II ECE EMWTL electro magnetic wave transform line

prepared by :- ANR. SHORT ANSWER QUESTIONS (PART-A)

UNIT-1 ELECTROSTATICS :-

1) State the coulomb's law?

A) The coulomb's law states that the force between two point charges Q_1 and Q_2 is
→ Along the line Joining them
→ Directly proportional to product of two charges
→ Inversely proportional to square of distance between them.

$$F \propto \frac{Q_1 Q_2}{R^2} \Rightarrow F = K \frac{Q_1 Q_2}{R^2}$$

$$\therefore \boxed{F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}} \text{ Newtons.} \quad \left(\because K = \frac{1}{4\pi\epsilon_0}\right)$$

2. Write the Applications of coulomb's law?

A) By using coulomb's law we have to find
→ Force between two charges
→ Electric field at a point due to a fixed Charge
→ Distance between two charges.

3) Write the limitation of coulomb's law?

A) → The coulomb's law doesn't applicable for arbitrary surfaces. (charged bodies)

4) Define electric field?

A) A region around a charged particle, within which a force would be exerted on other charged particles (or) objects.

5. Define electric field intensity.

The electric field Intensity is defined as force per unit charge, when placed in an electric field. It is denoted by $\bar{E}$.

$$\boxed{\bar{E} = \frac{\bar{F}}{Q}} \quad \text{Newton/coulombs}$$

In scalar form $\boxed{E = \frac{V}{d}}$ Volts/meter.

(OR) $\boxed{\bar{E} = -\nabla V}$

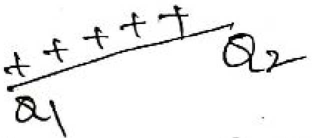
6. Define point charge?

If the dimensions of a surface carrying charge are very less then this type of charge is called as "point charge"

Q_1 + Q_2 −

7. Define line charge? If the charge is distributed uniformly along a line is called a "line charge".

Line charge density is $\rho_L = \frac{\text{Total charge}}{\text{Total Length}}$



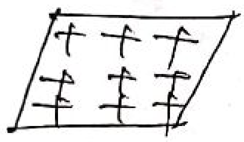
$$\boxed{\rho_L = \frac{Q}{L} \text{ (or) } \rho_L = \frac{dQ}{dL}} \text{ C/m}$$

8. Define surface charge?

If the charge is distributed uniformly along a two dimensial surface then it is called as a "surface charge".

The surface charge density is

$$\rho_S = \frac{\text{Total charge}}{\text{Total surface area}} \text{ C/m}^2$$



$$\rho_S = \frac{Q}{S} = \frac{dQ}{dS}$$

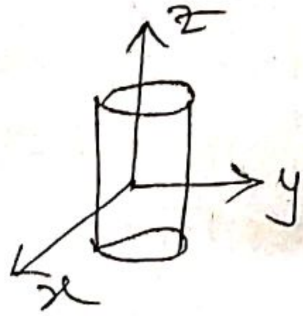
$\Rightarrow dQ = \rho_S dS$, $\boxed{Q = \int_S \rho_S dS}$

Define Volume charge?

If the charge is uniformly distributed over a volume then it is called a "volume charge"

The Volume charge density is

$$\rho_v = \frac{\text{Total charge}}{\text{Total volume}} = \frac{Q}{V} \ C/m^3$$



$$\rho_v = \frac{dQ}{dV}$$

$$dQ = \rho_v dV \Rightarrow \boxed{Q = \int_V \rho_v dV}$$

Define Electric displacement? (Electric flux)

The electric flux represents total number of direction of electric lines of force.

→ The electric flux is numerically equal to the electric charge (ie) $\boxed{\Psi = Q}$ coulombs

Define Electric flux density [Electric displacement density]

The electric flux density is defined as the net flux passing through unit surface area is called as "electric flux density"

scalar form $\boxed{D = \frac{\Psi}{S}}$ C/m^2

Vector form $\boxed{\bar{D} = \frac{d\Psi}{dS} \bar{a}_n}$ C/m^2

12. State gauss' law? give the limitations?

The gauss' law states that the net electric flux passing through any closed surface is equal to total charge enclosed by that surface.

$$\boxed{\psi = Q_{enc} = \oint_S \bar{D} \cdot d\bar{S}}$$

ψ = electric flux
$\bar{D}$ = flux density
$d\bar{S}$ = differential surface area
Q_{enc} = charge enclosed

Limitation :-

→ gauss' law cannot be applicable for non-gauss ian surfaces (or) un-symmetrical surfaces.

13. What are the applications of gauss' law?

By using gauss' law we have to calculate

→ Electric flux density ($\bar{D}$)
→ Electric field Intensity ($\bar{E}$)
→ charge enclosed by surface (Q)

gauss' law can be applied to point charge, line-charge, surface charge, volume charge

14. Define Workdone?

Workdone is defined as the product of force and the distance.

W = force x distance.

$$\boxed{dW = \bar{F} \cdot \bar{dl}} \Rightarrow dW = (-E \cdot Q_t)\, dl$$

(OR)

$$\boxed{W = -Q_t \int_A^B \bar{E} \cdot \bar{dl}}$$

Q. derive relation b/w $\bar{E}$ and V?

The potential differences at A and B are given by $V_{AB} = -\int_B^A \bar{E}\cdot\bar{dl} \rightarrow (1)$

$$V_{BA} = -\int_A^B \bar{E}\cdot\bar{dl} \rightarrow (2)$$

$$\Rightarrow V_{BA} = -\left[-\int_B^A \bar{E}\cdot\bar{dl}\right] = -V_{AB}.$$

$$\Rightarrow V_{BA} = -V_{AB} = \int \bar{E}\cdot\bar{dl}$$

$$\therefore V_{BA} + V_{AB} = \oint_l \bar{E}\cdot\bar{dl} = 0 \rightarrow (3)$$

Applying stoke's theorem ~~to~~ eq (3)

$$\oint_l \bar{E}\cdot\bar{dl} = \int_s (\nabla\times\bar{E})\,\bar{ds} = 0$$

$$\therefore \boxed{\nabla\times\bar{E} = 0} \rightarrow \text{maxwell's 2nd equation for point form.}$$

(OR)

We know that $\bar{E} = -\nabla V$

Taking curl on bothsides

$$\nabla\times\bar{E} = \nabla\times(-\nabla V) \quad (\because \nabla\times\nabla V = 0)$$

$$\therefore \boxed{\nabla\times\bar{E} = 0} \rightarrow \text{maxwell's 2nd equation}$$

Q. Write maxwell's two equations for Electrostatic fields?

first equation.	second equation
$\nabla\cdot\bar{D} = \rho_V \rightarrow$ point form	$\nabla\times\bar{E} = 0 \rightarrow$ point form
$\oint_S \bar{D}\cdot\bar{ds} = \int_V \rho_V\, dV \rightarrow$ Integral form	$\oint_l \bar{E}\cdot\bar{dl} = 0 \rightarrow$ Integral form

15. Define Electric potential with necessary equations?

The electric potential is defined as workdone per unit charge. It is denoted by 'V'.

$$\boxed{V = \frac{W}{Q} = -\int_{\infty}^{P} \bar{E} \cdot \overline{dl}} \quad J/c \text{ (or) Volts.}$$

(OR)

The potential is defined as the workdone in unit charge moving from ∞ to point P in electric field opposed to the force.

The potential difference is $\boxed{V_{BA} = \frac{W}{Q} = -\int_{A}^{B} \bar{E} \cdot \overline{dl}}$

$$V_{BA} = V_B - V_A$$

$$\boxed{V_B = -\int_{\infty}^{B} \bar{E} \cdot \overline{dl}}, \quad \boxed{V_A = -\int_{\infty}^{A} \bar{E} \cdot \overline{dl}}$$

Also $\boxed{V = \frac{Q}{4\pi\epsilon_0 R}}$

16. Define potential gradient?

The maximum value of rate of change of potential with respect to distance is called "potential gradient".

$$\boxed{\nabla V = \left.\frac{dV}{dl}\right|_{max.} = \text{grad } V}$$

Also $\boxed{\bar{E} = -\nabla V}$

→ negative gradient of potential gives Electric field Intensity ($\bar{E}$)

Q) define Energy density?

The energy density is defined as the energy stored per unit volume. for electric field, it is $W_E = \frac{1}{2}\int_V (\bar{D}\cdot\bar{E})\,dV = \frac{1}{2}\int_V \epsilon_0 E^2\,dV$ Joules.

$$\boxed{w_E = \frac{W_E}{V} = \frac{1}{2}\bar{D}\cdot\bar{E} = \frac{1}{2}\epsilon_0 E^2 \quad J/m^3}$$

define Current?

The flow of charge per unit time is called as current. It is denoted as I. Units are Amperes.

$$\boxed{I = -\frac{dQ}{dt}} \text{ Amp. (or) } \boxed{I = \frac{Q}{t}} \ C/sec \text{ (or) Amp.}$$

define Convection current? (Diffusion current)

The current flowing ~~through~~ due to movement of free electrons and free holes to make uniform charge density in semiconductor materials is called "convection current".

define Conduction current? (Drift current)

The current flowing due to the flow of free electrons (or) drifting of electrons in conductors is called as "conduction current" (or) drift current

·define Displacement current?

The current flowing due to flow of charges in dielectric under influence of time varying electric field is called "displacement current".

24 Define Current density?

The current density is defined as current per unit surface area. It is denoted by $\bar{J}$.

$$\boxed{\bar{J} = \frac{dI}{d\bar{S}}} \ A/m^2 \Rightarrow d\bar{I} = \bar{J}.\bar{dS}$$

$$\boxed{\bar{I} = \oint_S \bar{J}.\bar{dS}}$$

The relation between $\bar{J}$ and $\bar{E}$ is $\boxed{\bar{J} = \sigma \bar{E}}$

25 Define dielectric Constant?

It is the ratio between permittivity ϵ and free-space permittivity ϵ_0.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$$

26 State the continuity Equation?

The continuity equation states that current density is diverging from a small volume is equal to negative time derivative of volume charge density.

$$\boxed{\nabla.\bar{J} = -\frac{\partial \rho_v}{\partial t}}$$

27 define relaxation time?

Relaxation time is defined as the time taken by volume charge density to decrease ~~to~~ 36.8% of its initial value.

It is given as

$$\boxed{T_r = \frac{\epsilon}{\sigma}} \ sec$$

ϵ = permittivity of medium

σ = conductivity

Unit Define Poisson's equation?

It can be staty that In a homogeneous medium, there exists a volume charge density ρ_v.

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

? State the Laplace's equation?

It staty that volume charge density is zero in poisson's equation.

$$\boxed{\nabla^2 V = 0} \Rightarrow \boxed{\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0}$$

30 What is capacitance?

capacitance is the ratio between charge and potential. $\boxed{C = \frac{Q}{V}}$

→ for parallel plate capacitor $\boxed{C = \frac{\epsilon A}{d} \text{ (or) } \frac{\epsilon S}{d}}$

→ for coaxial capacitor $\boxed{C = \frac{2\pi\epsilon L}{\ln(b/a)}}$

→ for spherical capacitor $\boxed{C = \frac{4\pi\epsilon}{\frac{1}{a} - \frac{1}{b}}}$

31 Represent second maxwell's equation in Integral form

$\boxed{\oint_l \bar{E} \cdot d\bar{l} = 0}$ → static field

$\boxed{\oint_l \bar{E} \cdot d\bar{l} = \int_S \left(-\frac{\partial \bar{B}}{\partial t}\right) d\bar{s}}$ → for time varying field

32 Calculate the capacitance of parallel plate capacitor of 'N' dielectric slabs with different thickness?

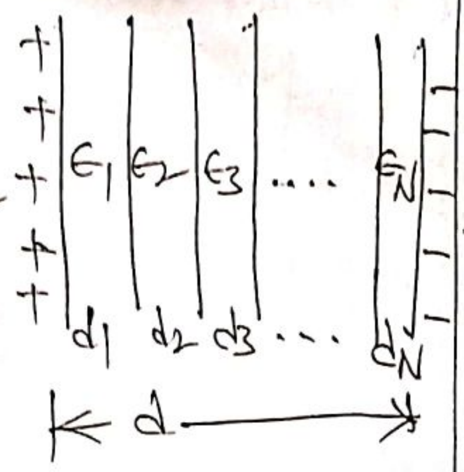
If capacitor with different dielectric materials then the capacitance is

$$C = \frac{Q}{V} = \frac{S}{\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \cdots + \frac{d_N}{\epsilon_N}\right)} \text{ farads.}$$

Where $\epsilon_1, \epsilon_2, \ldots \epsilon_N$ are different dielectric materials.

S = surface area = A

$d_1, d_2, \ldots d_N$ are thickness.



