SIGNALS & SYSTEMS

UNIT-I

SIGNAL ANALYSIS : Analogy between vectors and signals, – orthogonal signal space – Signal approximation using orthogonal functions – Mean square error – closed or complete set of orthogonal functions – orthogonality in complex functions – Exp. and sinusoidal signals – concepts of impulse function unit step function – signum function.

UNIT – II

FOURIER SERIES REPRESENTATION OF PERIODIC SIGNALS
Representation of Fourier Series – Continous time periodic signals – properties of Fourier Series – Dirichlet's conditions – Trignometric Fourier Series and Exponential Fourier Series – Complex Fourier Spectrum.

UNIT – III

FOURIER TRANSFORMS : Deriving Fourier transform from Fourier series – Fourier transform of arbitrary signal – Fourier transform of standard signals – Fourier transform of periodic signals – properties of Fourier transforms – Fourier transforms involving impulse function and signum function – Introduction to Hilbert Transform.

UNIT - IV

SIGNAL TRANSMISSION THROUGH LINEAR SYSTEMS:
Linear System - Impulse response - Response of a linear system - Linear time invariant (LTI) system - Linear time variant (LTV) System - Transfer function of a LTI System. Filter characteristics of linear systems. Distortion less transmission through a system - Signal bandwidth - System bandwidth - Ideal LPF - HPF and BPF characteristics - Causality and Poly - Wiener criterion for physical realisation - relationship between bandwidth and rise time.

UNIT - V

CONVOLUTION AND CORRELATION OF SIGNALS:
Concept of Convolution in time domain and frequen domain - Graphical representation of convolution - Convolution property of fourier transforms - Cross correlation and auto correlation of functions, propert of Correlation function - Energy density spectrum - Parseval's theorem - Power density Spectrum - Relation b/w auto correlation function & energy/power) Spectra density function - Relation b/w convolution and Correlation - Detection of periodic Signalsin the presence of noise by Correlation - Extraction of Signal from noise by filtering.

UNIT - VI

SAMPLING: Sampling theorem - Graphical and analytical proof for band limited signals - impulse Sampling - Natural and flat top Sampling -

Reconstruction of Signal from its Samples - effect of under Sampling - Aliasing - Introduction to Band Pass Sampling.

UNIT - VII

LAPLACE TRANSFORMS : Review of LT - Partial fraction expansion - Inverse LT - Concept of region of convergence (ROC) for LT - Constraints on ROC for various classes of signals - Properties of LT's relation between LT's and F.T. of a Signal - LT of certain Signals using wave form synthesis.

UNIT - VIII

Z-TRANSFORMS : Fundamental difference b/w Continous and discrete time Signals - discrete time Signal representation using complex exponentic and Sinosoidal components - Periodicity of discrete time using Complex exp. Signal - Concept of z-transform of a discrete sequence - Distinction between Laplace - Fourier and z transforms - Region of convergence in z-transform - Constraint on ROC for Various classes of Signals - Inverse Z-transform - Properties of z-transforms.

FORMULAE

Trignometric Identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B.$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

$$\sin 2A = 2\sin A \cos A.$$

$$\cos 2A = 2\cos^2 A - 1$$

$$= \cos^2 A - \sin^2 A.$$

$$e^{jx} = \cos x + j\sin x$$

$$\sin x = \frac{e^{jx} - e^{-jx}}{2}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

Taylor's Series:-

$$f(x) = f(a) + f'(a)\frac{(x-a)}{1!} + f''(a)\frac{(x-a)^2}{2!} + \cdots\cdots$$

$$f'(x) = \frac{d}{dx}\left[f(x)\right]$$

$$\lim_{x \to a} f'(x) = f'(a)$$

Maclaurin's Series Expansion :-

$$f(x) = f(0) + f'(0)\frac{x}{1!} + f''(0)\frac{x^2}{2!} + \cdots\cdots$$

Approximation :-

If $x << 1$, then $\frac{1}{1+x} \simeq 1 - x$

$$(1+x)^n \simeq 1 + nx \quad \therefore n \geq 1 \ \& \ x << 1$$

$$e^x \simeq 1 + x$$

$$\ln(1+x) \simeq x$$

$$\sin x \simeq x$$

$$\cos x \simeq 1 - \frac{x^2}{2}$$

$$\tan x \simeq x.$$

Indefinite Integrals

$$\int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \cos bx + b \sin bx\right]$$

$$\int e^{ax} \sin bx \, dx = \frac{e^{ax}}{a^2+b^2}\left[a \sin bx - b \cos bx\right]$$

Definite Integrals

$$\int_0^\infty e^{-r^2x^2} \, dx = \frac{\sqrt{\pi}}{2r}$$

$$\int_0^\infty x \, e^{-r^2x^2} \, dx = \frac{1}{2r^2}$$

$$\int_0^\infty x^2 e^{-r^2x^2}\cdot dx = \frac{\sqrt{\pi}}{4r^3}$$

$$\int_0^\infty x^n e^{-ax}\cdot dx = \frac{n!}{a^{n+1}}$$

$$\int_0^\infty \frac{\sin^2 x}{x}\cdot dx = \pi/2$$

$$\int_0^\infty \frac{\sin ax}{x}\cdot dx = \begin{cases} \pi/2 & \text{for } a>0 \\ 0 & \text{for } a=0 \\ -\pi/2 & \text{for } a<0. \end{cases}$$

$$\int_0^\infty \frac{\sin^2(ax)}{x^2}\cdot dx = |a|\,\pi/2$$

$$\int_0^\infty \sin mx\cdot\cos nx = \frac{2m}{m^2-n^2} \quad \text{if } (m+n) \text{ is odd}$$

$$= 0 \quad \text{if } (m+n) \text{ is even}$$

where m, n are integers.

Geometric Formulae :-

1. $$\sum_{n=0}^{N} n = \frac{N(N+1)}{2}$$

2. $$\sum_{n=0}^{N} n^2 = \frac{N(2N+1)(N+1)}{6}$$

3. $$\sum_{n=0}^{N} n^3 = \frac{N^2(N+1)^2}{4}$$

4. $$\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1$$

5. $$\sum_{n=0}^{N} a^n = \frac{a^{N+1} - 1}{a-1}$$

6. $$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a} \quad ; \quad |a| < 1$$

7. $$\sum_{n=0}^{\infty} n a^n = \frac{a}{(1-a)^2} \quad ; \quad |a| < 1$$

8. $$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

9. $$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

10. $$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

3/7/06

CLASSIFICATION OF SIGNALS

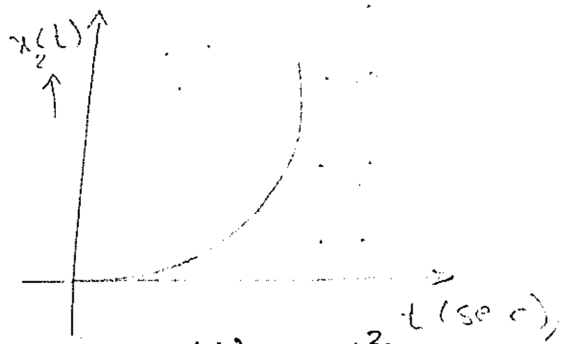
SIGNAL :-

Signal is defined as a physical quantity that varies with time, space or any other independent variable.

(or)

We define signal as a single value function of time that conveys information.

Ex :-



$x_1(t) = 5t$ $x_2(t) = 4t^2$

Here first signal $x_1(t)$ varies linearly with time 't' and the second signal $x_2(t)$ varies linearly with square of the time 't'.

* Time-axis is independent variable and $x(t)$, amplitude-axis is dependent variable.
* Practical signals are telephone signal, ECG signal, speech signal, noise signal etc.

SYSTEM :-

System is defined as a physical device that performs an operation on a signal.

Ex :- Filter, amplifier.

Filter :-

It is used to reduce the noise corrupting the desired information barring signal.

Amplifier :-

It is a device that performs amplification operation on a given input signal.

* Signals are classified into
  (1) Continous & discrete time signals.
  (2) Digital & Analog signals
  (3) Periodic & non-periodic signals.
  (4) Deterministic & random signals.
  (5) Symmetric (even) & anti-Symmetric (odd) Signals.
  (6.) Energy & Power signals.
  (7) Multichannel & multi-dimensional signals.

1. Continous time signals :-

Continous time Signal is defined at every time instant. i.e; It has values at all the time instants.

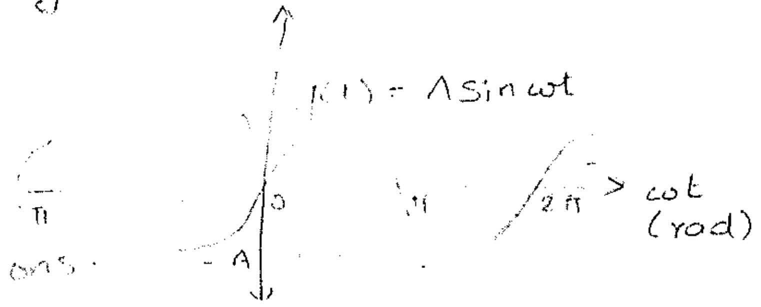
* It can be classified into 2 categories.
  (i) Continous time, Continous Amplitude Signal (or) Analog signal.
  (ii) Continous time, discrete amplitude Signal.

CT, CA Signals :-

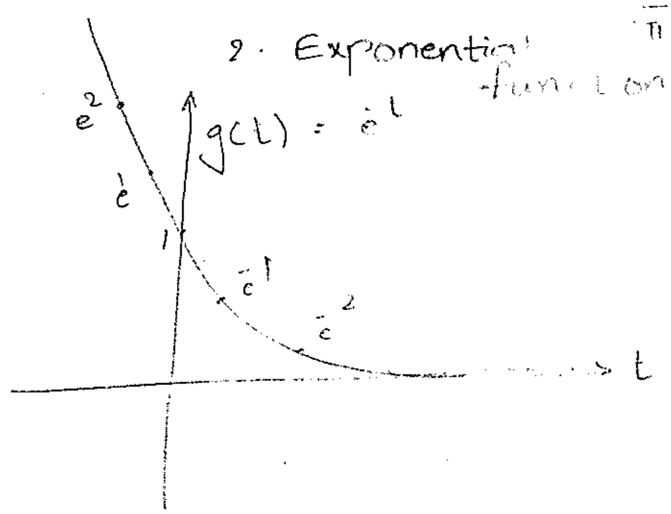
CT, CA Signal is that they are Continous in amplitude and it is defined at all the time instants. These Signals are called analog (or) CT, CA Signals.

Ex :- 1. Sinusoidal Signals.

$g(t) = A \sin \omega t$



2. Exponential functions.

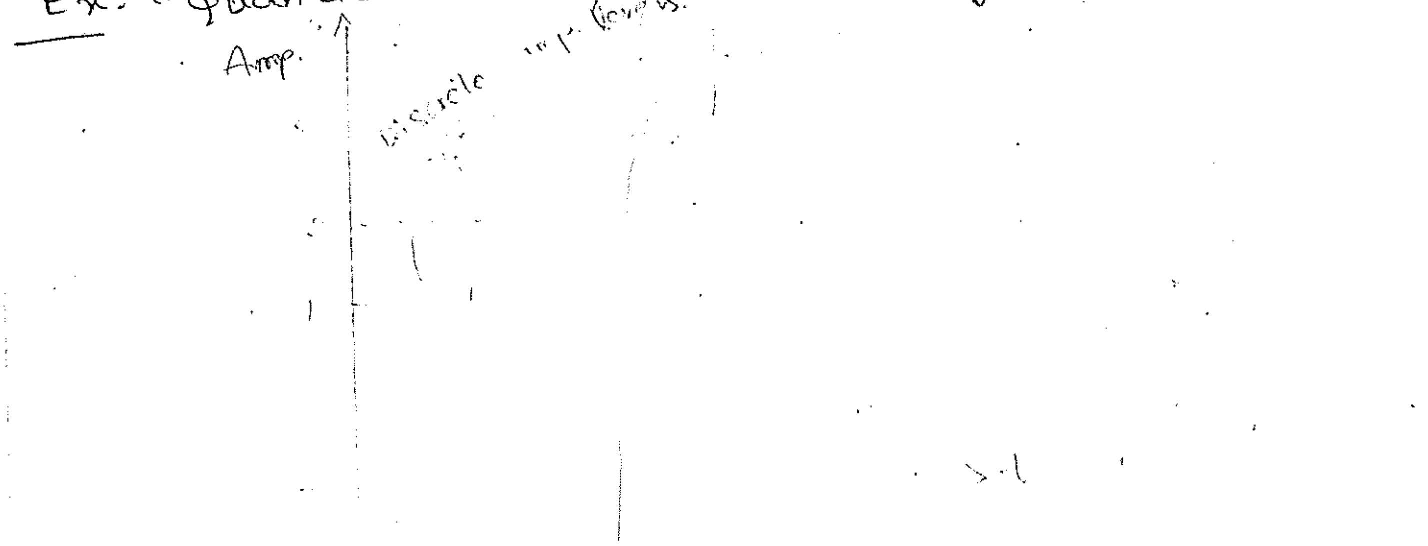


$g(t) = e^{-t}$

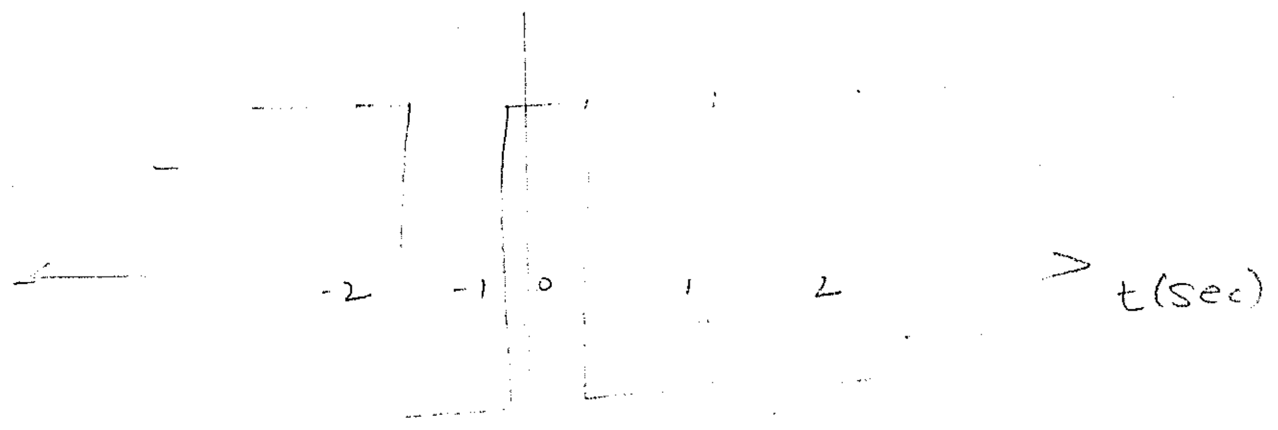
CT, DA Signals :-

CT, DA Signals is that they are discrete in amplitude and it is defined at all the time instants. Such Signals are called CT, DA Signals.

Ex :- 1. Quantised version of CT signal.



2. Square waves.



NOTE :- From this, we observe that all analog signals are CT signals but all CT signals are not analog signals.

4/7/06 **Discrete time signal** :-

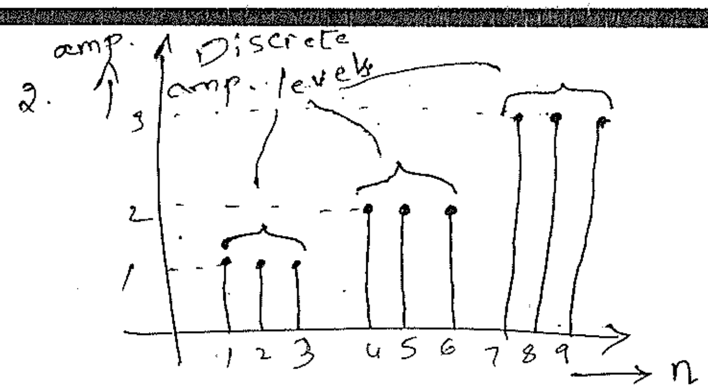
The signal that has values only at discrete instants of time.

* These signals can be classified into
  (i) Discrete time, discrete amplitude signals (or) Digital signals.
  (ii) Discrete time, continous amplitude signals.

DT, DA Signal :-

DT, DA signals is that they are discrete in amplitude and it is defined at discrete instants of time only. These signals are called DT, DA signals (or) digital signals.

Ex :- 1. Binary information



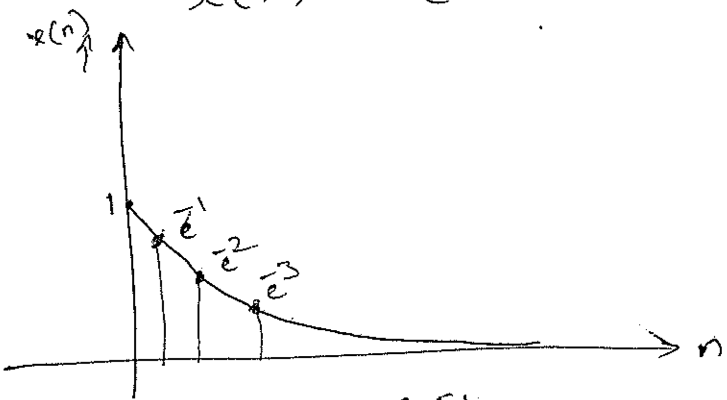
DT, CA Signals :-

DT, CA signals are that they are continous in amplitude and it is defined at discrete instants of time. Such signals are called DT, CA Signals.

Ex:- 1. Discrete exponential signal.

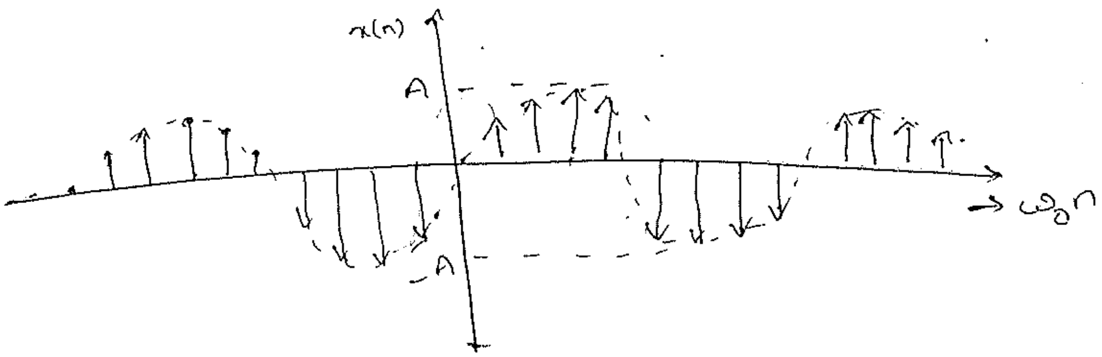
$$x(n) = e^{-n}$$

	representation
cont. time	t
Discrete time	n



2. Discrete ~~Sine~~ cosine (cos) wave.

$$x(n) = A\cos(\omega_0 n)$$



3. Periodic & non-Periodic signals :-

Periodic Signal :-

The Signal $g(t)$ is a periodic signal if it satisfies the Condition

$$g(t) = g(t + T_0) \quad \forall \text{ values of } t,$$

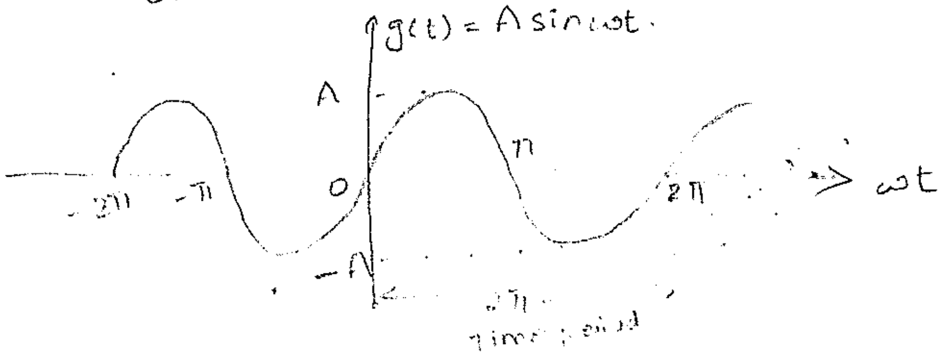
where 't' is Continous time in seconds.

'T_0' is Smallest value of time to satisfy the above Condition.

This is called timeperiod of the Signal $g(t)$.

Ex :- Sinusoidal Signals (sine, cos waves).

1. Sine wave.



$\cancel{g(t) = g(t+T_0)}$ $\quad T_0 = 2\pi$

$\cancel{= g(}$

$$g(t + T_0) = A \sin(\omega t + 2\pi)$$
$$= A \sin \omega t$$
$$= g(t).$$

Hence the Sine wave is a periodic wave

Non-periodic Signal :-

$g(t)$ is said to be non-periodic signal if it doesn't satisfy the condition,

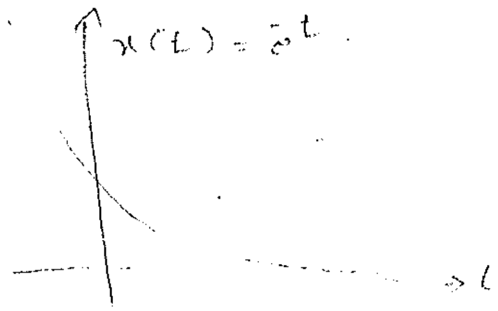
$$g(t) = g(t+T_0)$$

(or)

If $g(t) \neq g(t+T_0)$, then the signal is said to be non-periodic.

