2. Maxwell's Equations

Faraday's law :-

Introduction:- In the previous classes only static electric and magnetic fields. Static electric field was developed due to steady charges, static magnetic field developed due to D.C current (steady currents).

In this unit, we discuss time varying fields which are produced due to varying fields. Static EM fields are time varying currents. static fields are independent to each other where as dynamic fields are dependent to each other.

* static charges — static electric field.
  Steady currents — static magnetic field.
  time varying currents — Electromagnetic fields.

Faraday's law :- According to Faraday's experiment a static magnetic field produces, low current flow, but a time varying fields produces an induced voltage in a closed circuit, which causes flow of current.

25/2/15 The Induced emf (V_{emf}) in any closed circuit is equal to the time rate of change of magnetic flux linkage by the circuit. This is called faradays law.

$$V_{emf} = \frac{-d\lambda}{dt}$$

$$V_{emf} = -N\frac{d\psi}{dt} \text{ Volts } [N\psi = \lambda]$$

we know that, $\psi = \oint_S \bar{B}\cdot d\bar{S}$

$$\therefore V_{emf} = -N\oint \frac{d\bar{B}}{dt}\cdot d\bar{S}$$

The -ve sign shows Induced voltage acts in such a way as to oppose flux producing it. It is known as Lenz's law.

Transformer emf :-

Faradays law gives the relation b/w electric field and magnetic field. To produce transformer emf, let us take no. of turns

N = 1.

$$V_{emf} = \frac{-d\psi}{dt}$$

we know $\psi = \oint_S \vec{B}\cdot dS \Rightarrow \bar{B} = \frac{d\psi}{dS}$ w/m²

$$V_{emf} = -\frac{d}{dt}\left(\oint_S \vec{B}\cdot dS\right)$$

This emf induced by time varying current in a stationary loop is often refered to as transformer emf

$$V_{emf} = -\oint_s \frac{\partial \bar{B}}{\partial t} \cdot ds$$

But we know $V_{emf} = -\oint \bar{E} \cdot dl$

eq ① It shows time varying E field is not conservative $\nabla \times E \neq 0$. The workdone in taking a charge about a closed path in a time varying E for exam, is due to energy from time varying magnetic field.

From stocks theorem

$$-\oint_l \bar{E} \cdot dl = -\oint_S (\nabla \times \vec{E}) \cdot dS$$

$$\oint_S (\nabla \times \bar{E}) \cdot dS = -\oint_S \frac{\partial \bar{B}}{\partial t} \cdot ds$$

$$\boxed{(\nabla \times \vec{E}) = -\frac{\partial \bar{B}}{\partial t}} = -\mu \frac{\partial \bar{H}}{\partial t}$$

$$\boxed{\therefore \nabla \times \vec{E} = -\mu \frac{\partial \bar{H}}{\partial t}}$$

① equation is called one of maxwell's equation in time varing fields.

① **Displacement Current Density :-**

we know that $\nabla \times \vec{H} = \vec{J}$ (maxwell's 3rd equation)

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot \vec{J}$$

$\nabla \cdot J = 0$ {from this equation $I = 0$.

Amperes circuital law $\oint_l \vec{H} \, dl = I$ enclosed.

$\nabla \times \vec{H} = 0$.

From this condition amperes circuital law is inconsistance.

For consisting ampheres circuital law take another variable.

$$\nabla \times \vec{H} = \vec{J} + \vec{J_d}$$

$$\nabla \cdot (\nabla \times \vec{H}) = \nabla \cdot (\vec{J} + \vec{J_d})$$

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{J_d}$$

$$\nabla \cdot \vec{J} = -\nabla \cdot \vec{J_d}$$

From continuity equation $\nabla \cdot J = \dfrac{-\partial \rho_v}{\partial t}$

$$+\nabla \cdot \vec{J_d} = +\frac{\partial \rho_v}{\partial t}$$

$$\nabla \cdot \vec{J_d} = \frac{\partial (\nabla \cdot \vec{D})}{\partial t} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

$$\therefore \vec{J_d} = \frac{\partial \vec{D}}{dt}$$

$$\nabla \times \vec{H} = \bar{J} + \bar{J}_d$$

$$\boxed{\nabla \times \vec{H} = J_c + \frac{\partial \vec{D}}{\partial t}} \quad (1)$$

$$\left\{ \begin{array}{l} J_c = \sigma \vec{E} \\ J_d = \partial \vec{D} \\ J = \rho_v \cdot u \end{array} \right.$$

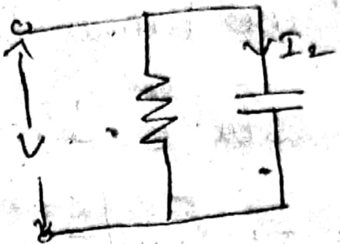
$$J_c = \sigma \vec{E}$$

$$\nabla \times \vec{H} = \sigma \vec{E} + \epsilon \frac{\partial \vec{E}}{\partial t}$$

This is maxwell's equation in time varing field

The term $J_d = \dfrac{\partial \bar{D}}{\partial t}$ is known as displacement current density, J is conduction current.

physical significance of displacement current density



$$C = \frac{\epsilon A}{d}$$

$$i_2 = \frac{\epsilon A}{d} \cdot \frac{\partial V}{\partial t}$$

$$i_2 = \frac{\epsilon A}{d} \frac{\partial(\bar{E} \cdot d)}{\partial t}$$

$$i_2 = \epsilon A \cdot \frac{\partial \vec{E}}{\partial t} \Rightarrow \frac{i_2}{A} = \frac{\partial \epsilon \vec{E}}{\partial t} = \frac{\partial \vec{D}}{\partial t}$$

$$\boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}}$$

$$J_c = \sigma \vec{E}, \quad J_d = \frac{\partial \vec{D}}{\partial t}$$

$\vec{E} = e^{j\omega t}$ in harmonic domain

$$J_d = \epsilon \cdot e^{j\omega t} \cdot j\omega = j\omega\epsilon \cdot \vec{E}$$

$$\frac{|J_c|}{|J_d|} = \frac{\sigma}{\omega\epsilon}$$

If the ratio $\frac{J_c}{J_d} << 1 \Rightarrow$ dielectric medium

$\frac{J_c}{J_d} >> 1 \Rightarrow$ conducting medium

$$\therefore \frac{J_c}{J_d} = \frac{1}{\omega \frac{\epsilon}{\sigma}} = \frac{1}{\omega T_r}$$

where T_r = Relaxation time.

$$\boxed{\therefore T_r = \frac{\epsilon}{\sigma}}$$

σ – S/m

Inconsistance of Ampere's Law

we know that $\nabla \times \bar{H} = J$.

take divergence on both sides

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J}$$

$$0 = \nabla \cdot J$$

But we know continuity equation

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}.$$

so, Amperes law is not consist here

to modify this law

$$\nabla \times \bar{H} = \bar{J} + \bar{N}$$

$$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot (J + \bar{N})$$

$$\nabla \cdot \bar{J} + \nabla \cdot \bar{N} = 0$$

$$\nabla \cdot \bar{N} = -\nabla \cdot J = -\left(\frac{-\partial \rho_v}{\partial t}\right) = \frac{\partial \rho_v}{\partial t}$$

from gauss law $\nabla \cdot \bar{D} = \rho_v$.

$$\nabla \cdot \bar{N} = \nabla \cdot \frac{\partial \bar{D}}{\partial t}$$

$$\therefore \bar{N} = \frac{\partial \bar{D}}{\partial t}.$$

$$\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}$$

$$\therefore J_d = \frac{\partial \bar{D}}{\partial t} \text{ A/m}^2$$

Maxwell equations for time varying fields

$\nabla \cdot \vec{D} = \rho_v$ (Gauss law) — $\int_S \vec{D} \cdot ds = \int_v \rho_v \cdot dv$

$\nabla \times \vec{E} = \frac{-\partial \bar{B}}{\partial t}$ (Faradays) — $\int_l \bar{E} \cdot dl = \int_S \frac{-\partial \bar{B}}{\partial t} \cdot ds$

$\nabla \times \bar{H} = \bar{J_c} + \frac{\partial D}{\partial t}$ (Amperes law) — $\oint_l \bar{H} \cdot dl = \int_S J\, ds + \int_S \frac{\partial D}{\partial t} \cdot ds$

$\nabla \cdot \bar{B} = 0$ (Due to non existance of isolated spheres) — $\oint_S \bar{B} \cdot ds = 0.$

for harmonic conditions

$\vec{E} = E_0 e^{j\omega t} \Rightarrow \frac{\partial \vec{E}}{\partial t} = j\omega \vec{E}.$