33 State the divergence Theorem?

It states that total outward flux of a vector $\bar{A}$ through closed surface is same as volume Integral of divergence of $\bar{A}$.

$$\boxed{\oint_S \bar{A} \cdot \bar{ds} = \int_V (\nabla \cdot \bar{A}) dV}$$

34 State the stoke's theorem?

It states that the net circulation of a vector $\bar{A}$ around a closed path 'l' is equal to surface Integral of curl of $\bar{A}$ over the open surface

$$\boxed{\oint_l \bar{A} \cdot \bar{dl} = \int_S (\nabla \times \bar{A}) \bar{ds}}$$

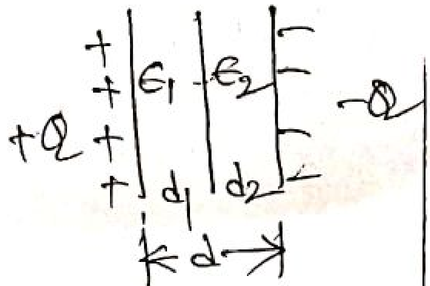
Find the capacitance of parallel plate capacitor containing two dielectrics each comprising $\epsilon_{r_1} = 2$ and $\epsilon_{r_2} = 3.3$, one-half of the volume, here $A = 4m^2$, $d = 10^{-3} m$.

Sol: given data

two dielectric constants $\epsilon_{r_1} = 2$, $\epsilon_{r_2} = 3.3$.

distance of seperation $d = 10^{-3} m$.

$\Rightarrow d = 1 mm$.



Each comprising one-half of volume

(ie) $d = d_1 + d_2$

$\therefore d_1 = 0.5 mm = 0.5 \times 10^{-3} m$

$\therefore d_2 = 0.5 mm = 0.5 \times 10^{-3} m$.

Area $A = 4 m^2$, capacitance $C = ?$

$$\boxed{C = \frac{Q}{V} = \frac{A}{\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \dots + \frac{d_N}{\epsilon_N}\right)}} \text{ farads.} \quad (\because S = A)$$

for two dielectrics

$$\boxed{C = \frac{Q}{V} = \frac{A}{\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2}}} = \frac{A}{\left(\frac{d_1}{\epsilon_0 \epsilon_{r_1}} + \frac{d_2}{\epsilon_0 \epsilon_{r_2}}\right)}$$

$$\Rightarrow C = \frac{4}{\left[\frac{0.5 \times 10^{-3}}{8.854 \times 10^{-12} \times 2} + \frac{0.5 \times 10^{-3}}{8.854 \times 10^{-12} \times 3.3}\right]}$$

$$C = \frac{4}{[\quad\quad]}$$

36. A sheet of charge lies in y-z plane at $x=0$ and has uniform surface charge density of $5.0\ pC/m^2$. Find the electric field at a point $P(-5,0,0)$ on x-axis.

Sol:- given data

surface charge density $\rho_s = 5.0\ pC/m^2$

$= 5.0 \times 10^{-12}\ C/m^2$

$\bar{E}$ at $P(-5,0,0) = ?$

We know that

Electric field due to surface charge is

$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_n}$$ (OR) for x-axis $$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0}\bar{a}_x}$$

$$\bar{E} \text{ at } P(-5,0,0) = \frac{-\rho_s}{2\epsilon_0}\bar{a}_x$$

$$= -\frac{5.0 \times \cancel{10^{-12}}}{2 \times 8.854 \times \cancel{10^{-12}}}\bar{a}_x$$

$$= -\frac{5}{2 \times 8.854}\bar{a}_x$$

$$\therefore \boxed{\bar{E} \text{ at } P(-5,0,0) = -0.282\,\bar{a}_x} \ V/m$$

Wave :- A wave is an oscillation that transmits the energy through space (or) matter.

Electro magnetic wave :- An electro magnetic wave is a wave it transmitts the long (or) short wave-length FM radio waves. These waves are used to propagate the signals through space.

> Generally electro magnetic waves are used to design the communication systems and analyse the system.

Applications of EM waves :-

1. Analog communications
2. Radar systems
3. Mobile communications
4. Digital communications.
5. Optical communications.
6. Radiating systems.

Basic Definitions :-

Scalar :- A scalar is a quantity, which has only magnitude and does not having direction.

Ex:- Mass, potential.

Vector :- A vector is a quantity, which has both magnitude and direction.

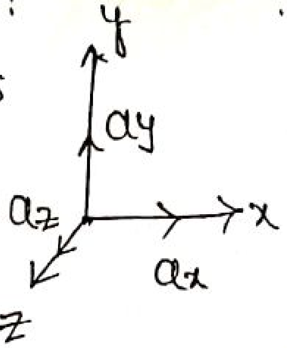
Ex:- Velocity, current, electric field, magnetic field.

A vector can be represented as

$$\bar{A} = A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z$$

where A_x, A_y, A_z are scalar components

$\bar{a}_x, \bar{a}_y, \bar{a}_z$ are unit vectors.



$\bar{A}$ is vector quantity

Magnitude of vector is

$$|\bar{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

<u>Unit vector</u> :- Unit vector is a vector whose magnitude is unity and direction is along $\bar{A}$.

It can be written as

$$\bar{a}_A = \frac{\bar{A}}{|\bar{A}|} = \frac{A_x \bar{a}_x + A_y \bar{a}_y + A_z \bar{a}_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

* <u>Co-ordinate systems</u>:-

→ The coordinate system can be defined as it is a system which can be used to represent a point in space.

→ The most commonly used coordinate systems are

(i) cartesian coordinate system [Rectangular system]

(ii) cylindrical (or) circular coordinate system (ρ, ϕ, z)

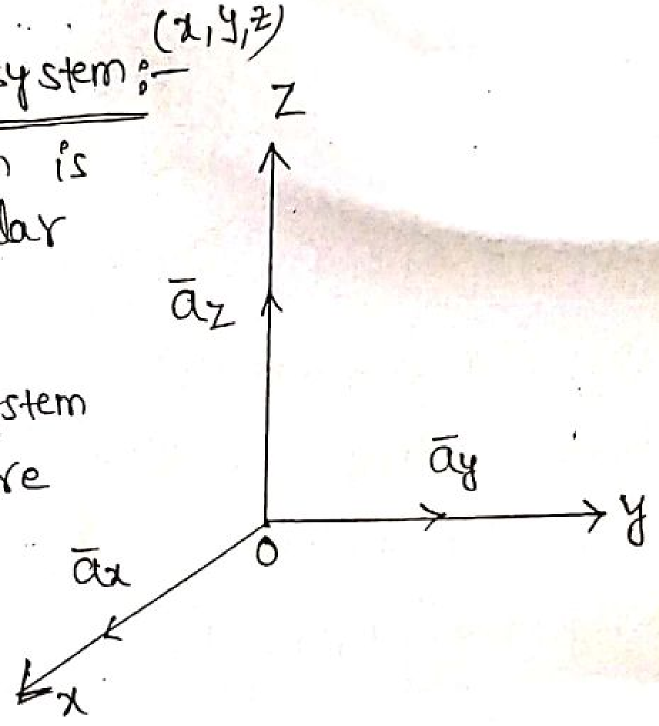
(iii) spherical coordinate system. (r, θ, ϕ)

(or)

poolar coordinate system.

Cartesian coordinate system :-

Cartesian coordinate system is also called as rectangular coordinate system.



In cartesian coordinate system the three axes x, y, z are perpendicular to each other.

The three planes are

xy - plane represented by $z=0$

yz - plane represented by $x=0$

xz - plane represented by $y=0$.

The range of variables are

for x ; $-\infty \leq x \leq \infty$

for y : $-\infty \leq y \leq \infty$

for z : $-\infty \leq z \leq \infty$

The vector can be represented as

$\bar{A} = A_x\bar{a}_x + A_y\bar{a}_y + A_z\bar{a}_z$

The properties of unit vectors are .

$$\left.\begin{array}{ll} \bar{a}_x \cdot \bar{a}_x = 1 & \bar{a}_x \cdot \bar{a}_y = 0 \\ \bar{a}_y \cdot \bar{a}_y = 1 & \bar{a}_y \cdot \bar{a}_z = 0 \\ \bar{a}_z \cdot \bar{a}_z = 1 & \bar{a}_z \cdot \bar{a}_x = 0 \end{array}\right\} \text{dot products.}$$

$\bar{a}_x \times \bar{a}_x = 0$ $\quad$ $\bar{a}_x \times \bar{a}_y = \bar{a}_z$

$\bar{a}_y \times \bar{a}_y = 0$ $\quad$ $\bar{a}_y \times \bar{a}_z = \bar{a}_x$

$\bar{a}_z \times \bar{a}_z = 0$ $\quad$ $\bar{a}_z \times \bar{a}_x = \bar{a}_y$

$$\left.\begin{aligned} \bar{a}_x \times \bar{a}_y &= \bar{a}_z \\ \bar{a}_y \times \bar{a}_z &= \bar{a}_x \\ \bar{a}_z \times \bar{a}_x &= \bar{a}_y \end{aligned}\right\} \text{cross product}$$

Differential elements :-

Differential length is given by

$$d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$$

Magnitude $|d\bar{l}| = \sqrt{(dx)^2 + (dy)^2 + (dz)^2}$

Differential surface vector is given by

$$d\bar{s}_x = dy\,dz\,\bar{a}_x$$

$$d\bar{s}_y = dz\,dx\,\bar{a}_y$$

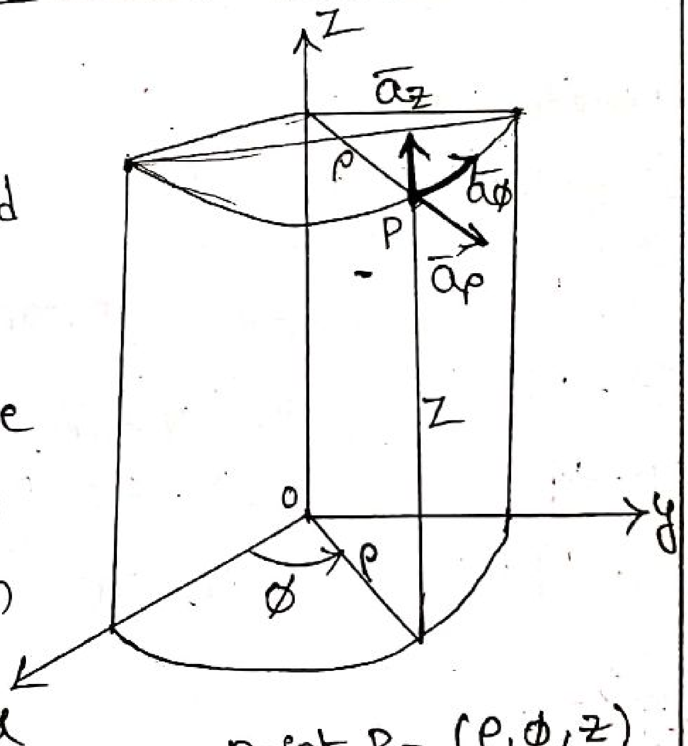
$$d\bar{s}_z = dx\,dy\,\bar{a}_z$$

Differential volume is given by

$$dV = dx\,dy\,dz.$$

cylindrical (or) circular coordinate system :- (ρ, ϕ, z)

Let us consider a point 'p' in cylindrical coordinates is represented by (ρ, ϕ, z).



Point $P = (\rho, \phi, z)$

ρ is the radius of the cylinder passing through 'p' (or) radial distance from z-axis.

ϕ is called the azimuthal angle is measured from the x-axis in the xy-plane.

z is the height of the cylinder.

The units of cylindrical coordinates are

$\rho \rightarrow$ meter.

$\phi \rightarrow$ degree (or) radian.

$z \rightarrow$ meter.

The ranges of the variables ρ, ϕ, z are

$$0 \le \rho \le \infty$$
$$0 \le \phi \le 2\pi$$
$$-\infty \le z \le \infty$$

The vector $\bar{A}$ in cylindrical coordinates can be written as

$$\bar{A} = A_\rho \bar{a}_\rho + A_\phi \bar{a}_\phi + A_z \bar{a}_z$$

The unit vectors are $\bar{a}_\rho$ in radial direction,
$\bar{a}_\phi$ in tangent to surface.
$\bar{a}_z$ in vertical direction (+ve z direction)

The magnitude of $\bar{A}$ is

$$|\bar{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$

Thus

$$\bar{a}_\rho \cdot \bar{a}_\rho = 1 \qquad \bar{a}_\rho \cdot \bar{a}_\phi = 0$$
$$\bar{a}_\phi \cdot \bar{a}_\phi = 1 \qquad \bar{a}_\phi \cdot \bar{a}_z = 0$$
$$\bar{a}_z \cdot \bar{a}_z = 1 \qquad \bar{a}_z \cdot \bar{a}_\rho = 0$$

dot products.

