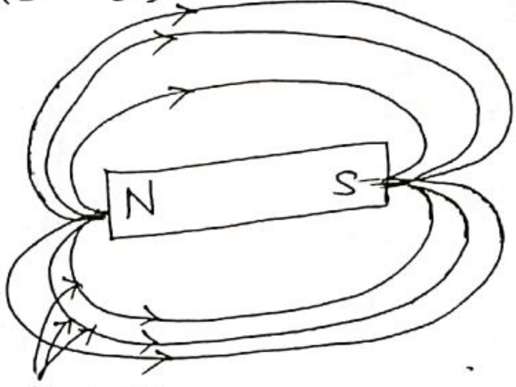
UNIT-2

Magneto Statics :- (PART-I)

Magnetic field :- It can be defined as the region around a magnet is called as "Magnetic field".



flux lines

Magnetic flux :- (ϕ)

The direction of magnetic lines of force is called as flux lines. The total no. of flux lines called as Magnetic flux. It is denoted by 'ϕ'. Units are Weber.

$$1 \text{ Weber} = 10^8 \text{ lines} = 100\text{M lines}.$$

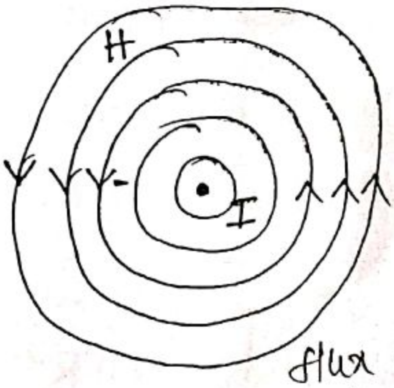
Magnetic field Intensity :- ($\bar{H}$) :

The magnetic field Intensity is defined as the force per unit north pole of 1 Weber strength. It is denoted by $\bar{H}$. Units are Newtons/Webers (or) Amp/m

$$\bar{H} = \frac{\text{Force}}{\text{Unit north pole}} \ N/Wb$$

(or)

$$\bar{H} = \frac{I}{l} \ \text{Amperes/meters}.$$



flux lines (ϕ)

Magnetic flux density :- ($\bar{B}$)

Magnetic flux density is defined as magnetic fi per unit surface area. It is denoted by $\bar{B}$.

$\boxed{\bar{B} = \frac{\phi}{S}}$ Wb/m² (or) $\boxed{\bar{B} = \frac{d\phi}{d\bar{s}}\,\bar{a}_n}$ Wb/m² (or) Tesla

The relation between $\bar{B}$ and $\bar{H}$ is given by

$\boxed{\bar{B} = \mu\bar{H}}$ $\mu = \mu_0\mu_r$

μ_r = relative permeability

μ_0 = free space permeability

$= 4\pi \times 10^{-7}$ (Henry/meter) H/m.

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Biot - savart's law :-

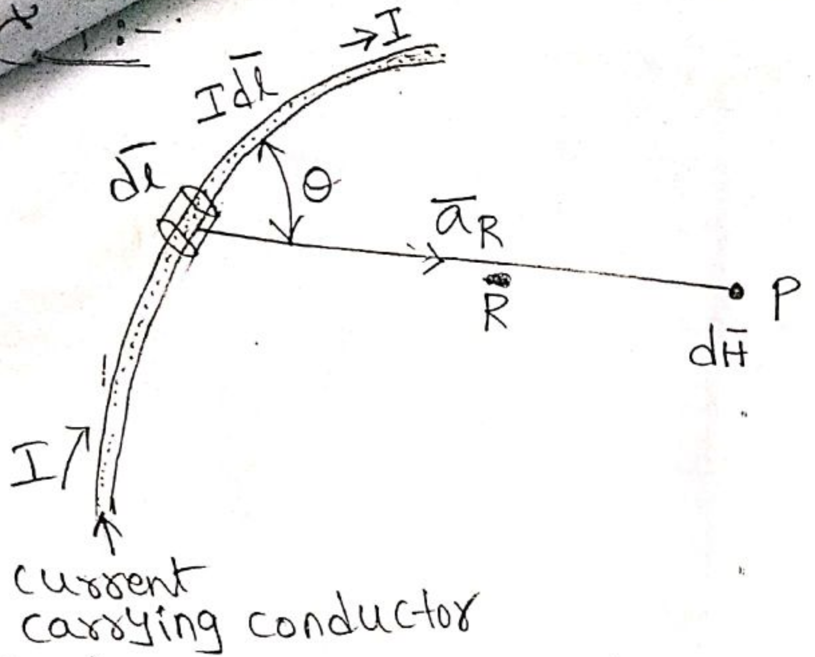
Biot-savart's law states that the differential magnetic field Intensity $d\bar{H}$ due to current element is

(i) directly proportional to the product of current I and differential conductor length dl

(ii) inversely proportional to square of the distance (R) between point 'p' and the element.

(iii) sine of the angle between current element and line joining from current element to point 'p

$$\boxed{d\bar{H} = \frac{I\,dl\,\sin\theta}{4\pi R^2}}$$ N/Wb (or) A/m.



I = current

$\bar{dl}$ = differential length

$I\bar{dl}$ = current element

$d\bar{H}$ = differential magnetic field Intensity (A/m)

According to Biot-savart's law

$$dH \propto Idl\sin\theta \rightarrow ①$$

$$dH \propto \frac{1}{R^2} \rightarrow ②.$$

from ① and ②

$$dH \propto \frac{Idl\sin\theta}{R^2}$$

$$\Rightarrow dH = K\frac{Idl\sin\theta}{R^2}$$

where K = constant $= \frac{1}{4\pi}$

$$\therefore dH = \frac{1}{4\pi}\frac{Idl\sin\theta}{R^2}$$

(ie) $\boxed{dH = \frac{Idl\sin\theta}{4\pi R^2}}$ N/Wb (or) A/m

where I = current carrying conductor

dl = differential length

Idl = current element

sinθ = angle

R = distance between point 'p' and element

In Vector form It can be expressed as

$$d\bar{H} = \frac{I\,|d\bar{l}|\sin\theta}{4\pi R^2} = \frac{I\,|d\bar{l}|\,|\bar{a}_R|\sin\theta}{4\pi R^2}$$

$(\because |\bar{a}_R| = 1$

$\bar{A}\times\bar{B} = |\bar{A}|\,|\bar{B}|\sin\theta)$

$$\Rightarrow \boxed{d\bar{H} = \frac{I\,d\bar{l}\times\bar{a}_R}{4\pi R^2}} \text{ N/Wb (or) A/m}$$

$$\boxed{\bar{H} = \oint \frac{I\,d\bar{l}\times\bar{a}_R}{4\pi R^2}} \text{ A/m.}$$

We know that $\bar{B} = \mu\bar{H}$

$$\Rightarrow \bar{B} = \mu\left[\oint \frac{I\,d\bar{l}\times\bar{a}_R}{4\pi R^2}\right]$$

$$\Rightarrow \boxed{\bar{B} = \frac{\mu}{4\pi}\oint \frac{I\,d\bar{l}\times\bar{a}_R}{R^2}} \text{ Wb/m}^2 \text{ (or) Tesla}$$

The Biot savart's law for different currents are given by

for line current $\bar{H} = \int_L \frac{I\,d\bar{l}\times\bar{a}_R}{4\pi R^2}$ A/m

for surface current $\bar{H} = \int_S \frac{\bar{K}\,ds\times\bar{a}_R}{4\pi R^2}$ A/m

for Volume Current $\bar{H} = \int_V \frac{\bar{J}\,dv\times\bar{a}_R}{4\pi R^2}$ A/m

ere's Circuit law (or) Ampere's work law :- (Maxwell's 3rd Eqn)

tatement :- The Ampere's circuit law states that the line integral of magnetic field intensity $\bar{H}$ around a closed path is equal to the net current enclosed by that path (Ienc)

It can be expressed as

$$\boxed{\oint_l \bar{H}\cdot d\bar{l} = I_{enc}} \rightarrow \text{Maxwell's Third Equation in Integral form.}$$

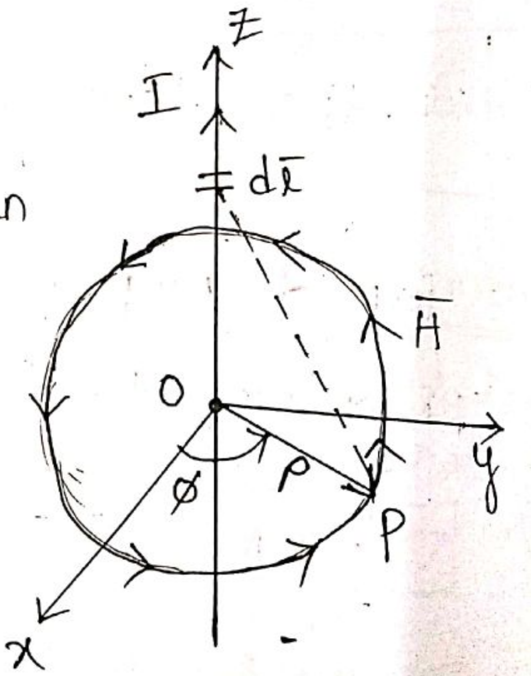
→ Ampere's circuit law applied to determine $\bar{H}$ when symmetrical current distribution exists.

Proof :- Let us consider a current carrying conductor 'I' along 'z' axis.

→ Take a closed circular path of radius 'ρ' called an Amperian path around the conductor, and a current element $Id\bar{l}$.

According to biot-savart's law

The $\bar{H}$ due to infinite length element of current is



current carrying conductor

$$\bar{H} = \frac{I}{2\pi\rho}\,\bar{a}_\phi \rightarrow ①$$

The $\bar{H}$ due to closed path is

$$\bar{H}\cdot d\bar{l} = \frac{I}{2\pi\rho}\,\bar{a}_\phi\cdot d\bar{l} \rightarrow ②$$

but $d\bar{l} = d\rho \bar{a}_\rho + \rho d\phi \bar{a}_\phi + dz \bar{a}_z$ ($\because$ cylindrical coordinate system)

$\therefore d\bar{l} = \rho d\phi \bar{a}_\phi$ ($\because$ Consider Closed circular path)

substitute $d\bar{l}$ in eq ②

$$\bar{H} \cdot d\bar{l} = \frac{I}{2\pi \cancel{\rho}} \bar{a}_\phi \cdot \cancel{\rho} d\phi \bar{a}_\phi \quad (\because \bar{a}_\phi \cdot \bar{a}_\phi = 1)$$

$$\therefore \bar{H} \cdot d\bar{l} = \frac{I}{2\pi} d\phi$$

Taking integral over closed path is

$$\oint_l \bar{H} \cdot d\bar{l} = \int_0^{2\pi} \frac{I}{2\pi} d\phi$$

$$= \frac{I}{2\pi} (\phi)_0^{2\pi}$$

$$= \frac{I}{\cancel{2\pi}} (\cancel{2\pi}) = I$$

$$\therefore \boxed{\oint_l \bar{H} \cdot d\bar{l} = I_{enc}}$$ Hence proved.

