# UNIT-3

## Angle Modulation

→ In amplitude modulation, we had seen the effect of slowly varying the amplitude of sinusoidal carrier wave in accordance with the baseband or information carrying or message signal i.e the carrier is amplitude modulated by the message signal m(t), hence the information content of m(t) is carried by the amplitude variations of the carrier.

→ There is another way of modulating a sinusoidal carrier wave namely "Angle modulation", in which the angle of the carrier wave is varied according to the baseband signal. The reason is any sinusoidal signal is described by 3 variables amplitude, frequency and phase.

→ Therefore, there exists a possibility of carrying the same information either by varying the frequency or phase of the carrier.

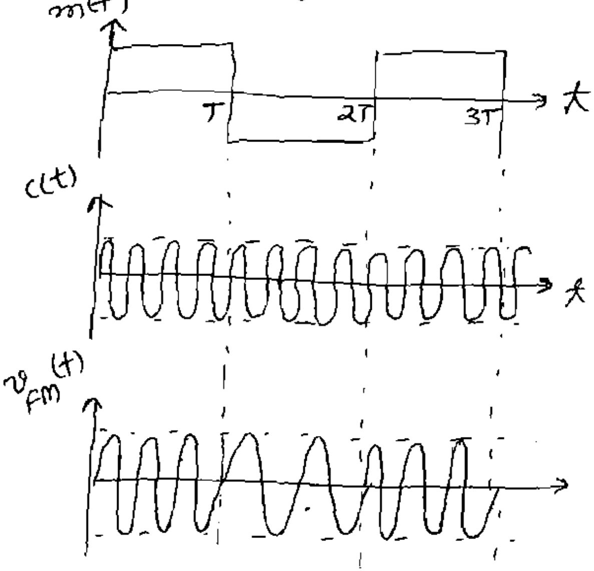



Fig (a)

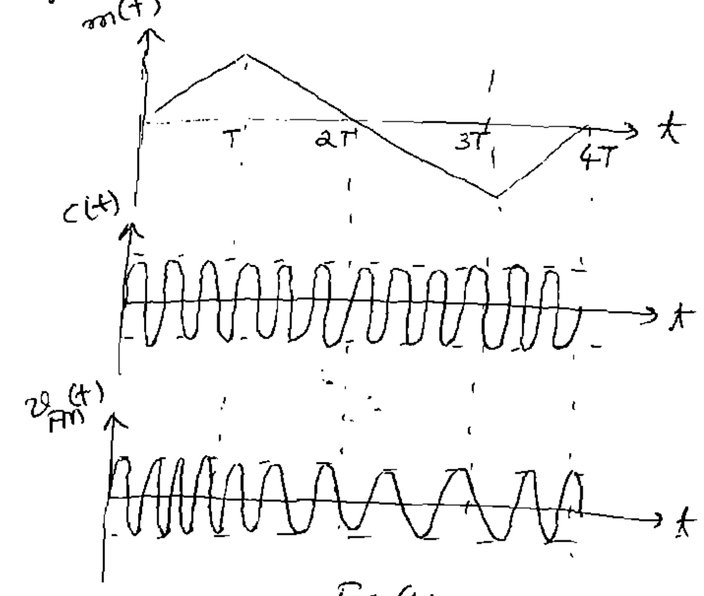



Fig (b)

from Fig (a)

→ maintaining same frequency $\omega_0$ between 0 and T as the amplitude of m(t) is constant in that interval

→ maintaining another frequency $\omega_1$ between T and 2T as the amplitude of m(t) is different in this interval

→ again the frequency is $\omega_0$ between 2T and 3T

from Fig (b):

→ we allowed a gradual variation in frequency as the amplitude is continuously varying in message m(t) at a uniform rate. Therefore, the frequency is different at every point

→ m(t) in Fig (b) can't be expressed by a simple sinusoidal expression, because, according it's definition, sinusoidal signal is a train of constant amplitude, frequency and phase.

→ Therefore, the new one should be defined as a generalized function whose amplitude, frequency and phase may vary with time.

→ The generalized sinusoidal signal is given by

$$f(t) = A \cos(\omega_c t + \phi) \rightarrow ①$$

where $A$ - amplitude

$\omega_c$ - angular frequency

$f_c$ - $\frac{\omega_c}{2\pi}$ frequency

$\phi$ - phase

$$= A \cos \theta(t) \rightarrow ②$$

I case :

$\theta(t)$ is varied linearly with time

$$\theta_i(t) = \omega_c t + \phi + k_p\, m(t)$$

where $k_p$ is constand called as phase deviation

The new carrier after modulation is given by

$$v_{PM}(t) = V_c \cos[\omega_c t + \phi + k_p\, m(t)]$$

represents the phase modulated carrier

→ the instantaneous frequency $\omega_i$ for a phase modulated carrier is given by

$$\omega_i = \frac{d\theta}{dt} = \frac{d}{dt}[\omega_c t + \phi + k_p\, m(t)]$$

$$= \omega_c + K_p \frac{d}{dt} m(t)$$

i.e the instantaneous frequency $\omega_i$ for a phase modulated carrier signal varies linearly with derivative of the modulating signal.

II case :

direct modulation of instantaneous frequency

$$\omega_i = \omega_c + K_f\, m(t)$$

$$\theta_i(t) = \int \omega_i\, dt$$

$$= \int [\omega_c + K_f\, m(t)]\, dt$$

$$= \omega_c t + K_f \int m(t)\, dt + \underset{\substack{\uparrow \\ \text{integration constant}}}{\phi}$$

comparing ① and ②, we have

$$\theta(t) = \omega_c t + \phi$$

differentiating both sides w.r.to $t$

$$\frac{d\theta(t)}{dt} = \omega_c + 0$$

$$\omega_c = \frac{d\theta}{dt} \rightarrow ③$$

→ normally, the angular frequency is constant and is given by derivative of $\theta(t)$.

→ $\frac{d\theta}{dt}$ represents the instantaneous frequency $\omega_i$ which may or may not be constant.

$$\omega_i = \frac{d\theta}{dt}$$

$$\theta = \int \omega_i \, dt$$

→ from equation Ⓐ, it is clear that, information in $m(t)$ can be sent by varying the angle of a carrier.

→ Therefore, the techniques of changing the angle of a carrier in the same manner according to the modulating / message signal $m(t)$ are called angle modulation.

$$\theta(t) = \underbrace{\omega_c t}_{\text{I term}} + \underbrace{\phi}_{\text{II term}}$$

→ by changing the I term or II term, we can change the value of angle $\theta(t)$.

where $\beta = \frac{\Delta\omega}{\omega_m}$ is called modulation index of the FM signal

it is the ratio of the frequency deviation to the modulating signal frequency.

$$\theta_i(t) = \omega_c t + \beta \sin\omega_m t$$

$\beta$ also represents the phase deviation of the FM signal. i.e maximum departure of the angle $\theta_i(t)$ from the angle $\theta(t) = \omega_c t$ of the unmodulated carrier.

$\beta$ is measured in radians.

→ finally the FM signal is given by

$$v_{FM}(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$$

(i) Single tone frequency Modulation :

→ Let the sinusoidal modulating signal be

$$m(t) = A_m \cos\omega_m t$$

Let the high frequency carrier signal be

$$c(t) = A_c \cos\omega_c t$$

the instantaneous frequency of FM signal is given by

$$\omega_i = \omega_c + K_f\, m(t)$$
$$= \omega_c + K_f A_m \cos\omega_m t$$
$$= \omega_c + \Delta\omega \cos\omega_m t$$

where $\Delta\omega$ is called frequency deviation

→ $\Delta\omega$ represents the maximum departure of the instantaneous frequency of FM signal from the carrier frequency $\omega_c$.

→ $\Delta\omega = K_f A_m$

i.e $\Delta\omega \propto A_m$

proportional to amplitude of the modulating signal but not depending on it's frequency

→ the instantaneous angle of FM signal is given by

$$\theta_i(t) = \int \omega_i \, dt$$
$$= \int (\omega_c + \Delta\omega \cos\omega_m t)\, dt$$
$$= \omega_c t + \frac{\Delta\omega}{\omega_m} \sin\omega_m t$$
$$= \omega_c t + \beta \sin\omega_m t$$

the new modulated carrier is given by

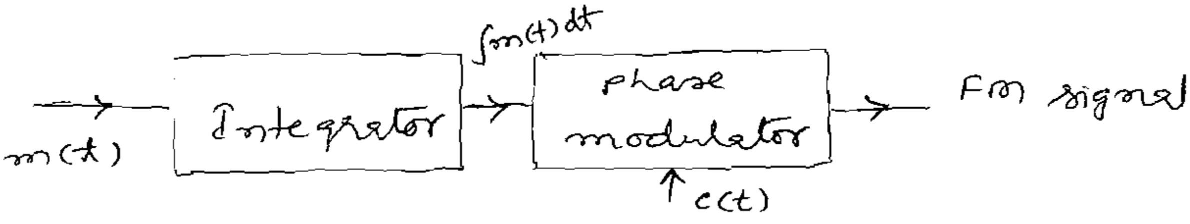
$$v_{Fm}(t) = V_c \cos[\omega_c t + k_f \int m(t)\, dt + \phi]$$

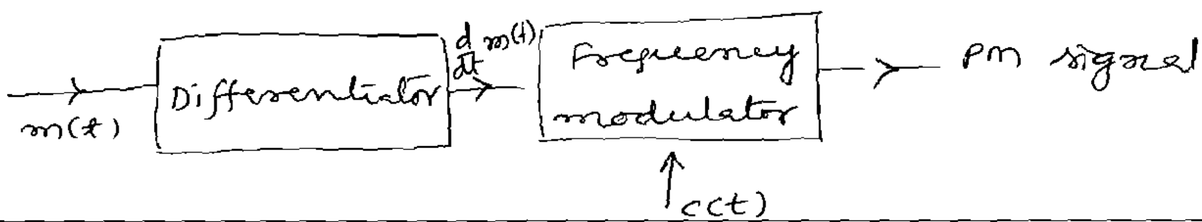
→ PM signal $v_{pm}(t) = V_c \cos[\omega_c t + \phi + k_p m(t)]$ → (I)