3

$\bar{a}_\rho \times \bar{a}_\rho = 0$ | $\bar{a}_\rho \times \bar{a}_\phi = \bar{a}_z$
$\bar{a}_\phi \times \bar{a}_\phi = 0$ | $\bar{a}_\phi \times \bar{a}_z = \bar{a}_\rho$
$\bar{a}_z \times \bar{a}_z = 0$ | $\bar{a}_z \times \bar{a}_\rho = \bar{a}_\phi$

} cross products for unit vectors.

the relationship between cartesian coordinate system and cylindrical coordinate system is given by

$$x = \rho\cos\phi \rightarrow (1)$$
$$y = \rho\sin\phi \rightarrow (2)$$
$$z = z \rightarrow (3)$$

from (1), (2)

$$x^2 = \rho^2\cos^2\phi$$
$$y^2 = \rho^2\sin^2\phi$$

$$x^2+y^2 = \rho^2\cos^2\phi + \rho^2\sin^2\phi$$
$$\Rightarrow x^2+y^2 = \rho^2(\cos^2\phi + \sin^2\phi)$$
$$\therefore x^2+y^2 = \rho^2(1) \qquad (\because \cos^2\phi + \sin^2\phi = 1)$$

$$\boxed{\rho = \sqrt{x^2+y^2}}$$

$$\frac{\text{Eq}(2)}{\text{eq}(1)} \Rightarrow \frac{y}{x} = \frac{\cancel{\rho}\sin\phi}{\cancel{\rho}\cos\phi} \Rightarrow \frac{y}{x} = \tan\phi$$

$$\boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$$

$$\boxed{z = z}$$

the differential length is

$$d\bar{l} = d\rho\,\bar{a}_\rho + \rho\, d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$$

...fferential surface Vector is

$$d\bar{S}_\rho = \rho \, d\phi \, dz \, \bar{a}_\rho$$

$$d\bar{S}_\phi = dz \, d\rho \, \bar{a}_\phi$$

$$d\bar{S}_z = (\rho \, d\phi)(d\rho) \, \bar{a}_z$$

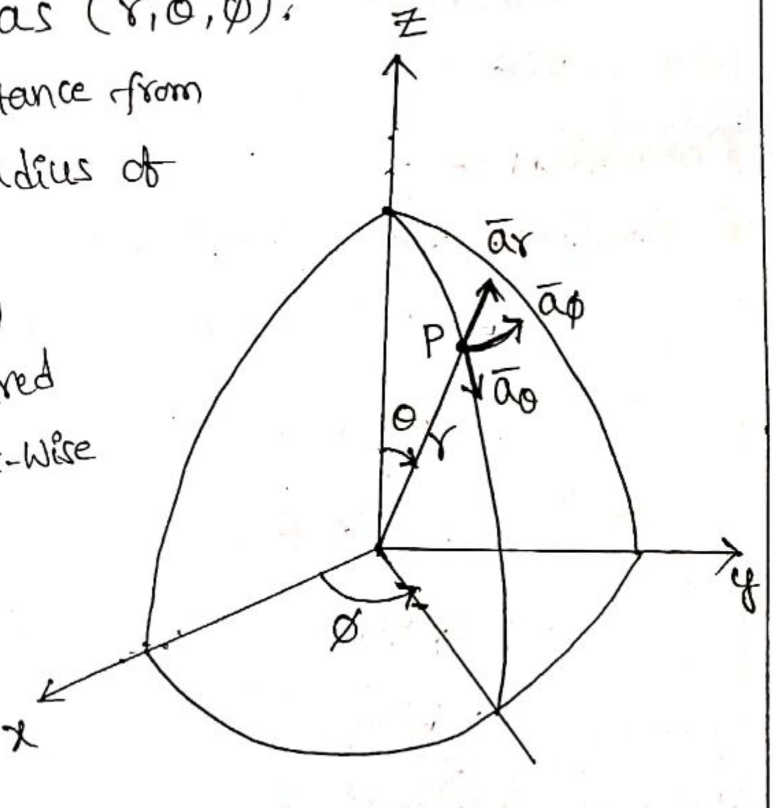
differential Volume is

$$dv = d\rho (\rho \, d\phi) \, dz$$

$$dv = \rho \, d\rho \, d\phi \, dz.$$

Spherical coordinate system (r, θ, ϕ)

Let us consider a point 'P' can be represented in spherical coordinates as (r, θ, ϕ).



- Where 'r' is the distance from origin to point 'P' (or) radius of the sphere
- 'θ' is Vertical angle (or) elevation angle measured from z-axis in clock-wise direction.
- 'φ' is horizontal angle (or) azimuthal angle measured from x-axis in anti-clockwise direction.

The ranges of Variables are

for $r \rightarrow 0 \le r \le \infty$

for $\theta \rightarrow 0 \le \theta \le \pi$

for $\phi \rightarrow 0 \le \phi \le 2\pi$

The Vector $\bar{A}$ in spherical coordinates is given by

$$\bar{A} = A_r \bar{a}_r + A_\theta \bar{a}_\theta + A_\phi \bar{a}_\phi$$

Magnitude $|\bar{A}| = \sqrt{A_r^2 + A_\theta^2 + A_\phi^2}$

The relation between cartesian system and spherical system is given by

$$x = r \sin\theta \cos\phi \Rightarrow x^2 = r^2 \sin^2\theta \cos^2\phi \rightarrow (1)$$

$$y = r \sin\theta \sin\phi \Rightarrow y^2 = r^2 \sin^2\theta \sin^2\phi \rightarrow (2)$$

$$z = r\cos\theta \Rightarrow z^2 = r^2 \cos^2\theta \rightarrow (3)$$

from above eqns

$$x^2+y^2+z^2 = r^2\sin^2\theta\cos^2\phi + r^2\sin^2\theta\sin^2\phi + r^2\cos^2\theta$$

$$= r^2\left[\sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta\right]$$

$$= r^2\left[\sin^2\theta\{\cos^2\phi + \sin^2\phi\} + \cos^2\theta\right]$$

$$= r^2\left[\sin^2\theta + \cos^2\theta\right]$$

$$= r^2(1)$$

$$= r^2$$

$(\because \sin^2\theta + \cos^2\theta = 1)$

$(\because \sin^2\phi + \cos^2\phi = 1)$

$$\therefore \boxed{r = \sqrt{x^2+y^2+z^2}}$$

from eq (3)

$$z = r\cos\theta \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{z}{r}\right)}$$

in eqns (1), (2)

$$\frac{y}{x} = \frac{\cancel{r\sin\theta}\sin\phi}{\cancel{r\sin\theta}\cos\phi} \Rightarrow \frac{y}{x} = \tan\phi$$

$$\therefore \boxed{\phi = \tan^{-1}\left(\frac{y}{x}\right)}$$

The unit vectors are

$$\left.\begin{array}{l|l} \bar{a}_r \cdot \bar{a}_r = 1 & \bar{a}_r \cdot \bar{a}_\theta = 0 \\ \bar{a}_\theta \cdot \bar{a}_\theta = 1 & \bar{a}_\theta \cdot \bar{a}_\phi = 0 \\ \bar{a}_\phi \cdot \bar{a}_\phi = 1 & \bar{a}_\phi \cdot \bar{a}_r = 0 \end{array}\right\} \text{dot products.}$$

$$\begin{array}{l|l} \bar{a}_r \times \bar{a}_r = 0 & \bar{a}_r \times \bar{a}_\theta = \bar{a}_\phi \\ \bar{a}_\theta \times \bar{a}_\theta = 0 & \bar{a}_\theta \times \bar{a}_\phi = \bar{a}_r \\ \bar{a}_\phi \times \bar{a}_\phi = 0 & \bar{a}_\phi \times \bar{a}_r = \bar{a}_\theta \end{array} \quad (\because r \to \theta \to \phi.)$$

<u>differential elements</u>

differential length vector is

$$d\bar{l} = dr\,\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi$$

differential surface vector is

$$d\bar{s}_r = (r\,d\theta)(r\sin\theta\,d\phi)\,\bar{a}_r \Rightarrow \boxed{d\bar{s}_r = r^2\sin\theta\,d\theta\,d\phi\,\bar{a}_r}$$

$$d\bar{s}_\theta = (r\sin\theta\,d\phi)(dr)\,\bar{a}_\theta \Rightarrow \boxed{d\bar{s}_\theta = r\sin\theta\,dr\,d\phi\,\bar{a}_\theta}$$

$$d\bar{s}_\phi = (dr)(r\,d\theta)\,\bar{a}_\phi \Rightarrow \boxed{d\bar{s}_\phi = r\,dr\,d\theta\,\bar{a}_\phi}$$

differential volume is

$$dv = (dr)(r\,d\theta)(r\sin\theta\,d\phi)$$

$$\therefore \boxed{dV = r^2\sin\theta\,dr \cdot d\theta \cdot d\phi}$$

Del operator :-

The del operator ∇ is the vector differential opera[tor]

In cartesian coordinates

$$\boxed{\nabla = \frac{\partial}{\partial x}\bar{a}_x + \frac{\partial}{\partial y}\bar{a}_y + \frac{\partial}{\partial z}\bar{a}_z}$$

This vector differential operator is known as "gradi-ent operator".

In cylindrical coordinates

$$\boxed{\nabla = \frac{\partial}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial}{\partial \phi}\bar{a}_\phi + \frac{\partial}{\partial z}\bar{a}_z}$$

In spherical coordinates

$$\boxed{\nabla = \frac{\partial}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial}{\partial \phi}\bar{a}_\phi}$$

gradient of a scalar :-

The gradient of a scalar field 'V' is a vector that represents both magnitude and direction of maximum space rate of increase of V

$$\text{Gradient } G = \left.\frac{dV}{dl}\right|_{max}$$

$$\boxed{\text{grad } V = \nabla V = \frac{\partial V}{\partial x}\bar{a}_x + \frac{\partial V}{\partial y}\bar{a}_y + \frac{\partial V}{\partial z}\bar{a}_z}$$

In cylindrical coordinates

$$\boxed{\nabla V = \frac{\partial V}{\partial \rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\bar{a}_\phi + \frac{\partial V}{\partial z}\bar{a}_z}$$

In spherical coordinates

$$\nabla V = \frac{\partial V}{\partial r}\bar{a}_r + \frac{1}{r}\frac{\partial V}{\partial \theta}\bar{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\bar{a}_\phi$$

Divergence of a Vector :-

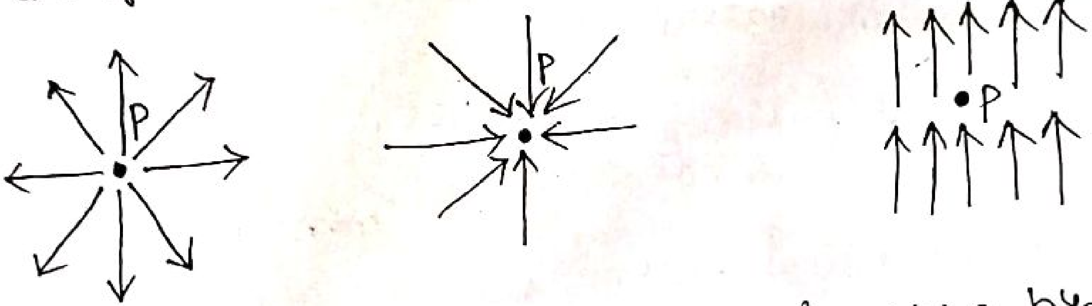
The divergence of $\bar{A}$ at a given point P is the outward flux per unit volume as the volume shrinks about 'P'.

$$\text{div}\,\bar{A} = \nabla\cdot\bar{A} = \lim_{\Delta V\to 0} \frac{\oint \bar{A}\cdot d\bar{s}}{\Delta V}$$

+ve divergence

-ve divergence

zero divergence



In Cartesian system divergence is given by

$$\text{div}\,\bar{A} = \nabla\cdot\bar{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

In cylindrical coordinates

$$\nabla\cdot\bar{A} = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\rho) + \frac{1}{\rho}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

In spherical coordinates

$$\nabla\cdot\bar{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

divergence Theorem :- It states that total outward flux of a vector $\bar{A}$ through the closed surface is same as volume integral of divergence of $\bar{A}$.

(6)

$$\oint_S \bar{A}\cdot d\bar{S} = \int_V (\nabla\cdot\bar{A})\,dv$$ → divergence theorem.

To convert surface Integral to Volume Integral, divergence theorem used.

Curl of a Vector :-

The curl of $\bar{A}$ is an axial or rotational vector whose magnitude is the maximum circulation of $\bar{A}$ per unit area as the area tends to '0' and whose direction is normal direction of area when the area is oriented to make circulation maximum.

$$\text{curl } \bar{A} = \nabla\times\bar{A} = \left(\lim_{\Delta S\to 0} \frac{\bar{a}_n \oint_l \bar{A}\cdot d\bar{l}}{\Delta S}\right)_{max}$$

In cartesian coordinate system

$$\nabla\times\bar{A} = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\bar{a}_x + \left[\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right]\bar{a}_y + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\bar{a}_z$$

In cylindrical coordinate system

$$\nabla\times\bar{A} = \frac{1}{\rho}\begin{vmatrix} \bar{a}_\rho & \rho\bar{a}_\phi & \bar{a}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

In spherical coordinate system

$$\nabla\times\bar{A} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \bar{a}_r & r\bar{a}_\theta & r\sin\theta\,\bar{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_r & rA_\theta & r\sin\theta\,A_\phi \end{vmatrix}$$

Stoke's Theorem :- It states that the circulation of a vector field $\bar{A}$ around a closed path L is equal to surface integral of curl of $\bar{A}$ over the open surface bounded by L.

$$\boxed{\oint_{l} \bar{A}\cdot d\bar{l} = \int_{S} (\nabla\times\bar{A})\cdot d\bar{s}}$$

(To convert Line Integral to surface Integral, stoke's theorem used.)

