* Check the following conditions.

1. Static, dynamic
2. Linear, Non-linear
3. Causal, non-causal
4. Time invariant, variant.

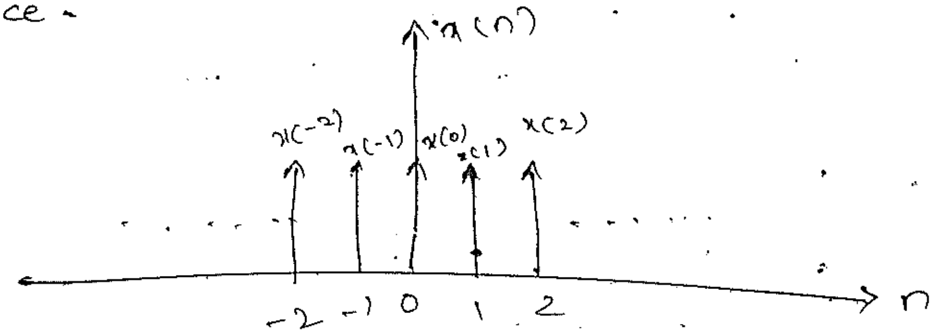
1. $y(n) = x(n) - 5x(n-1)$
2. $y(n) = x(2n)$
3. $y(n) = x(n/3)$
4. $y(n) = x(n) \cdot \cos\left(\frac{\pi n}{6}\right)$
5. $y(n) = a^{x(n)}$
6. $y(n) = |x(n)|$
7. $y(n) = \log(1 + |x(n)|)$

14/7/06.

Arbitrary Representation of a Sequence :-

Any sequence can be represented as sum of shifted version of unit sample sequences is called arbitrary representation of a sequence.

Ex :-



$x(n) = \ldots + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1)$
$+ x(2)\delta(n-2) + \ldots$

$$\boxed{x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)}$$

Linear Time-Invariant System (or) Discrete time linear Time-invariant System :-

LTI (or) DTLTI system :

A discrete time system it satisfies

the following properties

(i) Linearity Property

(ii) Time invariant Property.

Properties :-

① LTI System response (or) Convolution Sum :-

Any discrete time system can be represented mathematically as

$$x(n) \xrightarrow{\text{I/P}} \boxed{T} \xrightarrow{\text{O/P}} y(n) = T[x(n)]$$

where $x(n)$ is given as excitation and $y(n)$ is the response of the system.

- If the unit sample sequence is given to the input of the system i.e; $x(n) = \delta(n)$, then the response of the system is called impulse response, $(h(n))$

i.e; $y(n) = T[x(n)]$.

$$y(n)\Big|_{x(n)=\delta(n)} = h(n) = T[\delta(n)]$$

- Generally response of the system is

$$y(n) = T[x(n)]$$

w.k.T arbitrary rep. of ~~Summation~~ $x(n)$ is

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$y(n) = T\left[\sum_{k=-\infty}^{\infty} x(k).\,\delta(n-k)\right] \quad \text{—— ①}$$

∴ In LTI system, it satisfies linearity property.
By using linearity property, eq. ① reduces to

$$y(n) = \sum_{k=-\infty}^{\infty} x(n) \; T[\delta(n-k)]$$

An LTI system also satisfies time-invariant property
Impulse response due to shifted impulse sequence is

$$h[n, k] = T[\delta(n-k)].$$

$$\therefore \; y(n) = \sum_{k=-\infty}^{\infty} x(k).\, h(n,k)$$

It satisfies time in-variant property.

Then $h(n,k) = h(n-k)$

$$\therefore \boxed{y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)}$$ which is called as response of LTI system. ← $\boxed{y(n) = x(n) * h(n)}$

② Causality of LTI system :-

Causality :- A discrete time system is said to be causal if its output at any instant of time, n depends on present input, past input and output samples but does not depend on future input samples.

The necessary and sufficient condition for causality of LTI system is

$$\boxed{h(n) = 0 \quad \text{for} \quad n<0}$$

Pf :- w.k.T the $y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = x(n) * h(n)$

It satisfies commutative property.

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k) = h(n) * x(n).$$

$\Rightarrow$

$$y(n) = \ldots + h(-2)x(n+2) + h(-1)x(n+1) + h(0)x(n) + h(1)x(n-1) + \ldots$$

~~A system~~

The above equation satisfies causality only when

$$h(-1) = h(-2) = h(-3) = \ldots = 0$$

i.e; $h(n) = 0$ for $n < 0$.

$\therefore$ The above equation satisfies causaulity.

Response of causaulity of LTI system is

$$y(n) = h(0)x(n) + h(1)x(n-1) + \ldots$$

$$\boxed{y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)}$$

(3) Condition for stability of LTI system:-

STABILITY:- Any arbitrary relaxed system is said to be BIBO stable iff every bounded input yields bounded output.

The necessary and sufficient condition for stability of LTI system is

$$\boxed{\sum_{k=-\infty}^{\infty} |h(n)| < \infty}$$

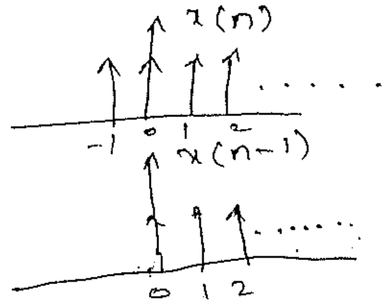
Pf:- we know that the response of LTI system is

$$y(n) = \sum_{k=-\infty}^{\infty} h(k) \cdot x(n-k).$$

for stable condition, the input is bounded such that the ~~magnitude~~.

$$|x(n)| = M_x$$

Ex:-



$$\therefore \quad |x(n-k)| = M_x$$

~~y(n)~~ =

Apply magnitude (or) absolute value on the both sides of absolute equation

$$|y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)\, x(n-k)\right|$$

w.k.T magnitude of sum of terms is always less than sum of magnitudes.

i.e., $|y(n)| = \left|\sum_{k=-\infty}^{\infty} h(k)\, x(n-k)\right| \leq \sum_{k=-\infty}^{\infty} |h(k)|\,|x(n-k)|$

$[\because |a+b| \leq |a| + |b|]$

$$|y(n)| \leq \sum_{k=-\infty}^{\infty} |h(k)|\, M_x$$

$$|y(n)| \leq M_x \sum_{k=-\infty}^{\infty} |h(k)|$$

The above eq. is stable and has finite value only

if $\sum_{k=-\infty}^{\infty} |h(k)| < \infty$

* Test whether the following systems are causal or not and stable or not.

1. $y(n) = \cos(x(n))$
2. $y(n) = a\,x(n)$
3. $y(n) = a^{x(n)}$
4. $y(n) = x(n)\,e^{n}$
5. $y(n) = \sum_{K=-\infty}^{\infty} x(K)$

→ (1) $y(n) = \cos(x(n))$

$$y(n)\Big|_{x(n)=\delta(n)} = h(n) = \cos(\delta(n))$$

$n=0$ | $h(0) = \cos(\delta(0)) = \cos(1) = 0.54$

$n=1$ | $h(1) = \cos(\delta(1)) = \cos(0) = 1$

$n=2$ | $h(2) = \cos(\delta(2)) = \cos(0) = 1$

$n=-1$ | $h(-1) = \cos(\delta(-1)) = \cos(0) = 1$

(i) Causality:

From this,

$h(-1) = h(-2) \ldots = 1$

$\therefore h(n) \neq 0$ for $n < 0$

So the system is non-causal.

(ii) Stability.

Condition for stability of LTI system is

$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

$$\sum_{n=-\infty}^{\infty} |h(n)| = \dots + |h(-1)| + |h(0)| + |h(1)| + \dots$$

$$= \dots + 1 + 1 + 0.54 + 1 + 1 + \dots$$

$$= \infty$$

$\therefore \sum_{n=-\infty}^{\infty} |h(n)|$ ... non-stable.

② $y(n) = a\,x(n)$

$$y(n)\Big|_{x(n)=\delta(n)} = h(n) = a\,\delta(n).$$

$n=0$	$h(0) = a\,\delta(0) = a$	$n=-1$	$h(-1) = a\,\delta(-1) = 0$
$n=1$	$h(1) = a\,\delta(1) = 0$		$\vdots$
$n=2$	$h(2) = a\,\delta(2) = 0$		
	$\vdots$		

(i) From this,

$$h(-1) = h(-2) = \dots = 0$$

$\therefore\ h(n) = 0$ for $n<0$

$\therefore$ system is causal.

(ii)
$$\sum_{n=-\infty}^{\infty} h(n) = \dots + h(-1) + h(0) + h(1) + \dots$$

$$= 0 + 0 + \dots + a + 0 + 0 + \dots$$

$$= \underline{\underline{a}} < \infty$$

$\therefore$ system is stable.

③ $y(n) = a^{x(n)}$

$$y(n)\Big|_{x(n)=\delta(n)} = h(n) = a^{\delta(n)}$$

$n=0$	$h(0) = a^{\delta(0)} = a$	$n=-1$	$h(-1) = a^0 = 1$
$n=1$	$h(1) = a^0 = 1$		$\vdots$
	$\vdots$		

$\therefore \quad h(-1) = h(-2) = \ldots\ldots = 1$

$\therefore \quad h(n) \neq 0$ for $n<0$ $\therefore$ It is non-causal.

$$\sum_{n=-\infty}^{\infty} h(n) = \ldots 1+1+\ldots+a+1+1+\ldots$$

$$= \infty$$

$\therefore$ It is unstable system.

(4) $y(n) = x(n)\, e^n$.

$\longrightarrow \left. y(n) \right|_{x(n)=\delta(n)} = h(n) = \delta(n)\, e^n$.

$n=0$	$h(0) = \delta(0)e^0 = 1$	$n=-1 \Rightarrow h(-1) = \delta(-1)e^{-1} = 0$
$n=1$	$h(1) = \delta(1)e^1 = 0$	$\vdots$
	$\vdots$	

$\therefore \quad h(n) = 0$ for $n<0$ $\therefore$ causal system.

$$\sum_{n=-\infty}^{\infty} h(n) = 1+0+0+\ldots$$

$$= 1 < \infty$$

$\therefore$ It is Stable system.

