HIGH FREQUENCY TRANSISTOR AND FET CIRCUITS

INTRODUCTION:-

At low frequencies we analyse transistor using h-parameters. But at high frequencies to analyse the transistor the h-parameter model is not suitable for following two reasons.

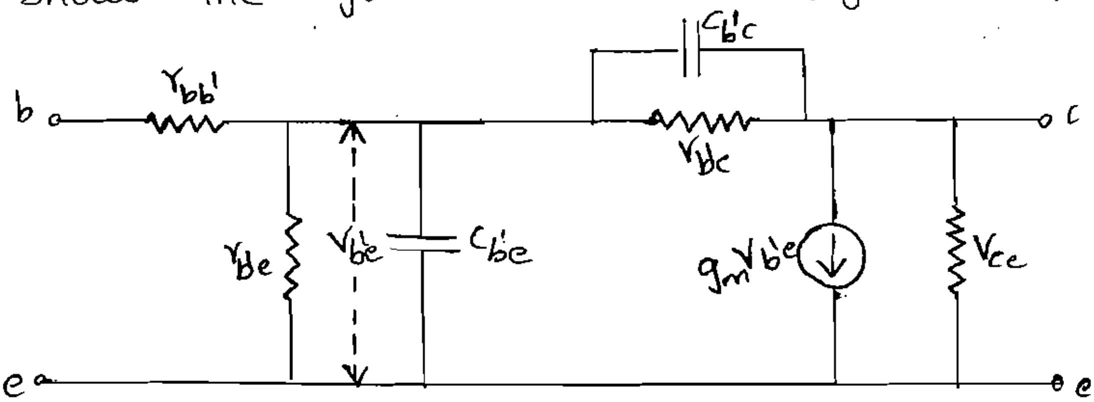
1. The values of h-parameters are not constant at high frequencies. Therefore, it is necessary to analyse transistor at each and every frequency, which is not possible practically.

2. At high frequencies the h-parameters become complex in nature.

So, in order to analyse the high frequency circuits we will use hybrid-π model.

HYBRID-π COMMON EMITTER TRANSCONDUCTANCE MODEL:-

Common emitter circuit is most important practical configuration. So, we choose this circuit for the analysis of the transistor using hybrid-π model. The following circuit shows the hybrid-π model configuration of a CE.



All parameters in this model are assumed to be independent of frequency.

PARAMETERS IN THE HYBRID-π MODEL :-

$C_{b'e}$ and $C_{b'c}$:-

$C_{b'e}$:- The forward biased PN junction exhibits a capacitive effect called as diffusion capacitance.

C_e is diffusion capacitance. This produces when the diode is in forward biased in the input.

$C_{b'c}$:- The reverse biased PN junction exhibits a capacitive effect called as transition capacitance.

C_c is transistor capacitance exists due to reverse bias of the diode in the output.

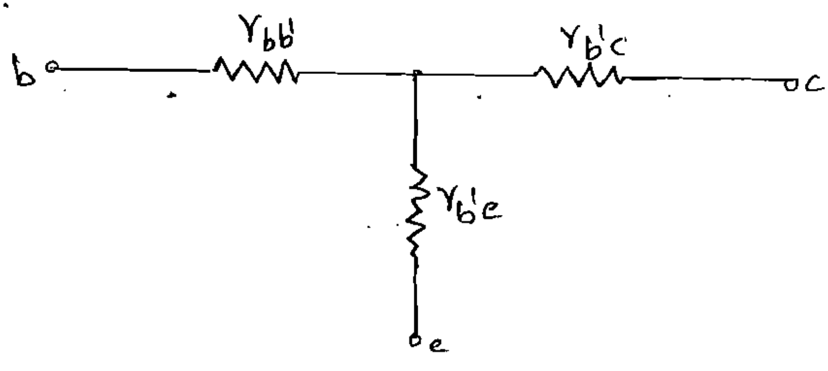
$r_{bb'}$:-

The base b' which is existing internally in the circuit is not practically possible to the use.

So, b' is spreaded to be with the help of resistor. So, $r_{bb'}$ is known as base spreading resistors

$r_{b'e}$:-

It is forward dynamic resistance existing in emitter diode.



$r_{b'c}$:-

Due to early effect, the varying voltages across the collector to emitter junction results in base width modulation. As V_{CE} is changing, the input current is also changing with the help of a feedback resistance $r_{b'c}$.

Trans conductance (gm) :-

It is the ratio of change in the ~~input~~ collector current to that of change in input voltage $V_{b'e}$ when V_{ce} is kept constant

$$\therefore g_m = \left. \frac{\partial I_c}{\partial V_{b'e}} \right|_{V_{ce} = \text{Constant}}$$

r_{ce} :-

It is the output resistance. It is also the result of the early effect.

HYBRID - π PARAMETER VALUES :-

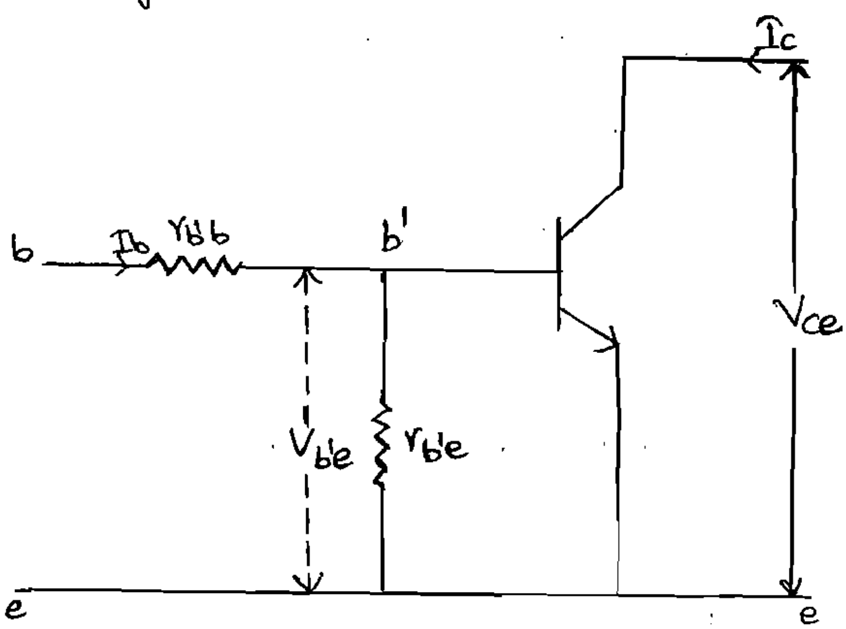
Parameter	Discription	Value
g_m	Mutual trans conductance of the transistor	50 mA/V
$r_{bb'}$	Base spreading resistance	100 Ω
$r_{b'e}$ (or) r_e	Resistance between base and emitter (or) Dynamic resistance of emitter diode.	1 kΩ

Parameter	Discription	Value.
$r_{b'c}$ (or) r_c	Feed back resistance due to early effect	$4M\Omega$
r_{ce}	Output resistance	$80k\Omega$
$C_{b'e}$ (or) C_e	Diffusion Capacitance	$100pF$
$C_{b'c}$ (or) C_c	Trans Capacitance	$3pF$.

DETERMINATION OF HYBRID-π CONDUCTANCES:-

TRANSISTOR TRANSCONDUCTANCE (G_M):-

Let us consider a p-n-p transistor in the CE configuration with V_{cc} bias in the collector circuit.



Now,

from the definition of transconductance we know that

$$g_m = \left.\frac{\partial I_c}{\partial V_{b'e}}\right|_{V_{ce} = \text{constant}}$$

The expression for the collector current in an n-p-n transistor is given as

$$I_c = I_{C_0} + \alpha I_E$$

Partial differentiate the above expression.

$$\partial I_c = \alpha\, \partial I_E \quad [\because I_{C_0} = \text{Constant}]$$

Substitute in g_m.

$$g_m = \frac{\alpha\, \partial I_E}{\partial V_{b'e}}$$

The emitter diode resistance $r_{b'e}$ is given as

$$r_{b'e} = r_e = \frac{V_{b'e}}{I_e}$$

$$\Rightarrow g_m = \frac{\alpha}{r_e}$$

The dynamic resistance of the emitter diode is given as

$$r_e = \frac{V_T}{I_e}$$

Where V_T = Volt equivalent temperature, which is given as $\frac{KT}{q}$.

K = Boltzmann's Constant.

q = Charge of electron.

Substitute above value in g_m

$$g_m = \frac{\alpha I_E}{V_T} = \frac{I_c - I_{C_0}}{V_T}$$

③

as $I_C >> I_{CO}$

$$g_m = \frac{I_C}{V_T}$$

$$g_m = \frac{I_C}{\left(\frac{KT}{q}\right)}$$

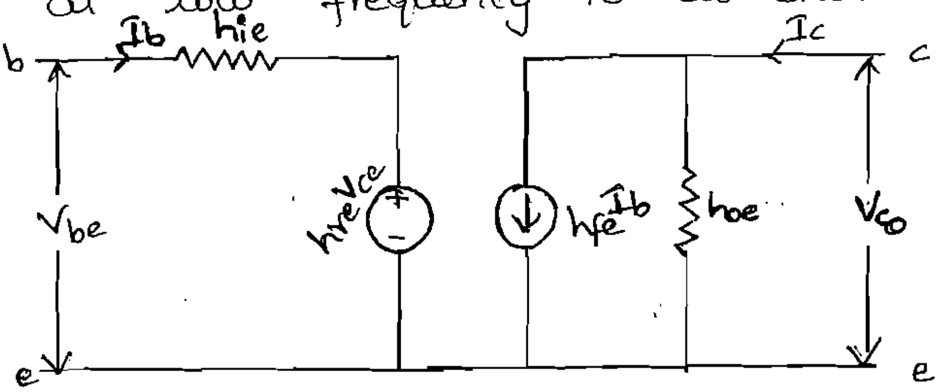
$$g_m = \frac{11,600 I_C}{T}$$

At room temperature $T = 300K$

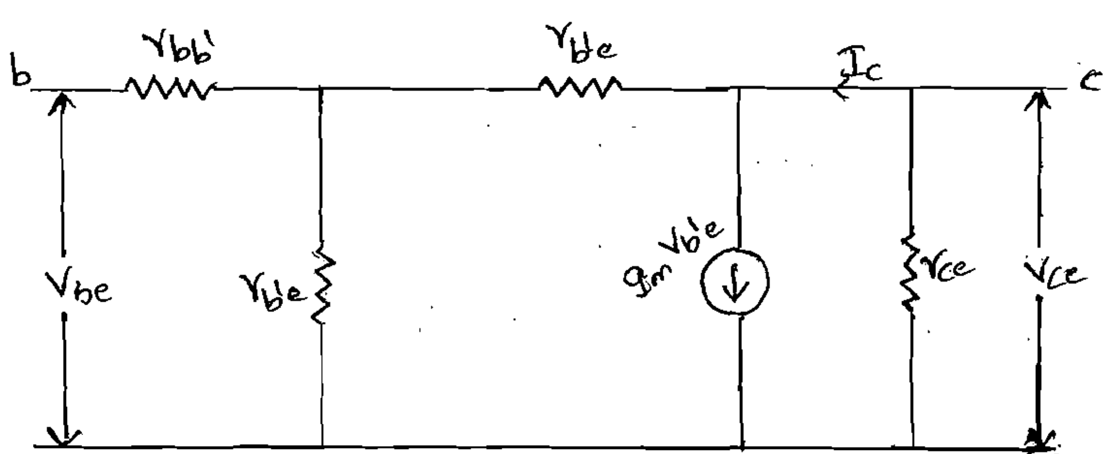
$$g_m = 38 \cdot 66 I_C$$

INPUT CONDUCTANCE ($g_{b'e}$):-

The hybrid - π model and h-parameter for CE configuration at low frequency is as shown.



h- Parameter Model



Here the capacitors acts as an open circuit.