Laplacian of a scalar :- ($\nabla^2 V$)

Laplacian $V = \nabla\cdot\nabla V = \nabla^2 V$ [divergence of gradient of scalar]

In cartesian coordinate system

$$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$$

In cylindrical coordinate system

$$\boxed{\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}}$$

In spherical coordinate system

$$\boxed{\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}}$$

<u>Electro statics:-</u>

Electro statics is a static electric field that means the electric field should not vary with time. This is called a time invarient electric field.

<u>Point Charge</u>:- A point charge is an electric charge which can be spread on a surface. which can be measured in coulomb's.

$1\bar{e} = 1.602 \times 10^{-19}$ coulombs.

$1 \text{ coulomb} = \frac{1}{1.602 \times 10^{-19}} = 6.2 \times 10^{18}\ \bar{e}$

* <u>Coulomb's law</u>:- The coulomb's law is basic electrostatic law. The main function of coulomb's law is to find static behavior of the electron.

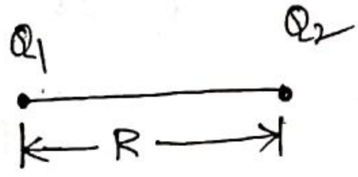
<u>statement</u>:- The coulomb's law states that the force between two point charges Q_1 and Q_2 is

(i) Along the line joining them

(ii) Directly proportional to the product $Q_1 Q_2$ of the charges. ($F \propto Q_1 Q_2$)

(iii) Inversely proportional to the square of the distance 'R' between them. ($F \propto \frac{1}{R^2}$)-

(ie) $F \propto \frac{Q_1 Q_2}{R^2}$



∴ The force between two charges is

$$\boxed{F = K \frac{Q_1 Q_2}{R^2}} \rightarrow (1)$$

that means
...ic

Where K is proportionality constant.

$$K = \frac{1}{4\pi\epsilon} \rightarrow (2).$$

ϵ = permittivity of medium. (or) dielectric constant

$\Rightarrow \epsilon = \epsilon_0 \cdot \epsilon_r$

ϵ_0 = permittivity of free space (F/m) (or) farads/meter

$= 8.854 \times 10^{-12}$ F/m

ϵ_r = relative permittivity

$= 1$

$\therefore \epsilon = \epsilon_0 \cdot 1 = \epsilon_0$ $\qquad \therefore \boxed{K = \frac{1}{4\pi\epsilon_0}} \rightarrow (3)$

from eqns (1), (3)

The force between two charges is

$$\boxed{F = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q_1 Q_2}{R^2}}$$ Newtons.

Where F is force between two charges

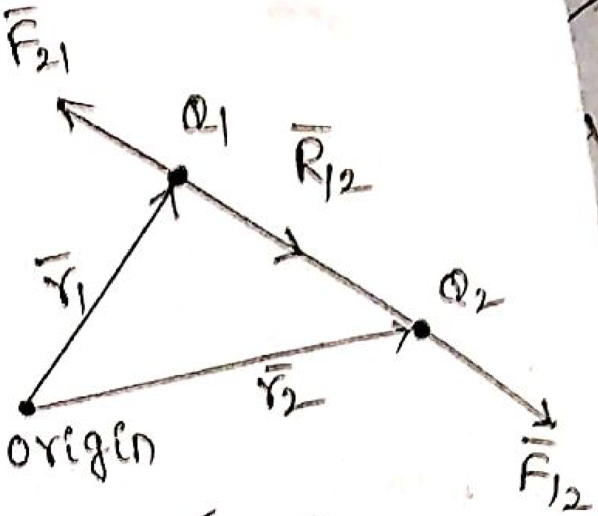
Q_1, Q_2 are two point charges.

ϵ_0 is permittivity of free space

R^2 is square of the distance b/w two charges.

a

Vector form of coulomb's law :-



The point charges Q_1 and Q_2 are located at position vectors $\bar{r}_1$ and $\bar{r}_2$ then force $\bar{F}_{12}$ on Q_2 due to Q_1 is given by

$$\boxed{\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\bar{a}_{R_{12}}} \rightarrow (1)$$

where $\bar{R}_{12} = \bar{r}_2 - \bar{r}_1 \rightarrow (2)$

$$R_{12} = |\bar{R}_{12}|$$

unit vector $\bar{a}_{R_{12}} = \frac{\bar{R}_{12}}{|\bar{R}_{12}|} = \frac{\bar{R}_{12}}{R_{12}} \rightarrow (3)$

$$\therefore \bar{a}_{R_{12}} = \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|} \rightarrow (4)$$

substituting eqns (2), (3), (4) in eq (1)

$$\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2} \cdot \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

$$= \frac{Q_1 \cdot Q_2}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^2} \cdot \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|}$$

$$\therefore \boxed{\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^3} \cdot (\bar{r}_2 - \bar{r}_1)}$$

similarly the force on Q_1 due to Q_2 is given by

$$\bar{F}_{21} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\bar{r}_1 - \bar{r}_2|^3} \cdot (\bar{r}_1 - \bar{r}_2)$$

$$\bar{F}_{21} = \frac{-Q_1 Q_2}{4\pi\epsilon_0 |\bar{r}_2 - \bar{r}_1|^3} (\bar{r}_2 - \bar{r}_1)$$

$$\boxed{\therefore \bar{F}_{21} = -\bar{F}_{12}}$$

The force due to 'n' no.of charges is

$$\boxed{\bar{F} = \frac{Q}{4\pi\epsilon_0} \sum_{k=1}^{n} \frac{Q_K(\bar{r} - \bar{r}_K)}{|\bar{r} - \bar{r}_K|^3}}$$

Applications of coulomb's law :-

By using coulomb's law we have to find

→ force between two charges

→ electric field at a point due to a fixed charge.

→ Distance between two charges.

Limitation :-

The coulomb's law doesn't applicable for arbitrary charged bodies.

Electric field Intensity :- ($\bar{E}$)

The electric field Intensity is defined as force per unit charge when placed in an electric field. It is denoted by $\bar{E}$. It can be expressed as

$$\boxed{\bar{E} = \frac{\bar{F}}{Q} \ N/C} \rightarrow \text{Vector form}$$

In scalar form $\boxed{E = \frac{V}{d}}$ Volts/meter

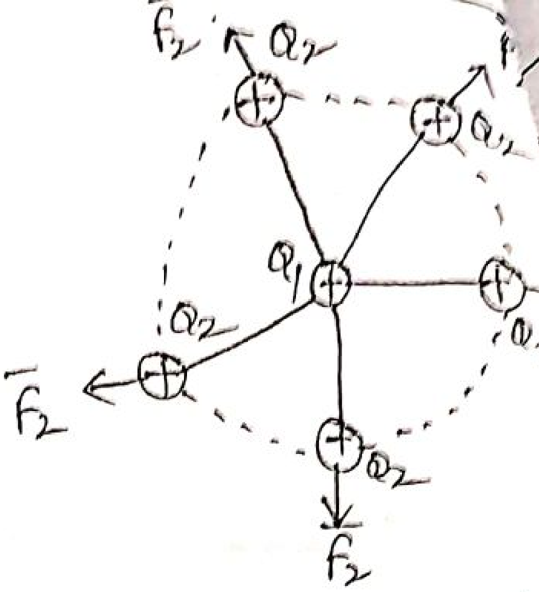
(OR)

The electric field Intensity is also defined as negative gradient of a potential due to charge.

$$\boxed{\bar{E} = -\nabla V}$$

9

According to coulomb's law



$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}$$

$$\boxed{E = \frac{F}{Q_2} = \frac{Q_1}{4\pi\epsilon_0 R^2}}$$

In vectorform the force on Q_2 due to Q_1 is

$$\bar{F}_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\,\bar{a}_{R12}$$

$$\boxed{\bar{E} = \frac{\bar{F}_{12}}{Q_2} = \frac{Q_1}{4\pi\epsilon_0 R_{12}^2}\,\bar{a}_{R12}}$$

For N point charges $Q_1, Q_2 \cdots Q_N$ located at $\bar{r}_1, \bar{r}_2, \bar{r}_3 \cdots \bar{r}_N$, the electric field intensity at point $\bar{r}$ is given by

$$\bar{E} = \frac{\bar{F}}{Q} = \frac{Q_1(\bar{r}-\bar{r}_1)}{4\pi\epsilon_0|\bar{r}-\bar{r}_1|^3} + \frac{Q_2(\bar{r}-\bar{r}_2)}{4\pi\epsilon_0|\bar{r}-\bar{r}_2|^3} + \cdots + \frac{Q_N(\bar{r}-\bar{r}_N)}{4\pi\epsilon_0|\bar{r}-\bar{r}_N|^3}$$

$$\therefore \boxed{\bar{E} = \frac{\bar{F}}{Q} = \frac{1}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{Q_K(\bar{r}-\bar{r}_K)}{|\bar{r}-\bar{r}_K|^3}}$$

Types of charge distributions :-

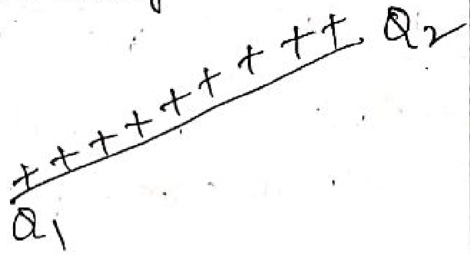
1) Point charge
2) line charge
3) surface charge
4) Volume charge:

1) **Point charge** :- If the dimensions of a surface carrying charge are very very less then this type of charge is called "point charge".

$+Q_1$ $-Q_2$ $+Q_3$

2) **line charge** :- If the charge is distributed uniformly along a line is called a line charge.

The line charge density is



$$\rho_L = \frac{\text{Total charge}}{\text{Total length}} \quad C/m \text{ (or) } \frac{\text{coulomb}}{\text{meter}}$$

$$\rho_L = \frac{Q}{L} = \frac{dQ}{dL}$$

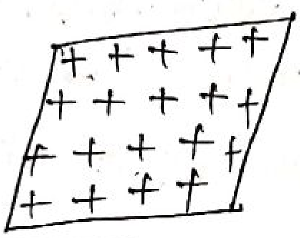
$$dQ = \rho_L dL$$

$$\boxed{Q = \int_L \rho_L dL}$$

Also $\boxed{\text{Electric field is } \bar{E} = \frac{Q}{4\pi\epsilon R^2}\bar{a}_R \quad \Rightarrow \bar{E} = \int_L \frac{\rho_L dL}{4\pi\epsilon R^2}\bar{a}_R \; N/C \text{ (or) } V/m}$

3) **Surface charge** :- If the charge is distributed uniformly over a two dimensional surface then it is a surface charge (or) sheet of charge.

The surface charge density is



$$\rho_S = \frac{\text{Total charge}}{\text{Total surface area}} \quad \text{coulombs}/m^2$$

$$\rho_S = \frac{Q}{S} = \frac{dQ}{ds} \Rightarrow dQ = \rho_S ds.$$

$$\therefore \boxed{Q = \int \rho_S ds}$$

Also $\boxed{\bar{E} = \int_S \frac{\rho_S ds}{4\pi\epsilon R^2}\bar{a}_R} \; N/C \text{ (or) } V/m$

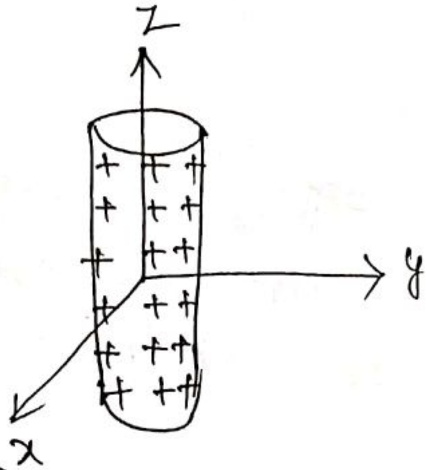
(10)

iv) <u>Volume charge</u> :- If the charge is uniformly distributed over a volume then it is called a "Volume charge".

the Volume charge density is

$$\rho_v = \frac{\text{Total charge}}{\text{Total Volume}} \text{ Coulombs}/m^3$$

$$= \frac{Q}{V}$$



$$\Rightarrow \rho_v = \frac{dQ}{dv}$$

$$\therefore dQ = \rho_v dv$$

$$\boxed{Q = \int_V \rho_v dv}$$

Also $\boxed{\bar{E} = \int_V \frac{\rho_v dv}{4\pi\epsilon R^2} \bar{a}_R}$ N/C (or) V/m

Electric flux :- (ψ)

In 1837 Michael faraday inverted electric flux. The electric flux represents direction of electric lines of force.