(5) $y(n) = \sum_{K=-\infty}^{n+1} x(K)$.

$\longrightarrow \left. y(n) \right|_{x(n)=\delta(n)} = h(n) = \sum_{K=-\infty}^{n+1} \delta(K)$

$= \ldots\ldots + \delta(-1) + \delta(0) + \delta(1) + \ldots$
$\ldots + \delta(n) + \delta(n+1)$.

$n=0$ | $h(0) = \cancel{\dots + \delta(-2) + \delta(-1) + \delta(0) + \delta(1) + \dots + \delta(0) + \delta(1)}$

$$= \sum_{k=-\infty}^{0+1} \delta(k)$$

$$= \sum_{k=-\infty}^{1} \delta(k) = 0 + 0 + 1 + 0 + \dots$$

$$= \underline{\underline{1}}$$

$$n=1 \Rightarrow h(1) = \sum_{k=-\infty}^{2} \delta(k) = \underline{\underline{1}}$$

$\vdots$

$$n=-1 \Rightarrow h(-1) = \sum_{k=-\infty}^{0} \delta(k) = \underline{\underline{1}}$$

$$n=-2 \Rightarrow h(-2) = \sum_{k=-\infty}^{-1} \delta(k) = 0$$

$\vdots$

$\therefore$ $h(n) = 1 \neq 0$ for $n<0$.

$\therefore$ The given system is non-causal system.

$$\sum_{n=-\infty}^{\infty} h(n) = \dots 1+1+1+0+\dots = \infty.$$

$\therefore$ It is unstable system.

(1) $h(n) = 2^n u(-n)$

(2) $h(n) = \sin\left(\frac{n\pi}{2}\right)$

(3) $h(n) = \delta(n) + \sin(n\pi)$

(4) $h(n) = e^{2n} u(n-1)$

→ (1) $h(n) = 2^n u(-n)$

$n=0 \Rightarrow h(0) = 2^0 u(0) = \underline{\underline{1}}$	$n=-1 \Rightarrow h(-1) = 2^{-1} u(1) = 1/2$
$n=1 \Rightarrow h(1) = 2\,u(-1) = 0$	$n=-2 \Rightarrow h(-2) = 2^{-2} u(2) = \frac{1}{4}$
$\vdots$	$\vdots$

$\therefore \quad h(n) = \frac{1}{2} + \frac{1}{4} + \cdots \neq 0$ for $n < 0$

$\therefore$ It is non-causal system.

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=-\infty}^{\infty} |2^n u(-n)|$$

$$= \sum_{n=-\infty}^{\infty} 2^n |u(-n)|$$

$$= \sum_{n=-\infty}^{0} 2^n |u(-n)| + \sum_{n=1}^{\infty} 2^n |u(-n)|$$

$$u(-n) = \begin{cases} 1 & \text{for } -n \geq 0 \\ & \phantom{\text{for }} n \leq 0 \\ 0 & \text{for } n > 0. \end{cases}$$

$$= \sum_{n=-\infty}^{0} 2^n (1) + 0$$

$$= \sum_{n=-\infty}^{0} 2^n$$

Put $n = -k$

$n \to -\infty \Rightarrow k \to \infty$

$n \to 0 \Rightarrow k \to 0$

$$\therefore \sum_{n=-\infty}^{\infty} h(n) = \sum_{k=\infty}^{0} 2^{-k}$$

$$= \sum_{k=0}^{\infty} 2^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$$

$$= \frac{1}{1 - 1/2} = \underline{\underline{2}}$$

$\therefore$ It is finite, Hence it is stable system.

② ~~$h(n) = \sin\left(\frac{n\pi}{2}\right)$~~

* Find the convolution b/w the following two sequences by the graphical method.

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 1\}$; $h(n) = \{1, \underset{\uparrow}{2}, 1, -1\}$

NOTE :- ① If $x(n)$ starts at $n = n_x$

$h(n)$ starts at $n = n_h$, then

the convolution output $y(n)$ starts at $n = n_x + n_h$

② If $x(n)$ ends at $n = n_{x1}$

$h(n)$ ends at $n = n_{h1}$, then

$y(n)$ ends at $n = n_{x_1} + n_{h_1}$

③ If the length of the sequence $x(n)$ is N_1, and the length of $h(n)$ sequence is N_2; then convolution output $y(n)$ is $N_1 + N_2 - 1$.

→ $y(n)$ starts at $n = 0 + (-1) = -1$

$y(n)$ ends at $n = n_{x_1} + n_{h_1} = 3 + 2 = 5$

Length of the o/p $y(n) = N_1 + N_2 - 1$

$= 4 + 4 - 1$

$= 7$

LTI System response is

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = x(n) * h(n)$$

$y(n)$ starts at $n = -1$

$\therefore$ At $n = -1$; $y(-1) = \sum_{k=-\infty}^{\infty} x(k)\, h(-1-k)$

$n = 0$; $y(0) = \sum_{k=-\infty}^{\infty} x(k)\, h(-k)$

$n = 1$; $y(1) = \sum_{k=-\infty}^{\infty} x(k)\, h(1-k)$

At $n=2$; $y(2) = \sum_{k=-\infty}^{\infty} x(k)\, h(2-k)$

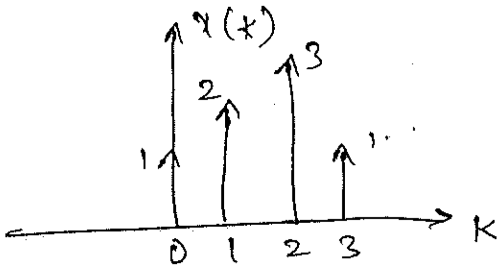
$n=3$; $y(3) = \sum_{k=-\infty}^{\infty} x(k)\, h(3-k)$

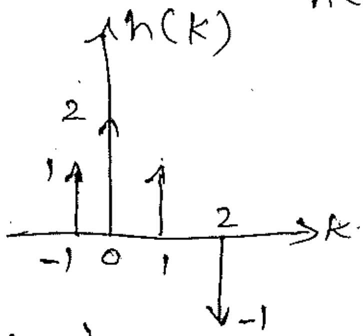
$n=4$; $y(4) = \sum_{k=-\infty}^{\infty} x(k)\, h(4-k)$

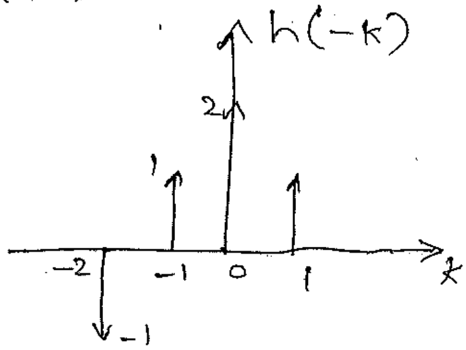
$n=5$; $y(5) = \sum_{k=-\infty}^{\infty} x(k)\, h(5-k)$.

Graphically,

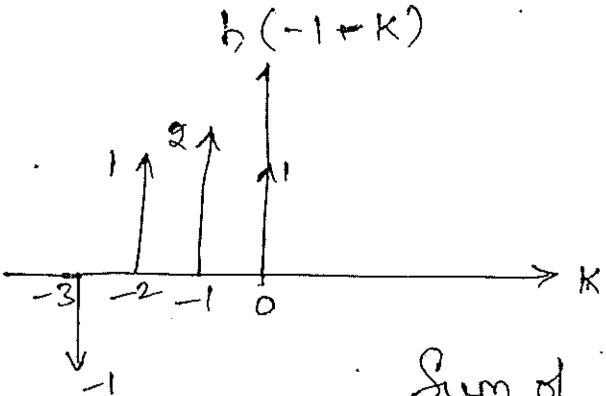
$h(-1-k) = h(-(1+k))$

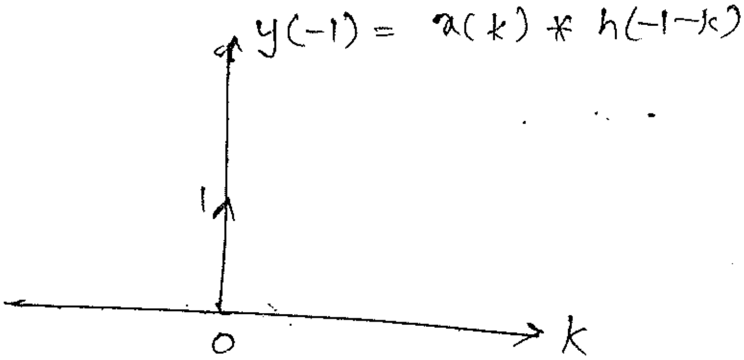






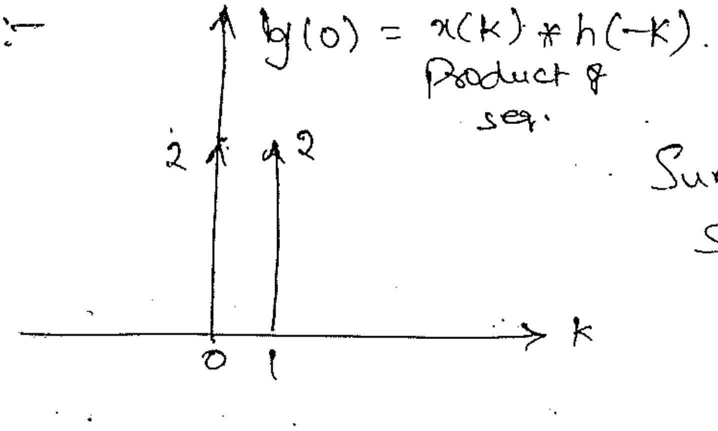
Computation of $y(-1)$ value





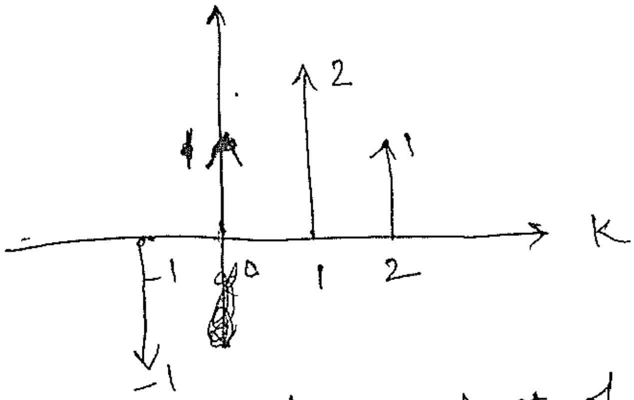
Sum of product of Sequence = $y(-1) = 1 = \underline{1}$

