy	**Co-energy**
In general,	$dW_f = i\,d\lambda - T\,d\theta$	$dW_f' = \lambda\,di + T\,d\theta$
	$W_f(\lambda,\theta) = \int_0^{\lambda} i(\lambda,\theta)\,d\lambda$	$W_f'(i,\theta) = \int_0^{i} \lambda(i,\theta)\,di$
	$i = \frac{\partial W_f(\lambda,\theta)}{\partial \lambda}$	$\lambda = \frac{\partial W_f'(i,\theta)}{\partial i}$
	$T = -\frac{\partial W_f(\lambda,\theta)}{\partial \theta}$	$T = \frac{\partial W_f'(i,\theta)}{\partial \theta}$
If the permeability is a constant,	$W_f(\lambda,\theta) = \frac{1}{2}\frac{\lambda^2}{L(\theta)}$	$W_f'(i,\theta) = \frac{1}{2} i^2 L(\theta)$
	$T = \frac{1}{2}\left[\frac{\lambda}{L(\theta)}\right]^2 \frac{dL(\theta)}{d\theta} = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$	$T = \frac{1}{2} i^2 \frac{dL(\theta)}{d\theta}$

Doubly Excited Rotating Actuator

The general principle for force and torque calculation discussed above is equally applicable to multi-excited systems. Consider a doubly excited rotating actuator shown schematically in the diagram below as an example. The differential energy and co-energy functions can be derived as following:

$$dW_f = dW_e - dW_m$$

where $dW_e = e_1 i_1 dt + e_2 i_2 dt$



A doubly excited actuator

$$e_1 = \frac{d\lambda_1}{dt}, \qquad e_2 = \frac{d\lambda_2}{dt}$$

and $$dW_m = Td\theta$$

Hence,

$$dW_f(\lambda_1, \lambda_2, \theta) = i_1 d\lambda_1 + i_2 d\lambda_2 - Td\theta$$

$$= \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_1} d\lambda_1 + \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \lambda_2} d\lambda_2$$

$$+ \frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} d$$

and

$$dW_f'(i_1, i_2, \theta) = d\left[i_1\lambda_1 + i_2\lambda_2 - W_f(\lambda_1, \lambda_2, \theta)\right]$$

$$= \lambda_1 di_1 + \lambda_2 di_2 + Td\theta$$

$$= \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_1} di_1 + \frac{\partial W_f'(i_1, i_2, \theta)}{\partial i_2} di_2$$

$$+ \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta} d\theta$$

Therefore, comparing the corresponding differential terms, we obtain

$$T = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta}$$

or $$T = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$

For magnetically linear systems, currents and flux linkages can be related by constant inductances as following

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

or

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix}$$

where $L_{12}=L_{21}$, $\Gamma_{11}=L_{22}/\Delta$, $\Gamma_{12}=\Gamma_{21}=-L_{12}/\Delta$, $\Gamma_{22}=L_{11}/\Delta$, and $\Delta=L_{11}L_{22}-L_{12}^2$. The magnetic energy and co-energy can then be expressed as

$$W_f(\lambda_1, \lambda_2, \theta) = \frac{1}{2}\Gamma_{11}\lambda_1^2 + \frac{1}{2}\Gamma_{22}\lambda_2^2 + \Gamma_{12}\lambda_1\lambda_2$$

and

$$W_f'(i_1, i_2, \theta) = \frac{1}{2}L_{11}i_1^2 + \frac{1}{2}L_{22}i_2^2 + L_{12}i_1 i_2$$

respectively, and it can be shown that they are equal.

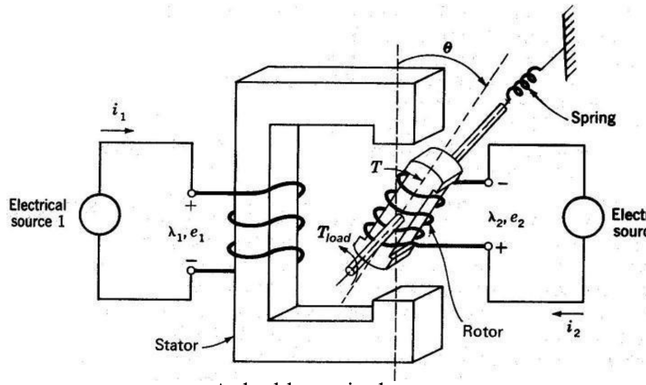
Therefore, the torque acting on the rotor can be calculated as

$$T = -\frac{\partial W_f(\lambda_1, \lambda_2, \theta)}{\partial \theta} = \frac{\partial W_f'(i_1, i_2, \theta)}{\partial \theta}$$
$$= \frac{1}{2} i_1^2 \frac{dL_{11}(\theta)}{\partial \theta} + \frac{1}{2} i_2^2 \frac{dL_{22}(\theta)}{\partial \theta} + i_1 i_2 \frac{dL_{12}(\theta)}{\partial \theta}$$

Because of the salient (not round) structure of the rotor, the self inductance of the stator is a function of the rotor position and the first term on the right hand side of the above torque expression is nonzero for that $dL_{11}/d\theta \neq 0$. Similarly, the second term on the right hand side of the above torque express is nonzero because of the salient structure of the stator. Therefore, these two terms are known as the reluctance torque component. The last term in the torque expression, however, is only related to the relative position of the stator and rotor and is independent of the shape of the stator and rotor poles.

Model of Electromechanical Systems

To illustrate the general principle for modeling of an electromechanical system, we still use the doubly excited rotating actuator discussed above as an example. For convenience, we plot it here again. As discussed in the introduction, the mathematical model of an electromechanical system consists of circuit equations for the electrical subsystem and force



A doubly excited actuator

or torque balance equations for the mechanical subsystem, whereas the interactions between the two subsystems via the magnetic field can be expressed in terms of the *emf*'s and the electromagnetic force or torque. Thus, for the doubly excited rotating actuator, we can write

$$
\begin{aligned}
v_1 &= R_1 i_1 + \frac{d\lambda_1}{dt} = R_1 i_1 + \frac{d(\lambda_{11} + \lambda_{12})}{dt} \\
&= R_1 i_1 + L_{11}\frac{di_1}{dt} + i_1 \frac{dL_{11}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{12}\frac{di_2}{dt} + i_2 \frac{dL_{12}(\theta)}{d\theta}\frac{d\theta}{dt} \\
&= \left[R_1 + \omega_r \frac{dL_{11}(\theta)}{d\theta}\right] i_1 + \omega_r \frac{dL_{12}(\theta)}{d\theta} i_2 + L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}
\end{aligned}
$$

$$
\begin{aligned}
v_2 &= R_2 i_2 + \frac{d\lambda_2}{dt} = R_2 i_2 + \frac{d(\lambda_{21} + \lambda_{22})}{dt} \\
&= R_2 i_2 + L_{12}\frac{di_1}{dt} + i_1 \frac{dL_{12}(\theta)}{d\theta}\frac{d\theta}{dt} + L_{22}\frac{di_2}{dt} + i_2 \frac{dL_{22}(\theta)}{d\theta}\frac{d\theta}{dt} \\
&= \omega_r \frac{dL_{12}(\theta)}{d\theta} i_1 + \left[R_2 + \omega_r \frac{dL_{22}(\theta)}{d\theta}\right] i_2 + L_{12}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}
\end{aligned}
$$

and $$T - T_{load} = J\frac{d\omega_r}{dt}$$

where $$\omega_r = \frac{d\theta}{dt}$$

is the angular speed of the rotor, T_{load} the load torque, and J the inertia of the rotor and the mechanical load which is coupled to the rotor shaft.

