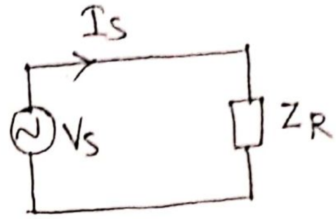
TRANSMISSION LINES :- II.

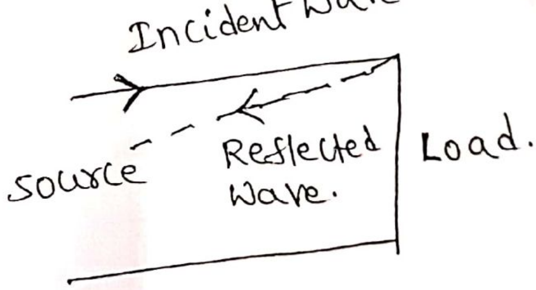
Incident and Reflected Waves :-



* When the source is applied on the line, the voltage and current components of travelling wave decrease exponentially along line with e^{-rx}. This wave is called as "Incident wave".
* At the destination end (far end), due to impedance mismatch, the waves reflects back and travels in opposite direction. This wave is called "reflected wave".
* The voltage and current components of reflected wave again decrease exponentially along line in negative x-direction (ie) $-x$. So the decreasing factor is $e^{-r(-x)} = e^{rx}$.
* The Incident and reflected waves combined on the line produce a "standing waves".

The voltage and current components of a standing wave are.

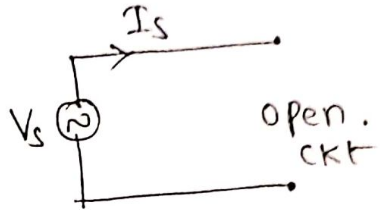
$$V = \underbrace{be^{-rx}}_{\text{Incident}} + \underbrace{ae^{rx}}_{\text{Reflected}}.$$



$$I = \frac{1}{Z_0}\left(\underbrace{be^{-rx}}_{\text{Incident wave}} - \underbrace{ae^{rx}}_{\text{Reflected wave}}\right)$$

Open and short circuited lines :- (OC & SC)

Open circuited line (OC) :- An open circuited line is a transmission line, whose far end (or) load end is open.



*** Standing waves on open ckt lines & short ckt lines**

→ In a transmission line, there will be two waves travelling in opposite directions between input and output ends.

Voltage V is maximum
Current I is minimum (almost '0')
Impedance is
$Z = \frac{V}{I} = \infty$.

→ At some points, the two waves will always be out of phase and will cancell, and at some other points, they will be in phase and will be added.

→ The places, where the two waves added is termed as "Antinodes."

→ The places, where the two waves cancelled (or) subtracted is termed as "Nodes".

→ At Anti node, there will be maximum voltage (or) current

→ At node, there will be minimum voltage (or) current.

For OC :- Vmax, Imin at even multiples of $\frac{\lambda}{4}$ distance from load

For SC :- Vmax, Imin at odd multiples of $\frac{\lambda}{4}$ distance from Load.

∴ for OC lines, the standing waves are shown below.

Loss less lines.

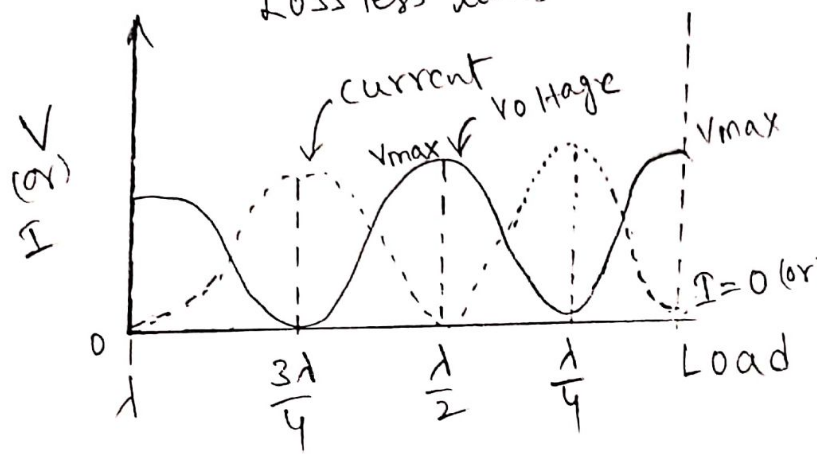
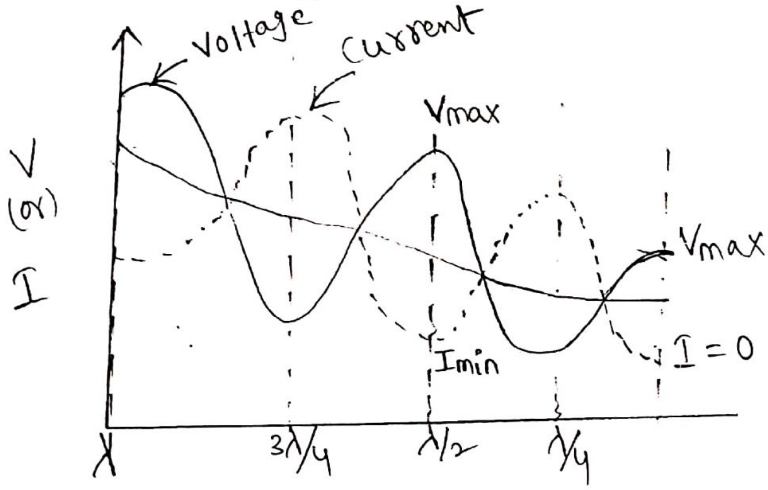


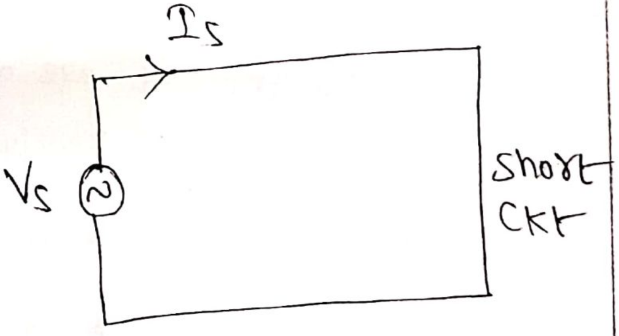
fig :- Voltage & Current distributions along OC line

Lossy lines.



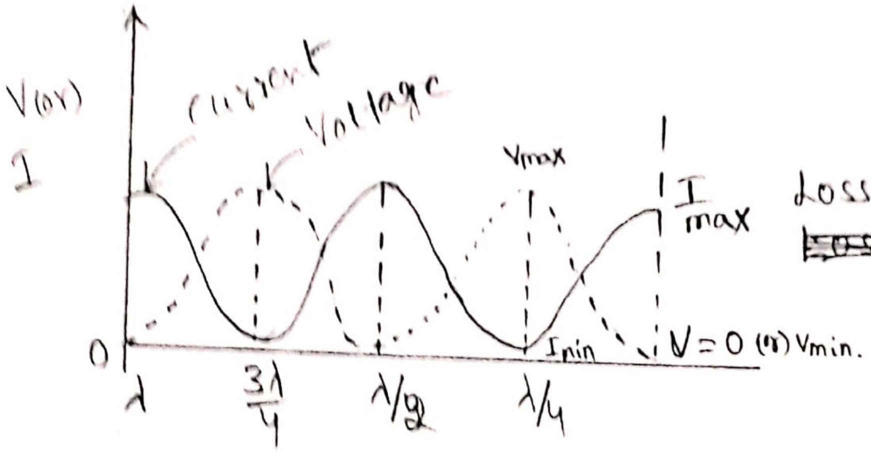
Short circuited lines:- A short circuited line is a line, whose far end (Load end) is shorted.

At short end ckt potential (or) voltage 'V' is '0' and current 'I' is maximum.

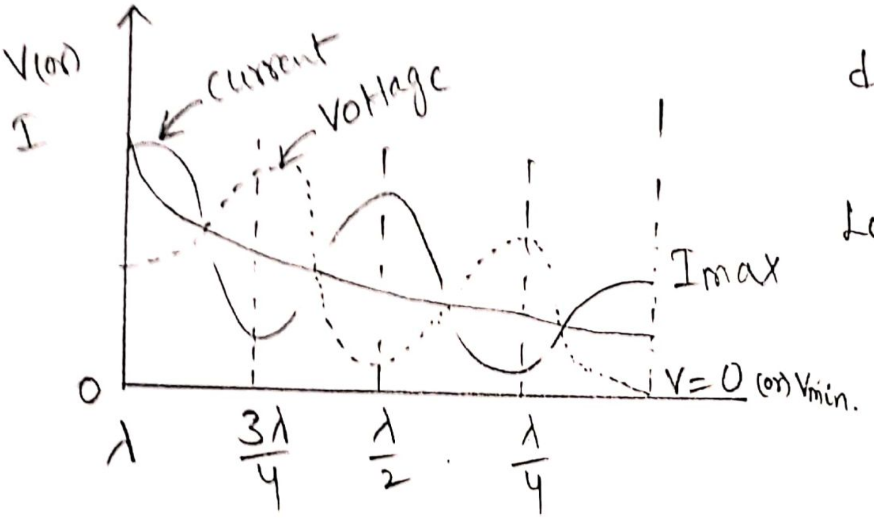


∴ Impedance at Load end is $Z = \frac{V}{I} = \frac{0}{I} = 0$.

The standing waves for short ckt line are shown below.



Lossless lines



Lossy lines

fig
Voltage & current distribution of SC lines.