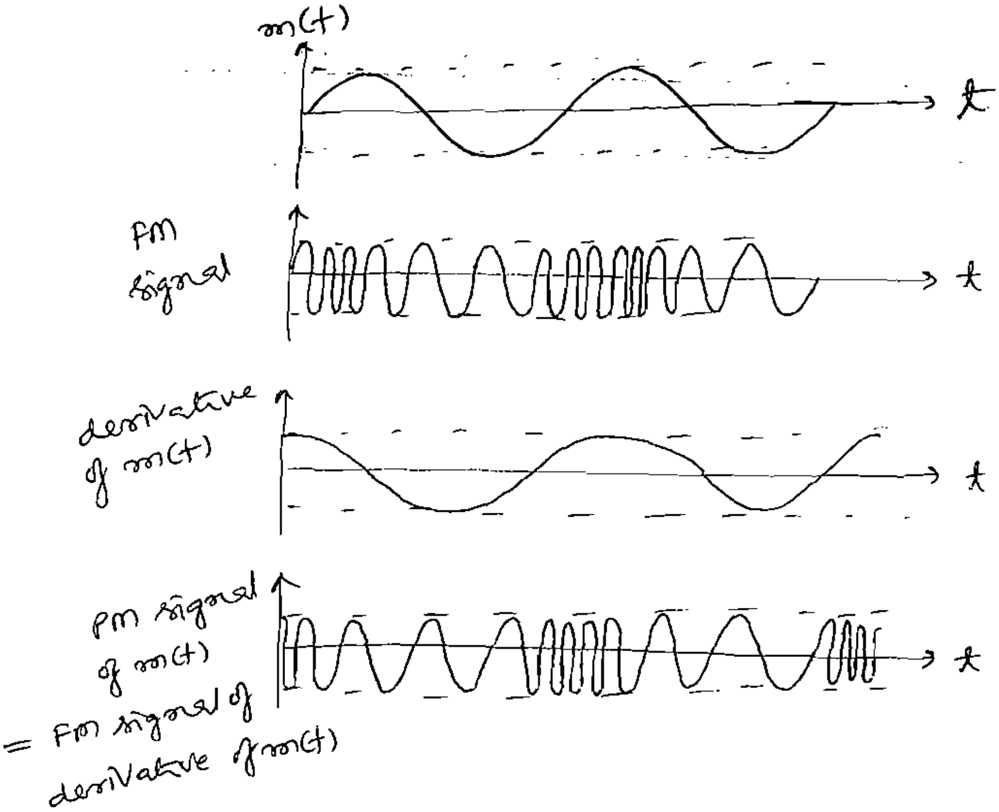
→ FM signal $v_{Fm}(t) = V_c \cos[\omega_c t + \phi + k_f \int m(t)\, dt]$ → (II)

Basic differences between PM and FM

| PM | FM |
|---|---|
| → angle is varied linearly with the modulating signal | → angle is varied linearly with the integral of the modulating signal. |
| → differentiate the modulating signal m(t) first and allow it to frequency modulation of the carrier, we obtain phase modulated signal | → Integrate the modulating signal m(t) first and allow it to phase modulation of the carrier, we obtain frequency modulated signal. |






→ from eqn (II), it is clear that, FM signal is a non linear function of the modulating signal m(t), which makes frequency modulation a non-linear modulation process.

∴ The spectrum of a FM signal is not related in a manner to that of the modulating signal. The analysis will be more difficult than amplitude modulation due to this non-linear behavior.

→ There are several methods of FM

(i) Single tone FM – NBFM

(ii) Single tone FM – WBFM

(iii) Multi tone FM –

# Narrow Band Frequency Modulation (NBFM) ⑤

$$v_{FM}(t) = A_c \cos[\omega_c t + \beta \sin\omega_m t]$$

$$= A_c \cos(\omega_c t)\cos(\beta \sin\omega_m t) - A_c \sin(\omega_c t)\sin(\beta \sin\omega_m t)$$

if $\beta$ is very small then,

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(\beta \sin\omega_m t) \cong 1$

$\sin(\beta \sin\omega_m t) \cong \beta \sin\omega_m t$

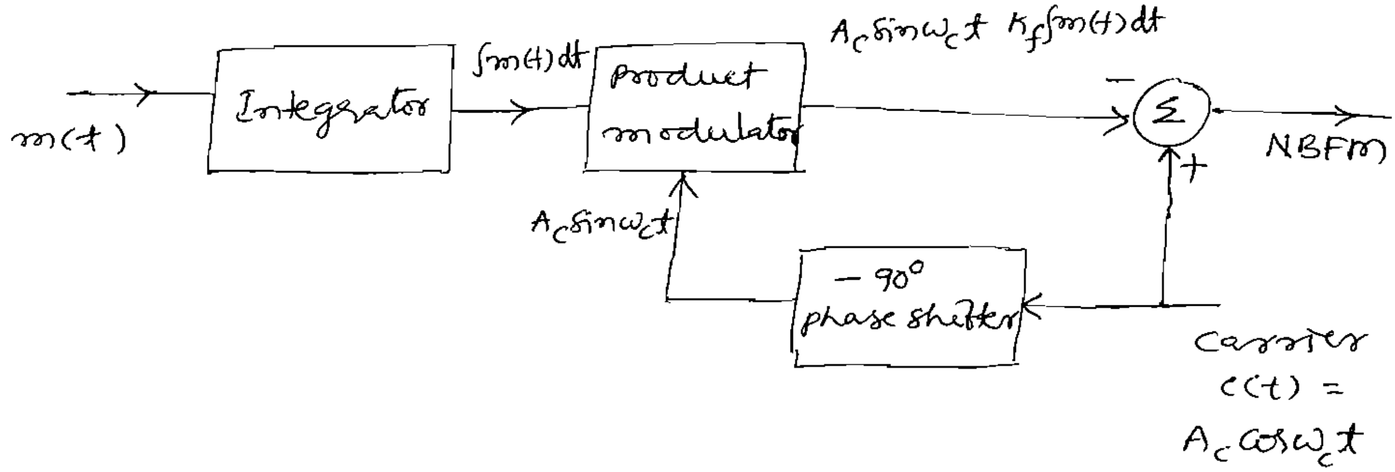
for small values of $\theta$

$\cos\theta = 1$ $\quad$ $\cos\theta = 1 - \frac{\theta^2}{2!} + \cdots$

$\sin\theta = \theta$ $\quad$ $\sin\theta = \theta - \frac{\theta^3}{3!} + \cdots$

$$= A_c \cos\omega_c t\,(1) - A_c \sin\omega_c t\,\beta \sin\omega_m t$$

$$= A_c \cos\omega_c t - \beta A_c \sin\omega_c t \sin\omega_m t \longrightarrow (A)$$




→ This arrangement produces some distortion. In FM, the amplitude should be constant and $\theta_i(t)$ should be sinusoidal with same frequency $\omega_m$, but the output from the above method, it is clear that the envelope is containing a residual amplitude modulation and that varies with time

→ the second problem is that, the instantaneous angle $\theta(t)$ is containing harmonic distortion in the form of third and higher order harmonics of the modulating frequency $\omega_m$

→ But, by selecting $\beta << 0.3$ radians, the above problems can be rectified.

$$v_{FM}(t) = A_c \cos\omega_c t - \beta A_c \sin\omega_c t \sin\omega_m t \; \frac{\times 2}{\times 2}$$

$$\because 2\sin A \sin B = \cos(A-B) - \cos(A+B)$$

$$= A_c \cos\omega_c t - \frac{\beta A_c}{2} \cos(\omega_c - \omega_m)t + \beta \frac{A_c}{2} \cos(\omega_c + \omega_m)t \rightarrow ①$$

↳ standard form of NBFM — $v_{NBFM}(t)$.

comparison with $v_{AM}(t)$

$$v_{AM}(t) = V_c \cos\omega_c t - \frac{\mu V_c}{2} \cos(\omega_c - \omega_m)t + \frac{\mu V_c}{2} \cos(\omega_c + \omega_m)t \rightarrow ②$$

$\mu$ and $\beta$ are modulation indices
$V_c$ and $A_c$ are amplitudes of carrier

→ hence eqns ① and ② are taking the same form except the sign of lower sideband

→ This form of frequency modulation is called Narrow Band Frequency modulation (NBFM) as the bandwidth is $2\omega_m$ which is same as bandwidth of AM signal.

## Wideband frequency modulation (WBFM)

→ The basic equation of frequency modulation is given by

$$s(t) = A_c \cos(\omega_c t + \beta \sin\omega_m t)$$

This FM signal is a non-periodic unless the carrier frequency $\omega_c$ is an integral multiple of the modulating signal frequency $\omega_m$.

→ Assume, the carrier frequency $\omega_c >>$ BW of FM signal, rewriting the above equation as given below

$$s(t) = R_e\{ A_c e^{j(\omega_c t + \beta \sin\omega_m t)}$$

$$= R_e\{ A_c e^{j\omega_c t + j\beta \sin\omega_m t} \}$$

$$= R_e\{ A_c e^{j\omega_c t} \cdot e^{j\beta \sin\omega_m t} \}$$

$$= R_e\{ \tilde{s}(t) e^{j\omega_c t} \}$$

where $\tilde{s}(t) = A_c e^{j\beta \sin\omega_m t}$ is the complex envelope of the signal $e^{j\omega_c t}$.

But, this complex envelope is a periodic function of time with fundamental frequency equal to modulating frequency $\omega_m$.

→ As we know that, any periodic signal can be represented as Fourier series.

∴ The complex Fourier series representation of $\tilde{s}(t)$ is as follows.

$$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{jn\omega_m t}$$

where
$$C_n = \frac{1}{T = \frac{1}{f_m}} \int_{-\frac{1}{2}f_m}^{+\frac{1}{2}f_m} \tilde{s}(t) \, e^{-jn\omega_m t} \, dt$$

$$= f_m \int_{-\frac{1}{2}f_m}^{+\frac{1}{2}f_m} A_c \, e^{j\beta \sin \omega_m t} \, e^{-jn\omega_m t} \, dt$$

$$= f_m A_c \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} e^{j\beta \sin \omega_m t - jn\omega_m t} \, dt$$

Limits

l.l $\to \omega_m(-\frac{1}{2}f_m)$

$\cancel{2}\pi \cancel{f_m} \times -\frac{1}{\cancel{2}\cancel{f_m}} = -\pi$

u.l $\to \omega_m(+\frac{1}{2}f_m) = \pi$

$\cancel{2}\pi \cancel{f_m} \left(\frac{1}{\cancel{2}\cancel{f_m}}\right)$

put $x = \omega_m t \Rightarrow x = 2\pi f_m t$

$$\frac{dx}{dt} = \omega_m$$

$$dx = \omega_m \, dt$$

$$= 2\pi f_m \, dt$$

$$dt = \frac{dx}{2\pi f_m}$$

$$f_m = \frac{x}{2\pi t}$$

$$-\frac{1}{2} f_m = \frac{x}{-4\pi t}$$

$$+\frac{1}{2} f_m = \frac{x}{4\pi t}$$

$$= \cancel{f_m} A_c \int_{-\pi}^{\pi} e^{j\beta \sin x - jnx} \, \frac{dx}{2\pi \cancel{f_m}}$$

$$= \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx$$

→ except scaling factor $A_c$, it is recognized as the $n^{th}$ order Bessel function of the first kind and argument $\beta$. ∴ it is denoted by $J_n(\beta)$.