Point form of Ampere's circuit law :-

The curl of magnetic field intensity is equal to current density.

(ie) $\boxed{\nabla \times \bar{H} = \bar{J}}$

proof :- The integral form of Ampere's law is

$$\oint_l \bar{H} \cdot d\bar{l} = I_{enc} \rightarrow ①$$

Applying Stoke's theorem for left side of eq ①

$$\oint_l \bar{H} \cdot d\bar{l} = \int_s (\nabla \times \bar{H}) \cdot d\bar{s} \rightarrow ②$$

equations. ①,②

$$I_{enc} = \int_S (\nabla \times \overline{H}) \cdot d\overline{S} \rightarrow ③$$

But $$I_{enc} = \oint_S \overline{J} \cdot d\overline{S} \rightarrow ④$$

From equations ③,④

$$\int_S (\nabla \times \overline{H}) \cdot d\overline{S} = \oint_S \overline{J} \cdot d\overline{S}$$

$$\therefore \boxed{\nabla \times \overline{H} = \overline{J}}$$ Hence proved.

Magnetic flux density:- (Maxwell's 4th Equation)

The relation between $\overline{B}$ and $\overline{H}$ is

$$\boxed{\overline{B} = \mu \overline{H}} \rightarrow ①$$

Where $\mu = \mu_0 \mu_r$

μ_0 = permeability of free space

$= 4\pi \times 10^{-7}$ Henry/meter

μ_r = relative permeability

For free space $\mu_r = 1$

$\therefore \mu = \mu_0$

$$\therefore \boxed{\overline{B} = \mu_0 \overline{H}} \rightarrow ②$$

We know that

magnetic flux density $\overline{B} = \frac{d\phi}{d\overline{S}}$ Wb/m^2

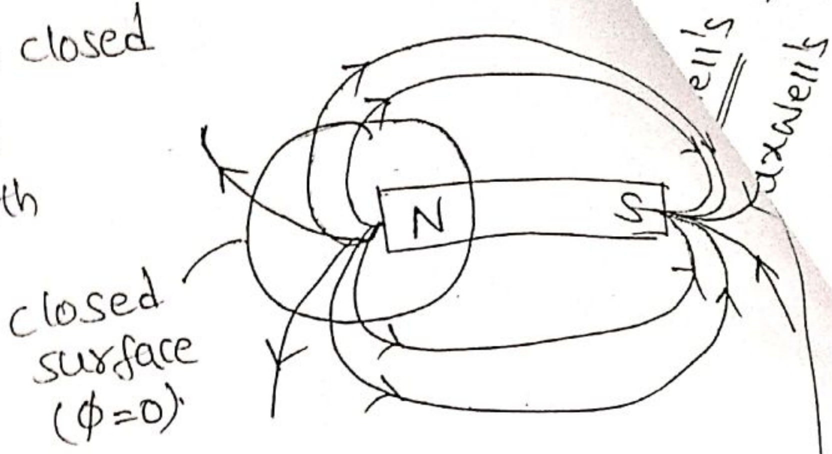
$$\Rightarrow d\phi = \overline{B} \cdot d\overline{S}$$

flux $$\boxed{\phi = \oint_S \overline{B} \cdot d\overline{S}} \rightarrow ③$$

Let us consider a closed surface.

* The total flux through closed surface should be '0'! because magnetism with monopole does not exists!

∴ $\phi = 0$.



from equation ③

$$\boxed{\phi = \oint_S \bar{B}\cdot d\bar{S} = 0} \rightarrow ④$$ maxwell's 4th equation in Integral form.

Point form (or) Differential form

Let us consider Integral form of magnetic flux density is

$$\oint_S \bar{B}\cdot d\bar{s} = 0$$

By Applying ~~stokes~~ divergence theorem on left side of above equation.

$$\therefore \text{We get } \oint_S \bar{B}\cdot d\bar{s} = \int_V (\nabla\cdot\bar{B})\cdot dV = 0$$

$$\Rightarrow \int_V (\nabla\cdot\bar{B}) dV = 0$$

$$\therefore \boxed{\nabla\cdot\bar{B} = 0}$$ → Law of Conservation of magnetic flux.

The divergence of magnetic flux density is zero.

well's two equations for magneto static fields:-

axwell's 3rd equation is $\nabla \times \bar{H} = \bar{J}$ → point form

$$\oint_{l} \bar{H} \cdot d\bar{l} = \int_{S} \bar{J} \cdot d\bar{s} \rightarrow \text{Integral form}$$

Maxwell's 4th Equation is

$$\nabla \cdot \bar{B} = 0 \rightarrow \text{point form}$$

$$\oint_{S} \bar{B} \cdot d\bar{s} = 0 \rightarrow \text{Integral form.}$$

Scalar and Vector magnetic potentials :-

Scalar magnetic potential :- (V_m)

We know that vector identities

$$\nabla \times \nabla V = 0 \rightarrow ①$$

$$\nabla \cdot (\nabla \times \bar{A}) = 0 \rightarrow ②$$

The scalar magnetic potential is defined as line integral of magnetic field intensity along a path where current density is zero.

(ie) $\boxed{\bar{J} = 0}$

($\because \nabla \times \bar{H} = \bar{J}$ if $\bar{J} = 0$ then $\nabla \times \bar{H} = 0$)

The relation is $\boxed{\bar{H} = -\nabla V_m}$ if $\bar{J} = 0$.

We know that

$$\nabla \times \bar{H} = \bar{J} \Rightarrow \nabla \times (-\nabla V_m) = \bar{J}$$

$$\therefore \bar{J} = 0$$

The scalar magnetic potential should satisfy laplace's equation. (ie) $\boxed{\nabla^2 V_m = 0}$ ($\because \bar{J} = 0$)

Vector magnetic potential :- ($\bar{A}$)

It can be defined as curl of the vector magnetic potential is equal to magnetic flux density.

It can be expressed as $\boxed{\bar{B} = \nabla \times \bar{A}}$

We can define

$$\boxed{\bar{A} = \int_L \frac{\mu_0 I \, d\bar{l}}{4\pi R}} \ \text{Wb/m} \quad [\text{for line current}]$$

$$\boxed{\bar{A} = \int_S \frac{\mu_0 \bar{K} \, ds}{4\pi R}} \ \text{Wb/m} \quad [\text{for surface current}]$$

$$\boxed{\bar{A} = \int_V \frac{\mu_0 \bar{J} \, dv}{4\pi R}} \ \text{Wb/m} \quad [\text{for volume current}]$$

Units of vector magnetic potential is Webers/meter.

Applications of Ampere's law :-

The ampere's circuit law is used to determine magnetic field Intensity $\bar{H}$ for

(i) Infinite long straight conductor
(ii) Infinite coaxial cable
(iii) Infinite sheet of currents.
(iv) Infinite solid conductor.

es due to magnetic fields :-

ere are 3 types of forces due to magnetic field.

) Force on a moving charge
(ii) Force on a differential element (or) current element
(iii) Force between two current elements.
(Ampere's Force law)

(i) Force on a moving charge :-

→ Let us consider the positive charge placed in a static electric field and static magnetic field.

The force exerted on a positive charge in electric field is given by

$$\bar{F}_e = Q\bar{E} \longrightarrow ①$$

→ If the charge is moving in a magnetic field with flux density $\bar{B}$ then magnetic force is given by

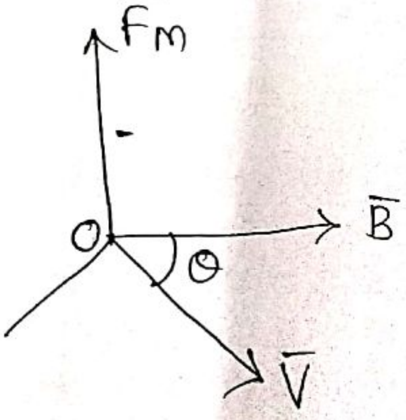
$$\bar{F}_M = Q(\bar{V} \times \bar{B}) \rightarrow ②$$

where $\bar{V}$ = Velocity, $\bar{B}$ = magnetic flux density.

The total force due to electric field and magnetic field is given by

$$\bar{F} = \bar{F}_e + \bar{F}_m$$

$$\Rightarrow \bar{F} = Q\bar{E} + Q(\bar{V} \times \bar{B})$$



*** $$\boxed{\bar{F} = Q(\bar{E} + \bar{V} \times \bar{B})}$$

↳ This Equation is Called as "Lorentz" Force Equation.

(ii) Force on a differential current element :

consider a differential charge dQ moving with a velocity $\bar{V}$ in a steady magnetic field of flux density $\bar{B}$.

The differential magnetic force is

$$d\bar{F} = dQ(\bar{V}\times\bar{B}) \text{ Newtons} \longrightarrow ①$$

If ρ_v is the volume charge density and $\bar{J}$ is the current density then.

$$\bar{J} = \frac{I}{A} = \frac{\frac{dQ}{dt}}{dy\,dz} \qquad \left(\because I = \frac{dQ}{dt}\right)$$

$$\Rightarrow \bar{J} = \frac{dQ}{dt}\times\frac{1}{dy\,dz} \rightarrow ② \qquad (\because A = \text{surface area} = dy\,dz)$$

But velocity is $\bar{V} = \frac{dx}{dt}$

$\therefore$ eq ② can be written as

$$\bar{J} = \left(\frac{dx}{dt}\right)dQ \times \frac{1}{(dx\,dy\,dz)} \qquad (\text{multiplying \& dividing by } dx)$$

$$\bar{J} = \bar{V}\frac{dQ}{dV}$$

$$\therefore \boxed{\bar{J} = \bar{V}\rho_V} \rightarrow ③ \qquad \left(\because \rho_V = \frac{dQ}{dV}\right)$$

We know that $dQ = \rho_V\,dV$

substitute dQ value in eq ①

$$d\bar{F} = \rho_V\,dv(\bar{V}\times\bar{B})$$

$$= \rho_V\,dV\,\bar{V}\times\bar{B}$$

$$\therefore \boxed{d\bar{F} = \bar{J}\,dV\times\bar{B}} \Rightarrow \boxed{\bar{F} = \int_V \bar{J}\,dV\times\bar{B}}$$

larly the force excerted on the surface rrent element is

$$\boxed{\bar{F} = \int_S \bar{K} ds \times \bar{B}}$$

and ! the force exerted on the line current element is

$$\boxed{\bar{F} = \int_l I d\bar{l} \times \bar{B}}$$

(iii) Force between two current elements :-

(OR)