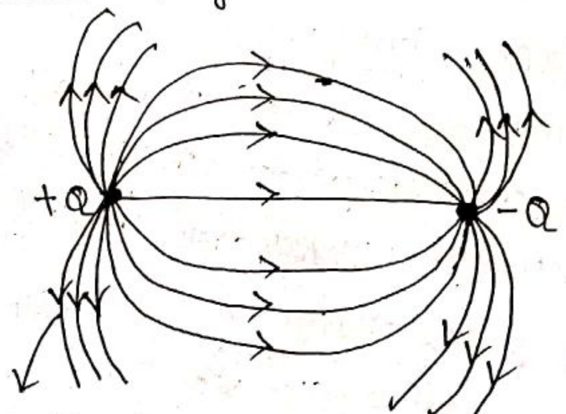
<u>Definition</u> :- the electric flux is numerically equal to the total electric charge. If charge increases the electric flux also increases.

$\therefore$ The electric flux is given by

$$\boxed{\psi = Q}$$

→ The flux starts at the positive charge and ends at the negative charge.

→ the flux lines are parallel to each other and do not dependent on the medium.



flux lines.

...tric flux density :- ($\bar{D}$)

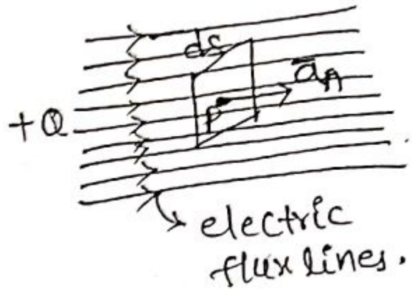
...electric flux density is defined as the net flux ...assing through unit surface area is called as "electric flux density. It is denoted as $\bar{D}$.

→ It is also called as Displacement flux density.

In scalar form $\boxed{D = \frac{\psi}{s}}$ coulombs/m^2

where ψ = flux in coulombs

s = surface area in m^2



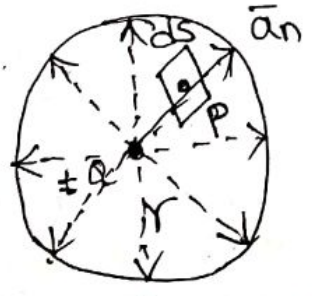
Let us consider ds be the differential surface area and $d\psi$ be the total flux passed through surface area.

The electric flux density is

$$\boxed{\bar{D} = \frac{d\psi}{ds}\bar{a}_n} \ C/m^2$$

Electric flux density due to a point charge:-

Let us consider a sphere of radius 'r' and $\bar{a}_n$ is the direction of flux normal to the surface area.



The electric flux density is

$$\bar{D} = \frac{d\psi}{ds}\bar{a}_n \ C/m^2 \rightarrow (1)$$

It can also be written as

$$\bar{D} = \frac{\text{Total flux}}{\text{surface area of sphere}}$$

$$\bar{D} = \frac{\psi}{4\pi R^2}\bar{a}_n \rightarrow (2) \qquad (\because \psi = Q)$$

If $\bar{a}_n$ is normal to surface area then

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\bar{a}_n}$$

(11)

If $\bar{a}_n = \bar{a}_r$ then

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r}$$

Relation between electric flux density $\bar{D}$ and field Intensity $\bar{E}$:-

We know that

Electric field Intensity is

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\bar{a}_r \rightarrow (A)$$

The electric flux density is

$$\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r \rightarrow (B)$$

$$\frac{(B)}{(A)} \Rightarrow \frac{\bar{D}}{\bar{E}} = \frac{\cancel{\frac{Q}{4\pi r^2}}\,\cancel{\bar{a}_r}}{\cancel{\frac{Q}{4\pi\epsilon_0 r^2}}\,\cancel{\bar{a}_r}} = \epsilon_0$$

electric [illegible] enclosed charge surface closed [illegible]

$$\therefore \frac{\bar{D}}{\bar{E}} = \epsilon_0$$

$$\boxed{\bar{D} = \epsilon_0 \bar{E}} \quad \text{(or)} \quad \boxed{\bar{D} = \epsilon \bar{E}}$$

$(\because \epsilon = \epsilon_0 \epsilon_r \cdot \; \epsilon_r = 1)$

Gauss Law :- (Integral form)

Statement :- The gauss law states that the net electric flux passed through any closed surface is equal to the total charge enclosed by that surface.

It can be expressed as

$$\boxed{\psi = Q_{enc} = \oint_s \bar{D}\cdot d\bar{s}}$$

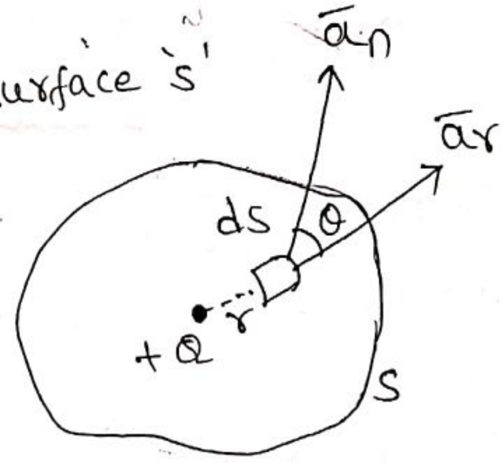
Here ψ = electric flux

$\bar{D}$ = flux density

$d\bar{s}$ = differential surface area

Q_{enc} = charge enclosed by surface.

Proof:- Let us consider arbitrary surface 's'



The differential surface area is

$$d\bar{s} = ds\,\bar{a}_n \rightarrow ①$$

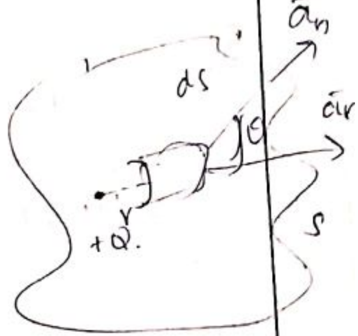
The electric flux density is

$$\bar{D} = \frac{Q}{4\pi r^2} \cdot \bar{a}_r \rightarrow ②$$

Taking dot product on both sides with $d\bar{s}$

$$\bar{D} \cdot d\bar{s} = \frac{Q}{4\pi r^2}\bar{a}_r \cdot d\bar{s}$$

$$= \frac{Q\bar{a}_r}{4\pi r^2} \cdot d\bar{s}$$



$$\Rightarrow \bar{D} \cdot d\bar{s} = \frac{Q\bar{a}_r}{4\pi r^2} \cdot ds\,\bar{a}_n \quad (\because \text{from eq } ①)$$

$$= \frac{Q}{4\pi r^2} ds\, \bar{a}_r \cdot \bar{a}_n$$

$$= \frac{Q}{4\pi r^2} ds\,(\cos\theta) \quad (\because \bar{a}_r \cdot \bar{a}_n = |\bar{a}_r||\bar{a}_n| \times \cos\theta)$$

$$= \cos\theta.$$

$$\therefore \bar{D} \cdot d\bar{s} = \frac{Q}{4\pi r^2} ds\cos\theta \rightarrow ③$$

Taking surface integral on both sides of eq ③.

$$\oint_S \bar{D} \cdot d\bar{s} = \oint_S \frac{Q}{4\pi r^2} ds\cos\theta = \frac{Q}{4\pi} \oint_S \frac{ds\cos\theta}{r^2}$$

But the solid angle of sphere is $\oint_S \frac{ds\cos\theta}{r^2} = 4\pi$

$$\oint_S \bar{D}\cdot d\bar{S} = \frac{Q}{\cancel{4\pi}} \times \cancel{4\pi}$$

$$\therefore \boxed{\oint_S \bar{D}\cdot d\bar{S} = Q_{enc} = \psi} \quad \text{Hence proved.}$$

Gauss law In point form:- [Maxwell's 1st Equation]

Statement:- "It states that the divergence of electric flux density is equal to volume charge density at a given point in a medium.

$$\boxed{\nabla\cdot\bar{D} = \rho_V} \quad (\text{or}) \quad \boxed{\text{div}\,\bar{D} = \rho_V}$$

Proof:-

We know that Gauss law in Integral form is

$$\oint_S \bar{D}\cdot d\bar{S} = Q_{enc} = \psi \rightarrow ①$$

But volume charge density is

$$\rho_V = \frac{dQ}{dV} \Rightarrow dQ = \rho_V\, dV$$

$$Q = \int_V \rho_V\, dV \rightarrow ②$$

from ① and ②

$$\oint_S \bar{D}\cdot d\bar{S} = \int_V \rho_V\, dV \rightarrow ③$$

Applying divergence theorem on eq ③

$$\therefore \oint_S \bar{D}\cdot d\bar{S} = \int_V (\nabla\cdot\bar{D})\, dV \rightarrow ④$$

Comparing eq ③ and ④ we get

$$\oint_S \bar{D}\cdot d\bar{S} = \int_V (\nabla\cdot\bar{D})\, dv = \int_V \rho_v\, dv$$

$\therefore \boxed{\nabla\cdot\bar{D} = \rho_v}$ Hence proved.

Applications of Gauss law :-

By using Gauss law we have to calculate

(i) Electric flux density ($\bar{D}$)

(ii) Electric field Intensity ($\bar{E}$)

(iii) charge enclosed by the surface (Q)

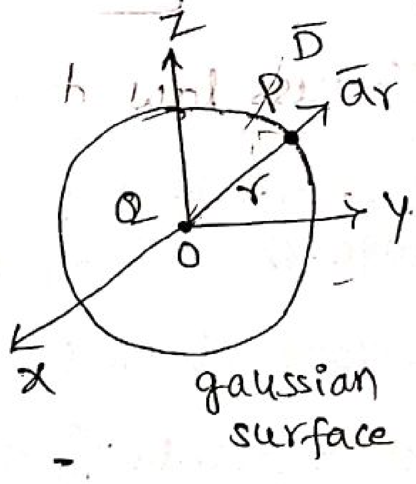
The gauss law can be applied to point charge, line charge, surface charge, Volume charge.

Limitation:- Gauss law can not be applicable for non-gaussian surface (or) un symmetrical surfaces.

Gauss law due to a point charge:-

Let us consider a point charge 'Q' located at origin

The integral form of gauss law is



$$Q = \oint_S \bar{D}\cdot d\bar{S} \rightarrow ①$$

$$\bar{D} = D_r \bar{a}_r \rightarrow ②$$

According to spherical coordinate system

$$d\bar{S}_r = (r\,d\theta)(r\sin\theta\, d\phi)\,\bar{a}_r$$

$$\therefore d\bar{S}_r = r^2\sin\theta\, d\theta\, d\phi\, \bar{a}_r \rightarrow ③$$

from eqns ①, ②, ③

$$Q = \oint_S D_r\bar{a}_r\cdot d\bar{S}_r = \oint_S D_r\bar{a}_r\cdot(r^2\sin\theta\, d\theta\, d\phi)\,\bar{a}_r$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} D_r\cdot r^2\sin\theta\, d\theta\, d\phi \qquad (\because \bar{a}_r\cdot\bar{a}_r = 1)$$

(13)

$$Q = D_r r^2 \int_{\phi=0}^{2\pi} d\phi \int_{\theta=0}^{\pi} \sin\theta \, d\theta$$

$$= D_r r^2 (\phi)_0^{2\pi} \left[-\cos\theta\right]_0^{\pi}$$

$$= D_r \cdot r^2 (2\pi) \left[(-\cos\pi) - (-\cos 0)\right]$$

$$= D_r \cdot r^2 \, 2\pi \left[-\cos\pi + \cos 0\right] \qquad (\because \cos\pi = -1)$$

$$= D_r \cdot r^2 \cdot 2\pi (1+1) \qquad (\cos 0^\circ = 1)$$

$$Q = 4\pi \cdot D_r \cdot r^2$$

$$\boxed{D_r = \frac{Q}{4\pi r^2}} \quad (or) \quad \boxed{\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r}$$

But $\bar{D} = \epsilon \bar{E}$

$$\bar{E} = \frac{\bar{D}}{\epsilon}$$

$$\therefore \boxed{\bar{E} = \frac{Q}{4\pi \epsilon r^2} \bar{a}_r}$$

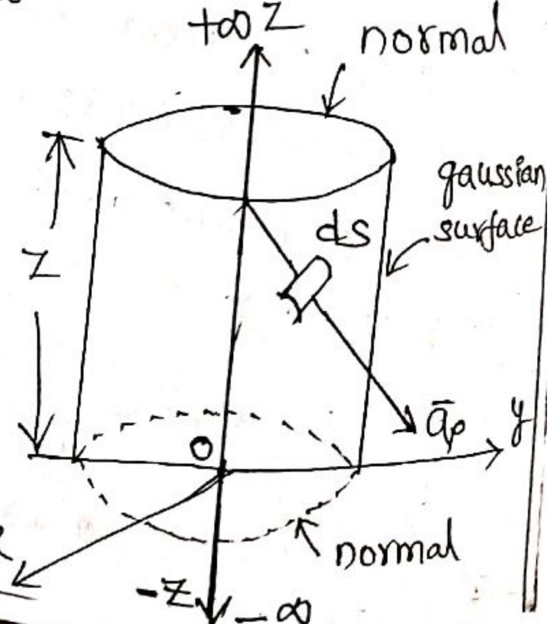
Gauss law due to an infinite line charge :- (ρ_L)