Input Impedance of a open and short circuited Lines:-

Consider a transmission line of length 'l'. Let V_s and I_s be the voltage and current applied at the source end, respectively.

Also let V_R be the voltage at Load end, I_R be the current at Load end. (Receiving end).

γ be the propagation constant.

The Transmission line equations are given by

$$V = V_s \cosh\gamma x - I_s Z_0 \sinh\gamma x \longrightarrow ①$$

$$I = I_s \cosh\gamma x - \frac{V_s}{Z_0} \sinh\gamma x \longrightarrow ②$$

At $x = l$, $V = V_R$, $I = I_R$

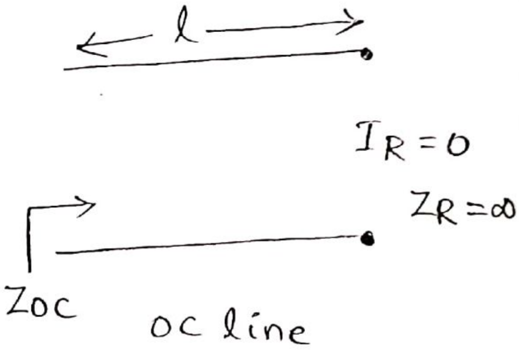
$$V_R = V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l \rightarrow ③$$

$$I_R = I_s \cosh\gamma l - \frac{V_s}{Z_0} \sinh\gamma l \rightarrow ④$$

For open circuited Line :- (OC line)

When the load is open, the load current is zero.

(ie) $I_R = 0$, so $Z_R = \infty$



from eq ④

$$I_s \cosh\gamma l - \frac{V_s}{Z_0} \sinh\gamma l = 0.$$

$$\Rightarrow I_s \cosh\gamma l = \frac{V_s}{Z_0} \sinh\gamma l.$$

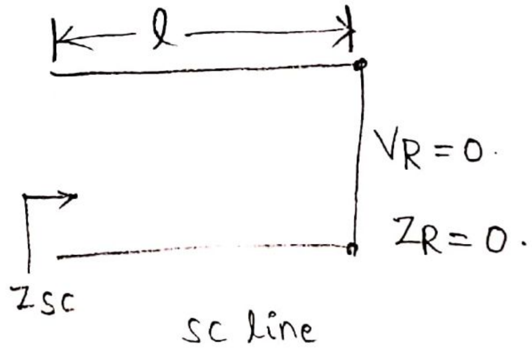
$$\therefore \frac{V_s}{I_s} = Z_0 \cdot \frac{\cosh\gamma l}{\sinh\gamma l}$$

assume Z_{oc} be input impedance of OC line.

$$\boxed{Z_{oc} = \frac{V_s}{I_s} = Z_0 \cdot \coth\gamma l} \rightarrow ⑤$$

Short Circuited line :- (SC line)

When the load is shorted, the load voltage is zero. (ie) $V_R = 0$.



from eq ③

$$V_s \cosh\gamma l - I_s Z_0 \sinh\gamma l = 0$$

$$\Rightarrow V_s \cosh\gamma l = I_s Z_0 \sinh\gamma l$$

$$\therefore \frac{V_s}{I_s} = Z_0 \frac{\sinh\gamma l}{\cosh\gamma l}$$

assume Z_{sc} be input impedance of SC line.

$$\boxed{Z_{sc} = \frac{V_s}{I_s} = Z_0 \tanh\gamma l} \rightarrow ⑥$$

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Secondary constants in terms of Z_{oc} and Z_{sc} :-

From equations (5), (6)

$$Z_{oc} = Z_0 \coth \gamma l \quad , \quad Z_{sc} = Z_0 \tanh \gamma l$$

$$Z_{oc} \cdot Z_{sc} = Z_0^2 \coth \gamma l \times \tanh \gamma l.$$

$$= Z_0^2 \frac{1}{\cancel{\tanh \gamma l}} \times \cancel{\tanh \gamma l}$$

$$\therefore Z_0^2 = Z_{oc} \times Z_{sc}$$

$$\boxed{Z_0 = \sqrt{Z_{oc} \times Z_{sc}}}$$

Also. $$\frac{Z_{sc}}{Z_{oc}} = \frac{\cancel{Z_0} \tanh \gamma l}{\cancel{Z_0} \coth \gamma l} = \frac{\tanh \gamma l}{\frac{1}{\tanh \gamma l}}$$

$$\therefore \frac{Z_{sc}}{Z_{oc}} = \tanh^2 \gamma l$$

$$\Rightarrow \tanh \gamma l = \sqrt{\frac{Z_{sc}}{Z_{oc}}}$$

$$\Rightarrow \gamma l = \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right)$$

$$\boxed{\gamma = \frac{1}{l} \tanh^{-1}\left(\sqrt{\frac{Z_{sc}}{Z_{oc}}}\right)}$$

Where Z_0, γ are secondary constants.

Z_0 = characteristic impedance.

γ = propagation constant

Z_{oc} = input impedance of OC line

Z_{sc} = input impedance of SC line

Input Impedance of Lossless OC and SC lines :-

We know that, for lossless line $\alpha=0$, $\gamma=\alpha+j\beta=0+j\beta$

Input impedance of OC line is $\gamma=j\beta$.

$$Z_{OC}=Z_0 \coth h\gamma l$$

$$=Z_0 \coth(j\beta)l$$

$$=Z_0 \coth(j\beta l) \qquad \left(\because \coth x=\frac{e^{x}+e^{-x}}{e^{x}-e^{-x}}\right)$$

$$=Z_0\cdot\left(\frac{e^{j\beta l}+e^{-j\beta l}}{e^{j\beta l}-e^{-j\beta l}}\right) \qquad \left(\because \begin{array}{l} e^{j\theta}=\cos\theta+j\sin\theta \\ e^{-j\theta}=\cos\theta-j\sin\theta \end{array}\right)$$

$$=Z_0\cdot\left(\frac{\cancel{2}\cos\beta l}{\cancel{2}j\sin\beta l}\right) \qquad (\because j^2=-1)$$

$$=-jZ_0\cot\beta l.$$

$$\therefore \boxed{Z_{OC}=-jZ_0\cot\beta l}$$

Input impedance of SC line is

$$Z_{SC}=Z_0 \operatorname{Tanh}\gamma l$$

$$=Z_0 \operatorname{Tanh}(j\beta l)$$

$$=Z_0\left[\frac{e^{j\beta l}-e^{-j\beta l}}{e^{j\beta l}+e^{-j\beta l}}\right] \qquad \left(\because \operatorname{Tanh} x=\frac{e^{x}-e^{-x}}{e^{x}+e^{-x}}\right)$$

$$=Z_0\left[\frac{\cancel{2}j\sin\beta l}{\cancel{2}\cos\beta l}\right]$$

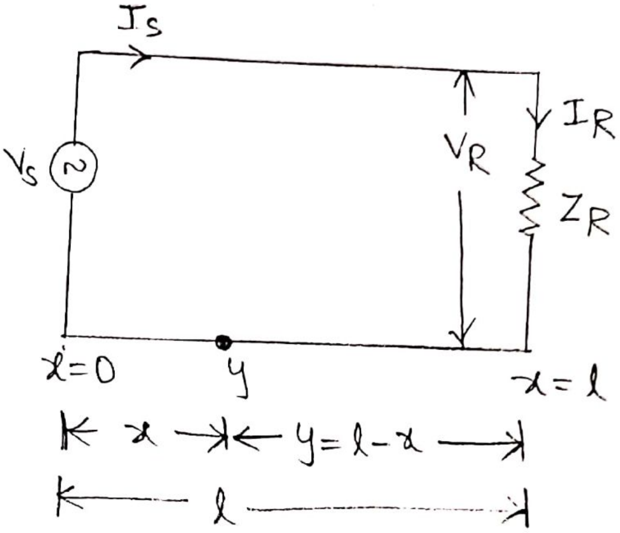
$$=jZ_0\left(\frac{\sin\beta l}{\cos\beta l}\right)$$

$$\therefore \boxed{Z_{SC}=jZ_0\cdot\operatorname{Tan}\beta l}$$

The Z_{OC}, Z_{SC} are purely reactive components

Transmission line Terminated With Load Impedance (Z_R)

* Consider a wave is applied on a Transmission line.



* If the line is of Infinite length (or) if the line is terminated with characteristic impedance 'Z_0', then there is no reflected wave.
* This is called as perfectly matched Transmission line. (ie) $Z_R = Z_0$, $Z_{in} = Z_0$.

* If the line is terminated with a load impedance 'Z_R', some part of the energy will transfer to Load, and other part of the wave will be reflected back.
* Thus, standing waves appear on the line, resulting in loss of power.

Voltage and current distributions :-

consider a transmission line of length 'l' terminating with an impedance Z_R. V_R be Voltage at Z_R and I_R be current through Z_R.

The Voltage and current at any point on Transmission line are

$$V = A\cosh\gamma x + B\sinh\gamma x \longrightarrow (1)$$

$$I = \frac{-1}{Z_0}[B\cosh\gamma x + A\sinh\gamma x] \longrightarrow (2)$$

Where A and B are constants.