$\vec{H} = H_0 e^{j\omega t} \Rightarrow \frac{\partial \vec{H}}{\partial t} = j\omega \vec{H}.$

Differential form:-

1) $\nabla \cdot \bar{D} = \rho_v$

2) $\nabla \times \vec{E} = -\mu \cdot \frac{\partial \vec{H}}{\partial t} = -\mu j\omega \vec{H}.$

3) $\nabla \times \vec{H} = \sigma \vec{E} + \varepsilon \frac{\partial \vec{E}}{\partial t} = \sigma \vec{E} + j\omega \varepsilon \vec{E}$

$= \vec{E}(\sigma + j\omega\varepsilon)$

4) $\nabla \cdot \vec{B} = 0.$

Intgral form:-

1) $\int_S \vec{D} \cdot ds = \int_v \rho_v \cdot dv$

2) $\oint_l \vec{E} \cdot dl = -j\omega\mu \int_S \vec{H} \cdot ds$

3) $\oint_l \vec{H} \cdot dl = (\sigma + j\omega\varepsilon) \cdot \int_S \vec{E} \cdot ds$

4) $\oint_S \vec{B} \cdot ds = 0.$

problems

① Find displacement current density within a parallel plate capacitor having $\epsilon = 100\epsilon_0$ and $A = 0.01\ m^2$, $d = 0.05\ mm$ and capacitor Voltage $100 \times \sin 200\pi t$.

② If $\vec{D} = 10x\,\vec{a_x} - 4y\,\vec{a_y} + kz\,\vec{a_z}\ \mu C/m^2$

$\vec{B} = 2\vec{a_y}\ m\,wb/m^2$. [wb/m² = Tesla]

Find value of 'K' which satisfy the maxwell eqn. $\sigma = 0$, $\rho_v = 0$.

①

sol:-

$\left[V = L\dfrac{di}{dt} \right]$

$J_d = ?$

$\epsilon = 100\epsilon_0$

$A = 0.01 m^2$

$d = 0.05\ mm = 0.05 \times 10^{-3} m$

Voltage of capacitor $= 100 \sin 200\pi t$.

$$i_d = \left(\frac{\epsilon A}{d}\right)\frac{dv}{dt} = \frac{100 \times 8.85 \times 10^{-12} \times 0.01}{0.05 \times 10^{-3}}\left(\frac{d(100\sin 200}{dt}\right.$$

$$i = C\frac{dV}{dt}$$

$$= 177 \times 10^{-9} \times 100\ \cos 200\pi t(200\pi)$$

$$i_d = 0.011 \cos 200\pi t \cdot A.$$

for parallel plate capacitor, $i_c = i_D$.

$$J_D = \frac{i_D}{A} = 1.12 \cos 200\pi t\ \ A/m^2$$

(2)

Sol:- $\overline{D} = 10x\,\overrightarrow{a_x} - 4y\,\overline{a_y} + kz\,\overline{a_z}$ $\mu C/m^2$

$\overline{B} = 2\,\overline{a_y}$ m.wb/m². flux - wb

$B = \frac{\psi}{S} = wb/m^2$

$\sigma = 0,\ \rho_v = 0.$

$\nabla \cdot \overline{D} = \rho_v = 0.$

$$\nabla \cdot \overline{D} = \left[\frac{\partial\, 10x}{\partial x} + \frac{\partial(-4y)}{\partial y} + \frac{\partial(kz)}{\partial z}\right] 10^{-6}$$

$$= (10 - 4 + k)\,10^{-6}$$

$$\nabla \cdot \overline{D} = (k+6)\,10^{-6}.$$

$$\nabla \cdot \overline{B} = 0 \Rightarrow \frac{\partial}{\partial y}(2)\ \text{m.wb/m}^2$$

$$= 0.$$

$$(k+6)\,10^{-6} = 0$$

$\therefore k = -6.$

27/2/15

Boundary conditions

1) Dielectric & Dielectric

2) conducting & Dielectric

3) conducting & free space.

By using boundary conditions we can determine field on one side of boundary if the field on other side is known.

1) For static fields :- we use two maxwell equations

1) $\oint_l \vec{E}\cdot dl = 0$

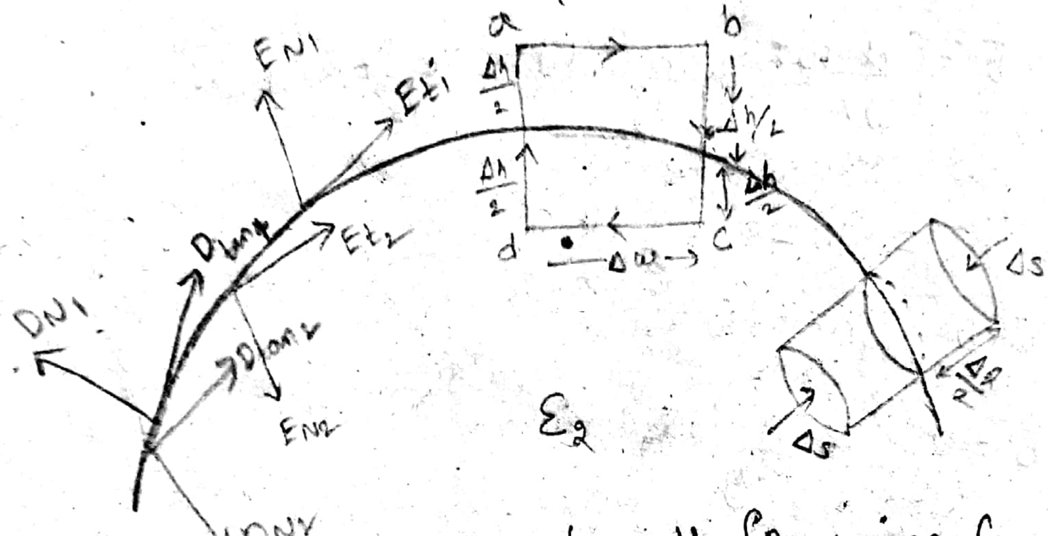
Dielectric – Dielectric

$\epsilon_1 = \epsilon_0 \epsilon_{r1}$

$\epsilon_2 = \epsilon_0 \epsilon_{r2}$

2) $\oint_s \bar{D}\cdot ds = 0.$

$E = E_t + E_n.$ ε_1



1) we take one closed path for using $\oint_l \bar{E}\cdot dl = 0$

$$\int_a^b E\cdot dl + \int_b^c \bar{E}\cdot dl + \int_c^d \bar{E}\cdot dl + \int_d^a \bar{E}\cdot dl = 0$$

$$E_{t1}\Delta w + \cancel{E_{N1}\frac{\Delta h}{2}} + \cancel{E_{N2}\frac{\Delta h}{2}} - \Delta w . E_{t2} - \cancel{E_{N2}\frac{\Delta h}{2}} - \cancel{E_{N1}\frac{\Delta h}{2}} = 0.$$

$$E_{t1}\Delta w - E_{t2}\Delta w = 0$$

$$\boxed{E_{t1} = E_{t2}}$$

The tangential components of field intensity at the boundary in both the dielectrics remain same, i.e electric field intensity continous across boundary.

E_t does not undergo any change

2) The tangential components of $\bar{D}$ undergoes some changes across boundary. Hence $\bar{D}$ is said to be discontinous across boundary.

$$\frac{\bar{D}t_1}{\epsilon_1} = \frac{\bar{D}t_2}{\epsilon_2}$$

$D = \epsilon E = D_t + D_n$

D_t – discontinous.

<u>normal component</u>:-

$$\int_{top} \bar{D}\cdot ds + \int_{bottom} \bar{D}\cdot ds + \int_{\substack{lateral\\side}} \bar{D}\, ds = Q_{enclosed}$$

$$\boxed{D_{N_1} - D_{N_2} = \rho_s}$$

For perfect dielectrics, $\rho_s = 0$. (freespace) there is no free charge.

$$D_{N_1} - D_{N_2} = 0.$$

$$\boxed{D_{N_1} = D_{N_2}}$$

electric flux densities are continous for normal components at the boundary

$$\epsilon_1 E_{N_1} = \epsilon_2 E_{N_2}$$

$$\boxed{\frac{E_{N_1}}{E_{N_2}} = \frac{\epsilon_2}{\epsilon_1}}$$

Electric field intensities are discontinous for normal components and inversly proportional at boundary

② Conductors & Dielectrics :- To determine the boundary conditions

Electric field inside the perfect conductor is always '0'. same procedure of before conditions

and $\sigma = \infty$.

