Jhansy
ECE Dept:

TRANSMISSION LINES-I

Transmission line:- A Transmission line is a conductor which can be used for transmitting electromagnetic waves from source to destination.

(OR)

A conductors (or) wave guides are used for transmitting electromagnetic waves over long distance between transmitter and Receiver. These lines are called as "Transmission lines".

The transmission line consists of two (or) more conductors through which the source can be connected to the load.

Applications:-

1. Transmission lines are used to transfer energy from one circuit to another.
2. They can be used as circuit elements like Inductors, capacitors.
3. Impedance matching devices.
4. They can be used as stubs.
5. They can be used as measuring devices.

Types of Transmission lines:- There are several types of transmission lines available depending on the propagation constant, distance, power lines.

1. open wire transmission line (parallel wire line)
2. Co-axial cable Transmission line
3. Wave guides
4. Optical fibres
5. Micro strip lines.

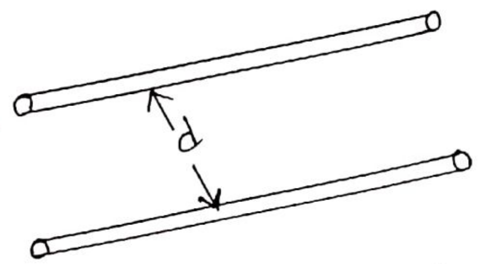
1. Open Wire Transmission lines:— (parallel line)

An open wire line is a pair of parallel conducting wires seperated by a distance in free space and mounted on towers.

> These lines are transmitts the waves upto 100MHz frequencies.

> The EM waves Transmitt in TEM mode. (Transverse electric and magnetic)

Ex:- Telephone lines, Telegraphy lines, power lines.



open wire Transmission line.

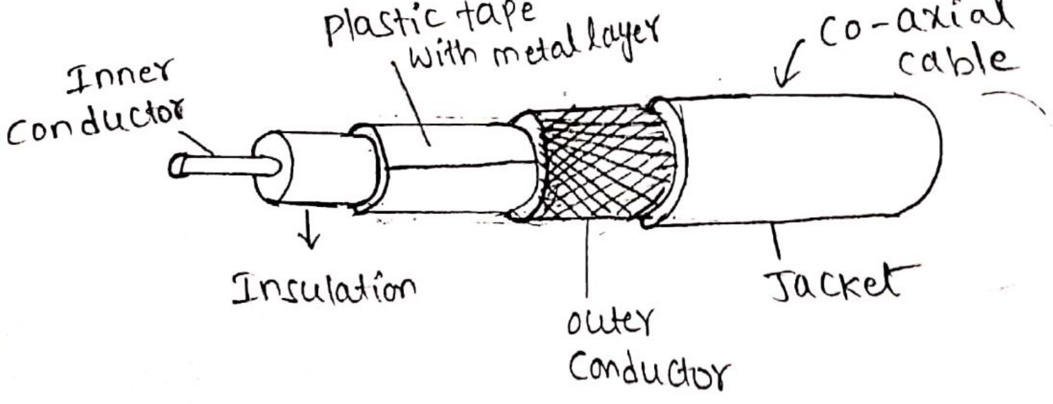
2. Co-axial Cable Transmission line:-

When two conductors are placed axially and filled with dielectric material, the cable is called as "co-axial cable".

It supports upto 1GHz Frequencies.

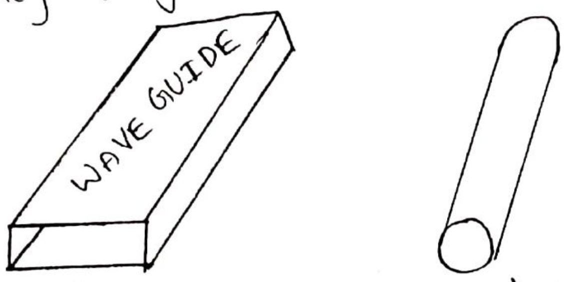
The waves Transmitt in TEM mode.

Examples:- TV Cables, Telephone Cables.



3. **Wave guides:-** Wave guides are hollow (or) dielectric filled conductors used to transmit electro magnetic waves at microwave frequencies.

* Waveguides are single conductor transmission lines, either rectangular (or) circular shapes.
* Waves are Transmitted in TE (or) TM mode
* Frequency range is 3 GHz.

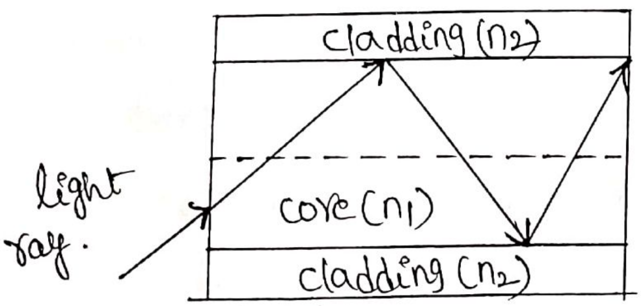


Rectangular wave guide

circular wave guide.

Optical fibres:- Optical fibres are non-conducting circular wave guides made up of silicon oxide (or) silica

* It consists of core and cladding.
* It Transmitts light waves at 10^5 GHz. frequencies.
* It travels in TE, TM, TEM, hybrid modes.
* It having small size, light weight,



n_1 = Core index

n_2 = cladding Index.

$n_1 > n_2$

Optical fibre.