At $x=l$, $V=V_R$, and $I=I_R$.

substitute these values in eq (1), (2)

$V_R = A\cosh\gamma l + B\sinh\gamma l \longrightarrow (3)$

$I_R = -\frac{A}{Z_0}\sinh\gamma l - \frac{B}{Z_0}\cosh\gamma l \rightarrow (4)$

Multiplying eq (3) with $\cosh\gamma l$ and eq (4) with $\sinh\gamma l$

$V_R\cosh\gamma l = A\cosh^2\gamma l + B\sinh\gamma l \cdot \cosh\gamma l \longrightarrow (5)$

~~$Z_0 I_R = -A\sinh\gamma l - B$~~

$Z_0 I_R \sinh\gamma l = -A\sinh^2\gamma l - B\cosh\gamma l \cdot \sinh\gamma l \longrightarrow (6)$

Adding eqns (5), (6), to get 'A' value

$V_R\cosh\gamma l + I_R Z_0 \sinh\gamma l = A(\cosh^2\gamma l - \sinh^2\gamma l)$

$\therefore \boxed{A = V_R\cosh\gamma l + I_R Z_0 \sinh\gamma l}$ $(\because \cosh^2\theta - \sinh^2\theta = 1)$

Similarly multiplying eq (3) with $\sinh\gamma l$ and eq (4) with $\cosh\gamma l$.

$V_R\sinh\gamma l = A\cdot\cosh\gamma l\cdot\sinh\gamma l + B\sinh^2\gamma l \longrightarrow (7)$

$Z_0 I_R\cosh\gamma l = -A\cdot\sinh\gamma l\cdot\cosh\gamma l - B\cdot\cosh^2\gamma l \rightarrow (8)$

Adding eq (7), (8) to get 'B' value

$V_R\sinh\gamma l + I_R Z_0\cosh\gamma l = B(\sinh^2\gamma l - \cosh^2\gamma l)$

$= -B(\cosh^2\gamma l - \sinh^2\gamma l)$

$= -B.$

$\therefore \boxed{B = -V_R\sinh\gamma l - I_R Z_0\cosh\gamma l}$

By substituting values of A and B in eq (1)

$V = (V_R\cosh\gamma l + I_R Z_0\sinh\gamma l)\cosh\gamma x - (V_R\sinh\gamma l + I_R Z_0\cosh\gamma l) \times \sinh\gamma x$

$= V_R(\cosh\gamma l\cosh\gamma x - \sinh\gamma l\cdot\sinh\gamma x) + Z_0 I_R(\sinh\gamma l\cdot\cosh\gamma x - \cosh\gamma l\cdot\sinh\gamma x)$

$\therefore \boxed{V = V_R\cosh\gamma(l-x) + Z_0 I_R\sinh\gamma(l-x)}$ 5

Similarly substituting A and B values in eq (2)

$$\boxed{I = I_R \cosh\gamma(l-x) + \frac{V_R}{Z_0}\sinh\gamma(l-x)}$$

If $y = l-x$ is distance from load point, then

$$V = V_R\cosh\gamma y + I_R Z_0 \sinh\gamma y \rightarrow (9)$$

$$I = I_R\cosh\gamma y + \frac{V_R}{Z_0}\sinh\gamma y \rightarrow (10)$$

general line equations at a point 'y' from Load end.

Input Impedance of a Transmission line :-

At $x=0$, $y = l-x = l$. , $V = V_s$, and $I = I_s$.

The input impedance is given by

$$Z_{in} = Z_s = \frac{V_s}{I_s} = \frac{V_R\cosh\gamma l + I_R Z_0 \sinh\gamma l}{I_R\cosh\gamma l + \frac{V_R}{Z_0}\sinh\gamma l}$$

($\because$ from general line equations at Load end)

$$\Rightarrow Z_{in} = \left[\frac{V_R\cosh\gamma l + I_R Z_0\sinh\gamma l}{\frac{I_R Z_0\cosh\gamma l + V_R\sinh\gamma l}{Z_0}}\right]$$

$$\Rightarrow Z_{in} = Z_0\left[\frac{V_R\cosh\gamma l + I_R Z_0\sinh\gamma l}{V_R\sinh\gamma l + I_R Z_0\cosh\gamma l}\right]$$

$$= Z_0\left[\frac{\frac{V_R}{I_R}\cosh\gamma l + Z_0\sinh\gamma l}{\frac{V_R}{I_R}\sinh\gamma l + Z_0\cosh\gamma l}\right]\frac{\cancel{I_R}}{\cancel{I_R}}$$

($\because$ Taken common as I_R of both numerator & denominator)

$$= Z_0\left[\frac{Z_R\cosh\gamma l + Z_0\sinh\gamma l}{Z_R\sinh\gamma l + Z_0\cosh\gamma l}\right]$$

$\because \left(Z_R = \frac{V_R}{I_R}\right)$

$$Z_{in} = Z_0 \left[\frac{\cancel{\cosh \gamma l}\left[Z_R + Z_0 \frac{\sinh \gamma l}{\cosh \gamma l}\right]}{\cancel{\cosh \gamma l}\left[Z_0 + Z_R \frac{\sinh \gamma l}{\cosh \gamma l}\right]}\right]$$

$$\therefore \boxed{Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right]} \; ***$$

For a Loss less line :-

Since for a lossless line $\alpha = 0$, so $\gamma = \alpha + j\beta$.

$$\therefore \boxed{\gamma = j\beta}$$

The input impedance is

$$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh(j\beta l)}{Z_0 + Z_R \tanh(j\beta l)}\right] \qquad (\because \tanh(j\theta) = j\tan\theta)$$

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right]}$$

Where $\beta = \frac{2\pi}{\lambda}$, $Z_R = \frac{V_R}{I_R} \rightarrow$ Load impedance.

$\beta \rightarrow$ phase constant

$$\beta l = \frac{2\pi}{\lambda} \times l = \omega\sqrt{LC}\, l.$$

* When the load impedance (Z_R) is equal to characteristic impedance (Z_0), the transmission line is said to be "perfectly matched" (or) proper Termination.

Reflection :- The phenomena of a wave reflected at load due to improper termination is called "Reflection".

* When $Z_R \neq Z_0$, the incident wave is reflected back at the load. So the power loss occurs due to reflection.

Reflection coefficient :- (Γ)

Reflection coefficient is defined as the ratio of the reflected voltage to the incident voltage. (OR) The ratio of Reflected current to Incident current.

→ It is a vector quantity.

$$\therefore \boxed{\Gamma = \frac{V_r}{V_i}} \quad (OR) \quad \boxed{\Gamma = -\frac{I_r}{I_i}}$$

($\because$ for Reflected current has 180° out of phase)

Where V_r, V_i are Reflected and Incident Voltages.

I_r, I_i are Reflected and Incident Currents.

$$\text{so} \quad \boxed{\Gamma = \frac{V_r}{V_i} = \frac{Z_R - Z_0}{Z_R + Z_0}}$$

Proof :- We know that, Voltage and current distributions are

$$V = a e^{\gamma x} + b e^{-\gamma x} \longrightarrow (1)$$

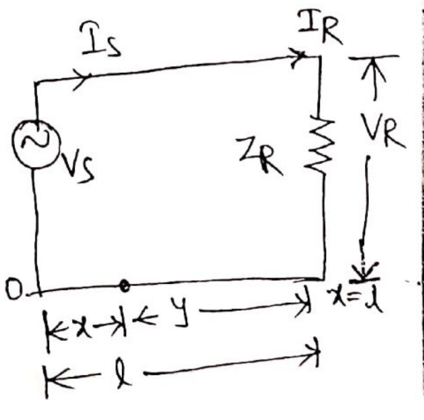
$$I = \frac{1}{Z_0}\left(b e^{-\gamma x} - a e^{\gamma x}\right) \longrightarrow (2)$$

If y is distance measured from Load end Z_R, then replacing x with $-y$. (ie). $\boxed{x = -y}$

so we get

$$V = b e^{\gamma y} + a e^{-\gamma y} \longrightarrow (3)$$

$$I = \frac{1}{Z_0}\left(b e^{\gamma y} - a e^{-\gamma y}\right) \longrightarrow (4)$$



From the definition of Reflection Coefficient,

$$\Gamma = \frac{V_r}{V_i} = \frac{a e^{-\gamma y}}{b e^{\gamma y}} = \left(\frac{a}{b}\right) e^{-2\gamma y}$$

But exactly at the Receiving side (or) load side $y=0$,

$V = V_R$,

$I = I_R$.

$$\Rightarrow \boxed{\Gamma = \frac{a}{b}} \rightarrow (5)$$

Now eqns (3), (4) at Receiving side becomes,

$V_R = a+b, \quad \Rightarrow V_R = b + \cancel{a}$

$I_R = \frac{1}{Z_0}(b-a) \Rightarrow I_R Z_0 = b - \cancel{a}$

$V_R + I_R Z_0 = 2b.$

$$\therefore \boxed{b = \frac{1}{2}(V_R + I_R Z_0)}$$

substitute 'b' value in $V_R = a+b$.

$$\Rightarrow V_R = a + \frac{1}{2}(V_R + I_R Z_0)$$

$$\Rightarrow a = V_R - \frac{1}{2}V_R - \frac{I_R Z_0}{2}$$

$$\therefore \boxed{a = \frac{1}{2}(V_R - I_R Z_0)}$$

from eq (5), Reflection Coefficient is

$$\Gamma = \frac{a}{b} = \frac{\cancel{\frac{1}{2}}(V_R - I_R Z_0)}{\cancel{\frac{1}{2}}(V_R + I_R Z_0)} = \frac{V_R - I_R Z_0}{V_R + I_R Z_0}$$

$$\therefore \Gamma = \frac{\cancel{I_R}\left(\frac{V_R}{I_R} - Z_0\right)}{\cancel{I_R}\left(\frac{V_R}{I_R} + Z_0\right)} = \frac{\frac{V_R}{I_R} - Z_0}{\frac{V_R}{I_R} + Z_0}.$$

$$\therefore \boxed{\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0}}$$ where $Z_R = \frac{V_R}{I_R}$ = Load impedance.