The above equations are nonlinear differential equations which can only be solved numerically. In the format of state equations, the above equations can be rewritten as

$$\frac{di_1}{dt} = -\frac{1}{L_{11}}\left[R_1 + \frac{dL_{11}(\theta)}{d\theta}\omega_r\right] i_1 - \frac{1}{L_{11}}\frac{dL_{12}(\theta)}{d\theta}\omega_r i_2 - \frac{L_{12}}{L_{11}}\frac{di_2}{dt} + \frac{1}{L_{11}} v_1$$

$$\frac{di_2}{dt} = -\frac{1}{L_{22}}\frac{dL_{12}(\theta)}{d\theta}\omega_r i_1 - \frac{1}{L_{22}}\left[R_2 + \frac{dL_{22}(\theta)}{d\theta}\omega_r\right] i_2 - \frac{L_{12}}{L_{22}}\frac{di_1}{dt} + \frac{1}{L_{22}} v_2$$

$$\frac{d\omega_r}{dt} = \frac{1}{J} T - \frac{1}{J} T_{load}$$

and
$$\frac{d\theta}{dt} = \omega_r$$

Together with the specified initial conditions (the state of the system at time zero in terms of the state variables):

$$I_1\big|_{t=0} = i_{10},\ i_2\big|_{t=0} = i_{20},\ \omega_r\big|_{t=0} = \omega_{r0},\ \text{and}\ \theta\big|_{t=0} = \theta_0,$$

the above state equations can be used to simulate the dynamic performance of the doubly excited rotating actuator.

Following the same rule, we can derive the state equation model of any electromechanical systems.

UNIT-I

Introduction to DC Machines:-

Basic principle ⇒ Electromagnetic induction (for any Electrical Device)

⇓

faraday's laws

1st law:- when a moving conductor kept in a magnetic field it induces some e.m.f.

2nd law:- the magnitude of induced emf is directly propotional to rate of change of flux linkages.

$$e \propto \frac{d\phi}{dt} \text{ (or) } \frac{d\psi}{dt}$$

ϕ → flux; ψ → flux linkages

Induced emf:- voltage (one of the force)

Electromotive force is the electrical action produced by a non-electrical source. A device that converts one form of energy into electrical energy it provides emf as its output.

Basic requirements:-

* set of conductors
* Magnetic field
* Time variation (or) Space variation b/n set of conductors and Magnetic field.

Types:-

⇒ statically induced emf

⇒ Dynamically induced emf

Statically induced emf:- when the stationary conductor is kept in the time varying magnetic field.

magnitude by faraday's law 2nd

ex:- transformer

direction by Lenz's law

Dynamically induced emf:- when a rotating set of conductors placed in a stationary Magnetic field.

direnction by fleming's right hand rule.

ex:- Motors, Generator.

fleming's Right hand rule:-

It shows the direction of induced current when a conductor attached to a circuit moves in a magnetic field.

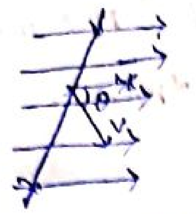
* Thumb is pointed in the direction of the motion of the conductor relative to the magnetic field.
* Middle finger represents the direction of induced current with in the conductor. (Induced e.m.f)
* the fore finger is pointed in the direction of the magnetic field.

Lenz's Law:-

the direction of the Current induced in a conductor by a changing magnetic field such that the magnetic field created by the induced current opposes the initial changing magnetic field.

* Lenz's law defined as direction of electromagnetically induced emf in such as to oppose very cause of producing it.

* flux cutting law:- $E_d = Blv \sin\theta$

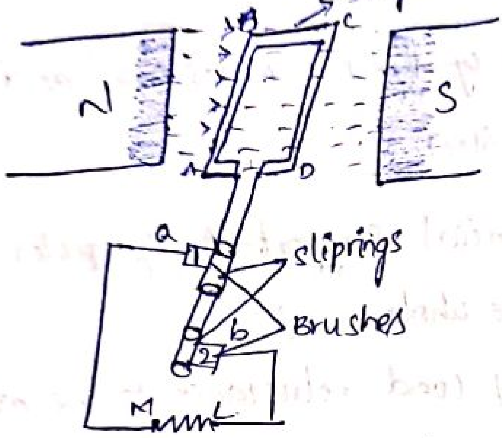


B = flux density

l = length of conductor

v = velocity

→ principle of operation:-

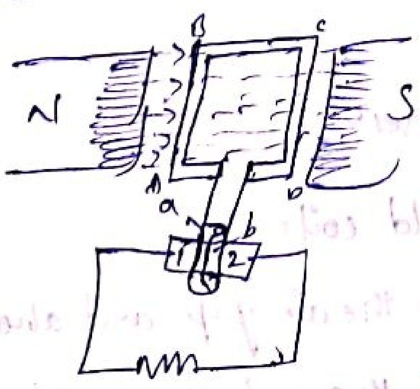


when it is rectangular coil is in vertical position total flux passing from N to S is maximum and rate of change of flux is mini..

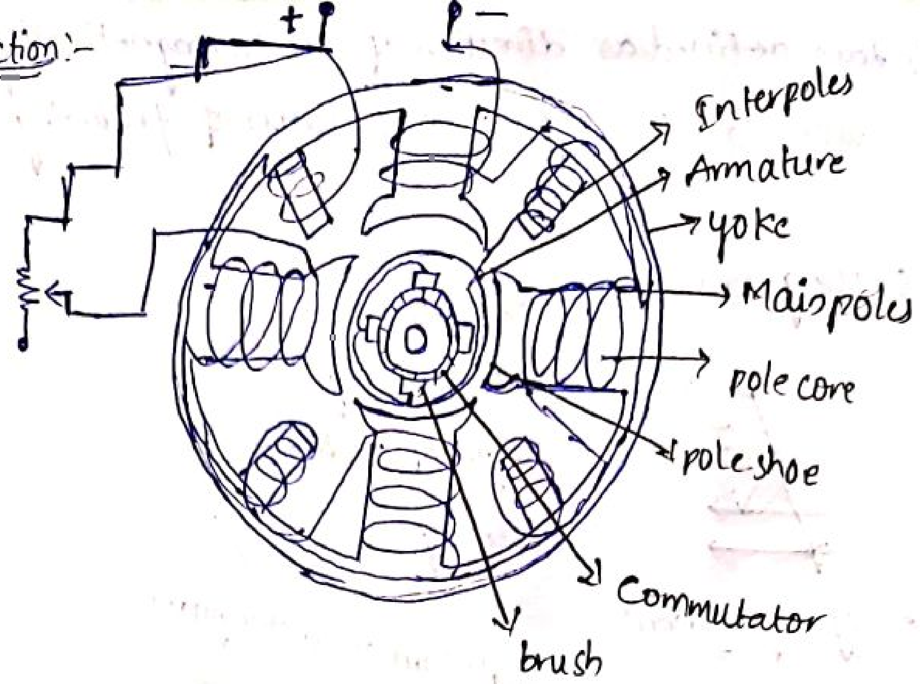
Brushes ⇒ collects current from the circuit and convey it to the external load.

Commutator: converts AC supply to DC supply.

also called as Mechanical rectifier.



Construction:-



yoke:-

yoke is the outer frame of the D.C Machine and is generally made of Iron (or) fabricated steel.

purpose:- 1. It provides Mechanical support for the poles and acts as a protective cover for the whole machine.

2. It provides a return path of Load reluctance to the magnetic flux produced by poles.

field coils (or) Main poles:-

There are 2 types of pole construction. the pole core itself maybe a solid piece made out of either cast iron or cast steel. the pole is laminated.

In modern design the pole core and pole shoe are of thin laminations of annealed steel.

The pole shoe serves 2 purposes:-

1. It acts as a support to field coils

2. To spread out the flux in the air gap and also being of large construction reduces the reluctance of magnetic path.

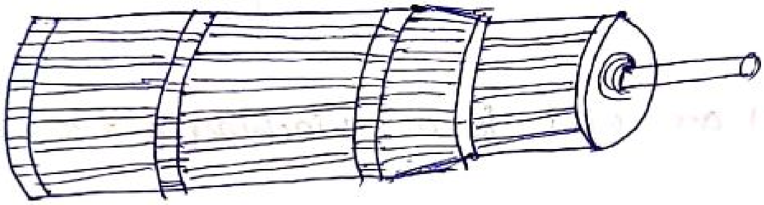
Enterpoles:- (or) commutating poles:

* It is used to reduce the disturbances b/n the Armature winding and field winding.

* To minimise the sparks in the DC Motor.