$J = \sigma \vec{E}$

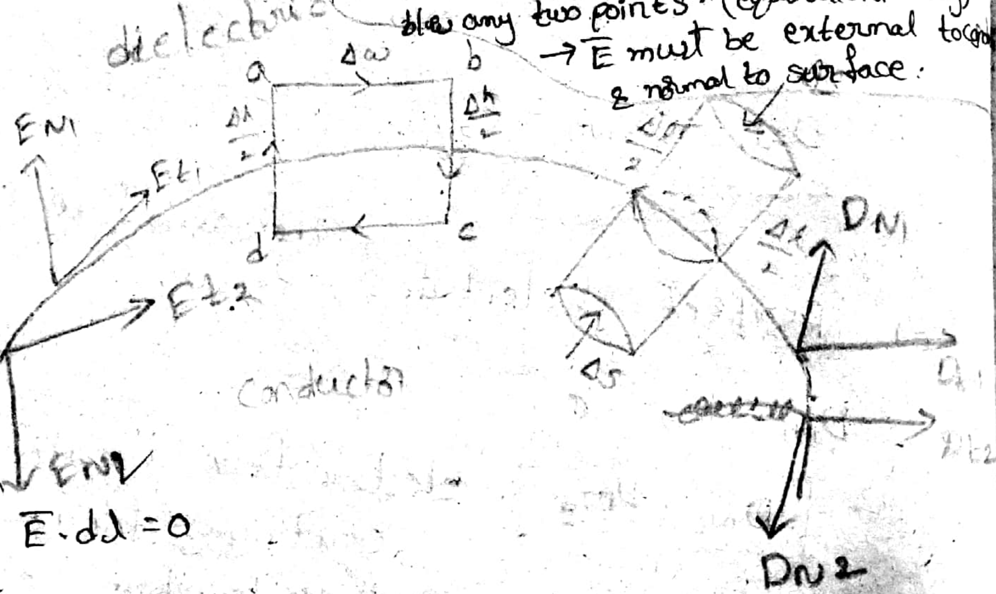
$\therefore \bar{E} = \frac{\bar{J}}{\sigma} = \frac{J}{\infty} = 0$

last conclusions

→ No electric field may exist within a conductor. $\rho_v = 0$.

→ $E = -\nabla V = 0$, there can be no potential diff b/w any two points. (equivalent body).

→ $\bar{E}$ must be external to conductor & normal to surface.



$\rightarrow \oint_L \bar{E} \cdot dl = 0$

$$\int_a^b \bar{E} \cdot dl + \int_b^c \bar{E} \cdot dl + \int_c^d \bar{E} \cdot dl + \int_d^a \bar{E} \cdot dl = 0$$

$$Et_1 \cdot \Delta w + E_{N_1} \frac{\Delta h}{2} + 0 + 0 + 0 - E_{N_1} \frac{\Delta h}{2} = 0$$

If $\Delta h \to 0$ for boundary conditions

$Et_1 \, \Delta w = 0$

$\boxed{\therefore E_{t_1} = 0}$ then $\boxed{D_{t_1} = 0}$

same in after also

★ An important application of the fact that $E = 0$ inside a conductor is in electrostatic screening or shielding. If cond 'A' keep at zero potential surrounds conductor B, then B is said to be screened by A. (last)

$$\int_{top} \bar{D}.ds + \int_{Bottom} \bar{D}.ds + \int_{later} \bar{D}.ds = Q_{enclosed}$$

$\Delta l = 0$ to get boundary conditions.

$$D_N = \frac{\Delta Q}{\Delta S} = \rho_s$$

$$D_{N_1} \cdot \cancel{\Delta S} = \rho_s \cancel{\Delta S} \Rightarrow$$

$$\boxed{D_{N_1} = \rho_s.} \Rightarrow E_{N1} \cdot \epsilon = \rho_s \Rightarrow \boxed{E_{N_1} = \frac{\rho_s}{\epsilon}}$$

(3) Conductor & free space (~~fo~~ perfect dielectrics).

For getting boundary conditions put $\epsilon_r = 1$. here

for free space $\epsilon = \epsilon_0 \epsilon_r [\epsilon_r = 1]$.

$\epsilon = \epsilon_0$.

$E_{t1} = 0$. tangentical component is zero.

$$\frac{D_{t1}}{\epsilon_0} = 0 \Rightarrow D_{t1} = 0.$$

$$\boxed{D_{N1} = \rho_s} \Rightarrow \boxed{\bar{E}_N = \frac{\rho_s}{\epsilon_0}}$$

28/2/15

Law of Refraction (or) snell's law :-

1) Dielectric & Dielectric

$$E_{t_1} = E_{t_2}$$

$$\sin\theta_1 = \frac{E_{t1}}{E_1} \Rightarrow E_{t_1} = E_1 \sin\theta_1$$

$$\sin\theta_2 = \frac{E_{t2}}{E_2} \Rightarrow E_{t2} = E_2 \sin\theta_2.$$

$\rightarrow D_{N2} = D_{N1}.$

$$\cos\theta_1 = \frac{D_{N1}}{D_1} \Rightarrow D_{N1} = D_1\cos\theta_1$$

$$\cos\theta_2 = \frac{D_{N2}}{D_2} \Rightarrow D_{N2} = D_2\cos\theta_2.$$

Equate these equations

$$E_1\sin\theta_1 = E_2\sin\theta_2 \quad \text{—— (1)}$$

$$D_1\cos\theta_1 = D_2\cos\theta_2 \quad \text{—— (2)}$$

divide these equations

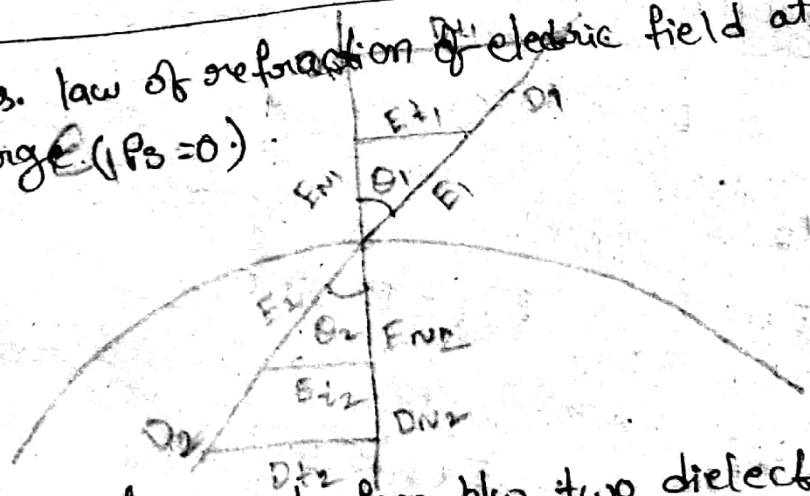
$$\frac{E_1}{D_1}\tan\theta_1 = \frac{E_2}{D_2}\tan\theta_2$$

$$\frac{\tan\theta_1}{\epsilon_1} = \frac{\tan\theta_2}{\epsilon_2}$$

$[\epsilon = \epsilon_0\epsilon_r]$

$$\boxed{\therefore \frac{\tan\theta_1}{\tan\theta_2} = \frac{\epsilon_1}{\epsilon_2} = \frac{\epsilon_{r1}}{\epsilon_{r2}}}$$

This is law of refraction of electric field at a boundary free of charge. ($\rho_s = 0$.)



Thus in general, an interface b/w two dielectrics produces bending of flux lines as a result of unequal polarization charges that accumulate on opposite sides problems

① If $\sigma = 0$, $\epsilon = 2.5\epsilon_0$, $\mu = 10\mu_0$, determine whether following pairs of fields satisfy maxwell equations or not.

(a) $\vec{E} = 2y\,\vec{a_y}$ V/m, $\vec{H} = 5x\,\vec{a_x}$ A/m

(b) $\vec{E} = 100 \sin 6\times10^{7}t \cdot \sin z \cdot \vec{a_y}$ V/m

$\vec{H} = -0.1328 \cos 6\times10^{7}t \cdot \cos z\, \vec{a_x}$ A/m.

② In freespace $\bar{D} = D_m \sin(\omega t + \beta z)\,\vec{a_x}$.

Find $\bar{B}$ by using maxwell equations.

③ Find conduction & displacement current densities in a material having conductivity $\sigma = 10^{3}$ ℧/m

$\epsilon_r = 2.5$, $\vec{E} = 5.8\times10^{-6}\sin(9\times10^{9}t)$ V/m.

