Introduction to noise :-

Noise is defined as any undesirable electrical energy that falls with in the passband of message signal. This gives rise to an audible noise in a system. The presence of noise degrades the performance of communication systems. In this chapter we analyze the noise in continuous wave modulation system. For such analysis we first define the receiver model. Then we analyze the noise in AM receivers namely DSBSC, SSB. Finally, we discuss the noise in FM receivers.

Receiver model :-

The Fig. shows the receiver model in its most basic form. Modulated signal is $s(t)$ and noise is $w(t)$. Signal $w(t)$ is known as front end receiver noise. The receiver input signal is the sum of $s(t)$ and $w(t)$. The output of band-pass filter is $x(t)$. The band-width of a bandpass filter is kept just wide enough to pass the modulated signal $s(t)$ without distortion.

The demodulation process represented by the block demodulator depends on the modulation used.

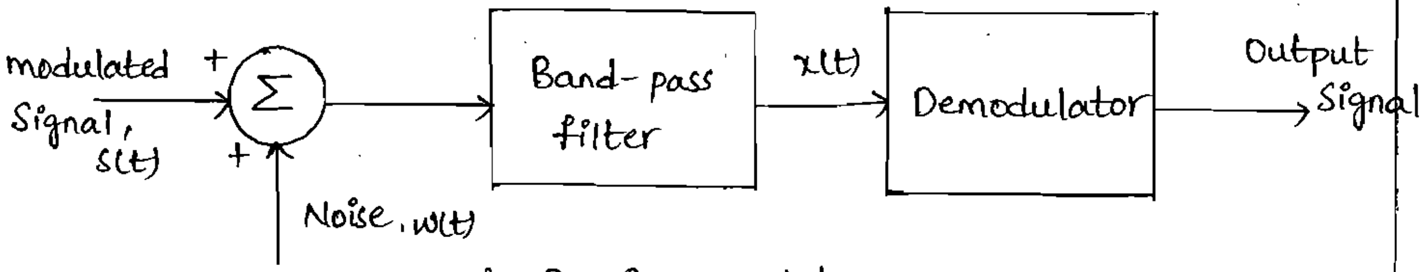


Fig. Receiver model.

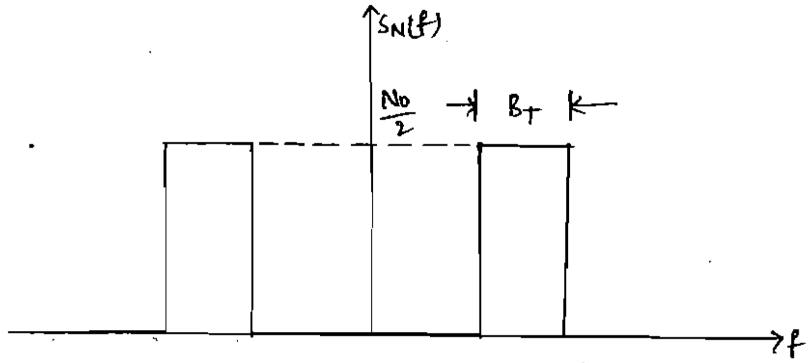


Fig. Idealized characteristic of bandpass filtered noise.

For receiver model, we may denote and define the following things.

* we denote $N_0/2$ as the power spectral density of noise $w(t)$ for both +ve and -ve frequencies.
* 'N_0' is the average noise power per unit bandwidth.
* Bandwidth of bandpass filter is equal to transmission bandwidth of the modulated signal and is denoted as 'B_T'.
* Midband frequency is equal to the carrier frequency and it is denoted as f_c.

* Typically, the carrier frequency $f_c >> B_T$ and therefore we may consider the filtered noise $n(t)$ as narrow band noise and it is defined in canonical form as

$$n(t) = n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t.$$

* The filtered signal available for demodulation is defined by $x(t) = s(t) + n(t)$

* The average noise power is given as,

avg. noise power = avg. noise power per unit bandwidth X Bandwidth

$= N_0 B_T$

* Input signal to noise ratio is given by

$$(SNR)_I = \frac{\text{Average power of the modulated signal, } s(t)}{\text{Average power of filtered noise, } n(t).}$$

* Output signal to noise ratio is given by

$$(SNR)_0 = \frac{\text{Average power of the demodulated msg signal}}{\text{Average power of the noise.}}$$

* Channel Signal to noise ratio is given by

$$(SNR)_c = \frac{\text{Average power of the modulated signal}}{\text{Avg. power of noise in message bandwidth}}$$

* Figure of merit $= \dfrac{(SNR)_0}{(SNR)_c}$

* Higher the value of figure of merit, better the performance of the receiver.
* The value of figure of merit also depends upon the type of modulation used.

Noise in DSBSC Receivers:-

The below figure shows the model of a DSBSC receiver using a coherent detector. As shown in the figure, the filtered signal is applied to coherent detector $x(t)$. It is multiplied with a locally generated sinusoidal wave $\cos 2\pi f_c t$ using product modulator. The product is then filtered using low pass filter.

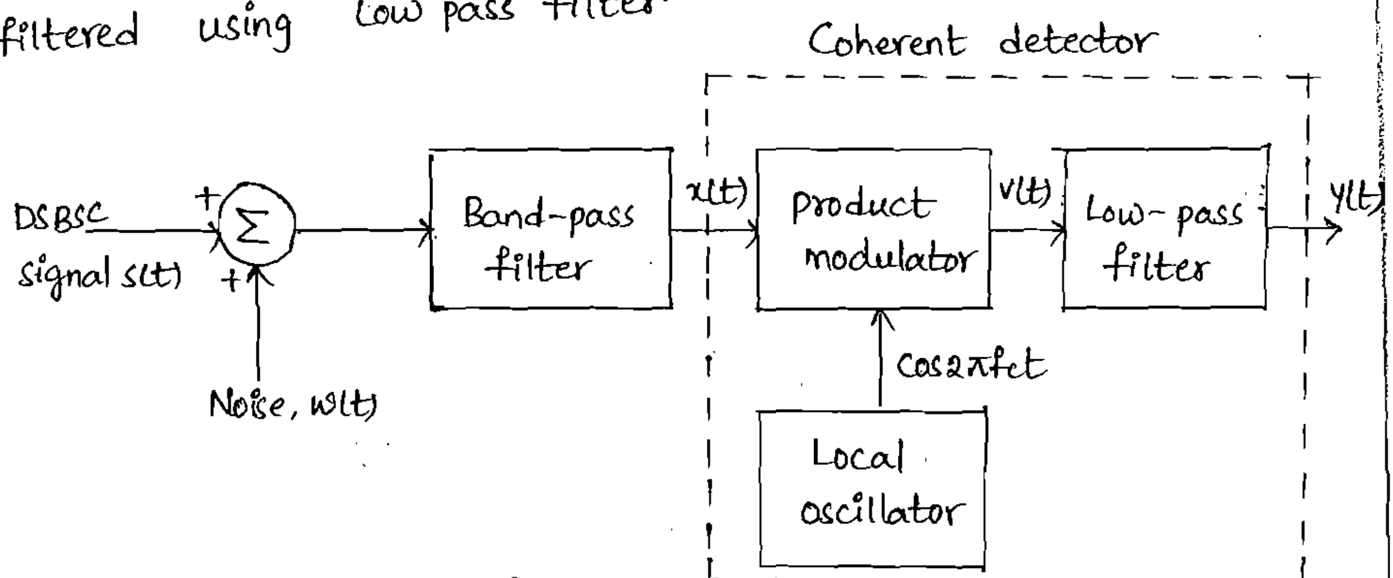


Fig. Model of DSBSC receiver

The time-domain expression of DSB-SC wave is given as,

$$s(t) = m(t)\, c(t)$$

$$s(t) = m(t)\, A_c \cos 2\pi f_c t$$

Channel Signal-to-noise ratio :-

It is given as,

$$(SNR)_c = \frac{\text{Avg. power of modulated signal } s(t)}{\text{Avg. power of noise in msg bandwidth}}$$

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of modulated signal $= \overline{s^2(t)}$

$$= \overline{[m(t) A_c \cos 2\pi f_c t]^2}$$

$$= \overline{m^2(t)}\ \overline{[A_c \cos 2\pi f_c t]^2}$$

$$= A_c^2 \cdot P \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2 P}{2}$$

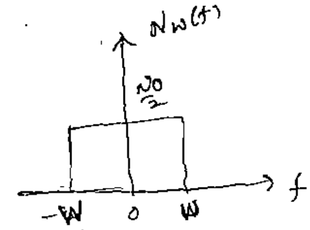
where, 'P' represents average power of $m(t)$.

Average power of noise in message bandwidth is given

as $\overline{n_w^2(t)}$ = avg. noise power per unit bandwidth x bandwidth

$$= \frac{N_0}{2} \cdot 2W$$

$$= N_0 W$$



$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}} = \frac{A_c^2 P}{2 N_0 W}$$

Output Signal-to-noise ratio :-

It is given as,

$$(SNR)_0 = \frac{\text{Average power of demodulated msg signal}}{\text{Average power of noise}}$$

$$(SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of band pass filter is

$$x(t) = s(t) + n(t)$$

$$x(t) = m(t) A_c \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

This $x(t)$ is passed through product modulator. Another input to product modulator is $\cos 2\pi f_c t$. The output of product modulator is

$$v(t) = x(t) \cos 2\pi f_c t$$

$$v(t) = m(t) A_c \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \cos 2\pi f_c t \sin 2\pi f_c t$$

$$v(t) = m(t) A_c \left(\frac{1+\cos 4\pi f_c t}{2}\right) + n_I(t) \left(\frac{1+\cos 4\pi f_c t}{2}\right) - \frac{n_Q(t)}{2} \sin 4\pi f_c t$$

$$v(t) = \frac{m(t) A_c}{2} + \frac{n_I(t)}{2} + \left(\frac{m(t) A_c}{2} + \frac{n_I(t)}{2}\right) \cos 4\pi f_c t - \frac{n_Q(t)}{2} \sin 4\pi f_c t.$$

This $v(t)$ is passed through Low-pass filter.

The output of LPF is

$$y(t) = \frac{m(t) A_c}{2} + \frac{n_I(t)}{2}$$

$$= m_d(t) + n_d(t)$$

where,

$m_d(t) = \frac{m(t) A_c}{2}$ is the desired signal component

$n_d(t) = \frac{n_I(t)}{2}$ is the noise component.

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[\frac{m(t) A_c}{2}\right]^2}$$

$$= \frac{A_c^2}{4} \overline{m^2(t)} = \frac{A_c^2 P}{4}$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

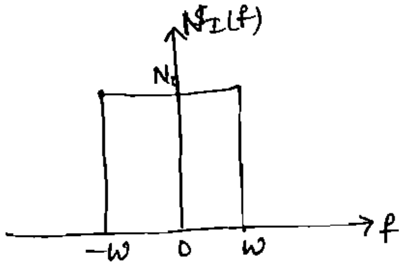
$$= \overline{\left[\frac{n_I(t)}{2}\right]^2}$$

$$= \frac{1}{4} \overline{n_I^2(t)}$$

$= \frac{1}{4} \times$ Area under PSD curve

$$= \frac{1}{4} \times N_0 \times 2W$$

$$= \frac{N_0 W}{2}$$



$$(SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}} = \frac{A_c^2 P/4}{N_0 W/2} = \frac{A_c^2 P}{2 N_0 W}$$

Figure of merit (γ) is given as,

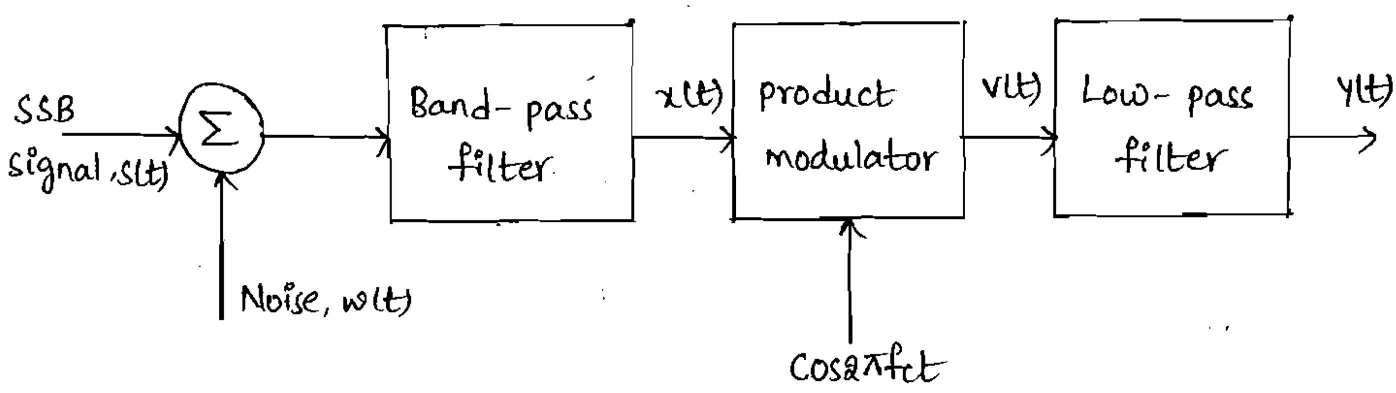
$$\gamma = \frac{(SNR)_o}{(SNR)_c}$$

$$\gamma = \frac{\frac{A_c^2 P}{2N_oW}}{\frac{A_c^2 P}{2N_oW}} = 1$$

Thus, the figure of merit of DSB-SC system is 1.

Noise in an SSB-SC system :-

The block-diagram of the SSB-SC system is just as like DSB-SC system except for the fact that bandwidth of BPF of SSB-SC receiver is exactly half of that required for DSB-SC.



Channel Signal-to-noise ratio,

$$(SNR)_c = \frac{\text{Avg. power of modulated signal}}{\text{Avg. power of noise in msg bandwidth.}}$$

$$= \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

The expression of s(t) for an SSB is given as

$$s(t) = \frac{A_c}{2} m(t)\cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t)\sin 2\pi f_c t$$

Average power of s(t) = $\overline{s^2(t)}$

$$= \overline{\left[\frac{A_c m(t)}{2}\cos 2\pi f_c t\right]^2} + \overline{\left[\frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t\right]^2}$$

$$\overline{s^2(t)} = \frac{A_c^2}{4}\overline{m^2(t)}\;\overline{\cos^2 2\pi f_c t} + \frac{A_c^2}{4}\overline{\hat{m}^2(t)}\;\overline{\sin^2 2\pi f_c t}$$

$$= \frac{A_c^2 P}{4}\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{A_c^2 P}{4}\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2 P}{4}$$

Average power of $n_\omega(t) = \overline{n_\omega^2(t)}$

= Power Spectral density x bandwidth

$= N_0 W$

$$\therefore (SNR)_c = \frac{A_c^2 P}{4 N_0 W}$$

Output Signal-to-noise ratio :-

$$(SNR)_o = \frac{\text{Avg. power of demodulated msg signal}}{\text{Average power of noise.}}$$

$$= \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of band pass filter is

$$x(t) = s(t) + n(t).$$

Here, the type of modulation used is SSB, So the center frequency of band pass filter changes to $(f_c - W/2)$

Therefore, noise signal is expressed as

$$n(t) = n_I(t)\cos 2\pi(f_c - W/2)t - n_Q(t)\sin 2\pi(f_c - W/2)t$$

The output of product modulator is

$$v(t) = x(t)\cos 2\pi f_c t$$

$$= [s(t) + n(t)]\cos 2\pi f_c t$$

$$= s(t)\cos 2\pi f_c t + n(t)\cos 2\pi f_c t$$

$$v(t) = \left[\frac{A_c}{2} m(t)\cos 2\pi f_c t + \frac{A_c}{2}\hat{m}(t)\sin 2\pi f_c t\right]\cos 2\pi f_c t +$$

$$\left[n_I(t)\cos 2\pi(f_c - W/2)t - n_Q(t)\sin 2\pi(f_c - W/2)t\right]\cos 2\pi f_c t.$$

$$v(t) = \frac{A_c m(t)}{2}\cos^2 2\pi f_c t + \frac{A_c \hat{m}(t)}{2}\sin 2\pi f_c t\cos 2\pi f_c t +$$

$$n_I(t)\cos 2\pi(f_c - W/2)t\cos 2\pi f_c t - n_Q(t)\sin 2\pi(f_c - W/2)t\cos 2\pi f_c t$$

$$v(t) = \frac{A_c m(t)}{2}\left\{\frac{1 + \cos 4\pi f_c t}{2}\right\} + \frac{A_c \hat{m}(t)}{4}\sin 4\pi f_c t +$$

$$\frac{n_I(t)}{2}\left\{\cos 2\pi(2f_c - W/2)t + \cos 2\pi(W/2)t\right\} -$$

$$\frac{n_Q(t)}{2}\left\{\sin 2\pi(2f_c - W/2)t - \sin 2\pi(W/2)t\right\}$$

$$v(t) = \frac{A_c m(t)}{4} + \frac{n_I(t)}{2}\cos\pi w t + \frac{n_Q(t)}{2}\sin\pi w t + \frac{A_c m(t)}{4}\cos 4\pi f_c t + \frac{A_c \hat{m}(t)}{4}\sin 4\pi f_c t + \frac{n_I(t)}{2}\cos 2\pi(2f_c - w/2)t - \frac{n_Q(t)}{2}\sin 2\pi(2f_c - w/2)$$

This $v(t)$ is passed through LPF, In LPF high frequencies are attenuated and only low frequencies are allowed. So, the output of LPF is

$$y(t) = \frac{A_c m(t)}{4} + \frac{n_I(t)}{2}\cos\pi w t + \frac{n_Q(t)}{2}\sin\pi w t$$

$$= m_d(t) + n_d(t)$$

where,

$$m_d(t) = \frac{A_c m(t)}{4}$$

$$n_d(t) = \frac{n_I(t)}{2}\cos\pi w t + \frac{n_Q(t)}{2}\sin\pi w t$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \overline{\left[\frac{A_c m(t)}{4}\right]^2}$$

$$= \frac{A_c^2}{16}\overline{m^2(t)} = \frac{A_c^2 P}{16}$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

$$= \overline{\left[\frac{n_I(t)}{2}\cos\pi w t\right]^2} + \overline{\left[\frac{n_Q(t)}{2}\sin\pi w t\right]^2}$$

$$\overline{n_d^2(t)} = \frac{\overline{n_I^2(t)}}{4}\ \overline{\cos^2 \pi w t} + \frac{\overline{n_Q^2(t)}}{4}\ \overline{\sin^2 \pi w t}$$

$$\overline{n_d^2(t)} = \frac{N_0 W}{4}\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{N_0 W}{4}\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{N_0 W}{4}$$

$$\therefore (SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}} = \frac{A_c^2 P/16}{N_0 W/4} = \frac{A_c^2 P}{4 N_0 W}$$

Figure of merit (r) is given as

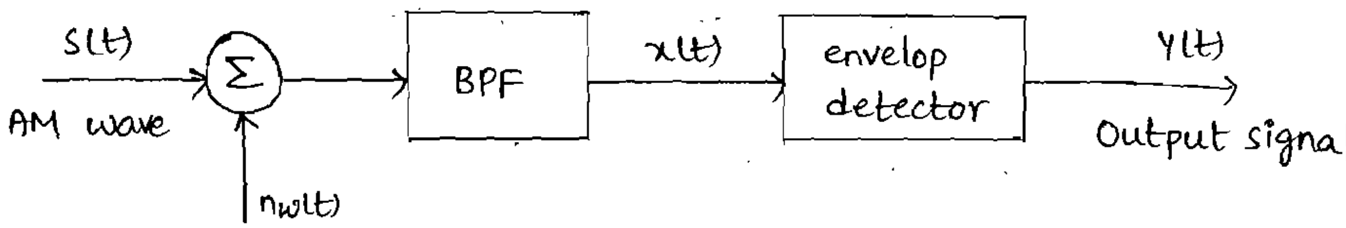
$$\text{Figure of merit} = \frac{(SNR)_0}{(SNR)_c}$$

$$= \frac{\frac{A_c^2 P}{4 N_0 W}}{\frac{A_c^2 P}{4 N_0 W}} = 1$$

Thus, the figure of merit of SSB-SC system is 1.

Noise in AM system :-

The block diagram of AM receiver is as shown.