Ex :- Exponential signals.

$x(t) = e^{-t}$.



4. Deterministic & random Signals :-

Deterministic signal :-

The signal which can be completely described by the mathematical model is called deterministic signal.

(or)

Deterministic signal is a signal about which there is no uncertainity before it actually occurs.

Ex :- 1. Sinusoidal waves

Sine wave $x(t) = A\sin(\omega t)$

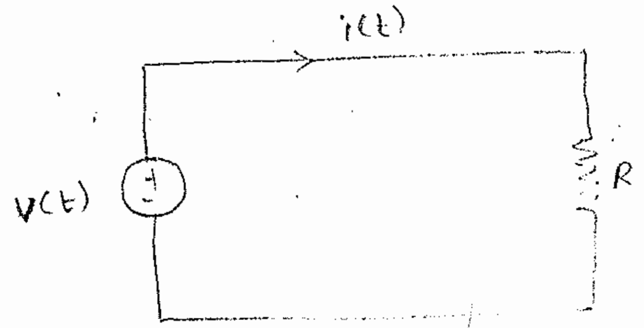
Cos wave $x(t) = A\cos(\omega t)$

2. Exponential functions

$x(t) = e^{-ax}$

$x(t) = e^{ax}$

6. Energy & Power Signals :-



Basic Electrical System.

* In an electrical system, ~~basic~~ instantaneous signals are $v(t)$, $i(t)$, where $v(t)$ ~~or~~ is voltage developed across resistor 'R' and producing current is $i(t)$ as shown in fig.

* It dessipates instantaneous power across

$$P(t) = \frac{v^2(t)}{R} \text{ (or) } (i(t))^2 R$$

* For $R = 1\Omega$; $p(t) = |v(t)|^2$ (or) $|i(t)|^2$

* In general for signal analysis, the instantaneous power is proportional to amplitude of square of signal.

i.e; $p(t) = |g(t)|^2$

* Total energy of a Signal ; $\boxed{E = \int_{-\infty}^{\infty} |g(t)|^2 . dt}$

(or)

$$\boxed{E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |g(t)|^2 . dt.}$$

* Total Power of the signal $g(t)$ is

$$\boxed{P = P_{avg.} = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 . dt}$$

Energy Signal :-

We say that the Signal $g(t)$ is energy signal iff the total energy of the Signal is finite and it has zero average power.

i.e; $\boxed{0 < E < \infty \quad \text{and} \quad P_{avg.} = 0}$

Ex :- non-periodic signals.

Power signals :-

We say that the signal $g(t)$ is power signal iff the total average power of the Signal is finite and it has infinite energy.

i.e; $\boxed{0 < P_{avg.} < \infty \quad \text{and} \quad E = \infty}$

Ex :- Periodic signals.

* Energy Signal having non-zero energy.
* Power Signal having non-zero power.

7. Multi-channel & multi-dimensional Signals :-

Multi-channel Signals :-

Different signals are recorded from a single Source. Such signals are called multichannel signals.

Ex :- 3 (or) 12 channel ECG Signals.

Single-dimensional Signals :-

The signal which is a single function of variable is called single-dimensional signal.

Ex :- $x(t) = A \sin(\omega t)$

Multi-dimensional signals :-

The signals which are ~~one~~ two or more function of variables are called multi-dimensional Signals.

Ex :- Image, picture,

Intensity calculation $I(x, y) = 5x + 6y^2$.

6/7/06 Standard continous time signals

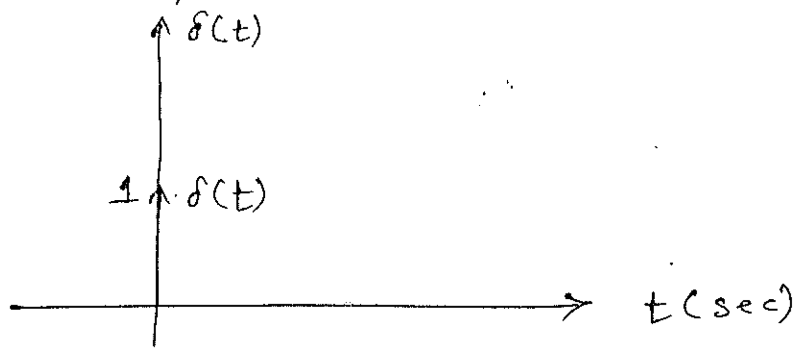
1. Unit Sample Signal (or) Unit Impulse signal (or) dirac delta functions.

Unit impulse sequence is denoted as $\delta(t)$.

Mathematical expression is

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

The graphical representation is



2. Unit Step Signal :-

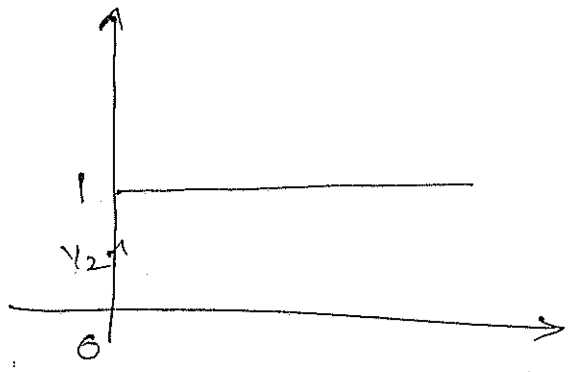
It is represented by $u(t)$.

Mathematical representation of unit step Signal is.

$$u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 1/2 & \text{for } t = 0 \\ 0 & \text{for } t < 0. \end{cases}$$

(or) $u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0. \end{cases}$

Graphical representation of u(t) is



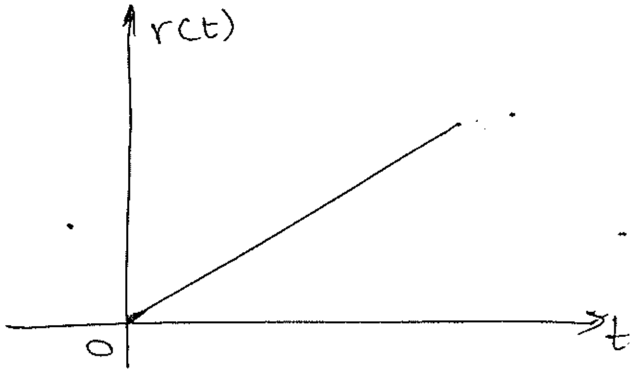
3. Unit ramp (sequence) signal :-

It is represented by r(t).

Mathematical representation is

$$r(t) = \begin{cases} t & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Graphical representation is



4. Unit Parabolic Signal :-

It is represented by $u_p(t)$.

Mathematical representation of parabolic Signal is

$$u_p(t) = \begin{cases} t^2 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0. \end{cases}$$

Graphical representation is

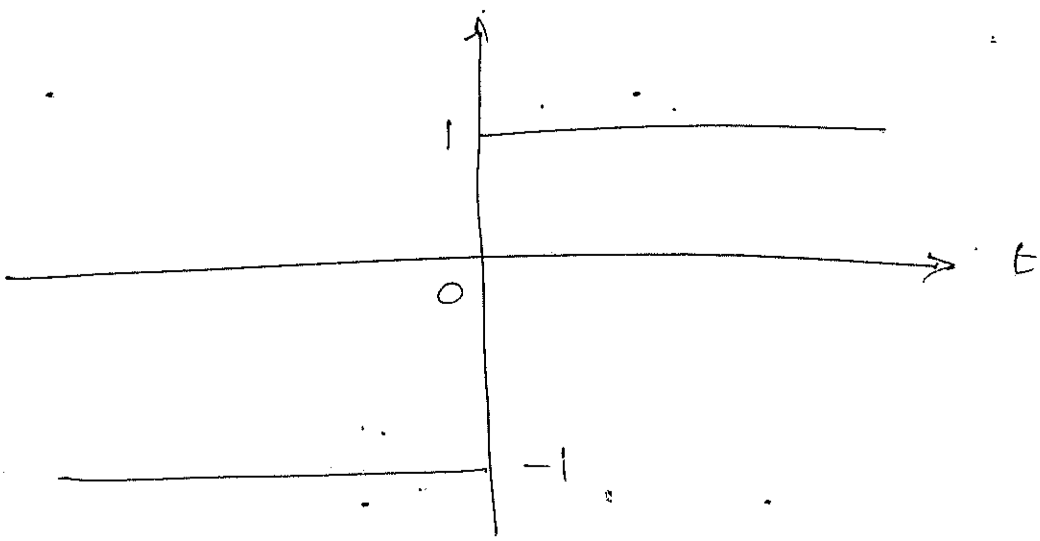
5. Signum function :-

It is represented by "$sgn(t)$."

Mathematical representation of signum function is

$$sgn(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & t = 0 \\ -1 & t < 0. \end{cases}$$

Graphical representation is

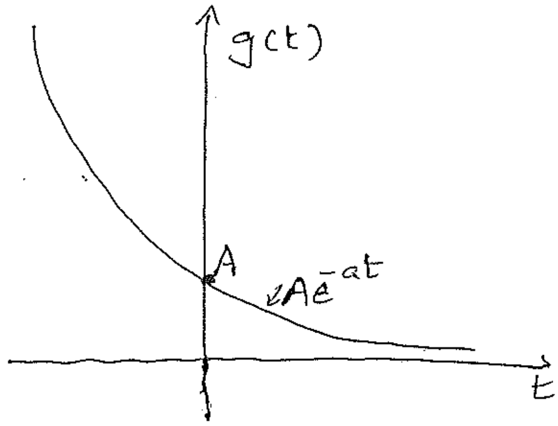


$\therefore$ $\boxed{sgn(t) = 2u(t) - 1}$

6. Exponential signals.

case (i) :-

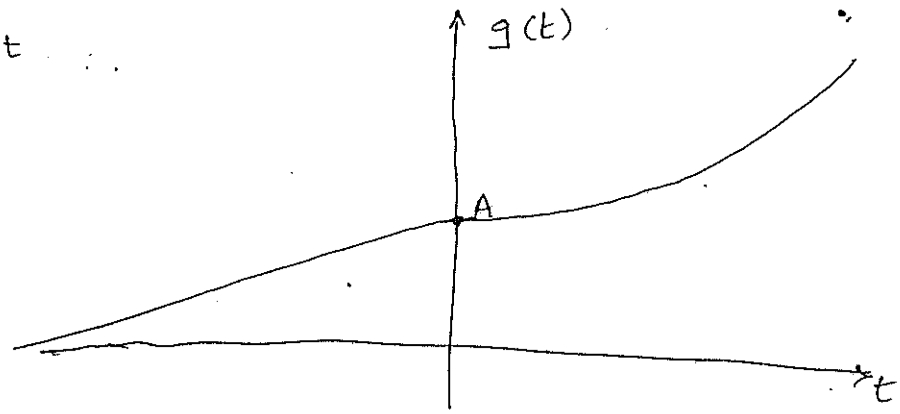
Decaying exp. signal.



case (ii) :-

Growing exponential signal
(or) Rising

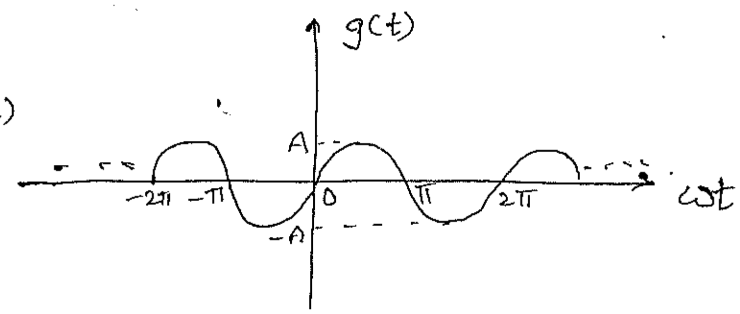
Ex: $g(t) = Ae^{at}$



7. Sinosoidal signals :-

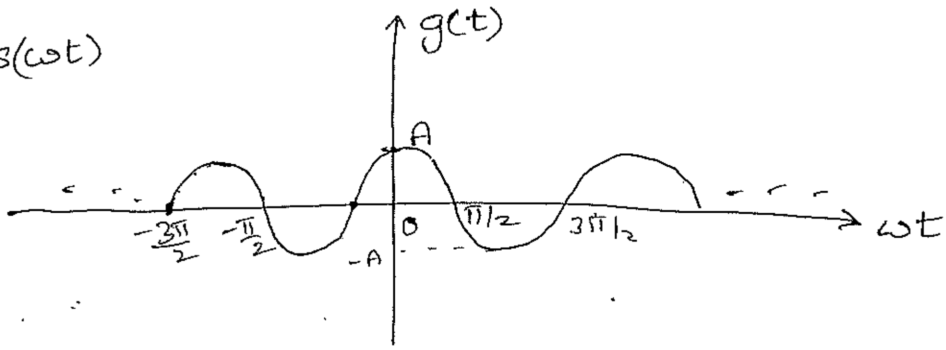
(i) Sine wave.

$g(t) = A\sin(\omega t)$



(ii) Cosine wave.

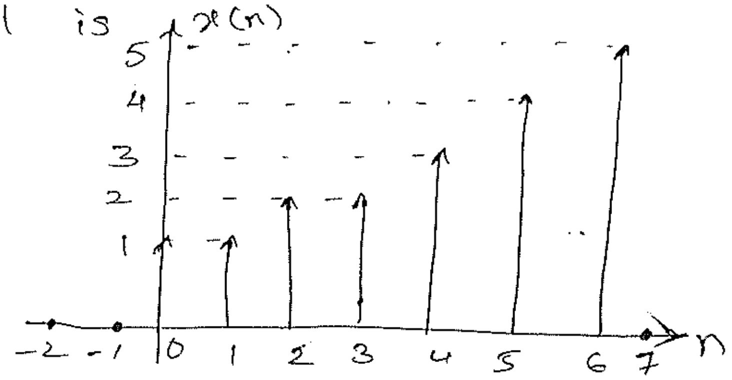
$g(t) = A\cos(\omega t)$



Representation of discrete time signals :-

1. Graphical Representation.

Ex :- The graphical representation of discrete time signal is



2. Functional (or) Mathematical representation.

Mathematical representation of above example is

$$x(n) = \begin{cases} 1 & \text{for } n=0,1 \\ 2 & \text{for } n=2,3 \\ 3 & \text{for } n=4 \\ 4 & \text{for } n=5 \\ 5 & \text{for } n=6 \end{cases}$$

3. Tabular representation.

Tabular representation of the above example is

n	...	-3	-2	-1	0	1	2	3	4	5	6	7.
$x(n)$	...	0	0	0	1	1	2	2	3	4	5	0.

4. Sequence Representation.

The Sequence representation of above example is

$$x(n) = \{\ldots\ldots, 0, 0, \underset{\uparrow}{1}, 1, 2, 2, 3, 4, 5, 0, 0, \ldots\ldots\}$$

$(n=0)$

* Example for finite duration Sequence representatio of

$$x(n) = \{1, 2, \underset{\uparrow}{3}, 4, 5\}$$

* finite duration sequence for $n<0$ is

$$x(n) = 0$$

$$x(n) = \{\underset{\uparrow}{1}, 3, 4, 5, 6, \ldots\}$$

* Ex. for $x(n) = 0$ for $n>0$ is

$$x(n) = \{\ldots, 4, 3, 2, -1, \underset{\uparrow}{1}\}$$

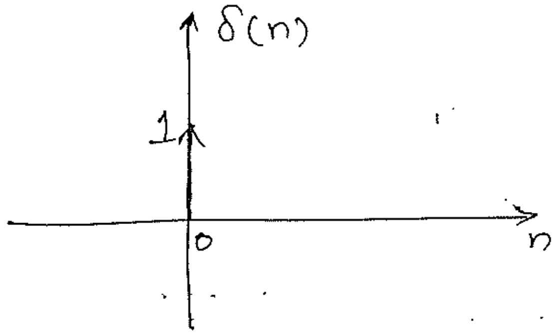
* Arrow mark indicates the instant at $n=0$.

Standard Discrete time Signals:-

1. Unit Sample Sequence
(or) Unit impulse Sequence
(or) Dirac delta function.

The impulse sequence represented by the symbol $\delta(n)$.

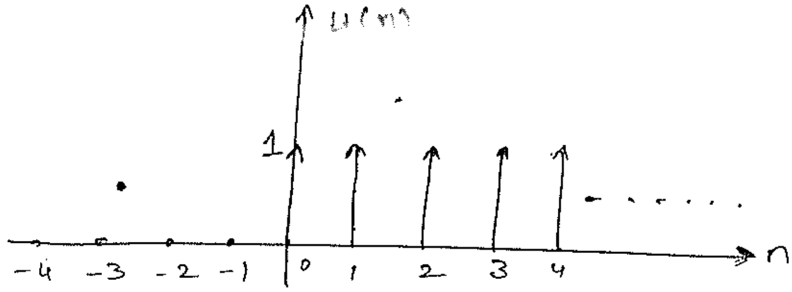
$$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n\neq 0 \end{cases}$$



2. Unit Step Sequence.

This is rep. by $u(n)$.

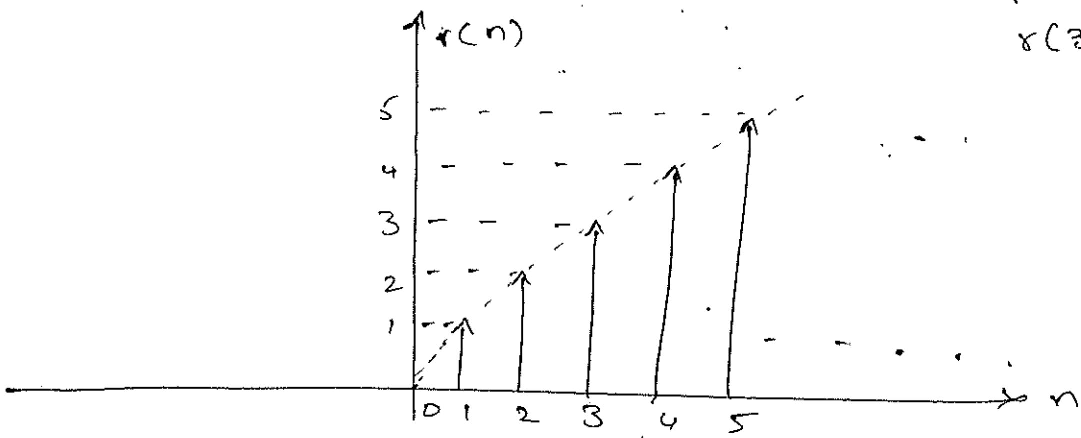
$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$



3. Unit Ramp Sequence.

$$r(n) = \begin{cases} n & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

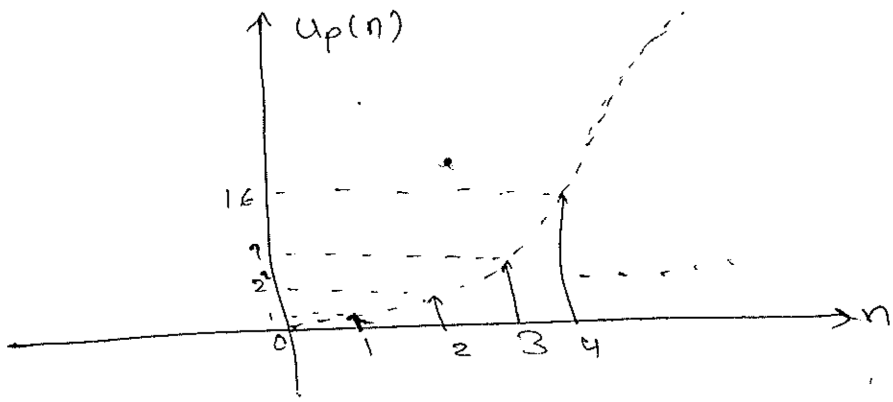
$r(0) = 0$ $r(-1) =$
$r(1) = 1$ $r(-2) =$
$r(2) = 2$ $r(-3) =$
$r(3) = 3$



4. Unit Parabolic Sequence

This is rep. by $u_p(n)$

$$u_p(n) = \begin{cases} n^2 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0. \end{cases}$$



5. Exponential Sequences.

Generally the exponential sequence in discrete domain is

$$x(n) = e^{a|n|} = \begin{cases} e^{an} & \text{for } n \geq 0 \\ e^{-an} & \text{for } n < 0. \end{cases}$$

(i) Exponentially ~~decaying~~ rising Sequence.

$$x(n) = e^{an} \quad \text{where} \quad 0 < a < 1.$$

Ex) Take $a = 0.5$

$$\therefore x(n) = e^{0.5n}$$

for $n = 0$ $\quad x(0) = 1$

$n = 1$ $\quad x(1) = e^{0.5}$

$n = 2$ $\quad x(2) = e^{1}$

$n = 3$ $\quad x(3) = e^{1.5}$

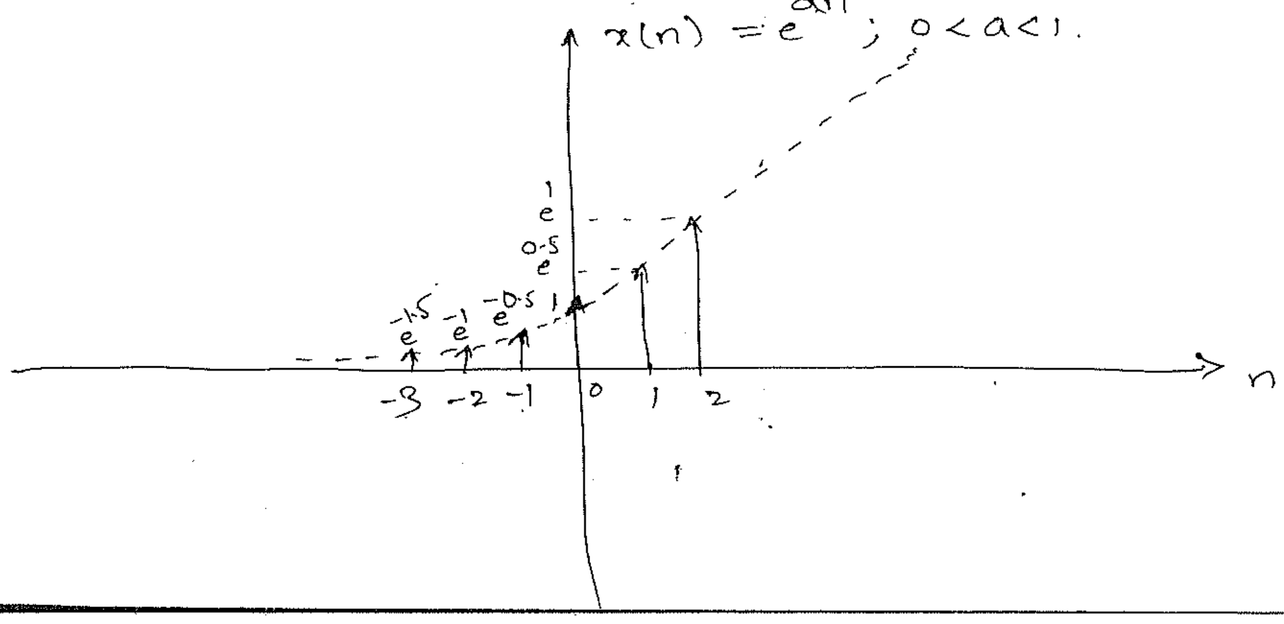
$\vdots$

for $n = -1$ $\quad x(-1) = e^{-0.5}$

$n = -2$ $\quad x(-2) = e^{-1}$

$n = -3$ $\quad x(-3) = e^{-1.5}$

$\therefore$ The graphical rep. is.