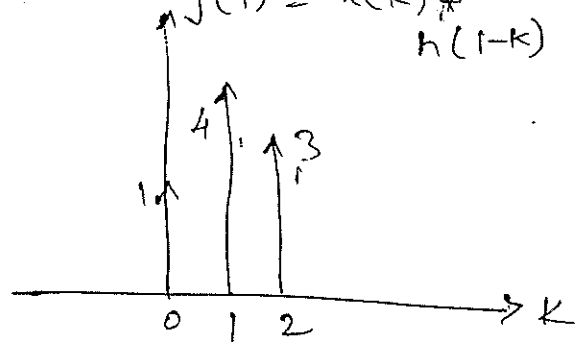
$\underline{y(0)}$:-



Sum of product of Sequences $= 2+2 = \underline{4}$

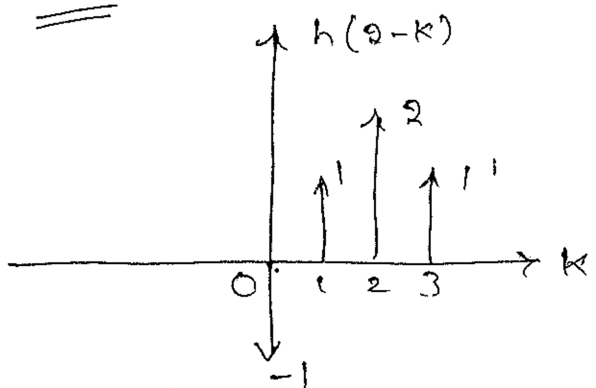
$\underline{y(1)}$:- $h(1-k) = h(-(k-1))$

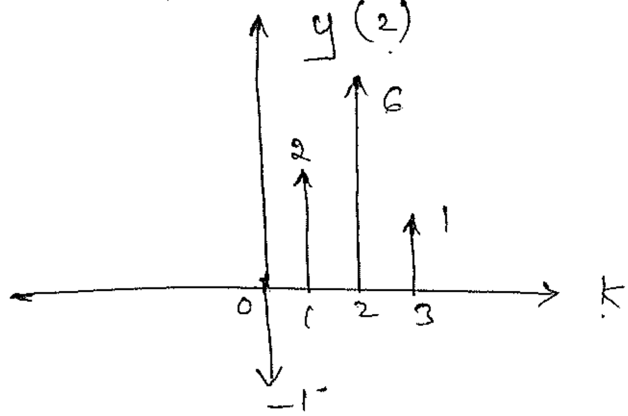




Sum of product of sequences = 1+4+3 = $\underline{8}$

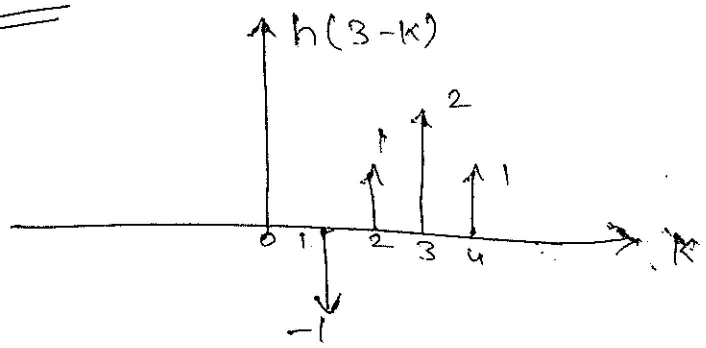
$\underline{y(2)}$:- $h(2-k) = h(-(k-2))$

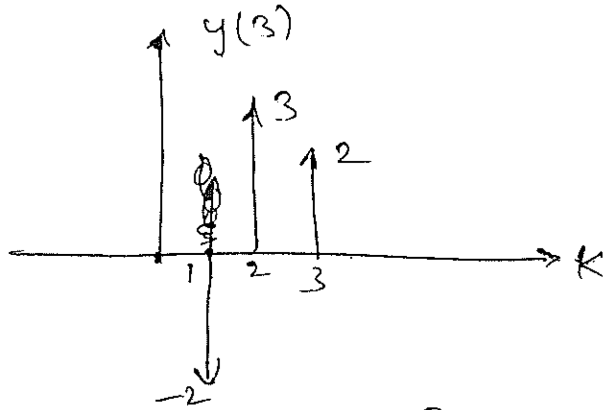




Sum of product of sequences = -1 + 2 + 6 + 1
= $\underline{8}$

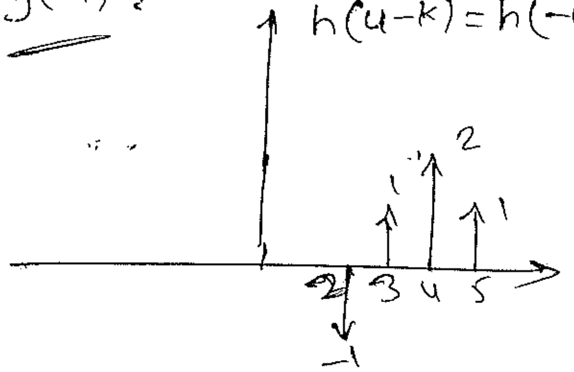
$\underline{y(3)}$:- $h(3-k) = h(-(k-3))$