The time-domain expression for AM wave is given by

$$s(t) = A_c[1+k_a m(t)] \cos 2\pi f_c t$$

Channel Signal-to-noise ratio :-

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of $s(t) = \overline{s^2(t)}$

$$= \overline{[A_c \cos 2\pi f_c t]}^2 + \overline{[A_c k_a m(t) \cos 2\pi f_c t]}^2$$

$$= A_c^2 \left(\frac{1}{\sqrt{2}}\right)^2 + A_c^2 k_a^2 P \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2}{2}\left[1 + k_a^2 P\right]$$

Average power of $n_w(t) = \overline{n_w^2(t)}$

$$= N_0 W$$

$$(SNR)_c = \frac{A_c^2(1+k_a^2 P)}{2N_0 W}$$

Output Signal-to-Noise ratio -

$$(SNR)_0 = \frac{\overline{m_d^2(t)}}{\overline{n_d^2(t)}}$$

The output of Band-pass filter is $x(t)$

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c[1+k_a m(t)]\cos 2\pi f_c t + n_I(t)\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t.$$

$$x(t) = [A_c + A_c k_a m(t) + n_I(t)]\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

$$e(t) = \sqrt{(\text{In phase comp.})^2 + (\text{Quadrature comp.})^2}$$

$$e(t) = \sqrt{[A_c + A_c k_a m(t) + n_I(t)]^2 + (n_Q(t))^2}$$

In this case,

$$A_c(1+k_a m(t)) >> n(t)$$

Thus,
$$A_c(1+k_a m(t)) >> n_I(t) \text{ (or) } n_Q(t)$$

$$\therefore e(t) = A_c(1+k_a m(t)) + n_I(t)$$

$$= A_c + A_c k_a m(t) + n_I(t)$$

The output of envelop detector is $y(t)$

$$y(t) = A_c k_a m(t) + n_I(t)$$

$$= m_d(t) + n_d(t)$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= \left(\overline{A_c k_a m(t)}\right)^2$$

$$= A_c^2 k_a^2 \cdot P$$

Average power of noise is given as $\overline{n_d^2(t)}$

$$= \overline{n_I^2(t)}$$

$$= N_0(2W)$$

$$= 2N_0W$$

$$(SNR)_0 = \frac{A_c^2 k_a^2 P}{2N_0W}$$

Figure of merit (r) is given as

$$r = \frac{(SNR)_0}{(SNR)_c}$$

$$r = \frac{\frac{\cancel{A_c^2} k_a^2 P}{2\cancel{N_0W}}}{\frac{\cancel{A_c^2}(1+k_a^2P)}{2\cancel{N_0W}}} = \frac{k_a^2 P}{1+k_a^2 P}$$

we know that 'p' is the average power of the message signal and it is given as

$$P = \frac{1}{2}A_m^2$$

$$r = \frac{\frac{k_a^2 A_m^2}{2}}{1+\frac{k_a^2 A_m^2}{2}} = \frac{k_a^2 A_m^2}{2+k_a^2 A_m^2}$$

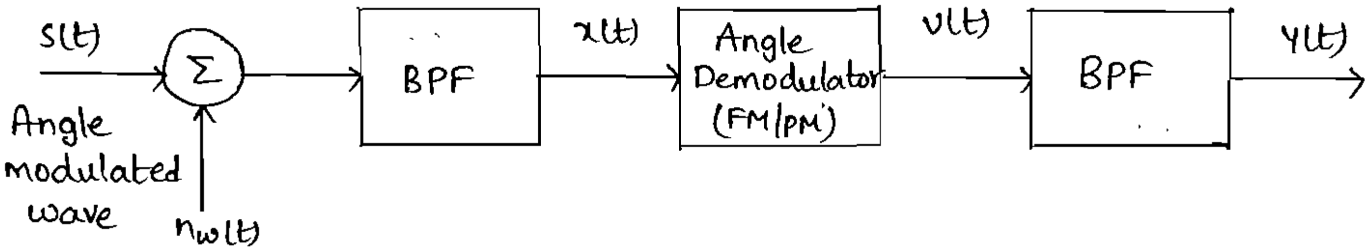
$$r = \frac{\mu^2}{2+\mu^2} \quad [\because \mu = k_a A_m]$$

For 100% modulation i.e $\mu = 1$ we get

$$r = \frac{1}{2+1} = \frac{1}{3}.$$

Noise in Angle modulated system :-

The block diagram of an angle modulated system is as shown,



The time-domain expression for an angle modulated carrier is given by,

$$s(t) = A_c \cos[2\pi f_c t + \phi(t)]$$

where $\phi(t)$ represents the instantaneous phase angle and is given as,

$$\phi(t) = k_p m(t) \quad [\text{For phase modulation}]$$

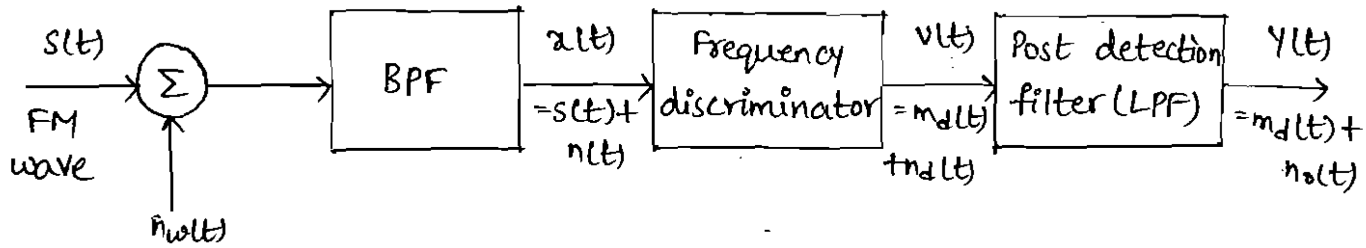
$$\phi(t) = 2\pi k_f \int_0^t m(t)\, dt \quad [\text{For frequency modulation}]$$

Here k_p and k_f represents the sensitivities of phase & frequency respectively. The transmission bandwidth B_T in angle modulated system determined by Carson's rule is

$$B_T = 2(\Delta f + W)$$

where W represents the Bandwidth of msg signal and Δf is the peak frequency deviation.

Noise in FM Receivers :-



The time domain expression for frequency modulated carrier is given by

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

The noise can be expressed in terms of inphase and quadrature components as,

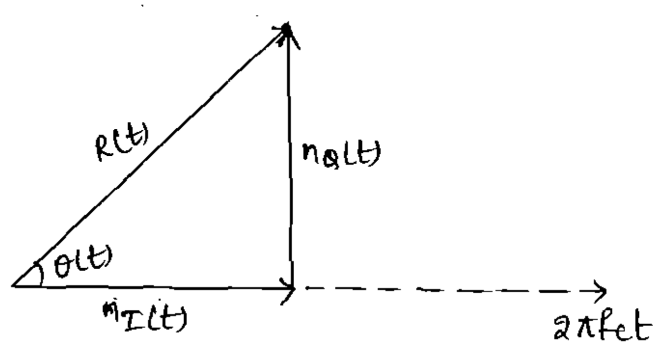
$$n(t) = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t$$

$$= R(t) \cos(2\pi f_c t + \theta(t))$$

where, $R(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

$$\theta(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

The phasor diagram of n(t) drawn by taking reference as phase of unmodulated carrier is as shown.



Channel Signal-to-noise ratio.

$$(SNR)_c = \frac{\overline{s^2(t)}}{\overline{n_w^2(t)}}$$

Average power of $s(t) = \overline{s^2(t)}$

$$= \overline{\left[A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt\right]\right]}^2$$

$$= A_c^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2$$

$$= \frac{A_c^2}{2}$$

Average power of noise $n_w(t) = \overline{n_w^2(t)}$

$$= N_0 W$$

Thus, $(SNR)_c = \dfrac{A_c^2}{2N_0W}$

Output Signal-to-noise ratio –

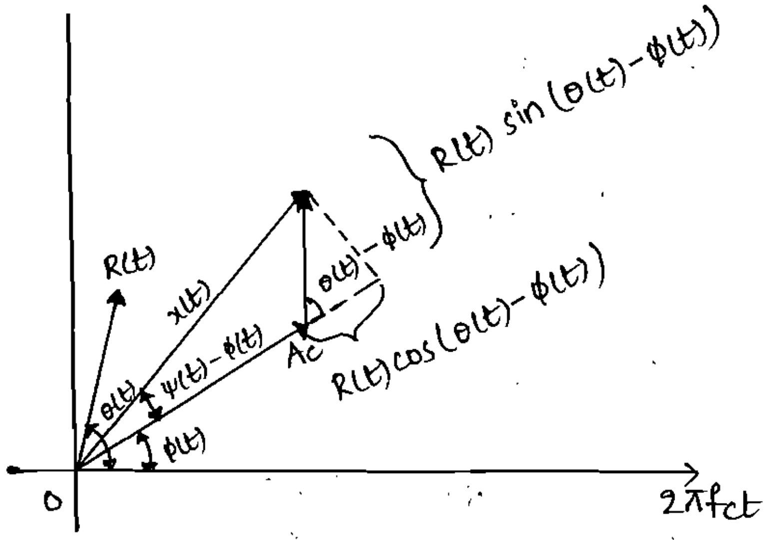
$$(SNR)_o = \frac{\text{Average power of demodulated signal}}{\text{Average power of the noise}}$$

The output of band-pass filter is

$$x(t) = s(t) + n(t)$$

$$= A_c \cos(2\pi f_c t + \phi(t)) + R(t) \cos(2\pi f_c t + \theta(t))$$

where, $\phi(t) = 2\pi k_f \int_0^t m(t)\, dt$



The relative phase $\psi(t)$ of the resulting $x(t)$ can be obtained from the figure as,

$$\psi(t) - \phi(t) = \tan^{-1}\left\{ \frac{R(t) \sin(\theta(t) - \phi(t))}{A_c + R(t)\cos(\theta(t) - \phi(t))} \right\}$$

While drawing phasor diagram, it is assumed that $\theta(t) > \phi(t)$ and $A_c > R(t)$.

We can write,

$$\tan(\psi(t) - \phi(t)) \approx \frac{R(t) \sin\theta(t)}{A_c}$$

Referring the phasor diagram of narrow band noise, we get

$$R(t) \sin\theta(t) = n_Q(t)$$

Hence,

$$\tan(\psi(t) - \phi(t)) = \frac{n_Q(t)}{A_c}$$

Since $A_c >> n_Q(t)$ we can write

$$\psi(t) - \phi(t) \approx \frac{n_Q(t)}{A_c}$$

$$\psi(t) = \phi(t) + \frac{n_Q(t)}{A_c}$$

The output of frequency discriminator is

$$v(t) = \frac{1}{2\pi}\frac{d\psi(t)}{dt}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[\phi(t) + \frac{n_Q(t)}{A_c}\right]$$

$$v(t) = \frac{1}{2\pi}\frac{d\phi(t)}{dt} + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= \frac{1}{2\pi}\frac{d}{dt}\left[2\pi k_f\int_0^t m(t)\,dt\right] + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= k_f\, m(t) + \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

$$= m_d(t) + n_d(t)$$

where,

$$m_d(t) = k_f\, m(t)$$

$$n_d(t) = \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}$$

Average power of $m_d(t) = \overline{m_d^2(t)}$

$$= k_f^2\, \overline{m^2(t)}$$

$$= k_f^2 \cdot P$$

Average power of $n_d(t) = \overline{n_d^2(t)}$

By applying fourier transformation, we get

$$N_d(f) = \frac{1}{\cancel{2\pi} A_c}\left[j\, \cancel{2\pi} f\; N_Q(f)\right]$$

$$= \frac{jf}{A_c} N_Q(f)$$

If $S_{nd}(f)$ and $S_{nQ}(f)$ be the power spectral densities of $N_d(f)$ and $N_Q(f)$ then relation is given as,

$$S_{nd}(f) = \frac{f^2}{A_c^2} S_{nQ}(f)$$

If the signal is passed through LPF, the value of $S_{nQ}(f)$ will be N_0.

$$\therefore S_{no}(f) = \frac{f^2}{A_c^2} N_0$$

The average power of noise signal in demodulated signal is given as

$$\overline{n_o^2(t)} = \int_{-W}^{W} S_{No}(f)\, df$$

$$= \int_{-W}^{W} \frac{N_0}{A_c^2} f^2\, df$$

$$= \frac{N_0}{A_c^2}\int_{-W}^{W} f^2\, df = \frac{N_0}{A_c^2}\left(\frac{f^3}{3}\right)_{-W}^{W}$$

$$= \frac{N_0}{A_c^2}\left[\frac{W^3}{3} + \frac{W^3}{3}\right] = \frac{2 N_0 W^3}{3 A_c^2}$$

$$\therefore (SNR)_o = \frac{K_f^2 P}{\frac{2 N_0 W^3}{3 A_c^2}} = \frac{3 A_c^2 K_f^2 P}{2 N_0 W^3}$$

$$\text{Figure of merit } (r) = \frac{(SNR)_o}{(SNR)_c}$$

$$r = \frac{\frac{3A_c^2 k_f^2 P}{2N_o W^3}}{\frac{A_c^2}{2N_o W}} = \frac{3k_f^2 P}{W^2}$$

$$\boxed{\therefore\ r = \frac{3k_f^2 P}{W^2}}$$

we know that P is the average power of message signal and it is given as,

$$P = A_m^2/2$$

$$r = \frac{3k_f^2}{W^2} \cdot \frac{A_m^2}{2} = \frac{3}{2W^2}\Delta f^2$$

$$r = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2$$

$$\boxed{r = \frac{3}{2}\beta^2}$$ where, $\frac{\Delta f}{W} = \beta$ (modulation index)

Now Let us compare the figure of merit of FM w.r.t AM

For 100% modulation the figure of merit of AM $= \frac{1}{3}$

The figure of merit of FM $= \frac{3}{2}\beta^2$

To have less noise in FM when compared to AM we have to take

$$\frac{3}{2}\beta^2 > \frac{1}{3}$$

$$\Rightarrow \beta > \frac{\sqrt{2}}{3}$$

$$\beta > 0.47 \approx 0.5$$

The value of $\beta = 0.47$ (or) $\beta = 0.5$ actually the transition point between the narrow-band FM and wide-band FM.

If $\beta < 0.5$, the FM is considered as narrow band FM in which there is no improvement in noise when compared to AM.

Capture Effect :-

In the frequency modulation, the signal can be affected by another frequency modulated signal whose frequency content is close to carrier frequency of the desired FM wave. The receiver may lock such an interference signal and suppress the desired FM signal when interference signal is stronger than desired signal.

When the strength of interference and desired signal are nearly equal, the receiver fluctuates back and forth between them i.e receiver locks interference signal for some time and desired signal for some time and this goes randomly. This phenomenon is Capture effect.

Threshold effect in angle modulation System:-

The threshold effect in FM is much more pronounced than in AM. The figure of merit of FM is valid if the carrier-to-noise is high compared to unity (i.e, CNR>>1).

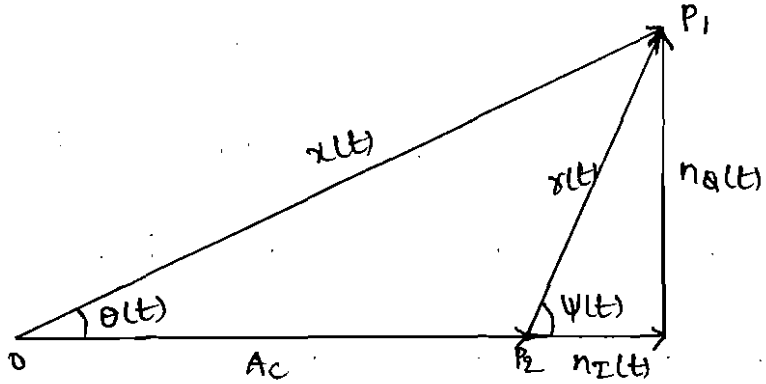
Suppose if the input noise power is increased or the carrier power is decreased, the CNR decreases consequently demodulator discriminator output becomes more and more corrupted by noise. Spikes comes out FM receiver and if CNR further decreases, continuous spikes comes out of FM receiver. The FM receiver is said to breakdown when clicks are heard. This phenomenon is called as threshold effect.

The threshold effect is defined as the minimum carrier to noise ratio that gives the output signal to noise ratio not less than the value predicted by the usual signal to noise formula assuming a small noise power.

At the frequency discriminator input is given by

$$x(t) = \left[A_c + n_I(t)\right]\cos 2\pi f_c t - n_Q(t)\sin 2\pi f_c t$$

where $n_I(t)$ and $n_Q(t)$ are in-phase and quadrature components of narrow band noise signal $n(t)$ w.r.t carrier respectively. The relationships defined by this equation are represented by phasor diagram as shown.



Let us derive the conditions for positive clicks to occur and negative clicks to occur are as follows:

Conditions for positive clicks:

$$r(t) > A_c$$

$$\psi(t) < \pi < \psi(t) + d\psi(t)$$

$$\frac{d\psi(t)}{dt} > 0$$

conditions for negative clicks:

$$r(t) > A_c$$

$$\psi(t) > -\pi > \psi(t) + d\psi(t)$$

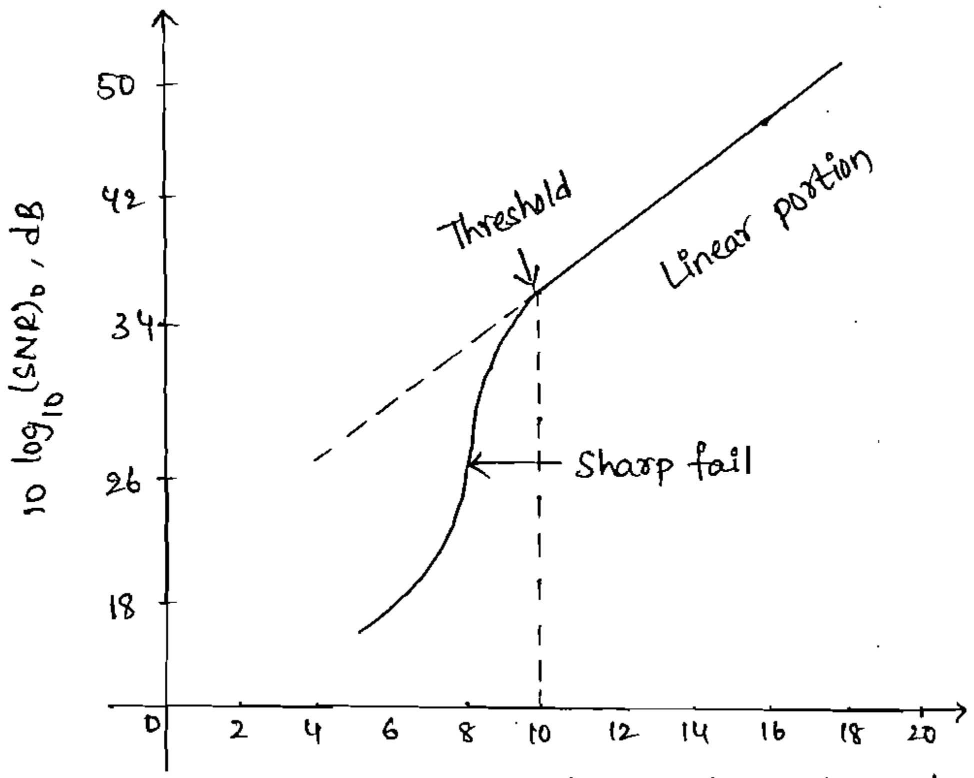
$$\frac{d\psi(t)}{dt} < 0$$

The conditions for positive clicks ensure that $\theta(t)$ changes by 2π radians and for negative clicks ensure that $\theta(t)$ changes by -2π radians during the time increment dt.

Carrier-to-noise ratio is defined by

$$P = \frac{A_c^2}{2 B_T N_0}$$

The average no. of clicks per unit time is inversely proportional to P. It is seen that $(SNR)_0$ ratio is a linear function of P when P is greater than 10 dB. However, it falls sharply for lower values of P than 10 dB. This is shown below.



Threshold can be avoided by keeping $P > 20$ i.e 13 dB

$$\frac{A_c^2}{2 B_T N_0} \geq 20 \quad \Rightarrow \quad \frac{A_c^2}{2} \geq 20 B_T N_0$$

$$\gamma = \frac{3k_f^2 P}{W^2}$$

As $m(t) = A_m \cos[2\pi f_m t]$

$$\Rightarrow P = \overline{m^2(t)} = A_m^2/2$$

$$\Rightarrow \gamma = \frac{3k_f^2 A_m^2}{2W^2}$$

The expression for frequency deviation is given by

$$\Delta f = |k_f\, m(t)|_{max} = |k_f A_m \cos(2\pi f_m t)|_{max}$$

$$\Rightarrow \cancel{\Delta f = k_f\, m(t)} \Rightarrow \Delta f = k_f A_m$$

$$\therefore \gamma = \frac{3}{2}\left[\frac{\Delta f}{W}\right]^2 \Rightarrow \boxed{\gamma = 1.5\beta^2}$$

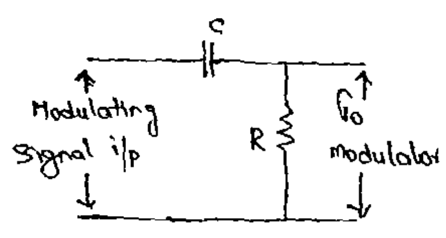
Where β is modulation index of FM given by $\beta = \frac{\Delta f}{W}$

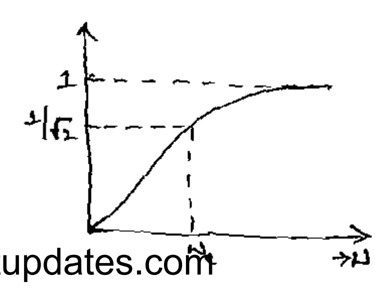
PRE-EMPHASIS AND DE-EMPHASIS

Noise produced in electronic circuits is low in low AF range, but at higher frequencies it increases. So, for information signals with a uniform signal level, a non-uniform signal-to-noise ratio is obtained. The higher modulating signal frequencies have a lower signal-to-noise ratio than the lower frequencies. To overcome this a high frequency modulating signals are emphasized (or) boosted in amplitude in the FM transmitter before modulation. This is known as pre-emphasis. The de-emphasis circuit restores the original amplitude-frequency characteristics of the information signal. Pre-emphasis and De-emphasis give a more (or) less uniform signal-to-noise ratio over the whole AF range.