(ii) Exponentially decaying sequence.

$$x(n) = e^{-an} \quad \text{where} \quad a > 1 \quad \text{i.e.;} \quad 1 < a < \infty$$

Ex:- Take $a = 2$. $\therefore x(n) = e^{-2n}$

for $n = 0 \Rightarrow x(0) = 1$

$n = 1 \quad x(1) = e^{-2}$

$n = 2 \quad x(2) = e^{-4}$

$n = 3 \quad x(3) = e^{-6}$

⋮

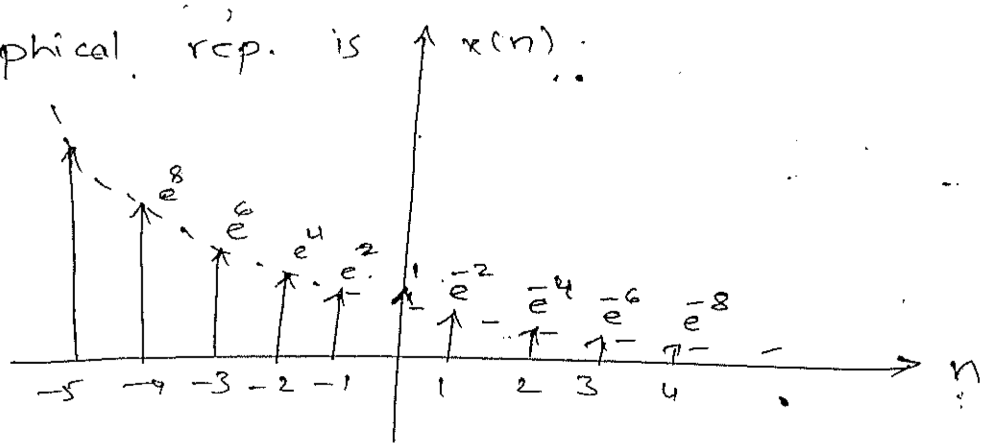
for $n = -1 \Rightarrow x(-1) = e^{2}$

$n = -2 \Rightarrow x(-2) = e^{4}$

$n = -3 \Rightarrow x(-3) = e^{6}$

⋮

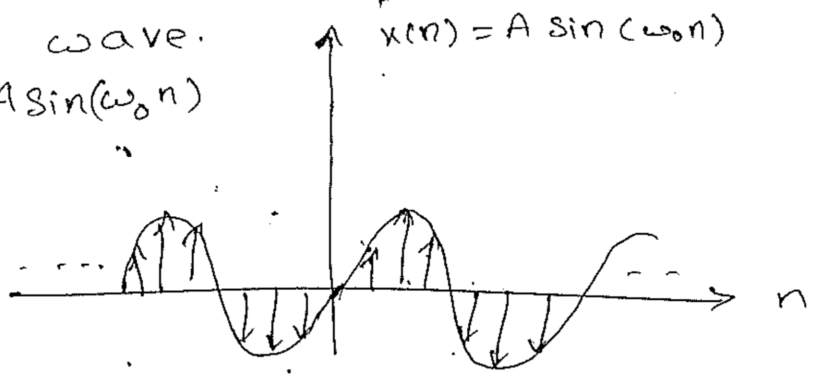
Graphical rep. is



6. Sinusoidal Sequences.

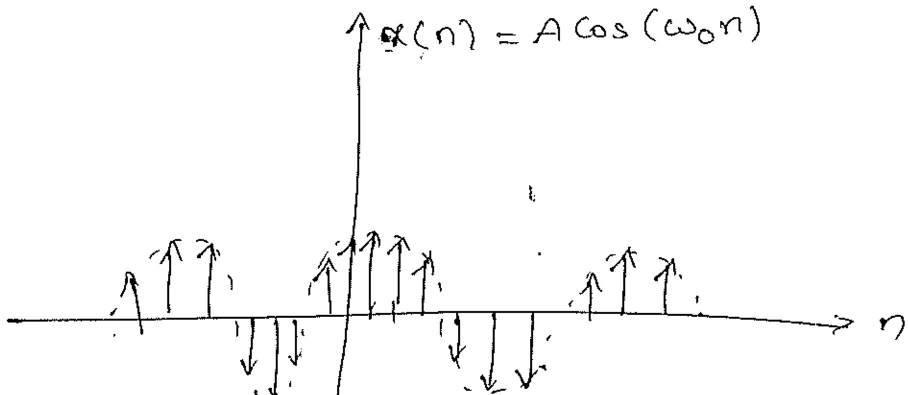
(i) Sine wave.

$x(n) = A\sin(\omega_0 n)$



(ii) Cosine wave

$$x(n) = A\cos(\omega_0 n)$$



OPERATION ON DISCRETE TIME SIGNALS :-

The mathematical transformation on one signal to another is represented by

$$x(n) \xrightarrow{I/P} \boxed{T(\,)} \xrightarrow{o/p} y(n) = T[x(n)]$$

(i) Signal multiplied by a Scaling factor :-
(or operation of amp. Scaling factor :-

$$x(n) \longrightarrow \boxed{a} \longrightarrow y(n) = a\,x(n)$$

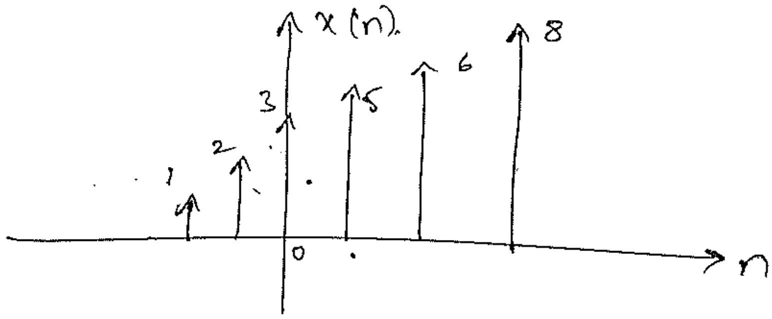
Ex :- $x(n) = \{1, 2, \underset{\uparrow}{3}, 5, 6, 8\}$

Find ~~3x(n)~~ $3 \cdot x(n) = y(n)$

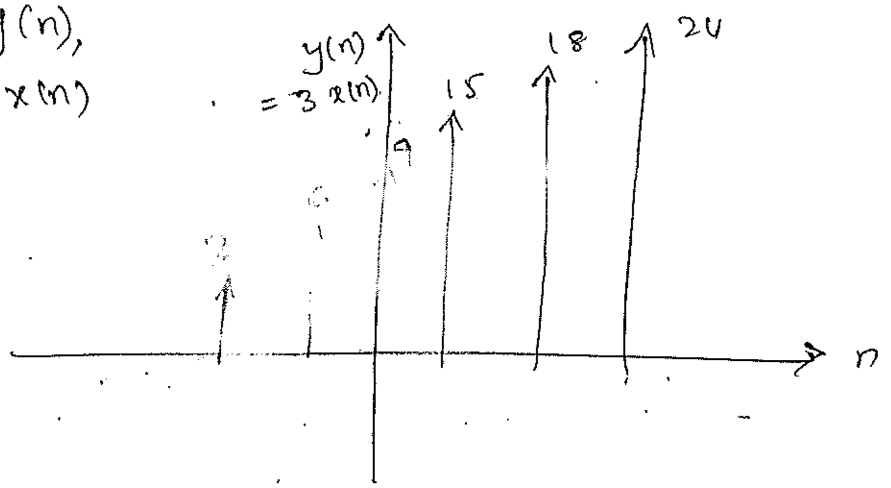
$\longrightarrow$ $y(n) = \{3, 6, \underset{\uparrow}{9}, 15, 18, 24\}$

Graphically.

for $x(n)$.

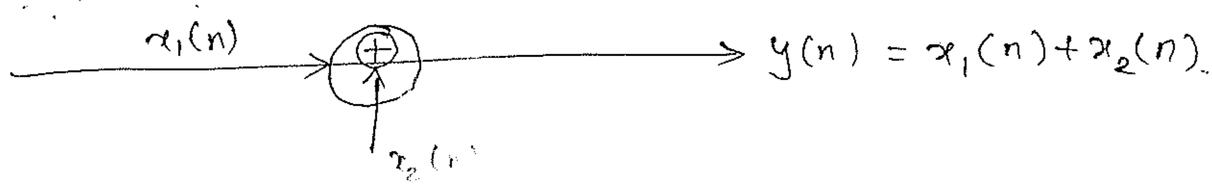


for $y(n)$,
$= 3x(n)$



(ii) Signals scaling factor :-

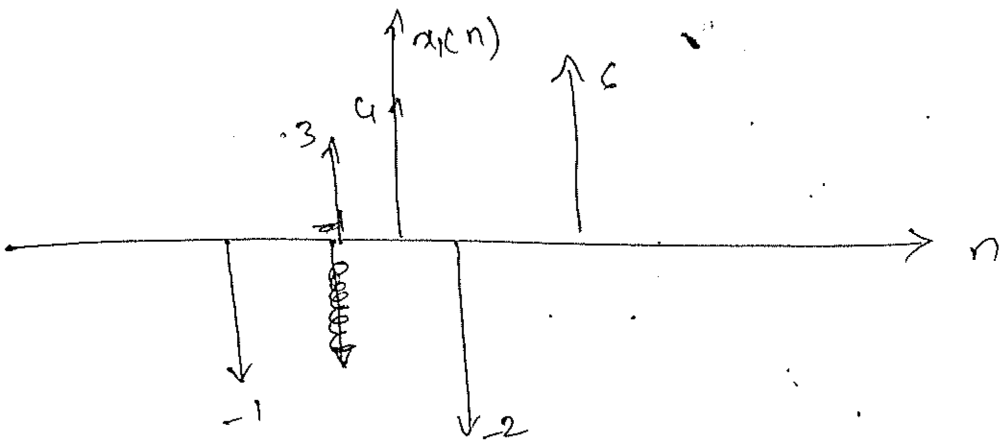
Mathematical rep. of signals scaling factor is

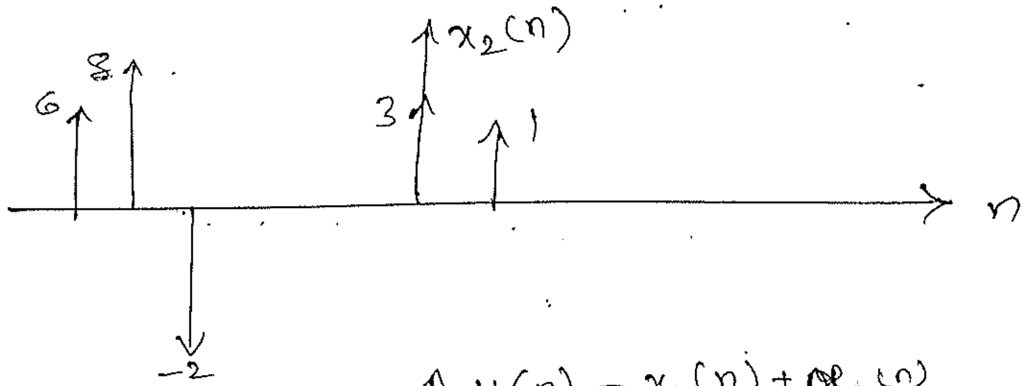


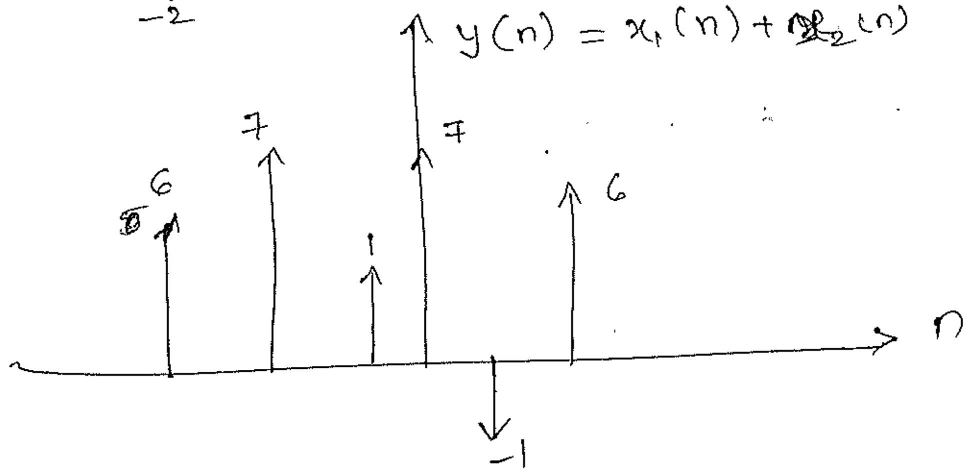
Ex :- $x_1(n) = \{-1, 3, \underset{\uparrow}{4}, -2, 6\}$; $x_2(n) = \{6, 8, -2, 3, \underset{\uparrow}{1}\}$

Find $y(n) = x_1(n) + x_2(n)$

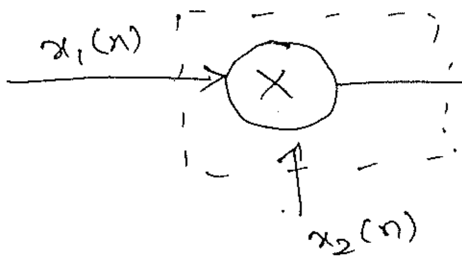
→





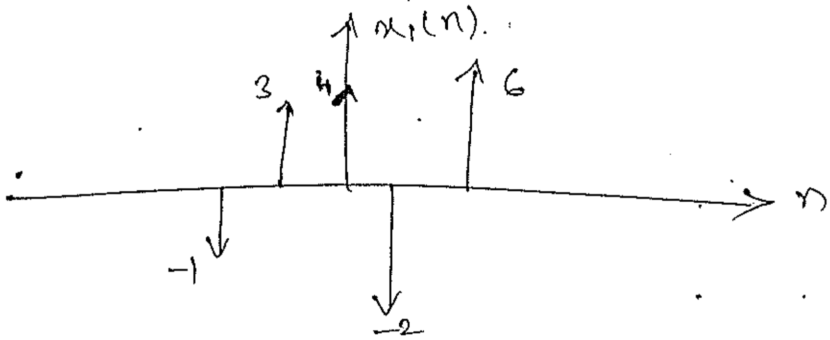


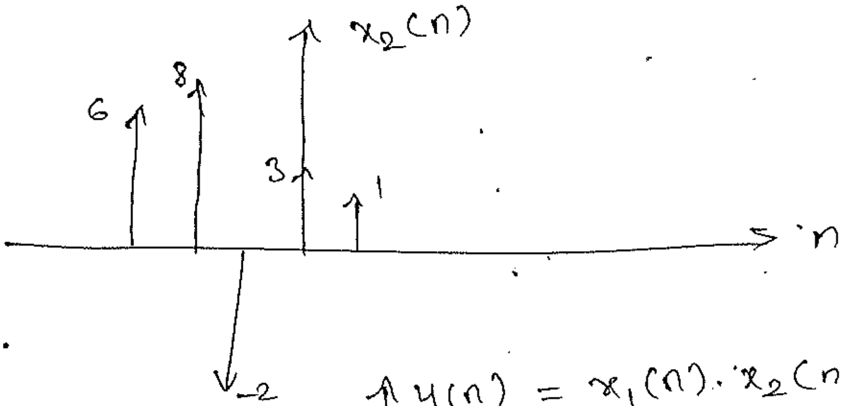
Signal Multiplier :-

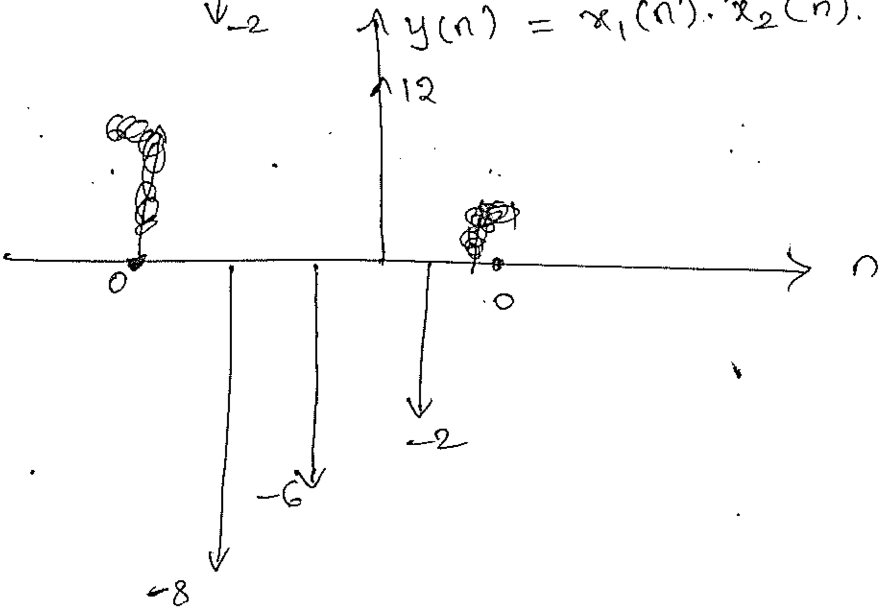


$y(n) = x_1(n) \cdot x_2(n)$

Ex :- $x_1(n) = \{-1, 3, \underset{\uparrow}{4}, -2, 6\}$; $x_2(n) = \{6, 8, -2, \underset{\uparrow}{3}, 1\}$







7/7/06.

Time shifting Operation :-

There are 2 types of time shifting operations.

1. Time delay operation
2. Time advance operation..

Generally the shifting operation is represented as

$\boxed{y(n) = x(n-k)}$ where 'k' is integer.

1. Time delay operation :-

The Sequence $x(n-k)$ is obtained for positive integer values of 'k'. The signal $x(n)$ is shifted to right by 'k' units of time. (or) delay by 'k' units of time.

Ex :- $x(n) = \{ \underset{\uparrow}{1}, 2, 3, 4 \}$. find $y(n) = x(n-2); k = 2$.

→ Mathematically

for $n=0$, ⇒ $y(0) = x(0-2) = x(-2) = 0$

for $n=1$ $y(1) = x(1-2) = x(-1) = 0$

$n=2$ $y(2) = x(2-2) = x(0) = 1$

$n=3$ $y(3) = x(3-2) = x(1) = 2$

$n=4$ $y(4) = x(4-2) = x(2) = 3$

$n=5$ $y(5) = x(5-2) = x(3) = 4$

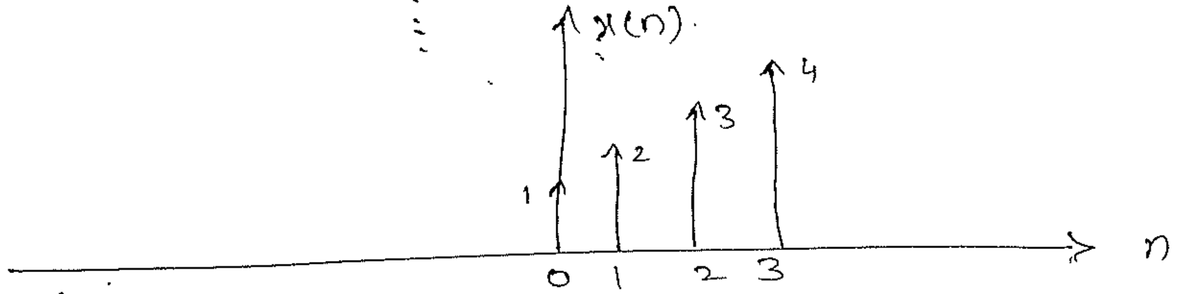
$n=6$ $y(6) = x(3) = 0$

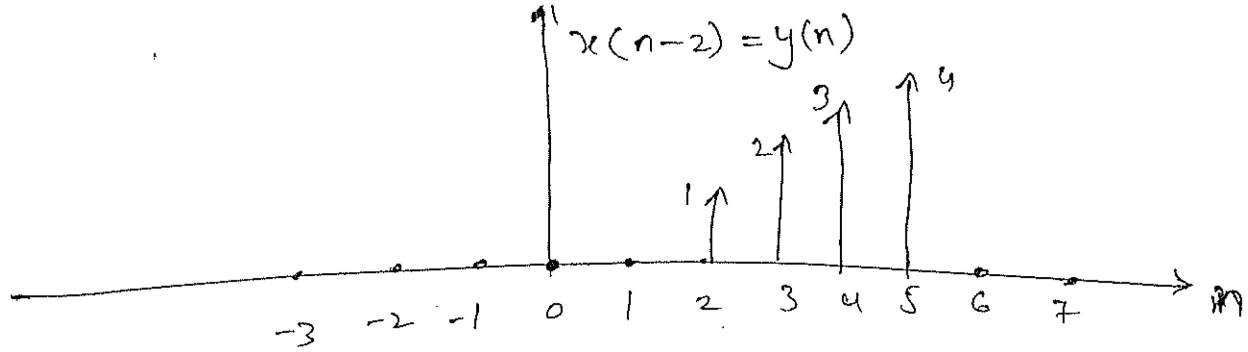
$= 0$

⋮ $= 0$

$n=-1$ $y(-1) = x(-1-2) = x(-3) = 0$

⋮





From this, we observe that $x(n-2)$ is obtained $x(n)$ is shifted to right by 2 units of time.