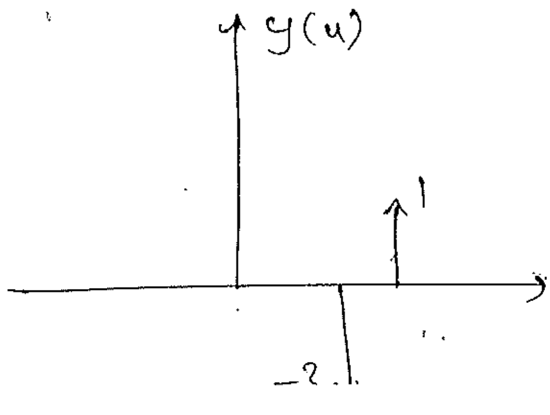




Product of sequences = -2+3+2 = $\underline{3}$

$\underline{y(4)}$:- $h(4-k) = h(-(k-4))$

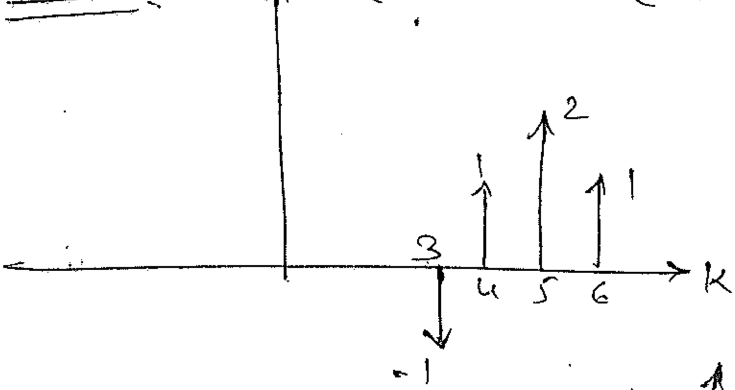


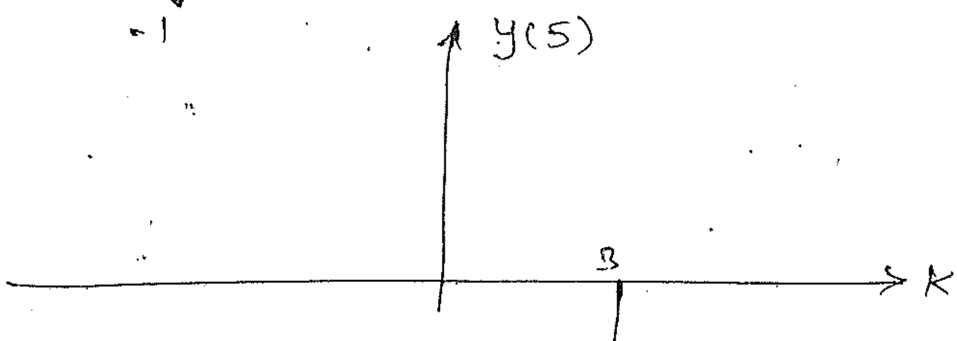


Product of Sequences $= -3+1 = \underline{\underline{-2}}$

$\underline{\underline{y(5)}}$:-

$h(5-k) = h(-(k-5))$

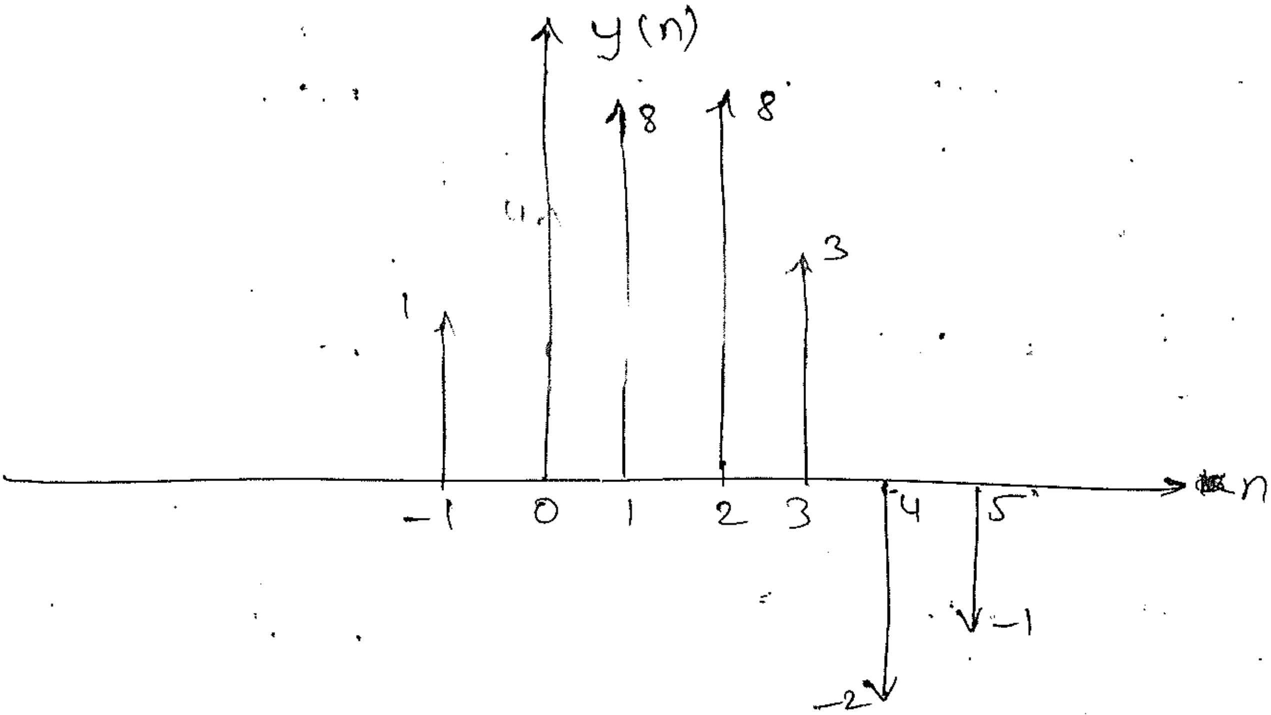




$\therefore$ Product of sequences $= \underline{\underline{-1}}$

$\therefore \quad y(n) = \{1, \underset{\uparrow}{4}, 8, 8, 3, -2, -1\}$

Graphically, $y(n)$ is



Verification method:-

$$\begin{array}{c|cccc}
h(n)\downarrow \;\backslash\; x(n)\rightarrow & 1 & 2 & 3 & 1 \\
\hline
1 & 1 & 2 & 3 & 1 \\
2 & 2 & 4 & 6 & 2 \\
1 & 1 & 2 & 3 & 1 \\
-1 & -1 & -2 & -3 & -1
\end{array}$$

$y(-1) = 1$

$y(0) = 2+2 = 4$

$y(1) = 1+4+3 = 8$

$y(2) = -1+2+6+1 = 8$

$y(3) = -2+3+2 = 3$

$y(4) = -3+1 = -2$

$y(5) = -1$

H.W

① $x_1(n) = \{1, \underset{\uparrow}{2}, -3, -2\}$

$x_2(n) = \{-1, \underset{\uparrow}{0}, -1, 2\}$

$$\begin{array}{c|cccc}
 & 1 & 2 & -3 & 2 \\
\hline
-1 & -1 & -2 & 3 & 2 \\
0 & 0 & 0 & 0 & 0 \\
-1 & -1 & -2 & 3 & 2 \\
2 & 2 & 4 & -6 & -4
\end{array}$$

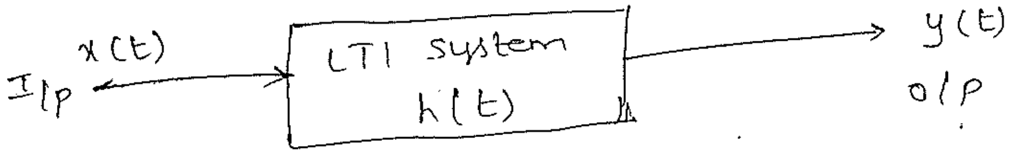
11/8/06

UNIT - IV

SIGNAL TRANSMISSION THROUGH SYSTEMS.

Transfer function of LTI System :-

If the Continous time signal $x(t)$ is given to the I/p of an LTI system, $h(t)$ is unit Sample response of LTI system, then the response of system is as shown in the figure.



$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau.$$

Apply F.T on both sides,

$$F[y(t)] = F[x(t) * h(t)]$$

W.K.T

$$y(t) \longleftrightarrow Y(\omega)$$
$$x(t) \longleftrightarrow X(\omega)$$
$$h(t) \longleftrightarrow H(\omega).$$

and

$$g_1(t) * g_2(t) \longleftrightarrow F[g_1(t)].F[g_2(t)]$$
$$\longleftrightarrow G_1(\omega)\, G_2(\omega).$$

$$Y(\omega) = F[x(t)].\, F[h(t)]$$
$$= X(\omega).\, H(\omega).$$

$$H[\omega] = \frac{Y[\omega]}{X[\omega]}$$

The transfer fn. of the system defined by ~~of~~ the ratio of F.T of o/p Signal to the F.T of I/p signal.

$$T.F = \boxed{H(\omega) = \frac{Y(\omega)}{X(\omega)}} = \frac{\text{o/p polynomial in '}\omega\text{'}}{\text{I/p Polynomial in '}\omega\text{'}}$$

where $H(\omega) = |H(\omega)| e^{j\angle H(\omega)}$

$|H(\omega)|$ — magnitude spectrum of the system.

$\angle H(\omega)$ — Phase " " "

$X(\omega) = |X(\omega)| e^{j\angle X(\omega)}$ — I/p freq. Spectrum.

$Y(\omega) = |Y(\omega)| e^{j\angle Y(\omega)}$ — o/p " "

$$|Y(\omega)| e^{j\angle Y(\omega)} = |X(\omega)| e^{j\angle X(\omega)} \cdot |H(\omega)| e^{j\angle H(\omega)}$$

$$= |X(\omega)| \cdot |H(\omega)| e^{j[\angle X(\omega) + \angle H(\omega)]}$$

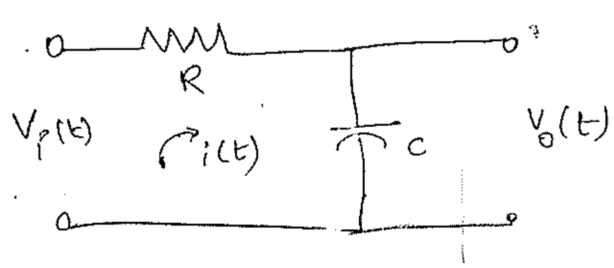
$\therefore$ $\boxed{|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|}$ and

$\boxed{\angle Y(\omega) = \angle X(\omega) + \angle H(\omega)}$

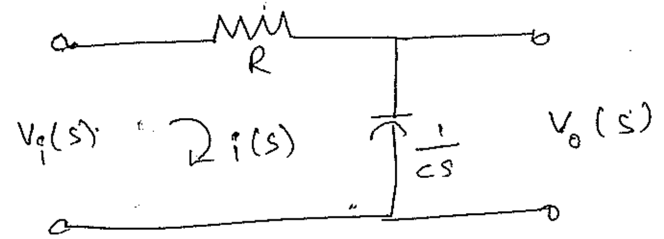
The system transfer fn.'s changes with input Spectral characteristics in amplitude and phase functions.

Filter Characteristics of linear Systems :-

Consider a RC-low pass filter as shown in figure.



Equivalent . Laplace ~~transform~~ ckt is



$$V_i(s) = \left(R + \frac{1}{cs}\right) I(s) \quad ; \quad V_o(s) = \frac{1}{cs} I(s).$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1/cs}{R + 1/cs} = \frac{1}{1+sCR}.$$

$$L^{-1}[H(s)] = L^{-1}\left(\frac{1}{Rc\left(\frac{1}{Rc}+s\right)}\right)$$

$$\Rightarrow \boxed{h(t) = \frac{1}{Rc} e^{-t/Rc} u(t)}$$

W.K.T
$$e^{-at}u(t) \longleftrightarrow \frac{1}{a+j\omega}.$$

$$e^{at}u(t) \longleftrightarrow \frac{1}{a-j\omega}.$$

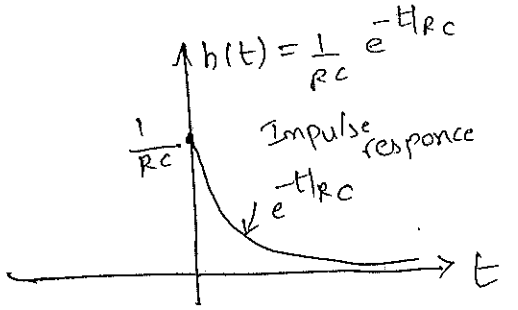
$$\frac{1}{Rc} e^{-t/Rc} u(t) \longleftrightarrow \frac{1}{Rc} \cdot \frac{1}{\frac{1}{Rc}+j\omega}.$$

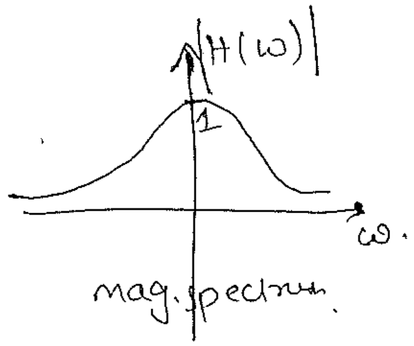
$$h(t) = \frac{1}{Rc} e^{-t/Rc} u(t) \longleftrightarrow \frac{1}{1+j\omega cR} = H(\omega).$$

$$\therefore \boxed{H(\omega) = \frac{1}{1+j\omega cR}}$$

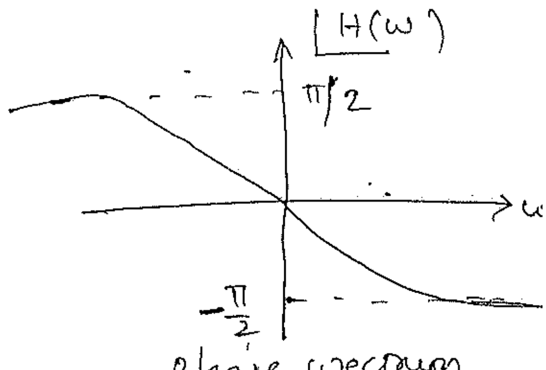
$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 c^2 R^2}} \longrightarrow \text{system magnitude spectrum}$$