Ampere's Force law :-

Ampere's force law states that the force between two current carrying elements $I_1 d\bar{l}_1$ and $I_2 d\bar{l}_2$ placed at a distance R_{12} is given by

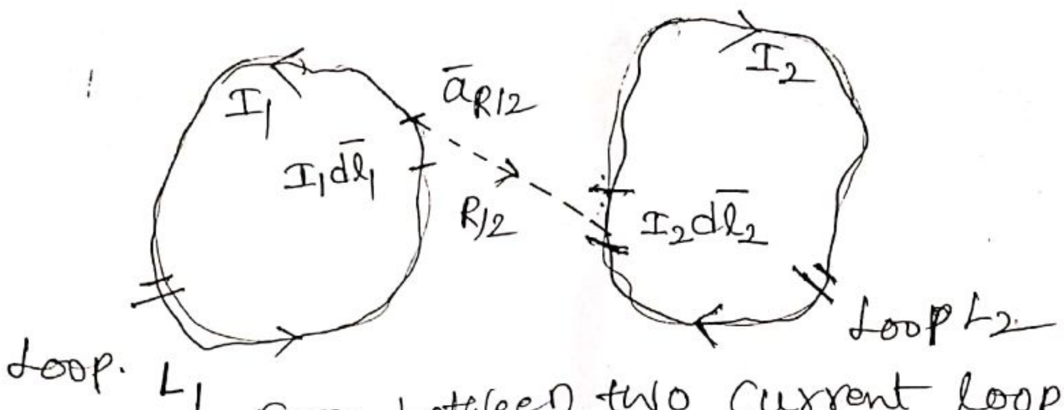
$$\bar{F} = \frac{\mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{I_2 d\bar{l}_2 \times (I_1 d\bar{l}_1 \times \bar{a}_{R_{12}})}{R_{12}^2}$$

Where $I_1 d\bar{l}_1$ is the current element of the first conductor

$I_2 d\bar{l}_2$ is the current element of the second conductor

R_{12} is the distance between the current elements of the two conductors.

Proof:- Let us consider two current elements $I_1 d\bar{l}_1$ and $I_2 d\bar{l}_2$. The current elements are pla at a distance R_{12} from each other.



Force between two current loops.

According to Biot-savart's law
The magnetic field intensity at $I_2 d\bar{l}_2$ due to the current element $I_1 d\bar{l}_1$ is

$$d\bar{H}_2 = \frac{I_1 d\bar{l}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2} \rightarrow ①$$

But we know that

$$\bar{B} = \mu\bar{H} \Rightarrow \bar{B}_2 = \mu H_2$$

$$\boxed{d\bar{B}_2 = \mu d\bar{H}_2}$$

$$\therefore d\bar{B}_2 = \frac{\mu I_1 d\bar{l}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2} \rightarrow ②$$

From force on differential current element

$$\bar{F} = \int_l I d\bar{l} \times \bar{B}$$

$$\bar{F}_2 = \int_l I_2 d\bar{l}_2 \times \bar{B}_2$$

$d\bar{F}_2 = I_2 d\bar{l}_2 \times \bar{B}_2$

Taking differential magnetic flux density

$d(d\bar{F}_2) = I_2 d\bar{l}_2 \times d\bar{B}_2 \rightarrow$ ③

substituting eq ② in eq ③ we get

$$d(d\bar{F}_2) = I_2 d\bar{l}_2 \times \frac{\mu I_1 d\bar{l}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2}$$ ~~$\mu I_2 d\bar{l}_2 \times$~~

$$\Rightarrow d(d\bar{f}_2) = \frac{\mu I_2 d\bar{l}_2 \times I_1 d\bar{l}_1 \times \bar{a}_{R12}}{4\pi R_{12}^2}$$

Taking double integration on closed paths.

∴ The total force $\bar{F}_2$ on current element 2 due to current element 1 is

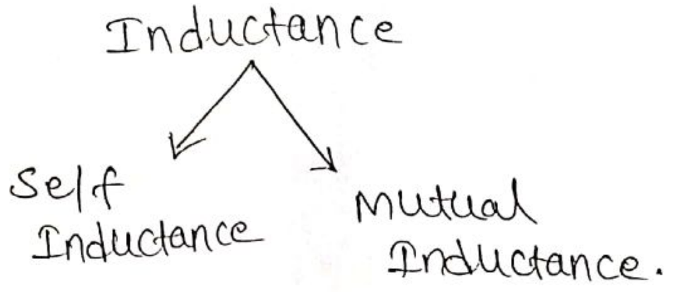
$$\boxed{\bar{F}_2 = \frac{\mu I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\bar{l}_2 \times d\bar{l}_1 \times \bar{a}_{R12}}{R_{12}^2}}$$

Hence proved

similarly the force $\bar{F}_1$ on current element 1 due to current element 2 is

$$\boxed{\bar{F}_1 = -\bar{F}_2}$$

Inductances :- The Inductance can be defined as the ratio of total flux linkage to the current flowing through the circuit. (or) coil.



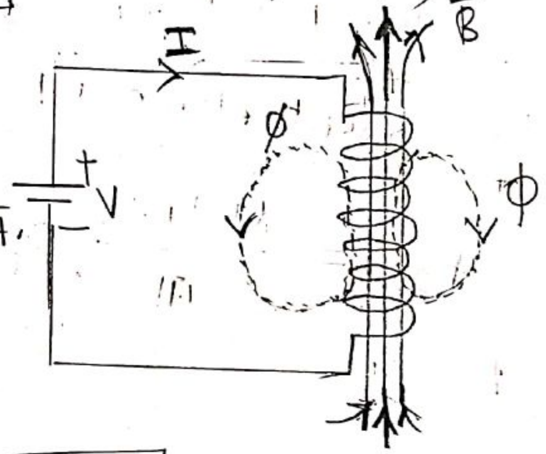
Self Inductance :- (L)

The self Inductance can be defined as the flux linkage produced by inductor itself.

It is denoted as $\boxed{L = \frac{N\phi}{I}}$ Henry $\left(\because \begin{matrix} N\phi \propto I \\ N\phi = LI \end{matrix} \right)$

Proof :-

Let us consider a closed circuit carrying current I, that produces magnetic field $\bar{H}$.



$\therefore \bar{B} = \mu \bar{H}$

$\therefore$ flux $\phi = \int_S \bar{B} \cdot d\bar{S}$

the flux linkage is $\boxed{\lambda = N\phi}$ Wb-Turn $\rightarrow$ ①

$\lambda \propto \phi \rightarrow$ ②

when the flux produces current I.

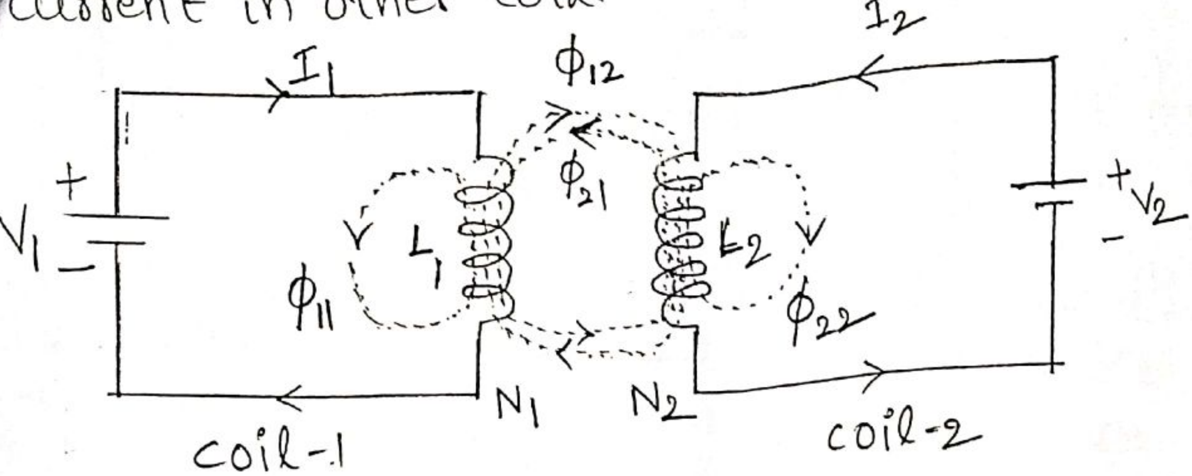
$\lambda \propto I \rightarrow$ ③

$\lambda = LI$, where $L =$ Inductance

$\Rightarrow L = \frac{\lambda}{I}$

$\therefore \boxed{L = \frac{N\phi}{I}}$ Henry (or) $\frac{\text{Wb-Turn}}{\text{Amp}}$.

tual Inductance :- (M) : The mutual Inductance between two coils is defined as the ratio of the total flux linkage in one coil to the current in other coil.



→ Let L_1 be the inductance of coil 1 of N_1 turns carrying current I_1 and producing a magnetic flux ϕ_{11}.

→ Let L_2 be the inductance of coil 2 of N_2 turns carrying current I_2 and producing a magnetic flux ϕ_{22}.

→ the flux ϕ_{12} be passing through coil 1 due to current I_2 of coil 2 and flux ϕ_{21} be passing through coil 2 due to current I_1 of coil 1

→ The total flux linkage of coil 1 due to flux produced by current I_2 of coil 2 is $N_1\phi_{12}$.

→ similarly the total flux linkage of coil 2 due to flux produced by current I_1 of coil 1 is $N_2\phi_{21}$

∴ The Mutual Inductance M_{12} is

$$\boxed{M_{12} = \frac{N_1\phi_{12}}{I_2} = \frac{\lambda_{12}}{I_2}} \text{ Henry}$$

The Mutual Inductance M_{21} is

$$\boxed{M_{21} = \frac{N_2 \phi_{21}}{I_1} = \frac{\lambda_{21}}{I_1}} \text{ Henry.}$$

Where $\phi_{12} = \int_{S_1} \bar{B}_2 \cdot d\bar{S}_2$, $\phi_{21} = \int_{S_2} \bar{B}_1 \cdot d\bar{S}_1$

Coefficient of coupling :- (K)

It can be defined as the ratio of total flux linkage between two coils to the flux produced by any one coil.

mathematically

$$K = \frac{\text{Total flux linkage between coil 1 and coil 2}}{\text{Flux produced by coil 1 (or) coil 2}}$$

$$\Rightarrow \boxed{K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}}$$

It can also be $K = \frac{M}{\sqrt{L_1 L_2}}$

M = mutual Inductance = $M_{12} = M_{21}$.