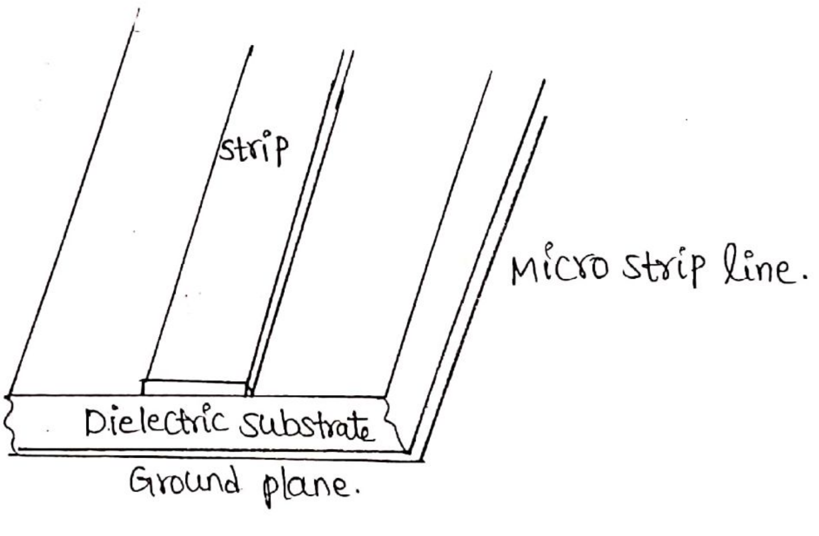
light waves are travel by total Internal Reflection.

Microstrip lines :-

Micro strip line is a parallel plate transmission line consisting of a conducting strip and ground plane seperated by a dielectric substrate.

→ A very thin copper sheet is used as a strip and the ground plane is fabricated using printed circuit board Technology.

→ Micro strip lines are lighter, more compact and flexible than other types of Transmission lines. These lines operate at microwave frequencies. (3GHz to 30 GHz).

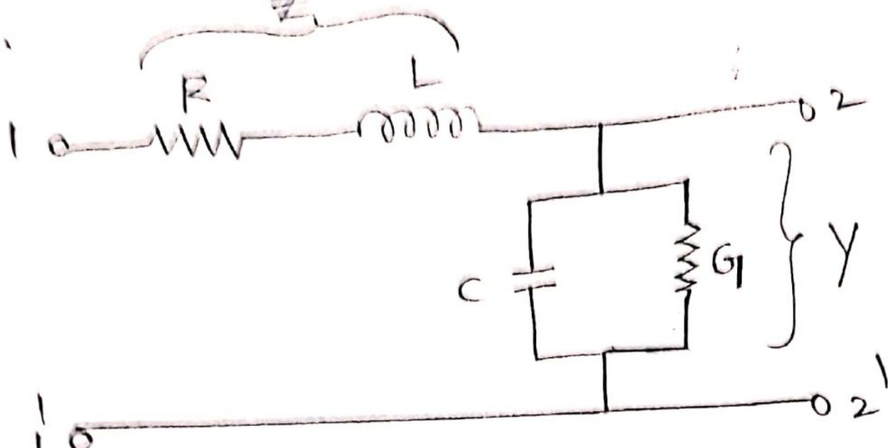


• Micro strip lines can be used as microwave components such as antennas, direction couplers, filters, circulators.

Ex :- High speed digital ckts, antennas in smart phones,

Transmission line parameters :-

The equivalent electric circuit of a transmission line consists of series resistance, series Inductance, shunt capacitance, and shunt conductance along the length.



Equivalent circuit of a Transmission line per unit length.

* The parameters R, L, G and C are called primary constants of a Transmission line. these are Independent of operating frequency.
* the parameters are measured in either /m (or) /km.

The series Impedance of a Transmission line is

$Z = R + j\omega L$

the shunt admittance of a Transmission line is

$Y = G + j\omega C$.

Resistance (R) :- A series resistance is due to the internal resistance of the conductors of a transmission line. It is uniformly distributed along the line and it is an ac resistance.

→ It depends on the conductivity and cross sectional area of conductors. units are Ω/m (ohms/m)

Inductance:-(L) A series (or) loop Inductance is due to the magnetic flux density produced around the conductors of a transmission line. It is uniformly distributed along the line. The flux linkages per unit current gives inductance of a transmission line. ($L = \frac{N\phi}{I}$)

units are H/m (Henry/meter)

Capacitance:-(C):- Two parallel conductors (or) co-axial conductors of a transmission line seperated by a distance 'd' acts as a capacitor.

→ A shunt capacitance is formed due to the electric field between the conductors. capacitance is also uniformly distributed along the line.

units are F/m (or) Farads/meter.

Conductance:-(G):- A shunt conductance is due to the leakage current between the conductors of a Transmission line. It is also uniformly distributed along the line.

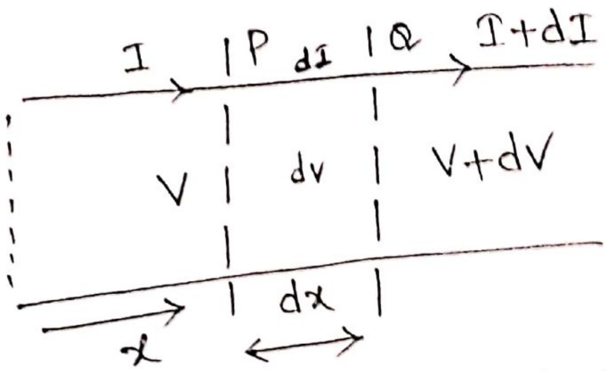
units are ℧/m (mhos/m) (or) (siemens/m)

* Transmission Line Equations:-

Consider a transmission line with two parallel conductors. Let R, L, C and G be the primary constants. Assume that these values donot vary with frequency.

consider a point 'p' on the line at a distance x from the source.