~~$\epsilon_r = 60$~~

④ Find frequency at which conduction current density & displacement have same magnitude

(i) In distilled water, for which $\epsilon_r = 60$ and $\sigma = 5\times10^{-4}$ ℧/m.

(ii) In sea water, for which $\epsilon_r = 1$ and $\sigma = 3$ ℧/m problems.

(1) $\sigma = 0, \quad \epsilon = 2.5\epsilon_0, \quad \mu = 10\mu_0$

sol:- Maxwell's equations

$\sigma = 0$, so $J = 0$, $\rho_v = 0$.

$\nabla \cdot \bar{D} = \rho_v$

$\nabla \cdot \bar{D} = 0$.

(i) $\nabla \cdot \epsilon \bar{E} = 0 \Rightarrow \epsilon(\nabla \cdot \bar{E}) = 0$

$\nabla \cdot \bar{E} = 0$

$\frac{\partial}{\partial y}(2y) = 0$

$2 = 0$

It does not satisfy Maxwell equation.

$\nabla \times \bar{H} = \bar{J}$

$\nabla \times \bar{H} = 0$

$$\begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 5x & 0 & 0 \end{vmatrix} = 0$$

$0 = 0$

'$\bar{H}$' equation ~~does not~~ satisfies the maxwell equation.

(ii) $\vec{E} = 100 \sin 6 \times 10^7 t \cdot \sin z \; \bar{a_y}$ v/m.

$\nabla \times \vec{E} = -\frac{\partial B}{\partial t}$

$$\nabla \times \vec{E} = \begin{vmatrix} x & y & z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 100\sin 6\times10^7 t \, \sin z & 0 \end{vmatrix} = \bar{x}(0 - 100\sin 6\times10^7 t \cdot \cos z) - \bar{y}(0) + \bar{z}(0)$$

$$\nabla \times \bar{E} = -100 \sin 6 \times 10^7 t \cdot \cos z \cdot \vec{a_x}$$

It does not satisfy maxwell equation.

$$\frac{\partial}{\partial t} \mu H = \mu \frac{\partial \bar{H}}{\partial t} = \frac{\mu \partial(-0.1328 \cos 6 \times 10^7 t \cdot \cos z \cdot \vec{a_x})}{\partial t}$$

$$= \mu \left[-0.1328 \times 6 \times 10^7 \sin 6 \times 10^7 t \cos z \cdot \vec{a_x} \right]$$

$$= -10 \times 4\pi \times 10^{-7} \times 0.1328 \times 6 \times 10^7 \sin 6 \times 10^7 t \cos z \cdot \vec{a_x}$$

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(2) In free space $\bar{D} = D_m \sin(\omega t + \beta z) \vec{a_x}$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

Sol:-

$$\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$$

$$\nabla \times \frac{\bar{D}}{\epsilon} = \frac{-\partial \bar{B}}{\partial t} \Rightarrow \nabla \times \bar{D} = -\epsilon \frac{\partial \bar{B}}{\partial t}$$

$$\nabla \times \bar{D} = \begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ D_m \sin(\omega t + \beta z) & 0 & 0 \end{vmatrix} = \bar{a_x}(0) - \bar{a_y}(0 - D_m \cos(\omega t + \beta z) \cdot \beta) + \bar{a_z}(0)$$

$$= \beta D_m \cos(\omega t + \beta z) \vec{a_y}.$$

$$\frac{\partial \bar{B}}{\partial t} = -\frac{\beta}{\epsilon} D_m \cos(\omega t + \beta z) \vec{a_y}$$

Integrate on both sides w.r.t "t"

$$\therefore \bar{B} = \frac{+\beta}{\epsilon} D_m \omega \sin(\omega t + \beta z) \vec{a_y}$$

(8)

Given $\sigma = 10^3$ ℧/m

$\epsilon_r = 2.5$

$\vec{E} = 5.8 \times 10^6 \sin(9 \times 10^9 t)$ V/m.

Conduction current density $J_c = \sigma E$.

$J_c = 10^3 \times 5.8 \times 10^6 \sin(9 \times 10^9 t)$ A.

~~convection~~

$J_c = 5.8 \times 10^9 \sin(9 \times 10^9 t)$ A.

convection current density $J_d = \frac{\partial D}{\partial t}$

$J_d = \frac{\partial}{\partial t}(\epsilon E) = \frac{\partial}{\partial t}(8.85 \times 10^{-12} \times 2.5 \times 5.8 \times 10^6 \sin(9 \times 10^9 t))$

$J_d = 128.3 \times 10^{-6} \times 9 \times 10^9 \cos(9 \times 10^9 t)$

$J_d = 1154.88 \times 10^3 \cos(9 \times 10^9 t)$ A.

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(9)

(i) Given $\sigma = 5 \times 10^{-4}$ ℧/m

$\epsilon_r = 60$

$\epsilon = \epsilon_r \epsilon_0 = 531 \times 10^{-12}$ F/m

$\left|\frac{J_c}{J_d}\right| = \frac{\sigma}{\omega \epsilon} = 1$ When $J_c = J_d$.

$\omega = \frac{\sigma}{\epsilon} = \frac{5 \times 10^{-4}}{531 \times 10^{-12}} = 9.4 \times 10^{-3} \times 10^8$

$= 9.4 \times 10^5$

$\omega = 2\pi f$

$\therefore f = \dfrac{9.4 \times 10^{5}}{2\pi} = 1.49 \times 10^{5}$ Hz.

(ii) $\varepsilon_r = 1$

$\sigma = 3 \mho/m$

$\varepsilon = \varepsilon_r \cdot \varepsilon_0 = 1 \times 8.85 \times 10^{-12}$ F/m.

$\omega = \dfrac{\sigma}{\varepsilon} = \dfrac{3}{8.85 \times 10^{-12}} = 0.338 \times 10^{12}$

$f = \dfrac{0.338 \times 10^{12}}{2\pi}$

$f = 0.053 \times 10^{12} = 53 \times 10^{9}$ Hz.

$\therefore f = 53$ ~~KHz~~ Tera Hz.

4/3/15

EM wave characterstics

* The existance of EM waves predicted by Maxwell's equa was first investigated by Heinrich Hertz.
ex :- Radio waves, TV signals, radar beams, light rays.
There are four types of mediums for propagat EM waves.

(1) Free space ($\sigma = 0$, $\epsilon = \epsilon_0$, $\mu = \mu_0$)

(2) Conducting medium ($\sigma = \infty$, $\epsilon = \epsilon_0$, $\mu = \mu_r \mu_0$). $\sigma >> \omega\epsilon$

(3) perfect (lossless) dielectrics ($\sigma = 0$, $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$) ($\sigma << \omega\epsilon$)

(4) Lossy dielectrics ($\sigma \neq 0$, $\epsilon = \epsilon_r \epsilon_0$, $\mu = \mu_r \mu_0$).

By using Maxwell's equations

Equation for EM wave in good conductors

$$\nabla \times \vec{E} = \frac{-\partial \bar{B}}{\partial t}$$

take curl on both sides

$$\nabla \times (\nabla \times \bar{E}) = \nabla \times \frac{-\partial \bar{B}}{\partial t}$$

$$\nabla(\nabla \cdot \bar{E}) - \nabla^2 \vec{E} = \frac{-\partial}{\partial t}(\nabla \times \bar{B}).$$

But $\nabla \cdot \bar{D} = 0$ in conductors

$\rho_v = 0$

$\nabla \cdot \epsilon \bar{E} = 0 \Rightarrow \nabla \cdot \bar{E} = 0.$

$$\nabla^2 \bar{E} = \frac{\partial}{\partial t}(\nabla \times \bar{B}) = \mu \frac{\partial}{\partial t}(\nabla \times \bar{H}).$$

$$= \mu \frac{\partial}{\partial t}\left(\sigma \vec{E} + \epsilon \frac{\partial \bar{E}}{\partial t}\right)$$

$$= \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}.$$

$$\boxed{\nabla^2 \bar{E} - \mu\sigma \frac{\partial \bar{E}}{\partial t} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0} \quad \text{—— (1)}$$

This is equation for EM wave in electric field.

for harmonic time varying fields

$$\bar{E} = E_0 \cdot e^{j\omega t}$$

$$\frac{\partial \bar{E}}{\partial t} = j\omega \vec{E}, \quad \frac{\partial^2 \bar{E}}{\partial t^2} = j\omega \cdot j\omega \vec{E} = -\omega^2 \vec{E}.$$

$$\nabla^2 \vec{E} + \mu\epsilon\omega^2 \vec{E} - \mu\sigma j\omega \vec{E} = 0.$$

$$\nabla^2 \bar{E} - j\omega\mu(\sigma + j\omega\epsilon) \cdot \vec{E} = 0.$$

$$\boxed{\nabla^2 \vec{E} - \gamma^2 \vec{E} = 0.} \quad \text{—— (2).}$$

where 'γ' is propagation constant.