Armature core:-

Armature core houses the Armature conductors or coils and causes them to rotate and hence cut the magnetic ~~field~~ flux of the field magnets



Armature is the rotating part of the D.C. Machine, Armature core is cylindrical in construction it is made up of high grade silicon steel. to reduce power loss.

Armature Winding:-

The Armature winding of a DC Machine is a closed circuit winding. the Armature winding is made up of copper. there are 2 types of Armature windings

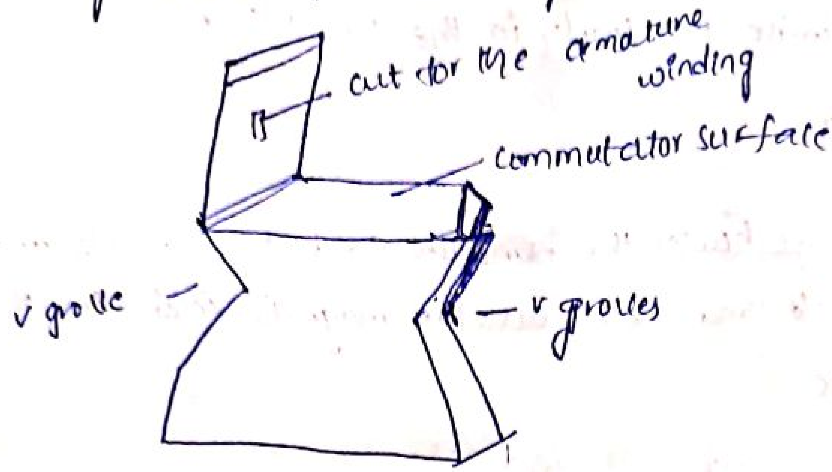
1. Lap winding → parallel connection of winding
2. Wave winding ⇒ series connection of winding

Commutator:-

It is a mechanical rectifier which converts alternating voltage which generated in the armature winding into direct voltage across the brushes. This is made up of copper segments insulated from each other.

the no. of commutator segments can be decided by no. of coils.

each commutator segment is connected to the Armature conductor by means of a copper lug or strip



Brushes:-

The brushes are used for DC Machines and are divided into 5 classes.

1. Metal Graphite
2. Copper Graphite
3. Electro Graphite
4. Graphite
5. carbon

It collects the current from the commutator and conveys it to external Load.

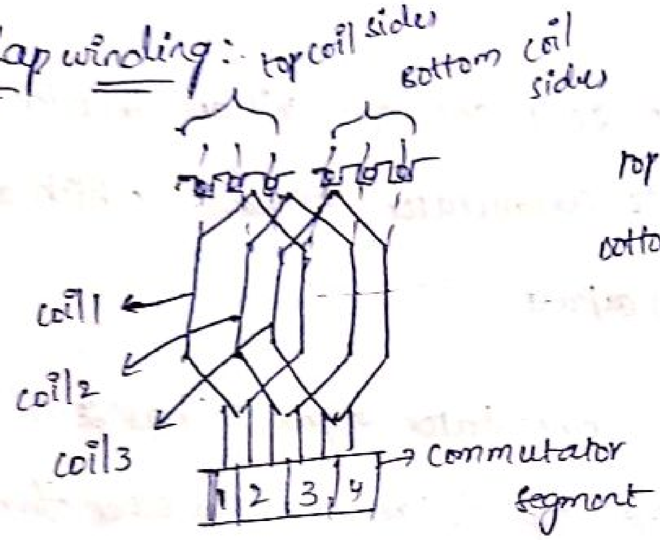
A flexible copper pigtail mounted at the top of the brush to carry the current

- Brushes are Made with carbon are used for small DC Machines.

= electro Graphite Carbon's are used for all types of DC Machines.

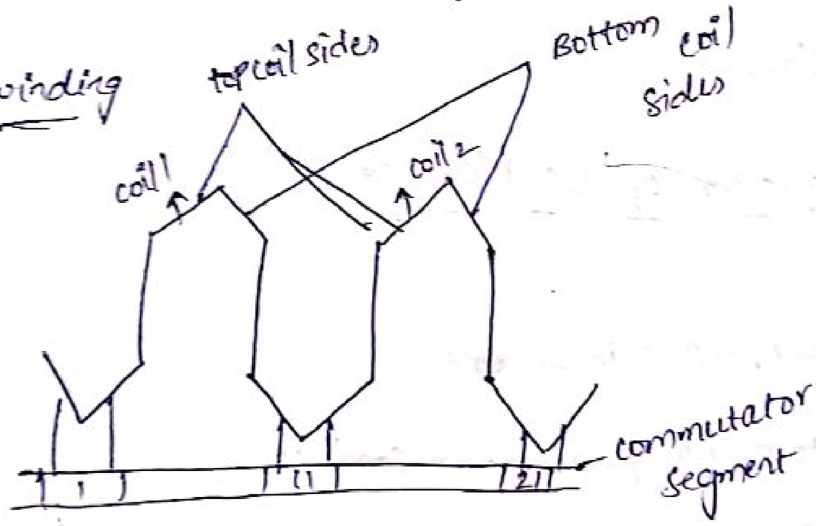
Armature Winding:-

Lap winding:



top coil sides - odd number
bottom " " - even number

Wave winding



Lapwinding:

In Lap winding the 2 ends ~~coil~~ ends of the coil are connected to 2 adjacent commutator segments as shown in the above figure i.e., one top coil side of the -- and the other from the Bottom coil - sides are connected to adjacent commutator segment.

the above figure shows that bottom coil side of coil 1 and top coil side of coil 2 are connected to segment 2, Bottom coil side of coil 2 and top coil side of coil 3 are connected to segment 3 and so on..

Wave Winding:-

In wave winding the 2 coil ends of a coil are bent in opposite directions and connected to commutator segments which are approximately 2 pole pitches apart.

- In wave winding each commutator segment has 2 coil ends connected to it one from the top coil side and the other from the bottom coil side.

* Emf equation of DC Machine

- let $\phi \rightarrow$ flux per pole in webers

$Z \rightarrow$ total no. of armature conductors

$P \rightarrow$ no. of Generator poles

$A \rightarrow$ no. of parallel paths in Armature

$N \rightarrow$ Armature rotation in revolutions per minute (speed)

$E \rightarrow$ Emf induced in any parallel path in Armature

$E_g \rightarrow$ Generated Emf in any one of the parallel paths

Average Emf generated per conductor $= \frac{d\phi}{dt}$ volt.

flux cut by one conductor in one revolution of the Armature $= d\phi = \phi p$ webers

no. of revolutions per second $= N/60$

time for one revolution $dt = \frac{60}{N}$ sec

Hence According to faraday's law of electromagnetic induction,

Emf generated per conductor $E = \frac{d\phi}{dt} = \frac{\phi P N}{60}$ volts

for wave windings,

No. of parallel paths $= 2$

no. of conductor's in one path $= Z/2$

emf Generated per path $= \frac{\phi P N}{60} \times \frac{Z}{2}$ volts

$$= \frac{\phi Z N}{60} \times \frac{P}{2}$$

for Lap winding:-

No. of parallel paths $= A$

No. of conductor's in one path $= Z/A$

emf Generated per path $= \frac{\phi P N}{60} \times \frac{Z}{A}$

$$= \frac{\phi Z N}{60} \times \frac{P}{A}$$

for wave winding $A = 2$

for Lap winding $A = P$

* The Lap wound Armature of a 4-pole Generator Armature has 51 slots. each slot contains 20 conductors: what will be the emf generated in machine when driven 1500 rpm. If the useful flux per pole is 0.01 wb

sol^n:- Given that

$P = 4$

no. of slots = 51

no. of conductors in each slot = 20

$\phi = 0.01$ wb

Total no of conductor = 51×20 = 1020

$$E_g = \frac{\phi Z N}{60}\left[P/A\right] = \frac{0.01 \times 1020 \times 1500}{60} \times \left[4/4\right]$$

$$= 255 \text{ volts}$$

* A 6-pole of lap wound Generator Armature has 720 conductors, a flux of 30 milliwebers and a speed of 600 rpm. calculate the emf generated. If the same Armature is wave wound at what speed it be driven to generate 600 volt.