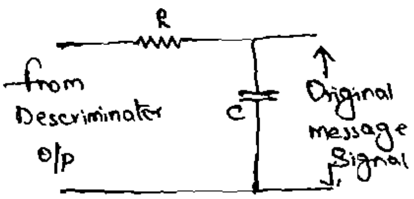
A Pre-emphasis circuit is a high pass filter i.e. ~~an~~ RC ~~circuit~~ ~~with a~~ high frequency components are boosted up at the output, because the capacitor C offers a low reactance at high frequencies.

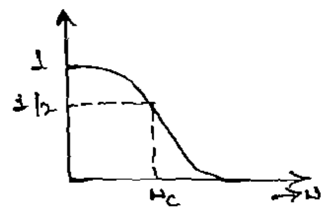
The circuit and frequency response curve of the Pre-emphasis are shown as





A de-emphasis circuit is low-pass filter, i.e high frequency components are attenuated because the capacitor short circuits such components. The circuit and frequency response curve of de-emphasis are shown as





UNIT-5
NOISE

Sources of noise:-

→ Natural
→ Manmade
→ fundamental

Classification

→ **Shot noise:-** It is produced due to shot effect. It produced in all the amplifying devices rather than in all the active devices.

* Shot noise is produced because of the random variations in the arrival of electrons & holes at the o/p electrode of an amplifying device. It sounds like a shower of lead shots falling on a metal sheet. The mean square shot noise current equation for diode is given as $I_n^2 = 2(I + 2I_0)qB$ (Amp²)

$I \to$ direct current across the junction (Amps)
$I_0 \to$ reverse saturation current (Amps)
$q \to$ electronic charge $= 1.6 \times 10^{-19}$ Columbs
$B \to$ effective noise Bandwidth (Hz)

for the amplying devices the shot noise is inversely proportion-al to the transconductance of the device & directly proportional to the direct current.

→ **Partition Noise:-** It is generated when the current gets divided b/w two or more paths it is generated due to the random fluctuations in the division. Therefore the partition noise in a transistor will be higher than that in a diode.

→ **Low frequency/flicker Noise:-** It will appear at frequencies below a few KHz. It is sometimes called as $\frac{1}{f}$ noise. In the semiconductor devices flicker noise is generated due to the fluctuations in the carrier density. These fluctuations in the carrier density will cause the fluctuations in the conductivity of the material. This will produce a fluctuation voltage drop when a direct current flows through a device.

This fluctuating voltage is known as flicker noise voltage. The mean square value of the flicker noise voltage is proportional to the square of direct current flowing through the device.

→ Thermal/Johnson/white noise:- The free electrons within a conductor are always in random motion. This random motion is due to the thermal energy received by them. The distribution of this free electrons with in a conductor at a given instant of time is not uniform. It is possible that an excess ~~ele~~ no of electrons may appear at one end or the other of the conductor. The average voltage resulting from this non-uniform distribution is zero, but the avg power is not zero. At this power results from the thermal energy. It is called as the thermal noise power.

Avg thermal noise power is given by $P_n = KTB$ watts.

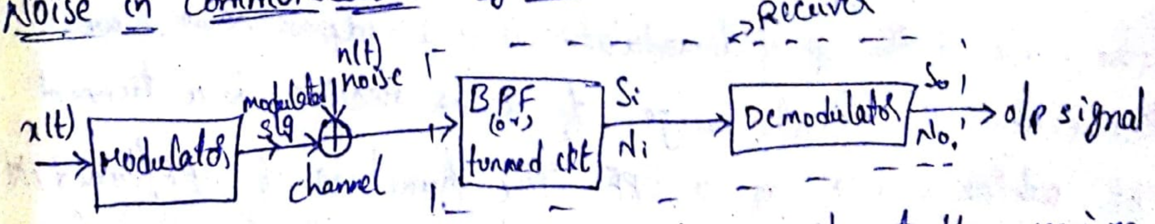
K – Boltzmann constant 1.38×10^{-23} J/K

B – B.W of noise spectrum (Hz)

T – temp of conductor °K

→ High frequency/Transit time noise:- If the time taken by an electron to travel from the emitter to the collector of a transistor becomes comparable to the period of the s/g which is being amplified then the transit time effect takes places. This effect is observed at very high frequencies. Due to transit time effect some of the carriers diffused back to the emitter. This gives rise to a i/p admittance, the conductance component of which increases with frequency. The minute currents induced in input of device by the random fluctuations in o/p current, will create ~~at~~ random noise at high frequencies.

Noise in Communication System:-



The message s/g travels from the transmitter to the receiver through a medium called channel. Noise is present in every communication system. The channel introduces a negative additive noise in the message s/g, and thus the msg which is received at the receiver is distorted.

Since the receiver detects both msg & noise signals. It will reproduce a msg s/g which contains noise. A noise calculation in a communication s/m is carried out in the form of a parameter called figure of merit. It is noted by letter (γ).

figure of merit is defined as the ratio of o/p SNR to i/p SNR of a receiver.

$$\gamma = \frac{(SNR)_o}{(SNR)_i}$$

→ few assumptions to calculate the figure of merit for various communications systems.

1) Channel noise is always white & Gaussian :- we assume that the noise of channel n(t) is always a white noise. This means that it is uniformly distributed over the entire band of frequencies hence the PSD of channel noise will be uniform.

The total noise power may be obtained by taking the product of noise power spectrum density $N_o/2$ with the Bandwidth ~~with the bandwidth~~.

Total noise power N = white noise PSD × B.W

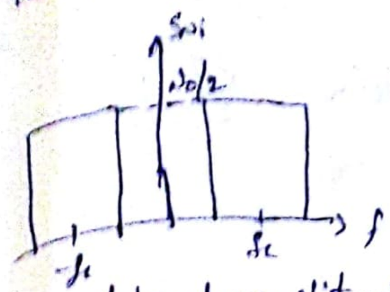
$$N = \frac{N_o}{2} \times B.W$$

Thus the noise has a Gaussian distribution.

2) Channel noise is always additive :- we assume that the disturbing effect of channel noise is always additive. This means that the effect of channel noise may be obtained by simple addition of s/g x(t) + noise n(t).

3) The noise at the i/p of demodulator is a bandpass noise :- we know that the first range of each receiver is a tuned ckt which works as a BPF. The function of BPF/tuned ckt is to allow only an narrow band s/g centered about $\pm f_c$ and reject all other frequencies. This means that the noise s/g lying ~~at~~ outside this range is also rejected. And thus the B.W of noise s/g at the i/p of detector as same as that of the incoming modulated s/g.

we assumed that the channel noise is white in nature so the PSD of white noise at i/p of demodulator is $S_{ni} = \frac{N_0}{2}$

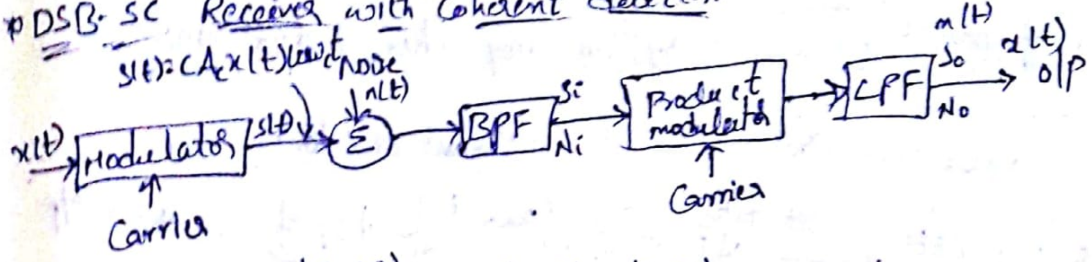


$r(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$

PSD of bandpass white noise

(i) The noise i/p power N_i to calculate the figure of merit we evaluate the noise i/p power for passband/B.W of incoming modulating signal. Total noise power $N = \frac{N_0}{2} \times 2f_m = N_0 f_m$ (for AM s/g)

* **DSB-SC Receiver with coherent detector:-**



$$\gamma = \frac{(SNR)_o}{(SNR)_i} \qquad (SNR)_i = (SNR)_c$$

$(SNR)_i = \frac{\text{Avg s/g power at the receiver i/p}}{\text{Avg noise power at the receiver o/p}}$

$$\text{Avg signal power} = \frac{c^2 A_c^2 P}{2}$$

$$\text{Avg noise power} = 2 \times \frac{N_0}{2} \times 2f_m = 2N_0 f_m$$

$$(SNR)_i = \frac{c^2 A_c^2 P}{2 \times 2N_0 f_m} = \frac{c^2 A_c^2 P}{4N_0 f_m} \quad \text{— (1)}$$

After LPF i.e., receiver o/p.

$$S_o = (CA_c x(t)\cos\omega_c t + n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t)\cos\omega_c t$$

$$= CA_c x(t)\cos^2\omega_c t + n_I(t)\cos^2\omega_c t - n_Q(t)\sin\omega_c t\cos\omega_c t$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin 2A = 2\sin A\cos A$$

$$= CA_c x(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) + n_I(t)\left(\frac{1+\cos 2\omega_c t}{2}\right) - \frac{n_Q(t)}{2}\sin 2\omega_c t$$

$$S_o = \frac{CA_c x(t)}{2} + \frac{CA_c x(t)}{2}\cos 2\omega_c t + \frac{n_I(t)}{2} + \frac{n_I(t)}{2}\cos 2\omega_c t - \frac{n_Q(t)}{2}\sin 2\omega_c t$$

After LPF

$$m(t) = \frac{CA_c x(t)}{2} + \frac{n_I(t)}{2}$$

$m(t) = \frac{CA_c}{2} x(t) + \frac{1}{2} n_I(t)$

Avg signal power $= \left(\frac{1}{2}\right)^2 \frac{C^2 A_c^2 P}{2} = \frac{C^2 A_c^2 P}{8}$

Noise $\left(\frac{1}{2}\right)^2 2 \times \frac{N_0}{2} \times 2f_m = \frac{1}{2} N_0 f_m = \frac{N_0 f_m}{2}$

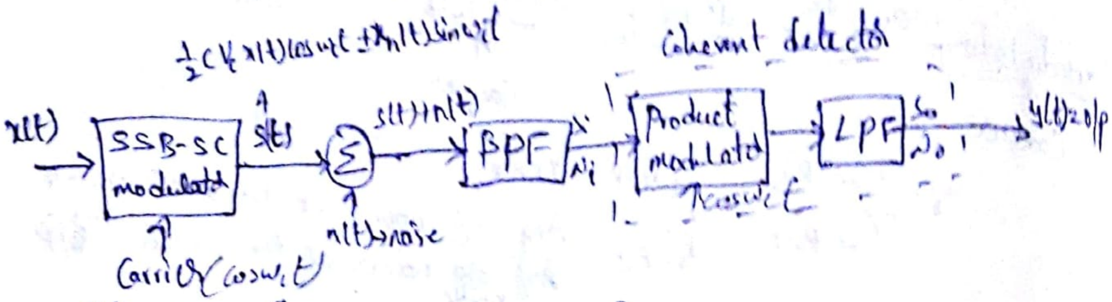
$$(SNR)_0 = \frac{2C^2A_c^2P}{4 \cdot 8 N_0 f_m} = \frac{C^2A_c^2P}{4N_0 f_m} \quad -\ (2)$$

Sub (1) & (2) in γ

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\frac{C^2A_c^2P}{4N_0 f_m}}{\frac{C^2A_c^2P}{4N_0 f_m}} = 1$$

$$\boxed{\gamma = 1}$$

* figure of merit for SSBSC system using coherent detection:-



$$s(t) = \frac{1}{2} CV_c \left[x(t)\cos w_c t \pm x_h(t)\sin w_c t\right]$$

$$S_i = \left(\frac{1}{2}CV_c\right)^2 \frac{P}{2} + \left(\frac{1}{2}CV_c\right)^2 \frac{P}{2}$$

$$= \frac{1}{4}C^2V_c^2 \frac{P}{2} + \frac{1}{4}C^2V_c^2\frac{P}{2}$$

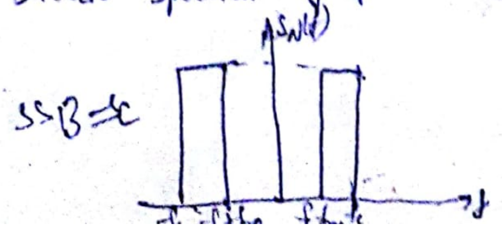
$$= \frac{C^2V_c^2P}{8} + \frac{C^2V_c^2P}{8}$$

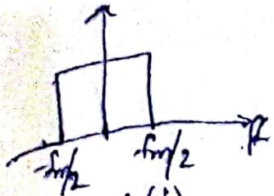
$$S_i = \frac{C^2V_c^2P}{4}$$

$$N_i = 2 \times \frac{N_0}{2} \times f_m = N_0 f_m$$

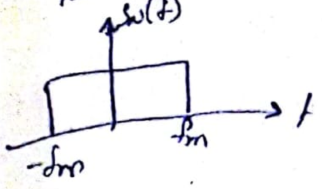
$$(SNR)_i = \frac{S_i}{N_i} = \frac{C^2V_c^2P}{4N_0 f_m} \quad -\ (1)$$

1) Shift the spectrum left $\frac{f_m}{2}$
2) Shift the spectrum right $f_m/2$
3) Divide spectrum by 4





$$\frac{f_m}{2} - f_m = -\frac{f_m}{2}$$
$$-\frac{f_m}{2} + f_m = \frac{f_m}{2}$$



$$n(t) = n_I(t)\cos\left(2\pi\left(f_c - \frac{f_m}{2}\right)t\right] + n_Q(t)\sin\left(2\pi\left(f_c - \frac{f_m}{2}\right)t\right]$$

$$\underbrace{s(t)+n(t)}_{m(t)} = \left[\left[\left(\frac{1}{2}cV_c x(t)\cos\omega_c t + \frac{1}{2}cV_c x(t)\sin\omega_c t\right) + \left[n_I(t)\cos 2\pi\left(f_c - \frac{f_m}{2}\right)t\right] + n_Q(t)\sin\left[2\pi\left(f_c - \frac{f_m}{2}\right)t\right]\right]\right] \cos\omega_c t.$$

$$\cos^2\theta = \frac{1+\cos 2\theta}{2}, \quad \sin 2A = 2\sin A\cos B$$

$$\cos A\cos B = \frac{1}{2}\left[\cos(A+B) + \cos(A-B)\right]; \quad \sin A\cos B = \frac{1}{2}\left(\sin(A+B) + \sin(A-B)\right)$$

$$m(t) = \frac{1}{2}V_c c x(t)\cos\omega_c t\cos\omega_c t + \frac{1}{2}cV_c x(t)\sin\omega_c t\cos\omega_c t + n_I(t)\cos 2\pi\left(f_c - \frac{f_m}{2}\right)t\cos\omega_c t + n_Q(t)\sin\left(2\pi f_c - \frac{f_m}{2}t\right)\cos\omega_c t.$$

$$= \frac{1}{4}V_c c x(t)\left[\cos(2\omega_c t) + \cos(0)\right] + \frac{1}{4}cV_c x(t)\left[\sin(2\omega_c t) + \sin(0)\right] + \frac{1}{2}n_I(t)\left[\cos\left(\omega_c - \frac{\omega_m}{2} + \omega_c t\right) + \cos\left(\omega_c - \frac{\omega_m}{2} - \cos\omega_c t\right)\right] + \frac{1}{2}n_Q(t)\left[\sin\left(\omega_c - \frac{\omega_m}{2} + \cos\omega_c\right)t + \sin\left(\omega_c - \frac{\omega_m}{2} - \omega_c t\right)\right]$$

$$\Rightarrow \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}V_c c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\left[\cos 2\omega_c - \frac{\omega_m}{2}\right] + \frac{1}{2}n_I(t)\cos\left(\frac{\omega_m}{2}\right) + \frac{1}{2}n_Q(t)\left[\sin\left(2\omega_c - \frac{\omega_m}{2}\right)\right] + \frac{1}{2}n_Q(t)\left[\sin\frac{\omega_m}{2}\right]$$

$$= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\cos 2\omega_c t - \frac{1}{2}n_I(t)\frac{2\pi f_m}{2} + \frac{1}{2}n_I(t)\cos\left|\frac{2\pi f_m}{2}\right| t + \frac{1}{2}n_Q(t)\sin 2\omega_c t + \frac{1}{2}n_Q(t)\frac{2\pi f_m}{2} - \frac{1}{2}n_Q(t)\sin\left(\frac{2\pi f_m}{2}\right)$$

$$\cancel{= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t - \frac{1}{2}n_I(t)\pi f_m t + \frac{1}{2}n_I(t)\cos\pi f_m t + \frac{1}{2}n_Q(t)\sin 2\omega_c t - \frac{1}{2}n_Q(t)\pi f_m t - \frac{1}{2}n_Q(t)\sin\pi f_m t}$$

$$= \frac{1}{4}V_c c x(t)\cos 2\omega_c t + \frac{1}{4}cV_c x(t) + \frac{1}{4}cV_c x(t)\sin 2\omega_c t + \frac{1}{2}n_I(t)\cos 2\omega_c t - \frac{1}{2}n_I(t)\pi f_m t + \frac{1}{2}n_I(t)\cos\pi f_m t + \frac{1}{2}n_Q(t)\sin 2\omega_c t - \frac{1}{2}n_Q(t)\pi f_m t - \frac{1}{2}n_Q(t)\sin\pi f_m t$$

~~After~~ The above expression is passed through a Low pass filter, the filter attenuates unwanted expression & allows wanted expression. After LPF o/p is

$$y(t) = \frac{1}{4}cV_c x(t) + \frac{1}{2}n_I(t)\cos(\pi f_m t) + \frac{1}{2}n_Q(t)\sin(\pi f_m t)$$

This is the ~~eq~~ required o/p of LPF.

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$y(t) = \frac{1}{4}C(V_c x(t) + \frac{1}{2} n_I(t)\cos(\pi f_m t) + \frac{1}{2} n_Q(t)\sin(\pi f_m t)$

S_0 = Avg signal power at the o/p

$S_0 = (\frac{1}{4}CA_c)^2 \frac{P}{2} = \frac{c^2}{16} V_c^2 \frac{P}{2} = \frac{c^2 V_c^2 P}{32}$ — ②

$N_0 = (\frac{1}{2})^2 \frac{N_0}{4} f_m + (\frac{1}{2})^2 \frac{N_0}{4} f_m$

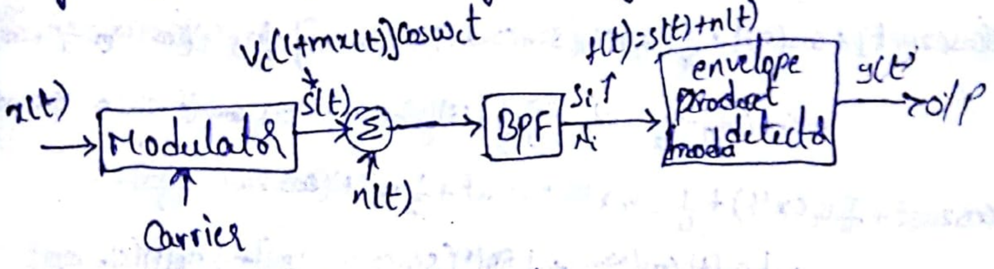
$N_0 = \frac{1}{4}\frac{N_0 f_m}{4} + \frac{1}{4}\frac{N_0}{4} f_m = \frac{N_0 f_m}{8}$

$(SNR)_0 = \frac{S_0}{N_0} = \frac{c^2 V_c^2 P}{32_4} \times \frac{8}{N_0 f_m} = \frac{c^2 V_c^2 P}{4 N_0 f_m}$ — ③

$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\frac{c^2 V_c^2 P}{4 N_0 f_m}}{\frac{c^2 V_c^2 P}{4 N_0 f_m}} = 1$

$\boxed{\gamma = 1}$

* figure of merit for AM system using ~~coherent~~ Envelope detection:-



$s(t) = V_c[1 + m x(t)]\cos\omega_c t$

$(SNR)_i = ?$ $\quad V_c\cos\omega_c t = \frac{V_c^2}{2}$

$V_c m x(t)\cos\omega_c t = \frac{V_c^2}{2}\frac{m^2}{V_m^2} P$

$S_i = \frac{V_c^2}{2} + \frac{V_c^2}{2}\frac{m^2}{V_m^2} P$

$S_i = \frac{V_c^2}{2}[1 + (\frac{m}{V_m})^2 P]$

$N_i = 2 \times \frac{N_0}{2} \times 2 f_m$

$N_i = 2 N_0 f_m$

$(SNR)_i = \frac{S_i}{N_i} = \frac{\frac{V_c^2}{2}[1 + (\frac{m}{V_m})^2 P]}{2 N_0 f_m}$ → ①

$(SNR)_0 = ?$

$f(t) = s(t) + n(t) = V_c[1 + m x(t)]\cos\omega_c t + \underbrace{n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t}_{n(t)}$

$f(t) = \cos\omega_c t[V_c + V_c m x(t) + n_I(t)] - n_Q(t)\sin\omega_c t$

i/p for envelope detector.

o/p $y(t) = \sqrt{(V_c + V_c m x(t) + n_I(t))^2 + n_Q(t)^2}$

$n_Q(t)$ is zero, because envelope detector allows only inphase components.