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \, dx$$

$$C_n = A_c \, J_n(\beta)$$

$$\tilde{s}(t) = \sum_{n=\infty}^{\infty} A_c J_n(\beta)\, e^{jn\omega_m t} \quad \text{and}$$

$$s(t) = Re\left\{ \sum_{n=-\infty}^{\infty} A_c J_n(\beta)\, e^{jn\omega_m t}\, e^{j\omega_c t} \right\}$$

$$= A_c\, Re\left\{ \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j(\omega_c + n\omega_m)t} \right\}$$

after interchanging order of summation and evaluating the real part in the RHS of above equation gives

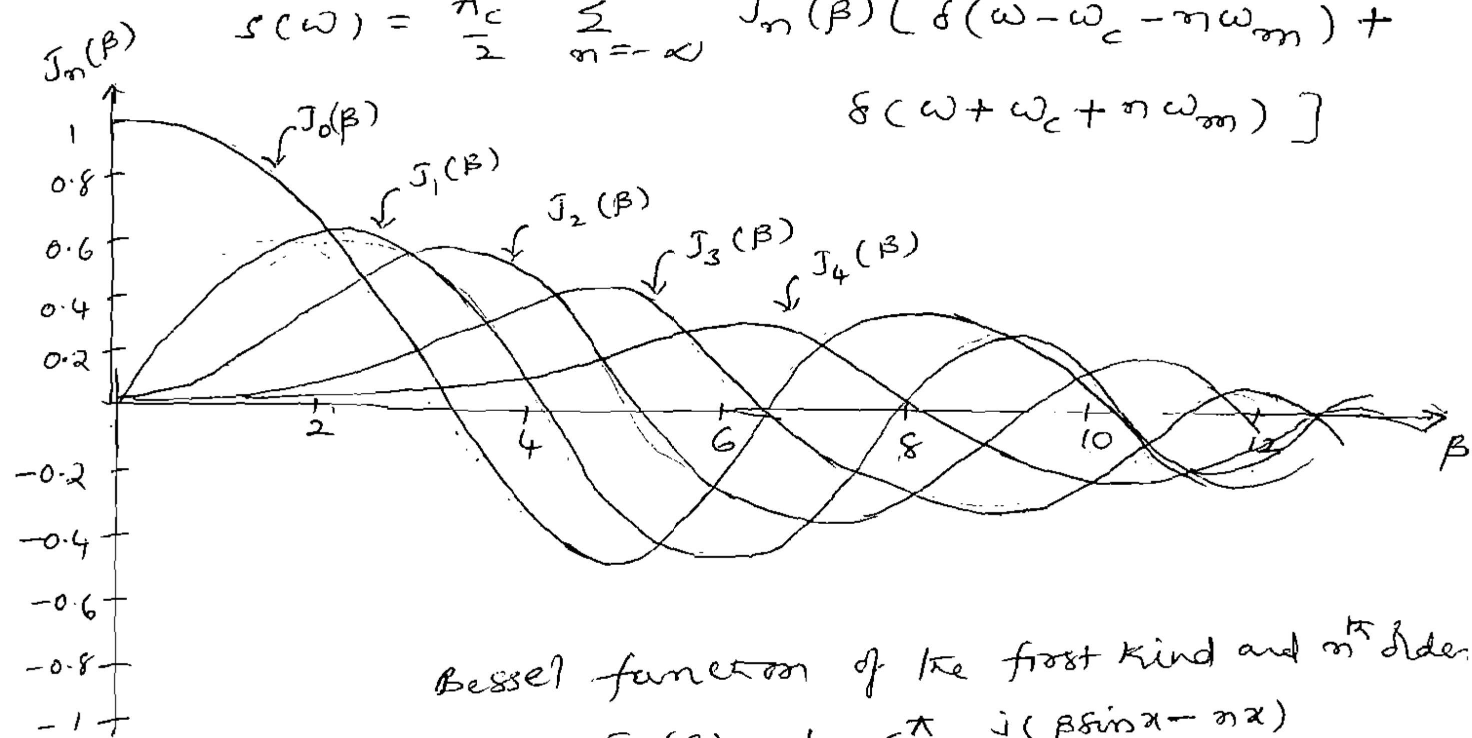
$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m) t$$

↳ standart form of WBFM $v_{WBFM}(t)$.

→ This is the required form of Fourier series representation of FM signal for single tone modulation for an arbitoary value of $\beta$.

→ The spectrum of $s(t)$ can be obtained by taking the fourier transform of on both sides

$$S(\omega) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[ \delta(\omega - \omega_c - n\omega_m) + \delta(\omega + \omega_c + n\omega_m) \right]$$




Bessel function of the first kind and $n^{th}$ order

$$J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\, dx$$

Properties of Bessel for $J_n(\beta)$

(i) $J_n(\beta) = J_{-n}(\beta)$ → for $n$ even

$J_n(\beta) = -J_{-n}(\beta)$ → for $n$ odd

$$J_n(\beta) = (-1)^n J_{-n}(\beta)$$

→ for all values of $n$

(ii) for small values of $\beta$

$J_0(\beta) = 1$

$J_1(\beta) = \beta/2$

$J_n(\beta) = 0$ for $n \geq 2$

(iii) $$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$$

→ The WBFM is given by (for $\beta$ is large value)

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(\omega_c + n\omega_m)t \quad \to ①$$

$$= A_c\{J_0(\beta)\cos\omega_c t + J_1(\beta)\cos(\omega_c+\omega_m)t + J_{-1}(\beta)\cos(\omega_c-\omega_m)t + J_2(\beta)\cos(\omega_c+2\omega_m)t + J_{-2}(\beta)\cos(\omega_c-\omega_m)t + - - - - \}$$

from property (i) of $J_n(\beta)$

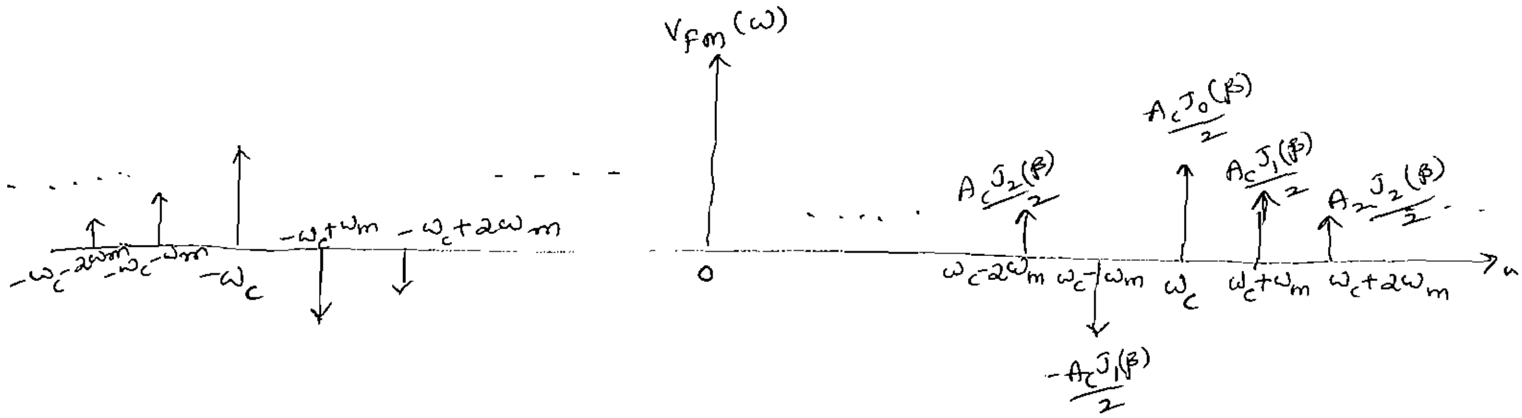
$J_1(\beta) = -J_{-1}(\beta)$

$J_2(\beta) = J_{-2}(\beta)$ and so on

$$= A_c\{J_0(\beta)\cos\omega_c t + J_1(\beta)\cos(\omega_c+\omega_m)t - J_1(\beta)\cos(\omega_c-\omega_m)t + A_c J_2(\beta)\cos(\omega_c+2\omega_m) + J_2(\beta)\cos(\omega_c+2\omega_m)t + - - - \}$$

$$= A_c J_0(\beta)\cos\omega_c t + A_c J_1(\beta)\{\cos(\omega_c+\omega_m)t - \cos(\omega_c-\omega_m)t\}$$
$$+ A_c J_2(\beta)\{\cos(\omega_c+2\omega_m)t + \cos(\omega_c-2\omega_m)t\}$$
$$+ \cdots \quad \rightarrow (2)$$

→ Thus, the frequency modulated signal contains carrier component and an infinite number of side frequency components at $\omega_c \pm \omega_m$, $\omega_c \pm 2\omega_m$ ---- an so on as shown below




$$J_n(\beta) = \left(\frac{\beta}{2}\right)^n \left[\frac{1}{n!} - \frac{(\beta/2)^2}{1!(n+1)!} + \frac{(\beta/2)^4}{2!(n+2)!} - \cdots\right] \rightarrow (3)$$

→ From eqns (1), (2) & (3), it is clear that

(1) FM has infinite number of sidebands as well as carrier and all they are separated from carrier by $\pm\omega_m$, $\pm 2\omega_m$, $\pm 3\omega_m$ ----. But, in AM there are only 3 components.

(2) The modulation index determines the how many sidebands components have significant amplitude i.e for large value of $\beta$, more number of significant sidebands are present. if $\beta$ is small, less no. of sidebands will exist.

(3) The sidebands are at equal distance from $\omega_c$ have equal amplitudes so that the sideband distribution is symmetrical about the carrier frequency.

(4) Theoritically, infinite sidebands are produced and the amplitude of each sideband is decided by the Bessel function.

(5) The presence of infinite no. of sidebands makes the bandwidth of FM infinite. However, the sidebands with negligible amplitudes are ignored, then the BW of FM becomes finite

(6) for small values of $\beta$ i.e $\beta << 1$ only the amplitudes of $J_0(\beta)$ and $J_1(\beta)$ are significant and other terms can be neglected.

This is equivalent to NBFM

(7) The amplitude of FM remains unchanged and hence the power of FM is same as that of the unmodulated carrier

(8) The total power of FM signal depends on the power of the unmodulated carrier, whereas in AM, the total power depends on the modulation index

(9) In AM, the increased modulation index increases the sideband power

In FM, the total power remains constant with increased modulation index and only the BW is increased.

## Multitone Modulation

→ modulation can be carried out with more than one message signal is called multi tone modulation.

Let us consider the message signal as

$$m(t) = A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t$$

Let the carrier signal be

$$c(t) = A_c \cos(\omega_c t + \phi)$$
$$= A_c \cos \theta$$

The frequency of carrier is changed according to instantaneous values of message signal

$\therefore$

$$\omega_i = \omega_c + K_f \, m(t)$$
$$= \omega_c + K_f (A_{m_1} \cos \omega_{m_1} t + A_{m_2} \cos \omega_{m_2} t)$$

the frequency deviation is maximum when

$\cos \omega_{m_1} t = \pm 1$ and $\cos \omega_{m_2} t = \pm 1$

the frequency deviation is proportional to the amplitude of the modulating signal, which is

$$\Delta\omega_1 = K_f A_{m_1} \quad \& \quad \Delta\omega_2 = K_f A_{m_2}$$

$$\omega_i = \omega_c + \Delta\omega_1 \cos \omega_m t + \Delta\omega_2 \cos \omega_{m_2} t$$

the instantaneous phase is given by

$$\theta_i = \int \omega_i \, dt$$
$$= \int [\omega_c + \Delta\omega_1 \cos \omega_{m_1} t + \Delta\omega_2 \cos \omega_{m_2} t] \, dt$$
$$= \omega_c t + \Delta\omega_1 \frac{\sin \omega_{m_1} t}{\omega_{m_1}} + \Delta\omega_2 \frac{\sin \omega_{m_2} t}{\omega_{m_2}}$$
$$= \omega_c t + \beta_1 \sin \omega_{m_1} t + \beta_2 \sin \omega_{m_2} t$$

where $\beta_1 = \frac{\Delta\omega_1}{\omega_{m_1}}$ & $\beta = \frac{\Delta\omega_2}{\omega_{m_2}}$