2. <u>Time advance operation</u> :-

~~For negative~~

$y(n) = x(n+k)$ for +ve value of 'k'.

$x(n+k)$ is obtained ~~for~~ +ve integer value of 'k', the sequence $x(n)$ is shifted to left by 'k' units of time (or) advance by 'k' units of time.

<u>Ex</u>:-

$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$. Find $y(n) = x(n+2)$; $k=2$

For $n=0 \Rightarrow y(n) = x(2) = 3$

$n=1 \Rightarrow y(n) = x(3) = 4$

$n=2 \Rightarrow y(n) = 0$

$\vdots$

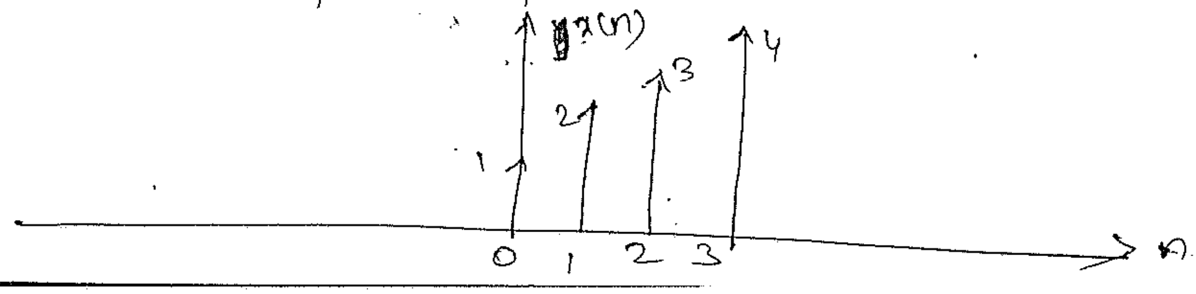
$n=-1 \Rightarrow y(n) = x(1) = 2$

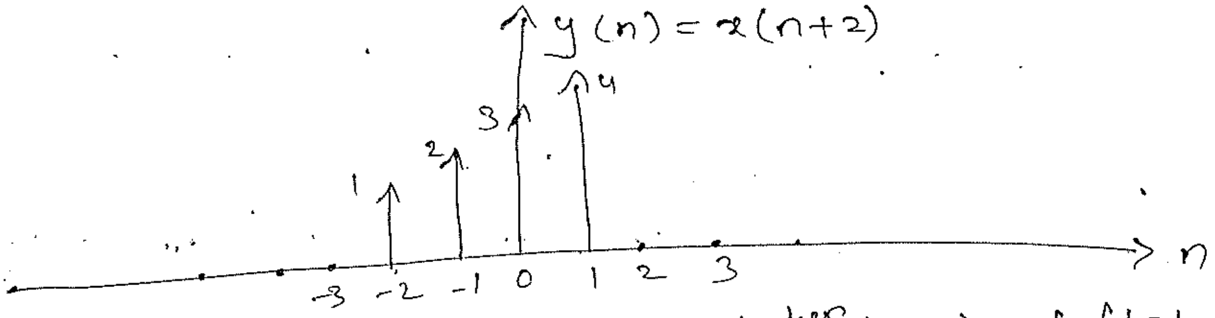
$n=-2 \Rightarrow y(n) = x(0) = 1$

$n=-3 \Rightarrow y(n) = x(-1) = 0$

$\vdots$

The Graphical representation is





From this, $x(n+2)$ is obtained when $x(n)$ is shifted to left by 2 units of time (or) advance by 2 units of time

Time reversal Sequence :-

Time reversal sequence of $x(n)$ is represented by $x(-n)$.

$$y(n) = x(-n).$$

The sequence $x(-n)$ is obtained when $x(n)$ is folded about $n=0$

Ex:- $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$. find $y(n) = x(-n)$.

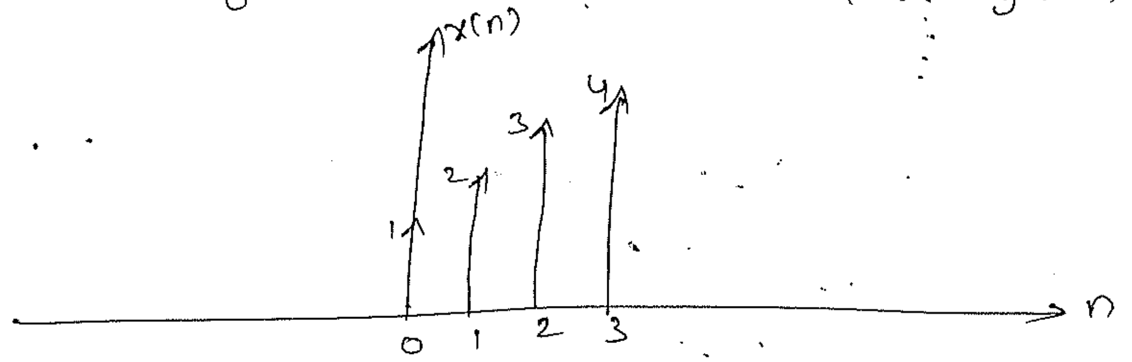
for $n=0 \Rightarrow y(n) = x(0) = 1$; $n=-1 \Rightarrow y(-1) = x(1) = 2$

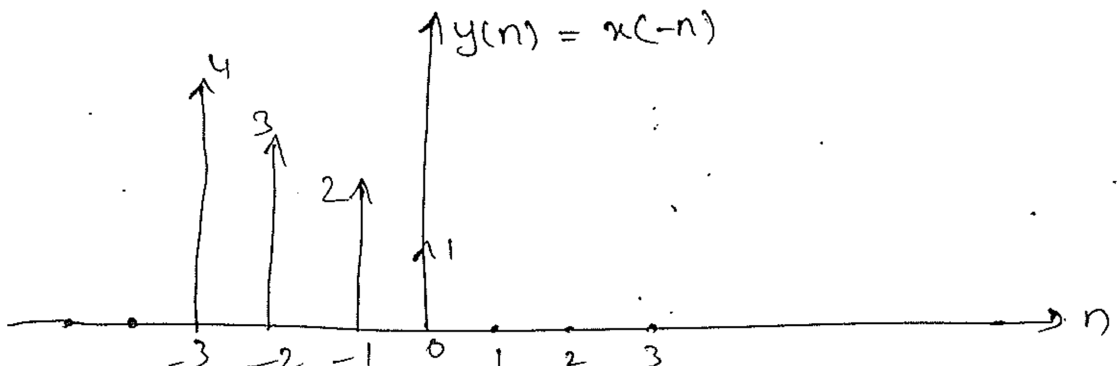
$n = 1 \Rightarrow y(1) = x(-1) = 0$; $n = -2 \Rightarrow y(-2) = 3$

$\vdots$ $n = -3 \Rightarrow y(-3) = 4$

0 $n = -4 \Rightarrow y(-4) = 0$

$\vdots$





From this, $x(-n)$ is obtained by simply folding about ~~x(n) = 0~~ $n=0$

NOTE :- $y(n) = x(-n+k)$; where 'k' is +ve integer.

$x(-n+k)$ is obtained when $x(-n)$ sequence is shifted to right by 'k' units of time (or) delay by 'k' units of time.

Ex :- $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$; find $y(n) = x(-n+2)$; $k=2$

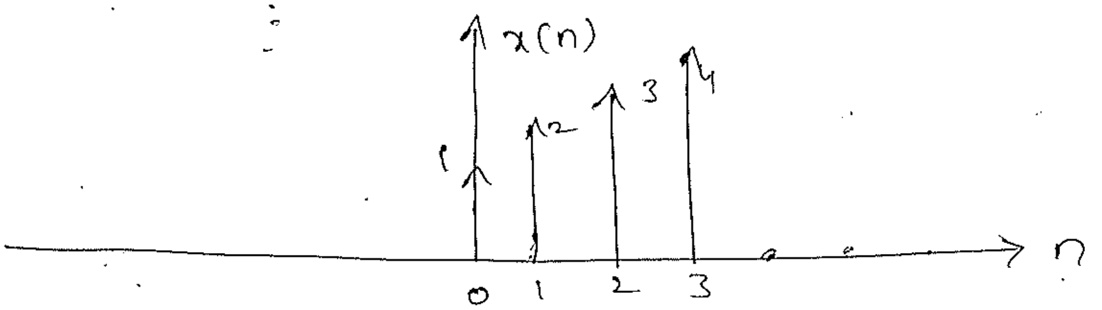
$\longrightarrow$ For $n=0 \Rightarrow y(0) = 3$; for $n=-1 \Rightarrow y(3) = 4$

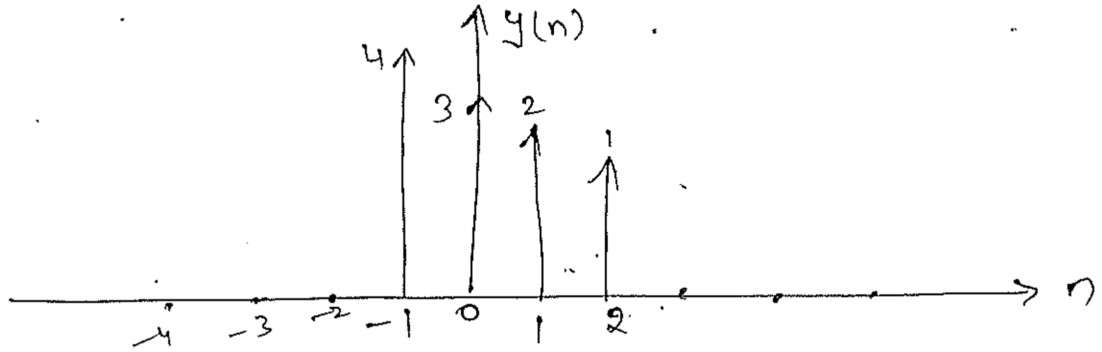
for $n=1 \Rightarrow y(1) = 2$; $n=-2 \Rightarrow y(4) = 0$

for $n=2 \Rightarrow y(0) = 1$;

for $n=3 \Rightarrow y(-1) = 0$;

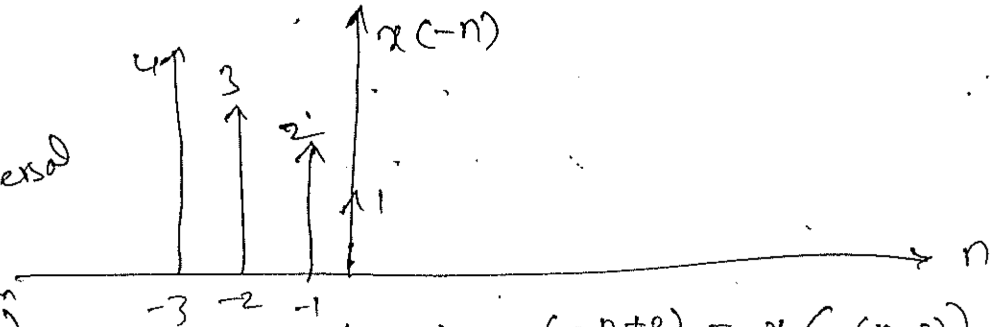
⋮





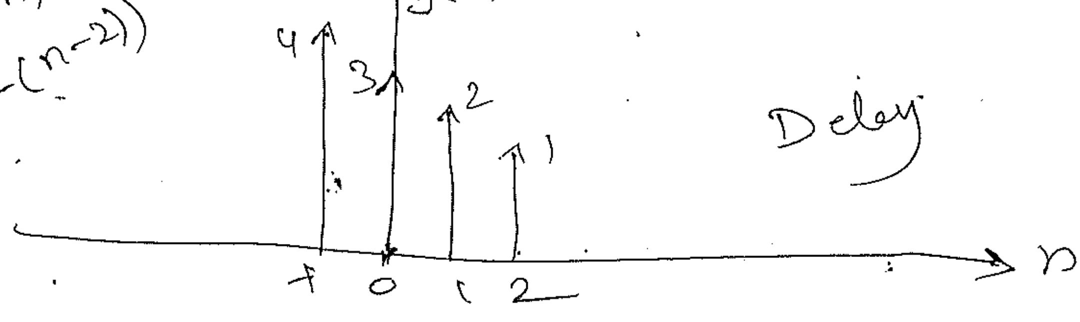
(or)

step 1 :-
$x(-n)$
Time reversal



Step 2 :-
Delay operation
$y(n) = x(-n+2)$
$= x(-(n-2))$

$y(n) = x(-n+2) = x(-(n-2))$



Delay

From this, $x(-n+2)$ is obtained by shifting $x(-n)$ to right by 2 units of time (or) delay by 2 units of time.

NOTE:

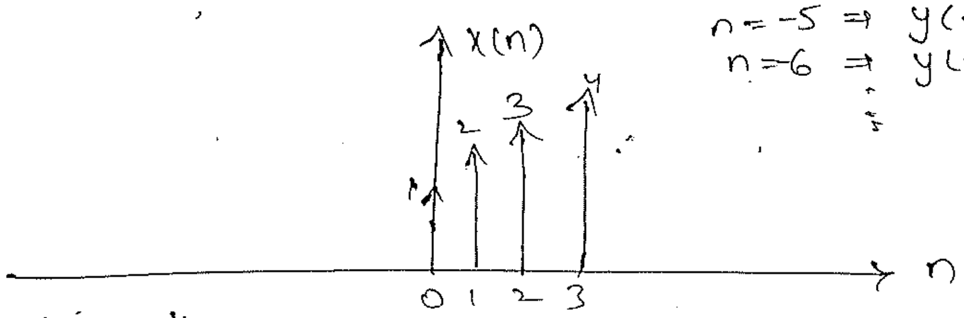
$$y(n) = x(-n-k) \text{ where 'k' is +ve integer.}$$

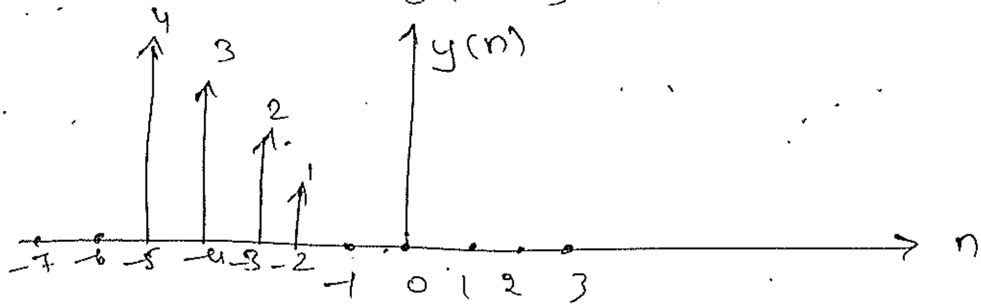
$x(-n-k)$ is obtained $x(-n)$ sequence shifted to left by 'k' units of time (or) advance by 'k' units of time.

Ex: $x(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$; find $y(n) = x(-n-2)$; $k = 2$

for $n=0 \Rightarrow y(0) = 0$
$n=1 \Rightarrow y(1) = 0$
⋮

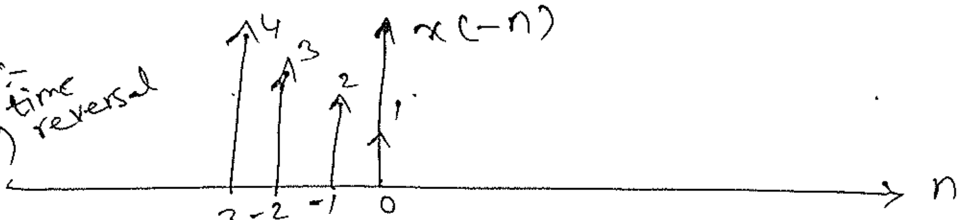
for $n=-1 \Rightarrow y(-1) = 0$
$n=-2 \Rightarrow y(-2) = 1$
$n=-3 \Rightarrow y(-3) = 2$
$n=-4 \Rightarrow y(-4) = 3$
$n=-5 \Rightarrow y(-5) = 4$
$n=-6 \Rightarrow y(-6) = 0$
⋮



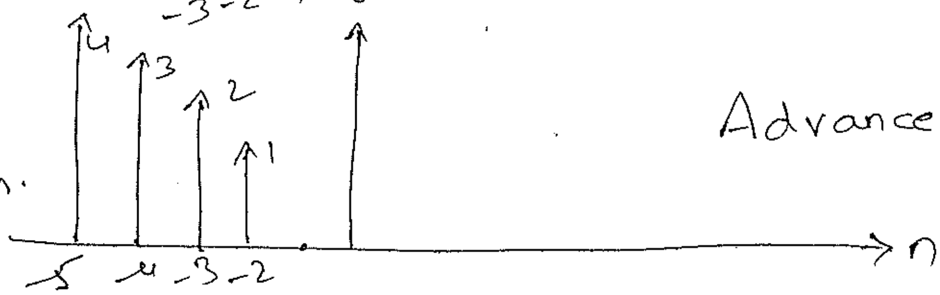


(or)

step 1: time reversal $x(-n)$



step 2: $x(-(n+2))$ Advance operation.



Advance

10/7/06.

* Sketch the following ~~Systems~~. Signals.

(i) $x(n) = \delta(n-3)$ (ii) $x(n) = \delta(n+4)$

(iii) $x(n) = 2\delta(n+2) - 3\delta(n+1) + 2\delta(n) + 5\delta(n-1) + 6\delta(n-2)$

(iv) $x(n) = u(n-5)$

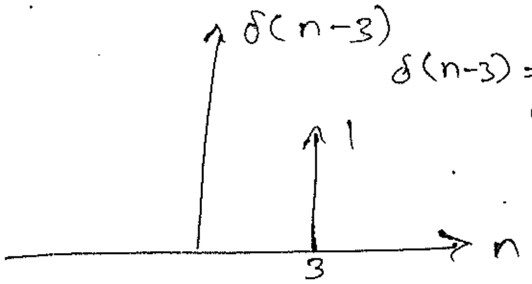
(v) $x(n) = u(n+7)$

(vi) $x(n) = u(-n-2)$

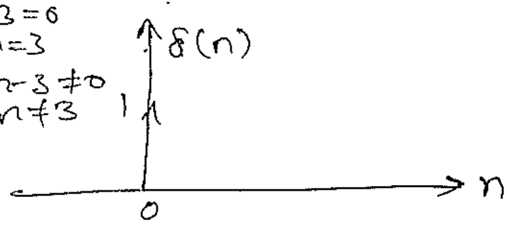
(vii) $x(n) = u(-n+3)$ (viii) $x(n) = u(n) - u(n-6)$.

→ (i) $x(n) = \delta(n-3)$

$$\delta(n) = \begin{cases} 1 & \text{for } n=0 \\ 0 & \text{for } n \neq 0 \end{cases}$$



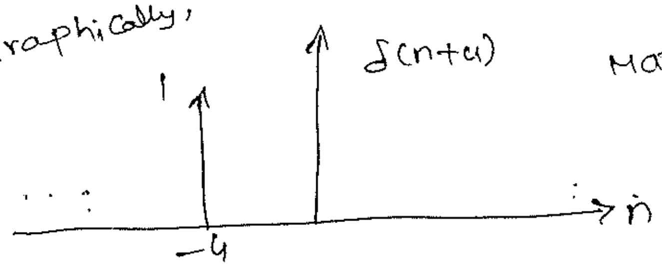
$$\delta(n-3) = \begin{cases} 1 & \text{for } n-3=0 \Rightarrow n=3 \\ 0 & \text{for } n-3 \neq 0 \Rightarrow n \neq 3 \end{cases}$$



(ii) $x(n) = \delta(n+4)$.

Shifted to left (or) advance by 4 units of time.