$y(t) \cong V_c + V_c m x(t) + n_I(t)$

$y(t) = V_c m x(t) + n_I(t)$ & $V_c = 0$

$$S_0 = \frac{V_c^2}{2} P \frac{m^2}{V_m^2}$$

$$N_0 = 2 \times \frac{N_0}{2} \times 2 f_m$$

$$N_0 = 2 N_0 f_m$$

$$\frac{S_0}{N_0} = (SNR)_0 = \frac{\frac{V_c^2}{2} P \left(\frac{m}{V_m}\right)^2}{2 N_0 f_m}$$

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\dfrac{\frac{V_c^2}{2} P \left(\frac{m}{V_m}\right)^2}{2 N_0 f_m}}{\dfrac{\frac{V_c^2}{2}\left[1 + \left(\frac{m}{V_m}\right)^2 P\right]}{2 N_0 f_m}}$$

$$\boxed{\gamma = \frac{P\left(\frac{m}{V_m}\right)^2}{1 + \left(\frac{m}{V_m}\right)^2 P}}$$

1) Calculate the figure of merit for AM using envelope detector for single tone AM.

A. Single tone AM.

$$x(t) = V_m \cos \omega_m t$$

$$x(t) = P$$

Avg power of $x(t) = P = \frac{V_m^2}{2}$

$$(SNR)_i = \frac{\frac{V_c^2}{2}\left(1 + \left(\frac{m^2}{V_m^2}\right)\frac{V_m^2}{2}\right)}{2 N_0 f_m} = \frac{\frac{V_c^2}{2} \cdot \left(1 + \frac{m^2}{2}\right)}{2 N_0 f_m}$$

$$(SNR)_0 = \frac{\frac{V_c^2}{2} \frac{V_m^2}{2} \frac{m^2}{V_m^2}}{2 N_0 f_m} = \frac{\frac{V_c^2}{2} \cdot \frac{m^2}{2}}{2 N_0 f_m}$$

for 100% modulation $m = 1$

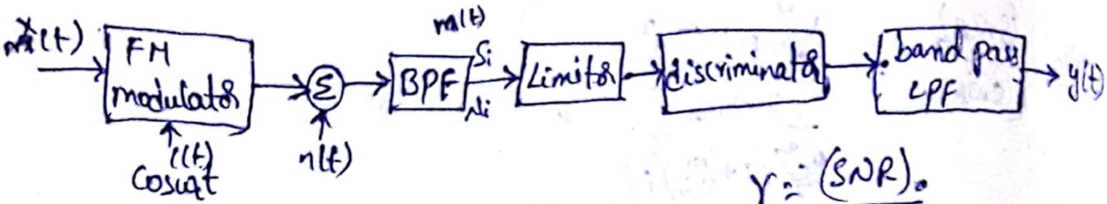
$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{\dfrac{\frac{V_c^2}{2} \frac{m^2}{2}}{2 N_0 f_m}}{\dfrac{\frac{V_c^2}{2} \cdot \left(1 + \frac{m^2}{2}\right)}{2 N_0 f_m}} = \frac{\frac{V_c^2}{2} \cdot \frac{m^2}{2}}{\frac{V_c^2}{2} \cdot \left(1 + \frac{m^2}{2}\right)}$$

$$\gamma = \frac{\frac{m^2}{2}}{1+\frac{m^2}{2}} \quad \text{Sub } m=1$$

$$= \frac{1/2}{1+\frac{1}{2}} = \frac{1}{2+1}$$

$$\boxed{\gamma = 1/3}$$

* Noise in FM Receivers:-



$$\gamma = \frac{(SNR)_o}{(SNR)_i}$$

$$n(t) = n_I(t)\cos\omega_c t - n_Q(t)\sin\omega_c t$$

$$\gamma(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$$

$$\psi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

$$n(t) = \gamma(t)\cos(\omega_c t + \psi(t)) \quad -①$$

$$s(t) = A_c\cos\left[\omega_c t + 2\pi k_f\int_0^t x(t)\,dt\right]$$

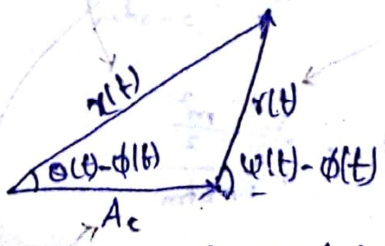
Assume $\phi(t) = 2\pi k_f \int_0^t x(t)dt$

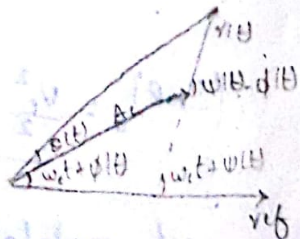
$$s(t) = A_c\cos(\omega_c t + \phi(t)) \quad -②$$

Add ① & ②

$$x(t) = s(t) + n(t)$$

$$x(t) = A_c\cos(2\pi f_c t + \phi(t)) + \gamma(t)\cos(2\pi f_c t + \psi(t))$$





$$\theta(t) - \phi(t) = \tan^{-1}\left[\frac{\gamma(t)\sin(\psi(t)-\phi(t))}{A_c + \gamma(t)\cos(\psi(t)-\phi(t))}\right]$$

$$= \frac{\gamma(t)}{A_c}\sin(\psi(t)-\phi(t))$$

independent of $x(t)$

$$\theta(t) = \frac{\gamma(t)}{A_c}\sin(\psi(t)-\phi(t)) + \phi(t)$$

$$\theta(t) = \frac{\gamma(t)}{A_c}\sin\psi(t) + \phi(t)$$

$$\theta(t) = 2\pi k_f \int_0^t x(t)\,d(t) + \frac{r(t)}{A_c} \sin\psi(t)$$

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = \frac{1}{2\pi} 2\pi k_f\, x(t) + \frac{1}{2\pi A_c} \underbrace{\frac{d}{dt} r(t)\sin\psi(t)}_{n_Q(t)}$$

$$\frac{1}{2\pi}\frac{d\theta(t)}{dt} = k_f x(t) + \frac{1}{2\pi A_c}[n_Q(t)]$$

$$v(t) = k_f x(t) + n_d(t)$$

$$S_0 = k_f^2 P \quad -③$$

$$\frac{d}{dt}x(t) \xleftrightarrow{FT} j2\pi f\, X(f)$$

$$H(f) = \frac{1}{2\pi A_c} \times j2\pi f = \frac{jf}{A_c}$$

$$|H(f)|^2 = \frac{f^2}{A_c^2}$$

$$P = \int_{-f_m}^{f_m} S_{N_0}(f)\,df$$

$$N_0 = P = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\int_{-f_m}^{f_m} f^2\,d(t) = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\left[\frac{f^3}{3}\right]_{-f_m}^{f_m} = \frac{N_0}{\cancel{2}}\frac{1}{A_c^2}\frac{\cancel{2}f_m^3}{3} \quad -④$$

sub ③ & ④ in $(SNR)_0$

$$(SNR)_0 = \frac{S_0}{N_0} = \frac{3k_f^2 P A_c^2}{2N_0 f_m^3} \quad -⑤$$

$$s(t) = A_c\cos(\omega_c t + \phi(t))$$

$$S = \frac{A_c^2}{2} \quad -⑥$$

$$n_i = \frac{N_0}{2}\times 2f_m = N_0 f_m \quad -⑦$$

sub ⑥, ⑦ in $(SNR)_i$

$$(SNR)_i = \frac{A_c^2}{2N_0 f_m} \quad -⑧$$

sub ⑤, ⑧ in γ

$$\gamma = \frac{(SNR)_0}{(SNR)_i} = \frac{3k_f^2 P A_c^2 / 2N_0 f_m^3}{\dfrac{A_c^2}{2N_0 f_m}}$$

$$= \frac{3k_f^2 P \cancel{A_c^2}}{\cancel{2N_0} f_m^3} \times \frac{\cancel{2N_0} f_m}{\cancel{A_c^2}}$$

$$\boxed{\gamma = \frac{3k_f^2 P}{f_m^2}}$$

Calculate figure of merit for single tone FM:-

$$A_c\cos\left(\omega_c t + \frac{\Delta f}{f_m}\sin\omega_m t\right)$$

$$\frac{\Delta f}{f_m}\sin\omega_m t = 2\pi k_f \int_0^t x(t)\,dt$$

Diff on b.s

$$\omega_m \frac{\Delta f}{f_m} \cos\omega_m t = 2\pi k_f x(t)$$

$$2\pi f_m \cdot \frac{\Delta f}{f_m} \cos\omega_m t = 2\pi k_f x(t)$$

$$\Delta f \cos\omega_m t = k_f x(t)$$

$$x(t) = \frac{\Delta f}{k_f} \cos\omega_m t$$

Avg power of the modulating signal across 1Ω resistor is

$$P = \left(\frac{\Delta f}{k_f}\right)^2 \frac{1}{2}$$

$$k_f^2 P = \frac{(\Delta f)^2}{2}$$

$$(SNR)_o = \frac{3}{2}\left(\frac{k_f^2 P A_c^2}{N_0 f_m^3}\right) = \frac{3}{2}\left[\frac{\frac{(\Delta f)^2}{2} \cdot A_c^2}{N_0 f_m^3}\right]$$

$$(SNR)_i = \frac{A_c^2}{2N_0 f_m}$$

$$\gamma = \frac{(SNR)_o}{(SNR)_i} = \frac{\frac{3}{2}\left[\frac{(\Delta f)^2}{2} \cdot A_c^2\right] / 2N_0 f_m^3}{\frac{A_c^2}{2N_0 f_m}} = \frac{3\left[\frac{(\Delta f)^2}{2} \cdot A_c^2\right]}{2N_0 f_m^3} \times \frac{2N_0 f_m}{A_c^2}$$

$$\gamma = \frac{3\frac{(\Delta f)^2}{2}}{\frac{f_m^2}{1}} = \frac{3(\Delta f)^2}{2} \times \frac{1}{f_m^2}$$

$$= \frac{3}{2}\left(\frac{\Delta f}{f_m}\right)^2$$

$$\boxed{\gamma = \frac{3}{2}(m_f)^2}$$

UNIT-V PULSE MODULATION

SAMPLING: The process of converting an analog signal into a discrete signal is known as sampling.

Figure 8.1shows how this conversion can be done. As shown in the figure 8.1a switch position is controlled by the sampling signal. The sampling signal is a periodic train of pulses of unit amplitude and of period T_s. The time T_s is known as sampling time and during this time switch is closed so that sampled signal is equal to the input signal. During remaining time switch is open and no input signal appear at the output.

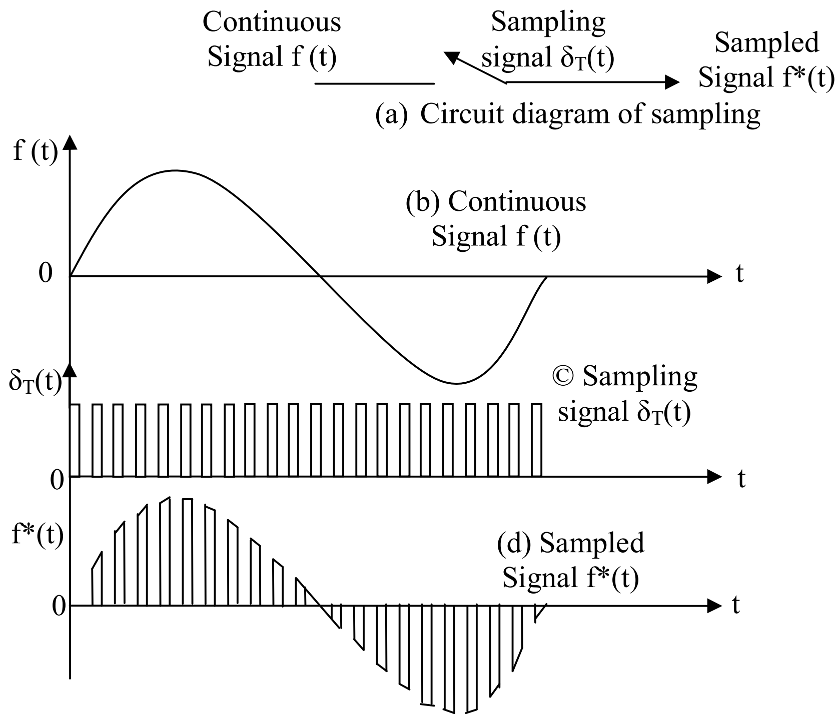
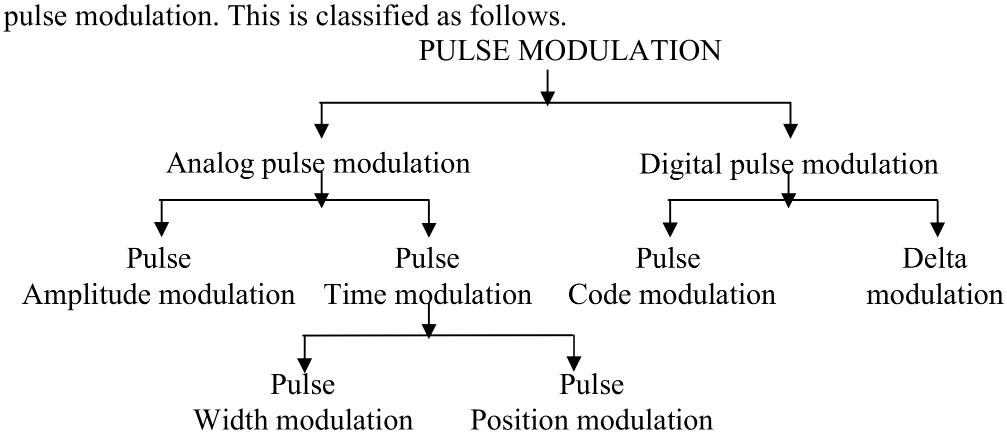


Fig 8.1: Sampling process

SAMPLING THEOREM: The sampling theorem states that if the sampling rate in any pulse modulation system exceeds twice the maximum signal frequency, the original signal can be reconstructed in the receiver with minimal distortion.

PULSE MODULATION: The process of changing any one of the characteristics of train of pulse in according to the amplitude of modulating signal at the time of sampling is called pulse modulation. This is classified as follows.



GENERATION OF PAM: The process of changing amplitude of the train of pulse in according to the amplitude of modulating signal at the time of sampling is called pulse amplitude modulation.

The figure 8.2a shows the block schematic of PAM generator. It consists of a low

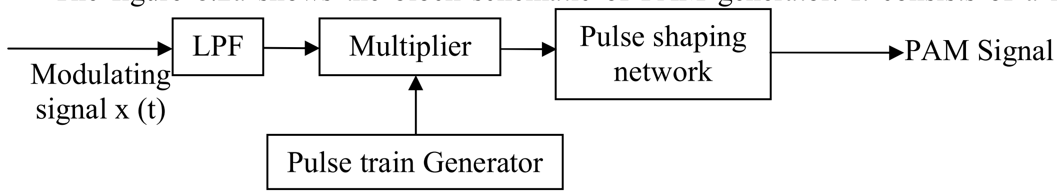


Fig 8.2a: Generation of PAM Signal

Pass filter, a multiplier and a pulse train generator. Initially, the modulating signal x (t) is passed through the low pass filter (LPF). The LPF removes all the frequency components which are higher than frequency f_m. This is known as band limiting. The band limiting is necessary to avoid the aliasing effect in the sampling process. The pulse train generator generates a pulse train at a frequency f_s, such that $f_s \geq 2f_m$. Thus the Nyquist criterion is satisfied. The pulse sampling network does the shaping work to give flat tops. The figure 8.2b, c, d and e show the waveforms related to the generation of PAM generator.

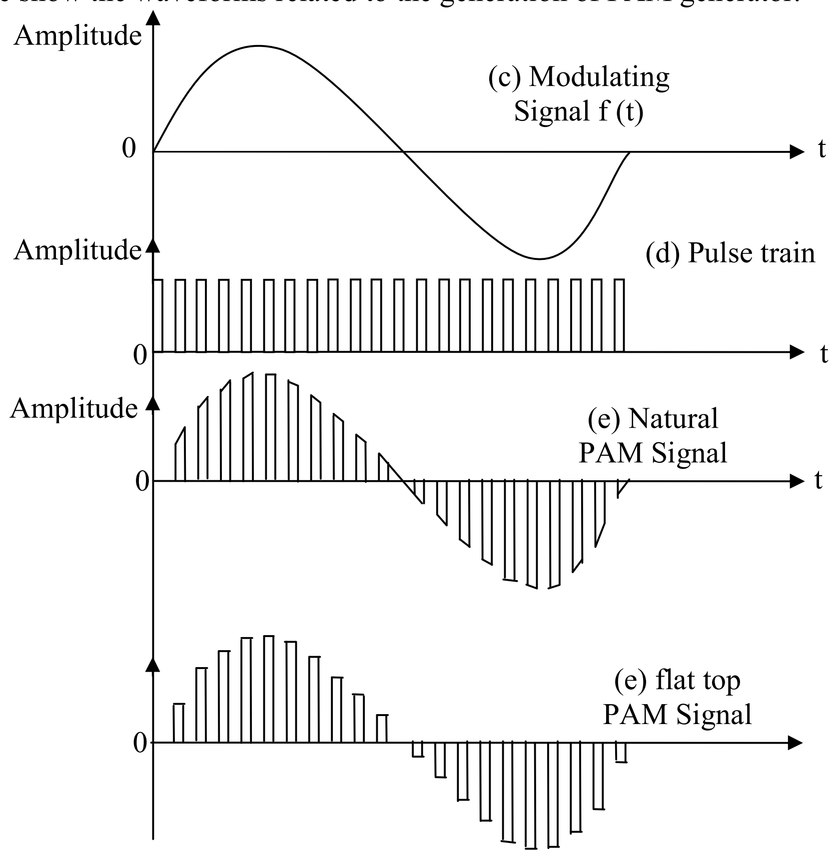


Fig 8.2b: waveforms of PAM

<u>DETECTION OF PAM</u>: The original modulating signal can be detected from the natural PAM by passing naturally modulated PAM through a diode detector and a low pass filter. The diode detector detects the envelope of the PAM signal. The low pass filter with cut-off frequency equal to f_m removes high frequency ripple and recovers the original modulating signal. This is illustrated in figure 8.3a.

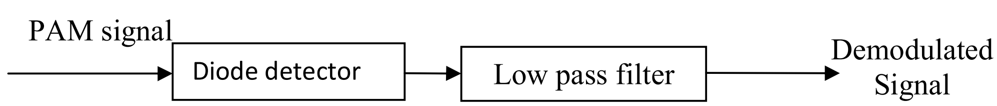


Fig 8.2a: Detection of natural PAM Signal

The demodulated output shown in figure 8.3b is close to the original modulating signal.

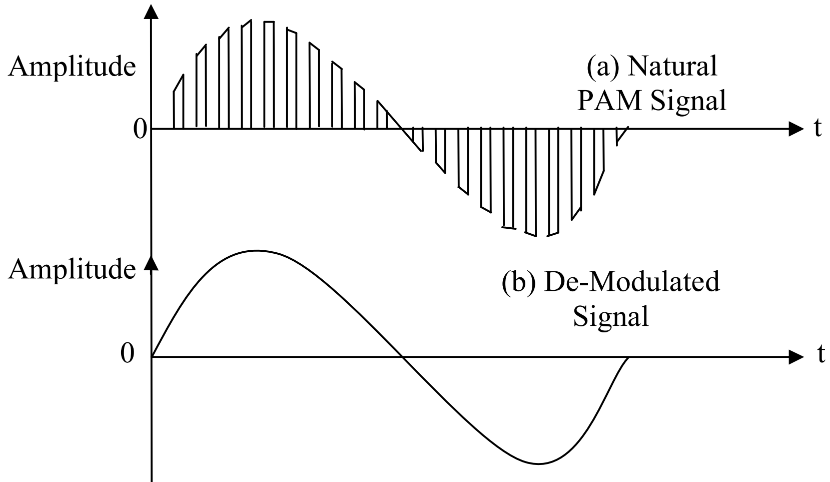


Fig 8.3b: waveforms of PAM detection

In case of flat top PAM to reduce aperture effect, an equalizer is used. As shown in figure 8.3c the receiver consists of low-pass reconstruction filter with cutoff frequency slightly higher than the maximum frequency of message signal. The equalizer compensates the aperture effect. It also compensates the attenuation by a low pas reconstruction filter.