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon).$$

np/m

and $\gamma = \alpha + j\beta$.

dB/m, neperts/m

α - attenuation constant

β - phase constant.
(radians/m)

$\Longrightarrow$ In magnetic field

$$\nabla \times \vec{H} = J_c + \frac{\partial \bar{D}}{\partial t}.$$

take curl on both sides.

$$\nabla \times (\nabla \times \bar{H}) = \nabla \times J_c + \nabla \times \frac{\partial \bar{D}}{\partial t} \quad [J_c = \sigma \bar{E}]$$

$$\nabla(\nabla \cdot \bar{H}) - \nabla^2 \bar{H} = \sigma(\nabla \times \bar{E}) + \frac{\partial}{\partial t}(\nabla \times \bar{D}).$$

$\therefore \nabla \cdot \bar{B} = 0$. then $\nabla \cdot \bar{H} = 0$.

$$-\nabla^2 \vec{H} = \sigma\left(\frac{-\partial \bar{B}}{\partial t}\right) + \epsilon \frac{\partial}{\partial t}\left(\frac{-\partial \bar{B}}{\partial t}\right)$$

$$\nabla^2 \vec{H} = \mu\sigma \frac{\partial \bar{H}}{\partial t} + \epsilon\mu \frac{\partial^2 \vec{H}}{\partial t^2}.$$

$$\nabla^2 \vec{H} - \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} - \mu\sigma \frac{\partial \vec{H}}{\partial t} = 0.$$

$$\nabla^2 \vec{H} - \mu\epsilon \overset{(-\omega^2)}{\vec{H}} - \mu\sigma \overset{(j\omega)}{\vec{H}} = 0. \Rightarrow \nabla^2 \vec{H} + \mu\epsilon\omega^2 \vec{H} - j\omega\mu\sigma \vec{H} = 0$$

For harmonic time varying fields

$$\boxed{\nabla^2 \vec{H} - \gamma^2 \vec{H} = 0.}$$

where $\gamma^2 = \alpha + j\beta = j\omega\mu(\sigma + j\omega\epsilon)$.

wave equation for perfect dielectrics:-

$(\sigma = 0,\ \epsilon = \epsilon_r \epsilon_0,\ \mu = \mu_r \mu_0)$.

$$\Rightarrow \nabla \times \vec{E} = -\frac{\partial \bar{B}}{\partial t} \quad \text{(In electric field)}$$

take curl on both sides

$$\nabla \times (\nabla \times \bar{E}) = \nabla \times \left(\frac{-\partial \bar{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \bar{B}).$$

$J = \sigma \bar{E} = 0$

$$\nabla \cdot (\nabla \cdot E) - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}(\nabla \times \bar{B})$$

$\rho_v = 0,\ J = 0$

$\nabla \cdot \bar{D} = 0,\ \nabla \cdot \bar{E} = 0$

$$\cancel{-}\nabla^2 \bar{E} = \cancel{-}\frac{\partial}{\partial t}(\nabla \times \bar{B})$$

$\sigma \bar{E} = 0$

$$\nabla^2 \bar{E} = \mu \frac{\partial}{\partial t}(\underbrace{\nabla \times \bar{H}}_{= \frac{\partial \bar{D}}{\partial t}})$$

$$\nabla^2 \bar{E} = \mu \frac{\partial}{\partial t}\left(+\frac{\partial}{\partial t} D\right)$$

$$\nabla^2 \bar{E} - \mu \epsilon \frac{\partial^2 \bar{E}}{\partial t} = 0.$$

$$\boxed{\nabla^2 \vec{E} - \mu \epsilon \frac{\partial^2 \vec{E}}{\partial t} = 0}$$

This is EM wave equation for perfect dielectrics.

$\bar{E} = E_0 e^{j\omega t}$

$\frac{\partial \bar{E}}{\partial t} = E_0 j\omega e^{j\omega t}$

$\frac{\partial \bar{E}}{\partial t} = \bar{E} j\omega$

$\frac{\partial^2 \bar{E}}{\partial t^2} = j^2\omega^2 \bar{E}$

for harmonic fields

$$\vec{E} = E_0 e^{j\omega t}.$$

$$\nabla^2 \bar{E} - \mu\epsilon(-\omega^2 \vec{E}) = 0.$$

where $\gamma^2 = \omega^2 \mu\epsilon$

$$\nabla^2 \bar{E} + \omega^2 \mu\epsilon \vec{E} = 0.$$

$\gamma = \alpha + j\beta.$

$$\boxed{\nabla^2 \bar{E} + \gamma^2 \vec{E} = 0} \Rightarrow \nabla^2 \bar{E} - \gamma^2 \bar{E} = 0$$

where $\gamma^2 = j^2\omega^2 \mu\epsilon$

$\gamma = \pm j\omega\sqrt{\mu\epsilon}$

$\Rightarrow$ From $\nabla \times \vec{H} = J_c + \frac{\partial \bar{D}}{\partial t}.$

take curl on both sides

$$\nabla \times (\nabla \times \vec{H}) = \nabla \times \left(J_c + \frac{\partial \bar{D}}{\partial t}\right).$$

$$\nabla(\nabla \cdot \vec{H}) - \nabla^2 \vec{H} = \sigma(\nabla \times E) + \frac{\partial}{\partial t}(\nabla \times D). \quad [\bar{E}\epsilon = \bar{D}]$$

we know $\nabla \cdot \bar{B} = 0$ then $\nabla \cdot \bar{H} = 0.$

$$-\nabla^2 \vec{H} = \sigma\left(\frac{-\partial \bar{B}}{\partial t}\right) + \epsilon \frac{\partial}{\partial t}\left(\frac{-\partial \bar{B}}{\partial t}\right)$$

here $\sigma = 0$

$$-\nabla^2 \vec{H} = +\epsilon \frac{\partial}{\partial t}\left(\frac{-\partial \bar{B}}{\partial t}\right)$$

$$+\nabla^2 \vec{H} = -\epsilon \frac{\partial^2 \bar{B}}{\partial t} = +\epsilon\mu \frac{\partial^2 \bar{H}}{\partial t^2}$$

$\vec{H} = H_0 e^{j\omega t}$

$\frac{\partial \bar{H}}{\partial t} = H_0 j\omega e^{j\omega t}$

$\frac{\partial \bar{H}}{\partial t} = j\omega \bar{H}$

$\frac{\partial^2 \bar{H}}{\partial t^2} = -\omega^2 \bar{H}$

$$\nabla^2 \bar{H} - \epsilon\mu(-\omega^2 \bar{H}) = 0.$$

$\nabla^2 \bar{H} - \gamma^2 \bar{H} = 0$

where $\gamma^2 = j^2\omega^2 \epsilon\mu \Rightarrow \gamma = \pm j\omega\sqrt{\epsilon\mu}$

$$\nabla^2 \bar{H} + \epsilon\mu\omega^2 \bar{H} = 0.$$

$$\boxed{\nabla^2 \vec{H} + \gamma^2 \bar{H} = 0}$$

where $\gamma^2 = \epsilon\mu\omega^2.$

$\gamma = \alpha + j\beta.$

7/3/15

uniform plane waves:-

Em wave is propagated in the direction which is perpendicular to both $\overline{E}$ and $\overline{H}$.

In uniform plane waves, $\vec{E}$ and $\vec{H}$ both are perpendicular to each other and have same magnitude.