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Reflection Coefficient for different Conditions:-

(a) for perfect matched termination, ie $\boxed{Z_R = Z_0}$.

$$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{Z_0 - Z_0}{Z_0 + Z_0} \Rightarrow \boxed{\Gamma = 0}$$

∴ Reflection is '0'.

(b) for Short Ckt Termination (ie) $\boxed{Z_R = 0}$

$$\Gamma = \frac{0 - Z_0}{0 + Z_0} = \frac{-\cancel{Z_0}}{\cancel{Z_0}} \Rightarrow \boxed{\Gamma = -1}$$

The entire Incident wave reflects back with 180° phase shift

(c) for open Ckt Termination (ie) $\boxed{Z_R = \infty}$

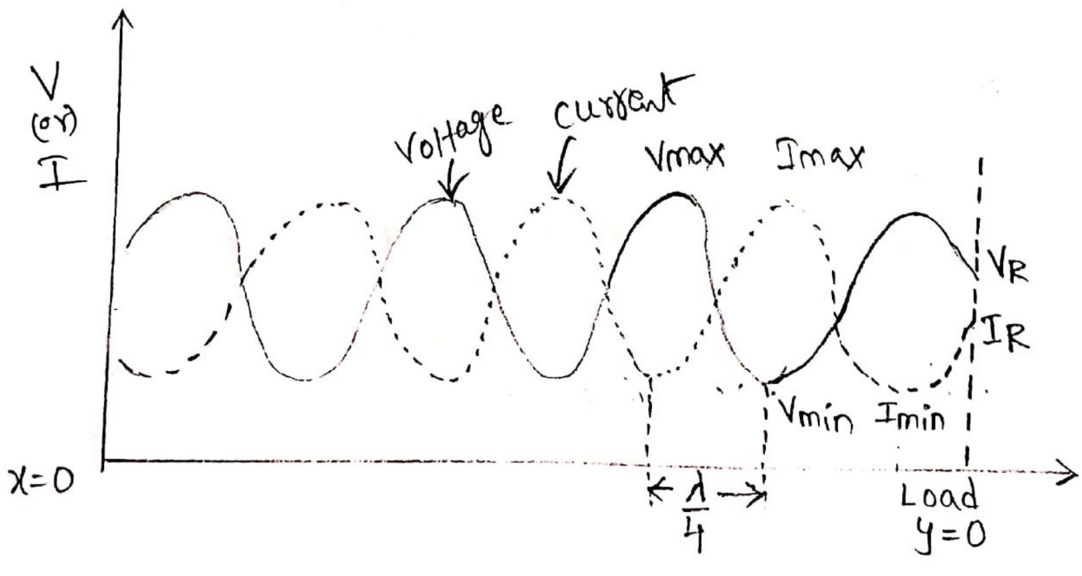
$$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} = \frac{\cancel{Z_R}\left(1 - \frac{Z_0}{Z_R}\right)}{\cancel{Z_R}\left(1 + \frac{Z_0}{Z_R}\right)} = \frac{1 - \frac{Z_0}{\infty}}{1 + \frac{Z_0}{\infty}} = \frac{1 - 0}{1 + 0}$$

$$\therefore \boxed{\Gamma = 1}$$

The entire Incident wave reflects back with same phase (0°) shift.

Standing Wave Ratio:- (S):-

The ratio of maximum magnitude to minimum magnitude is called "Standing Wave Ratio".



V_{max} be the maximum voltage, V_{min} be the minimum Voltage
I_{max} be the Maximum current, I_{min} be the Minimum Current.

→ The Maximum Values occur when the incident and reflected waves are added.

(ie) $|V_{max}| = |V_i| + |V_r| \rightarrow (1)$

$|I_{max}| = |I_i| + |I_r| \rightarrow (2)$

→ The minimum values occur when the incident and Reflected waves are subtracted.

(i.e) $|V_{min}| = |V_i| - |V_r| \rightarrow (3)$

$|I_{min}| = |I_i| - |I_r| \rightarrow (4)$

Voltage standing wave Ratio :- (VSWR)

The ratio of magnitude of maximum voltage to magnitude of minimum voltage is called a "Voltage standing wave Ratio" (VSWR)

Thus $\boxed{VSWR = S = \dfrac{|V_{max}|}{|V_{min}|}}$

Interms of Reflection coefficient :-

$$S = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$

$$\Rightarrow S = \frac{\cancel{|V_i|}\left[1 + \frac{|V_r|}{|V_i|}\right]}{\cancel{|V_i|}\left[1 - \left|\frac{V_r}{V_i}\right|\right]} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

reflection coefficient
$\left(\because \Gamma = \dfrac{V_r}{V_i}\right)$
$|\Gamma| = \left|\dfrac{V_r}{V_i}\right|$

$$\therefore \boxed{S = \frac{1 + |\Gamma|}{1 - |\Gamma|}} \quad (OR) \quad \boxed{|\Gamma| = \frac{S-1}{S+1}}$$

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Current Standing Wave Ratio :- (ISWR)

The ratio of maximum magnitude of current to minimum magnitude of current is called "current standing wave Ratio".

(ISWR)
$$\boxed{ISWR = \frac{|I_{max}|}{|I_{min}|}}$$

VSWR for different conditions :-

(a) for perfectly matched, $Z_R = Z_0$ and $|\Gamma| = 0$.

$\Rightarrow S = \frac{1+0}{1-0} = 1 \Rightarrow \boxed{S=1}$

(b) for an OC line, $Z_R = \infty$, $\Gamma = 1 \Rightarrow |\Gamma| = 1$

$\Rightarrow S = \frac{1+1}{1-1} = \frac{2}{0} = \infty$.

$\therefore \boxed{S = \infty}$

(c) for a SC line $Z_R = 0$, $\Gamma = -1 \Rightarrow |\Gamma| = 1$

$\Rightarrow S = \frac{1+1}{1-1} = \frac{2}{0} = \infty$.

$\therefore \boxed{S = \infty}$

→ The range of reflection coefficient is $-1 \leq \Gamma \leq 1$.

→ The range of VSWR is $0 \leq S \leq \infty$

Input Impedance interms of Reflection Coefficient :-

We know that the input impedance of a Transmission line terminated with Z_R is

$$Z_{in} = Z_0\left(\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l}\right)$$

$$\Rightarrow Z_{in} = Z_0\left[\frac{Z_R \cosh \gamma l + Z_0 \sinh \gamma l}{Z_0 \cosh \gamma l + Z_R \sinh \gamma l}\right]$$

In exponential form,

$$\cosh \gamma l = \frac{e^{\gamma l} + e^{-\gamma l}}{2}, \quad \sinh \gamma l = \frac{e^{\gamma l} - e^{-\gamma l}}{2}$$

Thus.

$$Z_{in} = Z_0\left[\frac{Z_R(e^{\gamma l} + e^{-\gamma l}) + Z_0(e^{\gamma l} - e^{-\gamma l})}{Z_0(e^{\gamma l} + e^{-\gamma l}) + Z_R(e^{\gamma l} - e^{-\gamma l})}\right] \cdot \frac{(\cancel{\tfrac{1}{2}})}{(\cancel{\tfrac{1}{2}})}$$

$$= Z_0\left[\frac{e^{\gamma l}(Z_R + Z_0) + e^{-\gamma l}(Z_R - Z_0)}{e^{\gamma l}(Z_0 + Z_R) - e^{-\gamma l}(Z_R - Z_0)}\right]$$

$$= Z_0\left[\frac{\cancel{e^{\gamma l}(Z_R + Z_0)}\left\{1 + \frac{e^{-\gamma l}(Z_R - Z_0)}{e^{\gamma l}(Z_R + Z_0)}\right\}}{\cancel{e^{\gamma l}(Z_R + Z_0)}\left\{1 - \frac{e^{-\gamma l}(Z_R - Z_0)}{e^{\gamma l}(Z_R + Z_0)}\right\}}\right]$$

$$\boxed{\therefore Z_{in} = Z_0\left[\frac{1 + e^{-2\gamma l} \cdot \Gamma}{1 - e^{-2\gamma l} \cdot \Gamma}\right]}$$

Where $\Gamma = \dfrac{Z_R - Z_0}{Z_R + Z_0}$.

9

UHF Lines as circuit elements :-

→ At ultra high frequency, the transmission line becomes lossless. The short length SC and O.C transmission lines can be used as circuit elements

SC (short ckt) lines at different lengths :-

1) for lengths $(0 < l < \frac{\lambda}{4})$

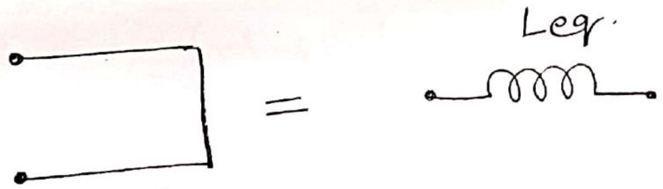
We know that $Z_{SC} = jZ_0 \tan\beta l$. (∵ lossless)

Z_{SC} is purely reactive. So if L_{eq} = equivalent Inductance then

$$\cancel{j}\omega L_{eq} = \cancel{j} Z_0 \tan\beta l.$$

$$\therefore \boxed{L_{eq} = \frac{Z_0}{\omega} \tan\beta l.}$$

Therefore The line acts as an Inductor.



