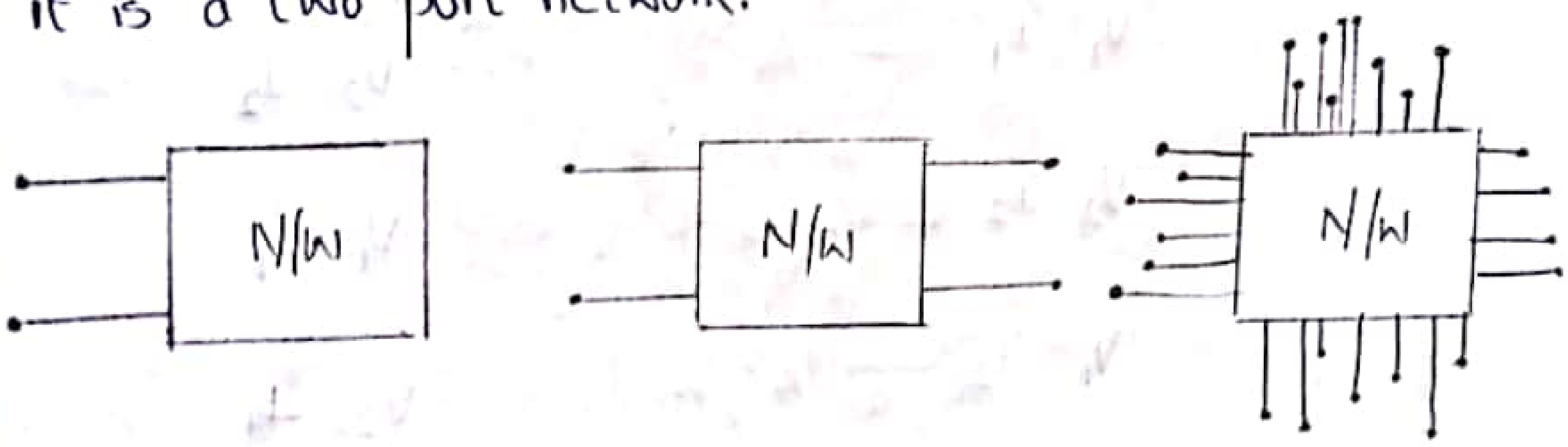
5. Two Port Networks

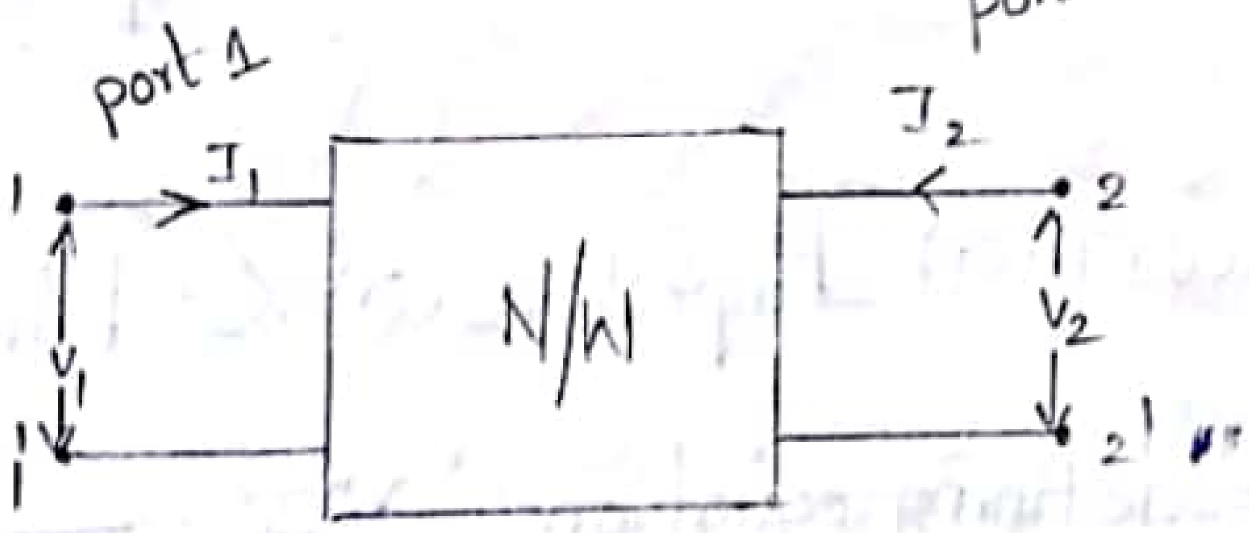
Port:-

A pair of terminals at which signal can enter or leave a network is known as port.