The range of K is $0 \leq K \leq 1$.

$$K = \begin{cases} 0 & \text{No coupling} \\ 1 & \text{Perfect coupling.} \end{cases}$$

gnetic energy :-

The energy stored in a magnetic field produced by an inductor is given by

$$\boxed{W_m = \frac{1}{2} L I^2} \text{ Joules}$$

Where L is the Inductance and I is the applied current

Proof :- If a current is applied on a coil having N Turns, it produces a magnetic field with flux ϕ the induced voltage across the coil is

$$V = -\frac{N\,d\phi}{dt} \rightarrow ①$$

Also

$$V = L\frac{dI}{dt} \rightarrow ②$$

(∵ Neglect -ve sign)

From ①, ②

$$-\frac{N\,d\phi}{dt} = L\frac{dI}{dt} \Rightarrow \boxed{N\phi = LI}$$

But the power is $P = VI = \left(\frac{N\,d\phi}{dt}\right) I$

$$\Rightarrow P = LI\frac{dI}{dt}$$

∴ The energy stored is

$$W_m = \int P\,dt = \int LI\frac{dI}{\cancel{dt}}\cdot\cancel{dt}$$

$$= \int LI\,dI \qquad \left(\because \int x\,dx = \frac{x^2}{2}\right)$$

$$= L\cdot\left(\frac{I^2}{2}\right)$$

$$\therefore \boxed{W_m = \frac{1}{2} L I^2 \text{ J}} \text{ (or) } \boxed{L = \frac{2W_m}{I^2} \text{ Henry}}$$

Energy density in the magnetic field :

Energy density is defined as energy per unit Volume.

The energy stored in the magnetic field of an inductor is given by

$$W_m = \frac{1}{2} L I^2 \rightarrow ①$$

Consider the case of a solenoid whose self Inductance is

$$L = \frac{\mu N^2 S}{l}.$$

Substituting 'L' value in eq ①

$$W_m = \frac{1}{2}\left(\frac{\mu N^2 S}{l}\right) I^2 = \frac{1}{2} \mu \times \frac{N^2 I^2}{l^2} \times lS$$

$$= \frac{1}{2} \mu \left(\frac{NI}{l}\right)^2 \times lS$$

But $lS = \text{Volume} = V$, $H = \frac{NI}{l}$.

$$\therefore W_m = \frac{1}{2} \mu H^2 V \rightarrow ②$$

The energy density is

$$\boxed{w_m = \frac{W_m}{V} = \frac{1}{2} \mu H^2 = \frac{1}{2} \bar{B} \cdot \bar{H}} \text{ J/m}^3$$

if dV is differential volume, the energy stored is $W_m = \int_V w_m \, dV$

$$\therefore \boxed{W_m = \frac{1}{2} \int_V \bar{B} \cdot \bar{H} \, dV \text{ Joules}}$$

MAXWELL'S EQUATIONS [TIME VARYING FIELDS]

1831 Michael Faraday discovered that a current is induced in a conducting loop when there is a change in the magnetic flux linkage.

Faraday's Law :- Faraday's law states that if the magnetic field is time varying then the induced emf is equal to the time rate of change of magnetic flux linkage by the circuit. (∵ emf = electro motive force)

It can be expressed as

Induced emf $\boxed{V = -N\frac{d\phi}{dt}}$ volts

Where N = no. of turns of circuit

ϕ = flux through each turn.

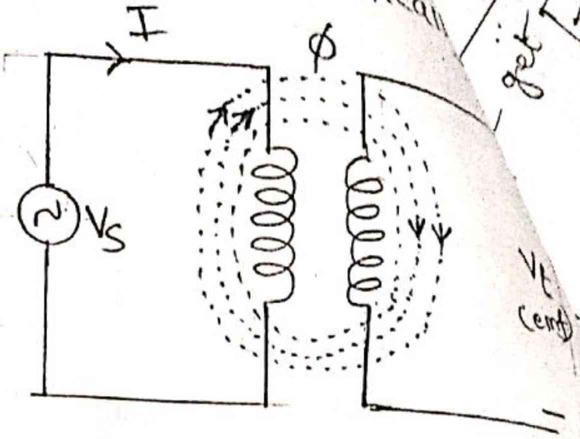
The negative sign (−) can be explained by Lenz's law.

Lenz's law :- It states that the direction of Induced emf is always equal to opposite of the flux linkages produced by current.

) Transformer emf :- Let us consider a transformer (or) stationary circuit.

Definition :- Transformer emf can be defined as when a time varying current is applied, it produces a time varying flux in the primary coil, that induces an emf in secondary coil.

This emf is called "Transformer emf" (or) statically Induced emf.



According to faraday's law

$$V = -N\frac{d\phi}{dt} \text{ volts} \rightarrow ①$$

for N = no. of Turns = 1

$$V_t = -\frac{d\phi}{dt} \rightarrow ②$$

We know that. flux is $\phi = \int_s \bar{B}\cdot d\bar{s}$ and

$$V_t = \oint_l \bar{E}\cdot d\bar{l} \rightarrow ③$$

from equations ② and ③

$$\boxed{V_t = \oint_l \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\left(\int_s \bar{B}\cdot d\bar{s}\right)} \rightarrow ④$$

This equation is Transformer emf equation.

Where 's' is surface area of circuit.

Applying stoke's theorem for eq ④

$$\oint_l \bar{E}\cdot d\bar{l} = \int_s (\nabla\times\bar{E})\cdot d\bar{s}$$

∴ from above equations

$$\int_s (\nabla\times\bar{E})\cdot d\bar{s} = -\frac{d}{dt}\left(\int_s \bar{B}\cdot d\bar{s}\right)$$

$$\Rightarrow \int_s (\nabla\times\bar{E})\cdot d\bar{s} = \int_s \left(-\frac{\partial\bar{B}}{\partial t}\right)\cdot d\bar{s}$$

By comparing the above equation,

(or)

Remove the surface integrals.

e get $\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$ *** → Maxwell's Eqn for Time Varying field

The curl of time Varying electric field is equal to time rate decrease of flux density.

③ Equation of Continuity for Time Varying fields :-

* this Equation of Continuity is same as continuity Equation for electro static fields.

We know that current $I = -\frac{dQ}{dt}$ → ①

also $I = \oint_S \bar{J} \cdot d\bar{S}$ → ②

from ①, ②

$$\oint_S \bar{J} \cdot d\bar{S} = -\frac{dQ}{dt}$$

But charge $Q = \int_V \rho_v dv$

$$\therefore \oint_S \bar{J} \cdot d\bar{S} = -\frac{d}{dt}\left[\int_V \rho_v dv\right] \rightarrow ③$$

Apply divergence theorem

$$\oint_S \bar{J} \cdot d\bar{S} = \int_V (\nabla \cdot \bar{J})\, dv \rightarrow ④$$

from ③, ④

$$\int_V (\nabla \cdot \bar{J})\, dv = \int_V \left(-\frac{\partial \rho_v}{\partial t}\right) dv$$

$$\therefore \boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}}$$ **

In-consistency of Ampere's Ckt law :- (OR)

Modified Ampere's Ckt law :-

From point form of Ampere's Ckt law

$\nabla \times \bar{H} = \bar{J} \rightarrow ①$

Take divergence of both sides

$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} \rightarrow ②$ $(\because \nabla \cdot (\nabla \times \bar{A}) = 0)$

$\therefore \nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \bar{J} = 0 \rightarrow ③$

divergence of curl of Vector is '0'

The continuity Equation is

$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \rightarrow ④$

if $\nabla \cdot \bar{J} = 0$ then it does not exist in the time varying field $\therefore$ it is In consistent.

The gauss law is

$\nabla \cdot \bar{D} = \rho_v$

from ④

$\Rightarrow \nabla \cdot \bar{J} = -\frac{\partial}{\partial t}(\nabla \cdot \bar{D}) = -\nabla \cdot \frac{\partial \bar{D}}{\partial t}$

$\Rightarrow \nabla \cdot \bar{J} + \nabla \cdot \frac{\partial \bar{D}}{\partial t} = 0$

$\Rightarrow \nabla \cdot \left[\bar{J} + \frac{\partial \bar{D}}{\partial t}\right] = 0 \longrightarrow ⑤$

from Equations ③ & ⑤

$\nabla \cdot (\nabla \times \bar{H}) = \nabla \cdot \left[\bar{J} + \frac{\partial \bar{D}}{\partial t}\right]$

compare above equation.

$$\therefore \boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}} \rightarrow$$ ****** Another Maxwell Eqn for Time Varying field

Where total current density is

$$\bar{J}_t = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \bar{J}_c + \bar{J}_d$$

$\bar{J}_c = \bar{J}$ = conduction current density (A/m²)

$\bar{J}_d = \frac{\partial \bar{D}}{\partial t}$ = Displacement current density (A/m²)

Integral form :-

using stoke's theorem

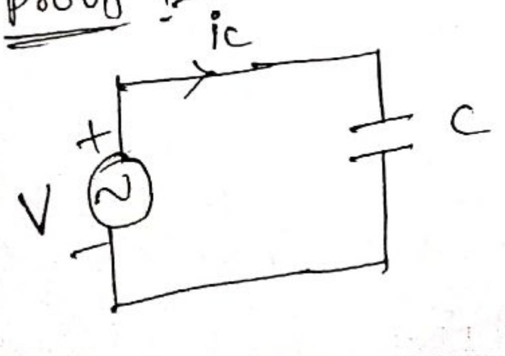
$$\oint_{l} \bar{H} \cdot d\bar{l} = \int_{S} \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right) \cdot d\bar{s} = I_{enc}$$

Displacement current Density $\left[\bar{J}_d = \frac{\partial \bar{D}}{\partial t}\right]$

Definition :- It can be defined as the ratio of displacement current to area of cross-section is called as "displacement current density".

$$\boxed{\bar{J}_d = \frac{i_d}{S} = \frac{\partial \bar{D}}{\partial t}} \text{ A/m}^2$$

proof :-



Let us consider a current is flowing through capacitor is id. ic = ckt current

∴ Conduction current density is

$$\bar{J} = \frac{i_c}{S} = \sigma \bar{E}$$

σ = conductivity = mho/m.