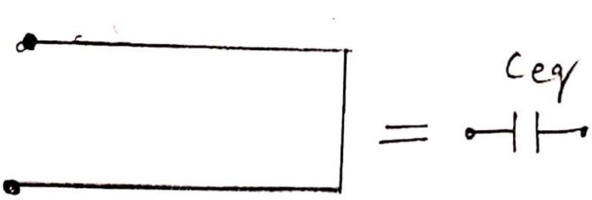
$0 < l < \frac{\lambda}{4}$

2) for lengths $(\frac{\lambda}{4} < l < \frac{\lambda}{2})$.

$$\frac{1}{j\omega C_{eq}} = jZ_0 \tan\beta l$$

$$\Rightarrow C_{eq} = \frac{1}{j^2 \omega Z_0 \tan\beta l}$$

$$\therefore \boxed{C_{eq} = \frac{-1}{\omega Z_0 \tan\beta l}}$$



$\frac{\lambda}{4} < l < \frac{\lambda}{2}$

the line acts as capacitor.

c) <u>for $l = \frac{\lambda}{4}$</u> :- We know that $Z_{sc} = jZ_0 \tan\beta l$.

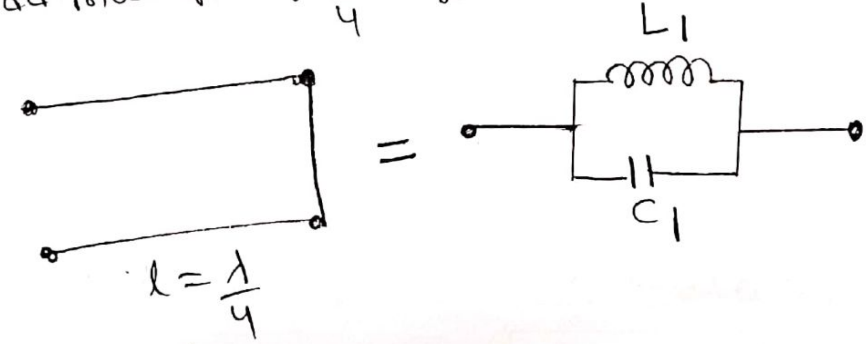
$\beta l = \frac{2\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{4_2} = \frac{\pi}{2}$ $\quad (\because \tan\frac{\pi}{2} = \infty)$

$\Rightarrow Z_{sc} = jZ_0 \tan\frac{\pi}{2}$

$\therefore \boxed{Z_{sc} = \infty}$

→ The sc line has infinite impedance (input impedance).

→ It acts as a parallel (or) anti resonance circuit at every odd multiple of $\frac{\lambda}{4}$ length.



d) <u>for $l = \frac{\lambda}{2}$</u> :-

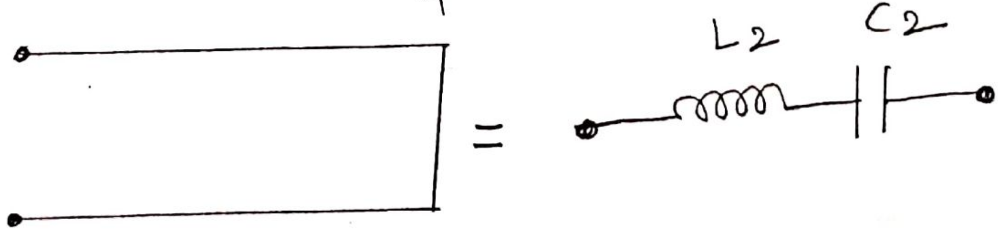
$Z_{sc} = jZ_0 \tan\beta l$, $\beta l = \frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} = \pi$

$Z_{sc} = jZ_0 \tan\pi$ $\quad (\because \tan\pi = 0)$

$= 0.$

$\therefore \boxed{Z_{sc} = 0}$

→ Therefore for $l = \frac{\lambda}{2}$ The sc line has zero input impedance. It acts as a series resonant ckt at every even multiples $\frac{\lambda}{4}$ length.



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Note:- For every $\frac{\lambda}{4}$ wavelength distance, the nature of reactance reverses.

* for every $\frac{\lambda}{2}$ wavelength distance, the same reactance values repeat.

i) OC (open ckt) lines at different lengths :-

(a) for lengths $\left(0 < l < \frac{\lambda}{4}\right)$:- Zoc is also purely reactive.

We know that,

$Z_{oc} = -jZ_0 \cot \beta l.$

If C_{eq} = Equivalent capacitance then

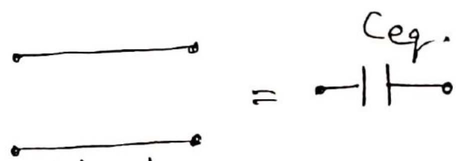
$$\frac{1}{j\omega C_{eq}} = -jZ_0 \cot \beta l.$$

$(\because j^2 = -1)$

$(-j^2 = 1)$

$$\therefore C_{eq} = \frac{1}{-j^2 \omega Z_0 \cot \beta l.}$$

$$\therefore \boxed{C_{eq} = \frac{1}{\omega Z_0 \cot \beta l}}$$



$0 < l < \frac{\lambda}{4}$

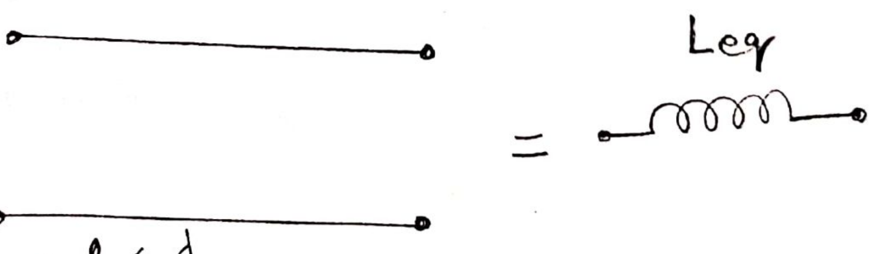
The line acts as capacitor

(b) for length $\left(\frac{\lambda}{4} < l < \frac{\lambda}{2}\right)$ We know that $Z_{oc} = -jZ_0 \cot \beta l$

$$\cancel{j}\omega L_{eq} = -\cancel{j}Z_0 \cot \beta l.$$

$$\boxed{L_{eq} = \frac{-Z_0}{\omega} \cot \beta l.}$$

The line acts as an Inductor.



$\frac{\lambda}{4} < l < \frac{\lambda}{2}$

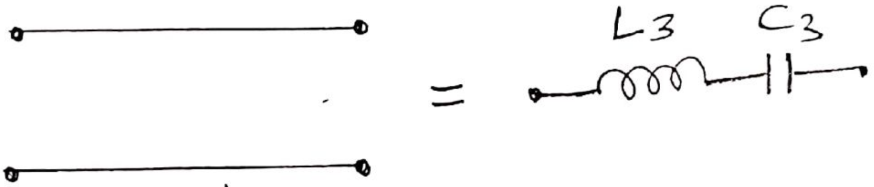
ⓒ **for length $l = \frac{\lambda}{4}$:-** $\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{4} = \frac{\pi}{2}$

$Z_{oc} = -jZ_0 \cot \beta l$.

$Z_{oc} = -jZ_0 \cot \frac{\pi}{2} = 0$. $\left(\because \cot \frac{\pi}{2} = \frac{\cos 90^\circ}{\sin 90^\circ} = 0\right)$

$\boxed{Z_{oc} = 0}$

→ The o.c line has zero input impedance. It acts as series resonant circuit at odd multiples of $\frac{\lambda}{4}$ length.



$l = \frac{\lambda}{4}$

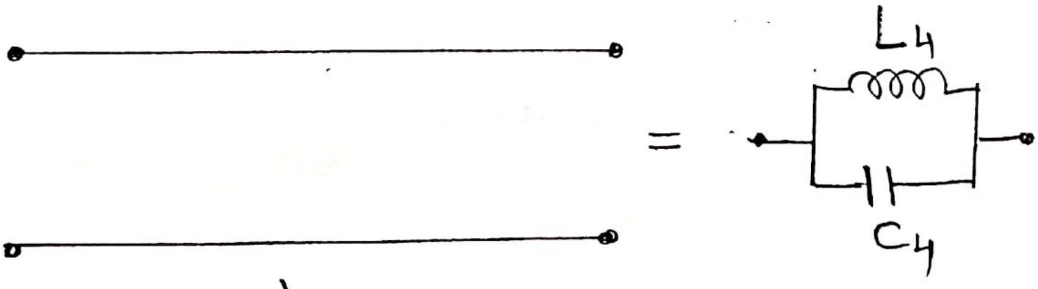
ⓓ **for length $l = \frac{\lambda}{2}$:-**

$\beta l = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \pi$

$Z_{oc} = -jZ_0 \cot \beta l = -jZ_0 \cot \pi$ $(\because \cot \pi = \infty)$

$\boxed{Z_{oc} = \infty}$

→ The oc line has Infinite input impedance. It acts as parallel resonance circuit at even multiples of $\frac{\lambda}{4}$ length



$l = \frac{\lambda}{2}$

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Impedance Transformations :-

→ Input impedance of a transmission line depends on its length. Thus a short length Transmission line is used as an "impedance matching device" (or) impedance Transformation.

→ If the load impedance is not equal to complex conjugate of source impedance, a short length Transmission line is added to the line to give "maximum power Transfer".