Let another point 'Q' be at a small distance dx from point 'p'.



Voltage and currents on a transmission line.

→ Let V and I be voltage and current respectively at 'P' point. V+dV and I+dI be voltage and current respectively at 'Q' point. Where voltage and currents are uniformly distributed along the line.

For a small length 'dx' the series impedance is given by

$(R+j\omega L)\,dx$

The shunt admittance at dx length is given by

$(G+j\omega C)\,dx.$

The potential difference between P and Q is

$$V-(V+dV) = I(R+j\omega L)\,dx \quad (\because V = IZ)(Z = R+j\omega L)$$

$$\Rightarrow \cancel{V}-\cancel{V}-dV = I(R+j\omega L)\,dx$$

$$\therefore -\frac{dV}{dx} = I(R+j\omega L) \longrightarrow (1)$$

The current difference between P and Q is

$$\left(\because I = \frac{V}{Z} = V\times Y,\ Y = G+j\omega C\right)$$

$$I-(I+dI) = V(G+j\omega C)\,dx$$

$$\Rightarrow \cancel{I}-\cancel{I}-dI = V(G+j\omega C)\,dx$$

$$\therefore -dI = V(G+j\omega C)\,dx$$

$$-\frac{dI}{dx} = V(G+j\omega C) \longrightarrow (2)$$

Taking derivative of eq (1) with respect to 'dx' on both sides.

$$\Rightarrow -\frac{d^2V}{dx^2} = \frac{dI}{dx}(R+j\omega L)$$

substituting eq ② in above equation.

$\Rightarrow \cancel{-}\frac{d^2V}{dx^2} = \cancel{-}V(G+j\omega C)(R+j\omega L)$

$\therefore \frac{d^2V}{dx^2} = (R+j\omega L)(G+j\omega C)V \rightarrow ③$

similarly for current

$\frac{d^2I}{dx^2} = (R+j\omega L)(G+j\omega C)I \rightarrow ④$

These equations are second order differential equations in Voltage (or) current.

Let the constant term be

$(R+j\omega L)(G+j\omega C) = \gamma^2$

(ie) $\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{ZY}$

Where α = attenuation constant, β = phase constant,

$\gamma = \sqrt{ZY}$ = propagation constant of Transmission line.

$$\left.\begin{aligned} \therefore \frac{d^2V}{dx^2} &= \gamma^2 V \rightarrow ⑤ \\ \frac{d^2I}{dx^2} &= \gamma^2 I \rightarrow ⑥ \end{aligned}\right\}$$ differential equations of the Transmission line.

The solutions for above equations are

$V = ae^{\gamma x} + be^{-\gamma x} \rightarrow ⑦$

$I = ce^{\gamma x} + de^{-\gamma x} \rightarrow ⑧$

Where a, b, c and d are the constants.

substitute eq ⑦ in eq ①, we get

$-\frac{d}{dx}(ae^{\gamma x} + be^{-\gamma x}) = (R+j\omega L)I$

$$-ae^{rx}(r) - be^{-rx}(-r) = I(R+j\omega L)$$

$$\Rightarrow r(be^{-rx} - ae^{rx}) = I(R+j\omega L)$$

$$\therefore I = \frac{r(be^{-rx} - ae^{rx})}{R+j\omega L} = \frac{\cancel{\sqrt{R+j\omega L}}\sqrt{G+j\omega C}\,(be^{-rx} - ae^{rx})}{\cancel{\sqrt{R+j\omega L}}\sqrt{R+j\omega L}}$$

$$\therefore I = \sqrt{\frac{G+j\omega C}{R+j\omega L}}\,(be^{-rx} - ae^{rx}) \rightarrow (9)$$

But

The characteristic Impedance is

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{Z}{Y}} \rightarrow (10)$$

$$\therefore I = \frac{1}{Z_0}(be^{-rx} - ae^{rx}) \rightarrow (11)$$

Therefore the voltage and currents are

$$V = ae^{rx} + be^{-rx} \rightarrow (12)$$

$$I = \frac{1}{Z_0}(be^{-rx} - ae^{rx}) \rightarrow (13)$$

Interms of Hyperbolic functions, Let

$$e^{rx} = \cosh rx + \sinh rx, \quad e^{-rx} = \cosh rx - \sinh rx$$

substitute above values in equations (12) + (13)

$$V = a[\cosh rx + \sinh rx] + b[\cosh rx - \sinh rx]$$

$$\Rightarrow V = (a+b)\cosh rx + (a-b)\sinh rx.$$

Let $a+b = A$, $a-b = B$

$$\therefore \boxed{V = A\cosh rx + B\sinh rx} \rightarrow (14)$$

$$I = \frac{1}{Z_0}\left[b(\cosh rx - \sinh rx) - a(\cosh rx + \sinh rx)\right]$$

$$I = \frac{1}{Z_0}\left[(b-a)\cosh rx - (a+b)\sinh rx\right]$$

$$\Rightarrow I = \frac{-1}{Z_0}\left[(a-b)\cosh\gamma x + (a+b)\sinh\gamma x\right]$$

$(\because a-b = B$
$a+b = A)$

$$\therefore \boxed{I = \frac{-1}{Z_0}\left(B\cosh\gamma x + A\sinh\gamma x\right)} \rightarrow (15)$$