→ The frequency modulated signal is given by

$$v_{FM}(t) = A_c \cos\theta_i t$$

$$= A_c \cos[\omega_c t + \beta_1 \sin\omega_{m_1} t + \beta_2 \sin\omega_{m_2} t]$$

$$= A_c \cos(\omega_c t + (\alpha_1 + \alpha_2))$$

where $\alpha_1 = \beta_1 \sin\omega_{m_1} t$

$\alpha_2 = \beta_2 \sin\omega_{m_2} t$

$$= A_c[\cos\omega_c t \cos(\alpha_1+\alpha_2) - \sin\omega_c t \sin(\alpha_1+\alpha_2)]$$

$$= A_c[\cos\omega_c t(\cos\alpha_1 \cos\alpha_2 - \sin\alpha_1 \sin\alpha_2) - \sin\omega_c t(\sin\alpha_1 \cos\alpha_2 + \cos\alpha_1 \sin\alpha_2)] \longrightarrow (1)$$

→ Bessel functions can be used to solve eqn (1)

$$v_{FM}(t) = A_c \sum_{n=\infty}^{\infty} J_n(\alpha_1) J_n(\alpha_2) \cos(\omega_c t \pm n\omega_{m_1} t + n\omega_{m_2} t)$$

Observations:

(i) a carrier frequency component with an amplitude $J_0(\alpha_2)\, J_0(\alpha_2)\, A_c$

(ii) A set of sidebands having amplitude $J_n(\alpha_1)\, J_n(\alpha_2)$ and frequencies $(\omega_{m_1} + n\omega_{m_2})$ where $n = 1, 2, 3 - - -$

## Transmission BW of FM

→ The no. of sidebands which are having significant amplitudes 'n' produced in FM waves can be obtained from the plot of Bessel fn $J_n(\beta)$

→ for $\beta < n$, the value of $J_n(\beta)$ are negligible particularly when $\beta >> 1$, ∴ the significant sidebands produced in WBFM may be considered to be an integer approximately equal to $\beta$.

i.e $n \cong \beta$ if $\beta >> 1$

→ The USB frequencies are separated by $\omega_m$ and form a frequency span $n\omega_m$, also it is similar by LSBs.

→ ∴ transmission BW of FM signal is

$$BW = 2n\omega_m \text{ rad/sec}$$

where $n$ = no. of sidebands

$$= 2\beta\omega_m$$

$$= 2\frac{\Delta\omega}{\cancel{\omega_m}}\cancel{\omega_m}$$

$$= 2\Delta\omega \text{ rad/sec}$$

approximately BW of FM is twice the frequency deviation for $\beta >> 1$

but, for smaller values of $\beta$, it may be greater than $2\Delta\omega$

→ this is given by approximate rule for transmission BW of FM signal

$$BW = 2(\Delta\omega + \omega_m) = 2\omega_m\left(1+\frac{\Delta\omega}{\omega_m}\right)$$

$$= 2\Delta\omega\left(1+\frac{\omega_m}{\Delta\omega}\right) = 2\omega_m(1+\beta)$$

$$= 2\Delta\omega\left(1+\frac{1}{\beta}\right)$$

This empirical relation is known as "Carson's rule".

## Power Content in FM

consider the signal

$$c(t) = A_c \cos\omega_c t$$

Frequency modulated signal (FM) is given by

$$v_{FM}(t) = A_c \cos\left[\omega_c t + K_f \int m(t)\, dt\right]$$

for $m(t) = A_m \cos\omega_m t$

$$= A_c \cos\left[\omega_c t + \beta \sin\omega_m t\right]$$

where $\beta = \frac{\Delta\omega}{\omega_m} = \frac{K_f A_m}{\omega_m}$

→ ∴ The envelope of the FM signal is constant by observing the above expression.

→ The power of the signal is calculated from it's amplitude

→ In FM, amplitude is constant which is also equal to the carrier amplitude, only frequency of the carrier is varied according to modulating signal.

∴ Power of the FM signal is

$$P = \frac{A_c^2}{2} \quad \text{per unit resistor}$$

power is independent of $\beta$

$$v_{FM}(t) = A_c J_0(\beta) \cos\omega_c t +$$
$$A_c J_1(\beta) \cos(\omega_c+\omega_m)t - A_c J_1(\beta) \cos(\omega_c-\omega_m)t +$$
$$A_c J_2(\beta) \cos(\omega_c+2\omega_m)t - A_c J_2(\beta) \cos(\omega_c-2\omega_m)t +$$
$$- - - -$$

$$\text{Power} = \left(\frac{A_c J_0(\beta)}{\sqrt{2}}\right)^2 + \left(\frac{A_c J_1(\beta)}{\sqrt{2}}\right)^2 + \left(\frac{A_1 J_1(\beta)}{2}\right)^2 +$$
$$\left(\frac{A_c J_2(\beta)}{\sqrt{2}}\right)^2 + \left(\frac{A_c J_2(\beta)}{\sqrt{2}}\right)^2 + - - -$$

$$= \frac{A_c^2}{2} J_0^2(\beta) + \frac{A_c^2}{2} J_1(\beta) + \frac{A_c^2}{2} J_1(\beta) +$$
$$\frac{A_c^2}{2} J_2^2(\beta) + \frac{A_c^2}{2} J_2^2(\beta) + - - -$$

$$= \frac{A_c^2}{2}\left[J_0^2(\beta) + 2J_1^2(\beta) + 2J_2^2(\beta) + - - -\right]$$

$$= \frac{A_c^2}{2}[1] \qquad \left\langle \sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1\right.$$

$$= \frac{A_c^2}{2} \text{ per unit resistor}$$

$$= \text{carrier power}$$

→ if modulation index $\beta$ increases, then BW of FM signal increases but power transmitted by FM is not changed.

→ As modulation index $\beta$ increases, sidebands increases, thereby most of the power located in sidebands. Thus, the efficiency is improved in FM system.

Problem : 1

Determine the instantaneous frequency in Hz of each of the following signal

(a) $10 \cos(200\pi t + \pi/3)$ (b) $10 \cos(20\pi t + \pi t^2)$

(c) $\cos 200\pi t \cos(5 \sin 2\pi t) + \sin 200\pi t \sin(5 \sin 2\pi t$.

Soln : (a) $\theta(t) = 200\pi t + \pi/3$

$$\omega_i = \frac{d\theta_i}{dt} = \frac{d}{dt}(200\pi t + \pi/3)$$

$$= 200\pi = 2\pi(100) = 2\pi f$$

$$f = 100 \text{ Hz}$$

(b) $\theta(t) = 20\pi t + \pi t^2$

$$\omega_i = \frac{d\theta_i}{dt} = \frac{d}{dt}(20\pi t + \pi t^2)$$

$$= 20\pi + 2\pi t = 2\pi(10 + t)$$

$$= 2\pi f$$

$$f = (10 + t) \text{ Hz}$$

(c) $\theta_i(t) = \cos(200\pi t - 5 \sin 2\pi t)$

$$\omega_i = \frac{d}{dt}\theta_i(t) = \frac{d}{dt}(200\pi t - 5 \sin 2\pi t)$$

$$= 200\pi - 5 \cos 2\pi t \, (2\pi)$$

$$= 2\pi(100 - 5 \cos 2\pi t)$$

$$f = 100 - 5 \cos 2\pi t$$

Problem : 2

consider the angle modulated signal $s(t) = 10 \cos[10^8 \pi t + 5 \sin 2\pi 10^3 t]$. Find maximum phase deviation and frequency deviation

Soln : $s(t) = 10 \cos(10^8 \pi t + 5 \sin 2\pi 10^3 t)$

$$v_{Fm}(t) = A_c \cos(\omega_c t + \beta \sin \omega_m t)$$

$c(t) = A_c \cos\theta(t)$

$\therefore \theta(t) = \omega_c t = 10^8 \pi t$

$\theta_i(t) = \omega_c t + \beta \sin \omega_m t$

$10^8 \pi t + 5 \sin 2\pi 10^3 t$

$\theta_i(t)$ is deviated from $\theta(t)$ by 5 radians

$$\omega_i = \frac{d\theta_i}{dt} = \frac{d}{dt}\left(10^8 \pi t + 5 \sin 2\pi 10^3 t\right)$$

$$= 10^8 \pi + 5 \cos 2\pi 10^3 t \; (2\pi 10^3)$$

$\omega_c = 10^8 \pi$

$\omega_{i\,(max)} = 10^8 \pi + 5(2\pi 10^3)$

instantantaneous frequency deviated by $5\, 2\pi 10^3$

Problem: 3

An angle modulated signal is described by

$$s(t) = 10 \cos(2\pi 10^6 t + 0.1 \sin 10^3 \pi t)$$

Find $m(t)$

(a) considering $s(t)$ as a PM signal with $K_p = 10$

(b) considering $s(t)$ as a FM signal with $K_f = 10$

Sol$^n$:

$$s_{PM}(t) = A_c \cos(\omega_c t + K_p m(t))$$

$$= 10 \cos(2\pi 10^6 + 0.1 \sin 10^3 \pi t)$$

$$= 10 \cos(2\pi 10^6 t + 10 \times 0.01 \sin 10^3 \pi t)$$

$$\therefore m(t) = 0.01 \sin 10^3 \pi t$$

$$s_{FM}(t) = A_c \cos\left(\omega_c t + K_f \int m(t)\, dt\right)$$

$$= 10 \cos(2\pi 10^6 t + 0.1 \sin 10^3 \pi t)$$

$$K_f \int m(t)\, dt = 0.1 \sin 10^3 \pi t$$

put $m(t) = A_m \cos \omega_m t$

$$K_f \int A_m \cos \omega_m t \, dt = 0.1 \sin 10^3 \pi t$$

$$K_f A_m \frac{\sin \omega_m t}{\omega_m} = 0.1 \sin 10^3 \pi t$$

$$\frac{K_f A_m}{\omega_m} = 0.1$$

$$\frac{10 A_m}{10^3 \pi} = 0.1 \Rightarrow A_m = 10\pi$$

$$\therefore m(t) = 10\pi \cos 10^3 \pi t$$

**Problem: 4**

Given an angle modulated signal as

$$s(t) = 10 \cos (2\pi 10^8 t + 200 \cos 2\pi 10^3 t)$$

what is it's BW

**Sol$^n$:** $BW = 2\Delta\omega = 2\beta\omega_m$

$$\omega_i = \frac{d\theta_i}{dt} = \frac{d}{dt}(2\pi 10^8 t + 200 \cos 2\pi 10^3 t)$$

$$= 2\pi 10^8 + 200 \sin 2\pi 10^3 t \, (2\pi 10^3)$$

$$= \omega_c + \Delta\omega \sin 2\pi 10^3 t$$

$$\Delta\omega = 200 (2\pi 10^3)$$
$$= 4\pi 10^5$$

$$\beta = \frac{\Delta\omega}{\omega_m} = \frac{4\pi 10^5}{2\pi 10^3} = 200$$

(i) From Carson's rule

$$BW = 2\Delta\omega (1 + \frac{1}{\beta})$$
$$= 2\omega_m (1+\beta)$$
$$= 2(2\pi 10^3)(1+200)$$
$$= 8.04\pi 10^5 \text{ rad/s}$$
$$= 2\pi (402) 10^3 \text{ rad/sec}$$
$$= 402 \text{ KHz}$$

(ii) from approximate meth

since $\beta >> 1$

$$BW = 2\Delta\omega$$
$$= 2(4\pi 10^5)$$
$$= 2\pi \times 400 \times 10^3 \text{ rad/s}$$
$$= 400 \text{ KHz}$$

# 3 - Angle Modulation

Def:-
It is defined as a modulation process in which the total phase angle of a carrier sgl is varied in accordance with the instantaneous values of modulating sgl $x(t)$ while keeping the amplitude of carrier constant

Mathematical representation of angle modulation:-
Let us consider an unmodulated carrier signal $c(t)$. i.e., $c(t) = A\cos(\omega_c t + \theta_0)$

where $A \rightarrow$ Amplitude
$\omega_c \rightarrow$ freq. of carrier sgl.
$\theta_0 \rightarrow$ same phase angle.