Let us consider an infinite line charge Q, charge density is ρ_L coulomb/meter

The range of z is from $-\infty$ to $+\infty$

From Gauss' law the net charge enclosed by cylindrical surface is

$$Q = \oint_s \bar{D} \cdot \bar{ds} \rightarrow (1)$$



...rge evaluated over closed surface of cylinder is

$$Q = \int_{side} \bar{D}\cdot d\bar{S} + \int_{top} \bar{D}\cdot d\bar{S} + \int_{bottom} \bar{D}\cdot d\bar{S} \longrightarrow (2)$$

But $\bar{D}$ has radial component $\bar{a}_\rho$

But $\bar{D}$ has radial component $\bar{a}_\rho$.
$\therefore$ No dimensions of top and bottom.

$$\Rightarrow \int_{top} \bar{D}\cdot d\bar{S} = \int_{bottom} \bar{D}\cdot d\bar{S} = 0.$$

$$\therefore Q = \int_{side} \bar{D}\cdot d\bar{S} = \int_{side} \bar{D}\cdot d\bar{S}_\rho$$

But $\bar{D} = D\bar{a}_\rho$

$d\bar{S}_\rho = \rho\, d\phi\, dz\, \bar{a}_\rho$

$$\Rightarrow Q = \int_{side} (D\bar{a}_\rho)\cdot(\rho\, d\phi\, dz)\bar{a}_\rho$$

$$= \int_0^z \int_0^{2\pi} D\rho\, d\phi\, dz\, (\bar{a}_\rho\cdot\bar{a}_\rho) \qquad (\because \bar{a}_\rho\cdot\bar{a}_\rho = 1)$$

$$= D\rho \int_0^z dz \int_0^{2\pi} d\phi$$

$$= D\rho\, (z)_0^z\, (\phi)_0^{2\pi}$$

$$\therefore Q = D\rho(z)(2\pi)$$

$$\therefore \boxed{D = \frac{Q}{2\pi\rho z}}$$

But $Q = \rho_L z$

$$\therefore D = \frac{\rho_L \cancel{z}}{2\pi\rho \cancel{z}} \Rightarrow \boxed{D = \frac{\rho_L}{2\pi\rho}}\ C/m^2 \qquad (\because \bar{D} = \epsilon\bar{E})$$

The electric flux density is

$$\boxed{\bar{D} = \frac{\rho_L}{2\pi\rho}\bar{a}_\rho\ C/m^2}$$

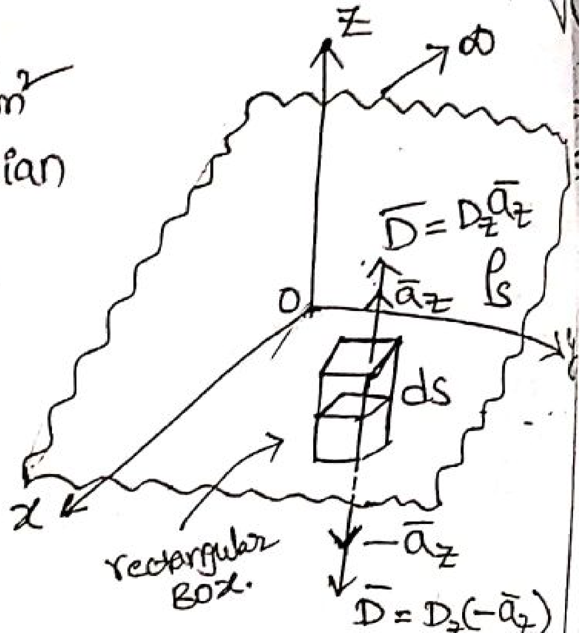
The electric field Intensity is

$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon\rho}\bar{a}_\rho\ N/C}$$

(14)

Gauss' Law due to an Infinite sheet charge (Surface ch...)

- consider an Infinite sheet of charge (surface charge) lying along x-y plane at $z=0$.
- The surface charge density is ρ_s C/m^2
- Let a rectangular box be a Gaussian surface, parallel to the z-axis.
- Let 'ds' be the differential surface area



From gauss' law, the charge enclosed by the surface is

$$Q = \oint_S \bar{D}\cdot d\bar{s} \longrightarrow (1)$$

The charge enclosed over rectangular box is

$$Q = \int_{side} \bar{D}\cdot d\bar{s} + \int_{top} \bar{D}\cdot d\bar{s} + \int_{bottom} \bar{D}\cdot d\bar{s}$$

since $\bar{D}$ has no components in the x, y-directions, that is $\bar{D}$ is in z-direction. (Both +ve z direction, -ve z-direction)

$$\therefore \int_{side} \bar{D}\cdot d\bar{s} = 0.$$

$$\therefore Q = \int_{top} \bar{D}\cdot d\bar{s} + \int_{bottom} \bar{D}\cdot d\bar{s} \longrightarrow (2)$$

$$\int_{top} \bar{D}\cdot d\bar{s} = \int_{top} (D_z\bar{a}_z)\cdot(dx\,dy\,\bar{a}_z).$$

$$= D_z \int_{top} dx \int_{top} dy \cdot (\bar{a}_z\cdot\bar{a}_z) \qquad (\because \bar{a}_z\cdot\bar{a}_z = 1)$$

$$= D_z(x)(y)$$

$$= D_z(S) \longrightarrow (3)$$

$$\int_{bottom} \bar{D}\cdot d\bar{s} = \int_{bottom} (-D_z\bar{a}_z)\cdot(dx\,dy(-\bar{a}_z)) = D_z \int_{bottom} dx \cdot \int_{bottom} dy\,(\bar{a}_z\cdot\bar{a}_z)$$

$(\because \bar{a}_z \cdot \bar{a}_z = 1)$

$\int_{bottom} \bar{D} \cdot \bar{ds} = D_z(x)(y) = D_z S \rightarrow (4)$ $(\because S = \text{surface area})$

substitute eq (3),(4) in eq (2)

$\therefore Q = D_z(S) + D_z(S) = 2D_z(S)$

$\Rightarrow D_z = \frac{Q}{2S}$ $(\because \text{surface charge density } \rho_s = \frac{Q}{S})$

$\Rightarrow D_z = \frac{\rho_s}{2}$

$\bar{D} = D_z \bar{a}_z = \frac{\rho_s}{2} \bar{a}_z \ c/m^2$

$\therefore$ The Electric flux density $\boxed{\bar{D} = \frac{\rho_s}{2} \bar{a}_z}$ c/m^2 $(\because \bar{D} = \epsilon \bar{E})$

The electric field Intensity $\boxed{\bar{E} = \frac{\bar{D}}{\epsilon} = \frac{\rho_s}{2\epsilon} \bar{a}_z}$ N/c

Gauss' Law due to a Volume charge :-

We know that, from gauss Law due to a point charge,

The electric flux density is $\boxed{\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r}$ $c/m^2 \rightarrow (1)$

Volume charge density $\rho_v = \frac{dQ}{dv} \ c/m^3$

$\Rightarrow dQ = \rho_v dV$

$$\boxed{Q = \int_V \rho_v dV}$$

substitute 'Q' value in eq (1), we get

$\therefore \boxed{\bar{D} = \int_V \frac{\rho_v dV}{4\pi r^2} \bar{a}_r}$ c/m^2

Similarly Electric field Intensity $\boxed{\bar{E} = \frac{\bar{D}}{\epsilon} = \int_V \frac{\rho_v dV}{4\pi\epsilon r^2} \bar{a}_r}$ V/m (or) N/c

Workdone :-(W) The Workdone is defined as the product of Force and distance. It is denoted by 'W'.

The Workdone (W) = Force × distance

Differential Workdone $dW = \bar{F} \cdot d\bar{l}$

$$= -(\bar{E} \cdot Q_t) \cdot d\bar{l}$$

$$\therefore \boxed{dW = -Q_t \bar{E} \cdot d\bar{l}} \text{ Joules.}$$

Total work done is $W = \int_A^B dW = \int_A^B -Q_t \bar{E} \cdot d\bar{l}$

$$\therefore \boxed{W = -Q_t \int_A^B \bar{E} \cdot d\bar{l}} \text{ Joules.}$$

Electric potential :- The electric potential is defined as workdone per unit charge. It is denoted by 'V'.

$$\boxed{V = \frac{W}{Q} = -\int_{\infty}^{P} \bar{E} \cdot d\bar{l}} \text{ Joules/coulombs (or) Volts.}$$

(OR)

→ The potential is defined as the workdone in unit charge moving from ∞ to point 'p' in electricfield opposite to the force.

→ Electric potential at 'r' due to a point charge Q at r_1 is

$$\boxed{V = \frac{Q}{4\pi\epsilon |\bar{r} - \bar{r}_1|} + C}$$

where 'C' is potential at reference point. C = 0 if potential at ∞ is zero.

If the field is conservative, the potential around a closed path is equal to zero. $\boxed{V = \oint_l \bar{E} \cdot d\bar{l} = 0}$

The absolute potential is $\boxed{V = \frac{Q}{4\pi\epsilon r}}$ Volts

The potential due to different charge distributions.

$$V = \int_l \frac{\rho_L dL}{4\pi\epsilon r} = \int_S \frac{\rho_s ds}{4\pi\epsilon r} = \int_V \frac{\rho_v dv}{4\pi\epsilon r} \text{ Volts.}$$

Potential Difference :- The potential difference is defined as difference between potentials at given points A and B. (ie)

$$V_{BA} = \frac{W}{Q} = -\int_A^B \bar{E} \cdot d\bar{l}$$

$$V_{BA} = V_B - V_A.$$

$$V_B = -\int_{\infty}^{B} \bar{E}\cdot d\bar{l} \quad , \quad V_A = -\int_{\infty}^{A} \bar{E}\cdot d\bar{l}$$

The potential difference is

$$\boxed{V_{BA} = \frac{W}{Q} = V_B - V_A}$$

$$\Rightarrow \boxed{W = QV_{BA} = Q(V_B - V_A)}$$

Potential gradient :- (∇V)

gradient of potential is $\text{grad}V = \left.\frac{dV}{dl}\right|_{max} = \nabla V$

We know that $V = -\int \bar{E}\cdot d\bar{l} \rightarrow (1)$

$$dV = -\left[d\int \bar{E}\cdot d\bar{l}\right]$$

$$dV = -\bar{E}\cdot d\bar{l} \qquad (\because \bar{A}\cdot\bar{B} = AB\cos\theta)$$

$$dV = -(E\,dl\cos\theta)$$

$$\frac{dV}{dl} = -E\cos\theta$$

potential is maximum at $\cos\theta = 1$

$$\therefore \left.\frac{dV}{dl}\right|_{max} = -E$$

$$\Rightarrow \nabla V = -E$$

$$\boxed{E = -\nabla V}$$

(or)

$$\boxed{\bar{E} = -\nabla V}$$

$\therefore$ The Negative gradient of a potential gives the electric field intensity.

Relation between $\bar{E}$ and V [MaxWell's 2nd Equation]

The potential differences at 'A' and 'B' are given by

$$V_{AB} = -\int_{B}^{A} \bar{E}\cdot d\bar{l} \rightarrow ①$$

$$V_{BA} = -\int_{A}^{B} \bar{E}\cdot d\bar{l} \rightarrow ②$$

$$\Rightarrow V_{BA} = -\left[-\int_{B}^{A} \bar{E}\cdot d\bar{l}\right] = -V_{AB}$$

$$\therefore V_{BA} = -V_{AB} = \int \bar{E}\cdot d\bar{l}$$

$$V_{BA} + V_{AB} = \oint_{l} \bar{E}\cdot d\bar{l} = 0 \rightarrow ③$$

Applying stoke's theorem to eq ③

($\because$ stokes theorem $\oint_{l} \bar{A}\cdot d\bar{l} = \int_{S} (\nabla\times\bar{A})\, d\bar{s}$)

$$\oint_{l} \bar{E}\cdot d\bar{l} = \int_{S} (\nabla\times\bar{E})\, d\bar{s} = 0$$

$\therefore \boxed{\nabla\times\bar{E} = 0} \rightarrow$ MaxWell's 2nd equation.