$\bar{E}$ = electric field strength (V/m)

S = surface area of cross section. (m^2)

The flowing current through capacitor is

$$i_d = C\frac{dV}{dt} \rightarrow ①$$

But capacitance for parallel plates is

$$C = \frac{\epsilon S}{d} \rightarrow ②$$

substitute eq ② in ①

$$\Rightarrow i_d = \left(\frac{\epsilon S}{d}\right)\left(\frac{dV}{dt}\right) \rightarrow ③$$

We know that $V = \bar{E} d$

$$\Rightarrow V = \left(\frac{\bar{D}}{\epsilon} \cdot d\right) \quad \left(\because \bar{D} = \epsilon \bar{E} \Rightarrow \bar{E} = \frac{\bar{D}}{\epsilon}\right)$$

$$\therefore i_d = \left(\frac{\epsilon S}{d}\right)\frac{\partial}{\partial t}\left(\frac{\bar{D} \cdot d}{\epsilon}\right)$$

$$\Rightarrow i_d = \frac{\cancel{\epsilon} S}{\cancel{d}} \times \frac{\cancel{d}}{\cancel{\epsilon}} \times \frac{\partial \bar{D}}{\partial t}$$

$$\therefore i_d = S \cdot \frac{\partial \bar{D}}{\partial t}$$

$$\boxed{\frac{i_d}{S} = \frac{\partial \bar{D}}{\partial t}}$$ This i_d current is called as Displacement Current.

$$\therefore \boxed{\bar{J_d} = \frac{\partial \bar{D}}{\partial t}} \quad **$$

Total current density $\bar{J_t} = \bar{J_c} + \bar{J_d}$ $(\because \bar{J} = \sigma \bar{E})$

$$= \sigma\bar{E} + \frac{\partial \bar{D}}{\partial t}$$

tio between $\bar{J_c}$ and $\bar{J_d}$

Let $\bar{E}$ be time Varying field

$$\bar{E} = E e^{j\omega t} \rightarrow ①$$

Taking partial differentiation

$$\frac{\partial \bar{E}}{\partial t} = E\left[e^{j\omega t} \cdot j\omega\right] = j\omega\left[E e^{j\omega t}\right]$$

∴ ~~$\frac{\partial \bar{E}}{\partial t} = j\omega$~~ ∴ $\frac{\partial \bar{E}}{\partial t} = j\omega \bar{E} \rightarrow ②$

Conduction current density is $\boxed{\bar{J_c} = \sigma \bar{E}}$

displacement current density is $\bar{J_d} = \frac{\partial \bar{D}}{\partial t}$

$$\Rightarrow \bar{J_d} = \frac{\partial}{\partial t}(\epsilon \bar{E}) = \epsilon \frac{\partial \bar{E}}{\partial t}.$$

$$\therefore \boxed{\bar{J_d} = j\omega\epsilon \bar{E}}$$

$$\therefore \frac{\bar{J_c}}{\bar{J_d}} = \frac{\sigma \cancel{\bar{E}}}{j\omega\epsilon \cancel{\bar{E}}} = \frac{\sigma}{j\omega\epsilon}$$

$$\left|\frac{\bar{J_c}}{\bar{J_d}}\right| = \frac{\sigma}{\sqrt{\omega^2\epsilon^2}} = \frac{\sigma}{\omega\epsilon}$$

$$\therefore \boxed{\left|\frac{\bar{J_c}}{\bar{J_d}}\right| = \frac{\sigma}{\omega\epsilon}}$$

Here $\omega = 2\pi f$
= Angular frequency

if $\frac{\sigma}{\omega\epsilon} >> 1$ the medium is perfect Conductor

if $\frac{\sigma}{\omega\epsilon} << 1$ the medium is perfect dielectric

if $\frac{\sigma}{\omega\epsilon} = 0$ the medium is free space

Maxwell's Equations in different final forms:

Ⓐ Maxwell's Equations for static electric & magnetic fields:-

Point form (or) differential form	Integral form
1. $\nabla \cdot \bar{D} = \rho_V$	$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_V \, dV = Q$ → Gauss law
2. $\nabla \times \bar{E} = 0$	$\oint_l \bar{E} \cdot d\bar{l} = 0$ → conservative of Electric field
3. $\nabla \times \bar{H} = \bar{J}$	$\oint_L \bar{H} \cdot d\bar{l} = \int_S \bar{J} \cdot d\bar{s} = I_{enc}$ Ampere's ckt law
4. $\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{s} = 0$ → Law of conservation of flux
Continuity Eqn $\nabla \cdot \bar{J} = \frac{-\partial \rho_V}{\partial t}$	$\oint_S \bar{J} \cdot d\bar{s} = -\int_V \frac{\partial \rho_V}{\partial t} \, dV$

Word statements

1. Divergence of Electric flux density is equal to volume charge density.
2. curl of Electric field is '0'.
3. The curl of Magnetic field Intensity $\bar{H}$ is equal to conduction current density $\bar{J}$
4. The divergence of a magnetic flux density is zero. '0'.

Maxwell's Equations for Time Varying fields:-

Point form	Integral form
1. $\nabla\cdot\bar{D} = \rho_v$	$\oint_s \bar{D}\cdot d\bar{s} = \int_v \rho_v \, dv = Q$ (Gauss law)
2. $\nabla\times\bar{E} = -\frac{\partial \bar{B}}{\partial t}$	$\oint_l \bar{E}\cdot d\bar{l} = -\int_s \frac{\partial \bar{B}}{\partial t}\cdot d\bar{s}$ [faradays law]
3. $\nabla\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$	$\oint_l \bar{H}\cdot d\bar{l} = \int_s \left(\bar{J} + \frac{\partial \bar{D}}{\partial t}\right)\cdot d\bar{s} = I$ → Modified Amp. Ckt law
4. $\nabla\cdot\bar{B} = 0$.	$\oint_s \bar{B}\cdot d\bar{s} = 0$ → conservation of magnetic flux
Continuity Equation is $\nabla\cdot\bar{J} = -\frac{\partial \rho_v}{\partial t}$	$\oint_s \bar{J}\cdot d\bar{s} = -\int_v \frac{\partial \rho_v}{\partial t} dV$

Word statements:-

1. The divergence of Electric flux density is equal to the volume charge density
2. The curl of Electric field Intensity $\bar{E}$ is equal to the time rate of decrease of magnetic flux density
3. The curl of magnetic field Intensity is equal to the sum of conduction current density $\bar{J}$, and displacement current density $\frac{\partial \bar{D}}{\partial t}$.
4. The divergence of magnetic flux density is equal to zero '0'.

Ⓒ Maxwell's Equations for free space :-

for free space volume charge density $\rho_v = 0$

conductivity $\sigma = 0$

Current density $\bar{J} = \sigma\bar{E} = 0\cdot\bar{E}$

$\Rightarrow \bar{J} = 0$.

Point form	Integral form
1. $\nabla\cdot\bar{D} = 0$.	$\oint_s \bar{D}\cdot d\bar{s} = 0$
2. $\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$	$\oint_l \bar{E}\cdot d\bar{l} = \int_s \left(\frac{-\partial\bar{B}}{\partial t}\right)\cdot d\bar{s}$
3. $\nabla\times\bar{H} = \left(\frac{\partial\bar{D}}{\partial t}\right)$ $(\because \bar{J}=0)$	$\oint_l \bar{H}\cdot d\bar{l} = \int_s \frac{\partial\bar{D}}{\partial t}\cdot d\bar{s}$
4. $\nabla\cdot\bar{B} = 0$.	$\oint_s \bar{B}\cdot d\bar{s} = 0$
Continuity Eqn $\nabla\cdot\bar{J} = 0$.	$\oint_s \bar{J}\cdot d\bar{s} = 0$

Ⓓ Maxwell's Equations for Good Conductors

$\sigma >> \omega\epsilon$, $\rho_v = 0$.

ρ_v = charge density

σ = Conductivity

ω = angular frequency

ϵ = permittivity.

Point form	Integral form
1. $\nabla\cdot\bar{D} = 0$	$\oint_s \bar{D}\cdot d\bar{s} = 0$
2. $\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$	$\oint_l \bar{E}\cdot d\bar{l} = -\int_s \left(\frac{\partial\bar{B}}{\partial t}\right)d\bar{s}$
3. $\nabla\times\bar{H} = \bar{J}$	$\oint_L \bar{H}\cdot d\bar{l} = \int_s \bar{J}\cdot d\bar{s} = I$
4. $\nabla\cdot\bar{B} = 0$	$\oint_s \bar{B}\cdot d\bar{s} = 0$
Continuity Eqn $\nabla\cdot\bar{J} = 0$	$\oint \bar{J}\cdot d\bar{s} = 0$

xwell's Equations for sinusoidal (or) Harmonic time Varying fields:- ⑯

Let us consider electric and magnetic fields Varying harmonically with time.

Let $\bar{D} = D e^{j\omega t}$ → electric flux density

$\bar{B} = B e^{j\omega t}$ → Magnetic flux density.

Where $\omega = 2\pi f$ = angular frequency (rad/sec)

Taking partial differentiation of above equations.

$$\frac{\partial \bar{D}}{\partial t} = \frac{\partial}{\partial t}(D e^{j\omega t}) = D.[e^{j\omega t}.j\omega]$$

$$\therefore \frac{\partial \bar{D}}{\partial t} = j\omega \times D e^{j\omega t} = j\omega \bar{D} \rightarrow ①$$

similarly $$\frac{\partial \bar{B}}{\partial t} = \frac{\partial}{\partial t}(B e^{j\omega t}) = B.[e^{j\omega t}.j\omega]$$

$$\therefore \frac{\partial \bar{B}}{\partial t} = j\omega \bar{B} \rightarrow ②$$

For conductors $\bar{J} = \sigma \bar{E}$, for dielectrics $\bar{D} = \epsilon \bar{E}$,

For magnetics $\bar{B} = \mu \bar{H}$

$$\therefore \boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} = -j\omega \bar{B} = -j\omega \mu \bar{H}} \rightarrow ③$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t} = \sigma \bar{E} + j\omega \bar{D}$$

$$\therefore \boxed{\nabla \times \bar{H} = \sigma \bar{E} + j\omega \epsilon \bar{E} = (\sigma + j\omega \epsilon)\bar{E}} \rightarrow ④$$

These equations are called as maxwell's equations for sinusoidal (or) time harmonic fields.

Point form	Integral form
1. $\nabla \cdot \bar{D} = \rho_V$	$\oint_S \bar{D} \cdot d\bar{S} = \int_V \rho_V dV$
2. $\nabla \times \bar{E} = -j\omega\mu\bar{H}$	$\oint_l \bar{E} \cdot d\bar{l} = -j\omega\mu \int_S \bar{H} \cdot d\bar{S}$
3. $\nabla \times \bar{H} = (\sigma + j\omega\epsilon)\bar{E}$	$\oint_l \bar{H} \cdot d\bar{l} = (\sigma + j\omega\epsilon) \int_S \bar{E} \cdot d\bar{S}$
4. $\nabla \cdot \bar{B} = 0$	$\oint_S \bar{B} \cdot d\bar{S} = 0.$
continuity Equation is $\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} = -\nabla \cdot (j\omega\bar{D})$	$\oint_S \bar{J} \cdot d\bar{S} = \int_V (-\nabla \cdot j\omega\bar{D}) dV$

Conditions at a boundary surface :-

The conditions that the field satisfy at the interface which can be seperated by two media are called as "Boundary-conditions". Boundary conditions determined for

(i) Dielectric- Dielectric Interface

(ii) Dielectric - conductor Interface.