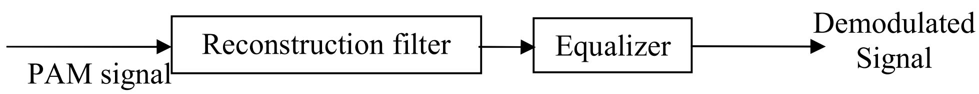


Fig 8.3c: Detection of flat top PAM Signal

TRANSMISSION BANDWIDTH OF PAM SIGNAL

The pulse duration τ is supposed to be very very small compared to time period T_s between the two samples. If the maximum frequency in the signal x (t) is W then by sampling theorem, f_s should be higher than Nyquist rate i.e. $f_s \geq 2W$

$$\frac{1}{fs} \leq \frac{1}{2W}$$

$$T_s \leq \frac{1}{2W} \qquad (\text{since } f_s = \frac{1}{Ts})$$

We know that $\tau \ll T_s$

Therefore $\tau \ll T_s \leq \frac{1}{2W}$ ------ (1)

If ON and OFF time of the pulse is same, then frequency of the PAM pulse becomes,

$f = \frac{1}{\tau+\tau} = \frac{1}{2\tau}$ ----- (2)

Thus figure 8.4 shows that if ON and OFF times of PAM signal are same. Then maximum frequency of PAM signal is given by equation 2 i.e.,

$f_{max} = \frac{1}{2\tau}$ ---- (3)

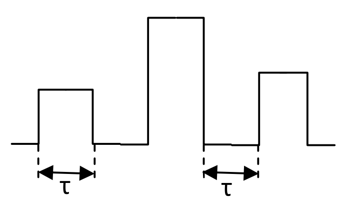


Fig 8.4: maximum frequency of PAM Signal

Therefore bandwidth required for transmission of PAM signal will be equal to maximum frequency f_{max} given by equation (3). This bandwidth gives adequate pulse resolution i.e.,

$$B_T \geq f_{max}$$

Therefore $\quad B_T \geq \frac{1}{2\tau} \quad$ Since $\tau \ll \frac{1}{2W}$

$$B_T \geq \frac{1}{2\tau} \gg W$$

Transmission bandwidth of PAM signal: $B_T \gg W$

Thus the transmission bandwidth B_T of PAM signal is very very large compared to highest frequency in the signal x(t).

ADVANTAGES OF PAM:

1. Generation and detection of PAM is simple

DISADVATAGES OF PAM:

1. PAM is less immune to noise.
2. It requires larger transmission power.

CLASSIFICATION OF PAM BASED ON SIGNAL POLARITY

The PAM signal can be classified according to signal polarity as Single polarity PAM and Double polarity PAM

The figure shows the single polarity PAM Here, a fixed d.c. level is added to the modulating signal x(t), such that the modulated output i.e. PAM signal is always positive.

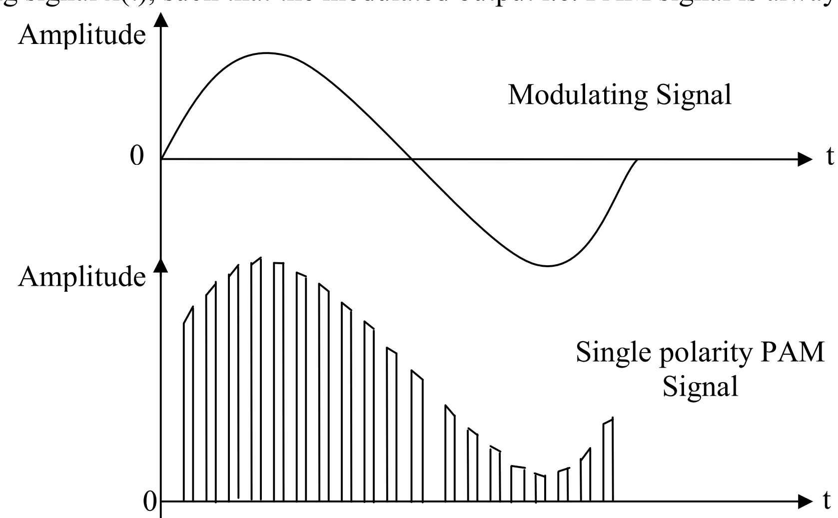


Fig 8.5a: waveforms of single polarity PAM

In double polarity PAM signal, signal has positive as well as negative polarity. It is shown in figure 8.5b.

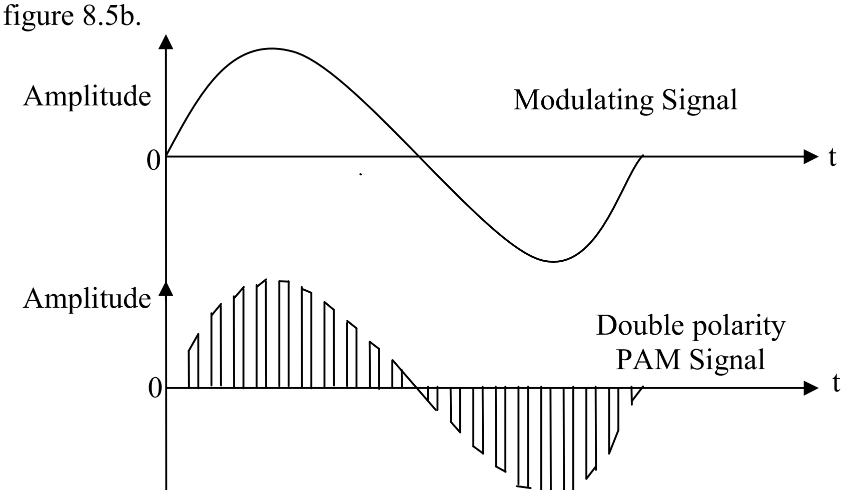


Fig 8.5b: waveforms of double polarity PAM

GENERATION OF PWM: The process of changing the width of the train of pulse in according to the amplitude of modulating signal at the time of sampling is called pulse width modulation. Figure 8.6a shows monostable multivibrator circuit to generate pulse width modulated wave.

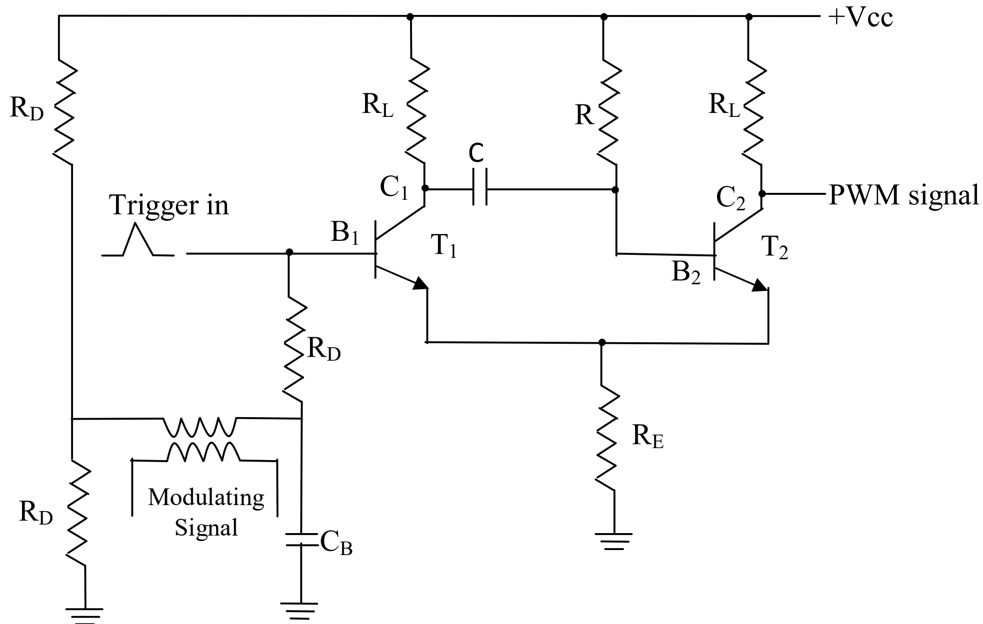


Fig 8.6a: monostable multivibrator circuit to generate pulse width modulated wave.

The stable state for above circuit is achieved when T_1 is OFF and T_2 is ON. The positive going trigger pulse at B_1 switches T_1 ON. Because of this, the voltage at C_1 falls as T_1 now begins to draw the collector current. As a result, voltage at B_2 also falls and T_2 is switched OFF, C begins to charge up to the collector supply voltage (V_{CC}) through resistor R. After a time determined by the supply voltage and the RC time constant of the charging network, the base of the T_2 becomes sufficiently positive to switch T_2 ON. The transistor T_1 is simultaneously switched OFF by regenerative action and stays OFF until the arrival of the next trigger pulse. To make T_2 ON, the base of the T_2 must be slightly more positive than the voltage across resistor R_E. This voltage depends on the emitter current I_E which is controlled by the signal voltage applied at the base of transistor T_1. Therefore, the changing voltage necessary to turn OFF transistor T_2 is decided by the signal voltage. If signal voltage is maximum, the voltage that capacitor should charge to turn ON T_2 is also maximum. Therefore, at maximum signal voltage, capacitor has to charge to maximum voltage requiring maximum time to charge. This gives us maximum pulse width at maximum input signal voltage. At minimum signal voltages, capacitor has to charge for minimum voltage and we get minimum pulse width at the output. With this discussion, we can say that pulse width is controlled by the input signal voltage, and we get pulse width modulated waveform at the output. The waveforms of PWM are shown in figure 8.6b.

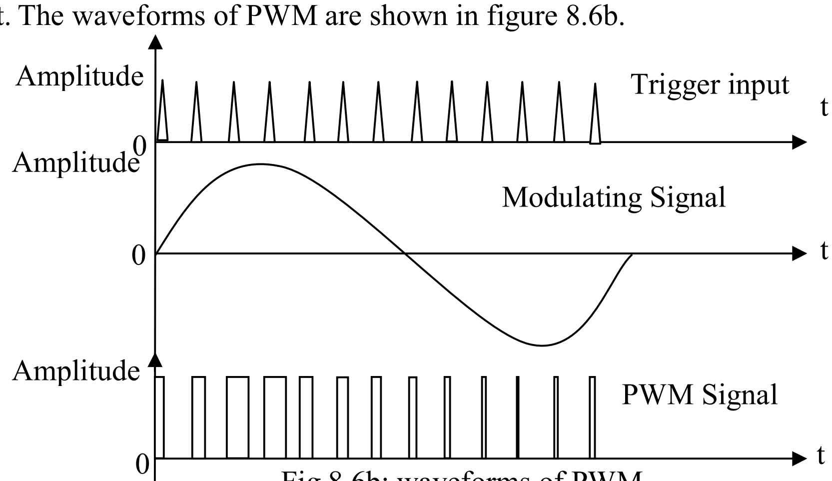


Fig 8.6b: waveforms of PWM

DEMODULATION OF PWM SIGNAL:

Figure 8.7a shows the block diagram of PWM detector.

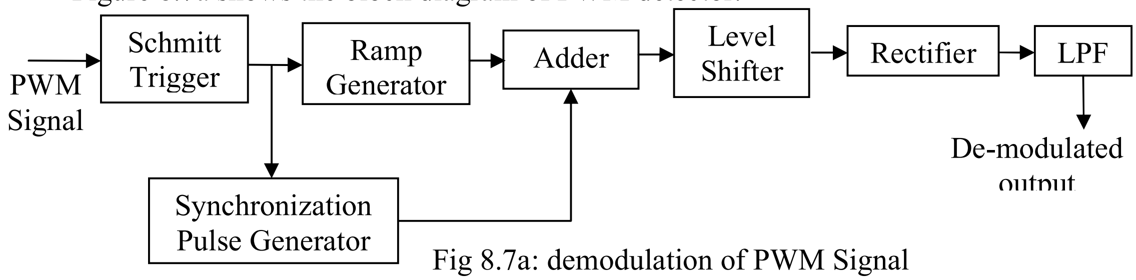


Fig 8.7a: demodulation of PWM Signal

The received PWM signal is applied to the Schmitt trigger circuit. This Schmitt trigger circuit removes the noise in the PWM waveform. The regenerated PWM is then applied to the ramp generator and the synchronization pulse detector. The ramp generator produces ramps for the duration of pulses such that heights of ramps are proportional to the widths of PWM pulses. The maximum ramp voltage is retained till the next pulse. The synchronous pulse detector produces reference pulses with constant amplitude and pulse width. These pulses are delayed by specific amount of delay as shown in the figure 8.7b. The delayed reference pulses and the output of ramp generator are added with the help of adder. The output of adder is given to the level shifter. Here, negative offset shifts the waveform as shown in the figure 8.7b. Then the negative part of the waveform is clipped by rectifier. Finally, the output of rectifier is passed through low-pass filter to recover the modulating signal, as shown in the figure 8.7b.

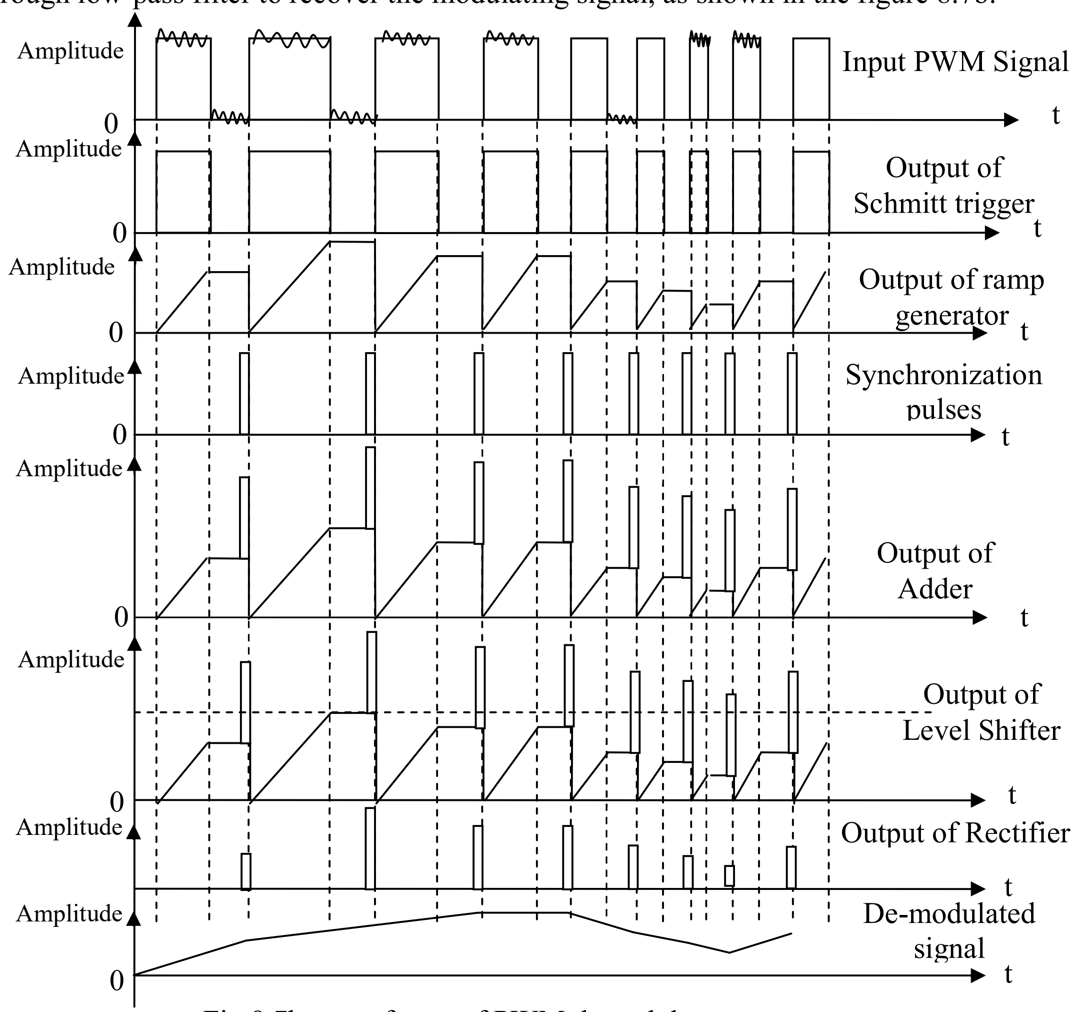


Fig 8.7b: waveforms of PWM demodulator

ADVANTAGES OF PWM:

1. noise is less
2. Signal and noise separation is very easy
3. PWM communication does not require synchronization between transmitter and receiver.

DISADVANTAGES OF PWM:

1. It requires larger transmission power.
2. Large bandwidth is required for the PWM communication as compared to PAM.

GENERATION OF PPM: The process of changing the position of the train of pulse in according to the amplitude of modulating signal at the time of sampling is called pulse position modulation. Figure 8.8a shows the block diagram to generate pulse width modulated wave.

Figure 8.8a shows the PPM generator. It consists of differentiator and a monostable multivibrator. The input to the differentiator is a PWM waveform. The differentiator generates positive and negative spikes corresponding to leading and trailing edges of the PWM waveform. Diode D_1 is used to bypass the positive spikes. The negative spikes are used to trigger the monostable multivibrator. The monostable multivibrator then generates the pulses of same width and amplitude with reference to trigger to give pulse position modulated waveform, as shown in the figure 8.8b.

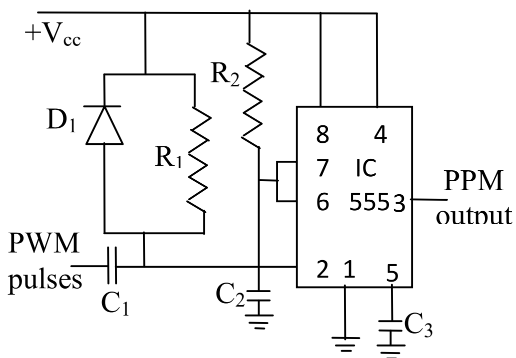


Fig 8.8a: Generation of PPM Signal

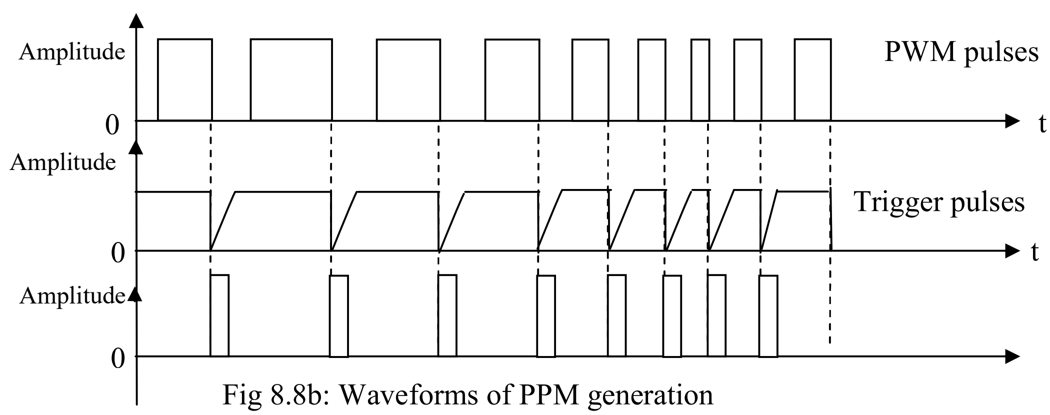


Fig 8.8b: Waveforms of PPM generation

DEMODULATION OF PPM:

In pulse position de-modulation, it is required to convert the received pulses that vary in position into pulses that vary in length. The block diagram of PPM demodulator is shown in figure 8.9a.

As shown in figure 8.9a, flip-flop circuit is set or turned 'ON' giving high output when the reference pulse arrives. This reference pulse is generated by reference pulse generator of the receiver with the synchronization signal from the transmitter. The flip-flop circuit is reset or turned 'OFF' giving low output at the leading edge of the position modulated pulse. This repeats and we get PWM pulses at the output of the flip-flop.

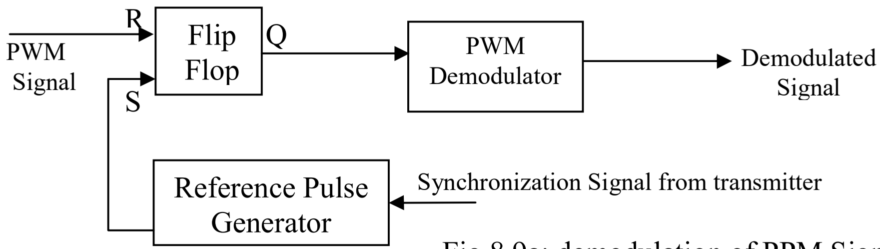


Fig 8.9a: demodulation of PPM Signal

The PWM pulses are then demodulated by PWM demodulator to get original modulating signal. The waveforms of PPM demodulation is shown in figure 8.9b.