<u>calculate A and B constants :-</u>

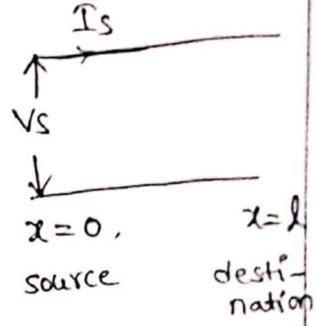
At sending end $x=0$, $V=V_s$, $I=I_s$.

from equations (14), (15)

$$V_s = A\cosh\gamma(0) + B\sinh\gamma(0)$$

$$\therefore V_s = A(1) + B(0)$$

$$\therefore \boxed{V_s = A} \text{ (or) } \boxed{A = V_s}$$



$$I_s = \frac{-1}{Z_0}\left(B\cosh\gamma(0) + A\sinh\gamma(0)\right)$$

$$\Rightarrow I_s = \frac{-1}{Z_0}\left[B(1) + A(0)\right] = -\frac{B}{Z_0}$$

$$\therefore \boxed{B = -I_s Z_0}$$

substituting constants A, B in eqns (14) & (15)

$$\therefore \boxed{V = V_s\cosh\gamma x - I_s Z_0\sinh\gamma x} \rightarrow (16)$$

$$I = \frac{-1}{Z_0}\left(-I_s Z_0 \times \cosh\gamma x + V_s\sinh\gamma x\right)$$

$$= \frac{I_s \cancel{Z_0}}{\cancel{Z_0}}\cosh\gamma x - \frac{V_s}{Z_0}\sinh\gamma x$$

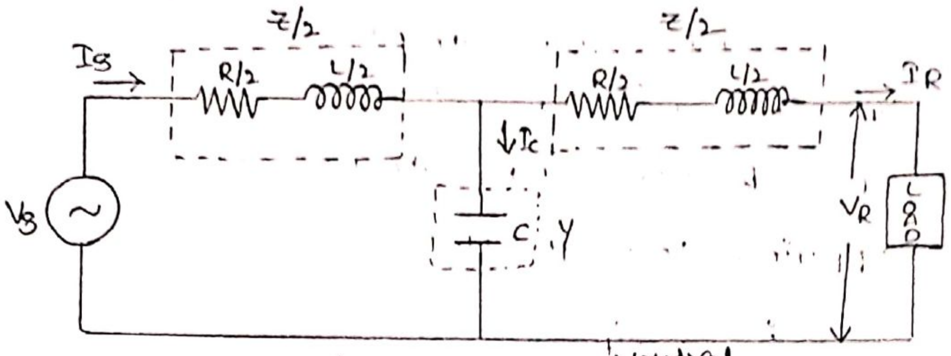
$$\therefore \boxed{I = I_s\cosh\gamma x - \frac{V_s}{Z_0}\sinh\gamma x.} \rightarrow (17)$$

$\therefore$ equations (16), (17) are called as "Transmission line equations".

T and Π - Equivalent circuits :-

T- Equivalent circuit :-

→ In T- Equivalent circuit, the shunt admittance is placed in the middle of the line, and the series Impedance can be divided equally in to two parts, placed at the either of the ends of the Transmission line.



T- Equivalent circuit

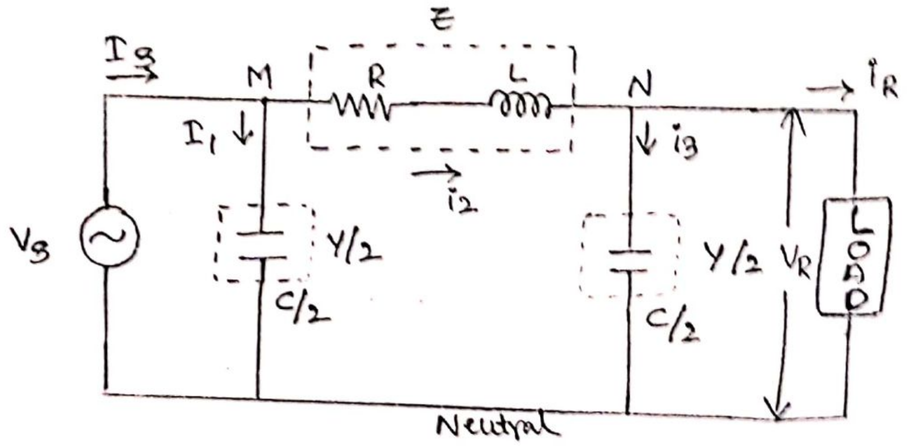
where :
V_S = sender end voltage
I_S = sender end current
V_R = Reciever end voltage
I_R = Reciever current

by applying KCL on above circuit

$$\boxed{I_S = I_C + I_R}$$

Π (Pi) - Equivalent circuit :-

→ In Pi(or)Π - equivalent circuit, the series Impedance is placed in the middle of the line and the shunt admittance can be divided equally into two parts, placed at the either of the ends of the Transmission line.



where: V_S = Sender end voltage

I_S = Sender end current

V_R = Reciever end voltage

I_R = Reciever end current.