$$\angle H(\omega) = -\tan^{-1}(\omega cR) \quad - \text{Phase spectrum}$$



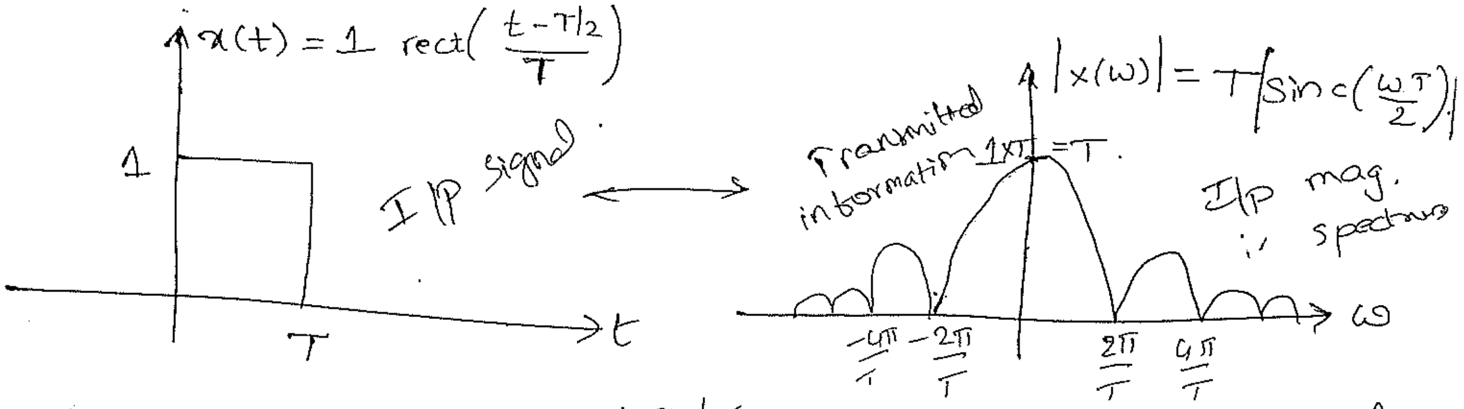


mag. spectrum.



phase spectrum

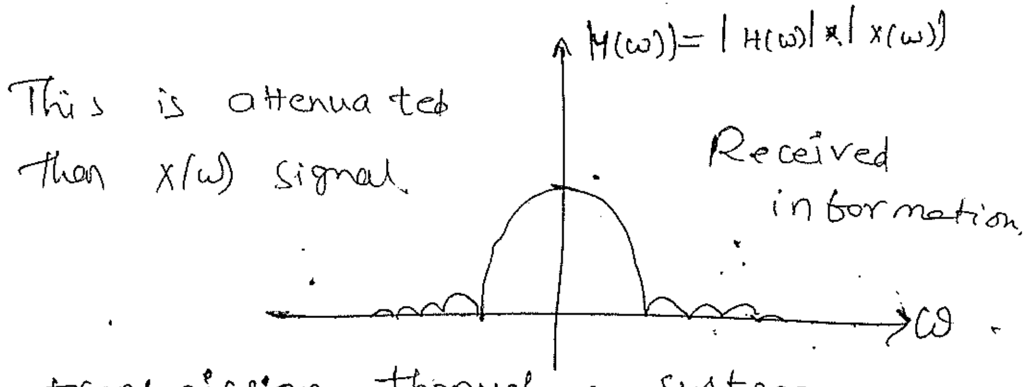
Let us discuss how
The filter characteristic $|H(\omega)|$ will be changes for the response of the ~~____~~ system due to rectangular I/P signal.



when we pass the rectangular i/p signal x through a low pass filter, lowpass filter allows only low frequency components of I/p signal and attenuates high frequency componen of I/P signal.

i.e; the I/p signal is filtered by the factor of $|H(\omega)|$ and the output phase is combination of input phas and system phase.

The o/p in time domain and freq. domains are as shown in below



<u>Distortion less transmission through a system:-</u>

The o/p of communication channel is exactly replica of the I/P signal except permissable change of constant amplitude and, constant time delay.

- the I/P signal $x(t)$ is transmitted through a system is distortion less, if the o/p of the system ~~is~~ satisfies the following conditions:-

$$y(t) = K\,x(t - t_0).$$

where 'K' is constant independent of frequency.

t_0 - Constant time delay.

$x(t)$ - Transmitted I/P signal ; $y(t)$ - received o/p signal

This ~~which~~ is condition for distortionless system in t-domain

In freq.-domain;

$$F[y(t)] = F[K\,x(t - t_0)]$$

w.k.T

$$y(t) \longleftrightarrow Y(\omega)$$

$$x(t) \longleftrightarrow X(\omega).$$

$$x(t - t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$$

$$Y(\omega) = K e^{-j\omega t_0} X(\omega)$$

w.k.T

$$Y(\omega) = H(\omega).\,X(\omega)$$

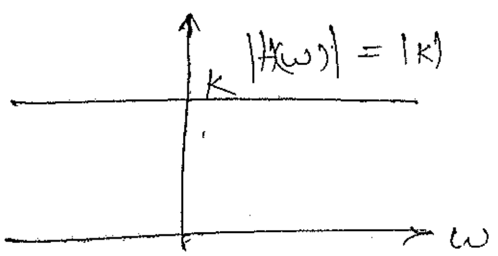
$\therefore$ $H(\omega) = K e^{-j\omega t_0}$ which is condition for distortionless system in f-domain

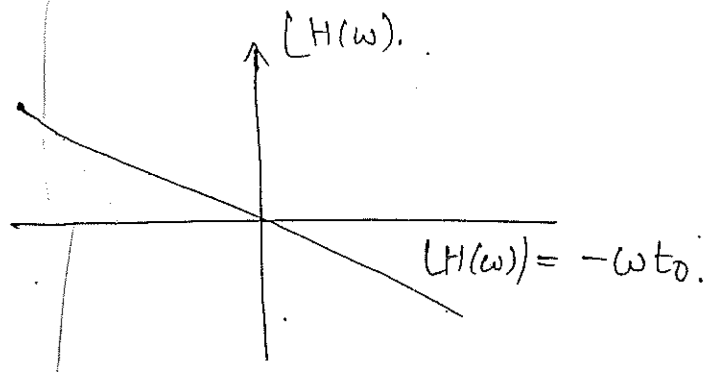
From this equation, it is seen that any signal is transmitted through distortionless system, the following conditions must be satisfied.

(i) $|H(\omega)| = |K||e^{-j\omega t_0}| = |K|$ — System mag. spectrum

(ii) $\angle H(\omega) = -\omega t_0$ — Phase spectrum.

Graphically,





Conditions :-

(i) The System is distortionless, System transfer function magnitude fn. must be Constant.

(ii) The system is distortionless, the system transfer fn, phase fn. must linearly vary with frequency.

- General distortionless transfer function 'H'

$$\boxed{H(\omega) = K \exp[j(-\omega t_0 + n\pi)]}$$

'n' is integer.

Causaulity and physical realisation — Paley Wiener Criteria :-

It is a test which distinguish b/w physical realisable char. from unrealisable one.

Condition for Causaulity and physical realisation in t-domain :-

"Condition for physically realisable system is that it must ~~have~~ be causal system".

~~An~~ LTI system is causal iff $h(t) = 0$ for $t < 0$.

So, ~~the~~ necessary & sufficient condition ~~for~~ physically realisable system is their impulse response $h(t)$ is zero for $t < 0$.

Condition for causality and physical realisation in f-domain :-

Paley wiener criteria, implies that the necessary and sufficient condition for the magnitude System funct. $H(\omega)$ to be physically realisable is satisfies the following Condition:-

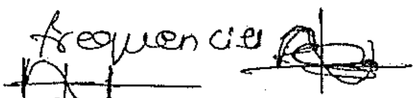
$$\int_{-\infty}^{\infty} \frac{\ln |H(\omega)|}{1+\omega^2} \, d\omega < \infty \quad - \text{ Paley wiener criteria.}$$

It is physically realisable, however the magnitude Square of transfer function of system is absolutely integrable before paley-wiener Criteria valid.

i.e; $$\int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega < \infty.$$

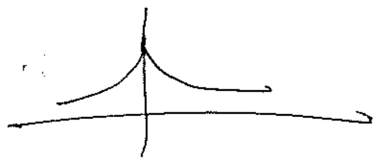
Conclusions: Significant conclusions drawn from paley-wiener criteria :—

(i) The transfer fn.'s of the system is zero at some discrete instants of frequencies but it cannot be zero over ~~which~~ within the band of frequencies.



(ii) The transfer function. magnitude function is not fall off to zero.

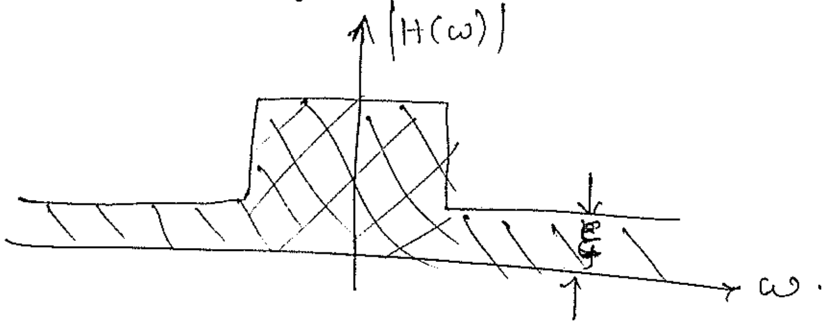
Thus $$|H(\omega)| = K e^{-\alpha|\omega|}$$



It is permissable.