Let $(\omega_c t + \theta_0) = \phi$ —— (1)

$\therefore \quad c(t) = A\cos\phi$ —— (2)

where $\phi = (\omega_c t + \phi_0)$ represents total phase angle of carrier - Differentiating on b.s of above equation

$$\frac{d\phi}{dt} = \omega_c \quad \text{—— (3)}$$

It may be noted at this point, for unmodulated carrier $\frac{d\phi}{dt}$ is a constant. However this derivative of $\frac{d\phi}{dt}$ may not be a constant with time. This mean vary with time. Hence this time dependent angular frequency is called instantaneous angular frequency and it is denoted by $\omega_i$:

$$\omega_i = \frac{d\phi}{dt}$$

$$(or) \quad \phi = \int \omega_i dt \quad \text{—— (4)}$$

If this total phase angle $\phi$ is varied linearly with the msg signal then the carrier wave is said to be angle modulated

$c(t) = A\cos\phi$

Types of angle modulation:-
It is classified into 2 types.

Angle modulation

Phase Modulation (phase of carrier changes)

Frequency modulation (freq. of carrier changes)

**Phase Modulation:**
The modulation process in which the phase of the carrier signal is changed according to instantaneous values of message signal is called as phase modulation.

**Frequency Modulation:**
The modulation process in which the frequency of carrier signal is changed according to instantaneous values of message signal is called as frequency modulation

**Advantages of Angle modulation:**
1. Noise reduction increases when compared to amplitude modulation.
2. Increases system Fidelity.

**Disadvantages:**
1. The transmission band width requirement more compared to amplitude modulation.
2. Complexity of circuit designing increases.

**Applications:**
1. Radio broad casting
2. Microwave communication
3. Satellite communication
4. point to point communication

**Phase Modulation:**
Phase Modulation is that type of modulation in which the phase of the carrier signal $\phi$ varies linearly with the base band signal (or) message signal $x(t)$. This means that, in phase modulation the instantaneous phase angle is equal to an modulated carrier phase angle plus a time varying component which is proportional to $x(t)$.

$\phi_i$ = unmodulated phase angle + time varying component proportional to $x(t)$.

Mathematical Representation of phase Modulation:

Let us consider an unmodulated carrier signal

$c(t) = A\cos(\omega_c t + \phi_0)$

$c(t) = A\cos\phi$ —(1)

where $\phi = \omega_c t + \theta_0$

by neglecting $\theta_0$ we can write $\phi = \omega_c t$ this phase angle $\phi$ is

According to phase modulation the message signal $x(t)$ to varied linearly with the

get instantaneous phase angle.

Let the instantaneous phase angle be denoted by $\phi_i$.

$\therefore \phi_i = \omega_c t + k_p x(t)$

where $k_p$ = proportionality constant, known as phase sensitivity of modulation, and that is expressed as rad/Volts.

Since the expression for carrier signal is $c(t) = A\cos\phi$

the expression for o/p phase modulator signal is

$s(t) = A\cos\phi_i$.

$\therefore \boxed{s(t) = A\cos[\omega_c t + k_p x(t)]}$

which is the required expression for phase modulated wave

Frequency Modulation:

FM modulation is the type of modulation in which the frequency of carrier signal is varied linearly with the message signal $x(t)$. This means that, in frequency modulation the instantaneous value of angular frequency is equal to unmodulated carrier frequency plus a time varying component which is proportional to $x(t)$.

$\omega_i$ = unmodulated carrier freq. + time varying component proportional to $x(t)$.

Mathematical representation of freq. modulation:

$$\boxed{\omega_i = \omega_c + k_f\, x(t)} \quad \text{--- (1)}$$

$k_f$ = proportionality constant which is known as frequency sensitivity → units: Hz/volts

Let us consider an unmodulated carrier signal

$c(t) = A\cos(\omega_c t + \theta_0)$

$\phi = \omega_c t + \theta_0$

$c(t) = A\cos\phi$

Let $\phi_i$ be the instantaneous phase angle of modulated signal. Now according to FM, the amplitude of the carrier signal remains constant and only total phase angle $\phi$ will changes. Hence the expression for freq. modulated wave will be

$S(t) = A\cos\phi_i$

wkT $\phi_i = \int \omega_i\, dt$ --- (2)

Sub. eq (1) in eq (2)

$\therefore \phi_i = \int [\omega_c + k_f\, x(t)]\, dt$

$\phi_i = \omega_c t + k_f \int x(t)\, dt$

$$\therefore \boxed{S(t) = A\cos(\omega_c t + k_f \int x(t)\, dt)}$$

Now with the phase angle of unmodulated carrier is taken at $t=0$, then the limits of integration in the above integration will be 0 to t

$$\therefore \boxed{S(t) = A\cos\left(\omega_c t + k_f \int_0^t x(t)\, dt\right)}$$

(For Multitone)

which is the required expression for freq. modulated wave.

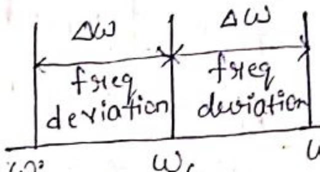
## Frequency Deviation ($\Delta\omega$) :-

In FM, the instantaneous frequency is given by unmodulated frequency ~~and~~ plus a time varying component which is proportional to $x(t)$.

$$\omega_i = \omega_c + k_f\, x(t)$$

This instantaneous frequency '$\omega_i$' varies with respect to time according to the message signal $x(t)$.
The maximum change in instantaneous frequency from the average frequency (or) carrier frequency is called as frequency deviation. and it depends on the magnitude and the sign of $k_f\, x(t)$. This means that the frequency deviation would be either +ve or -ve and it depends on the sign of $k_f\, x(t)$. However, the amount of frequency deviation in both the cases depends on maximum magnitude of $k_f\, x(t)$.
Frequency deviation is denoted by $\Delta\omega$.

$$\boxed{\Delta\omega = |k_f\, x(t)|_{max}}$$




Difference b/w phase modulation and frequency modulation.
We know that, the angle modulated wave is given by $s(t)$.

$$s(t) = A\cos\phi_i \;-\; TM$$

where $\phi_i$ is the total phase angle.
$\|^{ly}$, phase modulated and frequency modulated o/p's are expressed as

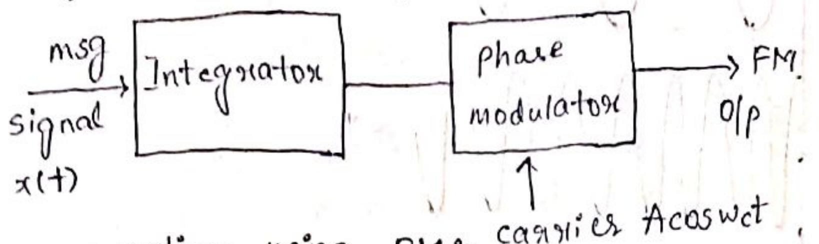
$$s(t) = A\cos[\omega_c t + k_p\, x(t)] \quad \text{- For PM}$$

$$s(t) = A\cos\left[\omega_c t + k_f \int_0^t x(t)\, dt\right] \quad \text{- For FM}$$

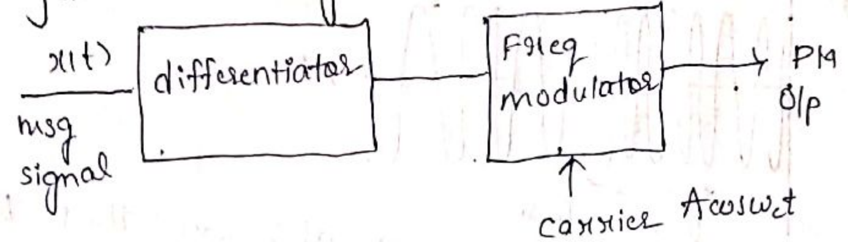
From the above equations, it may be noted that PM and FM waves are closely related to each other because in both the cases the total phase angle changes with respect to message signal.

In PM, the phase angle changes linearly with the message signal x(t) whereas in FM, the phase angle varies linearly with the integral of message signal x(t). So, a PM wave can be generated by using FM and an FM wave can be generated by using PM.

FM generation using PM :-




PM generation using FM :-




Modulation Index in FM :-

Modulation Index in FM is defined as the ratio of freq. deviation to the maximum frequency of message signal. It is denoted by $m_f$.

$$m_f = \frac{\text{freq. deviation}}{\text{freq. of msg sgl}}$$

$$\boxed{m_f = \frac{\Delta\omega}{\omega_m}} \quad \text{(or)} \quad m_f = \frac{\Delta f}{f_m}$$

Percentage of modulation in FM :-

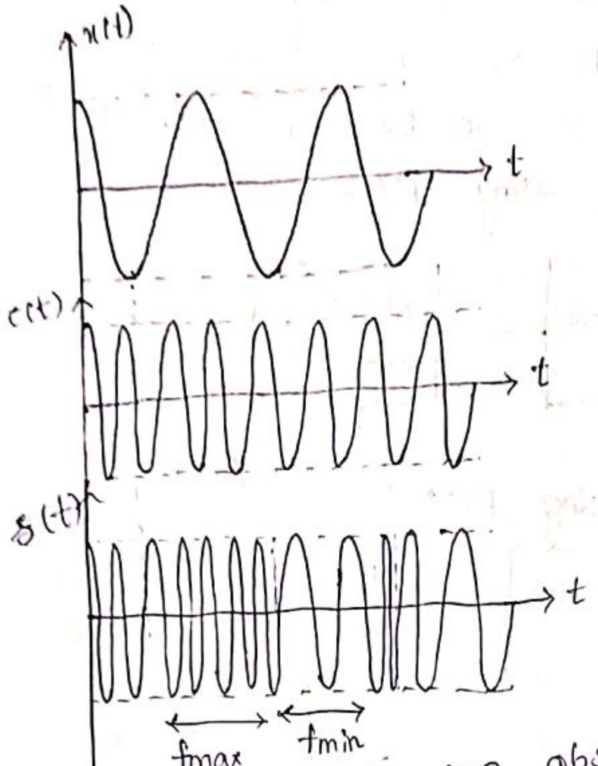
In FM, the percentage of modulation is the ratio of actual freq. deviation to the maximum allowable freq. deviation

$$\boxed{\% \text{ of mod} = \frac{\Delta f_{actual}}{\Delta f_{allowable}} \times 100}$$

* Single tone frequency modulation :-

Let us consider a carrier signal, $c(t) = A\cos\omega_c t$ and modulating signal $x(t) = V_m\cos\omega_m t$.

Now according to the definition of frequency modulation the frequency of the carrier signal is changed according to the instantaneous value of message signal. The figure below illustrates the process of frequency modulation.




From the above figure, we can observe that the frequency of the carrier signal shifts up and down from the center frequency / carrier frequency is $f_c$. This change (or) shift in carrier frequency is called as frequency deviation. and it is denoted by $\Delta\omega$. The frequency deviation always depends on the amount of amplitude of message signal which means that louder the music, greater the frequency deviation.