TE - Transverse electric field

TM - Transverse magnetic field

consider EMW $\longrightarrow z$

$\vec{E} \longrightarrow x$

$\vec{H} \longrightarrow y$.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

For harmonic fields $\vec{H} = H_0 \cdot e^{j\omega t}$

$$\frac{\partial \vec{H}}{\partial t} = j\omega \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = j\omega \vec{H}$$

$$\nabla \times \vec{E} = -j\omega\mu \vec{H}$$

$$\nabla \times \vec{E} = \begin{vmatrix} a_x & a_y & a_z \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix}$$

$a_x\left[0 + \frac{\partial}{\partial z}E_y\right] - a_y\left[0 - E_x\right]$

$= +a_x \frac{\partial}{\partial z} E_y - a_y \frac{\partial}{\partial z} E_x$

$$\frac{\partial}{\partial z}\left[+a_x E_y - a_y E_x\right] = -j\omega\mu \vec{H}$$

$$+\frac{\partial}{\partial z}E_y a_x - \frac{\partial}{\partial z}E_x a_y = -j\omega\mu[H_x a_x + H_y a_y]$$

$$\frac{\partial}{\partial z}E_y = -j\omega\mu H_x \quad \text{--- (1)} \Rightarrow$$

$$+\frac{\partial}{\partial z}E_x = j\omega\mu H_y \quad \text{--- (2)} \quad \frac{\partial E_x}{\partial z} = -\mu\frac{\partial B_y}{\partial t}$$

From $\nabla \times \vec{H} = (\sigma + j\omega\epsilon)\vec{E} = \sigma\vec{E} + j\omega\epsilon\vec{E}$

harmonic form (conductors)

$$+\frac{\partial}{\partial z}H_y a_x - \frac{\partial}{\partial z}H_x a_y = (\sigma + j\omega\epsilon)(E_x a_x + E_y a_y)$$

$$+\frac{\partial}{\partial z}H_y = (\sigma + j\omega\epsilon)E_x \quad \text{--- (3)}$$

$$-\frac{\partial}{\partial z}H_x = -(\sigma + j\omega\epsilon)E_y \quad \text{--- (4)}$$

from eq (3)

$$E_x = \frac{+1}{(\sigma + j\omega\epsilon)}\frac{\partial}{\partial z}H_y$$

Substitute in equation (2)

$$+\frac{\partial}{\partial z}\left(\frac{+1}{\sigma + j\omega\epsilon}\frac{\partial}{\partial z}H_y\right) = j\omega\mu H_y$$

$$\frac{\partial^2}{\partial z^2}H_y = j\omega\mu(\sigma + j\omega\epsilon)\cdot H_y$$

$$\frac{\partial^2}{\partial z^2}H_y = \gamma^2 H_y \Rightarrow \boxed{\frac{\partial^2}{\partial z^2}H_y - \gamma^2 H_y = 0}$$

from equation (2)

$$H_y = \frac{+1}{j\omega\mu}\frac{\partial}{\partial z}E_x \quad \text{put in eq (3)}$$

$$+\frac{\partial}{\partial z}\left(\frac{+1}{j\omega\mu}\frac{\partial}{\partial z}E_x\right) = (\sigma + j\omega\epsilon)E_x$$

$$\frac{\partial^2}{\partial z^2} E_x = j\omega\mu(\sigma + j\omega\epsilon) E_x$$

$$\boxed{\frac{\partial^2}{\partial z^2} E_x - \gamma^2 E_x = 0.}$$

from equation 1

$$H_x = \frac{-1}{j\omega\mu} \frac{\partial}{\partial z} \bar{E}_y$$

substitute in equation 4

$$\frac{\partial}{\partial z}\left(\frac{+1}{j\omega\mu}\frac{\partial}{\partial z} E_y\right) = +(\sigma + j\omega\epsilon) E_y.$$

$$\frac{\partial^2}{\partial z^2} E_y = j\omega\mu(\sigma + j\omega\epsilon) E_y.$$

$$\frac{\partial^2}{\partial z^2} \bar{E}_y = \gamma^2 E_y. \Rightarrow \boxed{\frac{\partial^2}{\partial z^2}\bar{E}_y - \gamma^2 \bar{E}_y = 0}$$

from equation 4

$$E_y = \frac{\partial}{\partial z} H_x \left[\frac{-1}{\sigma + j\omega\epsilon}\right]$$

substitute in eq^n 1

$$\frac{-\partial}{\partial z}\left[\frac{-\partial}{\partial z} H_x\left(\frac{1}{\sigma + j\omega\epsilon}\right)\right] = j\omega\mu H_x.$$

$$\frac{\partial^2}{\partial z^2} H_x = \gamma^2 H_x$$

$$\boxed{\frac{\partial^2}{\partial z^2} H_x - \gamma^2 H_x = 0}$$

$$\frac{\partial^2}{\partial z^2} E_x - \gamma^2 E_x = 0.$$

Let $\frac{\partial}{\partial z} = D.$

$$D^2 E_x - \gamma^2 E_x = 0 \Rightarrow D^2 - \gamma^2 = 0.$$

$$D = \gamma$$

$$E_x(z) = E_{x_1} e^{-\gamma z} + E_{x_2} e^{-\gamma(-z)}$$

$$E_{x_1} = E_{x_2} = E_0 \cdot e^{j\omega t}.$$

$$\boxed{\therefore E_x(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \bar{a}_x + E_0 e^{\alpha z} \cos(\omega t + \beta z) \bar{a}_x}$$

$$E_y(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \cdot \bar{a}_y + E_0 e^{\alpha z} \cos(\omega t + \beta z) \bar{a}_y$$

$$H_x(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \bar{a}_x + H_0 e^{\alpha z} \cos(\omega t + \beta z) \bar{a}_x.$$

$$H_y(z,t) = H_0 e^{-\alpha z} \cos(\omega t - \beta z) \bar{a}_y + H_0 e^{\alpha z} \cos(\omega t + \beta z) \bar{a}_y$$

12/3 Relations b/w $\vec{E}$ and $\vec{H}$:-

$$\Rightarrow \frac{\partial}{\partial z} E_y = -j\omega\mu H_x \quad eq(1)$$

Let $E_y(z) = E_1 e^{-\gamma z}$

$$\frac{\partial E_y}{\partial z} = E_1 e^{-\gamma z}(-\gamma)$$

put in equation(1)

$$+E_y \gamma = +j\omega\mu H_x$$

$$\frac{E_y}{H_x} = \frac{j\omega\mu}{\gamma}$$

$$\frac{+E_y}{H_x} = \frac{j\omega\mu}{\sqrt{j\omega\mu(\sigma+j\omega\epsilon)}} = \sqrt{\frac{j\omega\mu}{(\sigma+j\omega\epsilon)}} = \eta$$

$$\boxed{\therefore \frac{E_y}{H_x} = \eta}$$

$$\Rightarrow -\frac{\partial}{\partial z} E_x = j\omega\mu H_y \quad eq(2)$$

$$E_x = E_1 e^{-\gamma z}$$

$$-\frac{\partial}{\partial z} E_x = E_x \cdot (-\gamma)$$

$$-E_x \cdot (-\gamma) = j\omega\mu H_y$$

$$\frac{E_x}{H_y} = \sqrt{\frac{j\omega\mu}{(\sigma+j\omega\epsilon)}} = \eta$$

$$\boxed{\therefore \frac{E_x}{H_y} = \eta}$$

$$\boxed{\eta = \frac{E}{H} = \sqrt{\frac{\mu}{\epsilon}}}$$

Derivation for α & β :-

$$\gamma^2 = j\omega\mu(\sigma + j\omega\epsilon).$$

$$\gamma = \alpha + j\beta.$$

$$(\alpha + j\beta)^2 = j\omega\mu(\sigma + j\omega\epsilon) = j\omega\mu\sigma - \omega^2\mu\epsilon.$$

$$\alpha^2 - \beta^2 + 2j\beta\alpha = j\omega\mu\sigma - \omega^2\mu\epsilon.$$

$$\rightarrow \alpha^2 - \beta^2 = -\omega^2\mu\epsilon$$

$$\rightarrow 2\beta\alpha = j\omega\mu\sigma.$$

$$\beta = \frac{j\omega\mu\sigma}{2\alpha}.$$

wave velocity

$$\beta = \frac{2\pi}{\lambda}$$

$$v = \frac{\lambda}{t} = \lambda f$$

$$v = \frac{\omega}{\beta}.$$

$$\alpha^2 - \left(\frac{\omega\mu\sigma}{2\alpha}\right)^2 = -\omega^2\mu\epsilon.$$

$$4\alpha^4 - \omega^2\mu^2\sigma^2 = -4\alpha^2\omega^2\mu\epsilon$$

$$\alpha^2 = \frac{-4\omega^2\mu\epsilon \pm \sqrt{16\omega^4\mu^2\epsilon^2 + 16\omega^2\mu^2\sigma^2}}{8}$$

$$= \frac{-4\omega^2\mu\epsilon \pm 4\omega^2\mu\epsilon\sqrt{1 + \frac{\sigma^2}{\epsilon^2\omega^2}}}{8}$$

$$= \frac{4\omega^2\mu\epsilon\left(-1 + \sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2}\right)}{8}$$

$$= \frac{\omega^2\mu\epsilon}{2}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right].$$

$$\alpha = \sqrt{\frac{\omega^2\mu\epsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right)^{1/2}$$

$$\therefore \beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\cdot\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}$$