) **Dielectric - Dielectric Interface :-** The maxwell's equations are useful for determining conditions at a boundary surface of two different media.

For E-field :-

1. The tangential component of electric field Intensity is continuous at the surface.

$$\boxed{E_{tan1} = E_{tan2}}$$

2. The ~~tangent~~ normal component of the electric flux density is continuous at the boundary if ρ_s is 0.

$$\boxed{D_{n1} = D_{n2}}$$

the surface charge density is non Zero, then e normal component of electric flux density is discontinuous.

$$\boxed{D_{n_1} - D_{n_2} = \rho_s}$$

For H-field :- (Magnetic field)

1. The tangential component of magnetic field Intensity is continuous across the surface.

$$\boxed{H_{tan_1} = H_{tan_2}}$$

2. The tangential component of magnetic field Intensity is discontinuous at the boundary.

$$\boxed{H_{tan_1} - H_{tan_2} = K}$$

3. The normal component of magnetic flux density is continuous.

$$\boxed{B_{n_1} = B_{n_2}}$$

i) Dielectric - conductor Interface :-

For E-field :- (electric field)

1. $E_{tan} = 0$

2. $D_n = \rho_s$

For H-field :- (Magnetic field)

1. $H_{tan_1} = K$

2. $H_{tan_2} = 0$

3. $B_n = 0$.

Generator emf :- [Motional emf].

consider a stationary magnetic-field in a conducting circuit which is moving (or) revolving on its axis with respect to time. When the circuit cuts across the magnetic field, the electro magnetic field induced across the terminals. This EMF is called as "Generator emf" (or) Motional emf.

→ If a circuit is moving with a uniform velocity $\bar{V}$ in a magnetic-field of magnetic flux density ($\bar{B}$) then from the Lorentz's force equation, the force on free charge is $\bar{F} = Q(\bar{V}\times\bar{B}) \rightarrow ①$

When force applied on a charge, the electric-field induced in a circuit is $\bar{E} = \frac{\bar{F}}{Q} = \bar{V}\times\bar{B}$ V/m. → ②

The Induced emf ~~is~~ $V = \oint_l \bar{E}\cdot d\bar{l} = \oint_l (\bar{V}\times\bar{B})\cdot d\bar{l} \rightarrow ③$

Applying stoke's Theorem on eq ③

$$\oint_l \bar{E}\cdot d\bar{l} = \int_S (\nabla\times\bar{E})d\bar{S} = \int_S (\nabla\times(\bar{V}\times\bar{B}))\,d\bar{S}$$

By eliminating surface Integrals, we get

$$\boxed{\nabla\times\bar{E} = \nabla\times(\bar{V}\times\bar{B})}$$

Moving circuit in a Time varying fields :-

The moving circuit in a Time varying fields represents both Transformer emf and Generator emf.

(ie) $$\boxed{\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t} + \nabla\times(\bar{V}\times\bar{B})}$$

→•← THE END →•←

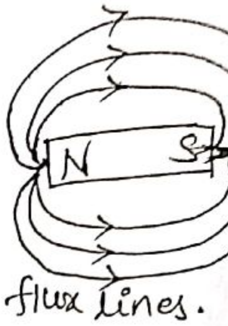
MWTL UNIT-2

II ECE SHORT ANSWER QUESTIONS

Define Magnetic Field?

The magnetic field can be defined as the region around a magnet is called as magnetic field.

Define magnetic flux? (ϕ)



flux lines.

The direction of magnetic flux lines of force is called magnetic flux.

It is denoted by 'ϕ'. Units are Webers.

$1 \text{ Weber} = 10^8 \text{ lines} = 100M\text{-lines}$.

What is Magnetic Field Intensity (H)?

The force per unit north pole of 1 Weber strength, is called as Magnetic field Intensity.

flux lines ϕ

$$\bar{H} = \frac{\text{Force}}{\text{unit North pole}} \quad N/Wb.$$

$$\boxed{H = \frac{I}{l}} \quad A/m.$$

What is Magnetic flux density?

The magnetic Flux per unit surface area is called as magnetic flux density ($\bar{B}$).

$$\boxed{B = \frac{\phi}{S}} \; Wb/m^2 \; (OR) \; \boxed{\bar{B} = \frac{d\phi}{dS}} \; \bar{a}_n \; Wb/m^2 \; (or) \text{ Tesla}$$

$$\boxed{\bar{B} = \mu\bar{H}}$$

state the Biot-savart's law?

Biot-savart's law states that differential magnetic field Intensity dH due to current element is

- Directly proportional to product of current I and differential conductor length $d\bar{l}$.

* proportional to sine of the angle between current and line Joining from current element to point p!

$$\boxed{dH = \frac{I\,dl \sin\theta}{4\pi R^2}}$$ N/wb (or) Amp/m

State Ampere's law (Ampere's Work law)

The Ampere's law states that the line Integral of magnetic field Intensity $\bar{H}$ around a closed path is equal to net current enclosed by that path.

$$\boxed{\oint_l \bar{H}\cdot\bar{dl} = I_{enc}} \rightarrow \text{Integral form.}$$

$$\boxed{\nabla \times \bar{H} = \bar{J}} \rightarrow \text{point form.}$$

Write the maxwell's 2 equations for magneto static fields?

Maxwell's 3rd equation $\nabla \times \bar{H} = \bar{J}$ → point form

$$\oint_l \bar{H}\cdot\bar{dl} = \int_S \bar{J}\cdot\bar{ds} \rightarrow \text{Integral form}$$

Maxwell's 4th equation $\nabla\cdot\bar{B} = 0$ → point form

$$\oint_S \bar{B}\cdot\bar{ds} = 0 \rightarrow \text{Integral form}$$

Define scalar and Vector magnetic potentials?

-The scalar magnetic potential is defined as line Integral of magnetic field Intensity along a path where current density is zero. $\boxed{\bar{J}=0}$

$$\boxed{\nabla^2 V_m = 0} \text{ if } \bar{J}=0$$

· Vector Magnetic potential is defined as curl of the Vector magnetic potential is equal to magnetic flux density

$$\boxed{\bar{B} = \nabla \times \bar{A}}$$

$$\bar{A} = \int_L \frac{\mu I \bar{dl}}{4\pi R} \text{ wb/m}, \quad \bar{A} = \int_S \frac{\mu K ds}{4\pi R} \text{ wb/m}$$

$$\bar{A} = \int_V \frac{\mu \bar{J} dV}{4\pi R} \text{ wb/m.}$$

ne Mutual Inductance?

Mutual Inductance between two coils is defined the ratio of total flux linkage in one coil to the urrent in other coil

$$M_{12} = \frac{N_1 \phi_{12}}{I_2} = \frac{\lambda_{12}}{I_2}, \quad M_{21} = \frac{N_2 \phi_{21}}{I_1} = \frac{\lambda_{21}}{I_1} \text{ Henry}$$

Write the expression for magnetic Energy?

The Energy stored in a magnetic field produced by an Inductor is given by

$$\boxed{W_m = \frac{1}{2} L I^2} \text{ Jouly}$$

Write the expression for Energy density in magnetic field?

$$\boxed{\omega_m = \frac{1}{2} \mu H^2 = \frac{1}{2} \bar{B} \cdot \bar{H} \text{ J/m}^3}$$

(OR)

if dv is differential Volume

$$\boxed{W_m = \frac{1}{2} \int_V \bar{B} \cdot \bar{H} \, dv \text{ Jouly}}$$

* * *

PART-2

State the faraday's law?

Faraday's law staty that if the magnetic field is time varying then the Induced emf is equal to time rate of change of magnetic flux linkage by the circuit.

Induced emf is $\boxed{V = -N \frac{d\phi}{dt}}$ volts.

N = no. of Turns, ϕ = magnetic flux.

State Lenz' Law?

It staty that the direction of Induced emf is always equal to opposite of the flux linkages produced by current

Write the Applications of Ampere's law?

The Ampère's circuit law is used to determine magnetic field Intensity $\bar{H}$ for

- Infinite long straight conductor
- Infinite Coaxial cable
- Infinite sheet of currents
- Infinite Solid conductor

Write the lorentz force equation?

The Lorentz force equation is given as

$$\boxed{\bar{F} = Q(\bar{E} + \bar{v} \times \bar{B})}$$ Newton's.

$\bar{F}$ = force, Q = charge, $\bar{E}$ = Electric field Intensity
$\bar{v}$ = Velocity, $\bar{B}$ = magnetic flux density

Write the Expression for force on a differential current element?

$$\boxed{\bar{F} = \int_{l} I d\bar{l} \times \bar{B}} \quad \boxed{\bar{F} = \int_{s} \bar{K} ds \times \bar{B}} \quad \boxed{\bar{F} = \int_{V} \bar{J} dv \times \bar{B}}$$

State Ampere's Force law? (Force b/w two current elements)

The Ampere's force law states that, the force b/w two current elements $I_1 d\bar{l}_1$, $I_2 d\bar{l}_2$ placed at a distance R_{12} is given by

$$\boxed{\bar{F} = \frac{\mu}{4\pi} \oint_{L_1} \oint_{L_2} \frac{I_2 d\bar{l}_2 \times (I_1 d\bar{l}_1 \times \bar{a}_{R_{12}})}{R_{12}^2}} \ N$$

Define Self Inductance?

The self Inductance is defined as the flux linkage produced by Inductor it self.

$$\boxed{L = \frac{N\phi}{I}}$$ Henry.

Transformer emf ?

former e.m.f can be defined as When a time ying current is applied, it produces a time Varying ux in the primary coil and that Induces an emf in the secondary coil."

$$\boxed{\nabla\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}} \quad (OR) \quad \boxed{\oint_l \bar{E}\cdot d\bar{l} = \int_s \left(\frac{-\partial\bar{B}}{\partial t}\right) d\bar{s}}$$

State the Inconsistency of Ampere's ckt law?

The curl of magnetic field Intensity is equal to sum of current density and displacement current density.

$$\boxed{\nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}} \Rightarrow \boxed{\nabla\times\bar{H} = \bar{J}_c + \bar{J}_d}$$

$$\boxed{\oint_l \bar{H}\cdot d\bar{l} = \int_s \left(\bar{J} + \frac{\partial\bar{D}}{\partial t}\right) d\bar{s}} \rightarrow \text{Integral form.}$$

Define displacement current density? ($\bar{J}_d$)

The ratio of displacement current to area of cross-section is called as "Displacement current density"

$$\boxed{\bar{J}_d = \frac{I_d}{S} = \frac{\partial\bar{D}}{\partial t}} \ A/m^2$$

Write Maxwell's equations for Time Varying fields?