If there is only 1-pair port then it is single port network and if there are 2-pair ports then it is a two port network.



Two Port Network :-



V, I are interdated and their relationships is explained (or) expressed by Parameters is known as N/W parameters.

1, 1'	:	port 1 Sending end point
2, 2'	:	port 2 Reciving end Point
V_1, I_1	:	Voltage and current at port 1
V_2, I_2	:	Voltage and current at port 2

The voltages and current at these ports are interlated and their relationships are expressed interms of parameters is known as <u>Network Parameters.</u>

The variables of the network are V_1, I_1, V_2, I_2, Out of these 4 variables only 2 are independent and the other 2 are dependent. The dependent variables are expressed interms of independent variables and network parameters. Out of these we have 6 combination. They are:

Dependent Variables — Independent Variables.

V_1 V_2 — Z parameters — I_1, I_2

I_1 I_2 — Y parameters — V_1 V_2

V_1 I_1 — ABCD parameters — V_2 I_2

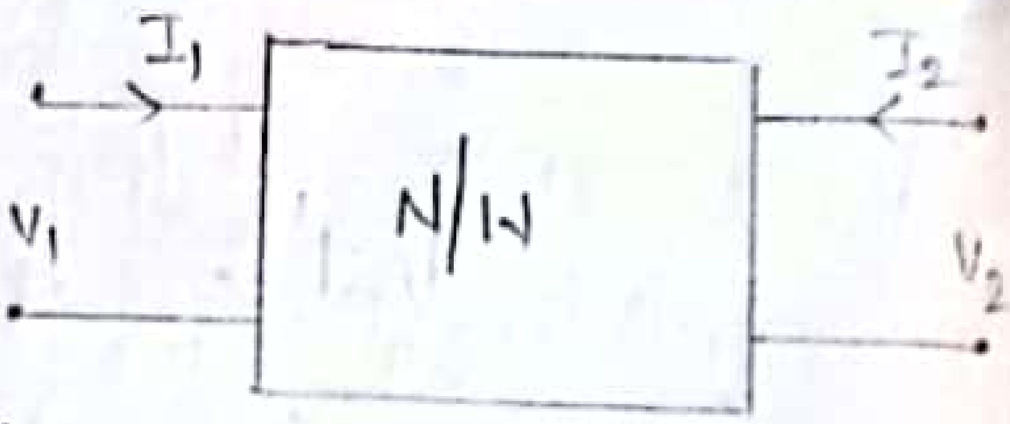
V_2 I_2 — Inverse ABCD Parameters — V_1 I_1

V_1 I_2 — h parameters — V_2 I_1

V_2 I_1 — Inverse H parameters. — V_1 I_2

Open CIRCUIT (or) Impedence (or) Z-Parameter:-

The defining equations can be written by considering any two independent variables and network parameters. The independent variables are I_1 and I_2 and network parameter is impedence i.e → Z.



Therefore the defining equation is

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2.$$

In Matrix form:-

$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}}_{\text{Dependent}} = \underbrace{\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}}_{\text{Network param}} \underbrace{\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}_{\text{Indep Variables.}}$$

Determination Of Z parameters :-

If I_2 is made equal to zero i.e., port 2 is open circuited we can determine Z_{11} and Z_{21}.

$$\therefore \quad V_1 = Z_{11} I_1 + 0.$$

port 2 o.c $\Rightarrow I_2 = 0$

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}.$$

$$V_2 = Z_{21} I_1 + 0.$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}.$$

If I_1 is made equal to zero i.e., port 1 is open circuited we can determine Z_{12} and Z_{22}.