Graphically,



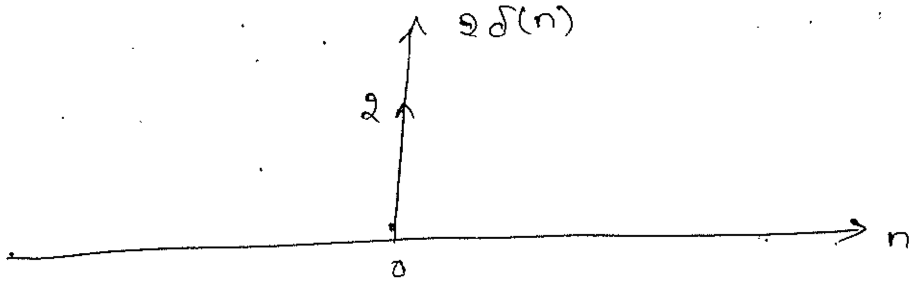
Mathematically,

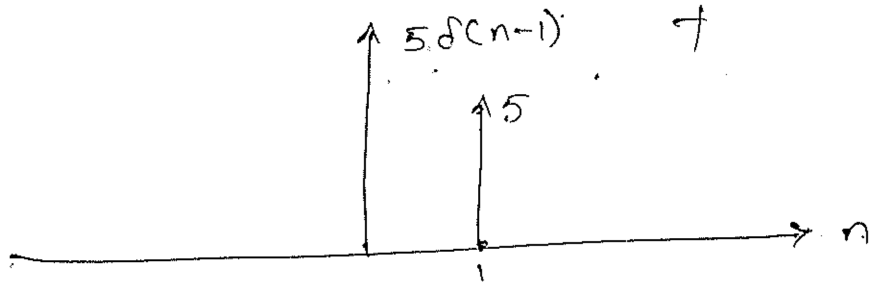
$$\delta(n+4) = \begin{cases} 1 & \text{for } n+4=0,\ n=-4 \\ 0 & \text{for } n+4 \neq 0,\ n \neq -4. \end{cases}$$

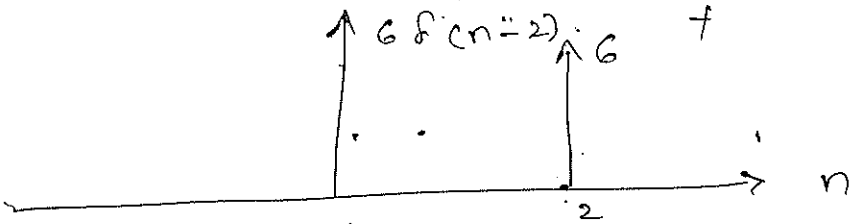
(iii) $x(n) = 2\delta(n+2) - 3\delta(n+1) + 2\delta(n) + 5\delta(n-1) + 6\delta(n-2)$.

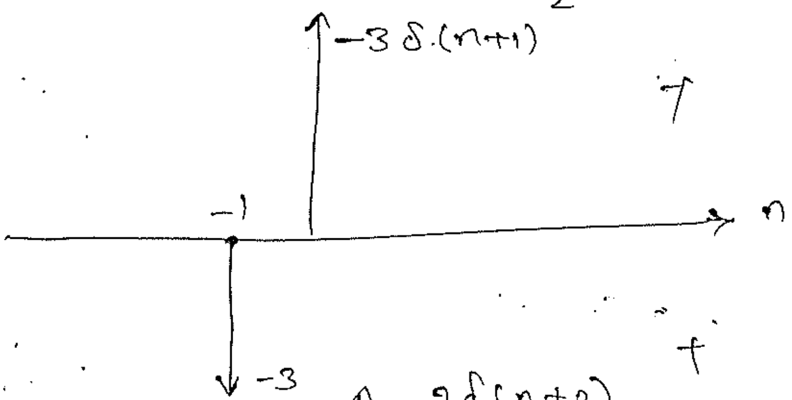
→ Eq. Sequence rep. of $\delta(n)$ is

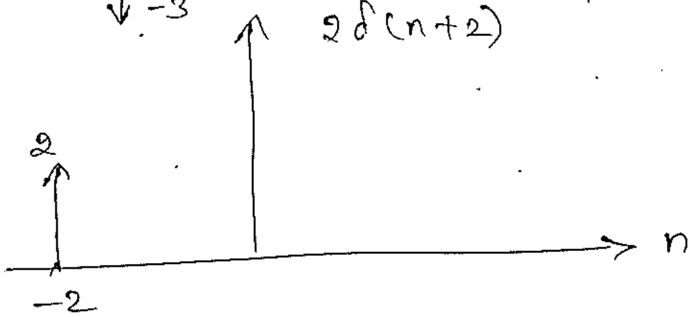
$$x(n) = \{2, -3, \underset{\uparrow}{2}, 5, 6\}$$

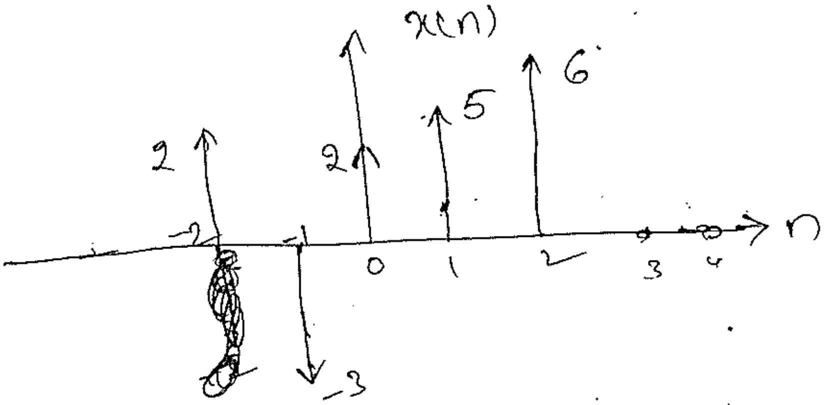

2δ(n)
2
0
n


5δ(n−1)
5
1
n


6δ(n−2)
6
2
n

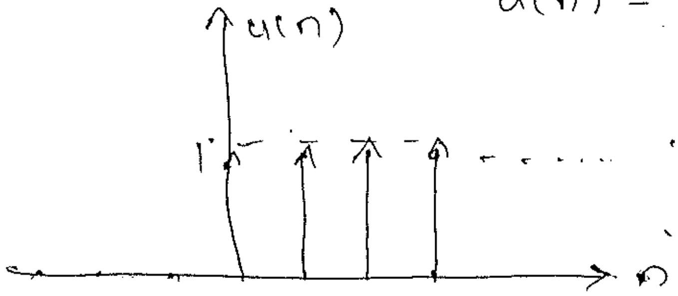

−3δ(n+1)
−1
−3
n

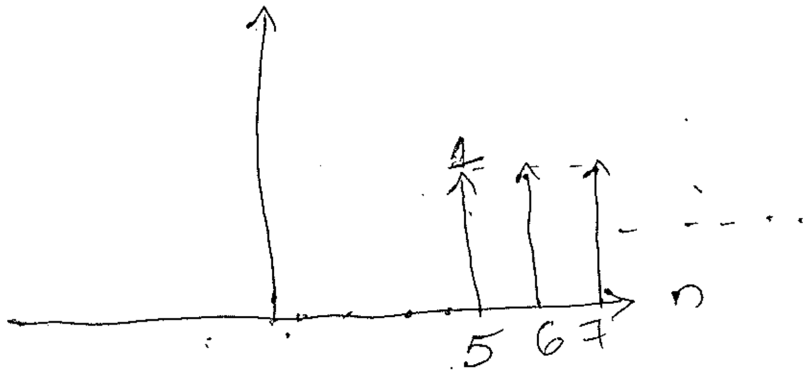

2δ(n+2)
2
−2
n


x(n)
2
2
5
6
−2
−1
0
1
2
3
4
−3
n

(iv) $x(n) = u(n-5)$.

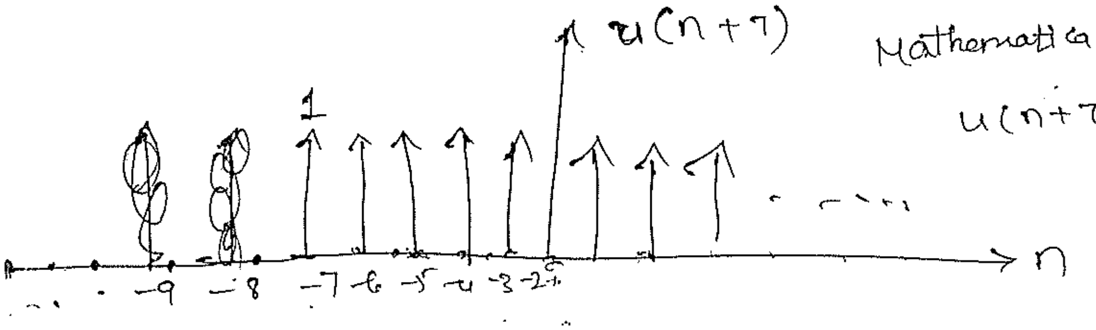
$\rightarrow$ $u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0. \end{cases}$





Mathematically, $u(n-5) = \begin{cases} 1 & \text{for } n-5 \geq 0 \\ & n \geq 5 \\ 0 & \text{for } n-5 < 0 \\ & n < 5. \end{cases}$

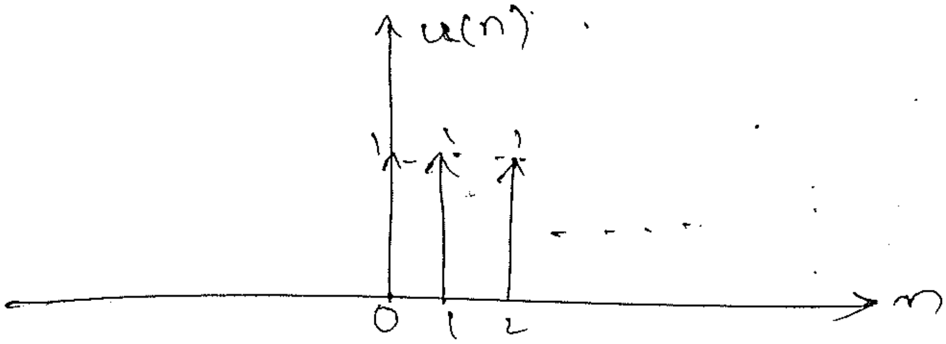
(V) $x(n) = u(n+7)$

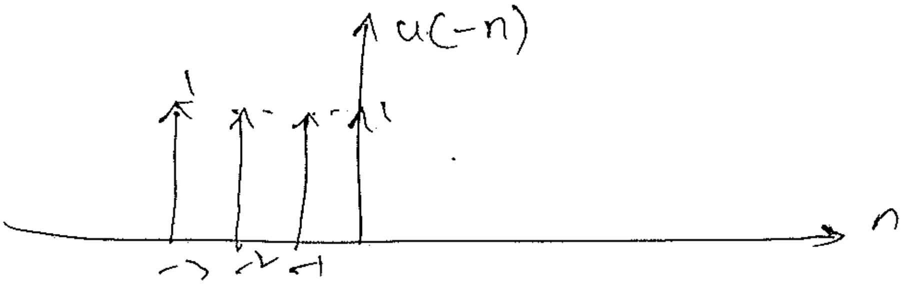


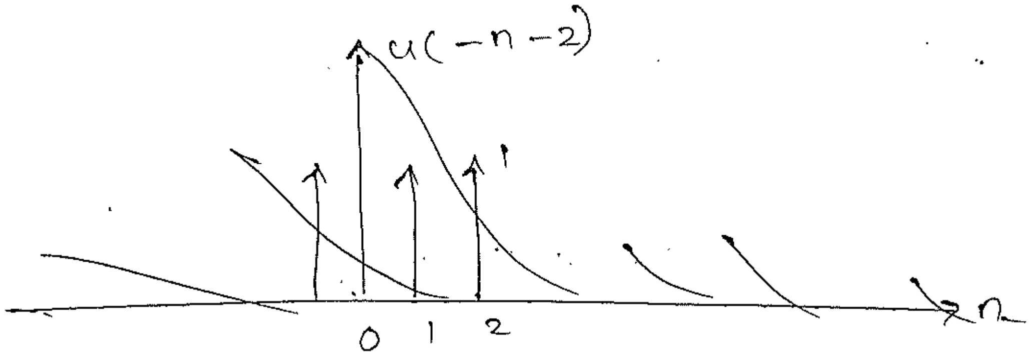
Mathematically,

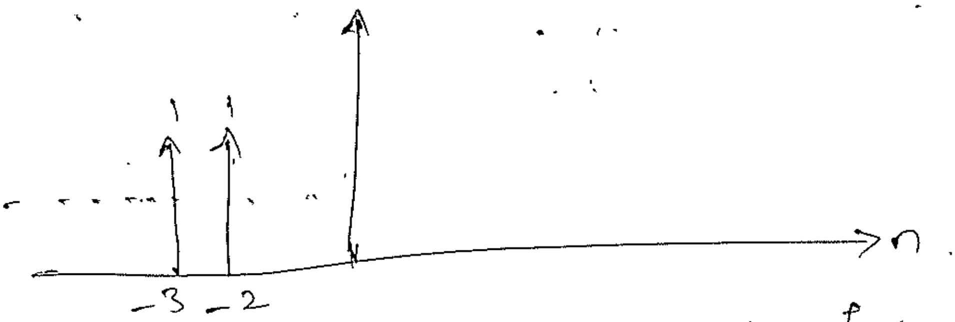
$u(n+7) = \begin{cases} 1 & \text{for } n+7 \geq 0 \\ & n \geq -7 \\ 0 & \text{for } n < -7 \end{cases}$

(Vi) $x(n) = u(-n-2)$.



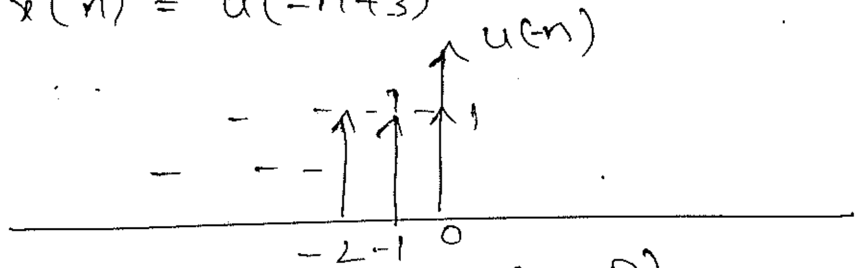


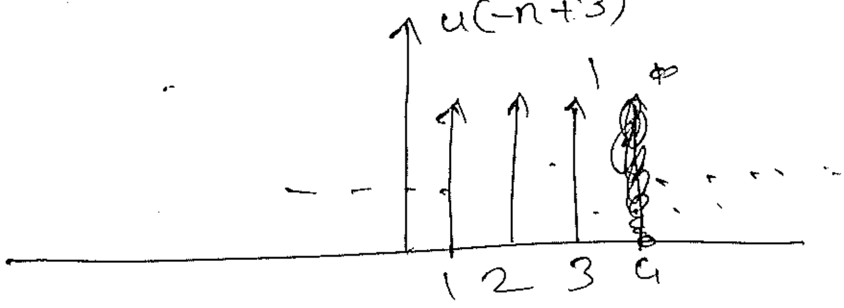




Mathematically, $u(-n-2) = \begin{cases} 1 & \text{for } -n-2 \geq 0 \\ & n+2 \leq 0 \\ & n \leq -2 \\ 0 & \text{for } -n-2 < 0 \\ & n+2 > 0 \\ & n > -2 \end{cases}$

(vii) $x(n) = u(-n+3)$



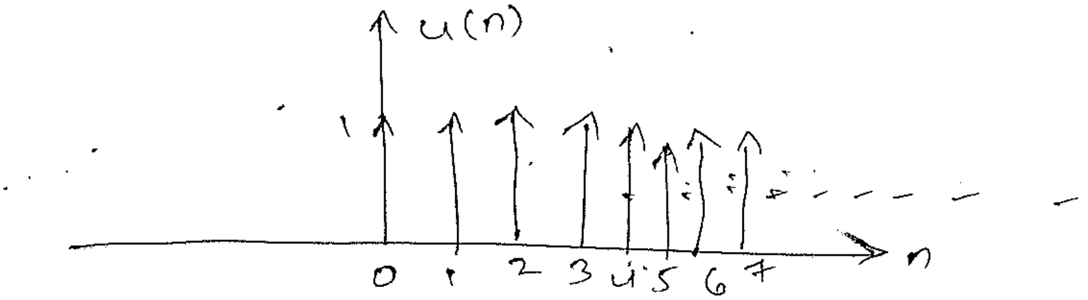


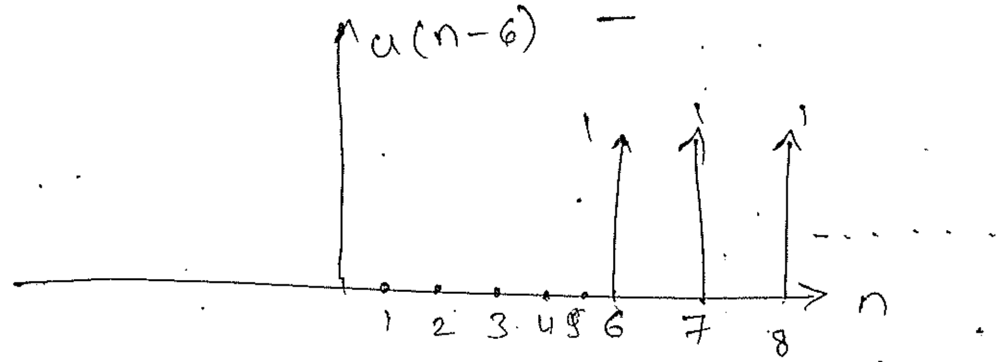
Mathematically,

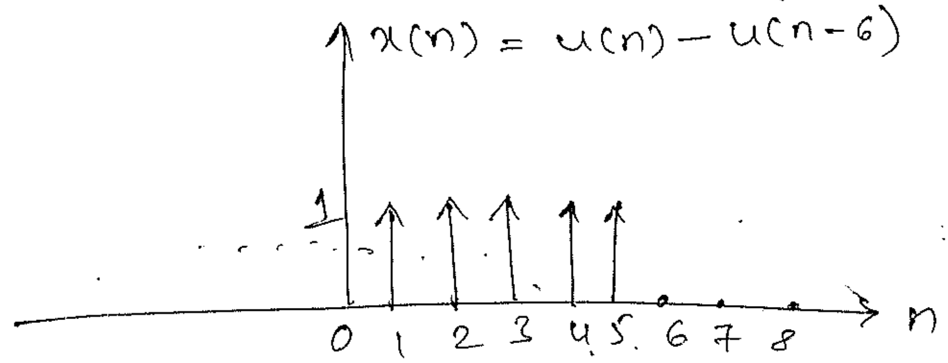
$$u(-n+3) = \begin{cases} 1 & \text{for } -n+3 \geq 0 \\ & -n \geq -3 \\ & n \leq 3 \\ 0 & \text{for } -n+3 < 0 \\ & n-3 > 0 \\ & n > 3 \end{cases}$$

(viii) $x(n) = u(n) - u(n-6)$

$\longrightarrow$







* Determine whether the following signals are energy signals (or) Power signals and compute their normalised energy and power. (8 marks)

(i) $x(t) = Ae^{-\alpha t}\, u(t)$

(ii) $x(t) = Ae^{\alpha t}\, u(-t)$

(iii) $x(t) = Ae^{-\alpha|t|}$; $\alpha > 0$

$\longrightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2\, dt$ (or) $\int_{-\infty}^{\infty} |x(t)|^2\, dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} |Ae^{-\alpha T} u(t)|^2\, dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 e^{-2\alpha T} |u(t)|^2 dt$$

As $u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0, \end{cases}$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 e^{-2\alpha T} (0)\, dt + \int_{0}^{T} A^2 e^{-2\alpha T} (1)\, dt.$$

$$= \underset{T\to\infty}{Lt} \int_{0}^{T} A^2 e^{-2\alpha T} dt$$

$$= \underset{T\to\infty}{Lt} \left. \frac{A^2 e^{-2\alpha T}}{-2\alpha} \right|_{0}^{T}$$

$$= \underset{T\to\infty}{Lt} \frac{-A^2}{2\alpha} \left(e^{-2\alpha T} - 1 \right)$$

$$= \underset{T\to\infty}{Lt} \frac{-A^2}{2\alpha} \left(e^{-2\alpha T} - 1 \right)$$

$$= \frac{A^2}{2\alpha}$$

$\therefore$ Energy of given Signal $= \frac{A^2}{2\alpha}$.

Average Power, $P_{avg} = P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{-T}^{T} A^2 e^{-2\alpha T} |u(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\int_{-T}^{0} (0)\, dt + \int_{0}^{\infty} A^2 e^{-2\alpha T} dt \right]$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\frac{A^2 e^{-2\alpha T}}{-2\alpha} \right]_{0}^{\infty}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\cancel{\ } \right] \left(e^{-2\alpha T} - 1 \right)$$

$$= \underline{0}.$$

$\therefore P_{avg} = P = \underline{0}$

$\therefore$ The signal is having finite energy i.e; $E = \frac{A^2}{2\alpha}$. and zero average power. So the signal is energy signal.

(ii) $x(t) = Ae^{\alpha t} u(-t)$.

$\rightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 e^{2\alpha t} |u(-t)|^2 dt$$

$$u(-t) = \begin{cases} 1 & \text{for } -t \geq 0 \Rightarrow t \leq 0 \\ 0 & -t < 0 \quad t > 0 \end{cases}$$

$$\therefore E = \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 e^{2\alpha T} |u(-t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 e^{2\alpha T} (1) dt + \int_{0}^{T} A^2 e^{2\alpha T} (0) dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 e^{2\alpha T} dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2 e^{2\alpha T}}{2\alpha} \Big|_{-T}^{0}$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2\alpha} (1 - e^{-2\alpha T})$$

$$= \frac{A^2}{2\alpha} (1 - 0) = \frac{A^2}{2\alpha}$$

$\therefore$ Energy of the given signal $= \frac{A^2}{2\alpha}$

Average power, $P_{avg.} = P = \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T}\int_{-T}^{0} A^2 e^{2\alpha T} dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \cdot \frac{A^2}{2\alpha} e^{2\alpha T}\Big|_{-T}^{0}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{A^2}{2\alpha}(1-e^{-2\alpha T})$$

$$= \underline{0}$$

$\therefore$ $\underline{\underline{P_{avg.} = 0}} = P$

$\therefore$ The signal is having finite energy, $E = \frac{A^2}{2\alpha}$ and zero avg. power. So the signal is energy signal.