II method :- (OR)

We know that

$$\bar{E} = -\nabla V$$

Taking curl on both sides

$$\nabla\times\bar{E} = \nabla\times(-\nabla V)$$

from vector identity ($\because \nabla\times\nabla V = 0$)

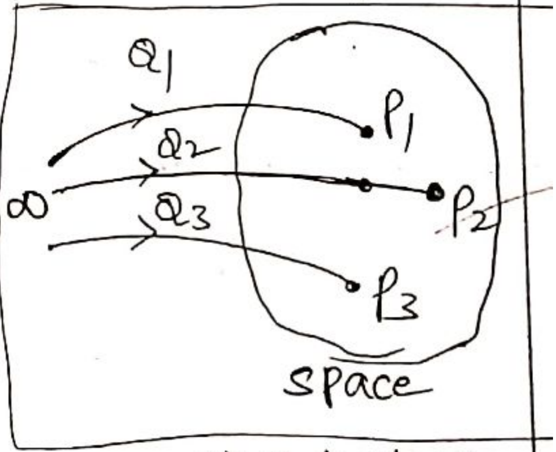
$\therefore \boxed{\nabla\times\bar{E} = 0} \rightarrow$ Represents conservative Nature of electrostatic field

Maxwell's Two Equations for electrostatic fields :-

1st Equation $\nabla \cdot \bar{D} = \rho_v$
2nd Equation $\nabla \times \bar{E} = 0$
Point form (or) differential form

1st eqn → $\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho_v \, dv$
2nd eqn → $\oint_l \bar{E} \cdot d\bar{l} = 0$
Integral form.

Energy Density :- energy density is the energy stored per unit volume.



Assembling of charges.

- When a unit positive charge is moved from '∞' to point in the field then the work is done by external source. and energy is expended.
- If the external source is removed then the unit positive charge will be subjected to the force exerted by the field and will be moved in the direction of force.
- thus to hold the charge at a point in an electro-static field an external source has to do work.
- This energy gets stored in the form of potential energy.
- When external source is removed, the potential energy gets converted to a kinetic energy.
- suppose we have to position three point charges Q_1, Q_2, Q_3 in an initially empty space.

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No work is required to transfer Q_1 from ∞ to P_1 because, inst the space is initially charge free and there is no electric field.

- The work done in transferring Q_2 from ∞ to P_2 is equal product of Q_2 and potential V_{21} at P_2 due to Q_1
- similarly the workdone in transferring Q_3 at P_3 is equal $Q_3(V_{32}+V_{31})$. Where V_{32}, V_{31} are potentials at P_3 due to Q_2 and Q_1.

$\therefore$ The total workdone in positioning three charges is

$$W_E = W_1 + W_2 + W_3$$

$$= 0 + Q_2V_{21} + Q_3(V_{31}+V_{32}) \rightarrow (1)$$

If the charges were positioned in reverse order

$$W_E = W_3 + W_2 + W_1$$

$$= 0 + Q_2V_{23} + Q_1(V_{12}+V_{13}) \rightarrow (2)$$

Where V_{23} is potential at P_2 due to Q_3.

V_{12}, V_{13} are potential at P_1 due to Q_2, Q_3

Adding eq (1) and (2)

$$2W_E = Q_2V_{21} + Q_3(V_{31}+V_{32}) + Q_2V_{23} + Q_1(V_{12}+V_{13})$$

$$\therefore 2W_E = Q_1(V_{12}+V_{13}) + Q_2(V_{21}+V_{23}) + Q_3(V_{31}+V_{32})$$

$$= Q_1V_1 + Q_2V_2 + Q_3V_3$$

$$\boxed{W_E = \frac{1}{2}(Q_1V_1 + Q_2V_2 + Q_3V_3)} \text{ Joules} \rightarrow (3)$$

If there are n point charges

$$\boxed{W_E = \frac{1}{2}\sum_{k=1}^{n} Q_kV_k} \text{ Joules} \rightarrow (4)$$

instead of point charges, the region has a continuous charge distributions, then

$$W_E = \frac{1}{2}\int_L \rho_L V\, dl \quad (\text{line charge}) \rightarrow (5)$$

$$W_E = \frac{1}{2}\int_S \rho_s V ds \quad (\text{surface charge}) \rightarrow (6)$$

$$W_E = \frac{1}{2}\int_V \rho_v V. dv \quad (\text{Volume Charge}) \rightarrow (7)$$

Since $\nabla.\bar{D} = \rho_v$ $\therefore$ eq (7) becomes

$$W_E = \frac{1}{2}\int_V (\nabla.\bar{D})V\, dV \rightarrow (8)$$

But from vector identity $\nabla. V\bar{A} = \bar{A}.\nabla V + V(\nabla.\bar{A})$

$$\Rightarrow \boxed{(\nabla.\bar{A})V = \nabla. V\bar{A} - \bar{A}.\nabla V}$$

substituting (or) applying above Identity in eq (8) we get

$$W_E = \frac{1}{2}\int_V (\nabla. V\bar{D})dv - \frac{1}{2}\int_V (\bar{D}.\nabla V)dv \rightarrow (9)$$

By applying divergence Theorem on first term of eq (9)

$$\therefore W_E = \frac{1}{2}\oint_S (V\bar{D}).d\bar{s} - \frac{1}{2}\int_V (\bar{D}.\nabla V)\, dV \rightarrow (10)$$

if the surface 'S' becomes large then $\frac{1}{2}\oint_S (V\bar{D}).d\bar{s} = 0$

$$\Rightarrow W_E = -\frac{1}{2}\int_V (\bar{D}.\nabla V)dV$$

$(\because \bar{E} = -\nabla V$
$\bar{D} = \epsilon_0 \bar{E})$

$$\therefore W_E = \frac{1}{2}\int_V (\bar{D}.\bar{E})dV \text{ Joules}$$

$$\boxed{W_E = \frac{1}{2}\int_V (\bar{D}.\bar{E})dV = \frac{1}{2}\int_V \epsilon_0 E^2 dV \text{ Joules.}}$$

Hence proved

$$\boxed{\text{Energy density } w_E = \frac{W_E}{V} = \frac{1}{2}\bar{D}.\bar{E} \text{ J/m}^3}$$

(11)

Convection and conduction currents:-

Current :- The flow of charge per unit time is called as "current". It is denoted by 'I'. units are "Ampere"

$$\therefore \boxed{I = \frac{Q}{t}} \text{ coulomb/sec (or) Ampere.}$$

$$\boxed{\bar{I} = \frac{-dQ}{dt} \text{ Amp}}$$

Convection current :- (Diffusion current) :-

The current flowing due to movement of free electrons and free holes to make uniform charge density in semi conductor materials is called " convection (or) diffusion current

Conduction current :- (Drift current).

The current flowing due to the flow of free electrons (or) the drifting of electrons in conductors is called as "conduction (or) drift current".

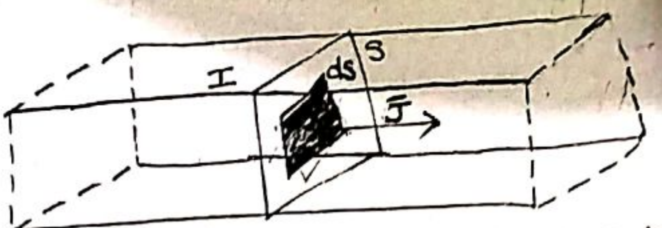
Displacement current :- The current flowing due to flow of charges in dielectrics under the influence of time Varying electricfield is called "displacement current".

Current density :- ($\bar{J}$) The current density is defined as current per unit surface area. It is denoted by $\bar{J}$.

$$\bar{J} = \frac{d\bar{I}}{d\bar{s}} \quad (A/m^2)$$

$$d\bar{I} = \bar{J} \cdot d\bar{s}$$

$$I = \oint_S \bar{J} \cdot d\bar{s}$$



$$\boxed{\bar{I} = \oint_S \bar{J} \cdot d\bar{s}}$$

Figure :- current density in a medium

for uniform surface area $I = Js$

$$\boxed{J = \frac{I}{S}}$$

Also the relation between $\bar{J}$ and $\bar{E}$ is

$$\boxed{\bar{J} = \sigma \bar{E}}$$

where $\bar{J}$ = conduction current density. A/m^2

σ = conductivity

$\bar{E}$ = electric field.

Dielectric Constant :- (ϵ_r)

It is the ratio between permittivity ϵ and free space permittivity (ϵ_0)

$$\therefore \boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$$

The flux density is $\bar{D} = \epsilon \bar{E}$

$$\epsilon = \frac{\bar{D}}{\bar{E}}$$

since $\epsilon = \epsilon_r \epsilon_0$.

$$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}} \qquad \therefore \boxed{\epsilon_r = 1 + \chi_e = \frac{\epsilon}{\epsilon_0}}$$

where χ_e is electric susceptibility

(18)

Continuity Equation :- The Continuity Equation states that the current density is diverging from a small volume is equal to negative time derivative of volume charge density.

It can be expressed as

$$\boxed{\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}}$$

proof:- We know that

$$\text{Current } \bar{I} = \oint_S \bar{J} \cdot d\bar{s} \rightarrow ①$$

$$\bar{I} = -\frac{dQ}{dt} \rightarrow ②$$

From ① and ②

$$\oint_S \bar{J} \cdot d\bar{s} = -\frac{dQ}{dt} \rightarrow ③$$

Applying divergence theorem to eq ③.

$$\oint_S \bar{J} \cdot d\bar{s} = \int_V (\nabla \cdot \bar{J}) dv = \frac{-dQ}{dt} \rightarrow ④$$

$$\text{But } -\frac{dQ}{dt} = \frac{-d}{dt}\left(\int_V \rho_v dv\right)$$

$$= -\int_V \left(\frac{\partial \rho_v}{\partial t}\right) dv$$

substitute above value in eq ④

$$\therefore \int_V (\nabla \cdot \bar{J}) dv = -\int_V \left(\frac{\partial \rho_v}{\partial t}\right) dv$$

$$\therefore \boxed{\nabla \cdot \bar{J} = \frac{-\partial \rho_v}{\partial t}}$$ Hence proved.

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...ation Time (T_r):-

Relaxation time is defined as the time taken by charge density to decrease to 36.8% of its initial value.

It is given by

$$\boxed{T_r = \frac{\epsilon}{\sigma} \text{ sec}}$$

where ϵ = permittivity of medium
σ = conductivity

Proof:- We know that

continuity Equation $\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \rightarrow (1)$

current density $\bar{J} = \sigma \bar{E} \rightarrow (2)$

The gauss law is $\nabla \cdot \bar{D} = \rho_v$ $\quad (\because \bar{D} = \epsilon \bar{E})$

$$\Rightarrow \nabla \cdot (\epsilon \bar{E}) = \rho_v$$

$$\nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon} \rightarrow (3)$$

From equations (2), (3)

$$\nabla \cdot \left(\frac{\bar{J}}{\sigma}\right) = \frac{\rho_v}{\epsilon} \qquad \left(\because \bar{J} = \sigma \bar{E},\ \bar{E} = \frac{\bar{J}}{\sigma}\right)$$

$$\Rightarrow \nabla \cdot \bar{J} = \frac{\sigma \rho_v}{\epsilon} \rightarrow (4)$$

From equations (1), (4)

$$-\frac{\partial \rho_v}{\partial t} = \frac{\sigma \rho_v}{\epsilon}$$

$$\Rightarrow \frac{\sigma \rho_v}{\epsilon} + \frac{\partial \rho_v}{\partial t} = 0 \rightarrow (5)$$

Solution of above equation is

$$\rho_v = A e^{-\frac{\sigma t}{\epsilon}} \rightarrow (6)$$

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For initial value $t=0$.

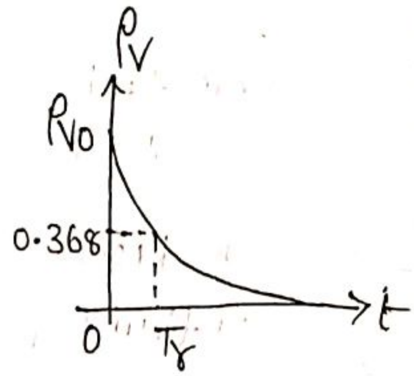
$$\Rightarrow \boxed{A = \rho_{v_0}}$$

substitute 'A' value in ⑥

Now $\rho_v = \rho_{v_0} e^{-\frac{\sigma t}{\epsilon}} \longrightarrow$ ⑦

$$\Rightarrow \rho_v = \rho_{v_0} e^{-\frac{t}{\left(\frac{\epsilon}{\sigma}\right)}}$$

$$\therefore \rho_v = \rho_{v_0} e^{-(t/T_r)}$$



where $\boxed{T_r = \frac{\epsilon}{\sigma} \text{ sec}}$

if $t = T_r$ in above equation.

$$\rho_v = \rho_{v_0} e^{-\cancel{T_r}/\cancel{T_r}} = \rho_{v_0} (e^{-1})$$

$$\therefore \rho_v = \rho_{v_0} (0.368)$$

$$\therefore \boxed{\rho_v = 0.368 \rho_{v_0}}$$

Poisson's Equation:- It states that there exist a volume charge density in a homogeneous medium.

It is given by

$$\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$$

Proof:- We know that gauss law is

$$\nabla \cdot \bar{D} = \rho_v$$

$$\nabla \cdot (\epsilon \bar{E}) = \rho_v$$

$$\Rightarrow \nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon} \rightarrow ①$$

relation between $\bar{E}$ and V is

$$\because \bar{E} = -\nabla V \rightarrow (2)$$

substitute eq (2) in eq (1)

$$\therefore \nabla\cdot(-\nabla V) = \frac{\rho_V}{\epsilon}$$

$$-\nabla^2 V = \frac{\rho_V}{\epsilon}$$

$$\therefore \boxed{\nabla^2 V = -\frac{\rho_V}{\epsilon}}$$ Hence proved.