port I_1 o.c $\Rightarrow I_1 = 0$

$$V_1 = 0 + Z_{12} I_2$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}.$$

$$V_2 = 0 + Z_{22} I_2$$

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}.$$

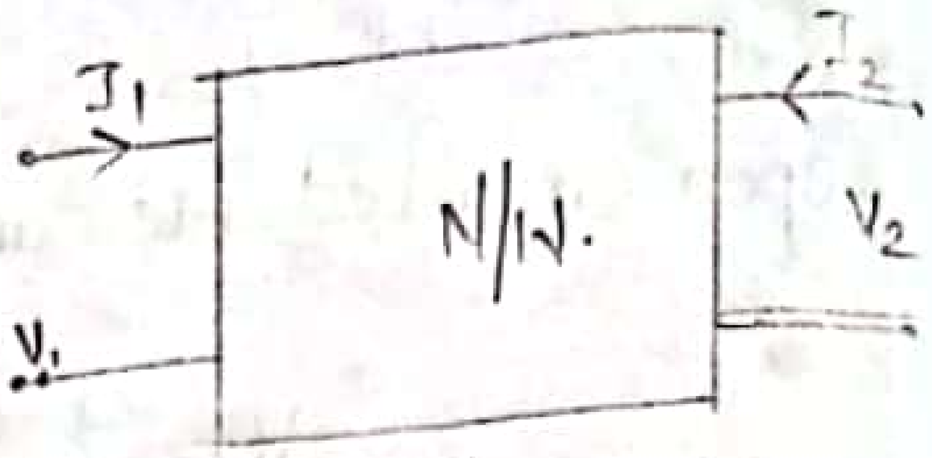
where Z_{11} : Driving point Impedence. (DPI)

Z_{21} : forward transfer impedence. (FTI)

Z_{12} : Reverse Transfer impedence. (RTI)

Z_{22} : Driving point Impedence. (DPI).

Short Circuit (or) Admittance (or) Y- Parameter :-



The defining equations can be written by using (or) considering any two independent variables and network parameters. The independent variables are V_1 and V_2 and network parameter is Admittance i.e., $y = \frac{1}{Z}$.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Determination Of Y - Parameters :-

If V_2 is made equal to zero i.e., port 2 is short circuited we can determine Y_{11} and Y_{21}

$$I_1 = Y_{11} V_1 + 0$$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2 = 0}.$$

$$I_2 = Y_{21} V_1 + 0$$

$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2 = 0}.$$

If V_1 is made equal to zero ie-- port 1 is short circuited we can determine Y_{21} and Y_{22}.

$$I_1 = 0 + Y_{12} V_2$$

$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$I_2 = 0 + Y_{22} V_2$$

$$Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Y_{11} : Input Admittance

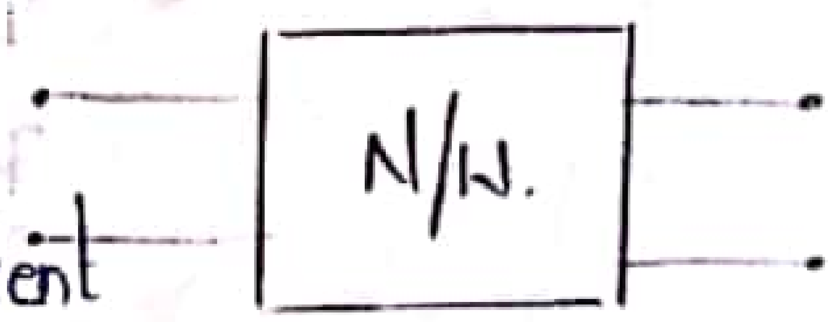
Y_{21} : Forward Transfer Admittance

Y_{12} : Reverse Transfer Admittance.

Y_{22} : Output Admittance.

TRANSMISSION LINES (or) ABCD PARAMETERS:-

In this voltage and current at the transmission end will be expressed with the voltage and current at the recieving end.



Generally, in power system engineering these parameters are used to study the performance of transmission lines.

The defining equations are

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2.$$

$-I_2$ is considered instead of $+I_2$ because the current direction of I_2 is opposite to I_1; the -ve sign is only for indicating the reversal of direction of current but not to the network parameters.