From hybrid model of collector current.

$$I_c = h_{fe} I_b + h_{oe} V_{ce}$$

As $V_{CE} = 0$, $h_{fe} = \frac{I_c}{I_b}$ — ①

From hybrid-π low frequency model

As the value of $(r_{b'c} = 4M\Omega) >> r_{b'e}$

∵ The base current I_b completely flows through $r_{b'c}$.

$$\because V_{b'e} = I_b r_{b'e} \quad — ②$$

As the value of $r_{b'c}$ is much more larger, the Collector current $I_c = g_m V_{b'e}$

From ① $\quad I_c = g_m I_b r_{b'e} \qquad [V_{b'e} = I_b r_{b'e}]$

$$\frac{I_c}{I_b} = g_m r_{b'e}$$

$$h_{fe} = g_m r_{b'e}$$

$$r_{b'e} = \frac{h_{fe}}{g_m}$$

$$r_{b'e} = \frac{h_{fe} \cdot V_T}{I_c}$$

$$\boxed{g_{b'e} = \frac{I_c}{h_{fe} \cdot V_T}}$$

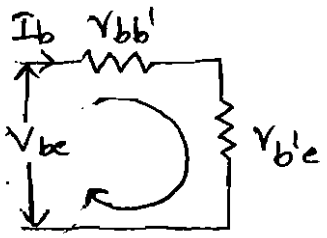
BASE SPREADING CONDUCTANCE ($g_{bb'}$):—

From the expression of h-parameter model

$$V_{be} = h_{ie} I_b + h_{re} V_{ce}$$

$$V_{be} = h_{ie} I_b \Big|_{V_{ce}=0} \quad — ①$$

$$h_{ie} = \frac{V_{be}}{I_b}$$



Now, applying KVL at input from the hybrid-π low frequency circuit.

$$V_{be} = I_b r_{bb'} + I_b r_{b'e}$$

$$V_{be} = I_b [r_{bb'} + r_{b'e}]$$

from (1)

$$I_b [r_{bb'} + r_{b'e}] = h_{ie} I_b$$

$$r_{bb'} = h_{ie} - \frac{h_{fe} \cdot V_T}{I_c}$$

$$g_{b'e} = \frac{1}{r_{bb'}}$$

$$\boxed{g_{bb'} = \frac{I_c}{I_c \cdot h_{ie} - h_{fe} V_T}}$$

FEED BACK CONDUCTANCE :- $[g_{b'c}]$:-

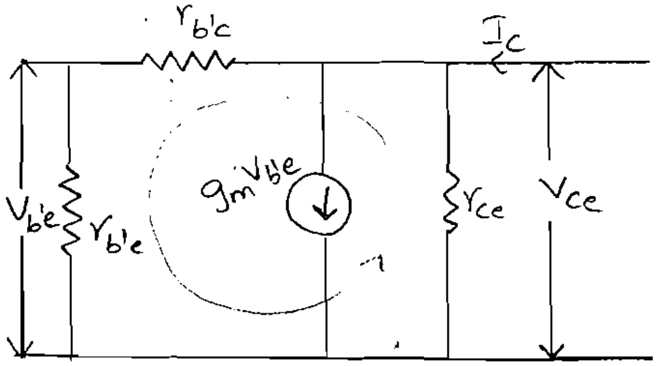
Let us consider the h-parameter model for CE configuration with input open circuit ($I_b = 0$),

W.K.T $\quad V_{be} = h_{ie} I_b + h_{re} V_{ce}$

As $I_b = 0$,

$$\Rightarrow V_{be} = h_{re} V_{ce} \quad \text{—} (1)$$

From hybrid-π model



Apply KVL to output loop

As the base current is zero the total current flowing through $r_{b'e}$ is i_1 and $V_{be} = V_{b'e}$

$$V_{ce} = I_1 r_{b'c} + I_1 r_{b'e}$$

$$\boxed{I_1 = \frac{V_{ce}}{(r_{b'c} + r_{b'e})}} \quad \text{— ②}$$

$$V_{b'e} = I_1 r_{b'e}$$

$$V_{b'e} = \frac{V_{ce} \cdot r_{b'e}}{(r_{b'c} + r_{b'e})}$$

We know that

$$V_{b'e} = V_{be}$$

$$V_{be} = \frac{V_{ce} \cdot r_{b'e}}{(r_{b'c} + r_{b'e})}$$

From ①

$$h_{re} \cancel{V_{ce}} = \frac{\cancel{V_{ce}} \cdot r_{b'e}}{[r_{b'c} + r_{b'e}]}$$

$$h_{re} = \frac{r_{b'e}}{[r_{b'c} + r_{b'e}]}$$

$$r_{b'e} + r_{b'c} = \frac{r_{b'e}}{h_{re}}$$

$$r_{b'c} = \frac{r_{b'e}}{h_{re}} - r_{b'e}$$

$$r_{b'c} = r_{b'e}\left[\frac{1}{h_{re}} - 1\right]$$

$$r_{b'c} = \frac{(1-h_{re})\, r_{b'e}}{h_{re}} \qquad [\because 1-h_{re} = 1]$$

$$r_{b'c} = \frac{r_{b'e}}{h_{re}}$$

$$g_{b'e} = \frac{h_{re}}{r_{b'e}}$$

$$g_{b'c} = h_{re} \cdot g_{b'e}$$

$$\boxed{g_{b'c} = h_{re}\left[\frac{I_c}{h_{fe} \cdot V_T}\right]}$$

OUTPUT CONDUCTANCE (g_{ce}) :-

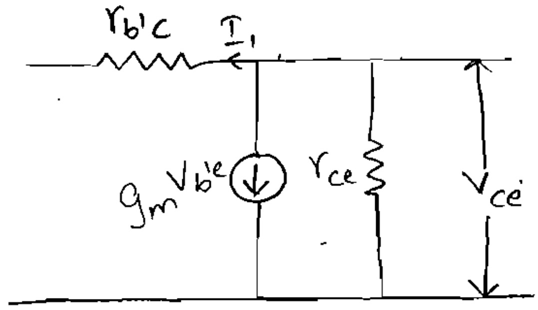
From hybrid parameter model

$$I_c = h_{fe} I_b + h_{oe} V_{ce}.$$

As $I_b = 0$

$$I_c = h_{oe} V_{ce}$$

$$\boxed{h_{oe} = \frac{I_c}{V_{ce}}} \quad \text{—} \ (1)$$



From hybrid -π low frequency model

$$I_c = \frac{V_{ce}}{r_{ce}} + g_m V_{b'e} + I_1 \qquad \left[\because V_{b'e} = \frac{V_{ce}\, r_{b'e}}{r_{b'c} + r_{b'e}}\right]$$

$$= \frac{V_{ce}}{r_{ce}} + g_m V_{b'e} + \frac{V_{ce}}{r_{b'c} + r_{b'e}}$$

$$I_c = \frac{V_{ce}}{r_{ce}} + \frac{g_m V_{ce} r_{b'e}}{r_{b'c}+r_{b'e}} + \frac{V_{ce}}{r_{b'c}+r_{b'e}}$$

Divide with V_{ce}

from ①

$$h_{oe} = \frac{I_c}{V_{ce}}$$

$$h_{oe} = \frac{1}{r_{ce}} + \frac{g_m r_{b'e}}{r_{b'c}+r_{b'e}} + \frac{1}{r_{b'c}+r_{b'e}}$$

$g_m r_{b'e} = h_{fe}$

$1+h_{fe} \approx h_{fe}$

$$h_{oe} = g_{ce} + \frac{1+h_{fe}}{r_{b'c}+r_{b'e}}$$

$$h_{oe} = g_{ce} + \frac{1+h_{fe}}{r_{b'c}+r_{b'e}}$$

$$= g_{ce} + \frac{h_{fe}}{r_{b'c}+r_{b'e}}$$

$$\boxed{g_{ce} = h_{oe} \cdot h_{fe}\, g_{b'c}}$$

HYBRID π - CAPACITANCE :-

There exists two types of capacitances in a transistor. They are diffusion capacitance and transition capacitance. The transition capacitance $C_c = C_{b'c}$ can be findout by using common base method in which the capacitance exists at the output. i.e between collector and base by making input current $I_e = 0$.

The diffusion capacitance is a combination of emitter diffusion capacitance and junction capacitance.

$$\because C_e = C_{De} + C_{Te}.$$

But the effect of emitter diffusion capacitance is more when ~~junction~~ compared to junction capacitance.

The expression of base width charge Q_B is given as

$$Q_B = \frac{1}{2} p'(0) \cdot (A)(W)(q) \quad \text{—①}$$

Where $\frac{1}{2} p'(0)$ is average minority charge distribution in base

$A \rightarrow$ Cross sectional area

$W \rightarrow$ volume of the base.

$q \rightarrow$ Charge in columbs.

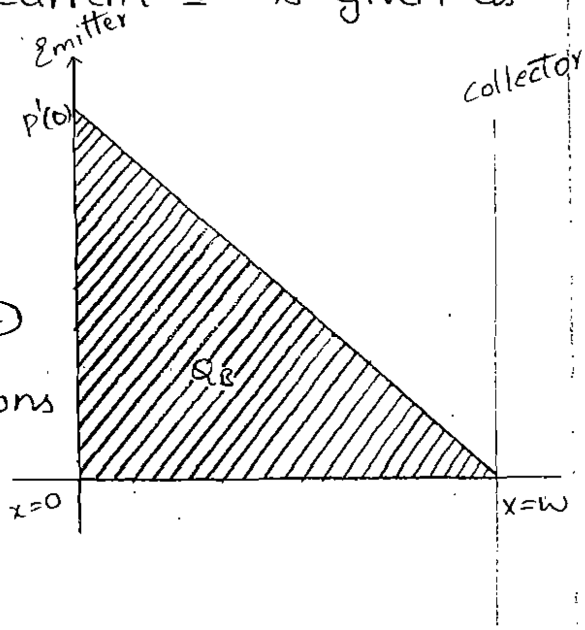
The expression of diffusion current I is given as

$$I = (A)(q)(D_B)\frac{dP}{dx}$$

Where

$$\frac{dP}{dx} = \frac{p'(0)}{W}$$

$$\therefore I = (A)(q)(D_B) \cdot \frac{p'(0)}{W} \quad \text{—②}$$



Now combine ① and ② equations

$$(A)(q)(p'(0)) = \frac{I \cdot W}{D_B}$$

$$Q_B = \frac{I}{2} \cdot \frac{W^2}{D_B}$$

The emitter diffusion capacitance is given as rate of change of Q_B with respect to voltage 'V'.

$$C_{DE} = \frac{dQ_B}{dV} = \frac{W^2}{2D_B} \cdot \frac{dI}{dV}$$

$$\Rightarrow C_{De} = \frac{W^2}{2D_B} \cdot \frac{1}{r_e} \qquad \left[\because r_e = \frac{dV}{dI} = \frac{V_T}{I_E}\right]$$

$$\Rightarrow C_{De} = \frac{W^2}{2D_B} \cdot \frac{I_E}{V_T}$$

$$\Rightarrow C_{DE} = \frac{\omega^2}{2D_B}[g_m] \qquad \left[\text{Since } g_m = \frac{I_E}{V_T}\right]$$

From the above expression we can say that the emitter diffusion capacitance is proportional to emitter current I_E. But practically the diffusion capacitance is found by the value of f_T, which is the frequency at which voltage gain drops to unity.

$$\boxed{\because C_e = \frac{g_m}{2\pi f_T}}$$

VALIDITY OF HYBRID-π PARAMETERS:-

Assuming that V_{BE} changes so slowly with time that the minority-carrier charge distribution in the base region is always triangular.