~~sol^n Given that~~

sol^n:- Given that

$P = 6$ $\quad A = 6$

$\phi = 30 \times 10^{-3}$

$N = 600$

$Z = 720$

$$E_g = \frac{\phi Z N}{60} \times \left[P/A\right] =$$

$$= \frac{30\times10^{-3}\times 720\times \cancel{600}^{10}}{\cancel{60}}\times[6/6]$$

$= 216$ volts.

* Given $Eg = 600$

$N = ?$

$$Eg = \frac{\phi Z N}{60}\times[P/2]$$

$$600 = \frac{\cancel{600}^{720}\times 30\times10^{-3}\times N}{\cancel{60}_{20}}\times[\cancel{6}^{6}/\cancel{2}]$$

~~12000~~ ~~40000~~ $\times 10^{3}$ ~~=N~~

$N = 555$

* The Armature of a 2 pole 220V Generature has 400 conductors and runs at 300 rpm. calculate the usefull flux per pole the no. of turns in each field coil is 1200, what is the Average value of the Emf induced in each coil on breaking the field, if the flux dies away completely in 0.10 secon.

Sol:-

$E = 220V$

$N = 300$ rpm

$Z = 400$

$P = 2$, $A = 2$

$$E = \frac{\phi Z N}{60}[P/A]$$

$$\phi = \frac{E\times60\times A}{P\times Z\times N} = \frac{2\cancel{20}\times 6\cancel{0}\times \cancel{2}}{\cancel{2}\times 4\cancel{00}\times 3\cancel{00}} = 0.1 \text{ wb}$$

$$\varepsilon \propto \frac{d\phi}{dt}$$

$$\varepsilon = -N \frac{d\phi}{dt}$$

$$= 1200 \times \frac{0.11}{0.10} = -1320 \text{ Vt}$$

* An 8-pole D.C Generator has per pole flux of 40mill wb and winding is connected in lap with 960 conductors. calculate the Generated emf on open circuit when it runs at 400rpm. If the Armature is wave wound at what speed must the machine be driven to generate the same voltage.

sol:-

$\phi = 40\times10^{-3}$

$Z = 960$

$P = 8$

$A = 8$

$N = 400$

$$\varepsilon = \frac{\phi Z N}{60} \times P/A = \frac{40\times10^{-3} \times 960 \times 400}{60} \times \frac{8}{8}$$

$$= 256 \text{ volt}$$

$$\varepsilon = \frac{\phi Z N}{60} \times \frac{P}{2} = N \Rightarrow \frac{256 \times 60 \times 2}{40\times10^{-3} \times 960 \times 8}$$

$$= 100 \text{ rpm}$$

* The Armature of a DC Generator is wave wound with 2 poles There are 40 slots on the armature Surface and 12 turns per coil. The armature winding is double layer winding. the resistance of each conductor is 10 milliohms. find the resistance of the Armature and emf generated, if flux per pole is 45 milli webers and generator is rotated at 350 rpm. repeat the calculation for lap winding.

Sol:- wave winding

$P = 2$

$A = 2$

$Z = 40 \times 12 \times 2$ (total No. of conductors)

$\phi = 45 \times 10^{-3}$

$N = 350$

$r = 10 \times 10^{-3}\ \Omega$

$= 10^{-2}$

$$E_g = \frac{\phi Z N}{60} \times P/2$$

$$= \frac{45 \times 10^{-3} \times 40 \times 12 \times 2 \times 350}{60} \times \frac{2}{2}$$

$= 252$ volts

Sol:- lap winding:- restistance of Armature $= 960 \times 10^{-2}$

$= 9.6\ \Omega$

lap winding

$$E_g = \frac{\phi Z N}{60} \times \frac{P}{A}$$

$$= \frac{45 \times 10^{-3} \times 40 \times 12 \times 2 \times 350}{60} \times \frac{2}{2}$$

$= 252$ volts

* calculate the Armature resistance of 6 pole lap wound Armature winding from the following data...

[1 turn contains 2 conductors]

No. of slots = 150

conductors per slot = 8

Mean length of 1 turn = 250 cm

cross section of each conductor = 10mm × 2.5mm

The resistance of 1mt of copper wire of $1mm^2$ in cross section is $0.0213\,\Omega$

sol:-

$Z = 150 \times 8$

$P = 6$, $A = 6$ $\qquad \rho = 0.0213\,\Omega$

$l = 250 \times 10^{-2} = 2.5$ mt

Area $A = 25 \times 10^{-6}$ mt

Mean length of Conductor $= \frac{250}{2} = 125$ cm $= 1.25$ mt

resistance of each conductor $= \frac{\rho l}{a} = \frac{0.0213 \times 1.25}{25 \times 10^{-6}}$

$= 1.065 \times 10^{-3}$

no. of conductors in series $Z/A = \frac{150 \times 8}{6} = 200$

resistance of each parallel path $= 200 \times 1.065 \times 10^{-3}$

$= 213 \times 10^{-3}$

Armature resistance $= \frac{0.213}{6}$

$= 0.0355\,\Omega$

Types of D.C Machines (Based on Excitation)

The method of providing the field current in called excitation of a DC Machine.

there are two types of Excitations:

field working is for generating flux

1. Seperately Exitated DC Machine

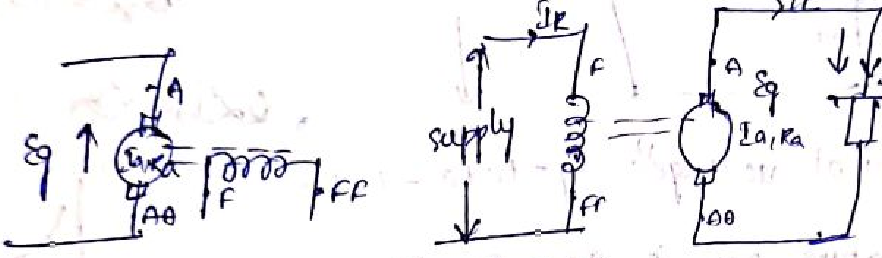
⇓ Separate supply to field winding

2. Self excitated DC Machine

⇓ supply to field winding is obtained from armature voltage

Seperately excited DC Machine:-

also called as servo motor



U - terminal voltage

Armature Current ($I_a = I_L$)

we will get constant supply in seperately exited machine, becz flux depends of I_f.

terminal voltage, $U = E_g - I_a R_a$

Electric power developed $P = E_g I_a$

Power delivered to load $P = U I_a = (E_g - I_a R_a) I_a$

U series $= E_g + I_a R_a + U_{brush}$

2. Self excited DC Machine:-

By residual Magnetism self voltage was developed. so self excitation held in the machine. The field poles must have residual Magnetism. so that when the armature rotates a residual voltage appears across the Brushes. This residual voltage should establish a current in the field winding. so, as to reinforce the residual flux.

Based on self excitation the field winding is connected to the armature in the manner of 3-different types.

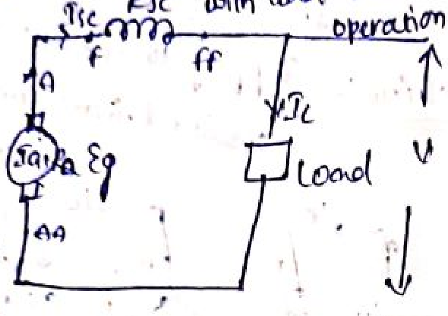
1. Series Generator
2. Shunt Generator
3. Compound Generator

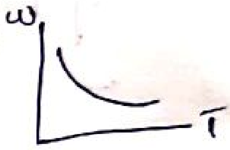
Series Generator:- In a series wound DC motor, the speed varies with load and field winding is in series with armature winding.

Operation wise this is its main difference from a shunt wound DC motor.