(iii) $x(t) = Ae^{-\alpha|t|}$; $\alpha > 0$.

$\longrightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 e^{-2\alpha|t|} dt$$

~~As given~~

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 e^{-2\alpha(-t)} dt + \int_{0}^{T} A^2 e^{-2\alpha t} dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 e^{2\alpha t} dt + \int_{0}^{T} A^2 e^{-2\alpha t} dt$$

$$= \underset{T\to\infty}{Lt} \frac{A^2}{2\alpha} e^{2\alpha T}\Big|_{-T}^{0} + \frac{A^2}{-2\alpha} e^{-2\alpha T}\Big|_{0}^{T}$$

$$= \underset{T\to\infty}{Lt} \left[\frac{A^2}{2\alpha}(1-e^{-2\alpha T}) - \frac{A^2}{2\alpha}(e^{-2\alpha T}-1)\right]$$

$$= \frac{A^2}{2\alpha}(1) - \frac{A^2}{2\alpha}(-1)$$

$$= \frac{A^2}{2\alpha} + \frac{A^2}{2\alpha} = \frac{A^2}{\alpha}$$

Power, $P = \underset{T \to \infty}{Lt} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T \to \infty}{Lt} \frac{1}{2T} \left[\frac{A^2}{2\alpha}(1 - e^{-2\alpha T}) - \frac{A^2}{2\alpha}(e^{-2\alpha T} - 1) \right]$$

$$= \underline{\underline{0}}$$

$\therefore$ As the signal is having finite energy i.e; $E = \frac{A^2}{\alpha}$ and zero average power, the given signal is an energy signal.

11/7/06.

4. $x(t) = A\, u(t)$
5. $x(t) = A \cos(\omega_0 t + \theta)$
6. $x(t) = 4 \sin(6\pi t + \pi/6)$
7. $x(t) = \text{rect}(t/T_0)$
8. $x(t) = \text{rect}(\ldots) \cos(\omega_0 t)$
9. $x(t) = e^{(2+j4\pi)t}$
10. $x(t) = \cos^2(\omega_c t)$

$\rightarrow$ (4) $x(t) = A\, u(t)$.

Energy calculation, $E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T \to \infty}{Lt} \int_{-T}^{T} A^2 |u(t)|^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{0} A^2 (0)\, dt + \int_{0}^{T} A^2 \, dt$$

$$= \underset{T\to\infty}{Lt} \int_{0}^{T} A^2 dt$$

$$= \underset{T\to\infty}{Lt} A^2 t \Big|_0^T = \underset{T\to\infty}{Lt} A^2 T$$

$$= \underline{\underline{\infty}}.$$

$$\text{Power} = P = \underset{T\to\infty}{Lt} \frac{1}{2T} \int_{0}^{T} A^2 dt$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2\cancel{T}} A^2 \cancel{T}$$

$$= \frac{A^2}{2}$$

$\therefore$ The signal is having infinite energy signal and finite avg. power i.e; $P_{avg.} = \frac{A^2}{2}$. Hence the given signal is power signal.

(5) $x(t) = A \cos(\omega_0 t + \theta)$

$\longrightarrow$ Energy, $$E = \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 \cos^2(\omega_0 t + \theta)\, dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} A^2 \left(\frac{1 + \sin 2(\omega_0 t + \theta)}{2} \right) dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} \left[\frac{A^2}{2} + \frac{A^2}{2} \sin 2(\omega_0 t + \theta) \right] dt$$

$$= \underset{T\to\infty}{Lt} \left[\frac{A^2}{2} t + \frac{A^2}{2} \frac{\cos 2(\omega_0 t + \theta)}{2(\omega_0 t + \theta)} \right]_{-T}^{T}$$

$$= \underset{T\to\infty}{Lt} \left[\frac{A^2}{2} T + \frac{A^2}{2} \frac{\cos 2(\omega_0 T + \theta)}{2(\omega_0 T + \theta)} + \frac{A^2}{2} T - \frac{A^2}{2} \frac{\cos 2(-\omega_0 T + \theta)}{2(-\omega_0 T + \theta)} \right.$$

$$= \underset{T\to\infty}{Lt} \left[A^2 T + \frac{A^2 \cos^2(\omega_0 T + \theta)}{4(-\omega_0 T + \theta)} - \frac{A^2}{4} \frac{\cos^2(-\omega_0 T + \theta)}{(-\omega_0 T + \theta)} \right]$$

$$= \underline{\underline{\infty}}$$

Power, $P_{avg.} = \underset{T\to\infty}{Lt} \frac{1}{2T} \left[\frac{A^2}{2} t \Big|_{-T}^{T} + \frac{A^2}{2}(0) \right]$

$$= \underset{T\to\infty}{Lt} \frac{1}{2T} \frac{A^2}{2} (T+T)$$

$$= \cancel{\underset{T\to\infty}{Lt}} \frac{A^2}{\cancel{4T}_2} \cancel{2T}$$

$$= \underline{\underline{A^2/2}}$$

$\therefore$ The given signal is having ∞ energy and finite power. So the given signal is power signal.

(6) $x(t) = 4\sin\left(6\pi t + \frac{\pi}{6}\right)$.

$\longrightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} 16 \sin^2\left(6\pi t + \frac{\pi}{6}\right) dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} 16 \left(\frac{1 - \cos\left(12\pi t + \frac{\pi}{3}\right)}{2} \right) dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} \left(8 - 8\cos\left(12\pi t + \frac{\pi}{3}\right)\right) dt$$

$$= \underset{T\to\infty}{Lt} \left[8t + \frac{8\cos\left(12\pi t + \frac{\pi}{3}\right)}{12\pi t + \pi/3} \right]_{-T}^{T}$$

$$= \underset{T\to\infty}{Lt}\ 8(2T) = \underline{\underline{\infty}}.$$

$$\text{Power, } P = \underset{T\to\infty}{Lt}\ \frac{1}{2T}(16T) = \underline{\underline{8}}.$$

$\therefore$ The given signal is having ∞ energy and finite power. So the given signal is power signal.

(7) $x(t) = rect\left(\frac{t}{T_0}\right)$.

$\rightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |x(t)|^2 dt$

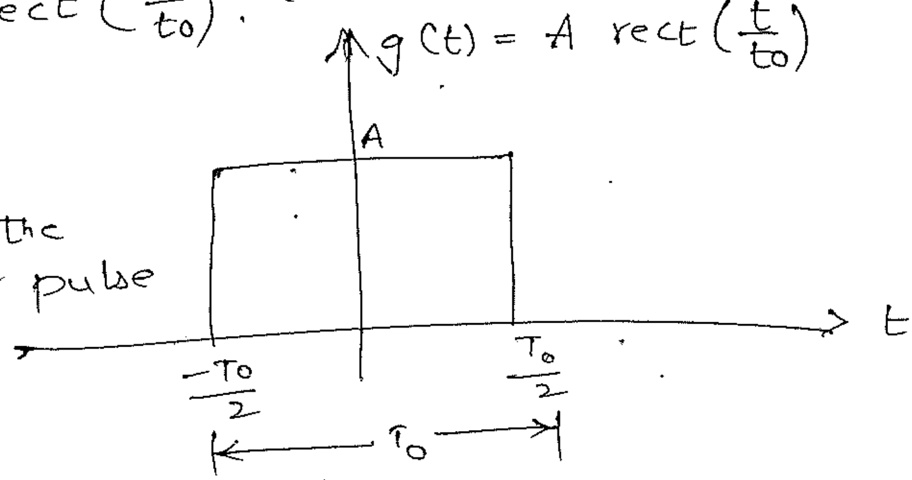
$$= \underset{T\to\infty}{Lt} \int_{-T}^{T} \left|rect\left(\frac{t}{T_0}\right)\right|^2 dt.$$

<u>Rectangular pulse</u> (or) Gate <u>function</u>:- It is defined mathematically as

$g(t) = A \cdot rect\left(\frac{t}{t_0}\right)$. (or) $g(t) = A\Pi(t/t_0)$

Graphically,

T_0 - width of the rectangular pulse



$$\therefore \text{ Energy, } E = \underset{T\to\infty}{Lt} \int_{-T}^{-T_0/2} 0 \cdot dt + \int_{-T_0/2}^{T_0/2} 1 \cdot dt + \int_{T_0/2}^{T} 0 \cdot dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T_0/2}^{T_0/2} dt = \underset{T\to\infty}{Lt}\ \frac{T_0}{2} + \frac{T_0}{2}$$

$$= \underline{\underline{T_0}}.$$

$$\boxed{g(t) = A \text{ rect}\left(\frac{t}{t_0}\right) = \begin{cases} A & \text{for } \frac{-T_0}{2} \le t \le \frac{T_0}{2} \\ 0 & \text{else.} \end{cases}}$$

$$\text{Power} = \underset{T\to\infty}{Lt} \frac{1}{2T}(T_0) = \underline{0}$$

$\therefore$ The given signal has finite energy (T_0) and zero power. So the given signal is energy signal.

(8) $x(t) = \text{rect}(t/T_0)\cos(\omega_0 t)$.

$\longrightarrow$ Energy, $E = \underset{T\to\infty}{Lt} \int_{-T}^{T} |\text{rect}(t/T_0)|^2 \cos^2(\omega_0 t)\, dt$

$$= \underset{T\to\infty}{Lt} \int_{-T}^{-T_0/2} 0.dt + \int_{-T_0/2}^{T_0/2} \cos^2(\omega_0 t) dt + \int_{T_0/2}^{T} 0.dt$$

$$= \underset{T\to\infty}{Lt} \int_{-T_0/2}^{T_0/2} \frac{1 + \cos 2\omega_0 t}{2} dt.$$

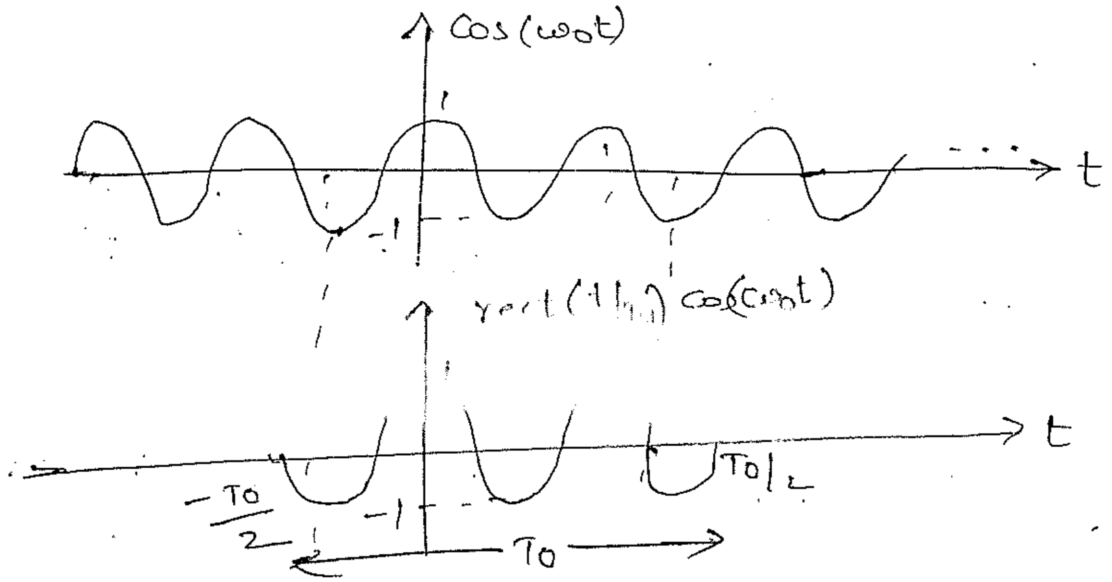
$$= \underset{T\to\infty}{Lt} \left[\frac{1}{2}t + \frac{\sin 2\omega_0 t}{4\omega_0 t}\right]_{-T_0/2}^{T_0/2}$$

$$= \underset{T\to\infty}{Lt} \frac{1}{2}\left(\frac{T_0}{2} + \frac{T_0}{2}\right)$$

$$= \frac{T_0}{2}$$

Avg. Power, $P = \underset{T\to\infty}{Lt} \frac{1}{2T}\left(\frac{T_0}{2}\right) = \underline{0}$

$\therefore$ The given signal is energy signal.



⑨ $x(t) = e^{(2+j4\pi)t}$

$\longrightarrow$ Energy, $E = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{(2+j4\pi)t} \right|^2 dt$

$= \lim_{T\to\infty} \int_{-T}^{0} e^{-2(2+j4\pi)t}\, dt + \int_{0}^{T} e^{2(2+j4\pi)t}\, dt$

$= \lim_{T\to\infty} \left. \frac{e^{-2(2+j4\pi)t}}{-4-8j\pi} \right|_{-T}^{0} + \left. \frac{e^{2(2+j4\pi)t}}{4+8j\pi} \right|_{0}^{T}$

$= \frac{1}{-4-8j\pi} + \frac{e^{2(2+j4\pi)T}}{4+8j\pi} + \frac{e^{2(2+j4\pi)T}}{4+8j\pi} - \frac{1}{4+8j\pi}$

$= \frac{-2}{4+8j\pi} = \frac{-1}{2+4j\pi}$

$E = \lim_{T\to\infty} \int_{-T}^{T} \left| e^{2t} \cdot e^{j4\pi t} \right|^2 dt$

$= \lim_{T\to\infty} \int_{-T}^{T} e^{4t} \left| e^{j4\pi t} \right|^2 dt$

$e^{j\theta} = \cos\theta + j\sin\theta$

$\Rightarrow \left| e^{j\theta} \right| = \sqrt{\cos^2\theta + \sin^2\theta} = 1$

$$= \underset{T \to \infty}{Lt} \int_{-T}^{T} e^{4t}.(1)\,dt$$

$$= \underset{T \to \infty}{Lt} \frac{e^{4t}}{4} \Big|_{-T}^{T} = \underset{T \to \infty}{Lt} \frac{1}{4}\left(e^{4T} - e^{-4T}\right)$$

$$= \underline{\underline{\infty}}$$

Average Power, $P_{avg.} = \underset{T \to \infty}{Lt} \frac{1}{2T}\left(\frac{e^{4T} - e^{-4T}}{4}\right)$

$$= \underset{T \to \infty}{Lt} \frac{e^{4T} - e^{-4T}}{8T} \quad \left(\frac{\infty}{\infty} \text{ doesn't exist.}\right)$$

Hence apply L-Hospital's rule,

$$= \underset{T \to \infty}{Lt} \frac{4e^{4t} + 4e^{-4t}}{8}$$

$$= \frac{1}{2}\left(e^{\infty} + e^{-\infty}\right) = \underline{\underline{\infty}}.$$

$\therefore$ The Signal is having infinite energy and infinite avg. power. So the Signal is neither an energy signal nor power Signal.

(10) $x(t) = \cos^2(\omega_0 t)$.

$P = 3/8$
$\underline{\underline{E = 0}}$

$\longrightarrow$ Energy, $E = \underset{T \to \infty}{Lt} \int_{-T}^{T} |\cos^2(\omega_0 t)|^2\,dt$

$$= \underset{T \to \infty}{Lt} \int_{-T}^{T} \cos^4(\omega_0 t)\,dt.$$

* P.T the following signals are neither energy signals nor power signals.

1. $x(t) = t^{-1/4}\, u(t-1)$

2. $x(t) = (\alpha^2 + t^2)^{-1/4}$. [Hint use $\int \frac{1}{\sqrt{a^2+x^2}}\,dx = \frac{\log(x+\sqrt{a^2+x^2})}{a}$]

$\longrightarrow$ (1) $x(t) = t^{-1/4}\, u(t-1)$

$$u(t-1) = \begin{cases} 1 & \text{for } t \geq 1 \\ 0 & \text{for } t < 1. \end{cases}$$

Energy,

$$E = \underset{T\to\infty}{Lt} \int_{-T}^{T} t^{-2/4} \left(u(t-1)\right)^2 dt$$

$$= \underset{T\to\infty}{Lt} \int_{1}^{T} t^{-2/4} (1)\, dt$$

$$= \underset{T\to\infty}{Lt} \left. \frac{t^{-2/4+1}}{-\frac{2}{4}+1} \right|^{T} = \underset{T\to\infty}{Lt} \left. \frac{t^{-2/4}}{-2/4} \right|_{1}^{T}$$

$$\underset{T\to\infty}{Lt}\, 2\left[T^{1/2} - 1\right]$$

$$\underline{\underline{\infty}}$$

$$\text{Power} = \underset{T\to\infty}{Lt} \frac{1}{2T} \left[2\left(T^{1/2}-1\right)\right]$$

$$= \underset{T\to\infty}{Lt} \frac{\frac{1}{2}T^{-1/2}}{1} = \underline{\underline{0}}$$

(Use L-hospital rule)

$\therefore$ It is neither energy signal nor power signal.

(2) $x(t) = (\alpha^2 + t^2)^{-1/4}$

$\longrightarrow$ Energy, $E = \underset{T \to \infty}{Lt} \int_{-T}^{T} \left|(\alpha^2+t^2)^{-1/4}\right|^2 dt$

$$= \underset{T \to \infty}{Lt} \int_{-T}^{T} (\alpha^2+t^2)^{-1/2} dt$$

$$= \underset{T \to \infty}{Lt} \int_{-T}^{T} \frac{1}{\sqrt{\alpha^2+t^2}} \cdot dt$$

$$= \underset{T \to \infty}{Lt} \left. \frac{\log(t + \sqrt{\alpha^2+t^2})}{\alpha} \right|_{-T}^{T}$$

$$= \underset{T \to \infty}{Lt} \frac{1}{\alpha}\left[\log(T + \sqrt{\alpha^2+T^2}) - \log(-T + \sqrt{\alpha^2+T^2})\right]$$

$$= \underset{T \to \infty}{Lt} \frac{1}{\alpha} \log\left(\frac{T + \sqrt{\alpha^2+T^2}}{-T + \sqrt{\alpha^2+T^2}}\right)$$

$$= \underset{T \to \infty}{Lt} \frac{1}{\alpha} \log\left[\frac{\frac{T}{\sqrt{\alpha^2+T^2}} + 1}{\frac{-T}{\sqrt{\alpha^2+T^2}} + 1}\right]$$

$$= \frac{1}{\alpha}$$

Energy & Power of discrete time Signals :-

Energy & power of discrete time Signal $x(n)$ is defined by

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |x(n)|^2 \quad (\text{or}) \quad \sum_{n=-\infty}^{\infty} |x(n)|^2$$

Avg. power of $x(n)$ is

$$P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

* Determine whether the given Samples are energy (or) power signals.

1. $x(n) = \delta(n)$
2. $x(n) = u(n)$
3. $x(n) = r(n)$
4. $x(n) = A e^{j\omega_0 n}$

$$\underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |x(n)|^2 = E$$

$$P_{avg} = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

→ ① $x(n) = \delta(n)$

$$E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |\delta(n)|^2$$

$$= \underset{N\to\infty}{Lt} |\delta(-N)|^2 + |\delta(-(N-1))|^2 + \cdots + |\delta(0)|^2 + |\delta(1)|^2 + \cdots + |\delta(N)|^2$$

$$= \underset{N\to\infty}{Lt} \; 0 + 0 + \cdots + 1 + 0 + \cdots + 0$$

$$= \underline{\underline{1}}$$

$$\text{Power, } P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |\delta(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} (1)$$

$$= \underline{\underline{0}}$$

$\therefore$ The signal is <u>energy</u> signal.

② $x(n) = u(n)$.

$\longrightarrow$ $$\text{Energy, } E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \underset{N\to\infty}{Lt} |u(-N)|^2 + |u(-(N-1))|^2 + \cdots + |u(0)|^2 + |u(1)|^2 + |u(2)|^2 + \cdots + |u(N)|^2$$

$$= \underset{N\to\infty}{Lt} (0+0+\cdots+1+1+\cdots+1)$$

$$= \underset{N\to\infty}{Lt} N+1$$

$$= \underline{\underline{\infty}}$$

$$\text{Power, } P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \sum_{n=-N}^{N} |u(n)|^2$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2N+1} (N+1)$$

$$= \underset{N\to\infty}{Lt} \frac{N+1}{2N+1}$$

$$= \underset{N\to\infty}{Lt} \frac{1}{2} \quad (\because \text{ use L-hospital rule})$$

$$= \underline{\underline{1/2}}$$

$\therefore$ The given signal is power signal.

(3) $x(n) = r(n)$.

$\longrightarrow$ Energy, $E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} |r(n)|^2$

$$= \underset{N\to\infty}{Lt} \sum_{n=-N}^{0} |r(n)|^2 + \sum_{n=0}^{N} |r(n)|^2$$

$$= \underset{N\to\infty}{Lt} \left(0 + \sum_{n=0}^{N} n^2\right)$$

$0+1+2+\cdots+N$.

$$= \underset{N\to\infty}{Lt} \frac{N(N+1)(2N+1)}{6}$$

$$= \underline{\underline{\infty}}$$

~~Power, P = $\underset{N\to\infty}{Lt} \frac{1}{2N+1}(N+1) = \underset{N\to\infty}{Lt} \frac{1+1/N}{2+1/N}$~~

~~$= \underline{\underline{1/2}}$~~

~~$\therefore$ The given signal is power signal.~~

Power, $P = \underset{N\to\infty}{Lt} \frac{N(N+1)(2N+1)}{6(2N+1)}$

$$= \underline{\infty}$$

$\therefore$ The given signal is neither energy signal nor power signal.