While by observing the expression of hybrid-π parameters we can say that these parameters are independent of frequencies. If they satisfies

$$2\pi f \cdot \frac{\omega^2}{6D_B} << 1 \quad \text{—①}$$

We know that

$$C_{DE} = \frac{\omega^2}{2D_B} \cdot g_m$$

$$\frac{\omega^2}{6D_B} = \frac{C_{DE}}{3g_m} \quad \text{—②}$$

and

$$C_e = \frac{g_m}{2\pi f_T}$$

$$\Rightarrow \frac{C_e}{g_m} = \frac{1}{2\pi f_T} = \frac{1}{6\pi f_T}$$

Substituting ② in ①

$$2\pi f \cdot \frac{C_{De}}{3g_m} << 1$$

$$\cancel{2\pi f} \cdot \frac{1}{\cancel{6\pi} 3 f_T} << 1$$

$$\frac{f}{3f_T} << 1$$

$$\boxed{f << 3f_T}$$

The hybrid π parameters are valied upto $3f_T$.

VARIATION OF HYBRID-π PARAMETERS WITH RESPECT TO I_c, V_{CE}, AND TEMPERATURE :-

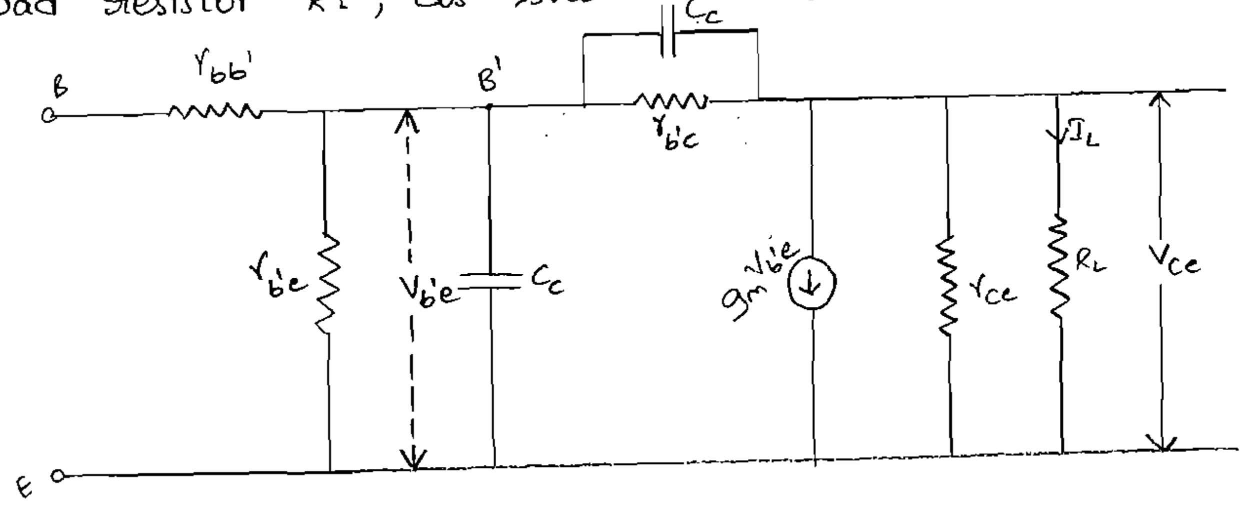
The hybrid parameters are dependent on the magnitudes of I_c and V_{ce} and the temperature. The dependence of hybrid parameters such as g_m, $r_{bb'}$, $r_{b'e}$, C_e, C_c, h_{fe} and h_{ie} on the collector current magnitude $|I_c|$, the collector to emitter voltage magnitude $|V_{CE}|$, and the temperature is shown in the following table.

Parameter	Variation with Increasing		
	$\lvert I_c \rvert$	$\lvert V_{CE} \rvert$	T
g_m	Increases - proportional to $\lvert I_c \rvert$	Independent	Decreases - Inversely proportional to T i.e $1/T$.
$r_{bb'}$	Decreases - due to conductivity modulation of the base		Increases due to decrease in conductivity as a result of decrease in the mobility of majority and minority carriers.

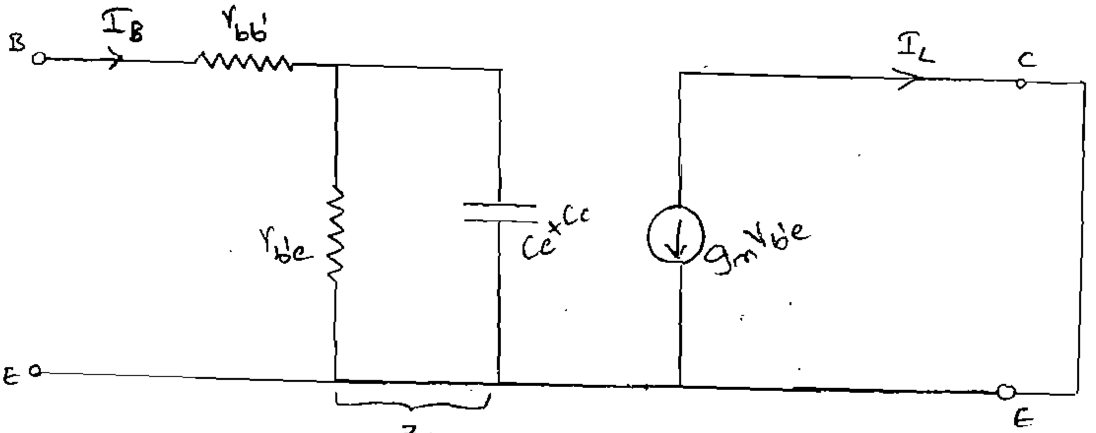
Parameter	$\lvert I_c \rvert$	$\lvert V_{CE} \rvert$	T
$r_{b'e}$	Decreases - inversely proportional to $\lvert I_c \rvert$ i.e $1/\lvert I_c \rvert$.	Increases.	Increases.
C_e	Increases - proportional to $\lvert I_c \rvert$	Decreases.	
C_c	Independent	Decreases	Independent
h_{fe}	Increases for smaller values of $\lvert I_c \rvert$ and decreases with higher values of $\lvert I_c \rvert$	Increases due to the increase of transistor 'α' as a result of decrease of the base width and the reduction in re-combination	Increases.
h_{ie}	Decreases - inversely proportional to $\lvert I_c \rvert$ i.e $1/\lvert I_c \rvert$	Increases	Increases.

COMMON EMITTER SHORT CIRCUIT CURRENT GAIN:-

Consider a single stage CE transistor amplifier with load resistor R_L, as shown below.



For the analysis of the short circuit current gain we have to assume $R_L = 0$. With $R_L = 0$, i.e output short circuited r_{ce} becomes zero, $r_{b'e}$, C_e and $C_{b'c}$ appear in parallel. When C_c ($C_{b'c}$) appear between base and emitter it is known as '<u>Miller Capacitance</u>'.



SIMPLIFIED HYBRID -π MODEL FOR SHORT CIRCUIT CE TRANSISTOR.

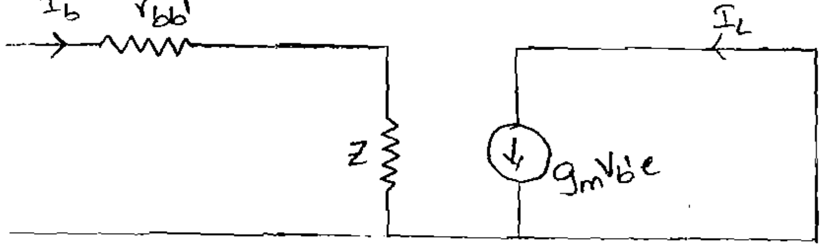
Here ($r_{b'c}$) and (C_c+C_e) are in shunt there exists an equivalent Impedance 'z'.

$$z = r_{b'e} \parallel (C_c+C_e)$$

$$= \frac{r_{b'e} \times \frac{1}{j\omega (C_c+C_e)}}{r_{b'e} + \frac{1}{j\omega (C_c+C_e)}}$$

$$= \frac{r_{b'e}}{1+j\omega r_{b'e}(C_e+C_c)}$$

Then the circuit can be redrawn as



$$A_I = \frac{I_L}{I_b} = \frac{-g_m V_{b'e}}{I_b}$$

Where $V_{b'e} = I_b Z$

$$\therefore A_I = -g_m Z$$

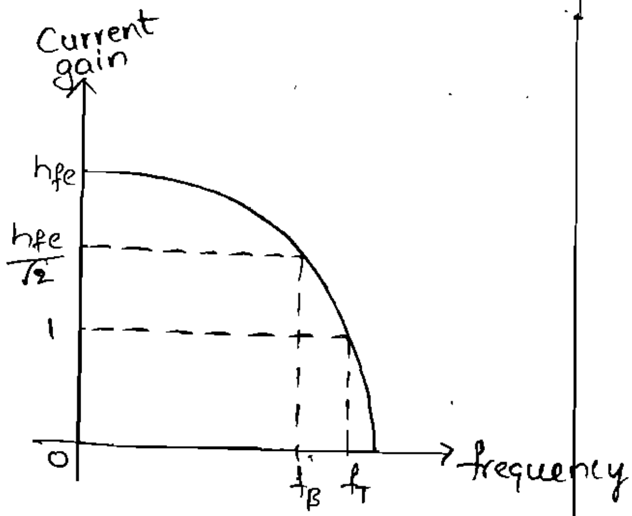
$$A_I = \frac{-g_m r_{b'e}}{1+j2\pi f\, r_{b'e}(C_e+C_c)}$$

$$A_I = \frac{-h_{fe}}{1+j2\pi f\, r_{b'e}(C_c+C_e)} \quad [\text{Since } g_m r_{b'e} = h_{fe}]$$

As current gain A_I is inversely proportional to frequency.

At low frequency, the value of denominator is approximately equal to 1

Then current gain



$$A_I = -h_{fe}$$

$$|A_I| = h_{fe}$$

A_I can be written as

$$A_I = \frac{-h_{fe}}{1+j(f/f_\beta)}$$

Where $f_\beta = \dfrac{1}{2\pi r_{b'e}(C_e+C_c)}$

Magnitude of A_I

$$|A_I| = \frac{h_{fe}}{\sqrt{1+(f/f_\beta)^2}}$$

When $f = f_\beta$

$$|A_I| = \frac{h_{fe}}{\sqrt{2}}$$

PARAMETER f_β :-

It is the frequency obtained when a short circuit CE current gain drops to 3dB (or) $1/\sqrt{2}$ times of its value.

$$\because\ f_\beta = \frac{1}{2\pi r_{b'e}(C_e+C_c)} = \frac{g_{b'e}}{2\pi(C_e+C_c)}$$

$$= \frac{g_m}{h_{fe}\, 2\pi(C_e+C_c)} \qquad \left[\because g_{b'e} = \frac{I_c}{h_{fe}V_T} = \frac{g_m}{h_{fe}}\right]$$

PARAMETER (f_α) :-

It is the frequency obtained when a short circuit common base current gain drops to 3dB (or) $\frac{1}{\sqrt{2}}$ times of its value.

Current gain of CB amplifier can be given as

$$|A_I| = \frac{h_{fb}}{\sqrt{1+(f/f_\alpha)^2}}$$

Where f_α is given as

$$\boxed{f_\alpha = \frac{1}{2\pi(1+h_{fb})C_e}}$$

PARAMETER (f_T) :-

It is a frequency when a short circuit CE current gain is equal to 1. (or) unity.