Torque = $k a \phi \omega$

$\phi \propto I_{se}$





Structure of large wire is to carry total current from armature

Terminal voltage $V = E_g - I_a R_a - I_{sc} R_{se}$

char:- It develops a large amount of starting torque

Armature current $I_a = I_{sc} = I_L$

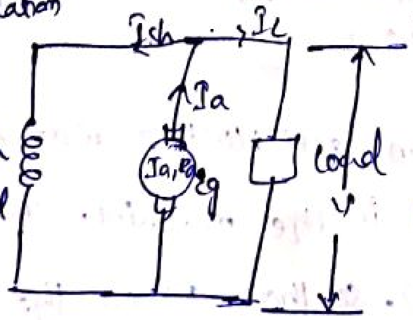
Electrical power generated $P = E_g I_a$ (in armature)

power delivered to load $P = V I_a$

$= (E_g - I_a R_a - I_{sc} R_{sc}) I_a$

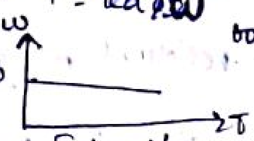
Shunt Generator:- field winding is in parallel with Armature winding

* good speed regulation
* The shunt connected motor offers simplified control for reversing. This is especially beneficial in regenerative drives



The torque equation of DC motor resembles

$T = k a \phi \omega$



torque speed characteristic

Armature current $I_a = I_L + I_{sh}$ $I_{sh} = \frac{V}{R_{sh}}$

Terminal voltage $V = E_g - I_a R_a$ $\phi \propto I_{sh}$

It constant b/t V supply const.

here $I_{sh} R_{sh} = V$ [parallel connection] power developed = $E_g I_a$

power delivered to load = $V I_L$

compound Generator:-

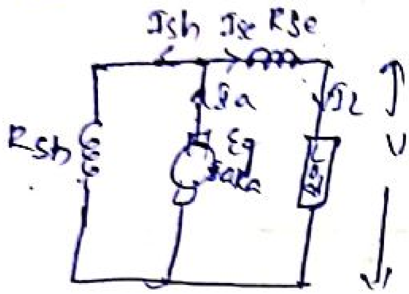
compound winding → series winding + ~~parallel~~ shunt winding

⇓

2 types → short shunt winding, long shunt winding

Generators.

short shunt



⇓

shunt field winding is parallel with only Armature winding

Armature current $I_a = I_{sh} + I_{se}$

$I_{sh} + I_L$

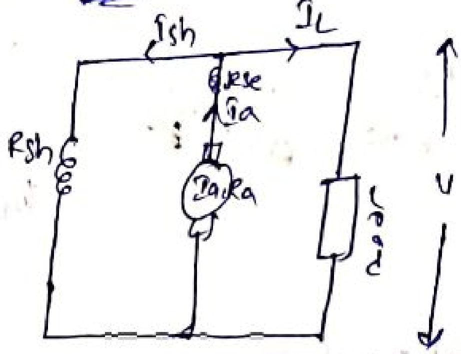
$I_{sh} = \frac{V + I_{se}R_{se}}{R_{sh}}$

terminal voltage $V = E_g - I_a R_a - I_{se} R_{se}$

power developed $P = E_g I_a$

power delivered to load $P = (V) I_L$

longshunt



⇓

shunt field winding is parallel with both Armature and series winding

Armature current $I_a = I_L + I_{sh}$

terminal voltage $V = E_g - I_a R_a - I_{se} R_{se}$

$I_{sh} = \frac{V}{R_{sh}}$

power developed $P = E_g I_a$

power delivered to load $P = V I_L$

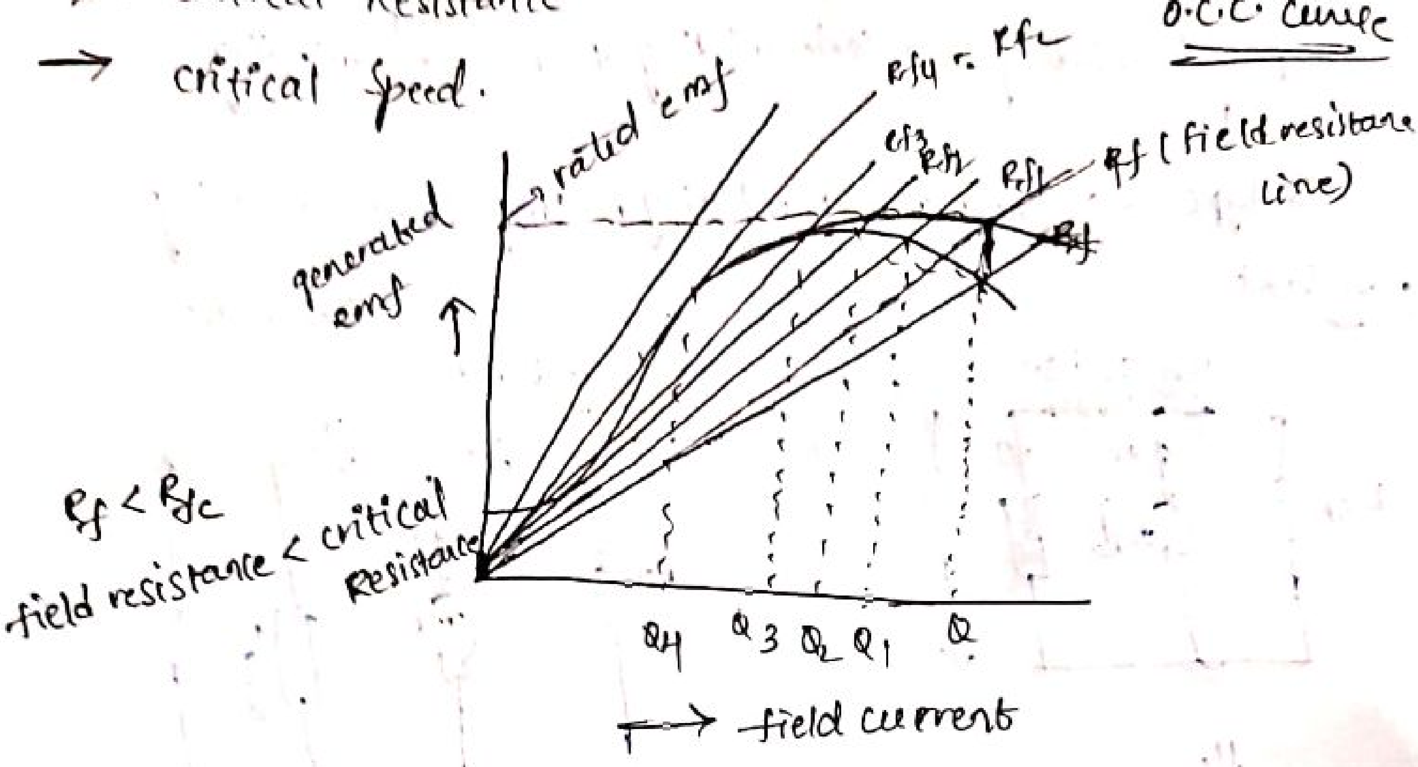
Building up of the emf:- [self excitation]