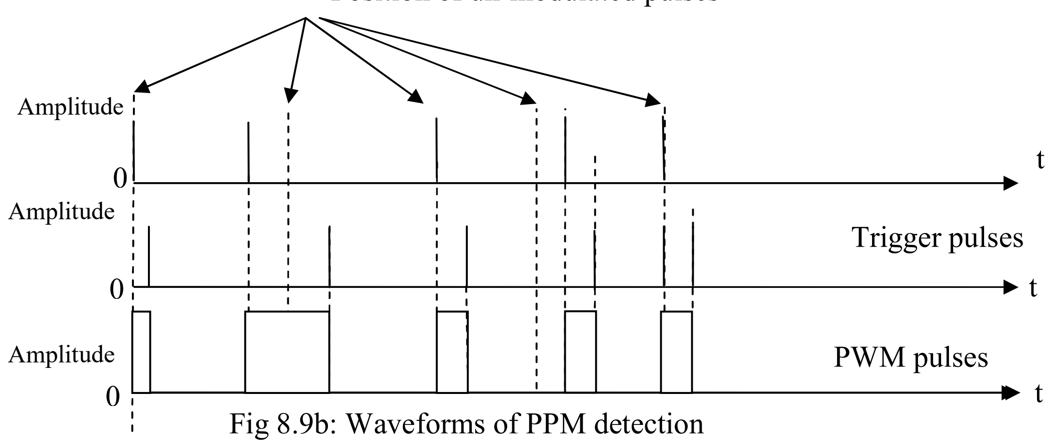


Fig 8.9b: Waveforms of PPM detection

ADVANTAGES OF PPM:
1. Noise is less.
2. Signal and noise separation is very easy.
3. Transmission power for each pulse is same.

DISADVANTAGES OF PPM:
1. Synchronization between transmitter and receiver is required.
2. Large bandwidth is required as compared to PAM.

PERFORMANCE COMPARISON OF VARIOUS PULSE ANALOG MODULATION METHODS:

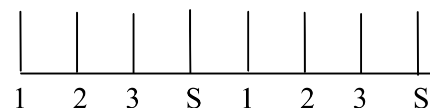
S.NO.	PAM	PWM	PPM
1			
2.	Amplitude of the pulse is proportional to the amplitude of the modulating signal.	Width of the pulse is proportional to amplitude of modulating signal.	The relative position of the pulse is proportional to the amplitude of modulating signal.
3	Bandwidth of the transmission channel depends on width of the pulse.	Bandwidth of the transmission channel depends on rise time of the pulse.	Bandwidth of the transmission channel depends on rising time of the pulse.
4	The instantaneous power of the transmitter varies.	The instantaneous power of the transmitter varies	The instantaneous power of the transmitter remains constant.
5	Noise interference is high.	Noise, interference is minimum	Noise, interference is minimum
6	Similar to amplitude modulation	similar to frequency modulation	similar to phase modulation.

SYNCHRONIZATION IN PULSE MODULATION:

Most pulse modulation systems require synchronization of the receiver to the transmitter. Generally start stop method of synchronization is used. We maintain synchronization on a per frame bases. This method involves transmitting some information in addition to the message bearing pulses, to serve as a time mark with in each frame interval so that certain gates in the receiver structure may be made to open and close at the appropriate instant of time. In some cases the necessary time mark is established by transmitting a distinctive marker per frame, where as in other cases it is established by omitting a pulse in that particular time slot. When markers are used, they must differ from the message bearing pulses in some recognizable fashion.

In PAM system the marker pulse may be identified by making its amplitude more than that of all possible message pulses as shown in figure 8.10a for a PAM system involving three independent message sources. Figure 8.10b shows the sampling times of the message sources and the times of synchronization or marker pulses. Such a marker can be located at the receiver by applying the received pulses to a slicer. With a slicing level that is just in excess of the maximum amplitude of the message pulses so that these pulses produce zero output. An ideal slicer has the property that its output is zero whenever the input exceeds this level as shown in figure 8.11. The pulses observed at the slicer output will thus be due to the markers only.

Fig 8.10a: sampling times of message sources 1, 2 and 3 and times of synchronizing pulses



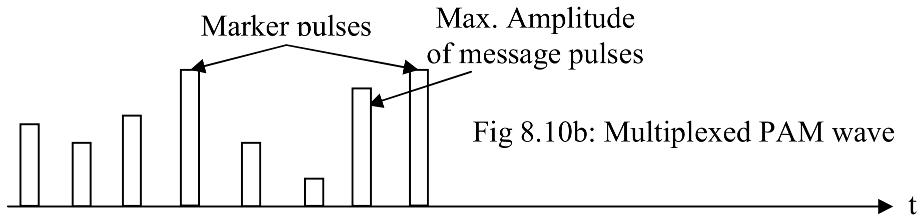


Fig 8.10b: Multiplexed PAM wave

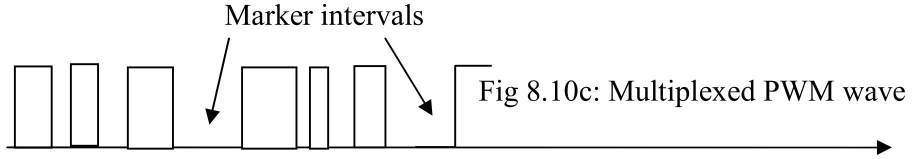


Fig 8.10c: Multiplexed PWM wave

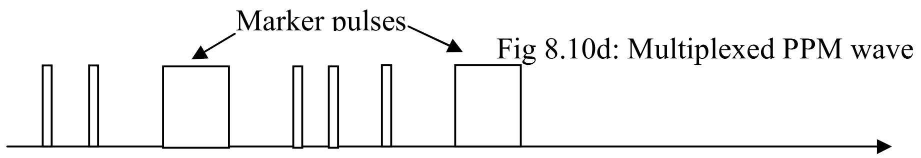


Fig 8.10d: Multiplexed PPM wave

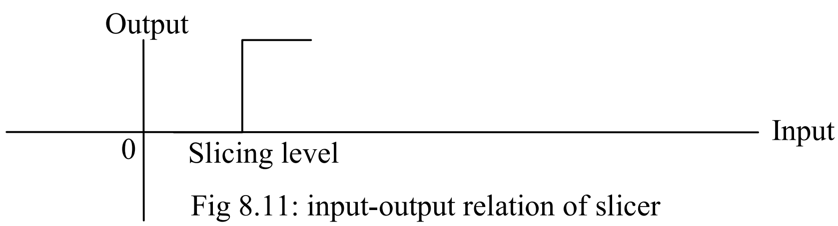


Fig 8.11: input-output relation of slicer

In PWM systems the marker may be identified by omitting a pulse as in figure 8.10c. One method of identifying such a marker in the receiver is to utilize the charging time of a simple resistor capacitor circuit to measure the duration of the intervals between PWM pulses. The time constant of the circuit is chosen so that, during a marker interval, the voltage across the capacitor rises to a value considerable higher than that during the normal charging interval. Thus, by applying the output of the circuit to a slicer with an appropriate slicing level, the presence of a marker is detected.

In a PPM system, the marker pulse may be identified by making its duration several times longer than that of the message pulses, as shown in figure 8.10d. At the receiver, the marker pulses may be separated from the message pulses by using a procedure essentially similar to that described for the PWM system. In this case, the capacitor is charged during the time of occurrence of each pulse, and discharged during the intervening intervals. Accordingly, the voltage across the capacitor reaches its highest value during the presence of a marker pulse and the marker pulses are thereby separated from the message pulses.

SPECTRA OF PDM AND PPM WAVES:

The spectral analysis of a PDM or PPM wave is complicated. We present here only a qualitative description of the spectra of PDM and PPM waves. Let Ts denote the time separation between the leading edges of duration modulated pulses obtained by natural sampling, with the modulation superimposed on the trailing edges. Then assuming a sinusoidal modulating wave of frequency f_m, we find that the spectrum of a naturally sampled PDM wave consists of the following components:

1. A dc component equal to the average value of the pulses.
2. Sinusoidal components of frequencies equal to integer multiples of 1/Ts corresponding to spectral lines at ± n/Ts, where n= 1, 2, 3… These sinusoidal components as well as the dc component are by the un-modulated pulse train which may be regarded as the carrier of the PDM wave.
3. A sinusoidal component to frequency f_m and in phase with the modulating wave, corresponding to spectral lines at $\pm f_m$.
4. Sinusoidal components of frequencies equal to (n/Ts) ± L f_m, where n, L=1, 2, 3… corresponding to pairs of side frequencies centered around each spectral line of the un modulated pulse train, except the dc component. These components represent the cross-modulation products between the sinusoidal modulation and sampling frequencies.

The message signal may be recovered by passing the PDM wave through a low pass filter. However, the reconstruction is accomplished with a certain amount of distortion caused by the cross modulation products that fall in the signal band. The frequencies of the important in band distortion components are $(1/T_s)-2f_m$, $(1/T_s)-3f_m$, and so on. To prevent undue distortion of the reconstructed message signal, it is necessary to restrict the maximum excursion of the trailing edge of a duration-modulated pulse. The output of the low pass reconstruction filter contains not only the desired message wave, but also its harmonics. With natural sampling these harmonics are missing. As with natural sampling, the other in band distortion products of the form (1/Ts) - nf_m are present. Thus we may expect a net deterioration of quality in the reconstructed message signal when the sampling is uniform instead of natural.

In the case of a PPM wave, each pulse has a very small duration compared with the sampling interval T_s, so that it may be approximated as an impulse. Then it turns out that the spectrum of a PPM wave obtained by natural sampling, and with a sinusoidal modulating wave, is similar in form to that of a PDM wave, except that it contains a component proportional to the derivative of the modulating wave rather than the modulating wave itself. Thus we may demodulate a PPM wave by passing it through a low pass filter and then integrating it to restore the wanted signal component to its original waveform. Greater signal amplitude with less distortion can be obtained at the receiver output.

COMPARISON OF SAMPLING TECHNIQUES OF PAM

S.No.	Natural sampling	Flat top sampling
1	It uses chopping principle of sampling	It uses sample and hold circuit Principle
2.	This method is used practically	This method is used practically.
3.	Sampling rate satisfies Nyquist criteria	Sampling rate satisfies Nyquist criteria
4	Noise interference is minimum	Noise interference is maximum.

CROSS TALK:

The interference of the adjacent channels or overlapping of information between adjacent channels is called crosstalk. For faithful communication cross talk must be avoided. In TDM cross talk may occur due to insufficient transmission bandwidth to preserve the shape of the TDM pulses. In FDM the crosstalk may occur when frequency response of filter is not sharp enough. To eliminate or to reduce cross talk a guard band is provided between the adjacent channels.

TIME DIVISION MULTIPLEXING:

In TDM, each intelligence signal to be transmitted (voice or telemetry data) is sampled sequentially and the resulting pulse code is used to modulate the carrier. The same carrier frequency is used to transmit different pulse sequentially, one after other. Each intelligence, to be transmitted, has been allotted a given time slot. Since only one signal modulates the carrier at any time, no added equipment and no increase in bandwidth is needed when multiplexing. The number of sequential channels that can be handled is limited by the time span required by any one channel pulse and the interval between samples.

Thus, in TDM, each signal occupies the entire bandwidth of the channel. However, each signal is transmitted for only a short period of time, as shown below..

Time Slot

Signal 1	Signal 2	Signal 3	Signal 4	Signal 5	Signal 1	Signal 2	Signal 3

One Frame

Time

Here five signals are time division multiplexed. Each signal is allowed to use the channel for a fixed interval of time, called time slot. The five signals use the channel sequentially one after other.

One transmission of each channel completes one cycle of operation, called a ‘frame.’ Once all the signals have been transmitted, the cycle repeats again and again, at a high rate of speed.

The concept of TDM is illustrated by the block diagram shown in figure 8.12. Each input message signal is first restricted in bandwidth by a low pass filter to remove the frequencies that are nonessential to an adequate signal representation. The low pass filter outputs are then applied to a commutator. A commutator is a rotating switch which connects the output of each channel modulator to the communication channel input in turn. The commutator is realized with electronic switches since it has to rotate at high speed. The commutator remains at each contact for an interval of time, which is the time slot allotted for each channel. Following the commutation process, the multiplexed signal is applied to a pulse modulator, the purpose of which is to transform the multiplexed signal into a form suitable for transmission over the common channel.

At the receiving end of the system, the received signal is applied to a pulse demodulator which performs the reverse operation of the pulse modulator. The narrow samples produced at the pulse demodulator output are distributed to the appropriate low pass reconstruction filters by means of a decommutator, which operates in synchronism with the commutator in the transmitter. This synchronization is essential for a satisfactory operation of the system.

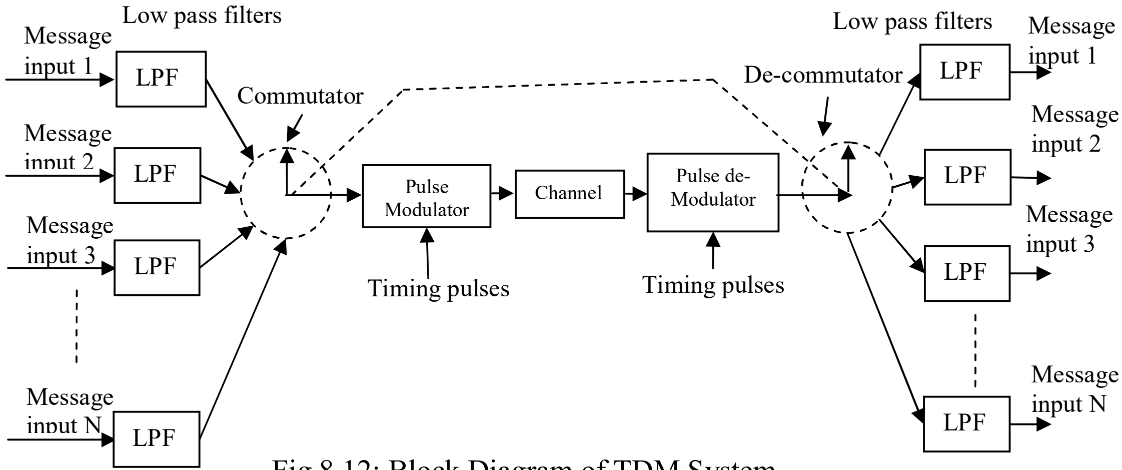


Fig 8.12: Block Diagram of TDM System

COMPARISON BETWEEN FDM AND TDM:

S.No.	FDM	TDM
1.	Signal separation is in frequency domain	Signal separation is in time domain.
2	Circuit is more complex.	Circuit is less complex
3	Cross talk is more	Cross talk is very less or nil.
4	Performance is medium	Performance is superior.
5	This is less flexible.	This is more flexible.
6	This is suitable for analog signals.	This is suitable for digital signals.
7	This is used in radio telephone, satellite communication.	This is used in digital telephone, satellite communication.
8	Synchronization between transmitter and receiver is not required	Synchronization between transmitter and receiver is required
9	It requires modulators, filters and de-modulators	It requires commutator at the transmitting end and de-commutator at the receiving end

UNIT – VI PULSE MODULATION

①

M. Ragunadh
Asst. proffessor

Sampling

The process of converting a continuous time signal into discrete-time signal is called sampling.

Sampling Techniques

There are three types of sampling techniques.

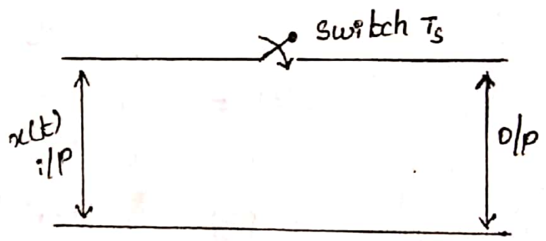
i) Instantaneous (or) ideal sampling

ii) Natural sampling.

iii) Flat top sampling

Ideal Sampling

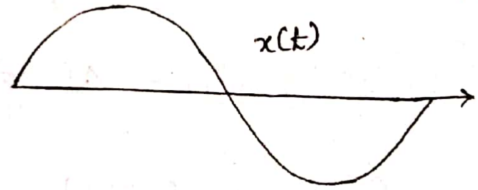
Let $x(t)$ be the msg signal and carrier be an impulse train with time period 'T_s'. (T_s must satisfy Nyquist rate $T_s \le T/2$)

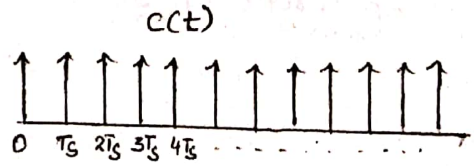


→ To produce ideal sampling we use circuit called switching sampler

→ The switch T_s is closed and opened when approaching zero.

→ Train of impulses are obtained which are having instantaneous value of $x(t)$



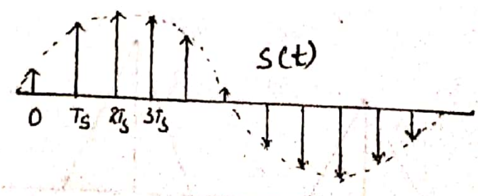


here $c(t) = \delta_{nTs}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$

$$\therefore s(t) = x(t) \cdot c(t)$$

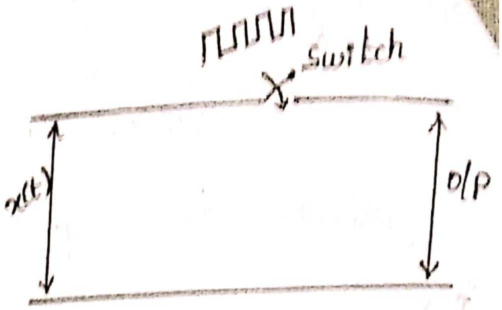
$$= \sum_{n=-\infty}^{\infty} x(t)\, \delta(t-nT_s)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)$$



Natural Sampling or Natural Sampled PAM

Let $x(t)$ be message signal and carrier be an rectangular pulse train with time period T_s



→ To produce natural sampling we use a circuit called natural sampler

→ The switch is closed for 'τ' and opened upto to 'T_s'. Train of pulses are obtained at output having instantaneous value of $x(t)$.

here $c(t) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} c_n e^{j2\pi f_s n t}$

where $c_n = \text{sinc}\left(\frac{nT}{T_s}\right)$

$\therefore s(t) = x(t) \cdot c(t)$

$= x(t) \cdot \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) e^{j2\pi n f_s t}$

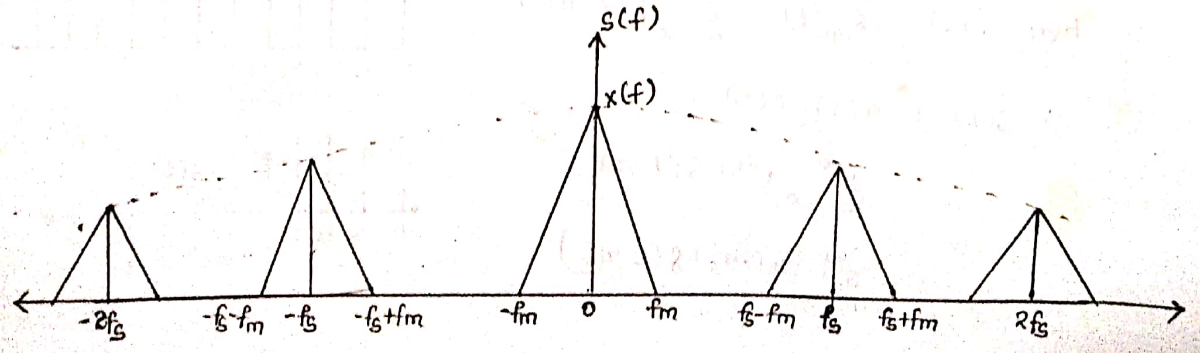
$s(t) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) e^{j2\pi n f_s t} x(t)$

Taking fourier transform on both sides

$S(f) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) F\left[x(t) e^{j2\pi n f_s t}\right]$ (∵ by frequency shifting property)

$S(f) = \frac{AT}{T_s} \sum_{n=-\infty}^{\infty} \text{sinc}\left(\frac{nT}{T_s}\right) X(f - nf_s)$

Spectrum of $S(f)$

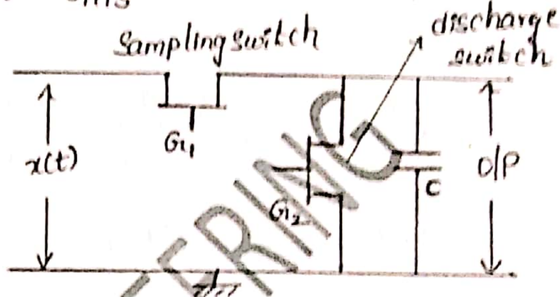


Flat top Sampling (or) Pulse amplitude modulation (or) Flat top PAM (or) Generation of PAM

(2)

In flat top PAM, the top of the samples remain constant and equal to instantaneous value of $x(t)$.