The important impedance Transformations are

(1) eight wave ($\frac{\lambda}{8}$) Transmission line

(2) The quarter wave ($\frac{\lambda}{4}$) Transmission line

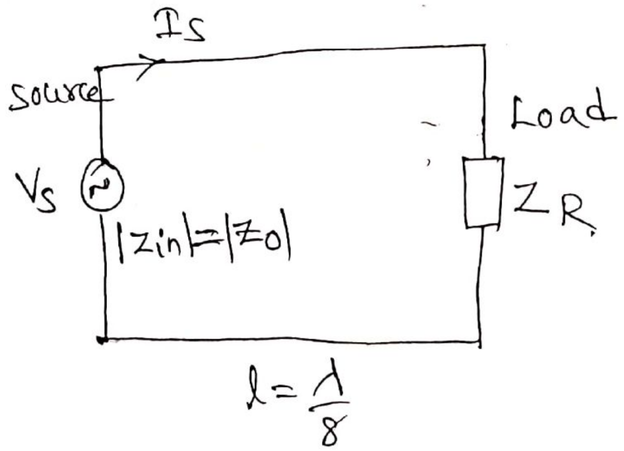
(3) The half wave ($\frac{\lambda}{2}$) Transmission line.

I) Eight Wave Transmission line ($\frac{\lambda}{8}$) :-

For eight wave length $l = \frac{\lambda}{8}$

λ = wave length,

We know that input impedance of Transmission line is



$$Z_{in} = Z_0 \left[\frac{Z_R + Z_0 \tanh \gamma l}{Z_0 + Z_R \tanh \gamma l} \right]$$

For lossless $\alpha = 0$, and $\gamma = j\beta$.

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan \beta l}{Z_0 + jZ_R \tan \beta l} \right]$$

For length $l = \frac{\lambda}{8}$

$$\Rightarrow \beta l = \frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{8}_4} = \frac{\pi}{4} \qquad \left(\because \tan\frac{\pi}{4} = 1\right)$$

$$\therefore Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\frac{\pi}{4}}{Z_0 + jZ_R \tan\frac{\pi}{4}}\right] = Z_0 \left[\frac{Z_R + jZ_0}{Z_0 + jZ_R}\right]$$

$$|Z_{in}| = |Z_0| \left|\frac{Z_R + jZ_0}{Z_0 + jZ_R}\right|$$

$$\Rightarrow |Z_{in}| = |Z_0| \times \frac{\cancel{\sqrt{Z_R^2 + Z_0^2}}}{\cancel{\sqrt{Z_0^2 + Z_R^2}}}$$

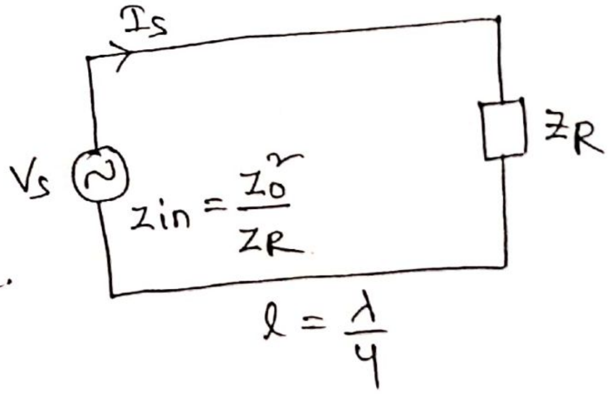
$$\therefore \boxed{|Z_{in}| = |Z_0|}$$

The magnitude of i/p impedance is equal to magnitude of characteristic impedance.

Quarterwave Transmission line :- $\left(\frac{\lambda}{4}\right)$

For quarter wave $l = \frac{\lambda}{4}$

Consider a $\frac{\lambda}{4}$ Transmission line Terminated with Z_R.



We know that, for lossless line.

$$Z_{in} = Z_0 \left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right]$$

for $l = \frac{\lambda}{4}$, $\beta l = \frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{4}_2} = \frac{\pi}{2}$

$$Z_{in} = Z_0 \left[\frac{\frac{Z_R}{\tan\beta l} + jZ_0}{\frac{Z_0}{\tan\beta l} + jZ_R}\right] = Z_0 \left[\frac{\frac{Z_R}{\tan\frac{\pi}{2}} + jZ_0}{\frac{Z_0}{\tan\frac{\pi}{2}} + jZ_R}\right]$$

$$\therefore Z_{in} = Z_0 \left[\frac{\frac{Z_R}{\infty} + jZ_0}{\frac{Z_0}{\infty} + jZ_R}\right] = Z_0\left[\frac{0 + jZ_0}{0 + jZ_R}\right]$$

$$\therefore Z_{in} = Z_0 \left[\frac{\cancel{j}Z_0}{\cancel{j}Z_R}\right]$$

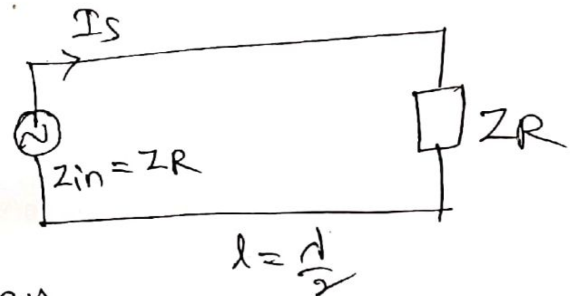
$$\boxed{Z_{in} = \frac{Z_0^2}{Z_R}} \text{ (or) } \boxed{Z_0 = \sqrt{Z_{in} \cdot Z_R}}$$

Thus a $\frac{\lambda}{4}$ length line is considered as a transformer that matches a load of Z_R to a source impedance of Z_{in}.

Half Wave Transmission line :- ($\frac{\lambda}{2}$)

for half wave $l = \frac{\lambda}{2}$

$$\beta l = \frac{\cancel{2}\pi}{\cancel{\lambda}} \times \frac{\cancel{\lambda}}{\cancel{2}} = \pi$$



We know that,
input impedance of a lossless Transmission line is

$$Z_{in} = Z_0\left[\frac{Z_R + jZ_0 \tan\beta l}{Z_0 + jZ_R \tan\beta l}\right] = Z_0\left[\frac{Z_R + jZ_0 \tan\pi}{Z_0 + jZ_R \tan\pi}\right]$$

$$\Rightarrow Z_{in} = Z_0\left[\frac{Z_R + 0}{Z_0 + 0}\right] = \cancel{Z_0} \times \frac{Z_R}{\cancel{Z_0}}$$

$$\therefore \boxed{Z_{in} = Z_R}$$

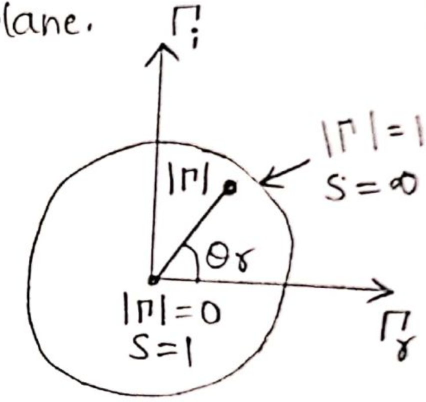
Thus the input impedance of a half wave line is equal to load impedance.

Smith chart :- phillip H. smith in the year 1939 developed a polar chart for calculating transmission line characteristics. this chart is called smith chart.

Definition :- smith chart is a polar plot of the reflection coefficient in terms of normalised impedance, $r+jx$.

(OR)

smith chart is a graphical plot of normalised resistance and reactance in the reflection coefficient plane.



- It consists of two sets of orthogonal circles, which represents the values of normalised impedance.
- one set of circles represents Resistive component 'r' called "r-circles". and other set of circles represent reactive component 'x' called "x-circles".

Applications of smith chart :-

The smith chart can be used to

1. Find the parameters of mismatched Transmission lines
2. Find normalised impedance from normalised admittance.
3. find normalised admittance from normalised impedance.
4. Find VSWR for a given load impedance.
5. Design stubs for impedance matchings
6. Find the Reflection coefficient.

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Stub matching :- "A stub is a piece of Transmission line. It can be short circuited (or) open circuited at the Load end (far end)".

→ stub has a pure reactance (or) susceptance. It is used to cancel out reactance (or) susceptance of a Transmission line.

→ A Stub is used for Impedance Matching.

→ When a UHF line is terminated with a load impedance, which is not equal to the characteristic impedance, so mismatch occurs.

→ Mismatch reduces efficiency and increases power loss.

· To avoid mismatching we have to add impedance matching devices between load and line.

Advantages :-

1. The length and characteristic impedance of line will be same.
2. since the stub is added in shunt, there is no need to cut the line.
3. The susceptance (or) reactance of stub can be adjusted for "perfect matching".