→ Residual Magnetism

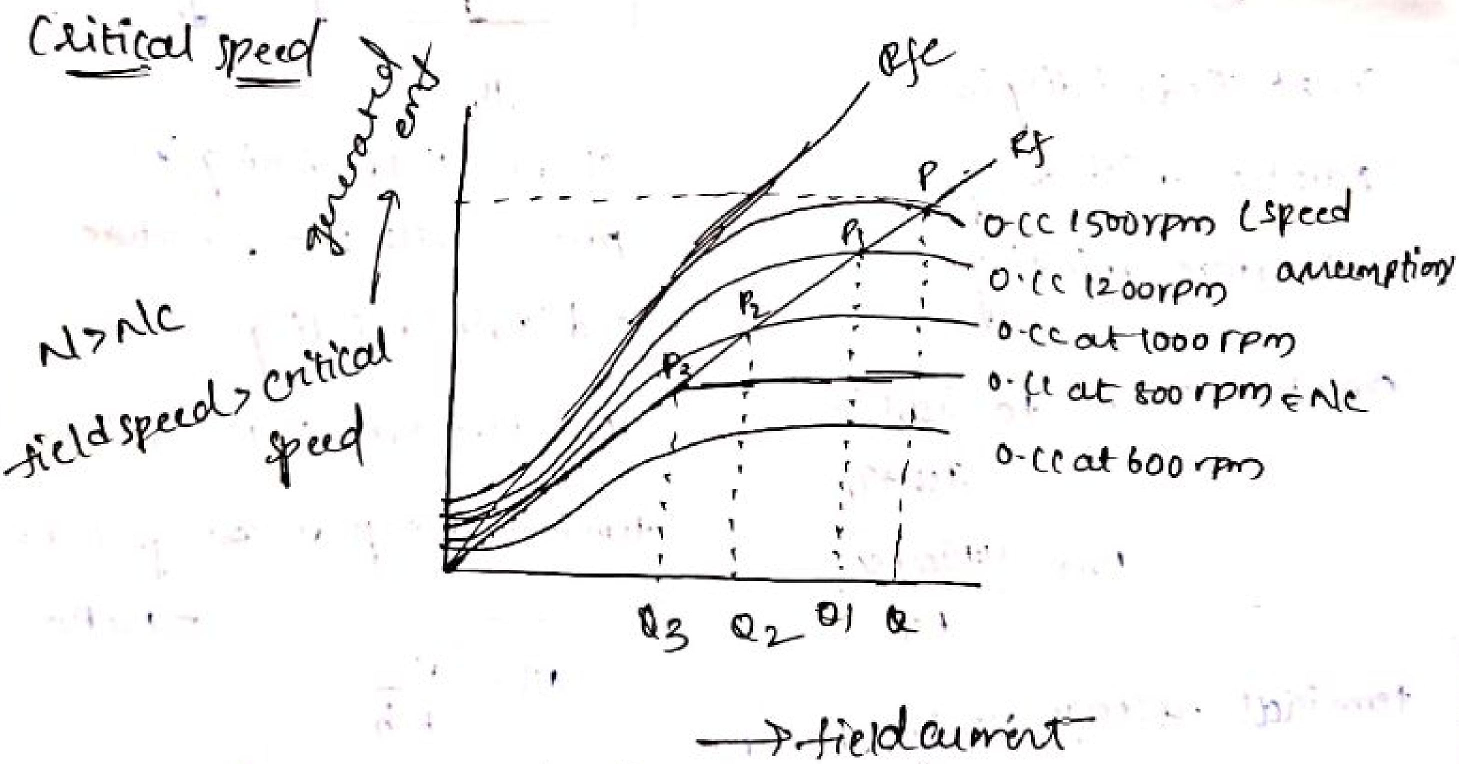
→ connection's of field terminals

→ critical Resistance

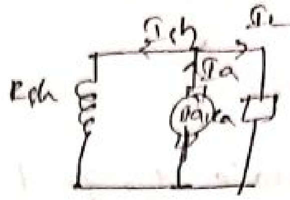
→ critical speed.



Critical speed



A 100 kW, 240 V shunt generator has a field resistance of 55Ω and the armature resistance of 0.067Ω. Find the full load generated voltage.



Sol:- power $P = 100$ kW

voltage $V = 240$V

$R_{sh} = 55\Omega$

Armature resistance $R_a = 0.067\Omega$

$$V = E_g - I_a R_a$$

$$E_g = V + I_a R_a$$

$$I_a = I_L + I_{sh}$$

$$I_L = P/V \quad , \quad I_{sh} = \frac{V}{R_{sh}}$$

$$= \frac{100 \times 1000}{240} \qquad = \frac{240}{55}$$

$$= 416.66 \qquad = 4.3636$$

$$\therefore I_a = I_L + I_{sh} = 421.023$$

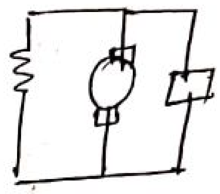
$$\cancel{E_g = 240 + 421.023}$$

$$\cancel{= 661.023\ V}$$

$$E_g = 240 + 421.023 \times 0.067$$

$$= 268.208 \text{ Volts}$$

A four pole DC-shunt generator with a wave wound Armature has to supply a load of 500 lamps each of 100 w at 250V allowing 10V for the voltage drop in the connecting leads b/n the generator and the load and drop of 1 volt per brush, calculate the speed at which the generator should be driven. The flux per pole is 30milli w and the Armature and shunt field resistances are 0.05Ω and 65Ω respectively. the no. of Armature conductors is 390.



sol:-

$P = 500 \times 100$ w no. of poles

$V = 250V$

voltage across shunt $= 250+10 = 260$

flux $\phi = 30 \times 10^{-3}$ w no. of conductors $Z = 390$

$R_A = 0.05$, $R_{sh} = 65\Omega$

$$E_g = \frac{\phi Z N}{60}\left[P/A\right]$$

$$N = \frac{E_g \times 60 \times A}{\phi \times Z \times P}$$

$E_g = V + I_a R_a$, $I_a = I_L + I_{sh}$

$P = 500 \times 100 = 50,000$

$I_L = \frac{P}{V} = \frac{50,000}{V}$, $I_{sh} = \frac{V}{R_{sh}}$

$= 200$ V $= \frac{260}{65} = 4A$

$V_{sht} = 250+10 = 260$ [10V drop at load and generation)

E_g =

$I_a = I_L + I_{sh}$

$= 204$ amp

$E_g = V + I_a R_a +$ Brush drop

$= 260 + 204 \times 0.05 \times 2 \times 1$

$= 272.2V$

generally

[If they not mention the no. of brushes (for every machine we are taking 2 brushes only)

(1V drop)

$$N = \frac{272.2 \times 60 \times 2}{30 \times 10^{-3} \times 390 \times 4}$$

$= 697.929$

≈ 698 rpm

* A DC shunt generator run's at 400 rpm and delivers 500 kW to bus bar having a constant voltage of 400 V. Assuming the field excitation to be constant at 5Amp, calculate the speed at which the generator must run if the load on it is to be reduced to 300kW. The armature resistance is 0.015Ω.

Sol:

$N_1 = 400$ rpm

$P_1 = 500 \times 1000$ W

$V = 400$ V

$R_a = 0.015\,\Omega$

$P_2 = 300$ kW

I_{sh} = field current = 5Am

$N_2 = ?$

$E_g = V + I_a R_a$

$I_a = I_{sh} + I_L$

$$I_L = \frac{P}{V} = \frac{500 \times 1000}{400}$$

$= 1250$ amp

$I_a = 1250+5$

$= \cancel{1300\,Amp} = 1255$

$E_g = V + I_a R_a$

$= 400 + \overset{1255}{\cancel{1300}} \times 0.015$

$= 418.825\ V$

$E_{g2} = V + I_a R_A$

$I_a = I_L + I_{sh}$

$I_L = \frac{P}{V} = \frac{300 \times 1000}{400} = 750\ Amp$

$I_a = 750+5 \quad = 755\,Amp$

$E_{g2} = 400 + 755 \times 0.015$

$= 411.325\ V$

$$\frac{E_{g1}}{E_{g2}} = \frac{N_1}{N_2}$$

$$N_2 = \frac{E_{g2}}{E_{g1}} \times N_1$$

$$= \frac{411.325}{418.825} \times 400$$

$= 392.8 \approx 393\ rpm$

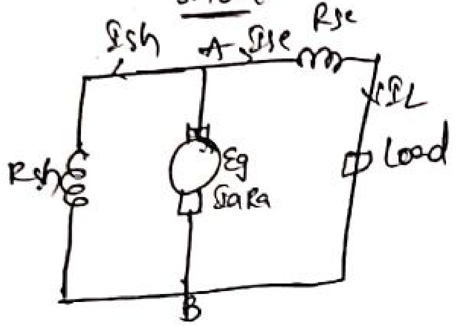
* A compound generator is to supply a load of 250 lamps each rated at 100 watt at 250V. The armature, series and shunt windings have resistances of 0.06 Ω and 0.04 Ω, 50 Ω respectively. Determine the generated emf, when the machine is connected in