1. $\nabla\cdot\bar{D} = \rho_v$	$\oint_s \bar{D}\cdot d\bar{s} = \int_v \rho_v \, dV$
2 $\nabla\times\bar{E} = \frac{-\partial\bar{B}}{\partial t}$	$\oint_l \bar{E}\cdot d\bar{l} = \int_s \left(-\frac{\partial\bar{B}}{\partial t}\right) d\bar{s}$
3 $\nabla\times\bar{H} = \bar{J} + \frac{\partial\bar{D}}{\partial t}$	$\oint_l \bar{H}\cdot d\bar{l} = \int_s \left(\bar{J} + \frac{\partial\bar{D}}{\partial t}\right) d\bar{s}$
4. $\nabla\cdot\bar{B} = 0$.	$\oint_s \bar{B}\cdot d\bar{s} = 0$.

Write the Boundary conditions for Dielectric - Dielectric Interface?

for Electric field :-	for Magnetic field
1. $E_{tan1} = E_{tan2}$ → continuous	1. $H_{tan1} = H_{tan2}$ → continuous
2. $D_{n1} \mp D_{n2}$ → continuous	2. $H_{tan1} - H_{tan2} = K$ → discontinuous
3. $D_{n1} - D_{n2} = \rho_s$ → discontinuous	3. $B_{n1} = B_{n2}$ → continuous

Write the Boundary conditions for Dielectric - Conductor Interface?

for Electric field	for magnetic field
1. $E_{tan} = 0$	1. $H_{tan1} = K$
2. $D_n = \rho_s$	2 $H_{tan2} = 0$
	3. $B_n = 0$.

What is Generator emf (Motional Emf)

consider a stationary magnetic field is applield on a conducting circuit which is moving on its axis with respect to time, When the circuit cuts across the magnetic field, an electro magnetic field is induced across the circuit terminals. This emf is called "Generator emf"(or) Motional emf

$$\boxed{\nabla \times \bar{E} = \nabla \times \bar{v} \times \bar{B}}$$

Write the Equation for Moving circuit in a time Varying field?

This moving circuit in a time Varying field represents both Transformer emf and Generator emf

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} + \nabla \times \bar{v} \times \bar{B}}$$

# of Boundary conditions :-

II ECE-(EMTL)

ielectric - Dielectric Interface :- [Two perfect dielectric media].

For Electric field $\bar{E}$:-

To determine the Boundary conditions, We need to use maxwell's equations.

From maxwell's equations $\oint_l \bar{E} \cdot d\bar{l} = 0 \longrightarrow ①$

and $\oint_s \bar{D} \cdot d\bar{s} = Q_{enc} \longrightarrow ②$

Where Q_{enc} is the free charge enclosed by the surface S.

Also we need to decompose the electricfield Intensity $\bar{E}$ into two orthogonal components.

$$\bar{E} = \bar{E}_{tan} + \bar{E}_{norm} \text{ (or) } \bar{E} = \bar{E}_t + \bar{E}_n \longrightarrow ③$$

Where E_{tan} and E_{norm} are the tangential and normal components of $\bar{E}$ to the interface. A similar decomposition can be done for the electric flux density $\bar{D}$.

→ Consider the $\bar{E}$ field existing in a region that consists of two different dielectrics characterized by $\epsilon_1 = \epsilon_0 \epsilon_{r_1}$ and $\epsilon_2 = \epsilon_0 \epsilon_{r_2}$ as shown in figure below.

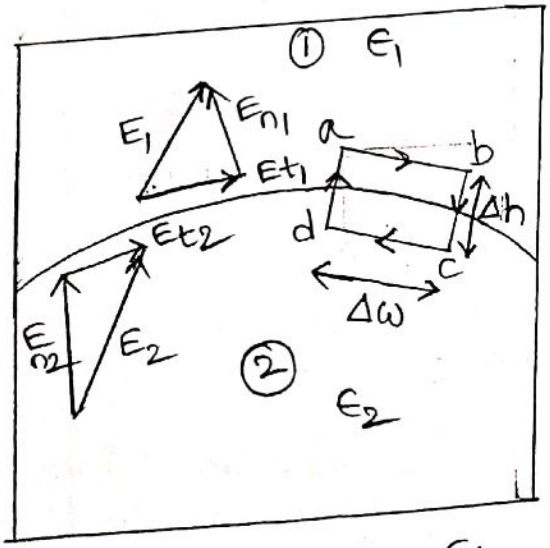


fig.(a) determining $E_{t_1} = E_{t_2}$ (or) $E_{tan_1} = E_{tan_2}$

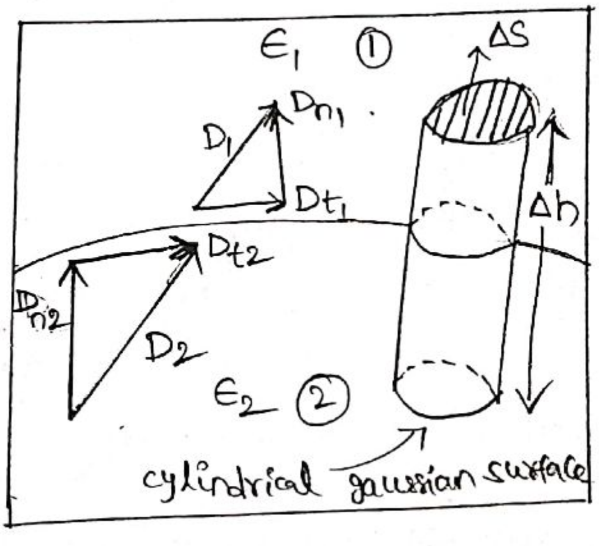


fig.(b) determining $D_{n_1} = D_{n2}$ (or) $D_{norm_1} = D_{norm2}$

The fields E_1 and E_2 in media 1 and 2 respectively can be decomposed as

$$\bar{E}_1 = \bar{E}_{t_1} + \bar{E}_{n_1} \rightarrow ④$$

$$\bar{E}_2 = \bar{E}_{t_2} + \bar{E}_{n_2} \longrightarrow ⑤$$

→ We apply eq ① to the closed path abcda of figure (a) assuming that the path is very small with respect to the spatial variation of $\bar{E}$. We obtain

$$0 = E_{t_1}\Delta w - \cancel{E_{n_1}\frac{\Delta h}{2}} - \cancel{E_{2n}\frac{\Delta h}{2}} - E_{2t}\Delta w + \cancel{E_{2n}\frac{\Delta h}{2}} + \cancel{E_{1n}\frac{\Delta h}{2}}$$

$$\Rightarrow 0 = E_{t_1}\Delta w - E_{t_2}\Delta w = \Delta w(E_{t_1} - E_{t_2})$$

$$\Rightarrow E_{t_1} - E_{t_2} = 0$$

$$\therefore \boxed{E_{t_1} = E_{t_2}} \text{ (or) } \boxed{E_{tan_1} = E_{tan_2}} \longrightarrow ⑥$$

where $E_t = |\bar{E}_t|$ and $E_n = |\bar{E}_n|$. $(\because \Delta h \rightarrow 0)$

Thus the Tangential components of $\bar{E}$ are same on the two sides of the boundary (or) E_t undergoes nochange on boundary.

∴ The Tangential component of Electricfield Intensity is continuous across the boundary.

since $\bar{D} = \epsilon\bar{E} = \bar{D}_t + \bar{D}_n$. eq ⑥ can be written as

$$\frac{D_{t_1}}{\epsilon_1} = \frac{D_{t_2}}{\epsilon_2} \rightarrow ⑦ \qquad \left(\because E_{t_1} = \frac{D_{t_1}}{\epsilon_1}, E_{t_2} = \frac{D_{t_2}}{\epsilon_2}\right)$$

that is D_t undergoes some changes across the Interface. Hence D_t is said to be discontinuous across the ~~Inter~~ Interface.

→ similarly We apply eq ② to the pillbox (cylindrical Gaussian surface) of figure (b). the contribution due to the sides vanishes. Allowing $\Delta h \rightarrow 0$ gives.

$$\Delta Q = \rho_s \Delta S = D_{n_1}\Delta S - D_{n_2}\Delta S. \qquad \left(\because \rho_s = \frac{Q}{S} = \frac{\Delta Q}{\Delta S}\right)$$

$$\Rightarrow \rho_s \cancel{\Delta S} = \cancel{\Delta S}(D_{n_1} - D_{n_2})$$

$$\therefore \boxed{D_{n_1} - D_{n_2} = \rho_s} \rightarrow ⑧$$

(where ρ_s is free charge density placed at the boundary)

If no free charges exist at Interface then $\rho_s = 0$.

$\boxed{D_{n_1} = D_{n_2}} \rightarrow ⑨$

thus the normal component of electric flux density $\bar{D}$ is continuous across the Interface. that is D_n undergoes no-change at the boundary. $\boxed{\epsilon_1 E_{n_1} = \epsilon_2 E_{n_2}} \rightarrow ⑩$ $(\because \bar{D}=\epsilon\bar{E})$

∴ Equations ⑥, ⑧ and ⑨ are collectively referred to as Boundary conditions for $\bar{E}$ field at Dielectric - Dielectric Interface. From eq ⑩ the normal component of $\bar{E}$ is discontinuous at the boundary.

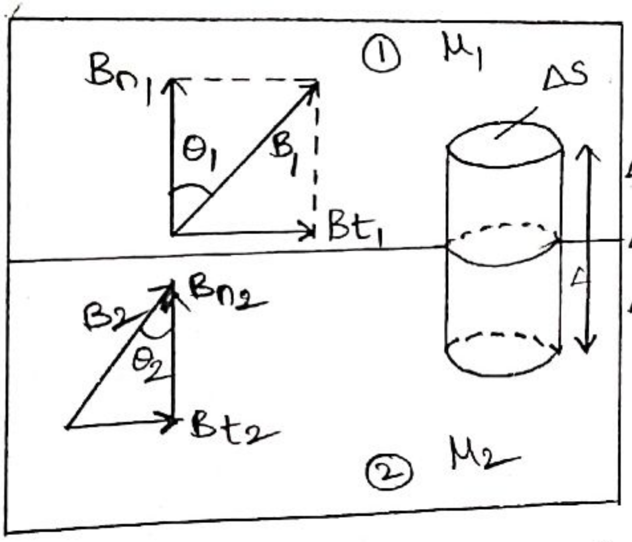
) FOR Magnetic field $\bar{H}$:-

We define magnetic boundary conditions as the conditions that $\bar{H}$ and $\bar{B}$ field must statisfy at the Boundary. between two media.