$\because$ The expression of current gain is given as

$$1 = \frac{h_{fe}}{\sqrt{1+(f_T/f_\beta)^2}}$$

Since $\frac{f_T}{f_\beta} >> 1$

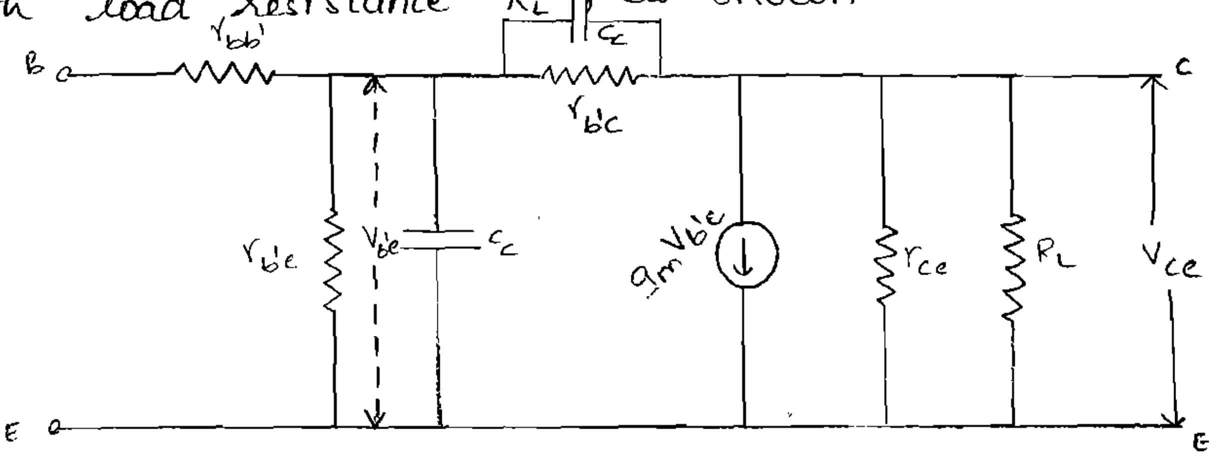
$$1 = \frac{h_{fe}}{(f_T / f_\beta)}$$

$$1 = \frac{h_{fe} \cdot f_\beta}{f_T}$$

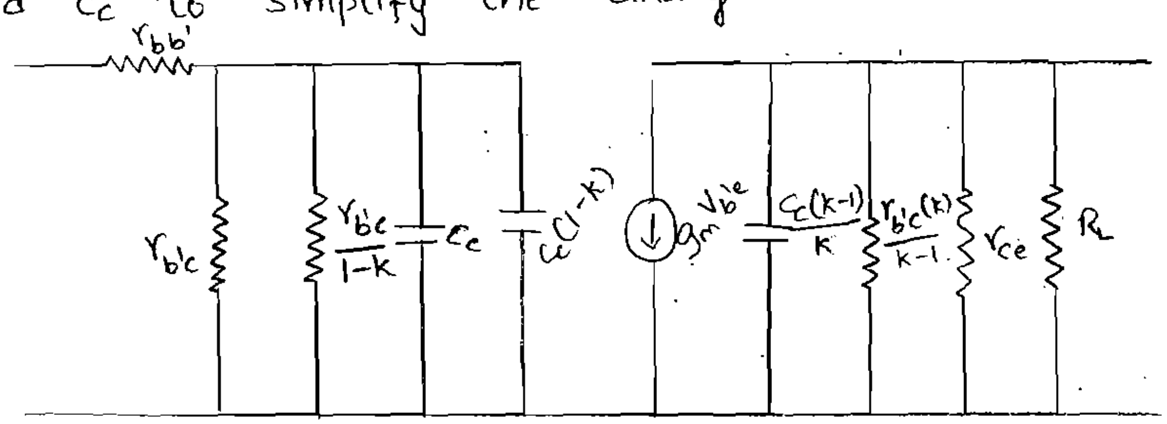
$$\Rightarrow \boxed{f_T = h_{fe} \cdot f_\beta}$$

CE CURRENT GAIN WITH LOAD RESISTANCE:-

Consider a single stage CE transistor amplifier with load resistance R_L as shown below.



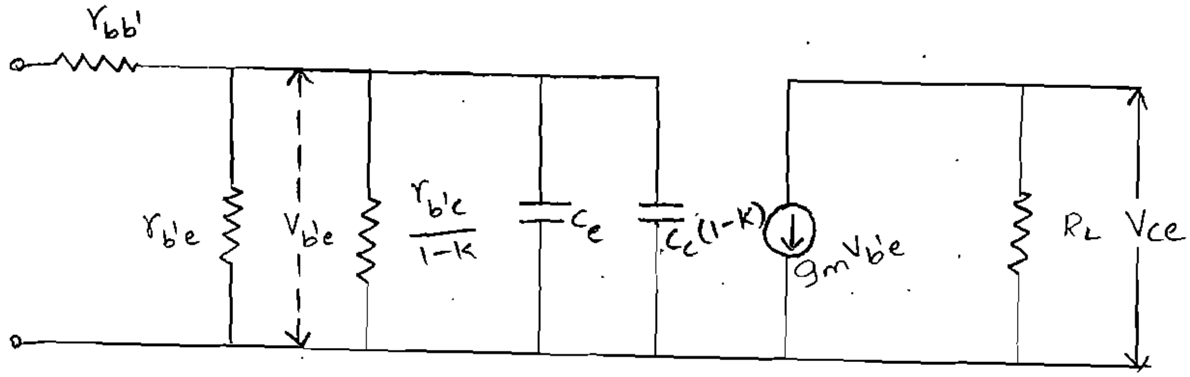
As $r_{b'c}$ and C_c are feedback resistance and capacitance. Using Millers theorem, we can split $r_{b'c}$ and C_c to simplify the analysis.



Considering the output circuit

$$\frac{r_{b'c} \cdot k}{k-1} \approx r_{b'c} \qquad \frac{C_c(k-1)}{k} \approx C_c$$

As we know that $R_{ce} = 80k\Omega$, $r_{b'c} = 4M\Omega$, $C_c = 3PF$
The load impedance R_L is considered to be very less when compared to r_{ce}, $r_{b'e}$. From the above approximations the circuit can be redrawn as



Voltage Gain (A_v):-

$$A_v = \frac{V_o}{V_i} = \frac{V_{ce}}{V_{b'e}}$$

$$V_{ce} = I_L R_L = -g_m V_{b'e} R_L$$

$$A_v = -g_m R_L$$

$$\Rightarrow k = -g_m R_L$$

Considering $g_m = 50 mA/v$, and $R_L = 2k\Omega$ then

$$k = -[(50)(2)]$$

$$\boxed{k = -100}$$

Considering the input circuit :-

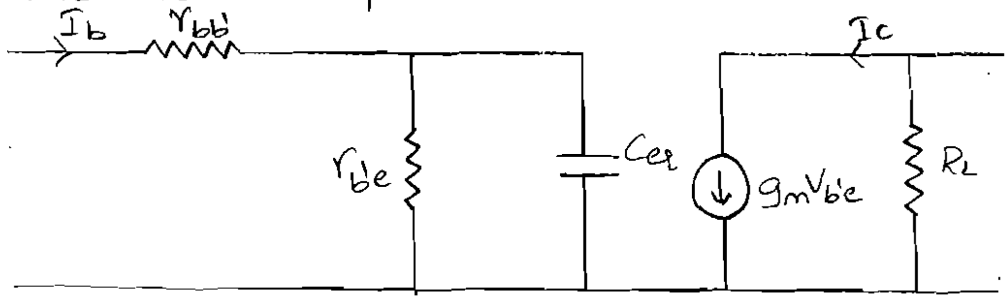
$$\frac{r_{b'c}}{1-k} = \frac{r_{b'c}}{101} = \frac{4\times10^{+6}}{101} = 40K$$

$$C_c(1-k) = C_c[1+g_m R_L]$$

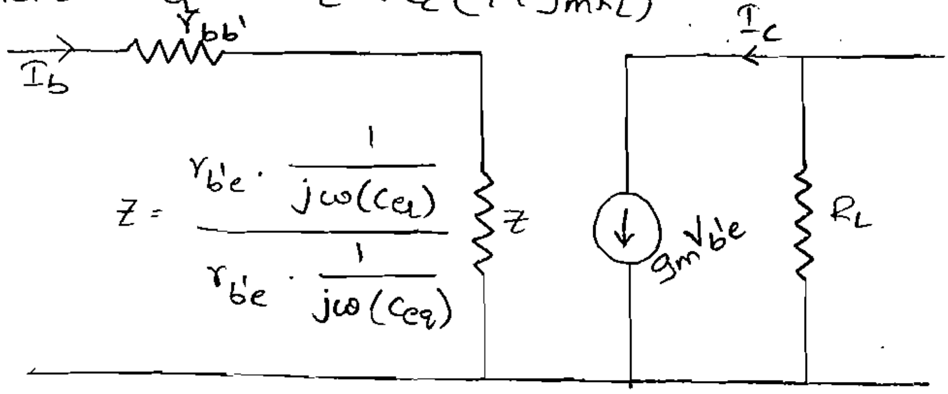
$$= (3\times10^{-12})(1+100)$$

$$=$$

From the above expression the ckt can be redrawn as



where $C_{eq} = C_e + C_c(1+g_m R_L)$



$$Z = \frac{r_{b'e} \cdot \frac{1}{j\omega(C_{eq})}}{r_{b'e} \cdot \frac{1}{j\omega(C_{eq})}}$$

$$Z = r_{b'e} \parallel C_e + C_c$$

$$Z = \frac{r_{b'e} \frac{1}{j\omega(C_c+C_e)}}{r_{b'e} + \frac{1}{j\omega(C_c+C_e)}} = \frac{r_{b'e}}{1+r_{b'e}(j\omega(C_c+C_e))}$$

Current Gain :-

$$A_I = \frac{I_o}{I_i} = \frac{I_L}{I_b} = \frac{-g_m V_{b'e}}{I_b}$$

where $V_{b'e} = I_b Z$

$$A_I = \frac{-g_m \cancel{I_b} Z}{\cancel{I_b}} = -g_m \cdot Z \qquad [\because g_m r_{b'e} = h_{fe}]$$

$$A_I = \frac{-g_m r_{b'e}}{1+r_{b'e}(j\omega(C_c+C_e))} \qquad \omega = 2\pi f$$

$$A_I = \frac{-g_m r_{b'e}}{1+j2\pi f r_{b'e}(C_c+C_e)}$$

From above expression

$A_I \propto 1/f$ at low frequencies the value of denominator is approximately equal $=1$

$A_I = -h_{fe}$, $|A_I| = h_{fe}$.

GAIN BANDWIDTH PRODUCT:-

FOR VOLTAGE:-

We know that

$$A_{vs} f_H$$

A_{vs} = Voltage source gain

f_H = Band width.