On by applying KCL at 'N'

$$i_2 = i_3 + i_R$$

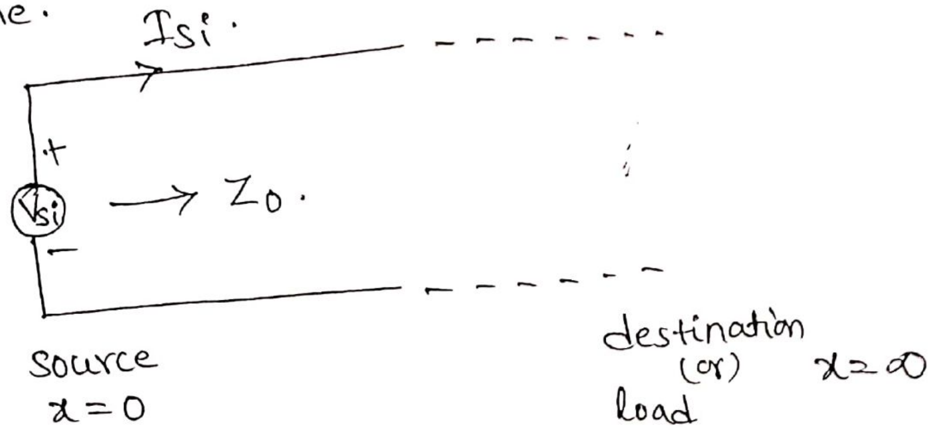
On applying KCL at 'M'

$$I_S = I_1 + I_2 \quad \because \text{from above eqn.}$$

$$\boxed{I_S = I_1 + I_3 + I_R}$$

Infinite Length Line:-

A Transmission line with Infinite length is called as "Infinite" line.



→ When a voltage source is applied to the line, a current will flow depending on primary constants. This current will not reach the destination (or) Load end due to Infinite attenuation.

→ In Infinite lines x varies from 0 to ∞.

The voltage and current equations for a Transmission line are

$$V = ae^{\gamma x} + be^{-\gamma x}$$
$$I = ce^{\gamma x} + de^{-\gamma x}$$

At sending side (or) source side $x=0$, so $V = V_{si}$

$I = I_{si}$

→ V_{si} is voltage in sending side of Infinite transmission line

I_{si} = current in sending side of Infinite Transmission line.

At Receiving (or) destination end $x=\infty$;

$V = 0$,

$I = 0$

$V = ae^{\gamma x} + be^{-\gamma x}$ | $I = ce^{\gamma x} + de^{-\gamma x}$

at $x = 0$; $V = V_{si}$ | $I = I_{si}$

$\Rightarrow V_{si} = ae^{0} + be^{-0}$ | $\Rightarrow I_{si} = ce^{0} + de^{-0}$

$\Rightarrow \boxed{V_{si} = a + b} \rightarrow (1)$ | $\Rightarrow \boxed{I_{si} = c + d} \rightarrow (2)$

At $x = \infty$, $V = 0$ | $I = 0$.

$0 = ae^{\infty} + be^{-\infty}$ | $0 = ce^{\infty} + de^{-\infty}$

$0 = ae^{\infty} + 0$. $(\because e^{-\infty} = 0)$ | $0 = ce^{\infty} + 0$. $(\because e^{-\infty} = 0)$

$\boxed{a = 0} \rightarrow (3)$ | $\boxed{c = 0} \rightarrow (4)$

substitute eq (3) in (1) | substitute eq (4) in (2)

$V_{si} = 0 + b$ | $I_{si} = 0 + d$

$\therefore \boxed{b = V_{si}} \rightarrow (5)$ | $\boxed{d = I_{si}} \rightarrow (6)$

Now place eqns (3), (4), (5) + (6) in Voltage and current equations we get

$$\boxed{V = V_{si}\, e^{-\gamma x}} \qquad \boxed{I = I_{si}\, e^{-\gamma x}}$$

These equations are propagation equations for Infinite Transmission line.

Primary and secondary constants :-

Primary constants :- The constants R, L, C and G are called as primary constants

R = series Resistance | C = shunt capacitance
L = series Inductance | G = shunt conductance.

Series Impedance $Z = R + j\omega L$

Shunt Admittance $Y = G + j\omega C$.

Secondary constants:- The constants Z_0, γ are called secondary constants.

Z_0 = characteristic Impedance.

$$= \sqrt{\frac{Z}{Y}}$$

$$= \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

γ = propagation constant

$$= \alpha + j\beta$$

$$= \sqrt{ZY}$$

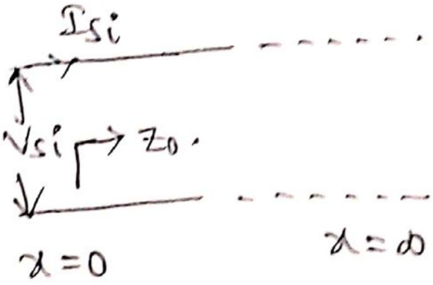
$$= \sqrt{(R + j\omega L)(G + j\omega C)}$$

Expression for characteristic Impedance:- (Z_0)

Definition:- characteristic Impedance is defined as the ratio of applied voltage to the current flowing through the infinite line.

It is denoted by Z_0.

$$Z_0 = \frac{V_{si}}{I_{si}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$



proof:- We know that, from Transmission line equations concept.