17/3/15

wave propagation in good conductors :-

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{\left(1+\left(\frac{\sigma}{\omega\epsilon}\right)^2\right)}-1\right]^{1/2}$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{1/2}$$

$$\gamma^2 = j\omega\mu\,(\sigma+j\omega\epsilon)$$

$$\gamma = \sqrt{j\omega\mu\,(\sigma+j\omega\epsilon)}$$

$$\gamma = \sqrt{j\omega\mu\sigma\left(1+j\frac{\omega\epsilon}{\sigma}\right)}$$

$$\left|\frac{J_c}{J_d}\right| = \frac{\sigma}{\omega\epsilon} >> 1 \Rightarrow \frac{\omega\epsilon}{\sigma} << 1.$$

$$\gamma = \sqrt{j\omega\sigma\mu}$$

$$\alpha + j\beta = \sqrt{\omega\mu\sigma}\cdot\sqrt{j} = \sqrt{\mu\omega\sigma}\sqrt{e^{j\pi/2}}$$

$$= \sqrt{\mu\omega\sigma}\cdot e^{j\pi/4}$$

$$\alpha + j\beta = \sqrt{\omega\mu\sigma}\; e^{j\pi/4}$$

$$\alpha + j\beta = \sqrt{\omega\mu\sigma}\;\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right).$$

$$= \sqrt{\frac{\mu\omega\sigma}{2}} + j\sqrt{\frac{\mu\omega\sigma}{2}}$$

$$u = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\sigma\mu}}$$

$$\lambda = \frac{2\pi}{\beta}$$

$$\therefore \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f \mu\sigma}$$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}} = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}}\, 1\angle 45^\circ$$

Intrinsic impedence $\therefore \eta = \sqrt{\frac{\omega\mu}{\sigma}} \angle 45^\circ$, $\bar{E}$ leads $\bar{H}$ by 45°.

wave propagation in lossy dielectrics medium ($\sigma \neq 0$)

$\nabla \cdot E_s = 0$

$\nabla \cdot H_s = 0$

$\nabla \times E_s = -j\omega\mu H_s$

$\nabla \times H_s = (\sigma + j\omega\epsilon) E_s$

$\nabla \times \nabla \times \bar{E}_s = -j\omega\mu (\nabla \times H_s)$

$\nabla \cdot (\nabla \cdot E) - \nabla^2 E = \text{''}$

$\therefore \nabla^2 E_s - \gamma^2 E_s = 0.$

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}} \left[\left(1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right)^{1/2} - 1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}} \left(\cancel{1} + \frac{1}{2}\frac{\sigma^2}{\omega^2\epsilon^2} - \cancel{1}\right)^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}} \left[\frac{1}{2}\frac{\sigma^2}{\omega^2\epsilon^2}\right]^{1/2}$$

$$= \frac{1}{2}\sqrt{\frac{\mu\sigma^2}{\epsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}.$$

$$\beta = \omega\sqrt{\frac{\mu\epsilon}{2}} \left[\left(1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2\right)^{1/2} - 1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}} \left[1 + \frac{1}{2}\left(\frac{\sigma}{\omega\epsilon}\right)^2 + 1\right]^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{2}} \left[2 + \frac{1}{2}\left(\frac{\sigma}{\omega\epsilon}\right)^2\right]^{1/2}$$

$$= \omega\sqrt{\frac{\mu\epsilon}{\cancel{2}}}\cdot\cancel{\sqrt{2}}\left[1+\frac{1}{\cancel{2}4}\frac{\sigma^2}{\omega^2\epsilon^2}\right]^{1/2}$$

$$= \omega\sqrt{\mu\epsilon}\cdot\left(1+\frac{1}{8}\left(\frac{\sigma^2}{\omega^2\epsilon^2}\right)\right)$$ ← neglect this term

$$= \omega\sqrt{\mu\epsilon}.$$

Intrinsic impedence $\eta = \sqrt{\frac{j\omega\mu}{(\sigma+j\omega\epsilon)}}$.

$$= \sqrt{\frac{\cancel{j\omega}\mu}{j\omega\epsilon\left(1+\frac{\sigma}{j\omega\epsilon}\right)}} = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\cdot\epsilon_0}}.$$

$$\left[\frac{\sigma}{\omega\epsilon} << 1\right]$$

$$\therefore \eta = \sqrt{\frac{\mu_r\mu_0}{\epsilon_r\epsilon_0}}.$$

wave propagation in ~~Lossless~~ medium ($\sigma=0$). (or) free space.
(~~perfect dielectrics~~)

$$\alpha = \omega\sqrt{\frac{\mu\epsilon}{2}}(1-1)^{1/2} = 0. \quad [\sigma = 0].$$

$$\beta = \omega\sqrt{\mu\epsilon}. = \omega\sqrt{\mu_0\epsilon_0}.$$

$$\eta = \sqrt{\frac{j\omega\mu}{(\sigma+j\omega\epsilon)}} = \sqrt{\frac{\cancel{j\omega}\mu}{\cancel{j\omega}\epsilon}} = \sqrt{\frac{\mu}{\epsilon}} = 373\,\Omega.$$

$$\boxed{V = \frac{\omega}{\beta}}$$

$$\therefore \eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega = 120\pi \quad [\mu_r = \epsilon_r = 1].$$

wave velocity $v_p = \frac{1}{\sqrt{\mu_0\epsilon_0}} = (3\times10^8)$ m/sec. $\Rightarrow c$ (or) $v = \frac{\omega}{\beta}$ $\quad \lambda = \frac{2\pi}{\beta}$

$c = v = f\lambda$

$\lambda = vT$

$\Rightarrow \mu$

$\Rightarrow \epsilon$

$$\boxed{V = \frac{1}{\sqrt{\mu\epsilon}}}$$

11/3/15

problems:

① calculate attenuation constant and phase con for a uniform plane wave with frequency of 10 GHz. $\mu = \mu_0$, $\epsilon_r = 2.3$, $\sigma = 2.5 \times 10^{-4}$ ℧/m

② A uniform plane wave is travelling at a velocity of 2.5×10^5 m/sec having a wavelength of $\lambda = 0.25$ m in a non-magnetic good conducting medium. calculate the frequency of wave and conductivity of the medium.

③ calculate skindepth, intrinsic impedence, propagation constant, for a medium having $\sigma = 10^{-2}$ s/m, $\epsilon_r = 15$, $\mu_r = 1$ at frequency 60 Hz

④ calculate intrinsic impedence 'η', propagation constant, wave velocity for a conducting medium in which $\sigma = 5.8$ M s/m, $\mu_r = 1$, ϵ_r = at 60 Hz.

⑤ A lossy dielectric medium has $\mu_r = 1$, $\epsilon_r = 50$ $\sigma = 60$ s/m (seimen/meter) at frequency of 15.9 MHz, find α, β, v, η. If the uniform plane wave travelling through this medium.

polarization

wave polarization is defined as orientation of E-field vector $\vec{E}$ at a fixed point with time varying along the direction of propagation.

$$\vec{E} = E_1 \cos(\omega t - \beta z)\vec{a_x} + E_2 \cos(\omega t - \beta z)\vec{a_y}.$$

$\phi = 0$, $\phi = 90^\circ$, $\phi = -90^\circ$.

when $\phi = 0^\circ$ then it is linear polarization.

$$E = E_1 \cos(\omega t - \beta z)\vec{a_x} + E_2 \cos(\omega t - \beta z + \phi)\vec{a_y}$$

$\phi = 0^\circ$

Then $|\vec{E_1}| = |\vec{E_x}|$

$|E_2| = |E_y|$.

case(i)

when $\phi = 90^\circ$.

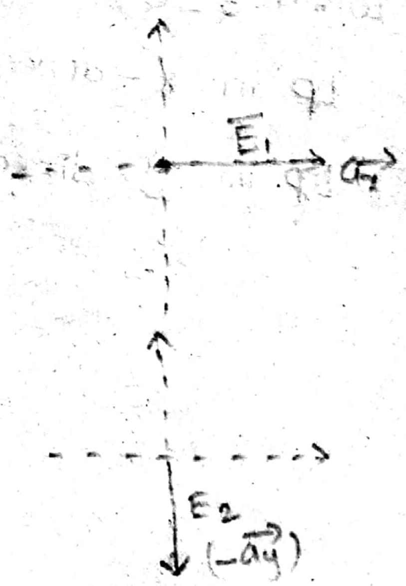
Assume $z = 0$, $\beta z = 0$.

At $\omega t = 0$, $\vec{E} = E_1(\vec{a_x})$

At $\omega t = 90^\circ$, $\vec{E} = E_2(-\vec{a_y})$.