We know that

$$A_{vs} = \frac{-h_{fe} R_L}{R_s + h_{ie}}$$

$$f_H = \frac{1}{2\pi r_{b'e} C_{eq}}$$

$$(R_s + r_{bb'}) \parallel r_{b'e} \Rightarrow R_{eq} = \frac{r_{b'e}[R_s + r_{bb'}]}{r_{b'e} + R_s + r_{bb'}}$$

$$R_{eq} = \frac{r_{b'e}[R_s + r_{bb'}]}{R_s + h_{ie}} \qquad [\because h_{ie} = r_{b'e} + r_{bb'}]$$

$$A_{vs} f_H = \frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi C_{eq} R_{eq}}$$

$$\frac{-h_{fe} R_L}{R_s + h_{ie}} \times \frac{1}{2\pi C_{eq} \left[\frac{r_{b'e}[R_s + r_{bb'}]}{R_s + h_{ie}}\right]}$$

$$A_{vs} f_H = \frac{-h_{fe} R_L}{2\pi C_{eq} r_{b'e}[R_s + r_{bb'}]}$$

$$= \frac{-g_m R_L}{2\pi C_{eq}[R_s + r_{bb'}]} \qquad \left[\because \frac{h_{fe}}{r_{b'e}} = g_m\right]$$

$$|A_{vs}f_H| = \frac{g_m}{2\pi C_{eq}} \cdot \frac{R_L}{R_s + r_{bb'}}$$

$$C_{eq} = C_e + C_c[1 + g_m R_L]$$

$$= C_e + C_c g_m R_L \quad [\because g_m R_L >> 1]$$

$$|A_{vs}f_H| = \frac{R_L}{R_s + r_{bb'}} \cdot \frac{g_m}{2\pi[C_e + C_c g_m R_L]}$$

We know that

$$g_m = 2\pi f_T C_e \quad [\text{from CE short circuit}]$$

Now substituting the value of g_m in above equation.

$$|A_{vs}f_H| = \frac{R_L}{R_s + r_{bb'}} \cdot \frac{2\pi f_T C_e}{2\pi[C_e + C_c 2\pi f_T C_e R_L]}$$

$$= \frac{R_L}{R_s + r_{bb'}} \cdot \frac{\cancel{2\pi C_e} f_T}{\cancel{2\pi C_e}[1 + C_c 2\pi f_T R_L]}$$

$$\boxed{|A_{vs}f_H| = \frac{R_L}{R_s + r_{bb'}} \cdot \frac{f_T}{[1 + C_c 2\pi f_T R_L]}}$$

FOR CURRENT:-

$A_{Is} f_H$

We know that

$$A_{Is} = \frac{-h_{fe} R_s}{R_s + h_{ie}}.$$

$$f_H = \frac{1}{2\pi r_{b'e} C_{eq}}$$

$$= \frac{-h_{fe} R_s}{R_s + h_{ie}} \cdot \frac{1}{2\pi r_{b'e} C_{eq}}$$

$$(R_s + r_{bb'}) \,//\, r_{b'e} \Rightarrow R_{eq} = \frac{r_{b'e}\left[R_s + r_{bb'}\right]}{r_{b'e} + R_s + r_{bb'}}$$

$$R_{eq} = \frac{r_{b'e}\left[R_s + r_{bb'}\right]}{R_s + h_{ie}} \qquad \left[\because h_{ie} = r_{b'e} + r_{bb'}\right]$$

$$A_{Is} f_H \Rightarrow \frac{-h_{fe} R_s}{R_s + h_{ie}} \cdot \frac{1}{2\pi C_{eq} r_{eq}}$$

$$= \frac{-h_{fe} R_s}{R_s + h_{ie}} \cdot \frac{1}{2\pi C_{eq}\left[\frac{r_{b'e}\left(R_s + r_{bb'}\right)}{R_s + h_{ie}}\right]}$$

$$= \frac{-h_{fe} R_s}{\cancel{R_s + h_{ie}}} \cdot \frac{\cancel{R_s + h_{ie}}}{2\pi C_{eq}\left[r_{b'e}\left(R_s + r_{bb'}\right)\right]}$$

$$A_{Is} f_H = \frac{-g_m R_s}{2\pi C_{eq}\left[R_s + r_{bb'}\right]}$$

$$\Rightarrow |A_{Is} f_H| = \frac{g_m}{2\pi C_{eq}} \cdot \frac{R_s}{R_s + r_{bb'}}$$

$$C_{eq} = C_e + C_c\left[1 + g_m R_L\right] \qquad \left[\because g_m R_L >> 1\right]$$

$$= C_e + C_c g_m R_L$$

$$|A_{Is} f_H| = \frac{R_s}{R_s + r_{bb'}} \cdot \frac{g_m}{2\pi\left[C_e + C_c g_m R_L\right]}$$

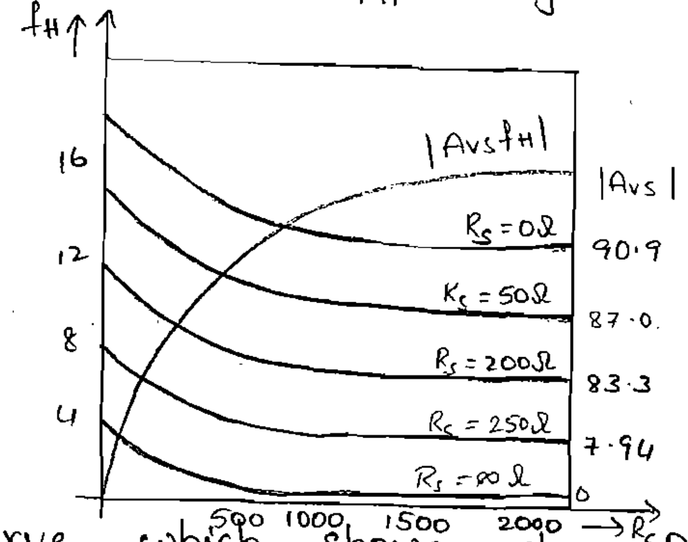
We know that $g_m = 2\pi f_T C_e$

$$|A_{Is} f_H| = \frac{R_s}{R_s + r_{bb'}} \cdot \frac{2\pi f_T C_e}{2\pi\left[C_e + C_c\, 2\pi f_T C_e R_L\right]}$$

$$= \frac{R_s}{R_s + r_{bb'}} \cdot \frac{\cancel{2\pi C_e} f_T}{\cancel{2\pi C_e}\left[1 + C_c\, 2\pi f_T R_L\right]}$$

$$\boxed{|A_{Is} f_H| = \frac{R_s}{R_s + r_{bb'}} \cdot \frac{f_T}{\left[1 + C_c\, 2\pi f_T R_L\right]}}$$

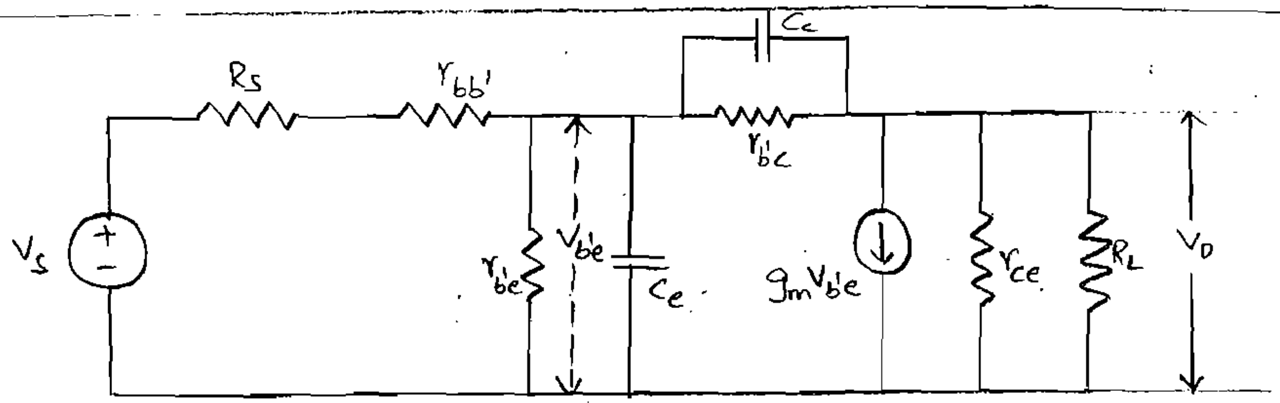
By considering the above gain bandwidth products the plot between R_L and f_H is given as



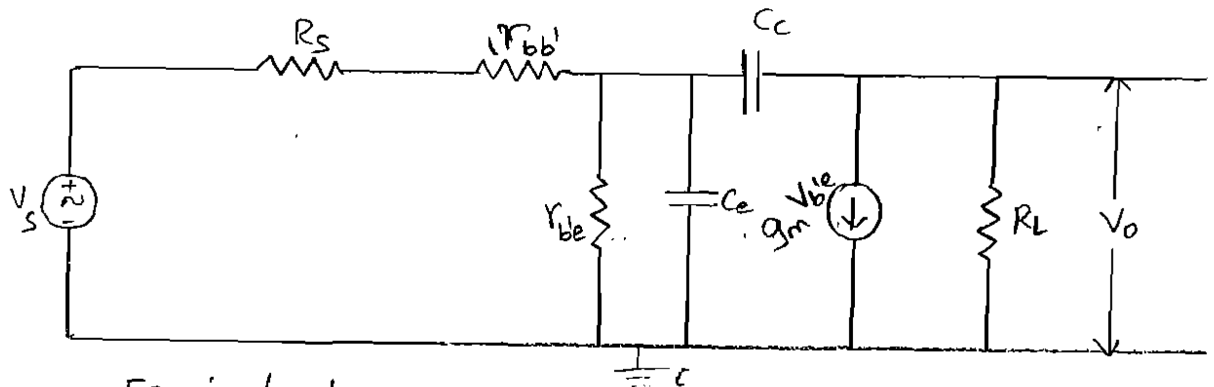
The curve which shows when $R_s = 0$ is an ideal transistor voltage gain, when $R_s = \infty$ is the ideal transistor current gain.

SINGLE STAGE TRANSISTOR CE AMPLIFIER RESPONSE:-

Let us consider a single stage CE amplifier with source V_s and finite resistance R_s. The equivalent circuit of such CE stage is shown below.



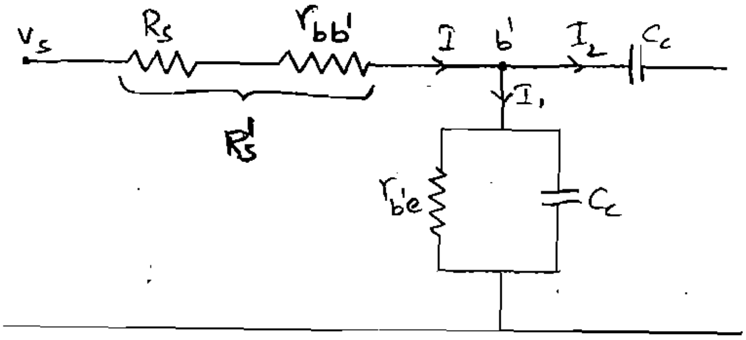
As the values of $r_{b'c}$ and r_{ce} are very large we can eliminate those two parameters from the circuit. Then circuit converts into



Equivalent Circuit for single stage CE Amplifier

NODAL ANALYSIS :-

Applying KVL at node b' by using Laplace transforms.



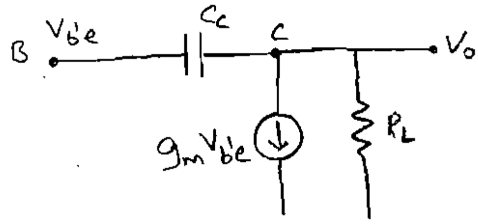
$$(V_s - V_{b'e})G'_s = V_{b'e}[g_{b'e} + sC_e] + [V_{b'e} - V_o]sC_c$$

$$V_sG'_s - V'_{be}G'_s = V_{b'e}g_{b'e} + V_{b'e}sC_e + V_{b'e}sC_c - V_osC_c.$$

$$V_sG'_s - V_osC_c = V_{b'e}\left[g_{b'e} + sC_e - sC_c + G'_s\right]$$

$$V_sG'_s = V_{b'e}\left[g_{b'e} + sC_e - sC_c + G'_s\right] - V_osC_c. \quad \text{—(1)}$$