Deteremination Of ABCD Parameters :-

If I_2 is made equal to zero i.e., port 2 is open circuited we can determine A and C.

port 2 is o.c $\Rightarrow I_2 = 0$.

$$V_1 = AV_2 - 0 \qquad I_1 = CV_2 - 0$$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0} \qquad C = \frac{I_1}{V_2}\Big|_{I_2=0}.$$

If V_2 is made equal to zero i.e., port 2 is short circuited we can determine B and D.

port 2 is S.C $\Rightarrow V_2 = 0$

$$V_1 = 0 - BI_2 \qquad I_1 = 0 - DI_2$$

$$B = \frac{V_1}{-I_2}\Big|_{V_2=0} \qquad D = \frac{I_1}{-I_2}\Big|_{V_2=0}.$$

A = Open Circuit voltage

C = Open circuit transfer admittance

B = Short circuit transfer impedence.

D = Short circuit current

INVERSE ABCD PARAMETERS [A'B'C'D']:-

These parameters gives the relationship as follows:

The sending reciving end voltage and currents are expressed interms of sending end voltage and current.

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Determination Of A'B'C'D' Parameters:-

If $I_1 = 0$ is made equal to zero i.e., port 1 is open circuited we can determine A' and C'

port 1 is o.c $\Rightarrow I_1 = 0$.

$$V_2 = A'V_1 - 0 \qquad\qquad I_2 = C'V_1 + 0$$

$$A' = \left.\frac{V_2}{V_1}\right|_{I_1=0} \rightarrow \text{forward voltage ratio} \qquad C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$$

If V_1 is made equal to zero i.e., port 1 is short circuited we can determine B' and D'

port 1 is s.c $\Rightarrow V_1 = 0$.

$$V_2 = 0 - B'I_1 \qquad\qquad I_2 = 0 - D'I_1$$

$$B' = -\frac{V_2}{I_1} \qquad\qquad D' = \left.\frac{I_2}{-I_1}\right|_{V_1=0}.$$

A': forward voltage ratio.

B': Transfer impedence

C': Transfer Admittance.

D': forward current ratio.

HYBRID PARAMETERS. (or) H PARAMETERS:-

These parameters gives the relationship between sending end voltage and recieving end current in terms of send ending end current and recieving end voltage.

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2$$

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Determination Of H-PARAMETERS:-

If V_2 is made equal to zero i.e., port 2 is short circuited we can deterimine h_{11} and h_{21}

port 2 is s.c $V_2 = 0$.

$V_1 = h_{11} I_1 + 0.$ $\qquad$ $I_2 = h_{21} I_1 + 0$

$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$ $\qquad$ $h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0.}$

If I_1 is made equal to zero i.e., port 1 is open circuited we can determine h_{12} and h_{22}

Port 1 is O.C $\Rightarrow I_1 = 0$.

$$V_1 = 0 + h_{12} V_2 \qquad I_2 = 0 + h_{22} V_2$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

h_{11} : short circuit impedence

h_{12} : reverse voltage gain

h_{21} : forward current transfer ratio.

h_{22} : Open circuit Output admittance.

H - parameters are useful in analysing the transistor characteristics

INVERSE HYBRID PARAMETERS (or) H^{-1} PARAMETER :-

These parameters gives the relationship between sending end current and reciving end voltage in terms of sending end voltage and reciving end current.

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2 .$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Determination Of H' Parameters:-

If V_2 is made equal to zero i.e→ port 2 is ~~short~~ open circuited we can determine g_{11} and g_{21}

port 2 is o.c , $I_2 = 0$

$I_1 = g_{11} V_1 + 0$ $\quad$ $V_2 = g_{21} V_1 + 0$

$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$ $\quad$ $g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$.

If. V_1 is made equal to zero i.e→ port 1 is short circuited we can determine g_{12} and g_{22}

port 1 is s.c $V_1 = 0$

$I_1 = 0 + g_{12} I_2$ $\quad$ $V_2 = 0 + g_{22} I_2$

$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$ $\quad$ $g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$.