Let $x(t)$ be the message signal and $\delta_{nT_s}(t)$ be the impulse train.



Working of circuit

Sample and hold consists of two gates and a capacitor. The gate G_1 is closed for short duration, the capacitor 'c' charges upto peak value of $x(t)$.

Now G_1 is opened and capacitor 'c' holds the charge. The discharge switch is closed i.e 'G_2' due to capacitor discharges to zero volts.

Analysis

Now $\delta_{nT_s}(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT_s)$

$$s(t) = x(t)\, \delta_{nT_s}(t)$$

$$s(t) = \sum_{n=-\infty}^{\infty} x(t)\, \delta(t-nT_s)$$

$$s(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, \delta(t-nT_s)$$

> Starting edge of the pulse represents instantaneous value of $x(t)$

> To obtain flat-top PAM convolution of instantaneous sample and a pulse $h(t)$.

$$\therefore\ g(t) = s(t) * h(t)$$

$$\therefore\ g(t) = \int_{-\infty}^{\infty} s(\tau)\, h(t-\tau)\, d\tau$$

but $s(\tau) = \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(\tau - nT_s)$

$$g(t) = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} x(nT_s)\,\delta(\tau-nT_s)\, h(t-\tau)\, d\tau$$

$$g(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \int_{-\infty}^{\infty} \underbrace{\delta(\tau-nT_s)}_{\delta(t-t_0)}\, \underbrace{h(t-\tau)}_{f(t)}\, d\tau$$

since $\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0) = f(t_0)$

$$\therefore\ g(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\, h(t-nT_s)$$

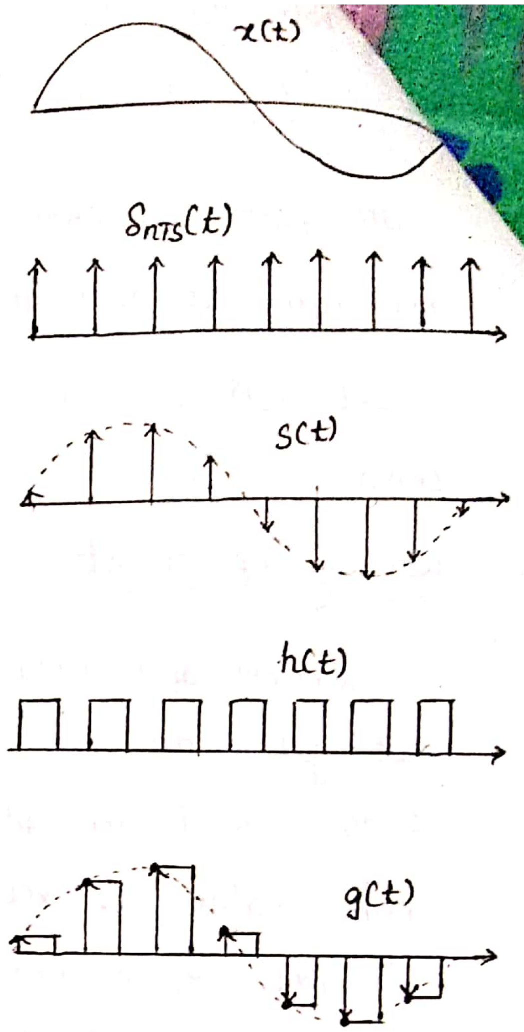
The above equation represent flat top in time domain. Taking fourier transform on both sides

$$g(t) = s(t) * h(t)$$

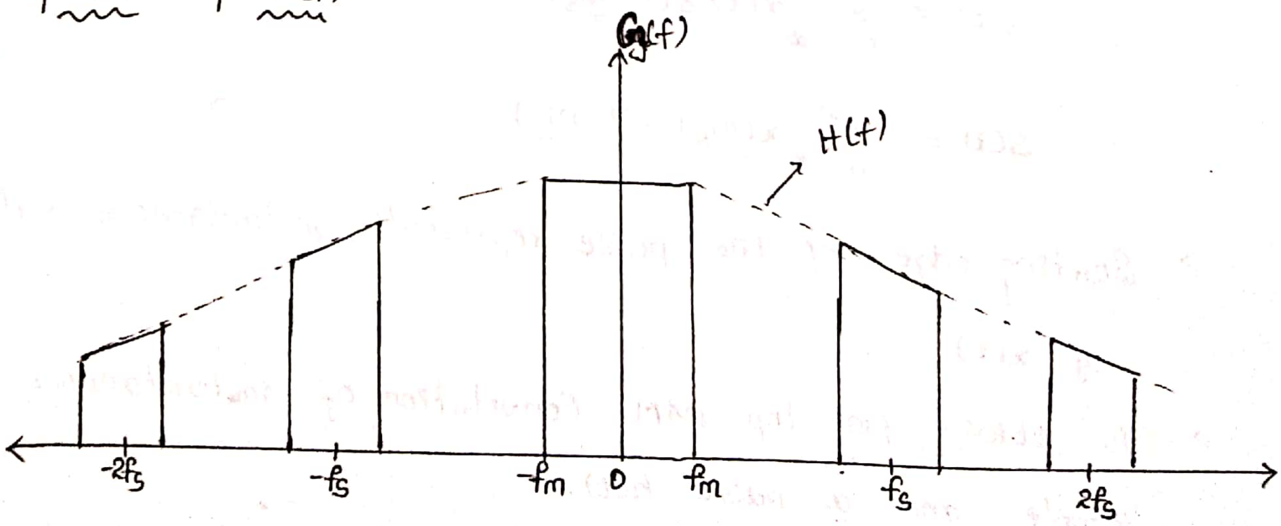
$$G(f) = S(f) \cdot H(f)$$

W.K.T $S(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s)$

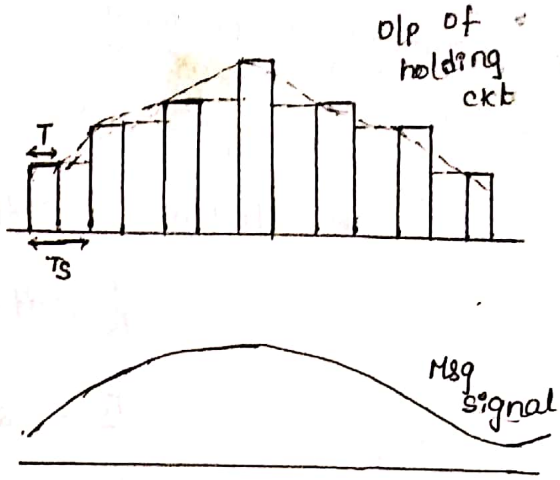
$$\therefore\ G(f) = f_s \sum_{n=-\infty}^{\infty} X(f-nf_s) \cdot H(f)$$



Spectrum of G(f)



Capacitor 'c' is charged to pulse amplitude value and it holds this value between two pulses.



> After holding circuit, output is applied to low pass filter. The output is smoothened in low pass filter.

Drawbacks of PAM

* B.W required for transmission of PAM signal is very large in comparison to maximum frequency present in modulating signal.
* Since PAM pulses varied in accordance with modulating signal. Therefore interference of noise is maximum.
* Amplitude of PAM signal varies, this also varies peak power required by transmitter.

PULSE TIME MODULATION

There are two types of PTM modulation
1. PWM
2. PPM

1. PWM (Pulse width modulation)

It is also known as pulse duration modulation. The width of the modulated pulses varied in accordance with amplitude of modulating signal

Bandwidth of PAM

$T << T_s \rightarrow (1)$

we follow $f_s \geq 2f_m$

$$\frac{1}{T_s} \geq 2f_m$$

$$T_s \leq \frac{1}{2f_m} \rightarrow (2)$$

$$f_{max} = \frac{1}{T+T} = \frac{1}{2T}$$

$$B.W \geq f_{max}$$

$$BW \geq \frac{1}{2T} \rightarrow (3)$$

from (1) and (2) $\quad T << T_s \leq \frac{1}{2f_m}$

$$T << \frac{1}{2f_m}$$

$$\frac{1}{T} >> 2f_m$$

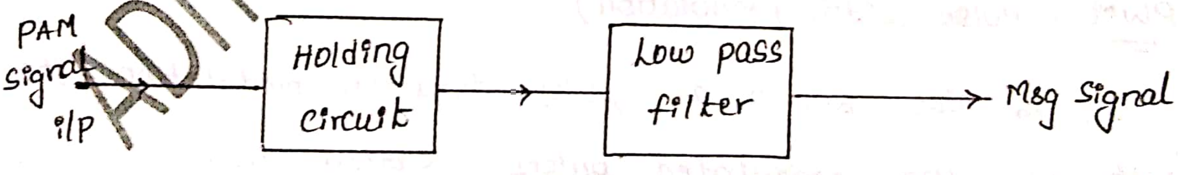
$$\frac{1}{2T} >> f_m \rightarrow (4)$$

from (3) and (4)

$$BW \geq \frac{1}{2T} >> f_m$$

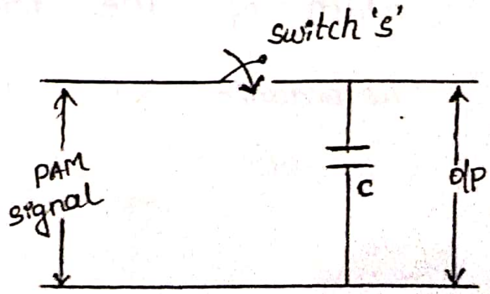
$$\boxed{BW >> f_m}$$

Demodulation of PAM



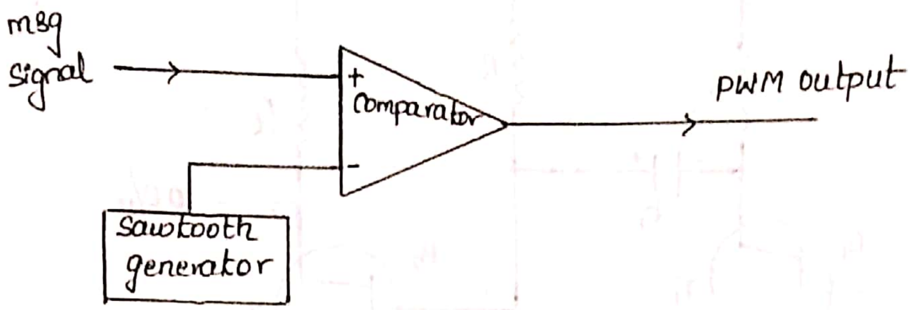
Working

> The switch 's' is closed after the arrival of the pulse and it is opened at the end of the pulse.

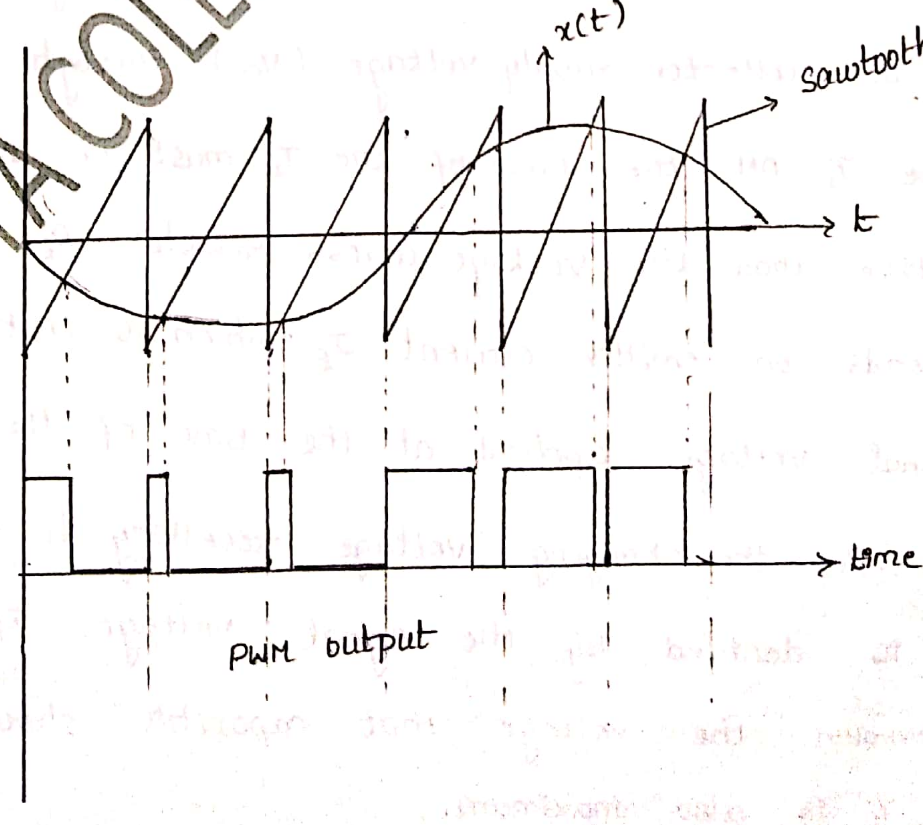


Holding circuit

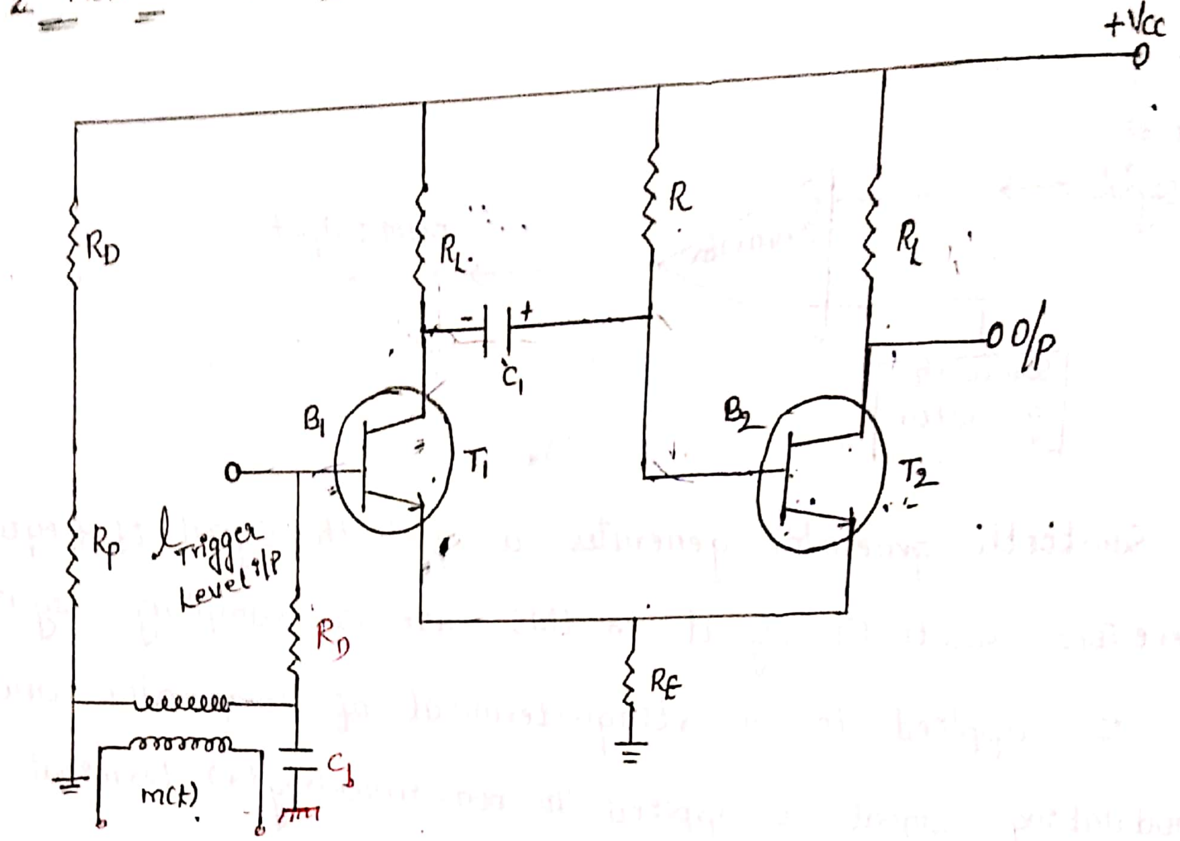
Generation of PWM



A Sawtooth generator generates a sawtooth signal of frequency fs. There fore sawtooth signal in this case is sampling signal, it is applied to inverting(-) terminal of comparator and modulating signal is applied to non-inverting(+) terminal of same comparator. Whenever x(t) amplitude is higher than sawtooth signal. The comparator output will be high otherwise output is low, The resultant output waveform is PWM wave form.



2nd Method of PWM generation

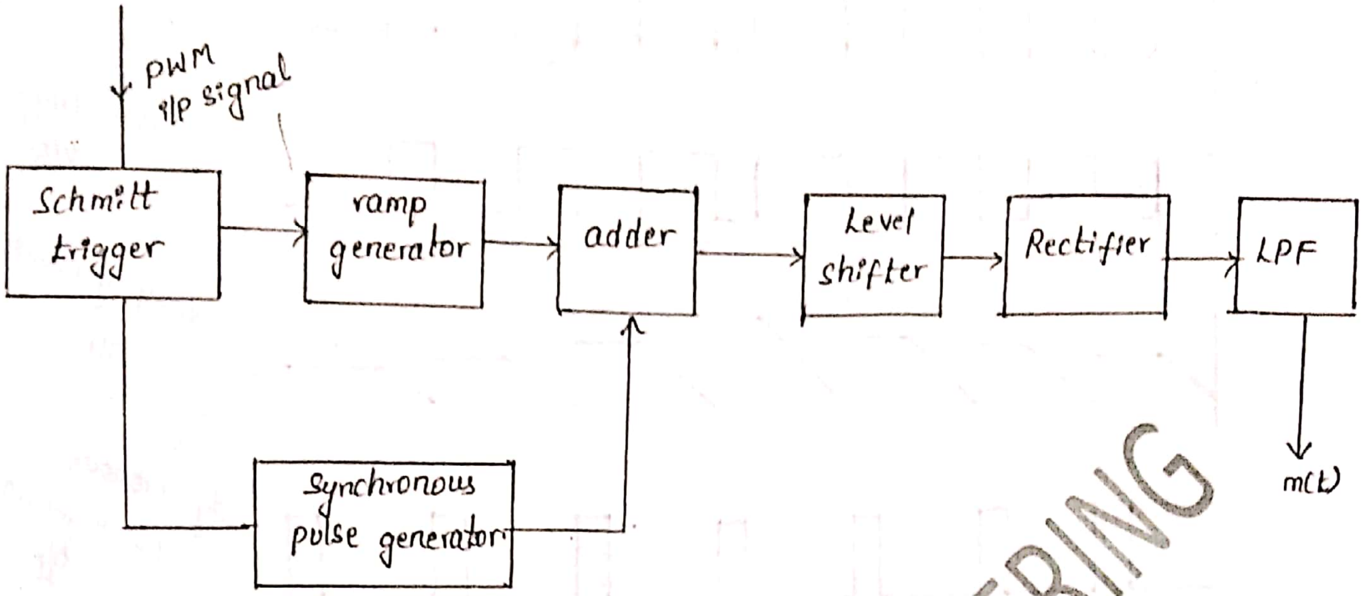


Operation

The output is obtained when T_2 is OFF. Initially T_1 is OFF and T_2 is ON. The positive going trigger pulse at B_1 switches T_1 ON. Because of this, the voltage at C_1 falls. As a result, voltage at B_2 also falls and T_2 is switched OFF, C begins to charge up to the collector supply voltage (Vcc) through resistor 'R'. To make T_2 ON, the base of the T_2 must be slightly more positive than the voltage across resistor R_E. This voltage depends on emitter current I_E which is controlled by the signal voltage applied at the base of the transistor T_1. Therefore the changing voltage necessary to turn OFF transistor T_2 is decided by the signal voltage. If signal voltage is maximum the voltage that capacitor should charge to turn ON T_2 is also maximum.