At node 'c':-



$$[V_{b'e}-V_0]sC_c = g_m V_{b'e} + \frac{1}{R_L}V_0$$

$$V_{b'e}sC_c - V_0 sC_c = g_m V_{b'e} + \frac{1}{R_L}V_0$$

$$g_m V_{b'e} + \frac{1}{R_L}V_0 + V_0 sC_c - V_{b'e}sC_c = 0$$

$$V_{b'e}[g_m - sC_c] + V_0\left[\frac{1}{R_L}+sC_c\right] = 0 \quad -(2)$$

From (2), we get

$$V_{b'e} = \frac{-V_0\left[\frac{1}{R_L}+sC_c\right]}{g_m - sC_c}$$

Now substitute this value of $V_{b'e}$ in (1)

$$V_s G_s' = \left[g_{b'e} + G_s' + s[C_e + C_c]\right]\left[\frac{-V_0\left[\frac{1}{R_L}+sC_c\right]}{g_m - sC_c}\right] - V_0 sC_c$$

$$V_s G_s' = \left[g_{b'e} + G_s' + s[C_e + C_c]\right]\left[\frac{-V_0 + R_L sC_c}{R_L g_m - sC_c R_L}\right] - V_0 sC_c$$

$$A_{vs} = \frac{V_0}{V_s}$$

$$V_s G_s'(g_m - sC_c) = -V_0\cdot(1/R_L + sC_c)(g_{b'e} + G_s' + s(C_e + C_c)) - [V_0 sC_c(g_m - sC_c)]$$

$$\frac{V_0}{V_s} = \frac{-G_s'(g_m - sC_c)}{\left[(1/R_L + sC_c)(g_{b'e} + G_s' + s(C_e+C_c)) + sC_c(g_m - sC_c)\right]}$$

$$\frac{V_o}{V_s} = \frac{-G_s' R_L [g_m - SC_c]}{G_s' + g_{b'e} + S[C_e + C_c] + G_s' R_L S C_c + g_{b'e} R_L S C_c + S^2 C_c [C_e + C_c] R_L + g_m R_L S C_c - S^2 C_c^2 R_L}$$

$$A_{VS} = \frac{-G_s' R_L [g_m - SC_c]}{S^2 R_L C_e C_c + S[C_e + C_c + R_L C_c [G_s' + g_{b'e} + g_m]] + G_s' + g_{b'e}} \quad \text{--- (3)}$$

The above expression seems like

$$A_{VS} = \frac{V_o}{V_s} = \frac{k(S - S_0)}{(S - S_1)(S - S_2)}$$

Where $k = \frac{G_s' \cancel{C_c R_L}}{C_e \cancel{C_c R_L}} = \frac{G_s'}{C_e}$

The above transfer function contains

1 – zeros $[\text{i.e. } S_0 = g_m / C_0]$

2 – poles

The typical values of hybrid -π model are given as

$R_s = 50\Omega$ $\quad$ $C_e = 100\,PF$

$r_{bb'} = 100\Omega$ $\quad$ $C_c = 3\,PF$

$r_{b'e} = 1K\Omega$ $\quad$ $R_L = 3k\Omega$

Then now find the value of k

$$k = \frac{G_s'}{C_e}$$

where $G_s' = \frac{1}{R_s'} = \frac{1}{R_s + r_{bb'}} = 6.66 \times 10^3 \text{ mho.}$

$$k = \frac{6.66 \times 10^3}{100 \times 10^{12}}$$

$$k = 66 \times 10^{6}$$

$$S_0 = g_m / C_c$$

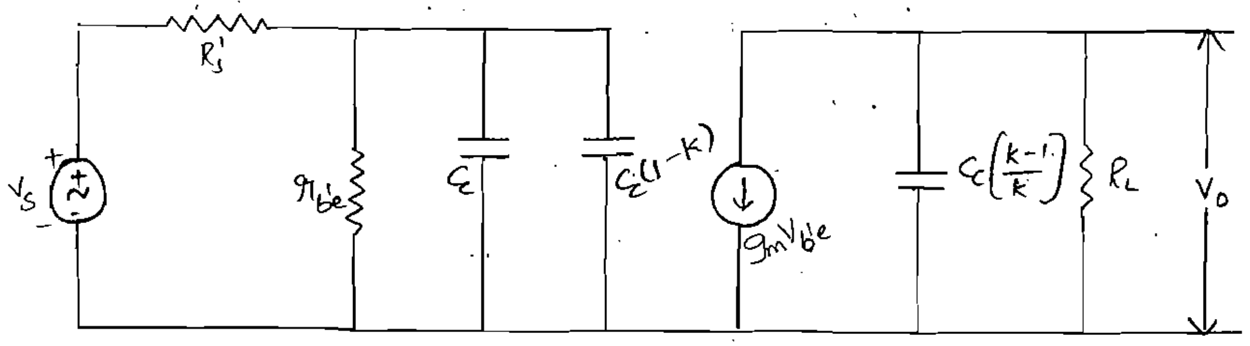
$$g_m = 50 \text{ milli amp} / \text{volt}$$

$$S_0 = \frac{50 \times 10^{-3}}{3 \times 10^{-12}} = 0.01 \times 10^{-12}$$

The poles of the transfer function can be found by finding the roots of the denominator.

SIMPLIFIED MODEL:-

Using Miller's theorem we can simplify the analysis of CE amplifier stage. The equivalent circuit is shown below.



As value of $k >> 1$, the output capacitance

$$C_c \left[\frac{k-1}{k}\right] \approx C_c$$

The time constant of output circuit is given as

$$\tau_0 = R_L C_L$$

$$= 3 \times 10^{-12} \times 3 \times 10^{-3} = 9\, n\,sec.$$

By eliminating C_c the value of k is given as

$$V_0 = -g_m V_{b'e} R_L$$

$$k = \frac{V_o}{V_{b'e}} = -g_m R_L$$

$$\boxed{k = -g_m R_L}$$

Equivalent resistance at the input $R = R_s' \parallel r_{b'e}$

$$R = \frac{R_s' \, r_{b'e}}{R_s' + r_{b'e}}$$

$$R_s' = R_s + r_{bb'}$$

$$= 50 + 100 = 150$$

$$R = \frac{150 \times 1 \times 10^3}{150 + 1 \times 10^3} = 130.43 \Omega$$

$$C = C_e + C_c[1-k] = C_e + C_c[1 + g_m R_L]$$

$$\Rightarrow 100 \times 10^{-12} + 3 \times 10^{-12}[1 + 50 \times 10^{-3} \times 3 \times 10^3]$$

$$\Rightarrow 5.53 \times 10^{-10} F.$$

Time constant of input circuit

$$T_{in} = RC \Rightarrow (130.43) \times (5.53 \times 10^{-10})$$

$$= 7.21 \times 10^{-8}$$

$$T_{in} = 72.12 \text{ nsec}$$

As the time constant of the input circuit is greater than output circuit. The bandpass single stage CE amplifier completely depends upon input time constants.

By eliminating the output time constants the eq ③ converts into

$$A_{vs} = \frac{V_o}{V_s} = \frac{-G_s' g_m R_L}{G_s' + g_{b'e} + s(C_e + C_c)} \qquad \left[\because C_e + C_c(1-k) = C\right]$$

$$= \frac{-G_s' \, g_m R_L}{G_s' + g_{b'e} + SC}$$

The above transfer function looks like

$$A_{vs} = \frac{k_2}{s - s_1}$$

It's a single pole transfer function, the pole of this transfer function is found by equating $s = s_1$

$$\because \; G_s' + g_{b'e} + s_1 C = 0$$

$$\boxed{s_1 = \frac{-(G_s' + g_{b'e})}{C}}$$

$$s_1 = \frac{-(G_s' + g_{b'e})}{C} = \frac{-1}{RC}$$

$$= \frac{1}{(130.4)(403 \times 10^{-12})} = -19 \times 10^6 \text{ rad/s}$$

The upper 3dB frequency of a single pole function is given by $1/2\pi\tau$, where τ is the time constant of the circuit; thus.

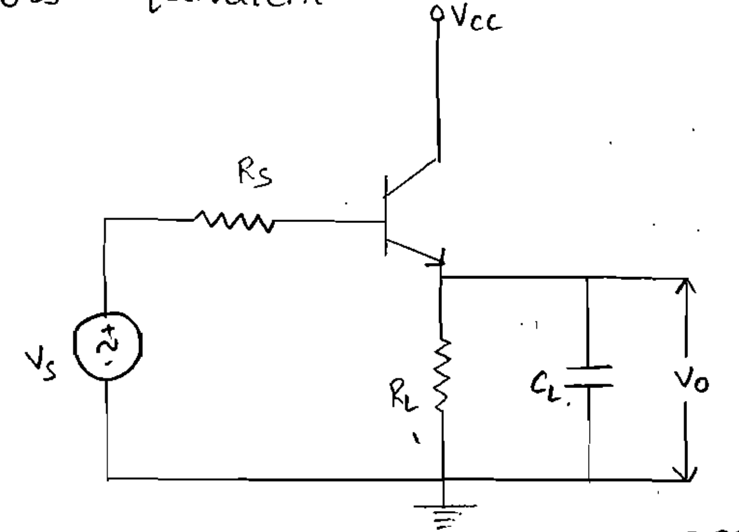
$$f_H = \frac{1}{2\pi RC} = \frac{|s_1|}{2\pi} \qquad \left(\because s_1 = \frac{-1}{RC}\right)$$

$$= \frac{19 \times 10^6}{2\pi} = 3.024 \text{ MHz}$$

$$\boxed{f_H = 3.024 \text{ MHz}}$$

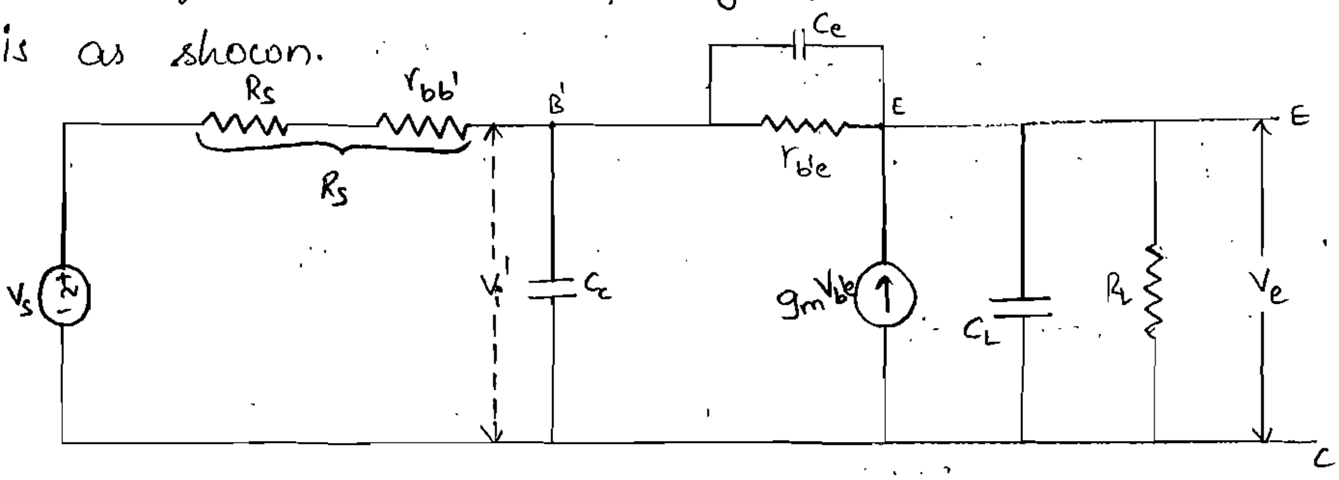
EMITTER FOLLOWER AT HIGH FREQUENCIES:-

Let us see the high frequency response of the emitter follower. A capacitor C_L is connected as load because of emitter follower is often used to drive capacitive loads. Its equivalent circuit is shown below.



EMITTER FOLLOWER AMPLIFIER

The equivalent circuit of high frequency emitter follower is as shown.