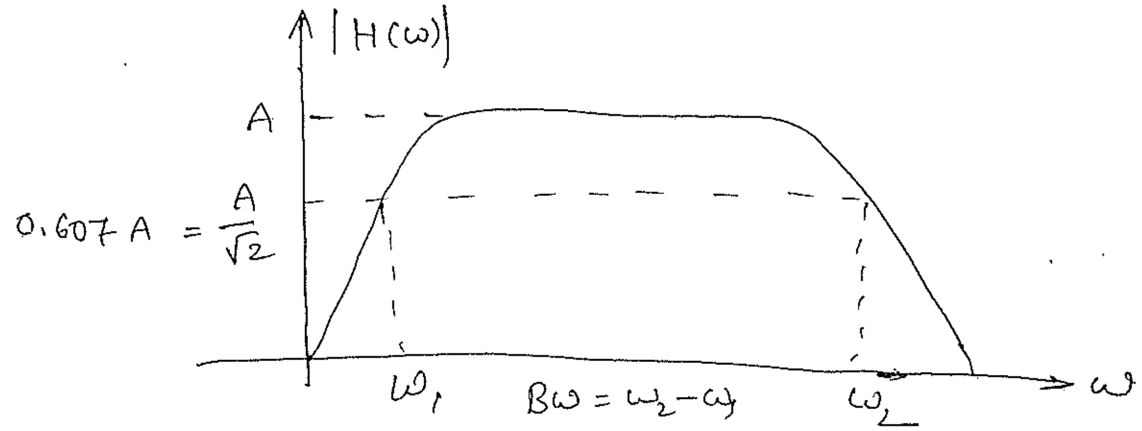
(iii) The system transfer fn. magnitude fn.'s have not high attenuation factor.

(iv) the characteristics of physically realisable low-pass filter which is permissable for small Values of 'ε' as shown in figure.



SYSTEM BAND WIDTH :-

Def :- The System bandwidth is arbitrarly defined a the band of frequency over which the magnitude fn., $|H(\omega)|$ is $\frac{1}{\sqrt{2}}$ times its mid band value.



i.e, Band width $= (\omega_2 - \omega_1)$ as shown in fig.

---

* Find the convolution b/w the following discrete signals

$u(n) = x(n)$; $h(n) = u(n)$.

→ $$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} u(k) \cdot u(n-k).$$

$u(k) = \begin{cases} 1 & \text{for } k \geq 0 \\ 0 & \text{for } k < 0 \end{cases}$ $$= \sum_{k=0}^{\infty} 1 \cdot u(n-k)$$

$u(n-k) = \begin{cases} 1 & \text{for } n-k \geq 0 \\ & \quad k \leq n \\ 0 & \text{for } k > n. \end{cases}$ $$= \sum_{k=0}^{n} 1 \cdot 1 = \sum_{k=0}^{n} 1.$$

$$= n - 0 + 1$$

$$= \underline{\underline{n+1}}$$

$\therefore$ $y(n) = n+1$

$*$ $x(n) = 2^n u(n)$; $h(n) = \frac{1}{2^n} u(n)$

$\rightarrow$ $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$

$$= \sum_{k=-\infty}^{\infty} 2^k u(k) \cdot \frac{1}{2^{n-k}} \cdot u(n-k)$$

$n+k$ $\sum_{k=0}^{\infty} 1 \cdot u(n-k)$

$$= 2^{2k-n} \sum_{k=0}^{n} 1$$

$$= \sum_{k=0}^{n} 2^{2k-n}$$

$$= \sum_{k=0}^{n} \frac{2^{2k}}{2^n}$$

$$= \frac{1}{2^n} \cdot \sum_{k=0}^{n} 2^{2k} = \frac{1}{2^n} \sum_{k=0}^{n} 4^k$$

$$= \frac{1}{2^n} \cdot \frac{4^{n+1}-1}{4-1}$$

$$= \frac{4^{n+1}-1}{3 \times 2^n}.$$

$*$ $x(n) = \delta(n)$; $h(n) = u(n)$.

$\longrightarrow$ $y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} \delta(k)\, u(n-k)$

$$= \cancel{\sum_{k=-\infty}^{\infty}} \cancel{u(n)\, \delta(n-k)}$$

$\because \delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{otherwise} \end{cases}$ $\Rightarrow$ $\sum_{k=0} u(n-k)$

$$= u(n)$$

Frequency Integration property :-

If $g(t) \longleftrightarrow G(\omega)$, then $\frac{g(t)}{-jt} \longleftrightarrow \int G(\omega)\, d\omega$.

Pf :- $g(t) \longleftrightarrow G(\omega)$.

Applying integration on both sides w.r.t '$d\omega$.'

$$\Rightarrow \cancel{\int g(t)\, d\omega} \longleftrightarrow \int \cancel{G(\omega)}\, d\omega.$$

$$G(\omega) = \int_{-\infty}^{\infty} g(t)\, e^{-j\omega t}\, dt$$

$$\int G(\omega)\, d\omega = \int_{-\infty}^{\infty} g(t) \int e^{-j\omega t}\, d\omega\, dt$$

$$= \int_{-\infty}^{\infty} g(t)\, \frac{e^{-j\omega t}}{-jt}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{g(t)}{-jt}\, e^{-j\omega t}\, dt$$

$$= F\left[g(t)/-jt\right]$$

$$\therefore \boxed{\frac{g(t)}{-jt} \longleftrightarrow \int G(\omega)\, d\omega}$$

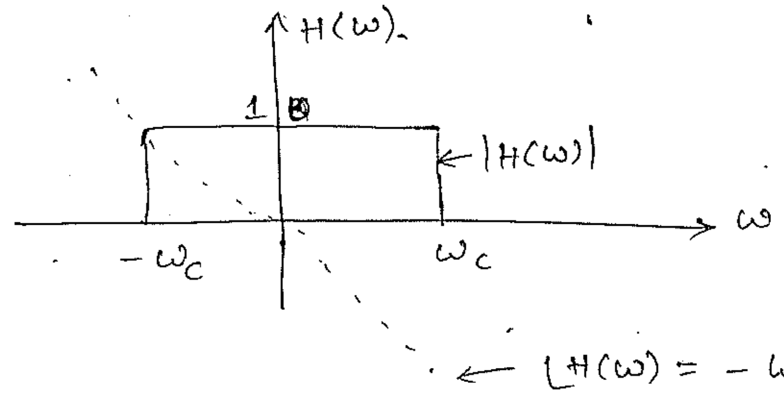
17/8/06 Ideal Filters :-

There are 3 types of ideal filters.

1. Ideal low pass filter.
2. Ideal high pass filter.
3. Ideal band pass filter.

1. Ideal low pass filter :-

The spectral characteristics of ideal low pass filter are as shown in figure.



$$H(\omega) = |H(\omega)|\, e^{j\angle H(\omega)}$$

$$|H(\omega)| = \begin{cases} 1 & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 & \text{elsewhere} \end{cases}$$

$$H(\omega) = \begin{cases} 1 \times e^{-j\omega t_0} & \text{for } -\omega_c \le \omega \le \omega_c \\ 0 \times e^{-j\omega t_0} = 0 & \text{elsewhere.} \end{cases}$$

Impulse response calculation $h(t)$:-

$$F^{-1}\left[H(\omega)\right] = h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t}\, d\omega$$

$$h(t) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{-j\omega t_0}\, e^{j\omega t}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega(t-t_0)}\, d\omega$$

$$= \frac{1}{2\pi} \cdot \frac{e^{j\omega(t-t_0)}}{j(t-t_0)} \Bigg|_{-\omega_c}^{\omega_c}$$

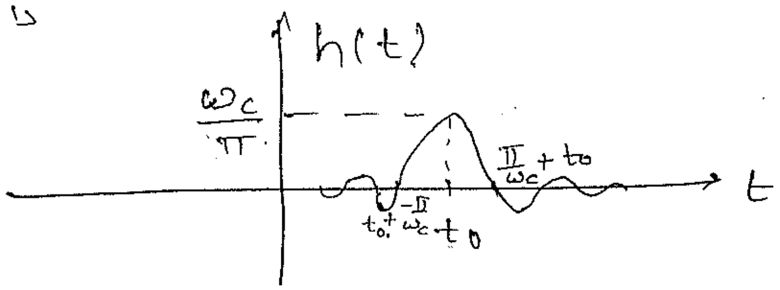
$$= \frac{1}{\pi(t-t_0)} \left[\frac{e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)}}{2j} \right]$$

$$= \frac{1}{\pi(t-t_0)} \sin[\omega_c(t-t_0)]$$

$$= \frac{\omega_c}{\pi} \times \frac{\sin[\omega_c(t-t_0)]}{\omega_c(t-t_0)}$$

$$= \frac{\omega_c}{\pi} \cdot \underline{\underline{\text{Sinc}[\omega_c(t-t_0)]}}.$$

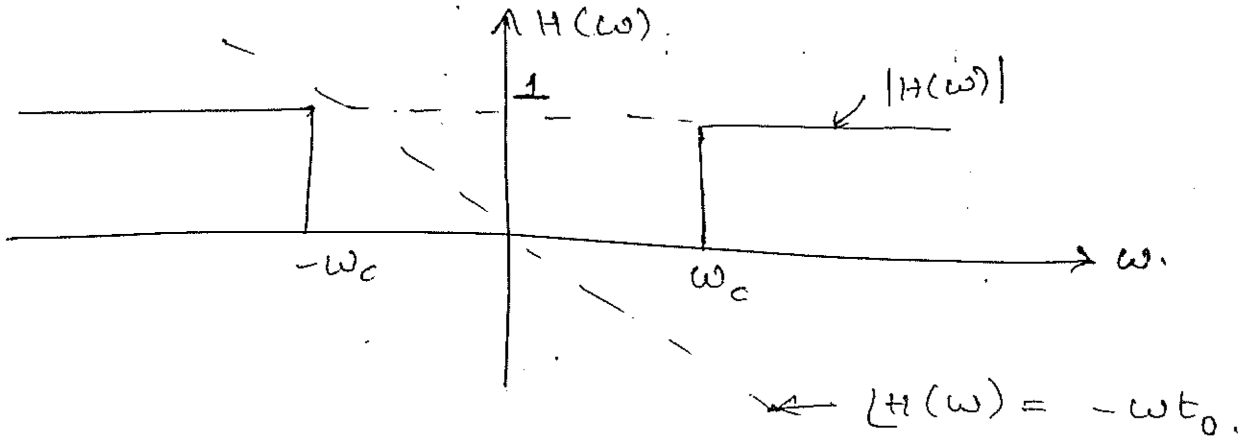
generalised
h(t) spectrum is



From this, the impulse response exist for -ve value of time i.e; $h(t) \neq 0$ for $t<0$. So, the system is non-Causal system. Hence it is not physically realisable.