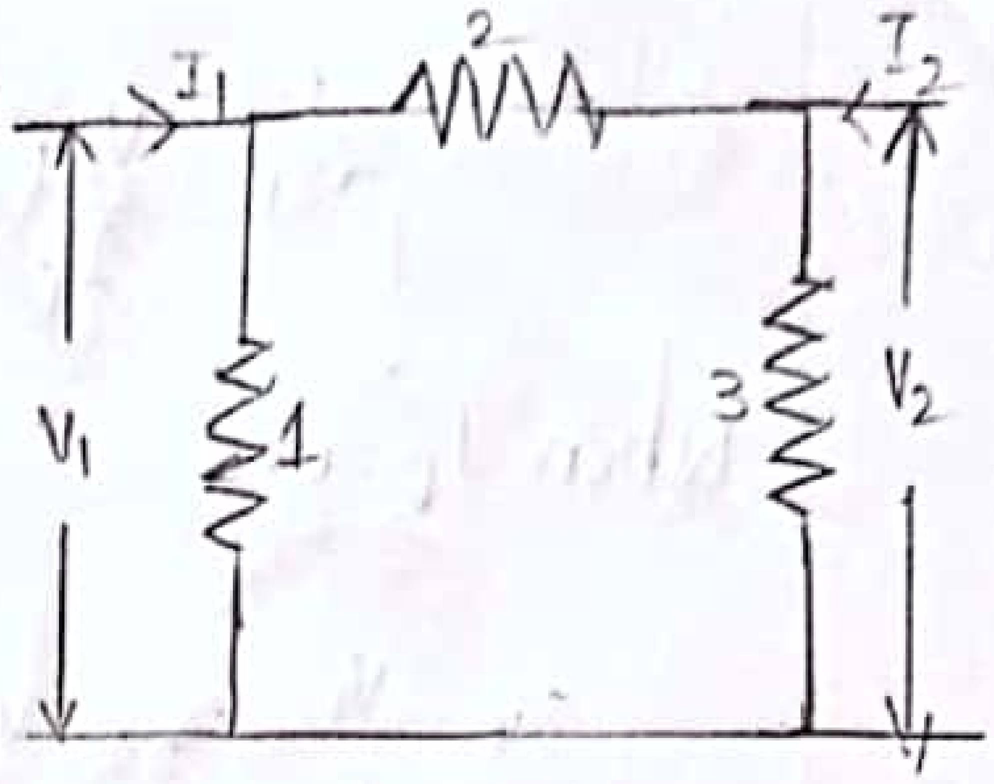
g_{11} : Impedence conductance.

g_{12} : reverse current ratio

g_{21} : open circuited forward voltage gain

g_{22} : Impedence

(e) Find the z-parameters for the given network.