Mathematical Expression :-

Let us consider the standard expression for angle modulated wave $s(t) = A\cos\phi$ —① where $\phi$ is the total phase angle $= \int \omega_i \, dt$ —②

here $\omega_i = \omega_c + k_f x(t)$

$= \omega_c + V_m \cos\omega_m t \; k_f$

WKT, frequency deviation $\Delta\omega = |k_f x(t)|_{max}$

$\omega_i = \omega_c + \Delta\omega \cos\omega_m t$

substitute '$\omega_i$' in eq ②

$$\phi = \int (\omega_c + \Delta\omega \cos\omega_m t) \, dt$$

$$= \omega_c t + \Delta\omega \frac{\sin\omega_m t}{\omega_m}$$

$\therefore \quad \frac{\Delta\omega}{\omega_m} = m_f / \beta$.

$\phi = \omega_c t + m_f \sin\omega_m t$ —③

sub. ③ in eq ①.

$\boxed{s(t) = A\cos(\omega_c t + m_f \sin\omega_m t)}$ (or) $s(t) = A\cos(\omega_c t + \beta \sin\omega_m t)$

1) A single tone FM is represented by the voltage eqn as $v(t) = 12\cos(6\times10^8 t + 5\sin 1250t)$ then find out the following:
i) Carrier freq. ii) Modulating freq. iii) Modulation index iv) maximum deviation v) what amount of power will this FM wave dissipate in 10 Ω resistor?

Given,
$v(t) = 12\cos(6\times10^8 t + 5\sin 1250t)$ is
The standard equation for single tone FM
$s(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$
Carrier frequency $\omega_c = 6\times10^8$ $f_c = 955$ MHz
modulating frequency $\omega_m = 1250$ $f_m = 198.9$ Hz.
Modulation index $m_f = 5$

wKT, $m_f = \frac{\Delta\omega}{\omega_m} = \frac{\Delta f}{f_m}$.

$\Delta f = m_f \cdot f_m = 5\times198.9 = 994.5$

$P = \frac{V_{rms}^2}{R} = \frac{\left(\frac{12}{\sqrt{2}}\right)^2}{10} = \frac{72}{10} = 7.2$ W

Types of FM:- the band width of an FM signal depends on frequency deviation i.e., if frequency deviation is high, band width will be large. similarly, if frequency deviation is low, the bandwidth is small.

Since, deviation is given by $\Delta\omega = |k_f\, x(t)|_{max}$, the band width will depend upon frequency sensitivity $k_f$. Hence when $k_f$ is quite small, the bandwidth will be narrow and when $k_f$ is large, the band width will be wide. Thus, depending upon the value of frequency sensitivity, FM may be classified as

i) Narrow band FM

ii) wide band FM

Narrow band FM :

For $k_f$ value less than 1, the FM is treated as narrow band FM. For narrow band FM, the band width is very small which means a pass band is very less in narrow band FM.

Let us consider the standard expression for multitone FM signal is

$$s(t) = A\cos\left(\omega_c t + k_f\int x(t)\,dt\right)$$

$$s(t) = A\cos\omega_c t\cos\left(k_f\int x(t)\,dt\right) - \sin\omega_c t\sin\left(k_f\int x(t)\,dt\right)$$

For $k_f < 1$, we can assume to realistic approximations for the simplicity of equation

$$\begin{cases}\cos\left(k_f\int x(t)\,dt\right) \simeq 1, & \because \cos\theta \simeq 1, \text{ for } \theta < 1,\\ \sin\left(k_f\int x(t)\,dt\right) \simeq k_f\int x(t)\,dt & \because \sin\theta \simeq \theta, \text{ for } \theta < 1\end{cases}$$

$$\therefore s(t) = A\cos\omega_c t - A\sin\omega_c t\, k_f\int x(t)\,dt$$

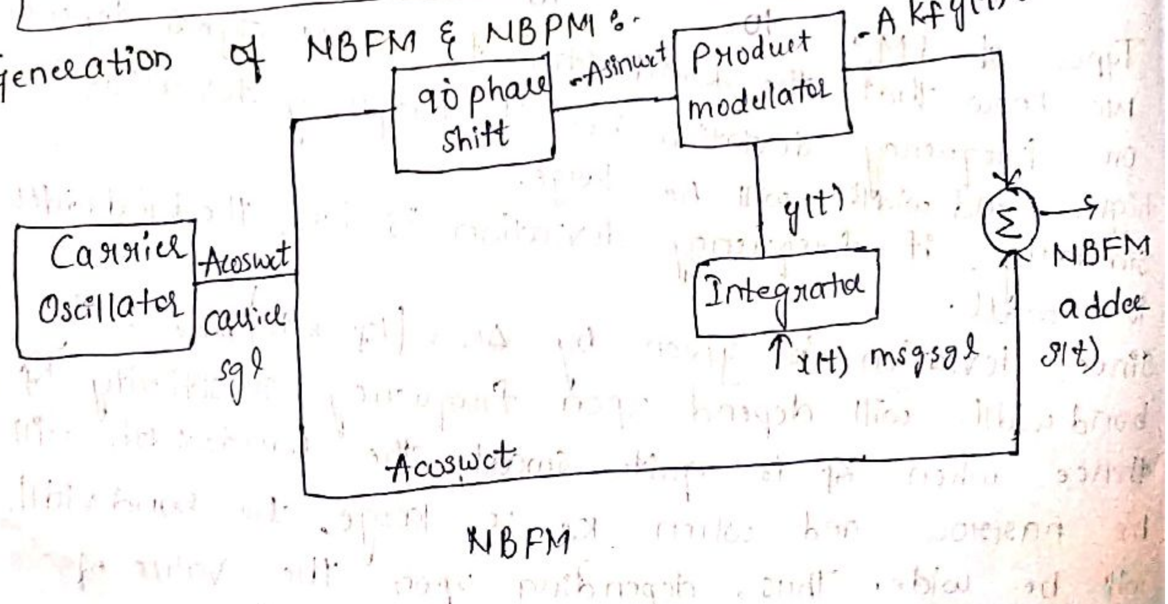
Let $\int x(t)\,dt = y(t)$

$$\boxed{s(t) = A\cos\omega_c t - Ak_f\, y(t)\sin\omega_c t}$$, which is the required standard expression for narrow band FM.

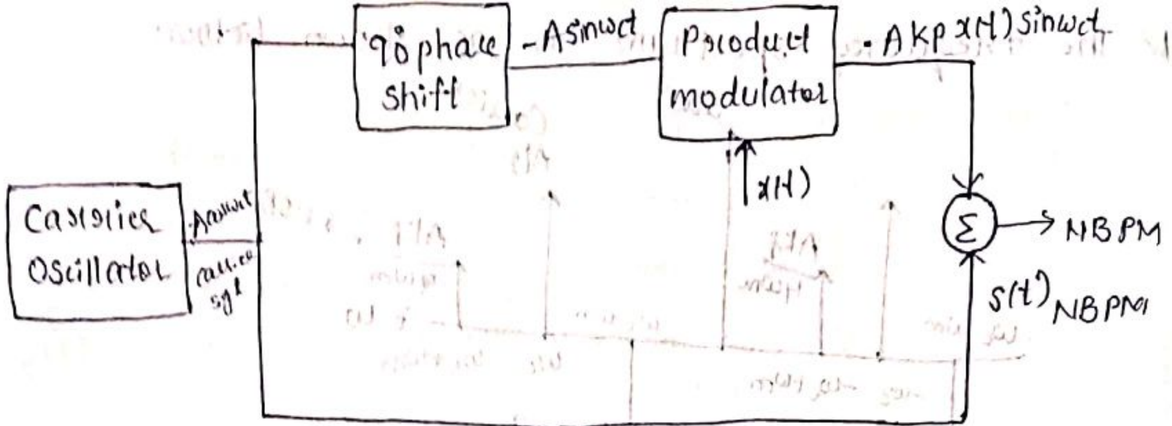
Similarly, for narrow band phase modulation the standard expression can be written as

$$\boxed{s(t) = A\cos\omega_c t - Ak_p\, x(t)\sin\omega_c t}$$

Generation of NBFM & NBPM :




NBFM




Frequency Spectrum of NBFM :-

Let us consider the standard expression for NBFM.

$s(t) = A\cos\omega_c t - AK_f y(t) \sin\omega_c t$

here $y(t) = \int x(t)\,dt$

Let $x(t) = \sin\omega_m t$

$\therefore y(t) = \dfrac{-\cos\omega_m t}{\omega_m}$

$$s(t) = A\cos\omega_c t + \frac{AK_f}{\omega_m}\cos\omega_m t \sin\omega_c t$$

$$= A\cos\omega_c t + \frac{AK_f}{2\omega_m}\left[\sin(\omega_c+\omega_m)t + \sin(\omega_c-\omega_m)t\right]$$

Let $x(t) = \cos\omega_m t$

$y(t) = \dfrac{\sin\omega_m t}{\omega_m}$

$$s(t) = A\cos\omega_c t - \frac{AK_f}{\omega_m}\sin\omega_m t \sin\omega_c t$$

$$= A\cos\omega_c t - \frac{AK_f}{2\omega_m}\left[\cos(\omega_c-\omega_m)t - \cos(\omega_c+\omega_m)t\right]$$

$$= \underbrace{A\cos\omega_c t}_{\text{Carrier}} - \underbrace{\frac{AK_f}{2\omega_m}\cos(\omega_c-\omega_m)t}_{\text{LSB}} + \underbrace{\frac{AK_f}{2\omega_m}\cos(\omega_c+\omega_m)t}_{\text{USB}}$$

The equation contains 3 components.

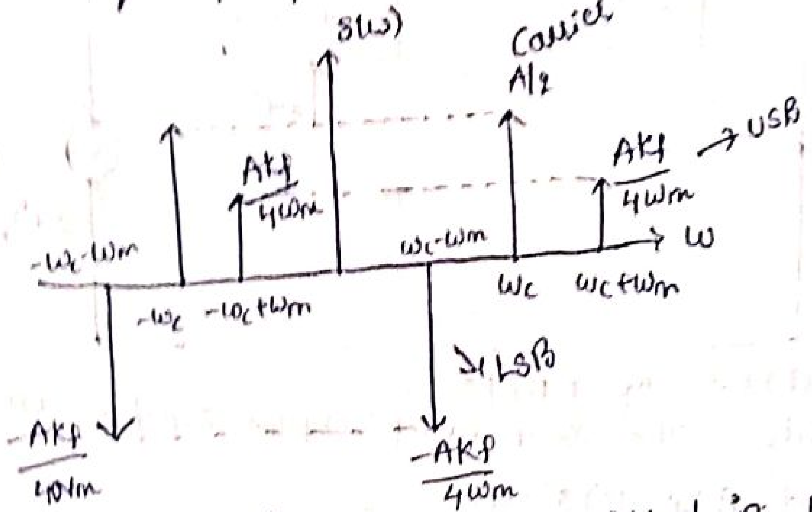
~~The first component~~

Apply F/T on b-s

$$s(\omega) = \frac{A}{2}\left[\delta(\omega-\omega_c) + \delta(\omega+\omega_c)\right] - \frac{AK_f}{4\omega_m}\left[\delta(\omega-(\omega_c-\omega_m)) + \delta(\omega+(\omega_c-\omega_m))\right]$$

$$+\frac{AK_f}{4\omega_m}\left[\delta(\omega-(\omega_c+\omega_m)) + \delta(\omega+(\omega_c+\omega_m))\right]$$

(iv) The frequency spectrum is as shown below:




* The amount of power transmitted in narrow band FM is $P_t = P_c + P_s = P_c + P_{LSB} + P_{USB}$

$$P_t = P_c \left[ 1 + \frac{m_f^2}{2} \right]$$

* The band width requirement in narrow band FM is The band width requirement of DSB-FC AM system. $2\omega_m$ = band width requirement of DSB-FC AM system.