Voltage equation is $-\frac{dV}{dx} = I(R + j\omega L) \rightarrow (1)$

For Infinite line, the propagation equations are

$V = V_{si}\, e^{-\gamma x} \rightarrow (2)$

$I = I_{si}\, e^{-\gamma x} \rightarrow (3)$

substitute eq ② & ③ in eq ①

$$\Rightarrow -\frac{d}{dx}\left(V_{si}\, e^{-\gamma x}\right) = I_{si}\, e^{-\gamma x}(R+j\omega L)$$

$$-V_{si}\cdot \cancel{e^{-\gamma x}}(-\gamma) = I_{si}\cdot \cancel{e^{-\gamma x}}(R+j\omega L)$$

$$V_{si}(\gamma) = I_{si}(R+j\omega L)$$

$$\Rightarrow \frac{V_{si}}{I_{si}} = \frac{R+j\omega L}{\gamma} = \frac{\cancel{R+j\omega L}\ \sqrt{R+j\omega L}}{\cancel{\sqrt{R+j\omega L}}\sqrt{G+j\omega C}}$$

$$\Rightarrow Z_0 = \frac{V_{si}}{I_{si}} = \frac{\sqrt{R+j\omega L}}{\sqrt{G+j\omega C}}$$

$$(\because \gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$ = propagation constant

$$\therefore \boxed{Z_0 = \frac{V_{si}}{I_{si}} = \sqrt{\frac{R+j\omega L}{G+j\omega C}}}\ \Omega$$

Propagation constant (γ) :-

The propagation constant $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{ZY}$

$$= \alpha + j\beta.$$

The propagation constant of a Transmission per unit length is defined as natural logarithm of ratio of steady state voltage entering and leaving the structure (or) steady state current entering and leaving the structure.

$$\boxed{\gamma = \ln\left(\frac{V_S}{V_R}\right)} \qquad \boxed{\gamma = \ln\left(\frac{I_S}{I_R}\right)}$$

(or)

$$\gamma = \log_e\left(\frac{V_S}{V_R}\right) \qquad \gamma = \log_e\left(\frac{I_S}{I_R}\right)$$

derivation of α and β :-

$$\gamma = \alpha + j\beta = \sqrt{(R+j\omega L)(G+j\omega C)} \longrightarrow (1)$$

$$\gamma^2 = (\alpha+j\beta)^2 = (R+j\omega L)(G+j\omega C)$$

$$\Rightarrow \gamma^2 = (\alpha^2-\beta^2) + j(2\alpha\beta) = (RG-\omega^2 LC) + j(\omega CR + \omega LG)$$

comparing real and Imaginary parts.

$$\left.\begin{aligned} \alpha^2-\beta^2 &= RG-\omega^2 LC \\ 2\alpha\beta &= \omega CR + \omega LG \end{aligned}\right\} \rightarrow (2)$$

since γ is a complex quantity

$$|\gamma| = \sqrt{\alpha^2+\beta^2} = \sqrt{\sqrt{R^2+\omega^2L^2}\sqrt{G^2+\omega^2C^2}}$$

Hence

$$\alpha^2 + \cancel{\beta^2} = \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$eq(2) \Rightarrow \alpha^2 - \cancel{\beta^2} = RG-\omega^2 LC$$

$$2\alpha^2 = (RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}$$

$$\alpha^2 = \frac{1}{2}\left[(RG-\omega^2LC) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}\right]$$

$$\boxed{\alpha = \frac{1}{\sqrt{2}}\left[\sqrt{RG-\omega^2LC + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right]}$$

similarly for β

substituting α^2 value in eq(2), we get

$$\boxed{\beta = \frac{1}{\sqrt{2}}\left[\sqrt{(\omega^2LC-RG) + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right]}$$

Phase and group velocities :-

Phase velocity :- The velocity at which a wave of a single frequency travels along a line is called as "phase velocity". It is denoted by V_p.

It can be expressed as

$$V_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{LC}} \quad \text{for lossless Transmission line.}$$

(OR)

$\boxed{V_p = \lambda \times f}$ where λ = wavelength, f = frequency (Hz)

$$\lambda = \frac{2\pi}{\beta}.$$

$$\therefore V_p = \frac{2\pi}{\beta} \times f = \frac{2\pi f}{\beta}$$

$$\therefore \boxed{V_p = \frac{\omega}{\beta} \text{ m/s}}$$

In free space phase velocity is approximately equal to velocity of light

$$V_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s}.$$

Group velocity :- (V_g)

Definition :- Group velocity is defined as the ratio of difference in angular frequencies to difference in phase constants of the wave. It is denoted by 'V_g'.

$$\boxed{V_g = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1} = \frac{d\omega}{d\beta} \text{ m/s}}$$

Relationship between phase velocity (V_p) and group velocity (V_g) :-

Phase velocity $V_p = \frac{\omega}{\beta} \rightarrow (1)$

Group velocity $V_g = \frac{d\omega}{d\beta} = \frac{\omega_2 - \omega_1}{\beta_2 - \beta_1} \rightarrow (2)$

differentiating eq (1) w.r.t 'ω' on both sides.

$$\frac{dV_p}{d\omega} = \frac{\beta \cdot 1 - \omega \cdot \frac{d\beta}{d\omega}}{\beta^2} = \frac{1}{\beta} - \frac{\omega}{\beta^2}\frac{d\beta}{d\omega}$$

$$= \frac{1}{\beta}\left(1 - \frac{\omega}{\beta} \times \frac{d\beta}{d\omega}\right)$$

$$\therefore \frac{dV_p}{d\omega} = \frac{1}{\beta}\left(1 - V_p \times \frac{1}{V_g}\right)$$

$$\beta \frac{dV_p}{d\omega} = 1 - \frac{V_p}{V_g} \Rightarrow \frac{V_p}{V_g} = 1 - \beta \frac{dV_p}{d\omega}$$

$$\therefore \boxed{V_p = V_g\left(1 - \beta \frac{dV_p}{d\omega}\right)}$$

If V_p is constant (or) Independent on 'ω' then

$$\frac{dV_p}{d\omega} = 0.$$

$$\therefore V_p = V_g\left(1 - \beta(0)\right)$$

$$\therefore \boxed{V_p = V_g}$$

Lossless Transmission line :-

Definition :- Lossless transmission line is a transmission line if $R=0$, and $G=0$.