(4) $x(n) = Ae^{j\omega_0 n}$

$\longrightarrow$ Energy, $E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} A^2 |e^{j\omega_0 n}|^2$

$N+N+1$
$2N+1$

$$= \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} A^2 |1|^2$$

$\because \boxed{\sum_{n=N_1}^{N_2} 1 = N_2 - N_1 + 1}$ $= \underset{N\to\infty}{Lt} A^2(2N+1) = \underline{A^2(\infty)}$

$$= \underline{\infty}$$

$$\text{Power, } P = \underset{N \to \infty}{Lt} \frac{1}{2N+1} \cdot \sum_{n=-N}^{N} A^2 (2N+1)$$

$$= \underset{N \to \infty}{Lt} \frac{A^2}{\cancel{2N+1}}$$

$$= \underline{\underline{\cancel{0}\, A^2}}$$

$\therefore$ The given signal is ~~energy~~ Power signal

---

⑤ $x(n) = u_p(n)$

$\rightarrow$ Energy, $E = \underset{N \to \infty}{Lt} \sum_{n=-N}^{N} |u_p(n)|^2$

$$= \underset{N \to \infty}{Lt} \left[\sum_{n=-N}^{0} (0) + \sum_{n=\cancel{0}}^{N} n^{\cancel{2}4} \right]$$

$$= \underset{N \to \infty}{Lt} \frac{0^4 + 1^4 + 2^4 + \cdots\cdots \cancel{N(N+1)(2N+1)}}{\cancel{6}} = \underline{\underline{\infty}}$$

$$\text{Power, } P = \underset{N \to \infty}{Lt} \frac{1^4 + 2^4 + 3^4 + \cdots}{2N+1} \quad \frac{\cancel{N(N+1)(2N+1)}}{\cancel{6}}$$

$$= \underline{\underline{\infty}}$$

$\therefore$ The given signal is neither energy signal nor power signal.

$$P = \underset{N \to \infty}{Lt} \frac{\sum_{n=0}^{\infty} n^4}{2N+1}$$

$$= \underset{N \to \infty}{Lt} \frac{0 + 1^4 + 2^4 + 3^4 + \cdots + N^4}{2N+1}$$

$$= \underset{N \to \infty}{Lt} \frac{\frac{1}{N} + \frac{2^4}{N} + \frac{3^4}{N} + \cdots + N^3}{2 + 1/N}$$

$$= \underline{\underline{\infty}}$$

(c) $x(n) = \left(\frac{1}{2}\right)^n u(n).$

$\longrightarrow$ Energy, $E = \underset{N\to\infty}{Lt} \sum_{n=-N}^{N} \left|\left(\frac{1}{2}\right)^n u(n)\right|^2$

$$= \underset{N\to\infty}{Lt} \sum_{n=0}^{N} \left(\frac{1}{2}\right)^{2n},$$

$$= \underset{N\to\infty}{Lt} \sum_{n=0}^{N} \left(\frac{1}{4}\right)^{n}$$

$$= \sum_{n=0}^{\infty} \frac{1}{4^n}$$

$$= \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\boxed{\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \; a<1}$$

$$\sum_{n=0}^{N} a^n = \frac{a^{N+1}-1}{a-1}$$

Power, $P = \underset{N\to\infty}{Lt} \frac{1}{2N+1} \cdot \frac{4}{3}$

$$= \underline{\underline{0}}$$

$\therefore$ The signal is energy signal.

~~classification~~ of

Signal transmission through Systems

classification of discrete ~~t~~ time systems :-

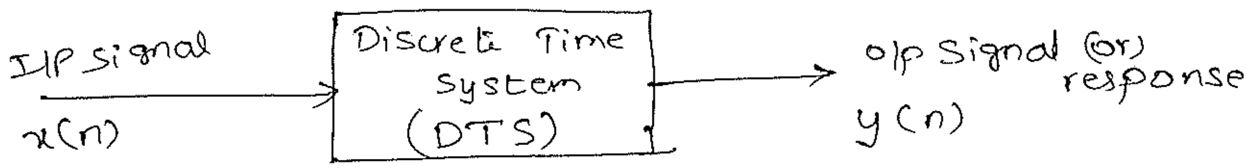
The properties of discrete time system are

(i) static & dynamic system.
(ii) causal & non-causal systems.
(iii) Linear & non-linear systems.
(iv) Time-variant & invariant systems.
(v) Stable & unstable systems
(vi) FIR & IIR Systems
(vii) Recursive & non-recursive systems.

Discrete time system :-

Discrete time system is a device which operates on a discrete time input signal, acc. to some well-defined rules, it produces another discrete time signal i.e; o/p signal.

It can be represented mathematically as



$$y(n) = T[x(n)]$$

(i) Static & dynamic systems :-

The discrete time system is Called a static (or) memoryless system if the response of discrete time system at any time 'n' depends on the present input Sample, but doesnot depend on past and future Samples of I/p & o/p's. $y(n) = F[x(n)]$

Ex :- $y(n) = 5x(n)$.

$n=0$	$y(0) = 5x(0)$	$n=-1$	$y(-1) = 5x(-1)$
$n=1$	$y(1) = 5x(1)$	$n=-2$	$y(-2) = 5x(-2)$
$n=2$	$y(2) = 5x(2)$	$\vdots$	$\vdots$
$\vdots$	$\vdots$		

- from this, $y(n)$ at any time 'n', it depends on present I/p sample only, so, This system is called memoryless (or) static system.

- Static system doenot require memory.

Dynamic System :

The discrete time system is Called the dynamic system (or memory) if the response of discrete time system at any time 'n' depends on the present I/p Samples, ~~futu~~ past I/p, O/p samples and future I/p Samples.

Ex :- $y(n) = 5x(n) + 6x(n-1)$

$n=0$	$5x(0) + 6x(-1)$	$n=-1$	$y(-1) = 5x(-1) + 6x(-2)$
$n=1$	$5x(1) + 6x(0)$	$n=-2$	$y(-2) = 5x(-2) + 6x(-3)$
$n=2$	$5x(2) + 6x(1)$	$\vdots$	$\vdots$
	$\vdots$		

This system $y(n)$ at any time 'n' depends on present & past i/p samples. So, this system is known as memory (or) dynamic system.

- Dynamic System requires memory.

$\therefore$ $y(n) = F[x(n), x(n-1), x(n-2), \ldots, y(n-1), y(n-2), \ldots, \& x(n+1), x(n+2), \ldots]$

Ex :- $y(n) = 6x(n) - 4x(n-2) + 6x(n+1) + 9y(n-1)$

Completed

11/06

13/7/06.

(ii) Causal & non-causal Systems :-

A discrete time System is said to be causal if its output at any instant of time 'n' depends on present input, past inputs and past output but doesnot depend on future input samples.

This can be represented mathematically as

$y(n) = F\{x(n), x(n-1), x(n-2), \ldots, y(n-1), y(n-2), \ldots\}$

where 'n' ~~F{}~~ is any arbitrary constant.

A discrete time system is said to be non-causal if its output at any instant of time 'n' depends not only on present input, past input & outputs, but also on future input samples.

Mathematically,

$$y(n) = f\{x(n), x(n-1), \ldots, y(n-1), y(n-2), \ldots, x(n+1), x(n+2), \ldots\}$$

Ex :- check whether the following systems are causal or not.

① $y(n) = a x(n) + b.x(n-4)$.

$\longrightarrow$

$n=0$	$y(0) = ax(0) + bx(-4)$	$n=-1$	$y(-1) = ax(-1) + bx(-5)$
$n=1$	$y(1) = ax(1) + bx(-3)$	$n=-2$	$y(-2) = ax(-2) + bx(-6)$
$n=2$	$y(1) = ax(2) + bx(-2)$		$\vdots$
	$\vdots$		

From this, we observe that the o/p, $y(n)$, at any instant of time, n depends on present and past input samples only. So, the system is causal system.

② $y(n) = x(-n)$.

$\longrightarrow$

$n=0$	$y(0) = x(0)$	$n=-1$	$y(-1) = x(1)$
$n=1$	$y(1) = x(-1)$	$n=-2$	$y(-2) = x(2)$
	$\vdots$	$\vdots$	$\vdots$

From this, o/p y(n) depends on both past I/p's & future I/p's. So, the system is non-causal system.

(iii) Linear & Non-linear Systems :-

A discrete time system satisfies the Superposition principle, then the system is said to be linear system.

Superposition Principle :-

Superposition principle states that response of weighted sum of ~~response~~ of input signals to be equal to weighted sum of responses to each of individual signals.

It is mathematically represented as

$$a_1 x_1(n) + a_2 x_2(n) \xrightarrow{T} a_1 y_1(n) + a_2 y_2(n)$$

i.e; $T\left[a_1 x_1(n) + a_2 x_2(n)\right] = a_1 T\left(x_1(n)\right) + a_2 T\left(x_2(n)\right)$

If any system doesnot satisfy the superposition principle, then the system is said to be non-linear system.

~~Ex~~ i.e; $T\left[a_1 x_1(n) + a_2 \cdot x_2(n)\right] \neq a_1 T\left[x_1(n)\right] + a_2 T\left[x_2(n)\right]$

Ex :- check whether the following systems are linear or not.

1. $y(n) = n\,x(n)$
2. $y(n) = A\,x(n) + B$
3. $y(n) = x^2(n)$
4. $y(n) = x(n^2)$
5. $y(n) = e^{x(n)}$

$\longrightarrow$ ① $y(n) = n.x(n)$

DTS System represented ~~by~~ mathematically by

$$x(n) \xrightarrow{T} y(n) = T[x(n)] = n\,x(n);$$

$$x_1(n) \xrightarrow{T} y_1(n) = T[x_1(n)] = n\,x_1(n);$$

$$x_2(n) \xrightarrow{T} y_2(n) = T[x_2(n)] = n\,x_2(n).$$

* Condition for linearity is

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

let $x_3(n) = a_1x_1(n) + a_2x_2(n)$

$$x_3(n) \xrightarrow{T} y_3(n) = T[x_3(n)]$$

$$\text{L.H.S} = y_3(n) = T[x_3(n)] = T[a_1x_1(n) + a_2x_2(n)]$$

$$= n[a_1x_1(n) + a_2x_2(n)]$$

$[\because T[x(n)] = n\,x(n)]$

$$= a_1n\,x_1(n) + a_2n\,x_2(n) \quad \text{—(1)}$$

$$\text{R.H.S} = a_1T[x_1(n)] + a_2T[x_2(n)] = a_1y_1(n) + a_2y_2(n)$$

$$= a_1n\,x_1(n) + a_2n\,x_2(n) \quad \text{—(2)}$$

From (1) & (2),

$$T[a_1x_1(n) + a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]$$

$\therefore$ It satisfies the condition for linearity & hence it is linear System.

(2) $y(n) = A\,x(n) + B.$

$$\rightarrow \quad x(n) \xrightarrow{T} y(n) = T[x(n)] = A\,x(n) + B$$

$$x_1(n) \xrightarrow{T} y_1(n) = T[x_1(n)] = A\,x_1(n) + B$$

$$x_2(n) \xrightarrow{T} y_2(n) = T[x_2(n)] = A\,x_2(n) + B$$

Condition for linearity is

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$\text{L.H.S} = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= A(a_1 x_1(n) + a_2 x_2(n)) + B \quad (\because T[x(n)] = A x(n) + B)$$

$=$ [illegible] $+ B$ —— (1)

$\text{R.H.S} = a_1 T[x_1(n)] +$ [illegible]

$$= a_1 (A x_1(n) + B) + a_2 (A x_2(n) + B)$$

$$= a_1 A x_1(n) + a_2 A x_2(n) + a_1 B + a_2 B \text{ —— (2)}$$

$\because$ (1) $\neq$ (2) i.e; L.H.S $\neq$ R.H.S

$\therefore$ The system is non-linear system.

(3) $y(n) = x^2(n)$.

$$x(n) \xrightarrow{T} y(n) = T[x(n)] = x^2(n)$$

$$x_1(n) \xrightarrow{T} y_1(n) = T[x_1(n)] = x_1^2(n)$$

$$x_2(n) \xrightarrow{T} y_2(n) = T[x_2(n)] = x_2^2(n)$$

Condition of linearity is

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$\text{L.H.S} = T[a_1 x_1(n) + a_2 x_2(n)]$$

$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$

$$\text{R.H.S} = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

$$= a_1 x_1^2(n) + a_2 x_2^2(n)$$

L.H.S $\neq$ R.H.S $\therefore$ The given system is not linear i.e, non-linear system.

(4) $y(n) = x(n^2)$

$\longrightarrow$ $x(n) \xrightarrow{T} y(n) = T[x(n)] = x(n^2)$

$x_1(n) \xrightarrow{T} y_1(n) = T[x_1(n)] = x_1(n^2)$

Condition for linearity is

$$T[a_1 x_1(n) + a_2 x_2(n)] = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

L.H.S $= T[a_1 x_1(n) + a_2 x_2(n)]$

$= a_1 x_1(n^2) + a_2 x_2(n^2)$

R.H.S $= a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 x_1(n^2) + a_2 x_2(n^2)$

L.H.S = R.H.S $\therefore$ It is linear system.

(5) $y(n) = e^{x(n)}$.

$\longrightarrow$ $x(n) \xrightarrow{T} y(n) = T[x(n)] = e^{x(n)}$

L.H.S $= T[a_1 x_1(n) + a_2 x_2(n)]$

$= e^{a_1 x_1(n) + a_2 x_2(n)}$

R.H.S $= a_1 T[x_1(n)] + a_2 T[x_2(n)]$

$= a_1 e^{x_1(n)} + a_2 e^{x_2(n)}$

As L.H.S $\neq$ R.H.S, the system is non-linear.

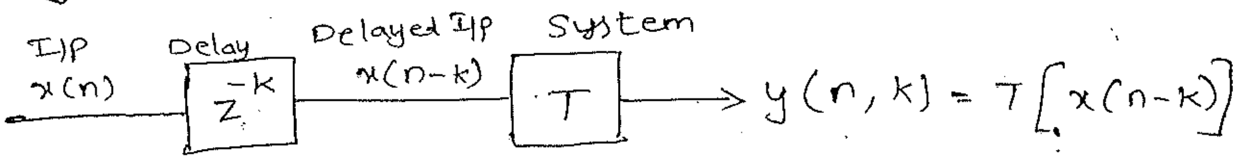
14/7/06.

(iv) Time Invariant & time variant systems :-

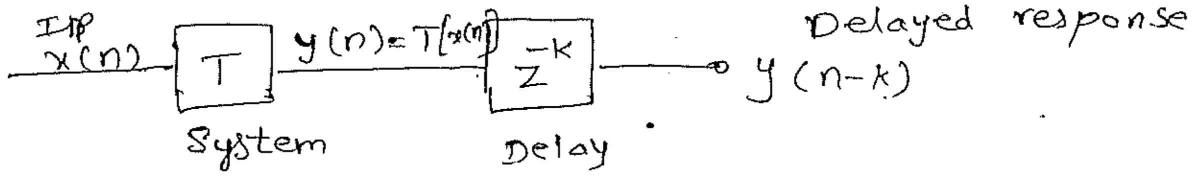
The output and input relation of discrete time system doesnot vary with ~~system~~ time, then the system is called time-invariant system.

Testing Condition for time-invariant system

step ①



step ②



Step ③ $\quad y(n, k) = y(n-k)$.

Step 1 : The input sequence $x(n)$ is delayed by 'k' units of time, then we get $x(n-k)$. Determine the response for delayed input sequence, $x(n-k)$. Let this be represented as $y(n, k)$.

Step 2 : Determine the delayed response for unshifted input sequence by 'k' unit of time. Let this be represented as $y(n-k)$.

Step 3 : Check the condition $y(n, k) = y(n-k)$. If this eq. is satisfied, then the system is time-invariant system, otherwise it is time-variant system.

Ex :- check whether the following systems are time-variant (or) time-invariant.

1. $y(n) = A\,x(n)$
2. $y(n) = n\,x(n)$
3. $y(n) = x(-n)$.

→ ① Step 1 : Response due to shifted (or) delayed input sequence by 'k' units of time.

$$y(n,k) = T[x(n-k)] = A\,x(n-k)$$

Step 2 : Delayed response due to unshifted input sequence.

$$y(n-k) = A\,x(n-k).$$

Step 3 :- $y(n,k) = y(n-k)$ from step ① & ②

∴ The system is time-invariant system.

② Step 1 :-

$$y(n,k) = T[x(n-k)] = \cancel{x(-(n-k))}$$
$$= x(-n-k)$$

Step 2 :- $y(n-k) = x(-(n-k)) = x(-n+k)$

Step 3 :- $y(n,k) \neq y(n-k)$

∴ The System is time-variant system.

③ Step 1 :- $y(n,k) = T[x(n-k)] = n\,x(n-k)$

Step 2 :- $y(n-k) = (n-k)\,x(n-k)$

Step 3 :- $y(n,k) \neq y(n-k)$

∴ The system is time-Variant system.

(V) Stable and unstable systems :-

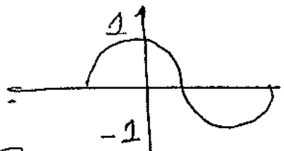
Any arbitrary relaxed system is said to be bounded input bounded output (BIBO) stable iff, every bounded input it yields bounded output. Then the system is called stable system.

- It can be mathematically represented as,

bounded I/p is $|x(n)| = M_x < \infty$

bounded O/p is $|y(n)| = M_y < \infty$

Ex :- ① $y(n) = \cos(\pi n)$



$|y(n)| = |\cos(\pi n)| < \infty$. $\therefore$ It is stable system.

② $y(n) = \delta(n)$

$|y(n)| = |\delta(n)| < \infty$ $\therefore$ System is stable.

③ $y(n) = u(n)$

$|y(n)| = |u(n)| < \infty$ $\therefore$ System is stable.

④ $y(n) = n\,u(n)$

$|y(n)| = |n(u(n))| \nless \infty$ $\therefore$ System is unstable

$\because$ $|y(n)|_{n=\infty} = |\infty\, u(n)| = \infty.$

(Vi) FIR, IIR Systems :-

FIR :- If a system having finite duration impulse sequence, then the system is called "finite impulse response" System (FIR).

Ex :- $h(n) = \{1, -1, \underset{\uparrow}{2}, 3, 2\}$

IIR :-

If a system having infinite duration impulse sequence, then the system is called "infinite impulse response" (IIR) System.

Ex :- $h(n) = u(n) = \{\underset{\uparrow}{1}, 1, 1, 1, \ldots\ldots\}$

$h(n) = \cos(\pi n)$.

(vii) Recursive & non-recursive systems :-

these are the classifications of causal system

Recursive :-

It is a system property that output at any time 'n' depends on present input, past input, past output samples.

~~Ex :-~~

It is mathematically represented as

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots, y(n-1), y(n-2), \ldots]$$

Non-recursive :-

Non-recursive system is a property that response at any time 'n' depends on present input and past input samples only

$$y(n) = F[x(n), x(n-1), x(n-2), \ldots\ldots]$$

Ex :- $y(n) = ax(n) - bx(n-1)$

It is non-recursive System,

* Check the following conditions.

1. Static, dynamic
2. Linear, Non-linear
3. Causal, non-causal
4. Time invariant, variant.

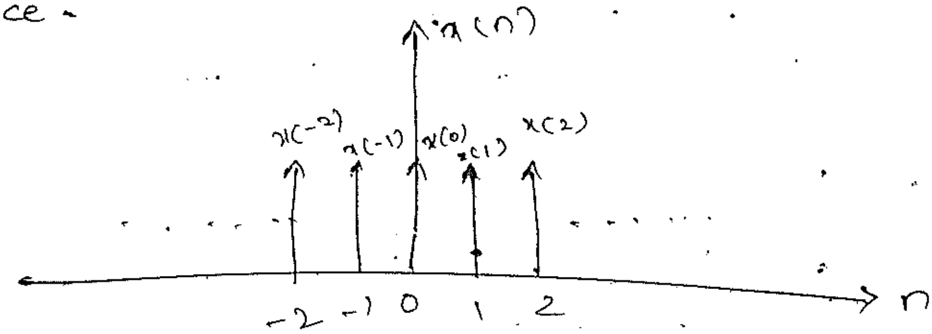
1. $y(n) = x(n) - 5x(n-1)$
2. $y(n) = x(2n)$
3. $y(n) = x(n/3)$
4. $y(n) = x(n) \cdot \cos\left(\frac{\pi n}{6}\right)$
5. $y(n) = a^{x(n)}$
6. $y(n) = |x(n)|$
7. $y(n) = \log(1+|x(n)|)$

14/7/06.

Arbitrary Representation of a Sequence:-

Any sequence can be represented as sum of shifted version of unit sample sequences is called arbitrary representation of a sequence.

Ex:-



$$x(n) = \ldots\ldots + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n- \\ + x(2)\delta(n-2) + \ldots\ldots$$

$$\boxed{x(n) = \sum_{K=-\infty}^{\infty} x(K)\,\delta(n-K)}$$

Linear Time-Invariant System (or) Discrete time linear Time-invariant System:-

LTI (or) DTLTI system:

A discrete time system it satisfies

14/8/08.