Laplace's equation :- It states that Volume charge density is zero. $\boxed{\nabla^2 V = 0}$ $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$

capacitance :- (C) It is the ratio between charge and potential between two plates

$$\boxed{C = \frac{Q}{V}} \text{ (or) } Q = CV$$

$$\therefore \boxed{Q \propto V}$$

We have to measure the capacitance for (i) parallel plates (ii) co-axial cable (iii) spherical conductors.

(i) Capacitance for parallel plates :-

Let us consider a capacitor consists of two conducting plates A and B seperated by free space (or) a dielectric medium of thickness d with permittivity ϵ.

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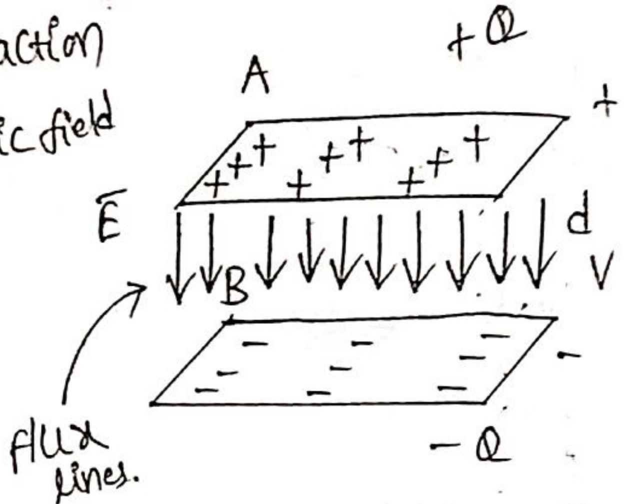
> Let the conducting platy carry equal and opposite charg

Due to the force of attraction between charges an electric field perpendicular to conducting surface.



parallel plate capacitor

> If the seperation between the plates is 'd' meters and area of conductor is 'S'(or)'A' then the capacitance of the parallel plate capacitor is

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon S}{d}} \text{ (or) } \boxed{C = \frac{\epsilon A}{d}} \text{ Farads.}$$

Where C = Capacitance
Q = Charge
V = voltage (or) potential
S = A = surface Area
d = seperation between platy

proof :-

Let two parallel plates in the x-y plane be seperated by a distance 'd' meters along z-axis. ϵ be the permittivity of dielectric medium, between platy

The flux density is $\bar{D} = \rho_s \bar{a}_z \rightarrow (1)$

$$\epsilon \bar{E} = \rho_s \bar{a}_z$$

$$\boxed{\bar{E} = \frac{\rho_s}{\epsilon} \bar{a}_z}$$

$$\Rightarrow \bar{E} = \frac{Q/S}{\epsilon} \bar{a}_z$$

$$\bar{E} = \frac{Q}{\epsilon S}\bar{a}_z$$

$\therefore$ potential difference is

$(\because d\bar{l} = dx\bar{a}_x + dy\bar{a}_y + dz\bar{a}_z)$ $dx, dy = 0$.)

$$V = \oint_l \bar{E}\cdot d\bar{l} = \oint_l \frac{Q}{\epsilon S}\bar{a}_z\cdot(dz\,\bar{a}_z)$$

$$\Rightarrow V = \frac{Q}{\epsilon S}\oint_l dz\,(\bar{a}_z\cdot\bar{a}_z) \qquad (\because \bar{a}_z\cdot\bar{a}_z = 1)$$

$$\Rightarrow V = \frac{Q}{\epsilon S}(z)_0^d = \frac{Q}{\epsilon S}(d) \qquad (\because z = d)$$

along z-axis.

$$\therefore \frac{Q}{V} = \frac{\epsilon S}{d}$$

The capacitance for parallel plate capacitor is

$$\boxed{C = \frac{Q}{V} = \frac{\epsilon S}{d}} \text{ farads.}$$

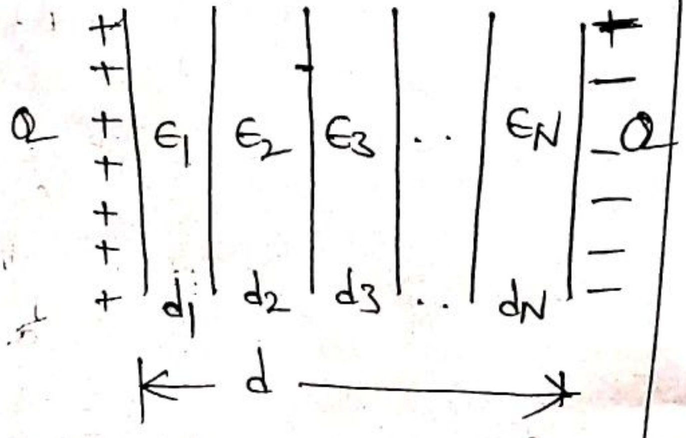
If the capacitor with different dielectric materials then the capacitance is

$$C = \frac{Q}{V} = \frac{S}{\left(\frac{d_1}{\epsilon_1} + \frac{d_2}{\epsilon_2} + \frac{d_3}{\epsilon_3} + \cdots + \frac{d_N}{\epsilon_N}\right)} \text{ farads.}$$

Where $\epsilon_1, \epsilon_2 \cdots \epsilon_N$ are different dielectric materials.

S = surface area

$d_1, d_2, d_3 \cdots d_N$ are thickness.



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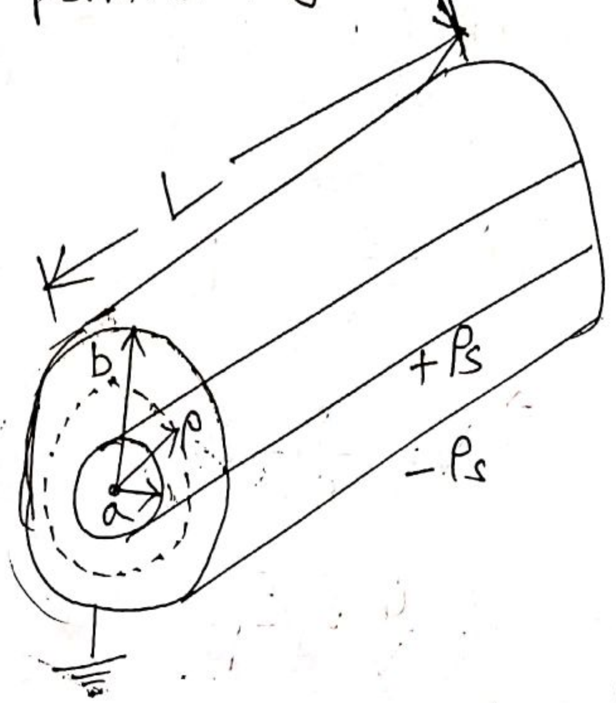
Capacitance of a Co-axial Cable :-

Consider two coaxial conducting cylinders seperated by a dielectric material of permittivity 'ϵ'.



→ The radius of the inner conductor is 'a' and radius of outer conductor is 'b'. height is 'L'.

- $+\rho_s$ be the charge density on the inner conductor, $-\rho_s$ be the charge density on the outer conductor.

The flux density is $\bar{D} = \rho_s \bar{a}_\rho$

$$\Rightarrow \epsilon \bar{E} = \rho_s \bar{a}_\rho$$

$$\boxed{\bar{E} = \frac{\rho_s}{\epsilon} \bar{a}_\rho} \rightarrow (1)$$

But charge $Q = \rho_s \cdot ds$

$$\Rightarrow Q = \rho_s \cdot 2\pi \rho L$$

$$\rho_s = \frac{Q}{2\pi \rho L} \rightarrow (2)$$

We know that potential is $V = \int_a^b \bar{E} \cdot d\bar{l} = \int_a^b \frac{\rho_s}{\epsilon} \bar{a}_\rho \cdot d\rho \, \bar{a}_\rho$

$$= \int_a^b \frac{\rho_s}{\epsilon} d\rho \, (\bar{a}_\rho \cdot \bar{a}_\rho) \qquad (\because \bar{a}_\rho \cdot \bar{a}_\rho = 1)$$

$$\cancel{\int_a^b \frac{\rho_s}{\epsilon}}$$

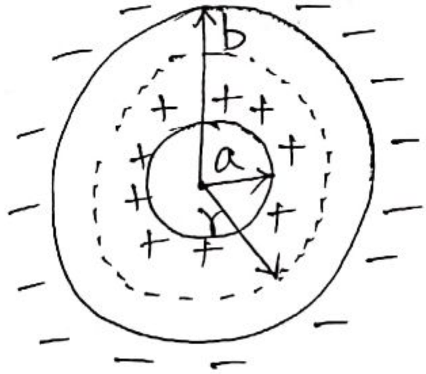
$$= \int_a^b \frac{Q}{2\pi \rho \epsilon L} d\rho$$

$$\therefore V = \frac{Q}{2\pi \epsilon L}\int_a^b \frac{1}{\rho}\,d\rho = \frac{Q}{2\pi\epsilon L}\left[\ln(b) - \ln(a)\right]$$

$$\therefore V = \frac{Q}{2\pi\epsilon L}\ln(b/a)$$

$$\therefore \boxed{C = \frac{Q}{V} = \frac{2\pi\epsilon L}{\ln(b/a)}} \text{ farads.}$$

capacitance of a spherical conductors :-



Let us consider two spheres of radii are a, b.

We know that electric field Intensity is

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \rightarrow (1)$$

The potential difference is

$$V = \int_a^b \bar{E}\cdot d\bar{l}$$

$(\because d\bar{l} = dr\bar{a}_r + r\,d\theta\,\bar{a}_\theta + r\sin\theta\,d\phi\,\bar{a}_\phi)$

$$= \int_a^b \frac{Q}{4\pi\epsilon_0 r^2}\bar{a}_r \cdot dr\bar{a}_r$$

$(\text{But } \theta, \phi = 0 \; \therefore d\bar{l} = dr\,\bar{a}_r)$

$$= \frac{Q}{4\pi\epsilon_0}\int_a^b \frac{1}{r^2}\,dr$$

$(\because \bar{a}_r \cdot \bar{a}_r = 1)$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{r^{-2+1}}{-2+1}\right)_a^b$$

$$= \frac{Q}{4\pi\epsilon_0}\left(\frac{r^{-1}}{-1}\right)_a^b$$

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$$\therefore V = \frac{Q}{4\pi\epsilon_0}\left(\frac{-1}{r}\right)_a^b = \frac{Q}{4\pi\epsilon_0}\left[\frac{-1}{b} - \left(\frac{-1}{a}\right)\right]$$

$$V = \frac{Q}{4\pi\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$\therefore \boxed{C = \frac{Q}{V} = \frac{4\pi\epsilon_0}{\frac{1}{a} - \frac{1}{b}}} \text{ Farads.}$$

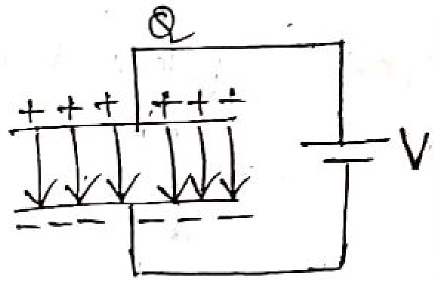
for $b = \infty$

$$\boxed{C = \frac{Q}{V} = 4\pi\epsilon a} \text{ Farads.}$$

Energy stored in a capacitor :-

When potential 'V' Volts is applied on a capacitor "C". the energy stored in the electric field of a capacitor is given by $\boxed{W_E = \frac{1}{2}CV^2}$ Joules.

proof :- Let 's' be the cross-sectional area, Q be the charge and "d" be the seperation between plates.



Energy stored in a capacitor

The flux density $\bar{D} = \frac{Q}{S}\bar{a}_x$ and the electric field Intensity $\bar{E} = \frac{V}{d}\bar{a}_x$

$$W_E = \frac{1}{2}\int_v \bar{D}\cdot\bar{E}\,dv = \frac{1}{2}\int_v \frac{Q}{S}\bar{a}_x \cdot \frac{V}{d}\bar{a}_x\,dv$$

$$\Rightarrow W_E = \frac{1}{2}\int_v \frac{QV}{Sd}(\bar{a}_x\cdot\bar{a}_x)\,dv = \frac{1}{2}\frac{QV}{Sd}\int_v dv \qquad (\because \bar{a}_x\cdot\bar{a}_x = 1)$$

$$\therefore W_E = \frac{1}{2}\frac{QV}{Sd}(v) = \frac{1}{2}\frac{QV}{\cancel{v}}\cancel{(v)}$$

($\because$ Volume $v = Sd$
S = surface area
d = distance)

$$\therefore W_E = \frac{1}{2}QV \text{ (OR) } \boxed{W_E = \frac{1}{2}CV^2}$$

($\because Q = CV$)