Condition :-

The propagation constant is $\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \rightarrow (1)$

Characteristic impedance $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \rightarrow (2)$

Substitute the condition for lossless is

$R=0, G=0$.

$\therefore \gamma = \alpha + j\beta = \sqrt{(0+j\omega L)(0+j\omega C)} = \sqrt{j^2\omega^2 LC}$

$\alpha + j\beta = j\omega\sqrt{LC}$

Equating real & imaginary parts

$$\boxed{\alpha = 0, \ \beta = \omega\sqrt{LC}} \rightarrow (3)$$

Similary characteristic impedance is

$Z_0 = \sqrt{\frac{0+j\omega L}{0+j\omega C}} = \sqrt{\frac{\cancel{j\omega} L}{\cancel{j\omega} C}}$

$$\boxed{\therefore Z_0 = \sqrt{\frac{L}{C}}} \rightarrow (4)$$

Phase velocity is $V_p = \frac{\omega}{\beta} = \frac{\cancel{\omega}}{\cancel{\omega}\sqrt{LC}}$

$$\boxed{\therefore V_p = \frac{1}{\sqrt{LC}}}$$

since $V_p = \frac{1}{\sqrt{\mu\epsilon}}$

$\therefore \frac{1}{\cancel{\sqrt{}}LC} = \frac{1}{\cancel{\sqrt{}}\mu\epsilon}$

$\therefore \boxed{LC = \mu\epsilon} \rightarrow$ Condition for Lossless Transmission line

Distortionless Transmission line :-

A line is said to be a distortionless transmission line if the attenuation constant 'α' is independent of frequency ω and 'β' (phase constant) varies with frequency in linearly.

condition:-

We know that, propagation constant is

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{L\left(\frac{R}{L}+j\omega\right)C\left(\frac{G}{C}+j\omega\right)}$$

$$\therefore \alpha + j\beta = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)}$$

To make 'α' in above equation independent of frequency Let $\frac{R}{L} = \frac{G}{C} \rightarrow (1)$

Then $\alpha + j\beta = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{R}{L}+j\omega\right)} = \sqrt{LC}\sqrt{\left(\frac{R}{L}+j\omega\right)^2}$

$$\therefore \alpha + j\beta = \sqrt{LC}\left(\frac{R}{L}+j\omega\right)$$

Equating Real and Imaginary terms.

$$\boxed{\alpha = \sqrt{LC}\,\frac{R}{L} = R\sqrt{\frac{C}{L}}} \rightarrow (2) \quad \text{and} \quad \boxed{\beta = \omega\sqrt{LC}} \rightarrow (3)$$

Also Z_0 = characteristic impedance

$$\Rightarrow Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L\left(\frac{R}{L}+j\omega\right)}{C\left(\frac{G}{C}+j\omega\right)}}$$

($\because$ equat (1)) $\left(\frac{R}{L} = \frac{G}{C}\right)$

$$= \sqrt{\frac{L\cancel{\left(\frac{R}{L}+j\omega\right)}}{C\cancel{\left(\frac{R}{L}+j\omega\right)}}}$$

$$= \sqrt{\frac{L}{C}} \rightarrow (4)$$

Therefore for distortionless Transmission

$$\alpha = R\sqrt{\frac{C}{L}} = G\sqrt{\frac{L}{C}} \rightarrow (5)$$

$\Rightarrow \boxed{\alpha = \frac{R}{Z_0} \text{ (or) } G Z_0}$ $\quad (\because Z_0 = \sqrt{\frac{L}{C}})$

and $\boxed{\beta = \omega\sqrt{LC}}$, phase velocity $V_p = \frac{1}{\sqrt{LC}} = \frac{\omega}{\beta} \rightarrow (6)$

$\therefore$ The α is independent of frequency, and β is Varies linearly with frequency.

so the condition for distortionless transmission line is $\boxed{\frac{R}{L} = \frac{G}{C}}$

Distortion:- When a wave propagates along the Transmission line, there was some fluctuations occur. This is called as "Distortion.

line distortion:- The presence of many components cause variation in attenuation and phase velocity of the wave. This phenomena is called "line distortion".

Frequency distortion:- Frequency distortion occurs due to variation of attenuation with frequency. At Receiver end the wave contains different amplitudes at different frequencies resulting in frequency distortion.

→ If α is independent of 'f' then no distortion exists.

Delay distortion:- Delay distortion occurs due to variation of time delay with frequency. If the time required to transmit various frequency components of wave is not same, then delay distortion occurs.

Condition for minimum attenuation:-

We know that attenuation constant α is expressed as

$$\alpha = \frac{1}{\sqrt{2}}\left[\sqrt{RG - \omega^2 LC + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right]$$

The attenuation constant depends on primary constants R, L, G, and C of line and frequency of wave.

Variation of attenuation with respect to Inductor:-

Let parameter 'L' be a variable.

The minimum attenuation occurs at $\frac{d\alpha}{dL} = 0$.