Appr. of fn. by a set of mutually orthogonal fn :-

Let us Consider a set of 'n' functions $g_1(t), g_2(t), \ldots, g_n(t)$ which are orthogonal to one another over the interval t_1 to t_2

i-e; $\displaystyle\int_{t_1}^{t_2} g_j(t)\, g_k(t)\, dt = 0$ for $j \neq k$.

$$\int_{t_1}^{t_2} g_j^2(t)\, dt = K_j \quad \text{for } j = k.$$

Let an arbitrary fn. $f(t)$ be approximated over (t_1, t_2) by a linear Combination of these 'n' mutually orthogonal functions.

$$f(t) \simeq c_1 g_1(t) + c_2 g_2(t) + \cdots + c_i g_i(t) + \cdots + c_n g_n(t).$$

(1).
$$\boxed{f(t) = \sum_{i=1}^{n} c_i\, g_i(t)}$$

For the best app., we must find the proper values of Constants $c_1, c_2, \ldots, c_n$ Such that e_r ~~for~~ (mean square error), the mean square of $f_e(t)$ is minimised.

By def.,

$$f_e(t) = f(t) - \sum_{i=1}^{n} c_i\, g_i(t)$$

Mean square error,

$$e_r = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} f_e^2(t)\, dt.$$

$$= \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left[f(t) - \sum_{i=1}^{n} c_i\, g_i(t) \right]^2 dt. \quad (2)$$

where e_r is a fn. of $c_1, c_2, \ldots, c_n$ and to minimise e_r, we must have

$$\frac{\delta e_r}{\delta c_1} = \frac{\delta e_r}{\delta c_2} = \cdots = \frac{\delta e_r}{\delta c_j} = \cdots = \frac{\delta e_r}{\delta c_n} = 0.$$

Let us Consider the equation, $\frac{\delta e_r}{\delta c_j} = 0$.

Diff. w.r.t c_j on both sides of eq. ①, we get.

$$\frac{\partial e_r}{\partial c_j} = \frac{\partial}{\partial c_j}\left[\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\left[f^2(t) + \sum_{i=1}^{n} c_i^2 g_i^2(t) - 2f(t)\sum_{i=1}^{n} c_i g_i(t)\right]\right]$$

Interchanging the differentiator & integrator Const

(a)

$$\frac{\partial e_r}{\partial c_j} = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2}\frac{\partial}{\partial c_j} f^2(t)\,dt + \int_{t_1}^{t_2}\frac{\partial}{\partial c_j}\left(\sum_{i=1}^{n} c_i^2 g_i^2(t)\right) - \int_{t_1}^{t_2}\frac{\partial}{\partial c_j}\left[2f(t)\sum_{i=1}^{n} c_i g_i(t)\right]d\right.$$

$$\Rightarrow \quad 0 = 0 + \int_{t_1}^{t_2}\frac{\partial}{\partial c_j}\left[c_1^2 g_1^2(t) + \cdots + c_j^2 g_j^2(t) + \cdots + c_n^2\right.$$

$$- \int_{t_1}^{t_2}\frac{\partial}{\partial c_j}\left[2f(t)\left(c_1 g_1(t) + \cdots + c_j g_j(t) + \cdots +\right.\right.$$

$$\Rightarrow \quad 0 = \int_{t_1}^{t_2} 2c_j g_j^2(t)\,dt - \int_{t_1}^{t_2} 2f(t)\,g_j(t)\,dt.$$

$$\Rightarrow \int_{t_1}^{t_2} 2c_j g_j^2(t)\,dt = \int_{t_1}^{t_2} 2f(t)\,g_j(t)\,dt$$

$$\Rightarrow \quad c_j = \frac{\int_{t_1}^{t_2} 2f(t)\,g_j(t)\,dt}{\int^{t_2} 2g_j^2(t)\,dt}$$

$$\Rightarrow \boxed{c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{K_j}} \quad \text{where} \quad K_j = \int_{t_1}^{t_2} 2\, g_j^2(t)\, dt.$$

Conclusion:-

we can conclude the result of this as follows:

Given a set of 'n' fn.'s $g_1(t), g_2(t), \ldots, g_n(t)$ mutually orthogonal over an interval (t_1, t_2), it is possible to approximate an arbitrary fn. $f(t)$ over this interval by linear combination of these 'n' fn.'s.

i.e; $f(t) \simeq c_1 g_1(t) + \cdots + c_j g_j(t) + \cdots + c_n g_n(t)$

$$= \sum_{i=1}^{n} c_i g_i(t).$$

for the best approximation, ϵ_r i.e; the mean square error over the interval should be minimum.

For this case,
$$c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{\int_{t_1}^{t_2} g_j^2(t)\, dt} = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{K_j}.$$

Evaluation of mean square error:-

we now try to find the value of ϵ_r when optimum values of coefficients $c_1, c_2, \ldots, c_n$ are choosen acc. to the following equation.

$$c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{\int_{t_1}^{t_2} g_j^2(t)\, dt} = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{K_j}.$$

$$\epsilon_r = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} \left(f(t) - \sum_{i=1}^{n} c_i g_i(t) \right)^2 dt.$$

$$= \frac{1}{t_2-t_1}\int_{t_1}^{t_2}\left[f^2(t) - 2f(t)\sum_{i=1}^{n} c_i\, g_i(t) + \sum_{i=1}^{n} c_i^2\, g_i^2(t)\right]dt$$

$$= \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - 2\sum_{i=1}^{n} c_i\int_{t_1}^{t_2} f(t)g_i(t)\,dt + c_i^2\sum_{i=1}^{n}\int_{t_1}^{t_2} g_i^2(t)\,dt\right]$$

We know that $c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{K_j} = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{\int_{t_1}^{t_2} g_j^2(t)\, dt}$

lly $c_i = \frac{\int_{t_1}^{t_2} f(t)\, g_i(t)\, dt}{K_i} = \frac{\int_{t_1}^{t_2} f(t)\, g_i(t)\, dt}{\int_{t_1}^{t_2} g_i^2(t)\, dt}$

$$\therefore\ c_i K_i = \int_{t_1}^{t_2} f(t)\, g_i(t)\, dt = c_i \int_{t_1}^{t_2} g_i^2(t)\, dt$$

$$\therefore\ e_r = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - 2\sum_{i=1}^{n} c_i \times c_i K_i + \sum_{i=1}^{n} c_i^2 K_i\right]$$

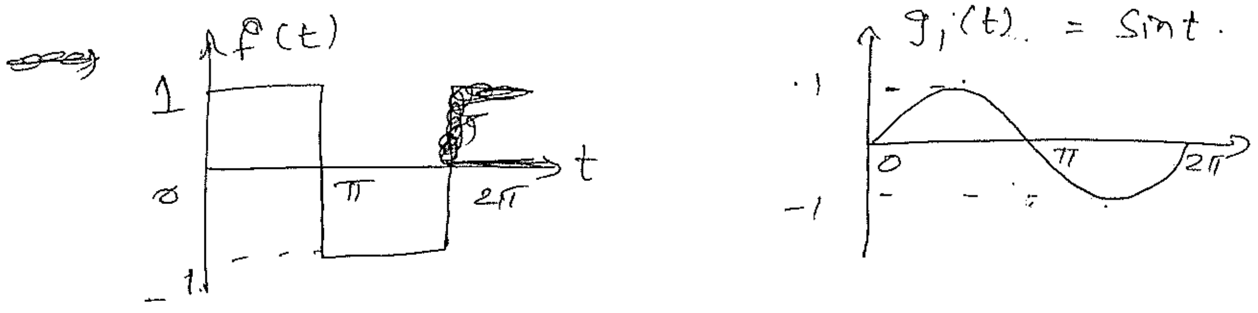
$$\therefore\ e_r = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \sum_{i=1}^{n} c_i^2 K_i\right]$$

$$= \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \left(c_1^2 K_1 + c_2^2 K_2 + c_3^2 K_3 + \cdots + c_n^2 K_n\right)\right]$$

This eq.ⁿ can be used to evaluate mean square error.

* The rect. fn. as shown in fig & approximate this fn. as a finite series of sine fn. P.T the no. of terms is added in their appro. then the mean square error is diminished (or) reduced

12m



$\rightarrow$ $f(t)$ is appro. by

$$f(t) \simeq c_1 g_1(t) + c_2 g_2(t) + \dots + c_n g_n(t).$$

$$\simeq c_1 \sin(t) + c_2 \sin(2t) + \dots + c_n \sin(nt)$$

where $f(t)$ is rect. fn.

$$f(t) = \begin{cases} 1 ; & \text{for } 0 \le t \le \pi \\ -1 ; & \pi \le t \le 2\pi . \end{cases}$$

$$c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{\int_{t_1}^{t_2} g_j^2(t)\, dt.} \qquad g_j(t) = \sin(jt) ; \quad 0 \le t \le 2\pi .$$

$$k_j = \int_{t_1}^{t_2} g_j^2(t)\, dt = \int_{t_1}^{t_2} \sin^2(jt)\, dt$$

$$= \int_0^{2\pi} \frac{1-\cos(2jt)}{2}\, dt$$

$$= \frac{1}{2}t - \frac{\sin(2jt)}{4j} \Big|_0^{2\pi}$$

$$= \frac{1}{2}(2\pi) - 0 = \underline{\pi}$$

$$\therefore c_j = \frac{\int_{t_1}^{t_2} f(t)\, g_j(t)\, dt}{\pi}.$$

$$\Rightarrow \frac{1}{\pi}\left[\int_0^{\pi} 1.\ \sin(jt)\,dt + \int_{\pi}^{2\pi} (-1)\sin(jt)\,dt\right]$$

$$\Rightarrow \frac{1}{\pi}\left(\left.\frac{-\cos(jt)}{j}\right|_0^{\pi} + \left.\frac{\cos(jt)}{j}\right|_{\pi}^{2\pi}\right)$$

$$= \frac{1}{\pi}\left(-\frac{\cos(\pi j)-1}{j} + \frac{\cos(2\pi j) - \cos(\pi j)}{j}\right)$$

$$= \frac{1}{\pi j}\left(-\left((-1)^j - 1\right) + 1 - (-1)^j\right)$$

$$= \frac{1}{\pi j}\left(-(+1-1) + 1 - 1\right) = 0 \text{ for } j \text{ is even}$$

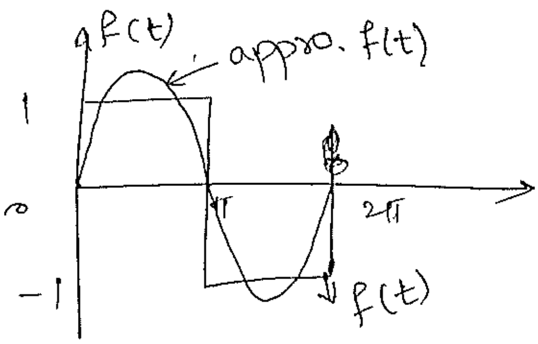
$$= \frac{1}{\pi j}(2+1+1) = \frac{4}{\pi j} \text{ for } j \text{ is odd.}$$

$$\therefore\ c_j = 4/\pi j \quad \text{for } j \text{ is odd}$$

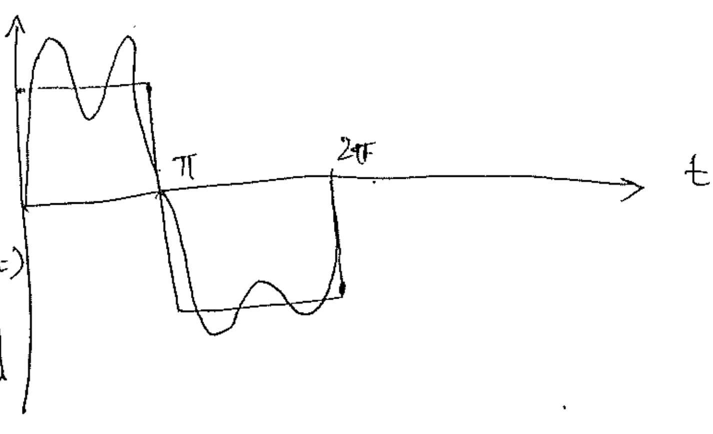
$$= 0 \quad \text{for even } 'j'.$$

Considering one ~~eq.~~ term in the appro:-

$$f(t) \simeq c_1 g_1(t) \simeq c_1 \sin(t) \simeq \frac{4}{\pi}\sin(t).$$



Adding 3 terms in their appro,



$$f(t) \simeq c_1 g_1(t) + c_2 g_2(t) + c_3 g_3(t)$$

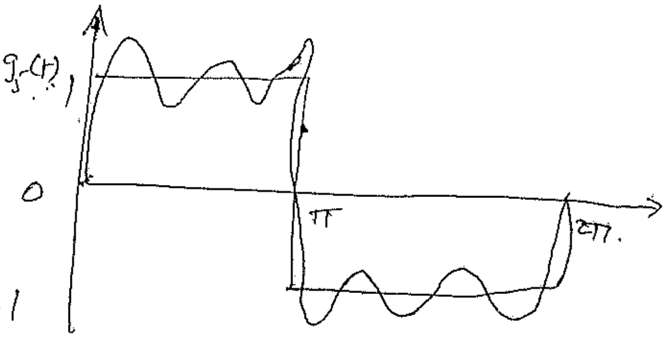
$$\simeq \frac{4}{\pi}\sin t + 0 + \frac{4}{3\pi}\sin(3t)$$

$$\simeq \frac{4}{\pi}\left(\sin t + \frac{1}{3}\sin 3t\right)$$

Adding 5 terms in the appro. :-

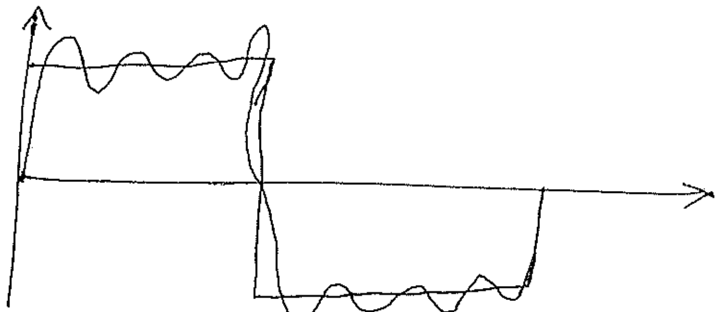
$f(t) \simeq c_1 g_1(t) + c_2 g_2(t) + c_3 g_3(t) + c_4 g_4(t) + c_5 g_5(t),$

$\simeq \frac{4}{\pi}\left[\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t\right]$



Adding 7 terms in the appr:-.

$f(t) \simeq \frac{4}{\pi}\left[\sin t + \frac{1}{3}\sin 3t + \frac{1}{5}\sin 5t + \frac{1}{7}\sin 7t\right].$



when no. of terms are added in the appro, the error decreases.

Mean square error values theoritically are.

$$e_r = \frac{1}{t_2 - t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - \left(c_1^2 k_1 + c_2^2 k_2 + \dots + c_n^2 k_n\right)\right]$$

$$C_i = \begin{cases} \frac{4}{\pi i} & ; \ i \text{ is odd} \\ 0 & ; \ i \text{ is even.} \end{cases}$$

$C_1 = \frac{4}{\pi}; \ c_2 = 0; \ c_3 = \frac{4}{3\pi}; \ c_4 = 0; \dots$

$k_i = \pi \Rightarrow k_1 = \pi, \ k_2 = \pi, \dots, k_7 = \pi.$

$$\int_{t_1}^{t_2} f^2(t)\,dt = \int_0^{\pi} 1^2\,dt + \int_{\pi}^{2\pi} (-1)^2\,dt.$$

$$= \pi + 2\pi - \pi = 2\pi$$

$t_2 - t_1 = 2\pi - 0 = 2\pi$

Add one term in their appro.; $e_r = \frac{1}{2\pi}\left[2\pi - \left(\frac{4}{\pi}\right)^2 \pi\right]$

$= \underline{0.19}.$

Add 3 terms in the appro;

$$e_r = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f_2^2(t)\,dt - c_1^2k_1 - c_2^2k_2 - c_3^2k_3\right]$$

$$e_r = \frac{1}{2\pi}\left[2\pi - \left(\frac{4}{\pi}\right)^2\pi - 0 - \left(\frac{4}{3\pi}\right)^2 \times \pi\right]$$

$$= 0.1$$

Add 5 terms in the app;

$$e_r = \frac{1}{t_2-t_1}\left[\int_{t_1}^{t_2} f^2(t)\,dt - c_1^2k_1 - c_2^2k_2 - c_3^2k_3 - c_4^2k_4 - c_5^2k_5\right]$$

$$= \frac{1}{2\pi}\left[2\pi - \left(\frac{4}{\pi}\right)^2\pi - 0 - \left(\frac{4}{3\pi}\right)^2\pi - 0 - \left(\frac{4}{5\pi}\right)^2\pi\right]$$

$$= 0.0675$$

Add 7 terms in the appro;

$$e_r = \frac{1}{2\pi}\left(2\pi - \left(\frac{4}{\pi}\right)^2\pi - 0 - \left(\frac{4}{3\pi}\right)^2\pi - 0 - \left(\frac{4}{5\pi}\right)^2\pi - 0 - \left(\frac{4}{7\pi}\right)^2\pi\right]$$

$$= 0.05$$

$\therefore$ we conclude that the no. of terms added in their app.., thus mean square error is reduced or diminished gradually.

* P.T Sinusoidal Signals are Orthogonal Signals.

→ Case(i) :- 1, Sin ωt are orthogonal over the interval (0, T)

$$\int_0^T 1 \times \sin(n\omega t)\,dt = 0; \quad n = 0, \pm1, \pm2, \ldots$$

we know that

$f_1(t)$, & $f_2(t)$ are orthogonal over the interval (t_1, t_2) is

$$\int_{t_1}^{t_2} f_1(t)\, f_2(t)\, dt = 0.$$

To find the orthogonal fn's of 1, Sin nωt over the interval $(0, T)$ is

$$\int_0^{T} 1 \times \sin(n\omega t)\, dt$$

$$\Rightarrow \left. -\frac{\cos(n\omega t)}{n\omega} \right|_0^T \qquad \omega = \frac{2\pi}{T}.$$

$$\Rightarrow -\frac{\cos(2\pi n)}{n\omega} + 1$$

$$= -1 + 1 = 0.$$

∴ 1, $\sin(n\omega t)$ are orthogonal

Case (ii) :-

$\sin(n\omega_0 t)$ & $\sin(m\omega_0 t)$ for $m \neq n$ two signals are orthogonal over the interval (t_0, t_0+T_0)

W.K.T two signals are orthogonal if

$$\int_{t_1}^{t_2} f_1(t)\, f_2(t)\, dt = 0.$$

we have to Proof :-

$$\int_{t_0}^{t_0+T_0} \sin(n\omega_0 t)\, \sin(m\omega_0 t)\, dt = 0 \text{ for } m \neq n$$

$$\int_{t_0}^{t_0+T_0} \sin(n\omega_0 t)\, \sin(m\omega_0 t)\, dt$$

$$= \int_{t_0}^{t_0+T_0} \frac{1}{2}\left(\cos(n-m)\omega_0 t - \cos(n+m)\omega_0 t\right) dt$$

$$= \frac{1}{2}\int_{t_0}^{T_0+t_0} \cos(n-m)\omega_0 t\, dt - \frac{1}{2}\int_{t_0}^{T_0+t_0} \cos(n+m)\omega_0 t\, dt$$

$$\Rightarrow \frac{1}{2}\frac{\sin(n-m)\omega_0 t}{(n-m)\omega_0}\Bigg|_{t_0}^{t_0+T_0} - \frac{1}{2}\frac{\sin(n+m)\omega_0 t}{(n+m)\omega_0}\Bigg|_{t_0}^{t_0+T_0}$$

$$= \frac{1}{2}\left[\frac{\sin(n-m)\omega_0(t_0+T_0) - \sin(n-m)\omega_0 t_0}{(n-m)\omega_0}\right] - \frac{1}{2}\left(\frac{\sin(n+m)\omega_0(t_0+T_0) - \sin(n+m)\omega_0 t_0}{(n+m)\omega_0}\right)$$

$$= \frac{1}{2}\left[\frac{\sin((n-m)\omega_0 t_0 + (n-m)2\pi) - \sin(n-m)\omega_0 t_0}{(n-m)\omega_0}\right] - \frac{1}{2}\left(\frac{\sin((n+m)\omega_0 t_0 + (n+m)2\pi) - \sin(n+m)\omega_0 t_0}{(n+m)\omega_0}\right)$$

$$\left(\because \omega = \frac{2\pi}{T_0}\right)$$

$$= \frac{1}{2}\left(\frac{\cancel{\sin(n-m)\omega_0 t_0} - \cancel{\sin(n-m)\omega_0 t_0}}{(n-m)\omega_0}\right) - \frac{1}{2}\left(\frac{\cancel{\sin(n+m)\omega_0 t_0} - \cancel{\sin(n+m)\omega_0 t_0}}{(n+m)\omega_0}\right)$$

$$= \underline{\underline{0}}$$

for $m = n$,

$$\int_{t_0}^{t_0+T_0} \sin(n\omega t)\sin(m\omega t)\,dt = \int_{t_0}^{t_0+T_0} \sin^2(n\omega t)\,dt \;;\; m=n$$

$$= \int_{t_0}^{t_0+T_0} \frac{1-\cos(2n\omega t)}{2}\,dt$$

$$= \int_{t_0}^{t_0+T_0} \frac{1}{2}dt - \int_{t_0}^{t_0+T_0} \cos(2n\omega t)\,dt$$

$$= \frac{1}{2}(t_0 + T_0 - t_0) - 0 = \frac{T_0}{2}$$

$\therefore$ for $m=n$, two signals are not orthogonal.

H.W.

* P.T $\cos(m\omega t)\cos(n\omega t)$ for $m \neq n$ are orthogonal over (t_0, t_0+T_0)

H.W.

* P.T $\cos n\omega_0 t$ & $\sin m\omega_0 t$ are orthogonal for all values of m & n.