Because of its much similarity with AM, NBFM is given least preference compared to WBFM.

Wide band frequency Modulation (Spectrum analysis of a sinusoidal wave in WBFM using Bessels function) is treated as:

For $K_f \gg 1$, (or) $\beta > 1$ the modulation is treated as wide band frequency Modulation. In wide band FM, the band width is very wide.

In order to find out, the frequency spectrum of WBFM first we need to know about the Bessels function and its properties.

The Bessels function is given by

$$J_n(x) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{J(x\sin\theta - n\theta)} \, d\theta$$

Properties :-

1. $J_n(x)$ decreases as n value increases

Eg. $J_0(x) > J_1(x) > J_2(x) > \ldots$

2. $J_{(-n)}(x) = (-1)^n J_n(x)$

i.e., $J_{-n}(x) = J_n(x)$, when $n$ = even
$= -J_n(x)$, when $n$ = odd

3) $\sum_{n=-\infty}^{\infty} J_n^2(x) = 1$

Let us derive the standard expression for NBFM wave.

WKT, the standard expression for single tone FM wave is given by $s(t) = A\cos(\omega_c t + m_f \sin\omega_m t)$ — ①

$$s(t) = A\cos(2\pi f_c t + \beta \sin 2\pi f_m t)$$

$$= A\,\mathrm{Re}\left[e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}\right]$$

$$= A\,\mathrm{Re}\left[e^{j2\pi f_c t} \cdot e^{j\beta \sin 2\pi f_m t}\right]$$

Let $e^{j\beta \sin 2\pi f_m t} = f(t)$, is a periodic sequence having time period $\frac{1}{f_m}$. So, this function can be expressed in exponential fourier series.

$$f(t) = e^{j\beta \sin 2\pi f_m t} = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \quad — ② \qquad \omega_0 = 2\pi f_m$$

where $C_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)\, e^{-jn\omega_0 t}\, dt$

$$C_n = \frac{1}{(1/f_m)}\int_{-T/2}^{T/2} e^{j\beta \sin 2\pi f_m t} \cdot e^{-jn\omega_0 t}\, dt$$

$$= f_m \int_{-1/2f_m}^{1/2f_m} e^{j(\beta \sin 2\pi f_m t - n 2\pi f_m t)}\, dt$$

Assume $2\pi f_m t = \theta$ $\therefore \frac{d\theta}{dt} = 2\pi f_m$

$t = -1/2f_m \Rightarrow 2\pi f_m t = \cancel{2}\pi \cancel{f_m} \cdot \frac{-1}{\cancel{2f_m}} = -\pi$

$t = 1/2f_m \Rightarrow \theta = \pi$

$$\therefore C_n = \cancel{f_m}\int_{-\pi}^{\pi} e^{j(\beta\sin\theta - n\theta)} \frac{d\theta}{2\pi \cancel{f_m}}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta\sin\theta - n\theta)}\, d\theta$$

$C_n = J_n(\beta)$ — ③

③ in ②

$$f(t) = \sum_{n=-\infty}^{\infty} J_n\beta\, e^{Jn2\pi f_m t}$$

substitute this eq. in eq ①

$$s(t) = A\, Re\left[e^{J2\pi f_c t} \cdot \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{Jn2\pi f_m t}\right]$$

$$= A\, Re\left[\sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{J2\pi (f_c+nf_m)t}\right]$$

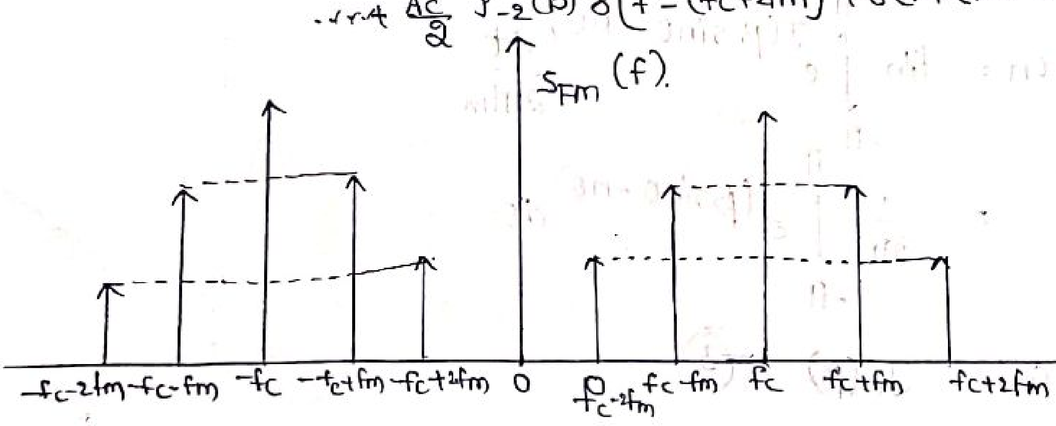
$$\boxed{s(t) = A \sum_{n=-\infty}^{\infty} J_n(\beta) \cos 2\pi (f_c+nf_m)t}$$ — WBFM standard exprsn

Spectrum :-
By giving the values of 'n' from [-2,2] the above eq becomes

$$S(t)_{FM} = A_c\, j_0(\beta) \cos 2\pi f_c t + j_1(\beta) \cos 2\pi (f_c+f_m)t$$
$$+ j_{-1}(\beta) \cos 2\pi (f_c-f_m)t + j_2\, \beta \cos 2\pi (f_c+2f_m)t$$
$$+ j_{-2}(\beta) \cos 2\pi (f_c-2f_m)t.$$

Taking fourier transform on both sides,

$$S(f)_{Fm} = \frac{A_c}{2} j_0(\beta)\left[\delta(f-f_c) + \delta(f+f_c)\right] +$$
$$\frac{A_c}{2} j_1(\beta)\, \delta[f-(f_c+f_m)) + \delta(f+(f_c+f_m))] +$$
$$\frac{A_c}{2} j_{-1}(\beta)\, \delta[f-(f_c-f_m)) + \delta(f+f_c+f_m)] + \ldots$$
$$\ldots + \frac{A_c}{2} j_{-2}(\beta)\, \delta[f-(f_c+2f_m)) + \delta(f+(f_c+2f_m))]$$




spectrum of FM.

Carson's rule (or) Practical B.W of FM:-

Carson's rule provides a formula to calculate the band width of a single tone FM. According to this rule, the band width of an FM sgl is twice the sum of frequency deviation and the highest modulating frequency. However, it must be remembered that this rule gives the approximation values of bandwidth

$$B.W = 2(\Delta\omega + \omega_m)$$

wKT, $m_f = \frac{\Delta\omega}{\omega_m} \Rightarrow \Delta\omega = m_f \omega_m$

$$B.W = 2(m_f\omega_m + \omega_m) = 2\omega_m(m_f+1)$$

we can analyse the above equation in two cases.

Case-(1):- For $m_f << 1$ i.e., the case for narrow band FM. we can neglect $m_f$.

$$\boxed{B.W_{NBFM} = 2\omega_m}$$

Case-(2):- For $m_f >> 1$ i.e., the case for wide band FM.

$$\boxed{B.W_{WBFM} = 2\omega_m m_f = 2\Delta\omega}$$

Ex:- Find the bandwidth of a commercial FM transmission if $\Delta\omega/\Delta f = 75KHZ$ and the modulating freq $f_m = 15KHZ$

Given, $\Delta f = 75KHZ$ ; $f_m = 15KHZ$

BW of a FM signal is given by $2(\Delta f + f_m)$

$$B.W = 2(\Delta\omega + \omega_m) = 2(75+15)\times 10^3$$

$$= 2(90)\times 10^3 = 180\times 10^3$$

$$B.W = 180KHZ.$$

Find out the B.W of a NBFM sgl which is generated by a 4KHZ audio sgl modulating a 125MHZ sgl

Given,

$f_m = 4KHZ.$

$f_c = 125MHZ$

$$B.W_{NBFM} = 2\omega_m = 2f_m = 8KHZ.$$

Scanned by CamScanner

Generation of FM :-

It can be done in 2 ways.

1. Direct Method
2. Indirect method

Direct Method :-

In Direct Method, the FM sgl is generated by using a device called VCO (Voltage control Oscillator) in which the frequency of an oscillator is directly controlled by the msg. sgl $x(t)$. i.e., VCO produces an o/p sgl which is proportional to i/p signal voltage

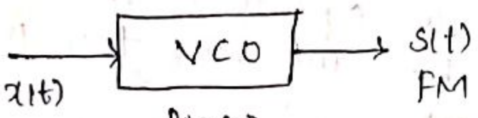



fig (a)

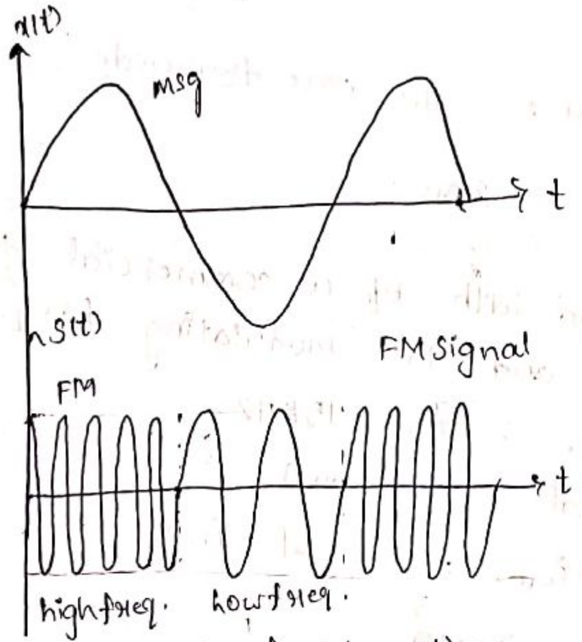
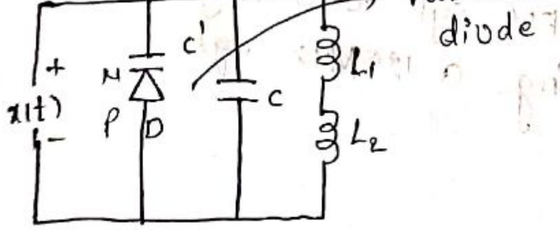



fig (b)

$$f_i = f_c + K_f x(t)$$

instantaneous freq.