(ie) $$\frac{d\alpha}{dL} = \frac{1}{\sqrt{2}} \times \frac{1}{2\left[\sqrt{RG - \omega^2LC + \sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right]} \times \left[-\omega^2C + \frac{2\omega^2L(G^2+\omega^2C^2)}{2\sqrt{(R^2+\omega^2L^2)(G^2+\omega^2C^2)}}\right] = 0.$$

$$\Rightarrow -\omega^2C + \frac{\cancel{2}\omega^2L\,\cancel{(G^2+\omega^2C^2)}^{\sqrt{G^2+\omega^2C^2}}}{\cancel{2}\sqrt{R^2+\omega^2L^2}\,\cancel{\sqrt{G^2+\omega^2C^2}}} = 0.$$

$$\therefore \frac{\cancel{\omega^2}L(\sqrt{G^2+\omega^2C^2})}{\sqrt{R^2+\omega^2L^2}} = \cancel{\omega^2}C.$$

$$\therefore L\sqrt{G^2+\omega^2C^2} = C\sqrt{R^2+\omega^2L^2}$$

squaring on both sides

$$L^2(G^2+\omega^2C^2) = C^2(R^2+\omega^2L^2)$$

$$\Rightarrow L^2G^2 + \cancel{L^2\omega^2C^2} = C^2R^2 + \cancel{C^2\omega^2L^2} \Rightarrow L^2G^2 = C^2R^2$$

$$\Rightarrow LG = CR. \quad (\text{or})$$

$$\boxed{\frac{R}{L} = \frac{G}{C}}$$

This condition is same as for distortion less line, so, minimum attenuation occurs at

$$\boxed{L = \frac{CR}{G} \ H/m}$$

Variation of attenuation with respect to capacitor :-

Let capacitor be a variable.

The minimum attenuation occurs at $\frac{d\alpha}{dC} = 0$.

similarly if capacitor is varying,

$$\boxed{C = \frac{LG}{R}} \ \text{farads}/m.$$

Variation of attenuation with respect to Resistor (or) Conductor :-

The minimum attenuation occurs at $\frac{d\alpha}{dR} = 0$ (or) $\frac{d\alpha}{dG} = 0$.

It is observed that $\alpha = 0$ occurs only when $R = 0$, and $G = 0$.

$\therefore$ For a loss less line $\boxed{\alpha = 0, \text{ if } R = 0, G = 0}$

The attenuation was minimized.

Loading :- Introduction of Inductance in series with the line is called "loading" and such a lines are called as "loaded" lines.

* By using loading the inductance of a transmission line can be increasing.

There are 3 types of loading a line.

1. continuous loading
2. patch loading
3. Lumped loading.

→ generally lumped Inductors (or) coils are placed at suitable intervals along the line to increase the inductance.

for a loaded line, 'α' is constant and very low upto a cut-off frequency fc.

$$\therefore \boxed{f_c = \frac{1}{\pi\sqrt{LCd}} \text{ Hz.}}$$

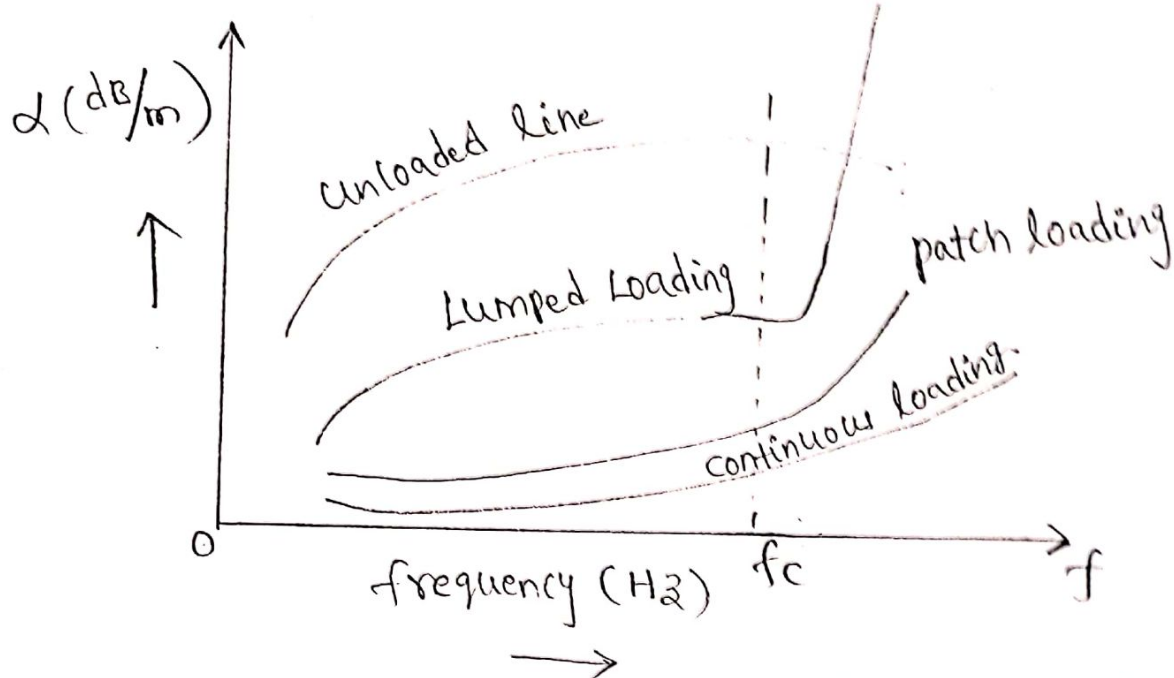
where L = Inductance (H/m)
C = capacitance (F/m)
d = length of loaded line (m)

① **Continuous loading** :- Here Loading is done by winding a type of Iron around the conductor. This Increases Inductance but It is expensive.

② **patch loading** :- This type of loading uses sections of continuous loaded cable seperated by unloaded cable. Hence cost is reduced.

③ **Lumped loading** :- Here loading is introduced at uniform intervals. It may be noted that hysteresis and eddy current losses are introduced by loading and hence design should be optimal.

frequency characteristics of loaded lines are given below.



The loaded lines provide very low distortion and perform much better than unloaded lines.