2. <u>Ideal high pass filter:-</u>

The spectral char. of ideal high pass filter is.



$$|H(\omega)| = \begin{cases} 1 & \text{for } \omega_c \le \omega \le \infty \text{ and } -\infty \le \omega \le -\omega_c \\ 0 & \text{elsewhere i.e; } -\omega_c \le \omega \le \omega_c. \end{cases}$$

$$\angle H(\omega) = -\omega t_0.$$

$$H(\omega) = \begin{cases} 1 \times e^{-j\omega t_0} & ; \omega_c \le \omega \le \infty \text{ and } -\infty \le \omega \le -\omega_c \\ 0 & ; -\omega_c \le \omega \le \omega_c. \end{cases}$$

Impulse response calculation, $F^{-1}[H(\omega)] = h(t)$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)\, e^{j\omega t}\, d\omega.$$

$$h(t) = \frac{1}{2\pi}\left[\int_{-\infty}^{-\omega_c} e^{-j\omega t_0} e^{j\omega t}\, d\omega + 0 + \int_{\omega_c}^{\infty} e^{-j\omega t_0} \cdot e^{j\omega t}\, d\omega\right].$$

$$= \frac{1}{2\pi}\left[\left.\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\right|_{-\infty}^{-\omega_c} + \left.\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\right|_{\omega_c}^{\infty}\right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega_c(t-t_0)}}{j(t-t_0)} - \frac{e^{j\omega_c(t-t_0)}}{j(t-t_0)}\right]$$

$$= \frac{-1}{\pi(t-t_0)}\left(\frac{e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)}}{2j}\right)$$

$$= \frac{-1}{\pi(t-t_0)} \sin[\omega_c(t-t_0)]$$

$$= \frac{-\omega_c}{\pi} \operatorname{Sinc}[\omega_c(t-t_0)]$$

$\therefore$ It cannot be physically realisable. Impulse values cannot exist for $-ve$ values of time.

(or) Alternative method :-

From the spectrum,

$$H(\omega) = e^{-j\omega t_0} u(-\omega-\omega_c) + e^{-j\omega t_0} u(\omega-\omega_c).$$

we know that

$$g(t) \longleftrightarrow G(\omega)$$

$$u(t) \longleftrightarrow \frac{1}{j\omega} + \pi\delta(\omega)$$

$$G(t) \longleftrightarrow 2\pi g(-\omega).$$

$$\frac{1}{jt} + \pi\delta(t) \longleftrightarrow 2\pi\, u(-\omega).$$

$$\left[\frac{1}{jt} + \pi\delta(t)\right] e^{-j\omega_c t} \longleftrightarrow 2\pi\, u(-\omega-\omega_c)$$

$$\left[\frac{1}{j(t-t_0)} + \pi\delta(t-t_0)\right] e^{-j\omega_c(t-t_0)} \longleftrightarrow 2\pi e^{-j\omega t_0} u(-\omega-\omega_c)$$

$$g(-t) \longleftrightarrow G(-\omega)$$

$$\frac{1}{-jt} + \pi\delta(-t) \longleftrightarrow 2\pi\, u(-(-\omega))$$

$$\frac{1}{-jt} + \pi\delta(t) \longleftrightarrow 2\pi\, u(\omega) \quad (\because \delta(t) \text{ satisfies even sym}).$$

$$\left[\frac{1}{-jt} + \pi\delta(t)\right] e^{j\omega_c t} \longleftrightarrow 2\pi\, u(\omega-\omega_c)$$

$$\left[\frac{1}{-j(t-t_0)} + \pi\delta(t-t_0)\right] e^{j\omega_c(t-t_0)} \longleftrightarrow 2\pi e^{-j\omega t_0} u(\omega-\omega_c)$$

$$\therefore F^{-1}[H(\omega)] = F^{-1}\left[e^{-j\omega t_0} u(-\omega-\omega_c)\right] + F^{-1}\left[e^{-j\omega t_0} u(\omega-\omega_c)\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{j(t-t_0)} + \pi\delta(t-t_0)\right] e^{-j\omega_c(t-t_0)} + \frac{1}{2\pi}\left[\frac{-1}{j(t-t_0)} + \pi\delta(t-t_0)\right] e^{j\omega_c(t-t_0)}$$

$$\Rightarrow h(t) = \frac{1}{2\pi}\,\pi\delta(t-t_0)\left[e^{j\omega_c(t-t_0)} + e^{-j\omega_c(t-t_0)}\right]$$

$$+ \frac{1}{2\pi}\cdot\cancel{\pi\delta}\;\frac{-1}{j(t-t_0)}\left[e^{j\omega_c(t-t_0)} - e^{-j\omega_c(t-t_0)}\right]$$

$$= \delta(t-t_0)\cos\left[\omega_c(t-t_0)\right] - \frac{1}{\pi(t-t_0)}\sin\left[\omega_c(t-t_0)\right]$$

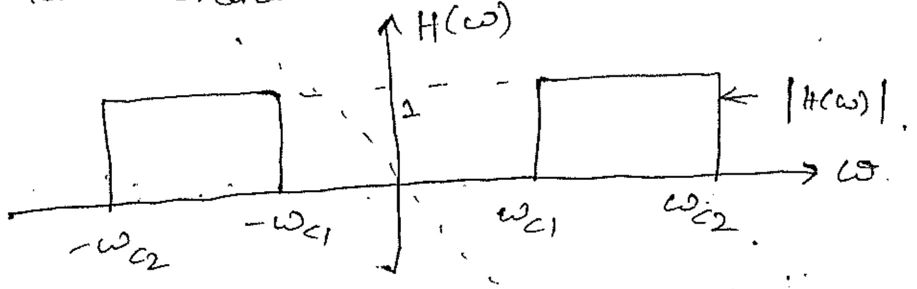
$$\boxed{h(t) = \delta(t-t_0) - \frac{\omega_c}{\pi}\,\mathrm{Sinc}\left[\omega_c(t-t_0)\right]}$$

$\left[\because \text{At } t = t_0,\ \cos(t-t_0) = 0\right]$.

∴ This is not physically realisable.

(iii) Ideal Band pass filter :-

The ideal characteristics of ideal band pass filter is



$\rightarrow \angle H(\omega) = -\omega t_0$.

$$\therefore H(\omega) = \begin{cases} 1\times e^{-j\omega t_0} ; & -\omega_{c2} \le \omega \le -\omega_{c1} ;\ \omega_{c1} \le \omega \le \omega_{c2} \\ 0 ; & \text{elsewhere.} \end{cases}$$

Impulse response is

$$F^{-1}\left[H(\omega)\right] = h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) e^{j\omega t}\, d\omega$$

$$h(t) = \frac{1}{2\pi}\left[\int_{-\omega_{c2}}^{-\omega_{c1}} e^{-j\omega t_0} e^{j\omega t}\, d\omega + \int_{\omega_{c1}}^{\omega_{c2}} e^{-j\omega t_0} e^{j\omega t}\, d\omega\right].$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\Bigg|_{-\omega_{c2}}^{-\omega_{c1}} + \frac{e^{j\omega(t-t_0)}}{j(t-t_0)}\Bigg|_{\omega_{c1}}^{\omega_{c2}}\right]$$

t−t0)

$$= \frac{1}{2\pi}\left[\frac{e^{-j\omega c_1(t-t_0)} - e^{-j\omega c_2(t-t_0)}}{j(t-t_0)} + \frac{e^{j\omega c_2(t-t_0)} - e^{j\omega c_1(t-t_0)}}{j(t-t_0)} \right]$$

$$= \frac{1}{2\pi} \cdot 2\left[\frac{e^{j\omega c_2(t-t_0)} - e^{-j\omega c_2(t-t_0)}}{2j} - \left(\frac{e^{j\omega c_1(t-t_0)} - e^{-j\omega c_1(t-t_0)}}{2j} \right) \right]$$

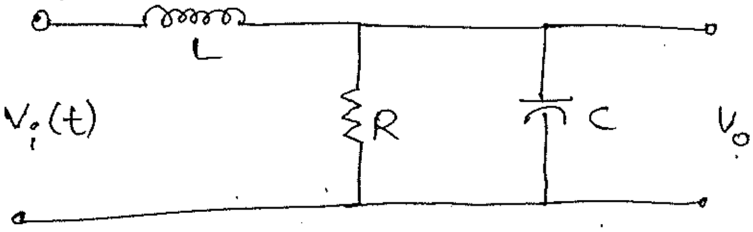
$$= \frac{1}{\pi(t-t_0)} \left[\sin(\omega c_2(t-t_0)) - \sin(\omega c_1(t-t_0)) \right]$$

$$\therefore h(t) = \frac{\omega c}{\pi} \left[\mathrm{sinc}(\omega c_2(t-t_0)) - \mathrm{sinc}(\omega c_1(t-t_0)) \right]$$

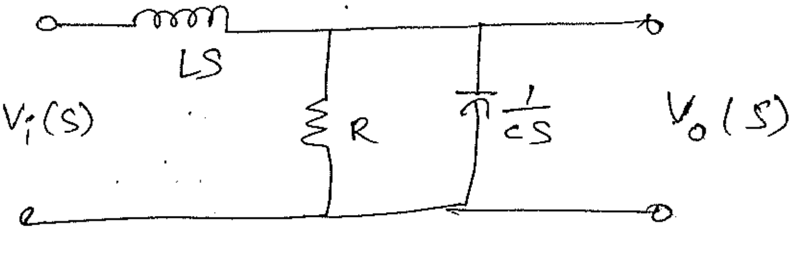
h(t) exist for -ve values of time. Hence $h(t) \neq 0$ for $t<0$. So it is not physically realisable.

Example for practical low pass (or) physically realisable low pass filter :-

The practical RLC - Low pass ckt is as shown in figure.