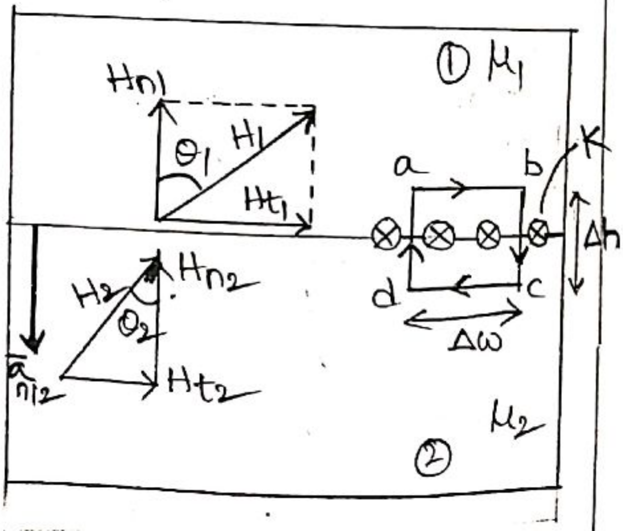
We know that $\oint_S \bar{B}\cdot d\bar{S} = 0 \rightarrow ①$

and Ampere's Ckt Law is $\oint_l \bar{H}\cdot d\bar{l} = I \rightarrow ②$

consider the boundary between two magnetic media 1 and 2 characterized respectively by μ_1 and μ_2 as shown in figure below.



fig(a) determining $B_{n_1} = B_{n_2}$ (or) determine Boundary conditions for $\bar{B}$



fig(b). determining $H_{t_1} = H_{t_2}$ (or) $H_{an_1} = H_{an_2}$ (or) determining Boundary Conditions for $\bar{H}$

→ Applying eq ① to the pillbox of figure (a) and allowing $\Delta h \to 0$ We obtain.

$$B_{n_1} \Delta S - B_{n_2} \Delta S = 0 .$$

$$\Rightarrow (B_{n_1} - B_{n_2}) \Delta S = 0$$

$$\therefore B_{n_1} - B_{n_2} = 0 .$$

$$\therefore \boxed{B_{n_1} = B_{n2}} \text{ (or) } \boxed{\mu_1 H_{n_1} = \mu_2 H_{n_2}} \quad (\because B = \mu H) \longrightarrow ③$$

eq ③ shows that the normal component of $\bar{B}$ is continuous at the boundary and normal component of $\bar{H}$ is discontinuous.

→ similarly, we apply eq ② to the closed path abcda of figure (b), where surface current density $\bar{K}$ on the boundary is assumed normal to the path. We obtain

$$K \cdot \Delta W = H_{t_1} \cdot \Delta W + \cancel{H_{n_1} \cdot \frac{\Delta h}{2}} + \cancel{H_{n_2} \cdot \frac{\Delta h}{2}} - H_{t_2} \cdot \Delta W - \cancel{H_{n_2} \cdot \frac{\Delta h}{2}}$$
$$- \cancel{H_{n_1} \cdot \frac{\Delta h}{2}} \longrightarrow ④$$

As $\Delta h \to 0$, then above equation ④ becomes

$$K \cdot \Delta W = H_{t_1} \cdot \Delta W - H_{t_2} \cdot \Delta W$$

$$\Rightarrow K \cdot \cancel{\Delta W} = \cancel{\Delta W} (H_{t_1} - H_{t_2})$$

$$\therefore \boxed{H_{t_1} - H_{t_2} = K} \text{ (or) } \boxed{H_{tan_1} - H_{tan_2} = K} \rightarrow ⑤ .$$

This shows that the Tangential component of $\bar{H}$ is also discontinuous. eq ⑤ may be written interms of $\bar{B}$ as

$$\frac{B_{t_1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K \longrightarrow ⑥$$

In general case eq ⑤ becomes $\boxed{(\bar{H}_1 - \bar{H}_2) \times \bar{a}_{n_{12}} = \bar{K}} \rightarrow ⑦$

where $\bar{a}_{n_{12}}$ is a unit vector normal to the Interface and is directed from medium 1 to medium 2.

If the boundary is free of current or the media are not conductors $K = 0$ and eq ⑤ becomes.

$$\boxed{H_{t_1} = H_{t_2}} \text{ (or) } \boxed{H_{tan_1} = H_{tan2}} \longrightarrow ⑧$$

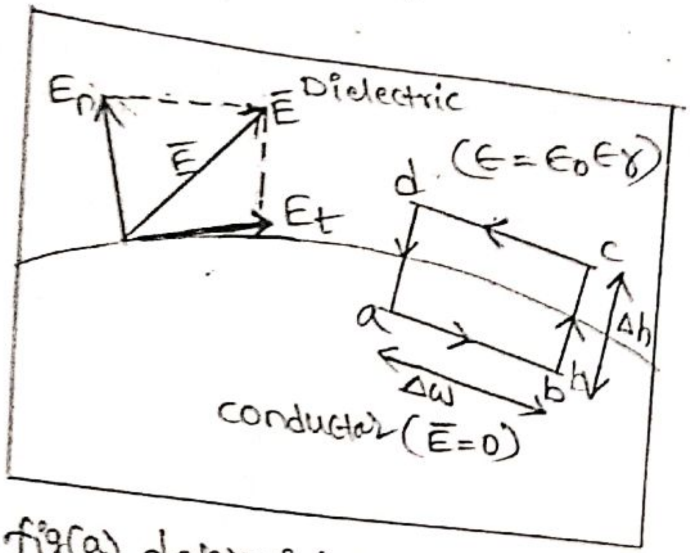
$$\text{(or) } \frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2} \longrightarrow ⑨$$

Thus the Tangential components of $\bar{H}$ is continuous, and tangential component of $\bar{B}$ is discontinuous.

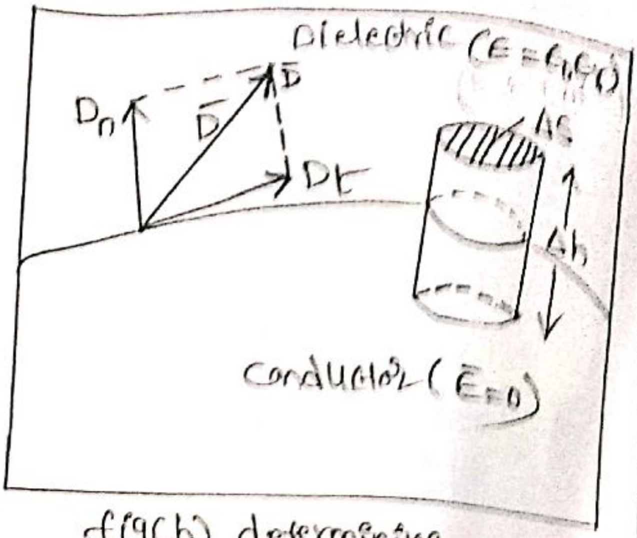
Boundary conditions for Dielectric- conductor Interface :-

(a) For electric field $\bar{E}$:-

The following figures shows the case of Dielectric - conductor Interface Boundary conditions.



fig(a) determining Boundary conditions for $\bar{E}$



fig(b) determining Boundary conditions for $\bar{D}$

We know that $\oint_l \bar{E} \cdot d\bar{l} = 0 \rightarrow ①$ and

$$\oint_s \bar{D} \cdot d\bar{s} = Q_{enc} \rightarrow ②$$

To determine boundary conditions for a conductor - dielectric Interface, We follow the same procedure used for the dielectric - dielectric Interface except that $\bar{E}=0$ inside the conductor.

⇒ Applying eq ① to the closed path abcda of figure (a), which gives

$$0 = 0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + \cancel{E_n \cdot \frac{\Delta h}{2}} - E_t \cdot \Delta w - \cancel{E_n \cdot \frac{\Delta h}{2}} - 0 \cdot \frac{\Delta h}{2}$$

$(\because \text{As } \Delta h \rightarrow 0)$

$$\Rightarrow 0 = -E_t \cdot \Delta w$$

$$\therefore \boxed{E_t = 0} \text{ (or) } \boxed{E_{tan} = 0} \rightarrow ③$$

similarly By applying eq ② to the cylindrical pillbox of figure (b), and letting $\Delta h \rightarrow 0$, We get

$$\Delta Q = D_n \cdot \Delta S - 0 \cdot \Delta S \cdot \quad (\because \bar{D}=0, \bar{E}=0 \text{ Inside the conductor})$$

$\Rightarrow \Delta Q = D_n \cdot \Delta S.$

$\Rightarrow D_n = \frac{\Delta Q}{\Delta S} = \rho_s$ $\quad\left(\because \rho_s = \frac{Q}{S} = \frac{\Delta Q}{\Delta S}\right)$

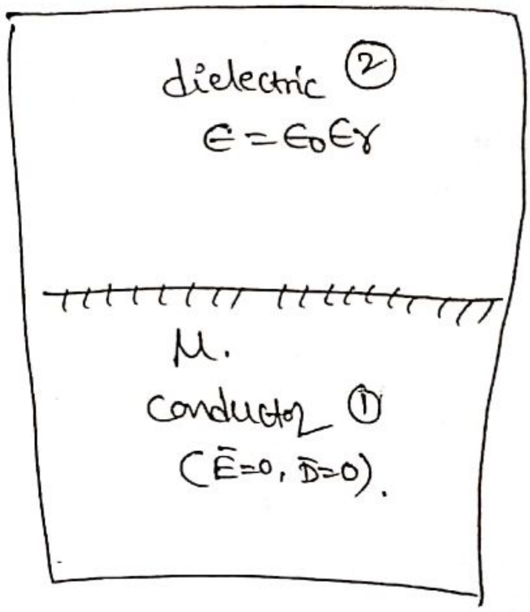
$\therefore \boxed{D_n = \rho_s} \rightarrow ④ \quad \Rightarrow \epsilon E_n = \rho_s$

$\therefore \boxed{E_n = \frac{\rho_s}{\epsilon}}$

∴ eq ③ and eq ④ called as Boundary conditions of Electric field for Dielectric-conductor Interface.

(b) For Magnetic field :-($\bar{H}$).

Assumption.



We know that $H_{tan_1} - H_{tan_2} = K$.

for dielectric $\boxed{H_{tan_2} = 0} \rightarrow ①$

$\therefore H_{tan_1} - 0 = K$

$\boxed{H_{tan_1} = K} \rightarrow ②$

Also We know that $\boxed{B_{n_1} = B_{n_2}}$

for dielectric $B_{n_2} = 0$.

$\therefore B_{n_1} = 0$.

$\boxed{B_n = 0} \rightarrow ③$

The normal component of magnetic fluxdensity ($\bar{B}$) is zero.