At $\omega t = 180^\circ$, $\vec{E} = E_1(-\vec{a_x})$

At $\omega t = 270^\circ$, $\vec{E} = E_2(\vec{a_y})$



$|E_1| = |E_2|$ then

Left hand circular polarization

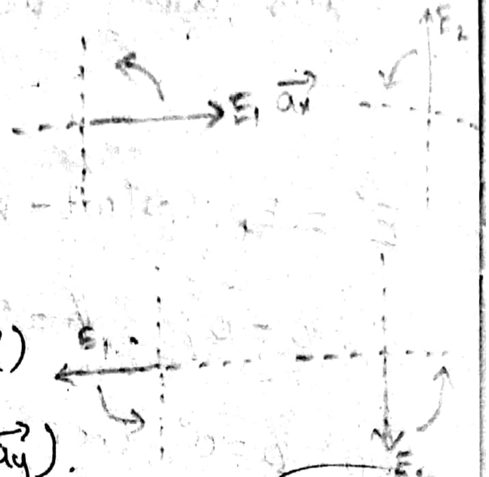
$|E_1| \neq |E_2|$ then LH EP.

case ii

when $\phi = -90^\circ$

$$E(z,t) = E_1 \cos(\omega t - \beta z)\,\vec{a_x} + E_2 \cos(\omega t - \beta z - 90^\circ)\,\vec{a_y}$$

at $z = 0$, $\beta z = 0$



$\omega t = 0^\circ$, $E(z,t) = E_1 \vec{a_x}$

$\omega t = 90^\circ$, $E(z,t) = E_2 \vec{a_y}$

$\omega t = 180^\circ$, $E(z,t) = E_1(-\vec{a_x})$

$\omega t = 270^\circ$, $E(z,t) = E_2(-\vec{a_y})$.

$|E_1| \neq |E_2| \Rightarrow$ RHEP

$|E_1| = |E_2| \Rightarrow$ RHCP.

case (iii)

$\phi = 0^\circ$.

when $z = 0$

Lp in x-direction

Lp in y-direction.

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right)$$

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- problems

(2)

sol:- The velocity of propagation is $v = f\lambda$

$$f = \frac{v}{\lambda} = \frac{2.5 \times 10^5}{0.25 \times 10^{-3}} = 1 \times 10^9 \text{ Hz} = 1 \text{ GHz}.$$

But $v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{\beta}$

$$\beta = \frac{2\pi \times 10^9}{2.5 \times 10^5} = 25.13 \times 10^3 \text{ rad/m}$$

For good conductor, phase constant is given by

$$\beta = \sqrt{\pi f \mu \sigma} = \sqrt{\pi f (\mu_0 \mu_r) \sigma}$$

But for a non-magnetic material, $\mu_r = 1$.

$$\beta = \sqrt{\pi \times 10^9 \times (4 \times \pi \times 10^{-7}) \times \sigma} = 25.13$$

$$\sqrt{39.47 \times 10^2 \times \sigma} = 25.13 \times 10^3$$

$$\sigma = \frac{(25.13)^2 \times 10^6}{39.47 \times 10^2} = 0.159 \times 10^6$$

$$\sigma = 1.6 \times 10^5 \text{ S/m}$$

(3)

$\sigma = 10^{-2}$ S/m

$\epsilon_r = 15$

$\mu_r = 1$

Frequency = 60 Hz.

If it is a conducting medium, $\frac{\sigma}{\omega\epsilon}$ is very high

$$= \frac{10^{-2}}{2\pi \times 60 \times 15 \times 8.85 \times 10^{-12}}$$

$$= 1.99 \times 10^{-5} \times 10^{10}$$

$= 2 \times 10^{5}$. The value is very high

(4)

sol:-

(a) propagation constant is given by.

$$\gamma = \sqrt{j\omega\mu\sigma} = \sqrt{\omega\mu\sigma}\ \angle 45^\circ$$

$$= \sqrt{2 \times \pi \times 60 \times 4\pi \times 10^{-7} \times 10^{-2}}\ \angle 45^\circ$$

$$= \sqrt{473.74 \times 10^{-8}}\ \angle 45^\circ$$

$$= 21.76 \times 10^{-4}\ \angle 45^\circ$$

$$= 2.176 \times 10^{-3}\ \angle 45^\circ$$

$$\gamma = \alpha + j\beta = 1.539 \times 10^{-3} + j\, 1.539 \times 10^{-3}$$

comparing real and imaginary terms.

$\alpha = 1.539 \times 10^{-3}$ NP/m

$\beta = 1.539 \times 10^{-3}$ rad/m.

(b) skin depth is given by $\delta = \frac{1}{\alpha} = \frac{1}{1.539 \times 10^{-3}}$

$= 649.75$ m

(c) Intrinsic impedence η is given by

$$\eta = \frac{1}{\sigma\delta} + j\frac{1}{\sigma\delta} = \frac{1}{649.75 \times 10^{-2}} + j\frac{1}{649.75 \times 10^{-2}}$$

$$\eta = 0.217\ \angle 45^\circ\ \Omega.$$

④

Sol:- For conducting medium $\sigma = 5.8$ ms/m. so using expressions of η, γ and v for good conductor.

The intrinsic impedence is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{j(2\pi f)\mu_0\mu_r}{\sigma}}$$

$$= \sqrt{\frac{2\pi \times 100 \times 10^6 \times 4\pi \times 10^{-7}}{5.8 \times 10^6}} \angle 90^\circ$$

$$= 3.68 \times 10^{-3} \angle 45^\circ \ \Omega$$

propagation constant

$$\gamma = \sqrt{j\omega\mu\sigma}$$

$$= \sqrt{2\pi f \mu_0 \mu_r \sigma} \angle 90^\circ = 2.13 \times 10^5 \angle 45^\circ$$

$$\gamma = \alpha + j\beta = 1.51 \times 10^5 + j\,1.51 \times 10^5$$

Then $\alpha = 1.51 \times 10^5$ np/m

$\beta = 1.51 \times 10^5$ rad/m.

$$\therefore v = \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 100 \times 10^6}{1.51 \times 10^5} = 4.15 \times 10^3 \text{ m/s}$$

⑤

Sol:- For Lossy dielectric medium, propagation constant

$$\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)}$$

$$\gamma = \sqrt{j(2\pi \times 15.9 \times 10^6)(4\pi \times 10^{-7}) + j(2\pi \times 15.9 \times 10^6)(8.854 \times 10^{-12} \times 50)}$$

$$\gamma = \sqrt{j(125.54)\cdot[60 + j\,0.044]}$$

$$= \sqrt{125.54 \angle 90^\circ (60 \angle 0.04^\circ)}$$

$$= 86.78 \angle 45.02^\circ$$

$$\gamma = \alpha + j\beta = 61.34 + j\,61.38.$$

velocity of propagation is given by

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon}}$$

$$= \sqrt{\frac{j(2\pi \times 15.9 \times 10^6)(4\pi \times 10^{-7})}{60 + j(2\pi \times 15.9 \times 10^6)(8.85 \times 10^{-12} \times 50)}}$$

$$= \sqrt{\frac{j(125.54)}{(60 + j\,0.044)}}$$

$$= \sqrt{\frac{125.54 \angle 90^\circ}{60 \angle 0.04^\circ}}$$

$$\therefore \eta = 1.44 \angle 44.98^\circ\ \Omega.$$

19/3/13

Depth of penatration :- (skin depth)

The distance at which the signal will reduces to 30% of it's max value or $\frac{1}{e}$, is called depth of penatration.

It is present in only in good conductors.

$$E(z,t) = E_1 e^{-\alpha z} \cos(\omega t - \beta z) + E_2 e^{\alpha z} \cos(\omega t - \beta z)$$

$$|E(z,t)| = e^{-1}$$

$$|\cancel{E_1} e^{-\alpha z}| = |\cancel{E_1} e^{-1}|$$

$$e^{-\alpha \delta} = e^{-1} \qquad [z = \delta)$$

$$\alpha \delta = 1$$

$$\delta = 1/\alpha$$

$$\delta = \frac{1}{\omega \sqrt{\frac{\mu \epsilon}{2}\left(\sqrt{1 + \left(\frac{\sigma}{\omega \epsilon}\right)^2} - 1\right)}^{1/2}}$$

for good conductors $\alpha = \sqrt{\frac{\omega \mu \sigma}{2}}$

$$\delta = \sqrt{\frac{2}{\omega \mu \sigma}} = \sqrt{\frac{1}{\pi f \mu \sigma}} \text{ m.}$$