Assuming that current leaving a node as positive and current entering a node as negative

Now applying a KCL at node B'.

$$(V_s - V_i')\,G_s' = V_i'\, j\omega C_c + (V_i' - V_{in})\left[g_{b'e} + j\omega C_e\right]$$

$$V_s G_s' = V_i'\left[G_s' + g_{b'e} + j\omega(C_c + C_e)\right] - V_e\left[j\omega C_e + g_{b'e}\right]$$

Applying KCL at Node E.

$$[V_i' - V_E][g_{b'e} + j\omega C_e] = -g_m V_{b'e} + V_e\left[\frac{1}{R_L} + j\omega C_L\right]$$

$$0 = -g_m V_{b'e} + V_e\left[\frac{1}{R_L} + j\omega C_L\right] - [V_i' - V_e][g_{b'e} - j\omega C_e]$$

$$= -V_i[g_{b'e} + j\omega C_e] + V_e\left[g_{b'e} + j\omega(C_e + C_L] + \frac{1}{R_L}\right] - g_m V_{b'e}$$

We know that $V_{b'e} = V_b' - V_e$

$$= V_i' - V_e$$

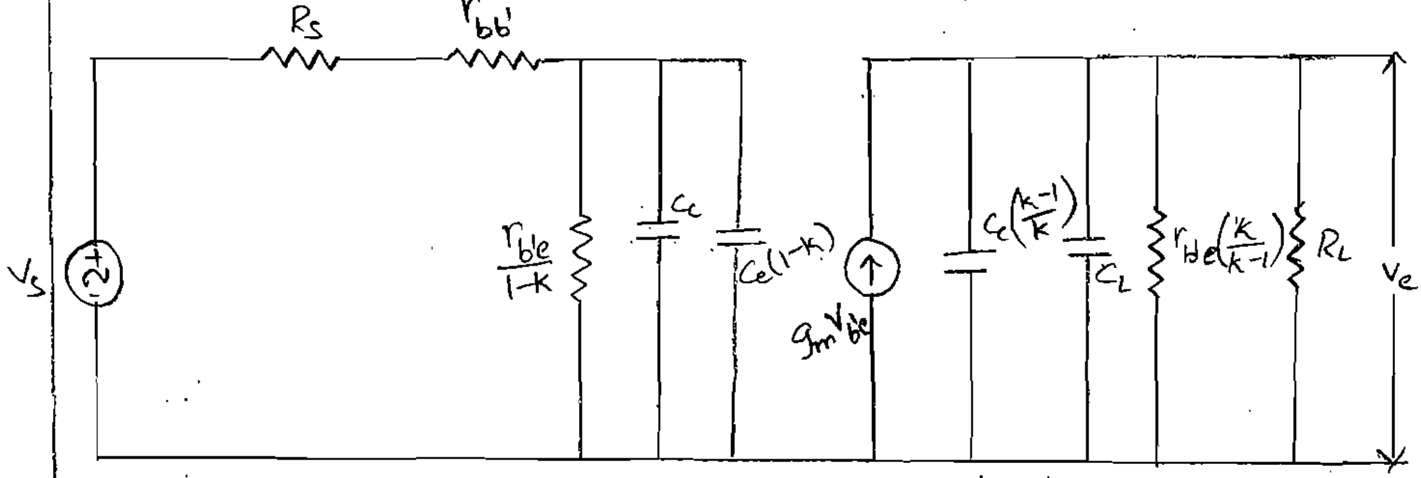
$$0 = -V_i'[g_{b'e} + g_m + j\omega C_e] + V_e\left[\underbrace{g_{b'e} + g_m}_{g} + \frac{1}{R_L} + j\omega(C_e + C_c)\right]$$

$$0 = -V_i'[g + j\omega C_e] + V_e\left[g + \frac{1}{R_L} + j\omega(C_e + C_c)\right]$$

By combining these two expressions we get a transfer function with 2 poles. To convert a transfer function into a single pole through transfer function we have to apply millers theorem for emitter follower circuit.

SINGLE POLE ANALYSIS:-

By applying Miller's theorem the C_c circuit converts into.



In a common collector the value of K (Voltage gain)

As 'K' is voltage gain of common collector amplifier the value of $K = 1$

$\therefore$ From the output circuit

$$r_{b'e}\left(\frac{K}{K-1}\right) = \infty \quad \text{[so, that resistance is open]}$$

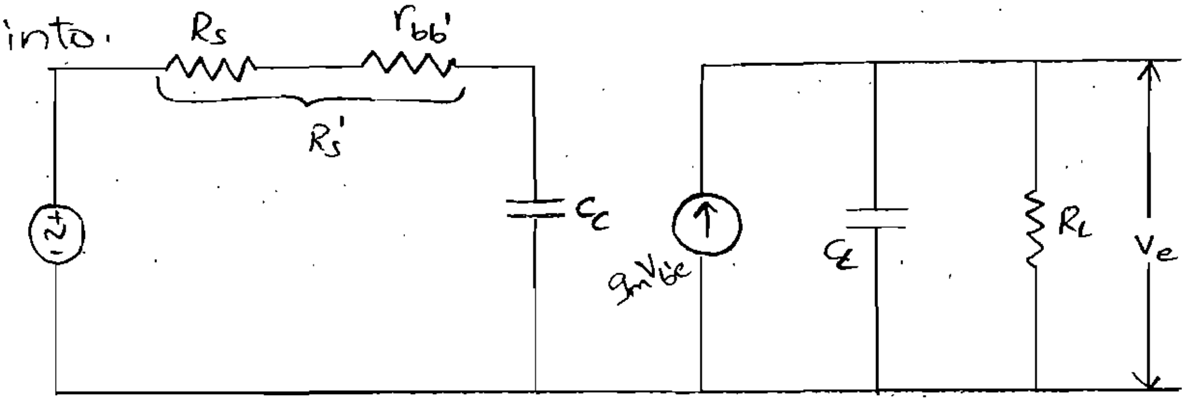
$$C_c \frac{K-1}{K} = 0 \quad \text{[so, } C_c \text{ is open]}$$

From the input circuit

$$C_e(1-K) = 0 \quad \text{[so, it is open]}$$

$$\frac{r_{b'e}}{1-K} = \infty \quad \text{[so, it is open]}$$

For these approximations the above circuit converts into.



Time constant of RC circuit

$$\tau_{in} = R_s' C_c$$

$$R_s' = R_s + r_{bb'}$$

$$= 150 \times 3 \times 10^{-2}$$

$$= 450 \times 10^{-12} = 450\,PS.$$

Time constant of output circuit is given as

$$\tau_o = R_L C_L$$

As the value of C_L is very large the time constant of output circuit is far greater than output

The bandpass of emitter follower depends on the output time constant.

$$\because V_e = I \cdot Z$$

$$Z = R_L \| C_L = \frac{R_L \cdot \frac{1}{j\omega C_L}}{R_L + \frac{1}{j\omega C_L}}$$

$$Z = \frac{R_L}{1 + j\omega R_L C_L}$$

$$Z = \frac{R_L}{R_L\left(\frac{1}{R_L} + j\omega C_L\right)} = \frac{1}{1/R_L + j\omega C_L}$$

$$V_e = \frac{I \cdot R_L}{1 + j\omega R_L C_L} \qquad [I = g_m V_{b'e}]$$

$$V_e = \frac{g_m V_{b'e} R_L}{1 + j\omega R_L C_L}$$

As $V_{b'e} = V_i' - V_e$

$$V_e = \frac{g_m R_L [V_i' - V_e]}{1 + j\omega R_L C_L}$$

$$V_e = \frac{g_m R_L V_i'}{1 + j\omega R_L C_L} - \frac{g_m R_L V_e}{1 + j\omega C_L R_L}$$

$$V_e [1 + j\omega R_L C_L + g_m R_L] = g_m R_L V_i'$$

$$\therefore K = \frac{V_e}{V_i'} = \frac{g_m R_L}{1 + j\omega R_L C_L + g_m R_L}$$

$$K = \frac{g_m R_L}{(1 + g_m R_L)\left[\frac{1 + j\omega R_L C_L}{1 + g_m R_L}\right]}$$

$$k = \left[\frac{g_m R_L}{1+g_m R_L}\right] \cdot \left[\frac{1}{1+\frac{j\omega R_L C_L}{1+g_m R_L}}\right]$$

$$= \frac{g_m R_L}{1+g_m R_L}\left[\frac{1}{1+\frac{j2\pi f R_L C_L}{1+g_m R_L}}\right]$$

$$= \frac{g_m R_L}{1+g_m R_L} \cdot \frac{1}{1+j\left(f/f_H\right)}$$

$$\left[\because f_H = \frac{1+g_m R_L}{2\pi R_L C_L}\right]$$

$$k = \frac{k_o}{1+j\left(f/f_H\right)} \qquad \left[k_o = \frac{g_m R_L}{1+g_m R_L}\right]$$

If $g_m R_L >> 1$ then k_o becomes 1.

$$K = \frac{1}{1+j\left(f/f_H\right)}$$

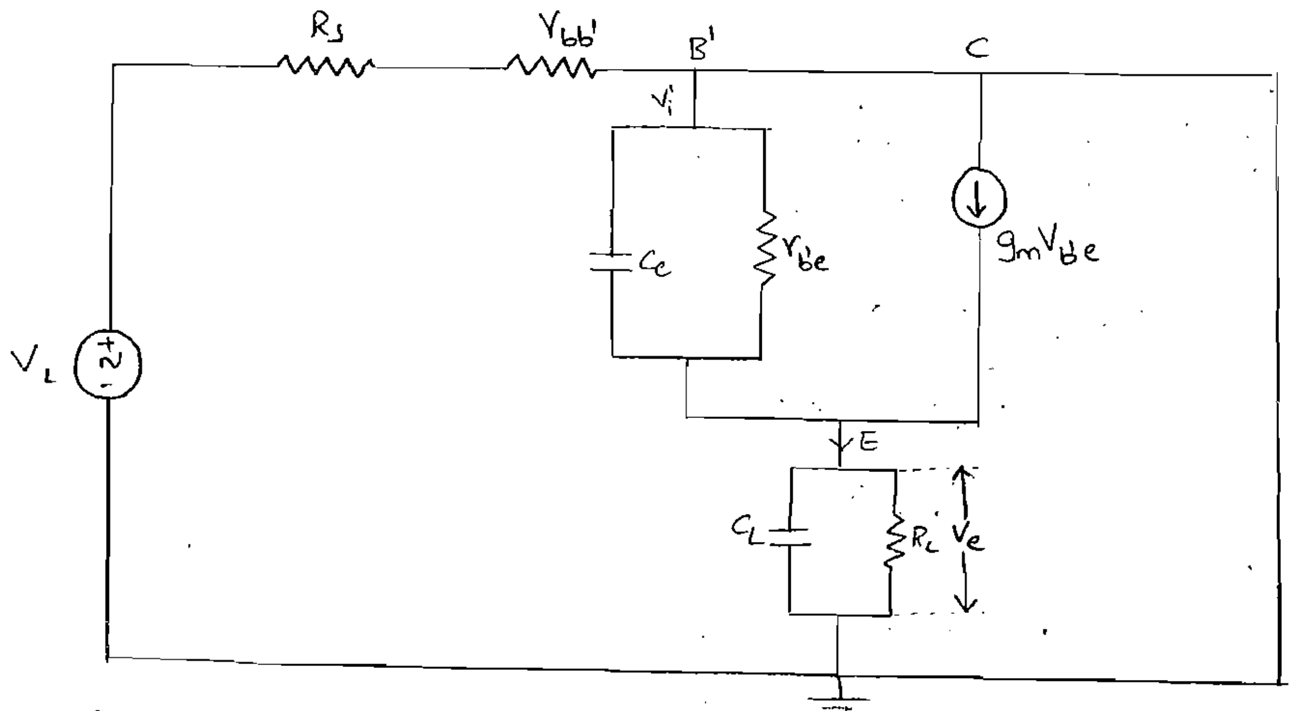
$$f_H = \frac{1+g_m R_L}{2\pi R_L C_L} = \frac{g_m R_L}{2\pi R_L C_L} = \frac{g_m}{2\pi C_L}$$

$$f_H = \frac{f_T \cdot C_e}{C_L} \qquad \left[\because f_T = \frac{g_m}{2\pi C_e}\right]$$

$$\boxed{f_H = \frac{f_T C_e}{C_L}}$$

BETTER APPROXIMATION FOR f_H:-

The circuit is as shown in below figure. The circuit connections are shown to make them suitable for the derivation of f_H.