1. Long shunt
2. short shunt

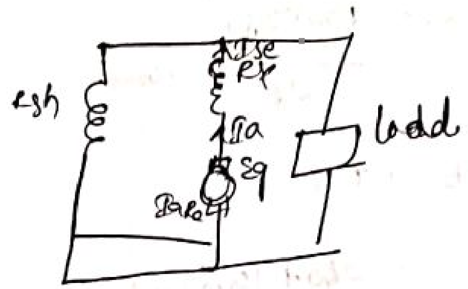
Take drop per brush as 1V.

sol:

short



long



Given $P = 250 \times 100$

$V = 250$

$R_a = 0.06$

$R_{se} = 0.04$

$R_{sh} = 50\Omega$

~~Eg = V + IaRa + IseRse + brush drop~~

$I_a = I_L + I_{sh}$

$\cancel{I_{sh}} = \frac{\cancel{V}}{\cancel{R_{sh}}}$ $\quad I_L = \frac{P}{V}$

$= \frac{\cancel{250}}{\cancel{0.04}\,50}$ $\quad = \frac{250 \times 100}{250}$

$= \cancel{50}$ $\quad = 100$

$I_a = I_{se}$

$E_g = V + I_aR_a + I_{se}R_{se} + \text{brush drop}$

$I_a = I_L + I_{sh}$

$I_L = \frac{P}{V}$

$= \frac{250 \times 100}{250} = 100\,\text{Amp}$

$I_{sh} = \frac{V}{R_{sh}}$

$= \frac{250}{50} = 5\,\text{Amp}$

$I_a = I_L + I_{sh}$

$= 100 + 5$

$= 105$

$I_a = I_L + I_{sh}$

$= 100 + 5$

$= 105$

$E_g = \cancel{400}\ 250 + 105\left[0.06 + \cancel{100} \times 0.04\right] + \text{Brushdrop}$ ~~$E_g = 250 + 105 \times 0.06$~~ $+ 100 \times 0.04$

$= 250 + 105\left[0.06 + 0.04\right] + 2 \times 1$ =

$= 262.5V$

Short shunt:-

$I_L = \frac{100 \times 250}{250} = 100A$

$V_{AB} = V + I_{se} R_{se}$

$= 250 + 100 \times 0.04$

$= 254V$

$I_{sh} = \frac{V_{AB}}{R_{sh}} = \frac{254}{50} = 5.08$

$I_a = I_L + I_{sh} = 105.08$

$E_g = V_{AB} + I_a R_a + \text{Brushdrop}$

$= 254 + 105.08 \times 0.06 + 2$

$= 262.3\ V.$

* In a long shunt Compound generator the terminal voltage is 230V. when generator delivers 150Amp. determine.

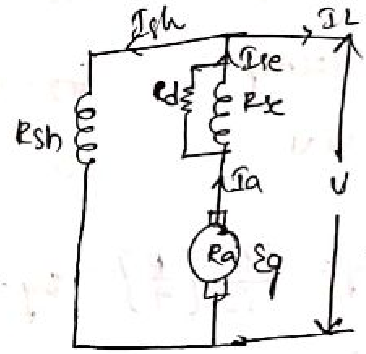
i) Induced Emf ii) total power generated

Given that shunt field, series field, diverter and Armature resistance are 92Ω, 0.15Ω, 0.03Ω, 0.032Ω respectively.

Sol:- $V_{Ter} = 230V$.

$I = 150$

$R_{sh} = 92Ω$, $R_{se} = 0.015Ω$, $R_d = 0.03Ω$, $R_a = 0.032Ω$



$E_g = V + I_a R_a$

$I_a = I_L + I_{sh}$

$I_L = 150A$

$I_{sh} = \frac{U}{R_{sh}} = \frac{230}{92} = 2.5A$

$I_a = I_L + I_{sh} = 152.5A$

Since Series field resistance and diverter resistance are in parallel. where combined resistance is,

$$= \frac{0.03 \times 0.015}{0.03 + 0.015} = 0.01Ω$$

Total Armature resistance $= 0.032 + 0.01$

$R_a = 0.042Ω$

$E_g = 230 + 152.5 \times 0.042$

$= 236.4V$

Total power generated $= E_g \times I_a = 236.4 \times 152.5$

$= 36051 W$

* A seperately exited DC generator has Armature circuit resistance of 0.1Ω and total drop at Brushes is 2V. when running at 1000 rpm, It delivers a current of 100Amp at 250 V to a load of constant resistance. If the generator speed drops to 700 rpm, with field current unaltered. find the current delivered to load.

Sol:

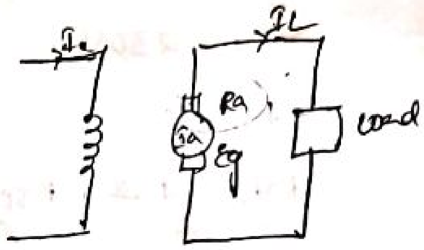
$R_a = 0.1\Omega$

Drop at Brushes $= 2V$

$N_1 = 1000 rpm$ $\quad N_2 = 700$

$I_1 = 100 Amp$ $\quad \cancel{I_{L2} = 100 Amp}$

$V = 250 V$ $\quad I_L = ?$



field current const. means flux also const.

for separate exitation

$I_L = I_a$

$E_g = \frac{\phi Z N}{60}\left[\frac{P}{A}\right]$ $\quad E_g = V + I_a R_a$

$E_g \propto N$ $\quad E_g \propto V + I_a R_a$

$\frac{V_1 + I_{a1} R_{a1} + \text{Brush}}{V_2 + I_{a2} R_{a2} + \text{Brush}} = \frac{N_1}{N_2}$ $\quad I_{a2} = ?$

$\frac{250 + 100(0.1) + 2}{250 + I_{a2}(0.1) + 2} = \frac{1000}{700}$

$\frac{262}{252 + 0.1 I_{a2}} = 1.428$

$252 + 0.1 I_{a2} = 183.473$

$0.1 I_{a2} = -68.6$

$I_{a2} = -686 A$

O.C.C. of DC shunt generator:-

load characteristics

Characteristics of DC Machine:-

1. O.C.C. (E_g vs I_f)
2. External characteristics — terminal (V vs I_L)
3. Internal characteristics. (E_g vs I_L)

↓

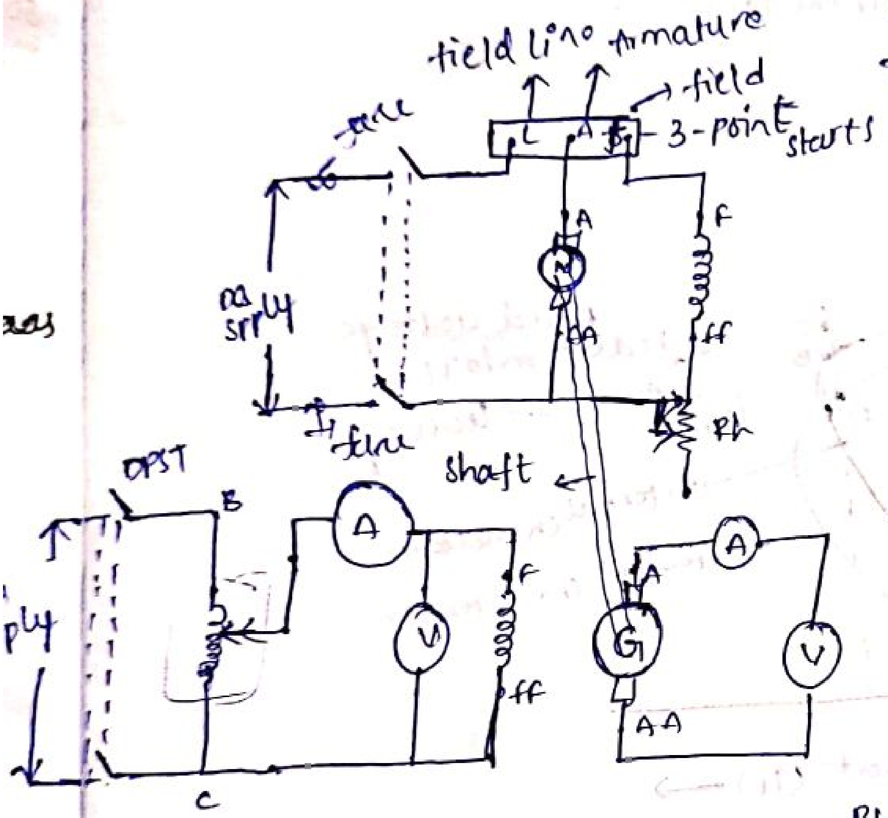
(open circuit characteristics / Magnetisation characteristics / no load characteristics)