Demodulation of PWM

(5)

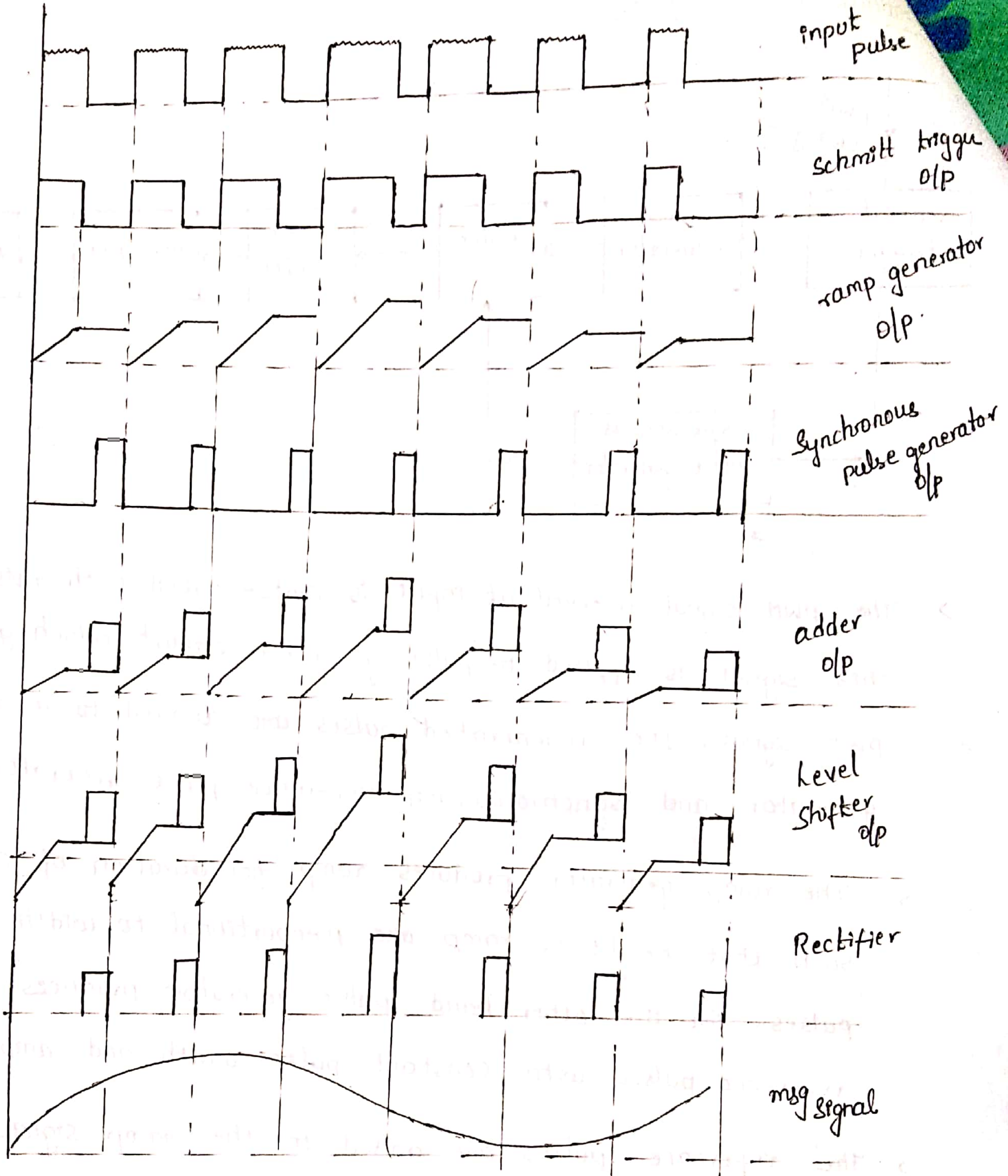


> The PWM signal received at input is contaminated with noise. This signal is applied to pulse generator circuit which generates PWM signal. The regenerated pulses are applied to a ramp generator and synchronization reference pulse generator.

> The ramp generator produces ramp for duration of pulses, such that height of ramp are proportional to width of pulses. On the other hand, pulse generator produces reference pulses with constant pulse width and amplitude.

> The reference pulses are added to the ramp signal. Then output of adder is clipped off in clipper ckt, a Low pass filter is used to recover the modulating signal back from the PAM signal.

> The waveform for the circuits shown below.



Advantages

- Less effect of noise and good noise immunity
- synchronization between transmitter & receiver is not essential

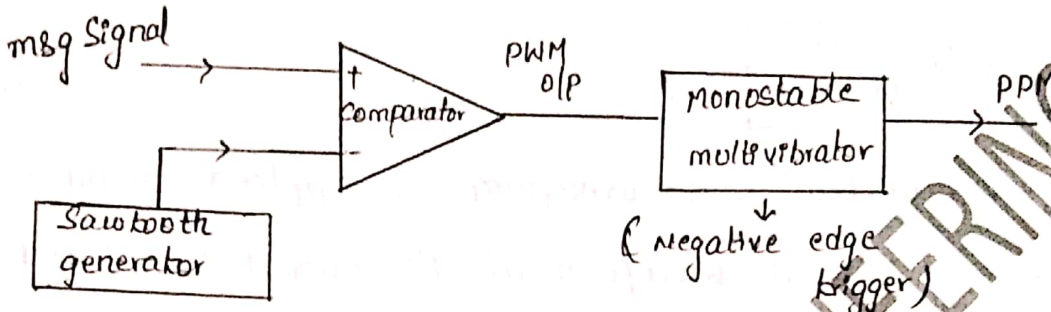
Disadvantages

- Due to variable pulse width, pulses have variable power contents, hence transmission must be powerful to handle maximum pulse width.
- Bandwidth required for PWM is large compared to PAM.

PULSE POSITION MODULATION (PPM)

The position of each pulse is varied in accordance with amplitude of sampled value of message signal is called PPM.

Generation of PPM



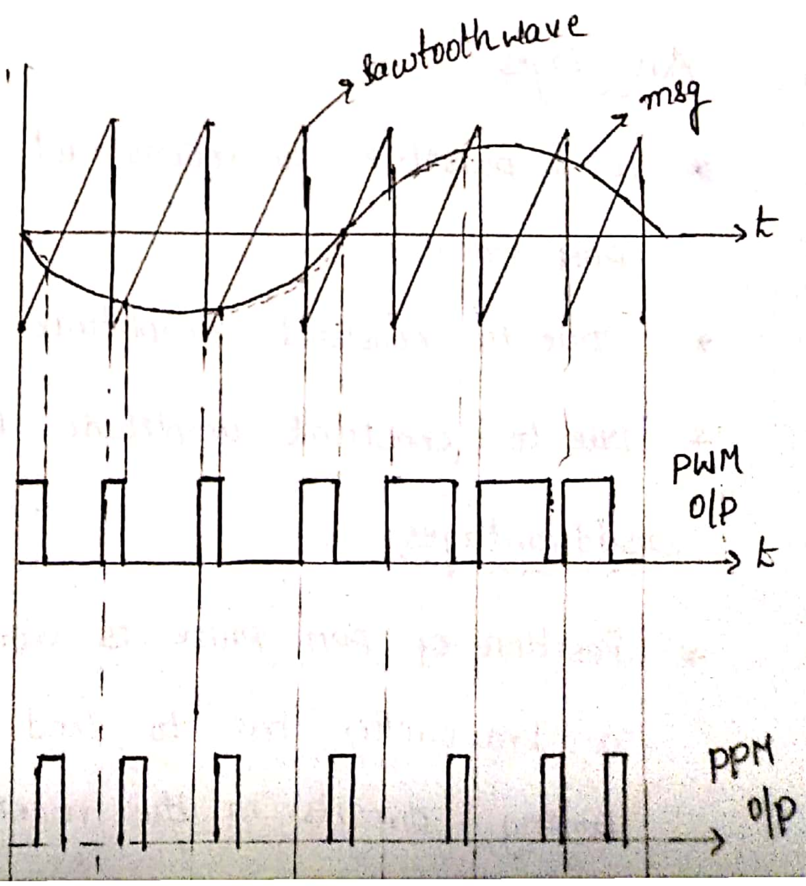
> PPM generator consists of comparator and monostable multivibrator

> The sawtooth generator generates a sawtooth signal and is applied to inverting(-) terminal of comparator and modulating signal to non-inverting(+) terminal.

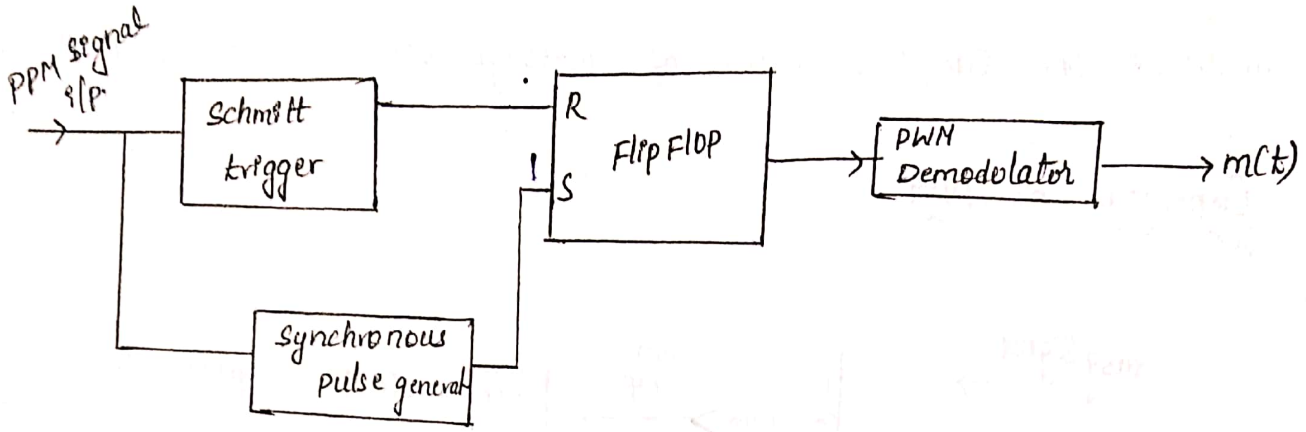
> The comparator output is PWM output and when it is given to monostable multivibrator with negative edge trigger.

> The pulses are obtained at the negative edge of PWM output.

> The waveforms of PPM are shown in here.



Demodulation of PPM



The noise corrupted PPM waveform is applied to pulse generator it develops a pulsed waveform at its output of fixed duration and applies these pulses to reset pin (R) of flip flop.. A fixed reference pulse is generated from the incoming PPM waveform and thus reference pulse is applied to set pin (S) of flip flop

Due to this Reset & set, we get PWM signal and can be demodulated using PWM demodulator we get original msg signal.

Advantages

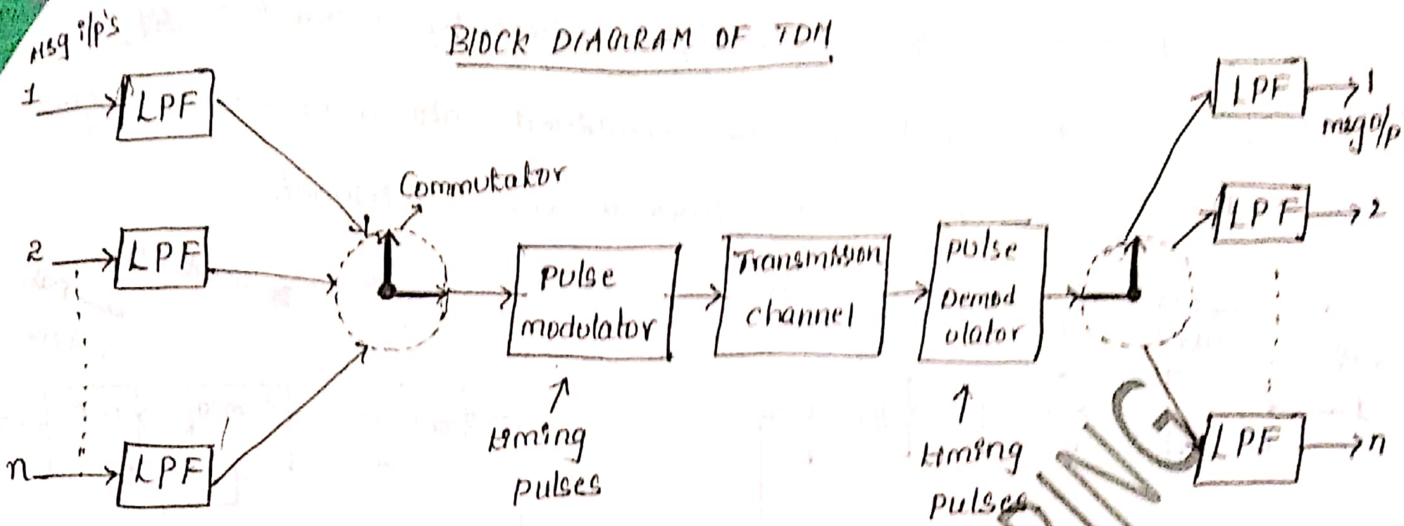
* It is possible to reconstruct PPM signal from noise contaminated PPM signal.
* Due to constant amplitude of PPM pulse, it has good immunity.
* Due to constant amplitude transmitted power always constant.

Disadvantages

* Position of PPM pulse is varied with respect to a reference pulse, a transmitter has to send synchronizing pulses to operate the timing circuits in the receiver without them demodulation not possible.
* Large bandwidth is required to ensure transmission of undistorted pulses.

TIME DIVISION MULTIPLEXING (TDM)

BLOCK DIAGRAM OF TDM



In this block diagram each input message signal is applied to low pass filter to remove the frequencies that are non-essential to an signal representation.

Low pass filter output is applied to Commutator which is implemented using electronic switching circuitry. The function of Commutator is

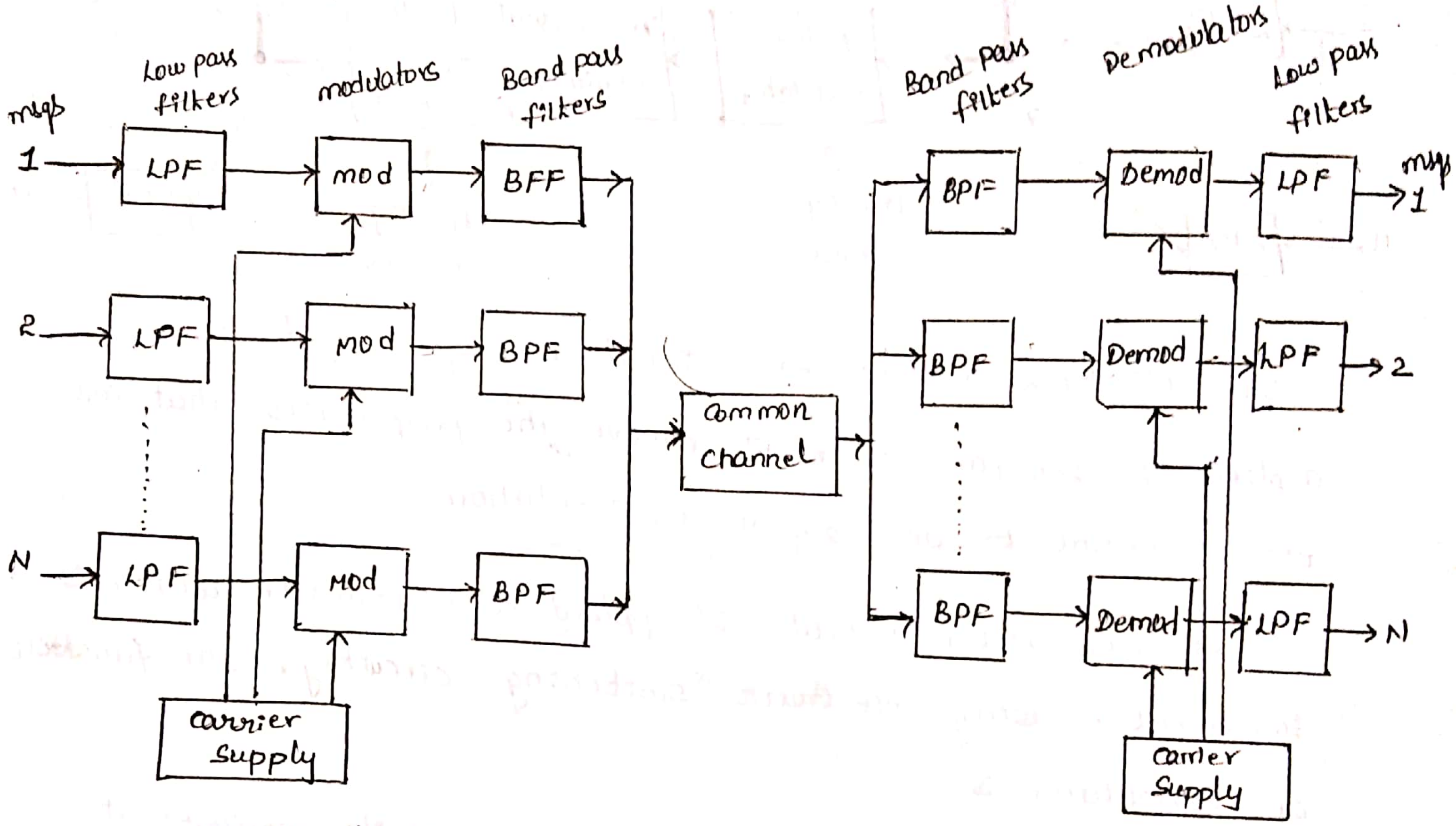
i) To take a narrow sample of each of N i/p messages at a rate $1/T_s$ that is slightly higher than $2W$, W-cut off frequency of lowpass filter.

ii) To sequentially interleave these N samples inside a sampling interval T_s

The multiplexed signal is applied to pulse modulator, the purpose of which is converting that signal into a form suitable for transmission over the common channel. at receiver pulse demodulator performs reverse operation of pulse modulator. The output at pulse demodulator are distributed to appropriate Low pass filter by means of a decommutator. **TDM** is a standard signal multiplexing formats used in telephony.

FREQUENCY DIVISION MULTIPLEXING (FDM)

Multiplexing is a technique where by a number of independent signals can be combined into a composite signal suitable for transmission over a common channel



Transmitter

Receiver

each input signal is applied to low pass filter to remove high frequency terms, that do not contribute to signal representation. The filtered signals are applied to modulators which shift the frequency range of signals as to occupy mutually exclusive frequency intervals. The necessary carrier signal for modulation is supplied by carrier supply. However most widely used modulation is SSB. The Band pass filters following the modulators are used to restrict the band of each modulated wave to its prescribed range.

The resulting band pass filter outputs are not combined in parallel to form input to common channel. at receiving a bank of B.P. filters with their inputs connected in parallel, is used to separate the message signal on frequency basis.

Finally original message signals are recovered by individual demodulators whose carrier frequency is supplied by carrier supply.

Comparision of TDM and FDM

TDM	FDM
* It is a technique for transmitting several messages on one channel by dividing time domain slots, one slot for each message.	* In this technique several messages on one channel, message signals are distributed in frequency spectrum such that they do not overlap.
* It requires commutator at the transmitting end and a distributor, working in perfect synchronization at receiving end.	* FDM requires modulators filters and demodulators.
* Perfect synchronization b/w transmitter and receiver is required.	* Synchronization between transmitter and receiver not required.
* Cross talk problem is not severe in TDM.	* FDM suffers from cross talk.

* Preferred for digital signal transmission

* It does not require any complex circuitry.

* Preferred for analog transmission

* It requires complex circuitry at transmitter & receiver.

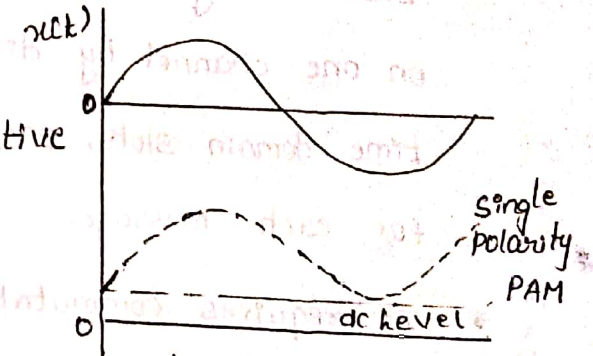
Single Polarity & Double Polarity PAM

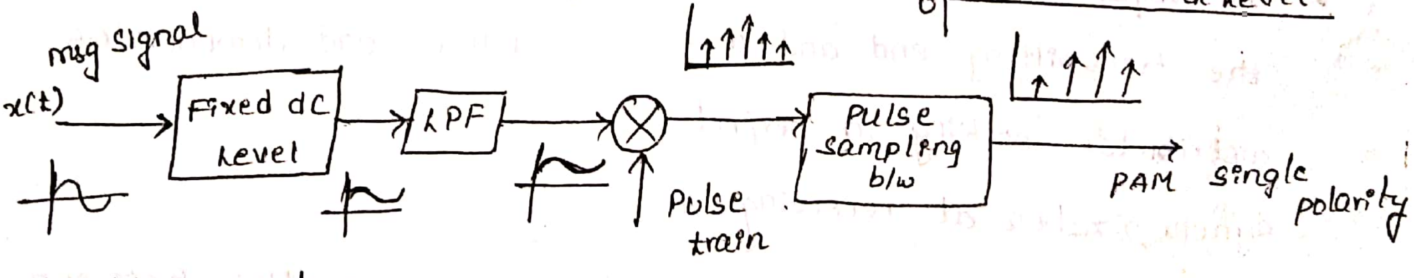
Single polarity

Single polarity PAM can be generated using a fixed DC level, low pass filter, multiplier, pulse train generator and pulse sampling network.

here PAM signal is always positive thats why called single polarity.

here we are shifting the negative value above '0' dc level.





Double polarity

Double polarity PAM signal has positive as well as negative polarity.

further refer natural PAM