From the above circuit we have

$$V_e = I \cdot Z$$

where $Z = R_L // C_L = \frac{R_L}{1+j\omega R_L C_L}$

$$I = (V_i' - V_e)\left[g_{b'e} + j\omega C_e\right] + g_m\left[V_i' - V_e\right]$$

$$I = (V_i' - V_e)\left[\underbrace{g_m + g_{b'e}}_{g} + j\omega C_e\right]$$

$$I = (V_i' - V_e)\left[g + j\omega C_e\right]$$

Where $g = g_m + g_{b'e}$

$$V_e = \left[V_i' - V_e\right]\left[g + j\omega C_e\right]\frac{R_L}{1+j\omega R_L C_L}$$

$$V_e = V_i'\left[\frac{R_L(g+j\omega C_e)}{1+j\omega R_L C_L}\right] - V_e\left[\frac{R_L(g+j\omega C_e)}{1+j\omega R_L C_L}\right]$$

$$V_e \left[1 + \frac{R_L\left[g+j\omega C_e\right]}{1+j\omega R_L C_L}\right] = V_i'\left[\frac{R_L(g+j\omega C_e)}{1+j\omega R_L C_L}\right]$$

$$\frac{V_e}{V_i} = \frac{R_L(g+j\omega C_e) / \cancel{1+j\omega R_L C_L}}{\frac{1+j\omega R_L C_L + R_L[g+j\omega C_e]}{\cancel{1+j\omega R_L C_L}}}$$

$$A_V = \frac{V_e}{V_i} = \frac{R_L[g+j\omega C_e]}{1+j\omega R_L C_L + R_L[g+j\omega C_e]}$$

$$A_V = \frac{gR_L\left[1+\frac{j\omega C_e}{g}\right]}{1+gR_L+j\omega R_L[C_L+C_e]}$$

$$= \frac{gR_L}{1+gR_L} \cdot \frac{1+\frac{j\omega C_e}{g}}{1+\frac{j\omega R_L(C_L+C_e)}{1+gR_L}}$$

Here $gR_L >> 1$ so, it becomes

$$A_V = \frac{\cancel{g R_L}}{\cancel{g R_L}} \cdot \frac{1+\frac{j\omega C_e}{g}}{1+\frac{j\omega R_L(C_L+C_e)}{1+gR_L}}$$

$$A_V = \frac{1+\frac{j\omega C_e}{g}}{\frac{1+j\omega(C_L+C_e)R_L}{1+gR_L}}$$

$$= \frac{R_L[g+j\omega C_e]}{gR_L+j\omega R_L[C_L+C_e]}$$

$$= \frac{g+j\omega C_e}{g+j\omega[C_L+C_e]}$$

By neglecting $j\omega C_e$ we get

$$A_V = \frac{g}{g+j\omega(C_L+C_e)}$$

$$A_V = \frac{1}{1 + \frac{j 2\pi f (C_L + C_e)}{g}}$$

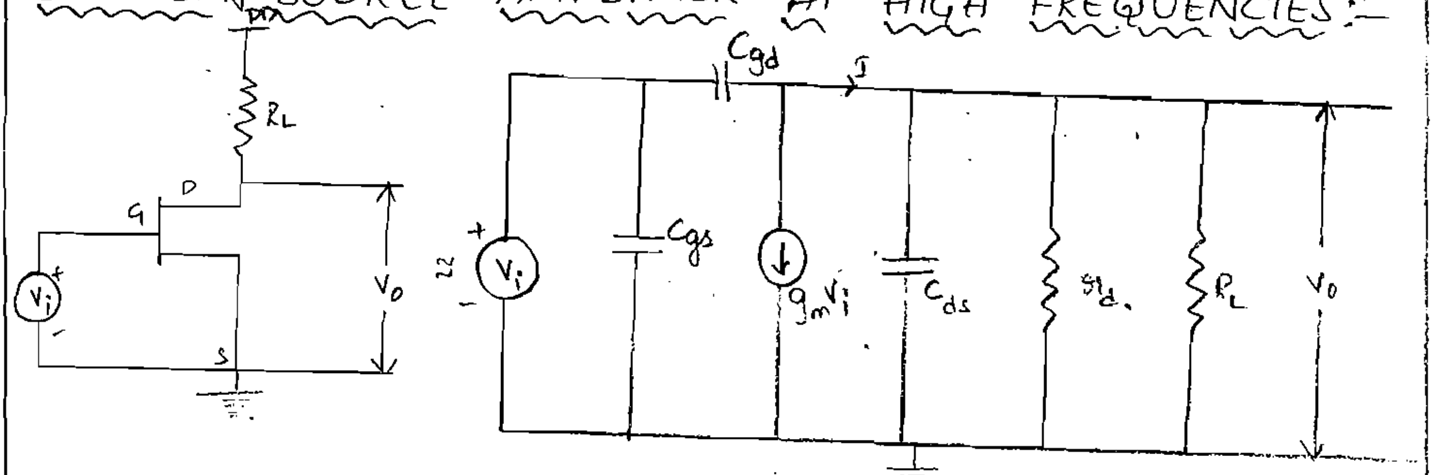
$$A_V = \frac{1}{1 + j\left(f/f_H\right)}$$

Where $f_H = \frac{g_m + g_{b'e}}{2\pi (C_L + C_e)}$

$$g = g_m + g_{b'e}$$

$$\Rightarrow f_H = \frac{g_m + g_{b'e}}{2\pi (C_L + C_e)}$$

COMMON SOURCE AMPLIFIER AT HIGH FREQUENCIES:-



The above circuit shows the common source amplifier circuit and the small signal equivalent circuit at high frequencies. The output voltage 'V_o' is given by

$$V_o = IZ$$

Where I = Short circuit current

Z = Impedance between the current terminals.

To find the 'Z' we have to make input voltage zero (i.e short). Then the current source $g_m V_i = 0$. Then the impedance Z is a parallel combination of R_L, r_d, C_{gs}, C_{gd}.

$$V_0 = IZ$$

$$Y = \frac{1}{Z} = Y_L + g_d + Y_{ds} + Y_{gd} \quad \left[\because g_d = \frac{1}{r_d} = Y_d\right]$$

Now from equivalent circuit

where $Y_L = \frac{1}{R_L}$

$g_d = 1/r_d$

$Y_{ds} = j\omega C_{ds}$

$Y_{gd} = j\omega C_{gd}$

$$I_2 = I_1 + I \Rightarrow I = I_2 - I_1$$

$$I = V_i Y_{gd} - g_m V_i$$

$$\boxed{I = V_i\left[Y_{gd} - g_m\right]}$$

VOLTAGE GAIN:-

$$A_V = \frac{V_o}{V_i} = \frac{I \cdot Z}{V_i} = \frac{I}{V_i Y} \qquad \left[\because Z = 1/Y\right]$$

$$A_V = \frac{V_i\left[Y_{gd} - g_m\right]}{V_i \cdot Y}$$

$$A_V = \frac{Y_{gd} - g_m}{Y_L + g_d + Y_{ds} + Y_{gd}}.$$

At low frequencies $Y_{ds} = Y_{gd} = 0$

$$A_V = \frac{-g_m}{Y_L + g_d}$$

$$A_V = \frac{-g_m}{\frac{1}{R_L} + \frac{1}{r_d}}$$

$$A_V = \frac{-g_m R_L r_d}{r_d + R_L} \qquad \left[\because Z' = \frac{r_d R_L}{r_d + R_L}\right]$$

$$\boxed{A_V = -g_m Z'}$$

INPUT ADMITTANCE:-

At high frequencies the gate and drain are connected to the capacitance C_{gd}. This circuit can be simplified by using Millers theorem.

∵ The impedance C_{gd} is replaced by an impedance at the input and the impedance at the output. The admittance of miller's capacitance at the input is $Y_{gd}[1-K]$.

The admittance of miller capacitance is at the output is $Y_{gd}\left[\frac{K-1}{K}\right]$

$$Y_i = Y_{gs} + Y_{gd}[1-K]$$

where K = Voltage gain.

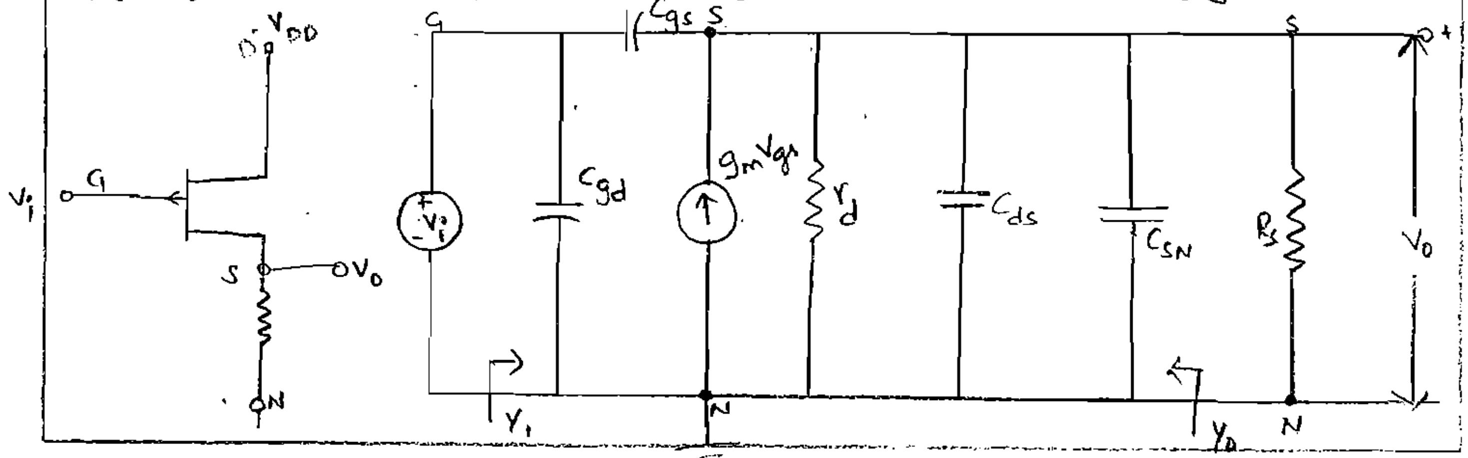
$$Y_i = Y_{gs} + Y_{gd}[1-A_v].$$

Output admittance:- (Y_o):-

From the circuit the output ~~admittance~~ impedance is obtained by "looking into the drain" with the input voltage becomes zero. If $V_i = 0$, then r_d, C_{ds}, C_{gd} in parallel. Hence the output admittance Y_o is given as

$$Y_o = g_d + Y_{ds} + Y_{gd}.$$

COMMON DRAIN AMPLIFIER AT HIGH FREQUENCIES:-



VOLTAGE GAIN:-

The output voltage V_o can be found from the product of the short circuit and the impedance between terminals S and N. Thus voltage gain is given by.

$$\frac{V_o}{V_i} = \frac{g_m + j\omega C_{gs}}{R_s + (g_m + g_d + j\omega C_T)}$$

Where

$$C_T = C_{gs} + C_{ds} + C_{sn}$$

$\because C_{sn}$ = Capacitance from source to ground.

$$A_V = \frac{(g_m + j\omega C_{gs}) R_s}{1 + (g_m + g_d + j\omega C_T) R_s}$$

At low frequencies the gain reduces to.

$$A_V = \frac{g_m R_s}{1 + (g_m + g_d) R_s}$$

Input Admittance:-

The input admittance Y_i can be obtained by applying Miller's theorem to C_{gs}. It is given by

$$Y_i = j\omega C_{gd} + j\omega C_{gs}(1 - A_V) = j\omega C_{gd}. \qquad [\because A_V = 1]$$

$$\boxed{Y_i = j\omega C_{gd}}$$

Output Admittance:-

The output admittance Y_o, with R_s considered external to the amplifier, is given by

$$Y_o = g_m + g_d + j\omega C_T$$

At lower frequencies, the output resistance R_o is

$$R_o = \frac{1}{g_m + g_d} = \frac{1}{g_m} \qquad \text{since } g_m >> g_d$$