Internal structure of VCO :-




$\omega \rightarrow$ width of depletion region

$$f_i = \frac{1}{2\pi\sqrt{(L_1+L_2)(C+C')}}$$

here $C' \propto \frac{1}{\omega}$ $\quad \because C' = \frac{\varepsilon_0 A}{\omega_0 (or) d}$

Operation :-

During +ve half cycle of $x(t) \rightarrow$ Diode is in F.B

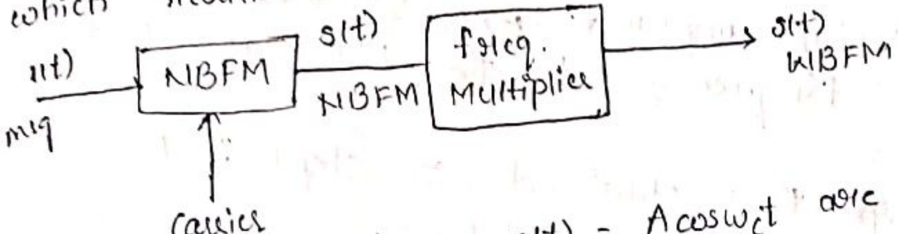
F.B $\uparrow$ $\omega \downarrow$ $c' \uparrow$ $f_i \downarrow$

During -ve half cycle of $x(t) \rightarrow$ diode is in RB

RB $\uparrow$ $\omega \uparrow$ $c' \downarrow$ $f_i \uparrow$

InDirect Method :-

In this method, first a NBFM wave is generated and this NBFM wave is passed through a frequency multiplier ckt to increase the frequency deviation which results in a wide band FM sgl.

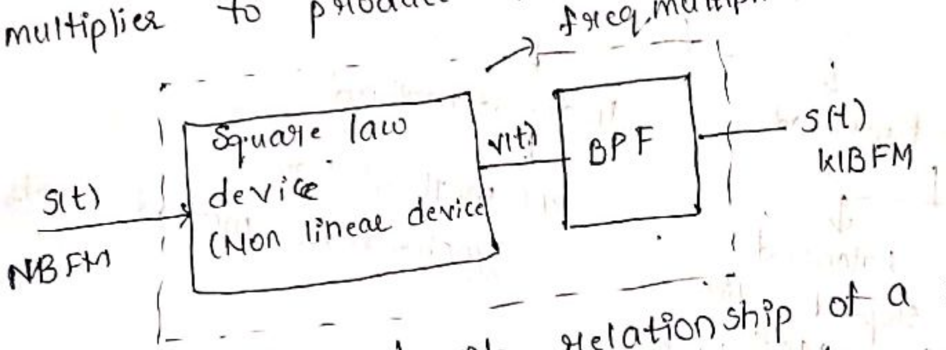



Let $x(t) = V_m \cos \omega_m t$ ; $c(t) = A \cos \omega_c t$ are applied as an inputs to NBFM modulator ckt. So, the o/p of NBFM modulator is

The instantaneous freq. of the sgl is $f_i = f_c + k_f x(t)$

$s(t) = A \cos(\omega_c t + m_f \sin \omega_m t)$ — ①

Now this sgl is allowed to pass through a frequency multiplier to produce desired WBFM sgl.




Here, the i/p and o/p relationship of a non linear device in general form is given by

$v(t) = a_1 s(t) + a_2 s^2(t) + a_3 s^3(t) + \cdots + a_n s^n(t)$ — ②

where $a_1, a_2 \cdots a_n$ are coefficients and 'n' is the highest order of non-linearity.

By substituting eq ① in eq ② and by simplifying the frequency modulated wave having carrier frequencies $f_1, 2f_1, 3f_1 \cdots n f_1$ with frequency deviations $\Delta f_1, 2\Delta f_1, 3\Delta f_1 \cdots n\Delta f_1$ the output of frequency multiplier produces the desired WBFM having the following time domain equation

$s(t)_{WBFM} = A\cos(n\omega_c t + n m_f \sin\omega_m t)$

By using proper mixer circuit, the centre frequency of resulting WBFM signal will be changed from $nf_c$ to $f_c$ only.

**Demodulation of FM (or) detection of FM :-**

Demodulation of FM (or) detection of
In the process of demodulation (or) detection of FM, the original message signal is recovered from the FM wave. FM detection is carried out in two steps.

1. A signal having amplitude proportional to the instantaneous frequency of the FM wave is first obtained.
2. The o/p sgl thus obtained in step 1 is then detected by using diode detector.

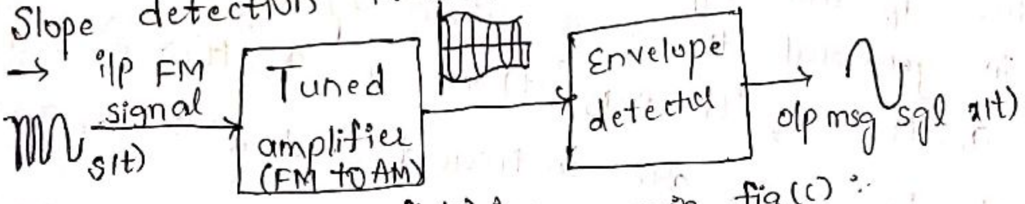
**Types of demodulation :**

FM detection can be done in 2 ways.

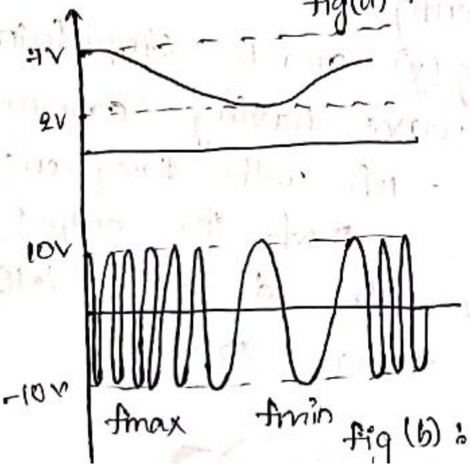
1. Direct Method (freq discrimination method)
2. Indirect method (phase discrimination method)

FM detection
- Direct method
  - slope detection
  - balanced slope detection
- Indirect method
  - PLL method (or) Synchronous method

**Slope detection Method :-**




fig(a) :-




fig(b) :-

gain fig(c) :-

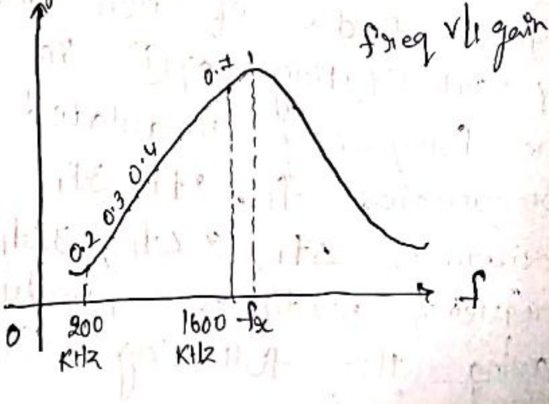
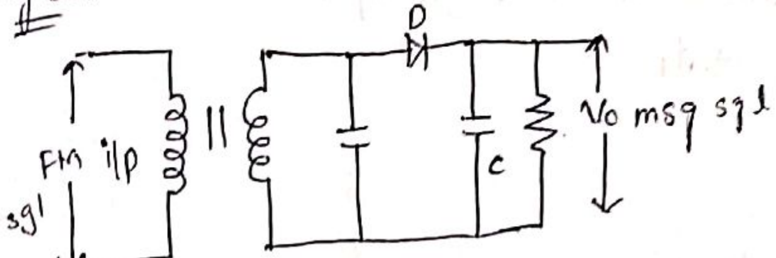


fig (d) :-



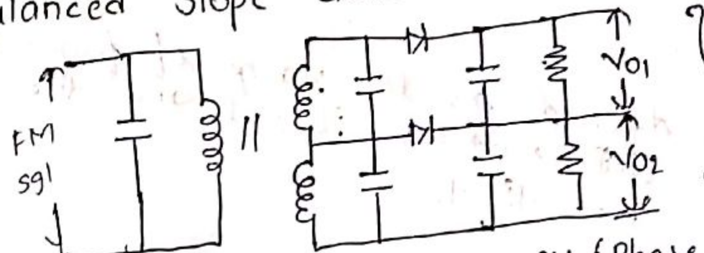

Tuned amplifier :-

As f becomes closer to fr → gain value ↑ → o/p voltage ↑.

As f moves away to fr → gain value ↓ → o/p voltage ↓

→ To increase the limited range of values, we go for balanced slope detection.

Balanced slope detection :-




final o/p

$V_0 = V_{01} - V_{02}$.

Demodulation of FM using PLL (Phase Locked Loop) :-

In synchronous detection method PLL is used for removing phase error.

A PLL is a non-linear feedback system that tracks the phase of input signal and minimize the phase error of carrier at local oscillator in receiver section.

PLL is used in synchronous demodulation (or) coherent demodulation of FM wave

fig (a) General structure of synchronous detection

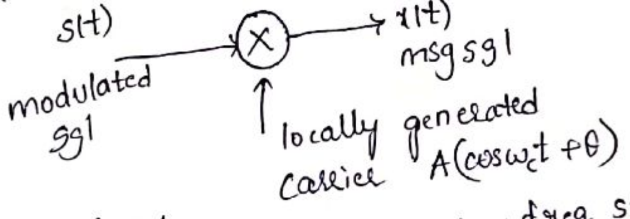
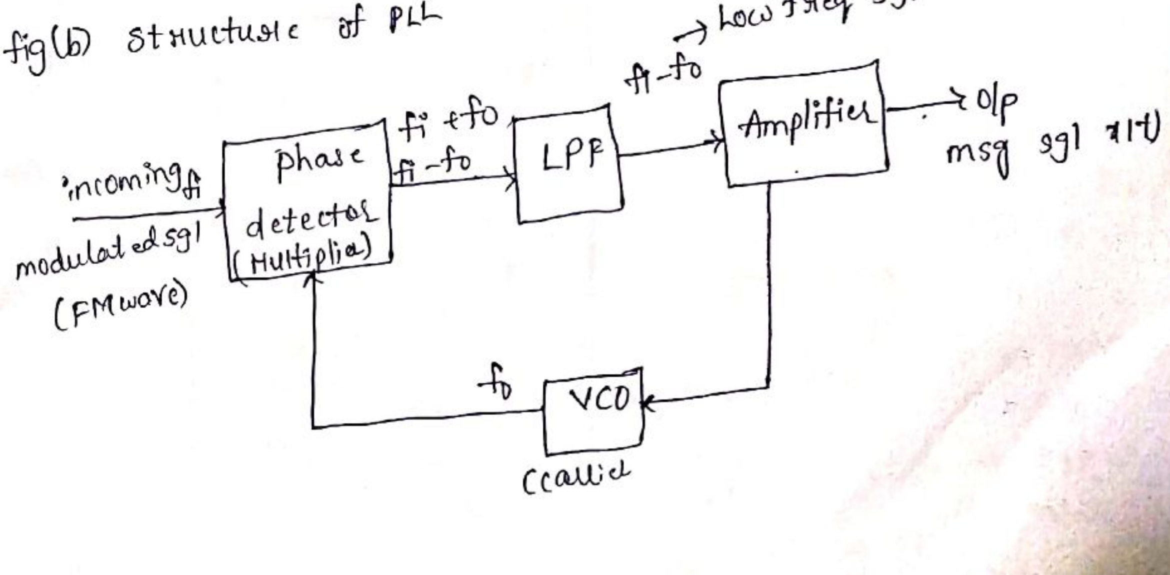



fig (b) structure of PLL



The circuit operates in 3 regions/states.

1) Free running state
2) Capture Range
3) Lock Range

Free running state :-
If no i/p sgl is applied then PLL is in free running state

Capture Range :-
Once the i/p frequency is applied the VCO frequency starts to change and PLL is said to be in capture mode

Lock Range :-
The lock range is defined as the range of frequency over which the phase difference b/w i/p sgl and the carrier sgl is zero. i.e., $f_i - f_o = 0$