Eq. Laplace S-domain ckt is



$$V_o(S) = \left(R \| \frac{1}{CS}\right) I(S) = \frac{R \cdot \frac{1}{CS}}{R + \frac{1}{CS}} \cdot I(S).$$

$$= \frac{R}{1+SCR} I(S).$$

$$V_i(S) = \left[LS + \left(R \| \frac{1}{CS}\right)\right] I(S)$$

$$= \left[LS + \frac{R}{1+SCR}\right] I(S).$$

$$= \frac{LS(1+SCR)+R}{1+SCR} \cdot I(S).$$

$$\frac{V_o(S)}{V_i(S)} = \frac{R/1+SCR}{LS(1+SCR)+R/(1+SCR)} = \frac{R}{LS(1+SCR)+R}.$$

$$\Rightarrow \frac{V_o(S)}{V_I(S)} = H(S) = \frac{R}{S^2LCR + LS + R}.$$

$$\Rightarrow H(S) = \frac{1}{S^2LC + S \cdot \frac{L}{R} + 1}$$

$$(\because S = j\omega)$$

$$H(S) = H(j\omega) = \frac{1}{-\omega^2 LC + j\omega\frac{L}{R} + 1}$$

$$H(j\omega) = \frac{1}{LC}\left[\frac{1}{-\omega^2 + j\omega \cdot \frac{1}{RC} + \frac{1}{LC}}\right]$$

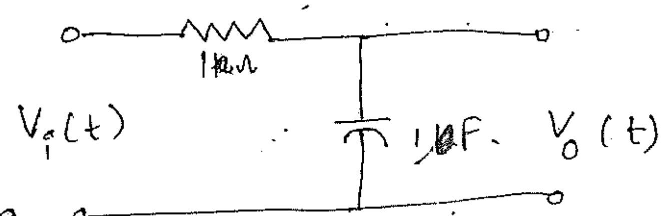
$$H(S) = \frac{1}{LC}\left(\frac{1}{S^2 + 2S \cdot \frac{1}{2RC} + \frac{1}{LC}}\right).$$

$$= \frac{1}{LC}\left(\frac{1}{S^2 + 2S \cdot \frac{1}{2RC} + \frac{1}{LC} + \left(\frac{1}{2RC}\right)^2 - \left(\frac{1}{2RC}\right)^2}\right)$$

$$= \frac{1}{LC}\left[\frac{1}{\left(S + \frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}\right]$$

$$= \frac{1}{LC}\left[\frac{1}{\left(S + \frac{1}{2RC}\right)^2 - \left(\frac{-1}{LC} + \left(\frac{1}{2RC}\right)^2\right)}\right].$$

* The practical low pass filter is as shown in the figure. Find the o/p power spectral density, o/p power, mean square value and RMS value of o/p signal.



Given I/p spectral densities are

(a) $S_i(\omega) = K$

(b) $S_i(\omega) = G_2(\omega)$ [i.e; gate fn. width is 'ω']

(c) $S_i(\omega) = \pi\left(\delta(\omega-1) + \delta(\omega+1)\right)$

$\longrightarrow$ Relation b/w i/p & o/p spectral densities :-

we know that I/p & o/p relation of LTI System is

$$y(t) = x(t) * h(t)$$

$$F[y(t)] = F(x(t) * h(t))$$

$$\Rightarrow \quad Y(\omega) = X(\omega) \cdot H(\omega), \text{ where}$$

$X(\omega)$ is I/p signal spectrum.

$Y(\omega)$ is o/p signal spectrum.

$H(\omega)$ is transfer fn. of the system (or) freq. response.

Applying both sides absolute value,

$$|Y(\omega)| = |X(\omega)| \cdot |H(\omega)|$$

$$= |H(\omega)| \cdot |X(\omega)|$$

Applying square on both sides

$$|Y(\omega)|^2 = |H(\omega)|^2 \, |X(\omega)|^2$$

Energy Spectral density of $g(t)$ is.

$$\boxed{\Psi_g(\omega) = |G(\omega)|^2 \quad (J/Hz)}$$

Energy spectral density of I/p signal is

$$\Psi_x(\omega) = S_i(\omega) = |X(\omega)|^2$$

Energy spectral density of o/p is

$$\Psi_y(\omega) = S_0(\omega) = |Y(\omega)|^2.$$

(or) $$\boxed{S_o(\omega) = |H(\omega)|^2 \, S_i(\omega)}.$$

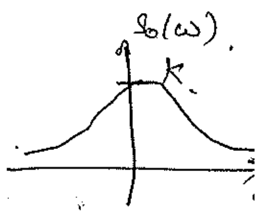
(a) $$H(\omega) \left(\frac{V_o(s)}{V_i(s)}\right) = \frac{1/j\omega}{1+\frac{1}{j\omega}} = \frac{1}{1+j\omega}.$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}} \Rightarrow |H(\omega)|^2 = \frac{1}{1+\omega^2}.$$

Output Spectral density is

$$S_o(\omega) = \Psi_y(\omega) = |H(\omega)|^2 \, S_i(\omega).$$

$$S_o(\omega) = \frac{1}{1+\omega^2} \times K = \frac{K}{1+\omega^2} \text{ watt/Hz}.$$



$$\text{Power} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_o(\omega)\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{K}{1+\omega^2}\, d\omega = \frac{K}{2\pi} \cdot \tan^{-1}(\omega)\Big|_{-\infty}^{\infty}$$

$$= \frac{K}{2\pi}\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$$

$$= \frac{K}{2} \text{ Watts.}$$

Mean Square Value :-

O/P Signal power is nothing but the mean square value of o/p signal.

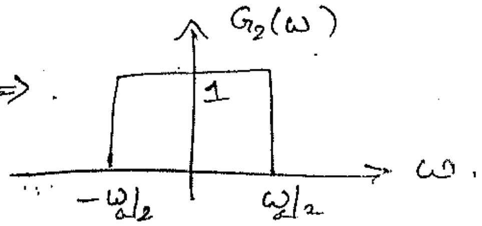
Mean Square value is $\frac{1}{T}\int_{-T}^{T} |g(t)|^2 dt$. watts

This is nothing but power $\left(\frac{1}{2\pi}\int_{-\infty}^{\infty} S_o(\omega)\, d\omega\right)$.

Root of mean Square value is rms value.

$\therefore$ Rms value $= \sqrt{\frac{K}{2}}$. //

(b) $S_i(\omega) = G_2(\omega) \Rightarrow$

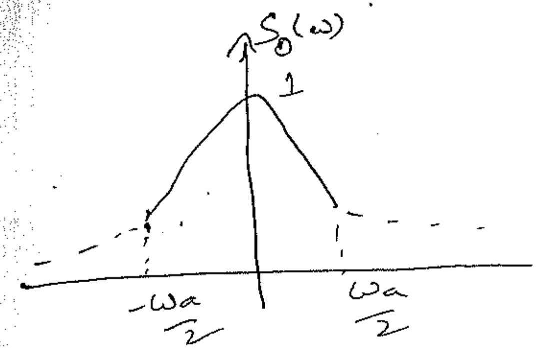


$\rightarrow$ $H(\omega) = \frac{1}{1+j\omega}$; $|H(\omega)|^2 = \frac{1}{1+\omega^2}$.

O/P Spectral density is

$$S_o(\omega) = |H(\omega)|^2 S_i(\omega)$$

$$= \frac{1}{1+\omega^2} \cdot |G_2(\omega)|^2$$



$$= \begin{cases} \frac{1}{1+\omega^2} \times 1^2 & ; -\frac{\omega_a}{2} \le \omega \le \frac{\omega_a}{2} \\ 0 & \text{else.} \end{cases}$$

Power, $P = \frac{1}{2\pi}\int_{-\frac{\omega_a}{2}}^{\frac{\omega_a}{2}} \frac{1}{1+\omega^2} d\omega = \frac{1}{2\pi} \operatorname{Tan}^{-1}(\omega)\Big|_{-\frac{\omega_a}{2}}^{\frac{\omega_a}{2}}$

$$= \frac{1}{\pi}\operatorname{Tan}^{-1}\left(\frac{\omega_a}{2}\right).$$

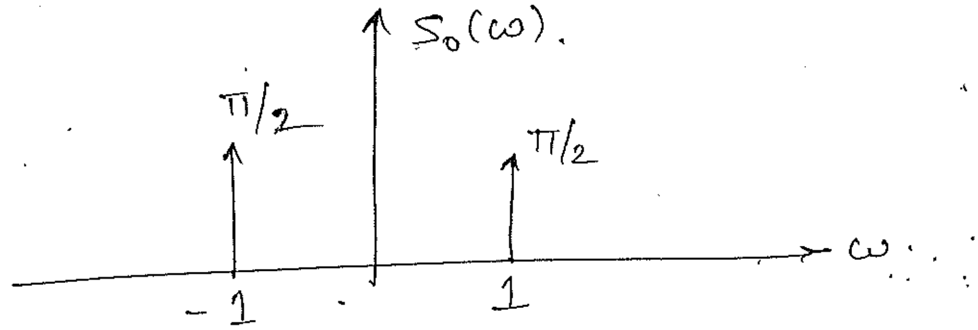
Mean Square = Power.

RMS = $\sqrt{\text{Power}} = \sqrt{\frac{1}{\pi}\operatorname{Tan}^{-1}\left(\frac{\omega_a}{2}\right)}$

(c) $S_i(\omega) = \pi\left[\delta(\omega-1) + \delta(\omega+1)\right]$

$$S_o(\omega) = \frac{1}{1+\omega^2} \cdot \pi\left[\delta(\omega-1) + \delta(\omega+1)\right].$$

$$= \frac{\pi}{1+\omega^2}\,\delta(\omega-1) + \frac{\pi}{1+\omega^2}\,\delta(\omega+1)$$



(a)

$$\text{Power} = \frac{1}{2\pi}\left[\int_{-\infty}^{\infty} \frac{\pi}{1+\omega^2}\,\delta(\omega-1)\,d\omega + \int_{-\infty}^{\infty} \frac{\pi}{1+\omega^2}\,\delta(\omega+1)\,d\omega\right]$$

$$= \frac{1}{2\pi}\left(\frac{\pi}{1+1^2} + \frac{\pi}{1+(-1)^2}\right)$$

$$= \frac{1}{2\pi}(\pi) = 1/2 \;//.$$

Mean Square Value = power = $\frac{1}{2}$

RMS Value = $\sqrt{\text{Power}} = \frac{1}{\sqrt{2}}$ //