Methods of stub matching :-

i) single stub matching
ii) Double stub matching.

i) single stub matching :-

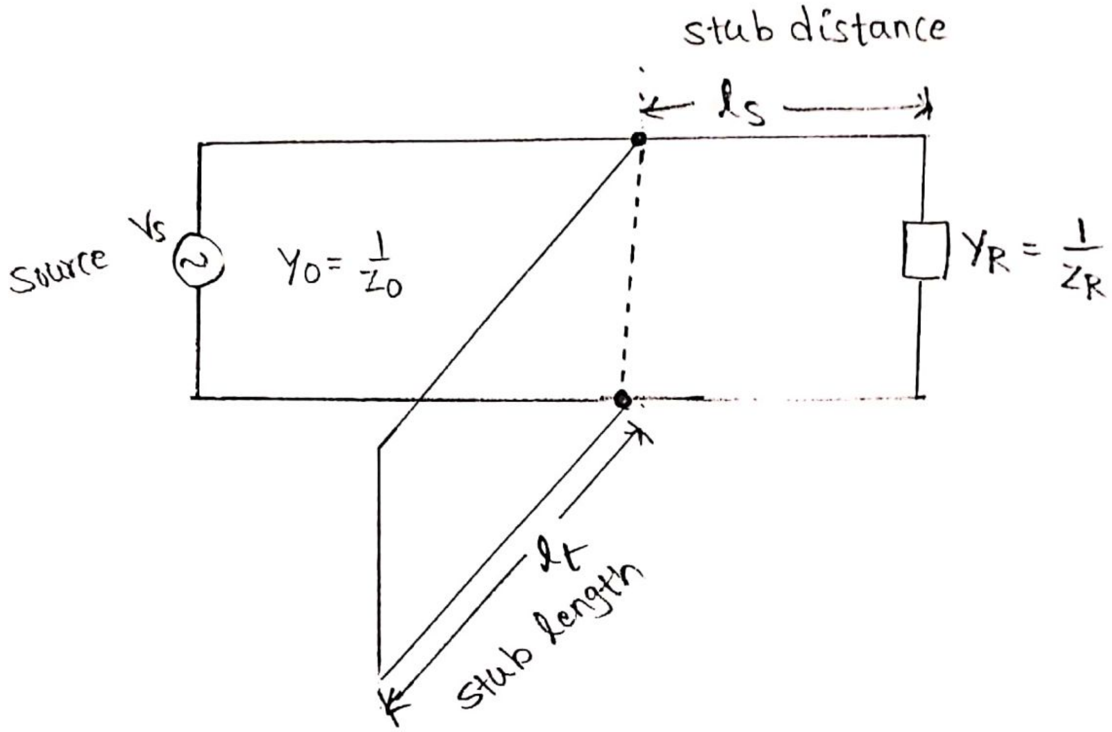
→ In this method, to achieve impedance matching, an open or short circuited short length transmission line (stub) is connected in parallel to the main line at a certain distance from Load.

→ since the stub is connected in parallel, it is easy to use admittance for analysis instead of impedance.

→ When Load admittance $\left(Y_R = \frac{1}{Z_R}\right)$ is connected to the line and if it is not equal to characteristic admittance $\left(Y_0 = \frac{1}{Z_0}\right)$ (i.e) $Y_R \neq Y_0$ the mismatch occurs.

→ so the standing waves exists on the line.

→ The construction of single stub matching shown below



→ When we move from load to source, the admittance on line varies from max. to min. (or) min. to max. value depends on the length of line.

→ At some point on the line, the real part of admittance (Y) equal to characteristic admittance (Y_0)

(ie) $Re\{Y\} = Re\{Y_0\}$.

→ generally a short ckt stub is preffered than open ckt stub. because (i) short ckt stub provides strong and supports to construction. and main line.

(ii) The short ckt stubs easily established, with metal plate

(iii) Radiation Loss is very less compared to open ckt stub.

Disadvantages :-

1. The location and length of single stub matching depend on frequency, if the frequency of wave changes, the length and location of stub should be changed.

2. It is very difficult to place a stub on co-axial cables.

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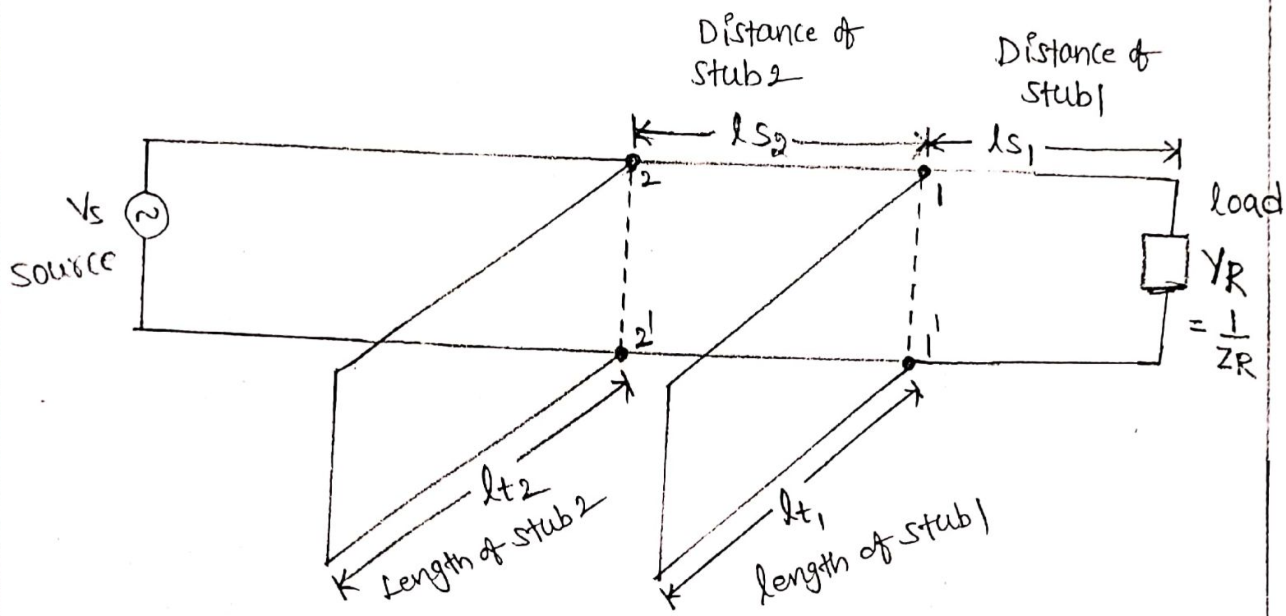
Double Stub matching :-

→ To overcome the disadvantages of single stub matching, two stubs can be used at different locations. This is called " Double Stub Matching".

→ consider a double stub matching system consists of two short ckt stubs connected in parallel to line near the load.



→ The characteristic admittance of stubs should be equal to characteristic admittance of line (Y_0).

- In this system, the locations of stubs have to be chosen arbitrarily. But It is very difficult to design stub matching with arbitrarily locations.
- so the locations of stubs should be restricted. The admittance repeats at every $\frac{\lambda}{2}$ length.

The total distance never be more than (or) equal to $\frac{\lambda}{2}$.

ie $\boxed{l_{S_1} + l_{S_2} < \frac{\lambda}{2}}$

- so l_{S_1} is in between 0.1λ and 0.15λ. Some times $l_{S_1} = 0$ (load) can be chosen.

The space between two stubs is generally taken as $\frac{\lambda}{8}, \frac{\lambda}{4}$ (or) $\frac{3\lambda}{8}$ distance.

The total distance of the double stub matching should be small.

∴ The Double Stub matching consists of fixed Stub locations.

The impedance matching is done by finding the lengths of the stubs.

Transmission line Losses :-

The Losses in Transmission lines are of three types.

Losses

- Copper losses
- Dielectric Losses
- Radiation Losses.

Copper losses :- The copper losses occur due to I^2R power loss and skin effect, crystallisation.

→ I^2R power loss due to dissipation of heating in pure resistance.

→ Skin effect is due to an AC signal at high frequency applied to the Transmission line, the current is restricted to surface ~~of~~ conductor.

→ ∴ The cross sectional area decreases and increase resistance.

→ So the power losses are Increases.

Dielectric Losses :- The dielectric losses due to improper characteristics of dielectric.

Radiation Losses :- This Radiation Losses due to when the spacing between lines is high.

→ At high frequency λ will be small and hence transmission lines are not useful at high frequencies

Low Loss Radio frequency Transmission line :-

→ A Low Loss radio frequency Transmission line is one for which $R << \omega L$, and $G << \omega C$.

Thus series Impedance $Z = R + j\omega L$

$$\Rightarrow Z \simeq j\omega L$$

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the shunt admittance $Y = G + j\omega C$

$$\Rightarrow Y \simeq j\omega C$$

→ characteristic Impedance is

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{j\omega L}{j\omega C}} \qquad (\because R << \omega L,\ G << \omega C)$$

$$\boxed{Z_0 = \sqrt{\frac{L}{C}}}\ \Omega$$

→ propagation constant is

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \qquad (\because R << \omega L,\ G << \omega C)$$

$$= \sqrt{j\omega L \times j\omega C}$$

$$= j\omega\sqrt{LC}$$

$$\Rightarrow \gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

By comparing real and imaginary parts.

$\boxed{\alpha = 0}$ → attenuation constant (NP/m)

$\boxed{\beta = \omega\sqrt{LC}}$ → phase constant (rad/m)

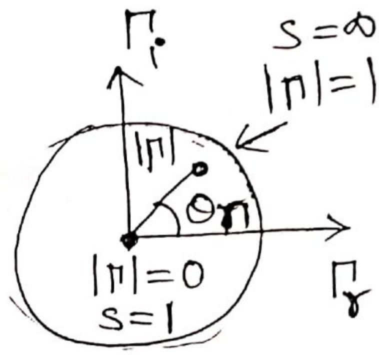
phase Velocity $V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ m/sec

Smith chart :- Phillip H. Smith in the year 1939 developed a polar chart for calculating Transmission line characteristics. This chart is called "smith chart"

Definition :- Smith chart is a polar plot of reflection coefficient in terms of normalised impedance $r+jx$.

(OR)

Smith chart is a graphical plot of normalised resistance and reactance in the reflection coefficient plane.



- It consists of two sets of orthogonal circles, which represents the values of normalised Impedance.
- One set of circles represents Resistive component 'r' called as "r- circles". and other set of circles represent reactive component 'x' called "x- circles".