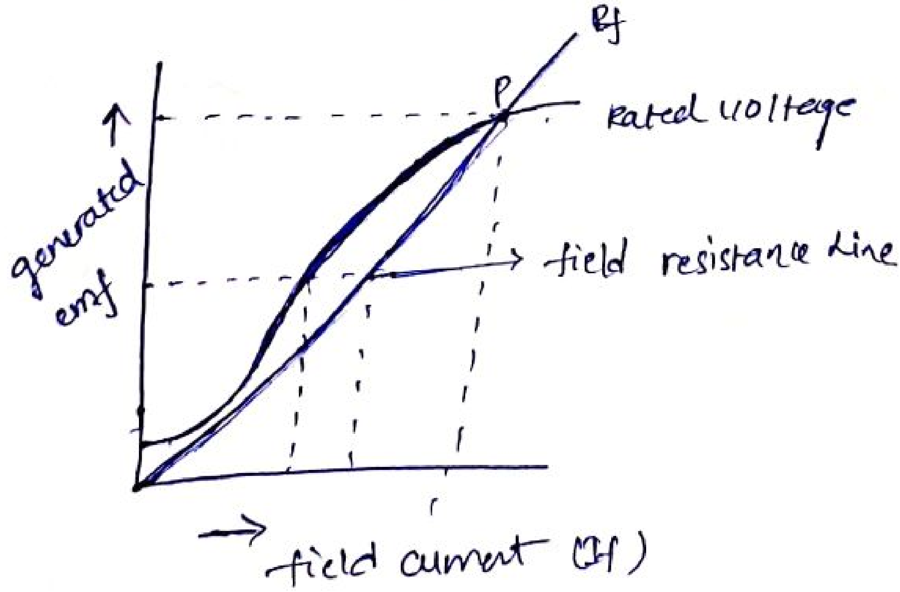
DC shunt generator:- [characteristics of shunt and seperated DC are same]

DPST - Double pole single through switch

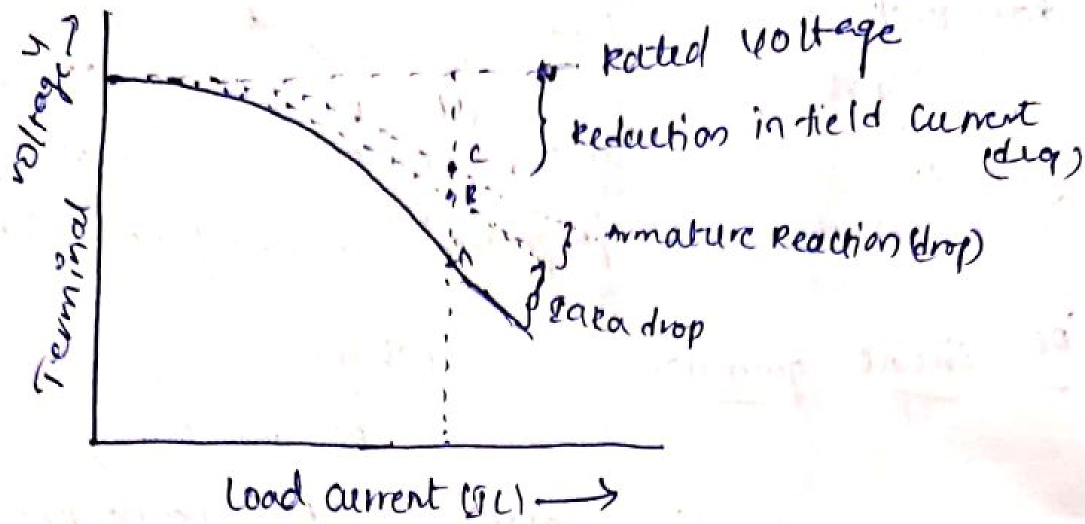
Tachometer - used to measure the speed

potential divider ↓ 3 fixed ends

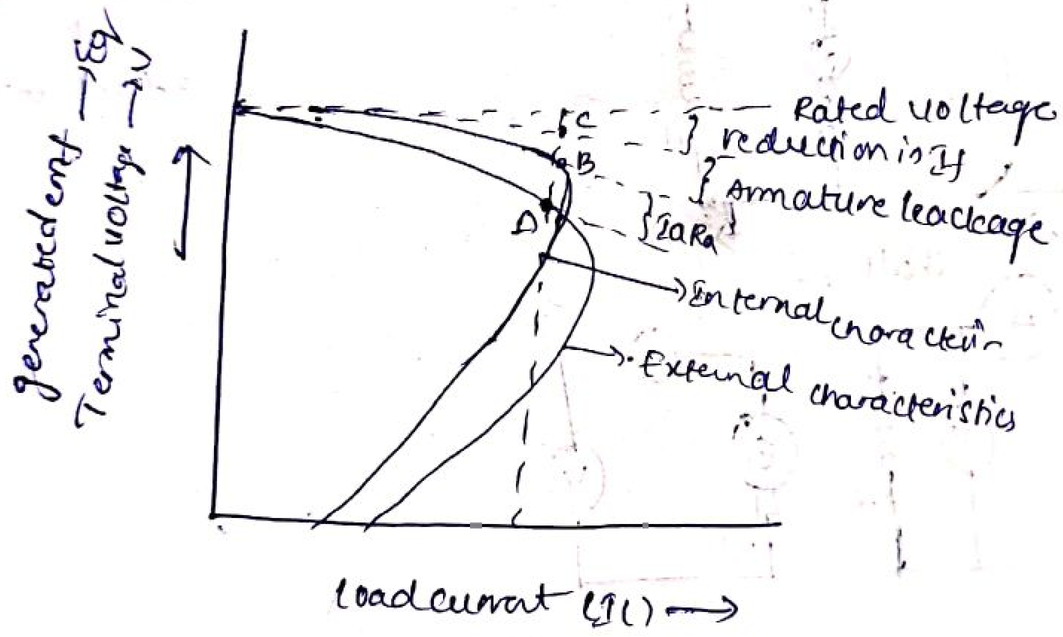




External characteristics: (drooping characteristics)
(dc shunt generator)



Internal characteristic:



* A DC shunt generator gives an open circuit voltage of 240V. when loaded the terminal voltage falls to 220V. Determine the load current in case of Armature circuit and field winding resistances are 0.1Ω and 50Ω respectively.

Sol:

$E = 240V \quad V = 220$

$R_a = 0.1\Omega \quad R_{sh} = 50\Omega$

$I_L = ?$

$I_a = I_L + I_{sh} \Rightarrow I_L = I_a - I_{sh}$

$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{50} = 4.4$

$E_g = V + I_a R_a$

$I_a = \frac{E_g - V}{R_a} = \frac{240-220}{0.1} = \frac{20}{0.1} = 200 Amp$

$I_L = 200 - 4.4$

$= 195.6A$

* find the resistance of the load which takes a power of 5kw from a DC-shunt generator whose external characteristics is given by the equation $V = 250 - 0.5 I_L$

Sol:

$pow = 5kw = 5000W \quad R_L = \frac{V}{I_L} = ~~500~~$

$V = 250 - 0.5 I_L$

$I_L = \frac{P}{V} = \frac{5000}{V} \Rightarrow V = 250 - 0.5\left[\frac{5000}{V}\right]$

$= \frac{5000}{239.56} \quad V = 250 - \frac{2500}{V}$

$= 20.87 \quad V^2 - 250V + 2500 = 0$

$V_1 = 239.564$ (single phase)

$V_2 = 10.435$ (neglected)

$R_{EQ} \frac{V_L}{I_L} = \cancel{270}$

$V = 250 - (0.5)(20.87)$

$V = +239.6$

$R_L = \frac{V}{I_L} = \frac{+239.6}{20.87} = 11.48\,\Omega$