Construction :-

- The smith chart is constructed within a circle of unit radius $(|\Gamma| \leq 1)$ as shown in the given figure.
- The construction of smith chart is based on the relation is

$$\Gamma = \frac{Z_R - Z_0}{Z_R + Z_0} \longrightarrow (1)$$

(OR)

$$\Gamma = |\Gamma| \angle\theta_\Gamma = \Gamma_r + j\Gamma_i \longrightarrow (2)$$

Where Γ_r and Γ_i are the real and imaginary parts of the reflection coefficient Γ.

For the Load impedance Z_R, the normalized Load impedance is given by

$$z_n = z_R = \frac{Z_R}{Z_0} = r + jx \longrightarrow (3) \qquad \left(\because r = \frac{R}{Z_0},\ jx = \frac{jX}{Z_0}\right)$$

from eq (1), (2)

$$\Gamma = \Gamma_r + j\Gamma_i = \frac{Z_R - Z_0}{Z_R + Z_0} \qquad \left(\because z_n = z_R = \text{normalised Impedance}\right)$$

$$\Rightarrow \Gamma = \Gamma_r + j\Gamma_i = \frac{\cancel{Z_0}\left(\frac{Z_R}{Z_0} - 1\right)}{\cancel{Z_0}\left(\frac{Z_R}{Z_0} + 1\right)} = \frac{z_n - 1}{z_n + 1} \longrightarrow (4)$$

$$\Rightarrow z_n - 1 = (\Gamma_r + j\Gamma_i)(z_n + 1)$$

$$\Rightarrow z_n - 1 = \Gamma_r z_n + \Gamma_r + j\Gamma_i z_n + j\Gamma_i$$

$$\Rightarrow z_n - \Gamma_r z_n - j\Gamma_i z_n = 1 + \Gamma_r + j\Gamma_i$$

$$\Rightarrow z_n(1 - \Gamma_r - j\Gamma_i) = 1 + \Gamma_r + j\Gamma_i$$

$$\therefore \boxed{z_n = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i}} \longrightarrow (5)$$

Multiplying and dividing by $(1-\Gamma_r) + j\Gamma_i$

$$\Rightarrow z_n = r + jx = \frac{(1+\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) - j\Gamma_i} \times \frac{(1-\Gamma_r) + j\Gamma_i}{(1-\Gamma_r) + j\Gamma_i} \qquad (\because j^2 = -1)$$

$$\Rightarrow r + jx = \frac{(1-\Gamma_r^2) + j\Gamma_i(1+\Gamma_r) + j\Gamma_i(1-\Gamma_r) + j^2\Gamma_i^2}{(1-\Gamma_r)^2 - j^2\Gamma_i^2}$$

$$\Rightarrow r + jx = \frac{(1-\Gamma_r^2) + j\Gamma_i + \cancel{j\Gamma_i\Gamma_r} + j\Gamma_i - \cancel{j\Gamma_i\Gamma_r} - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$\Rightarrow r + jx = \frac{(1-\Gamma_r^2-\Gamma_i^2)+j2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

Normalizing and equating real and imaginary parts. We obtain

$$r = \frac{1-\Gamma_r^2-\Gamma_i^2}{(1-\Gamma_r)^2+\Gamma_i^2}, \quad x = \frac{2\Gamma_i}{(1-\Gamma_r)^2+\Gamma_i^2}$$

Rearranging above equations, then we get

$$\boxed{\left[\Gamma_r - \frac{r}{1+r}\right]^2 + \Gamma_i^2 = \left[\frac{1}{1+r}\right]^2} \rightarrow (6)$$

r-circle

$$\boxed{\left[\Gamma_r - 1\right]^2 + \left[\Gamma_i - \frac{1}{x}\right]^2 = \left[\frac{1}{x}\right]^2} \rightarrow (7)$$

x-circle

The above equations are in the form of equation of circle. (ie) $(x-h)^2+(y-k)^2 = a^2$

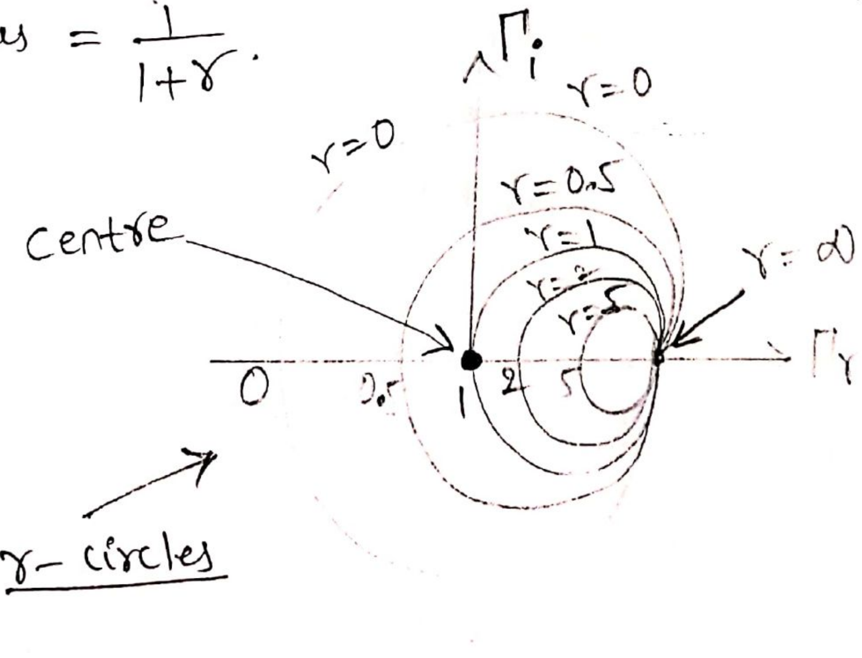
a = radius

(h,k) = center

$\therefore$ eq (6) is an 'r'-circle. (resistance circle)

center at $(\Gamma_r, \Gamma_i) = \left(\frac{r}{1+r}, 0\right)$

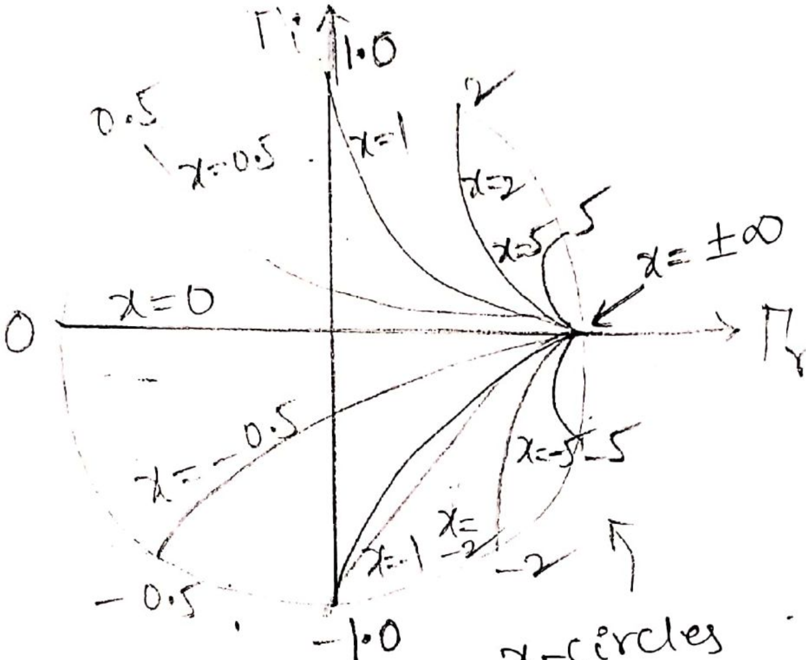
radius $= \frac{1}{1+r}$.



Similarly eq ⑦ is an x-circle (reactance circle).

center at $(\Gamma_r, \Gamma_i) = \left(1, \frac{1}{x}\right)$

radius $= \frac{1}{x}$



x-circles

Applications of smith chart :-

The smith chart can be used to

1. Find the parameters of mismatched Transmission lines.
2. Find the normalised impedance from normalized admittance
3. Find the normalized admittance from normalized Impedance.
4. Find Voltage standing Wave Ratio (VSWR) for a given Load impedance.
5 Design stubs for Impedance matchings
6. Find the Reflection coefficient (Γ)

Steps for solving smith chart problems:-

1. Find the normalized Load impedance. $z_n = \frac{Z_R}{Z_0} = r + jx$

   (or) $z_R = \frac{Z_R}{Z_0} = r + jx$

2. Mark the normalized Load impedance on the smith chart (r and x circles).

3. Assume point 'p' at an intersection of 'r' and 'x' circles.

4. Draw the line from origin 'o' to point 'p'. and extend point 'p' to point 'Q'.

5. observe the Reflection coefficient magnitude.

$$|\Gamma| = \frac{\text{OP distance}}{\text{OQ distance}}$$

6. Measure the Reflection angle θ_Γ

$$\angle\theta_\Gamma = \text{POS Angle} \quad \therefore \boxed{\Gamma = |\Gamma| \angle\theta_\Gamma}$$

7. Take the OP radius and draw the circle at centre 'o' with OP radius

8. Measure the standing Wave Ratio from point 'o' to point 'S'.

9. Find the normalized input impedance using the length of Transmission line. (Length must be interms of 'λ' (or) degrees $\lambda \rightarrow 720°$, $\frac{\lambda}{2} \rightarrow 360°$) ($\because \frac{\lambda}{2} = 0.5\lambda$)

10. To find the normalized input Impedance by moving from 'Q' in clockwise direction, with a given transmission line, Draw the line from 'o' to given length of line. mark the point 'A' at intersection of SWR circle and input impedance line.