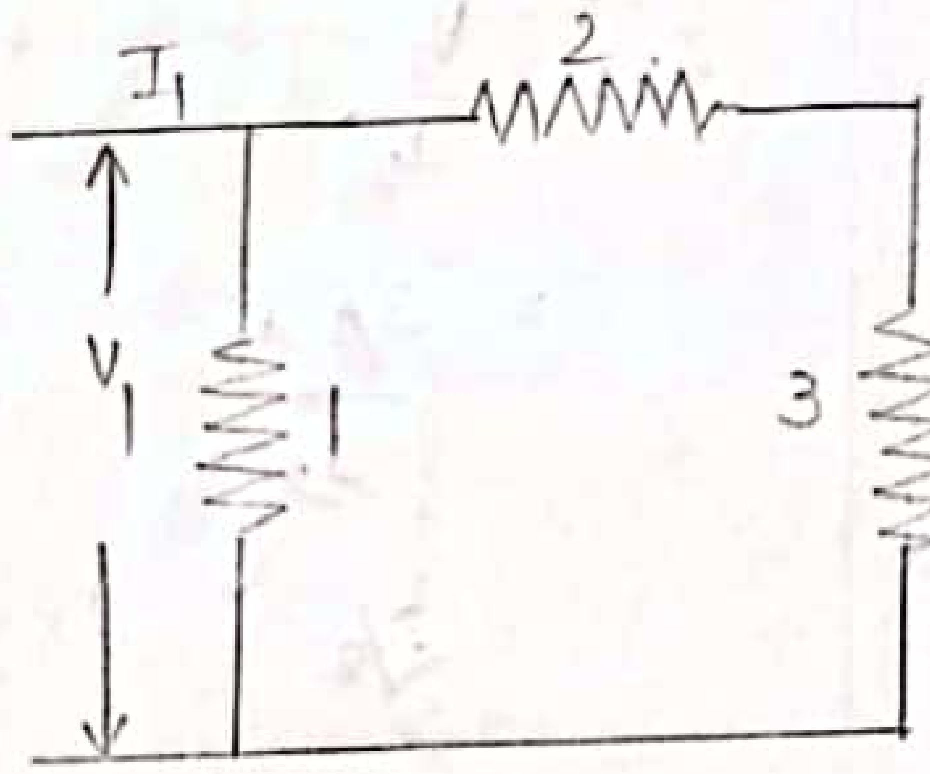


Sol: W.k.T

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

When $I_2 = 0$. O.C

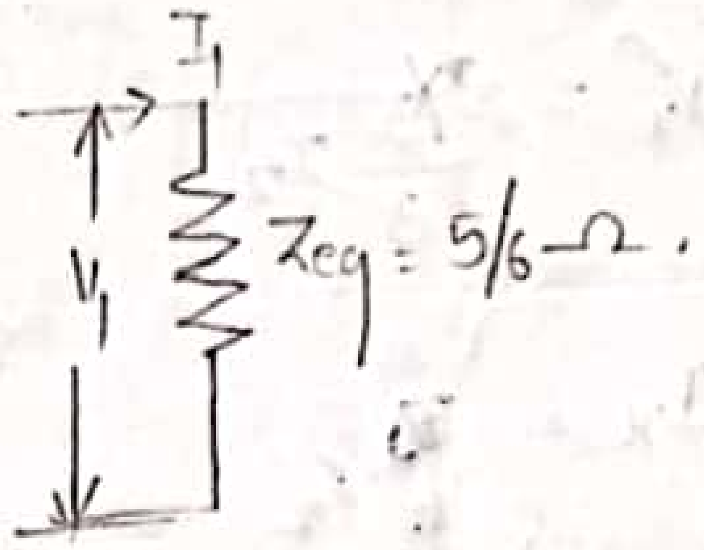


$$Z_{11} = \frac{V_1}{I_1} \quad \& \quad Z_{21} = \frac{V_2}{I_1}$$

I_2 is open circuited.

$$V_1 = Z_{eq} \cdot I_1$$

$$= 5/6 \cdot I_1$$



Now, $Z_{11} = \frac{5/6 \cdot \cancel{I_1}}{\cancel{I_1}}$

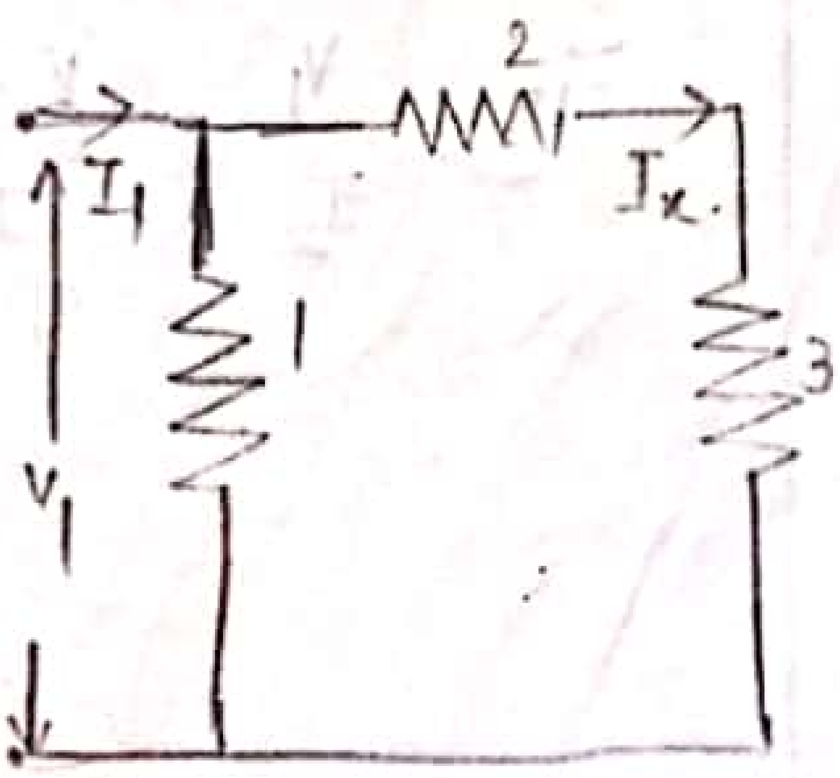
$$= 5/6\ \Omega.$$

$$I_x = \frac{1}{6} I_1$$

$$V_{3\Omega} = \frac{1}{6} I_1 \times 3$$

$$= 1/2\ I_1$$

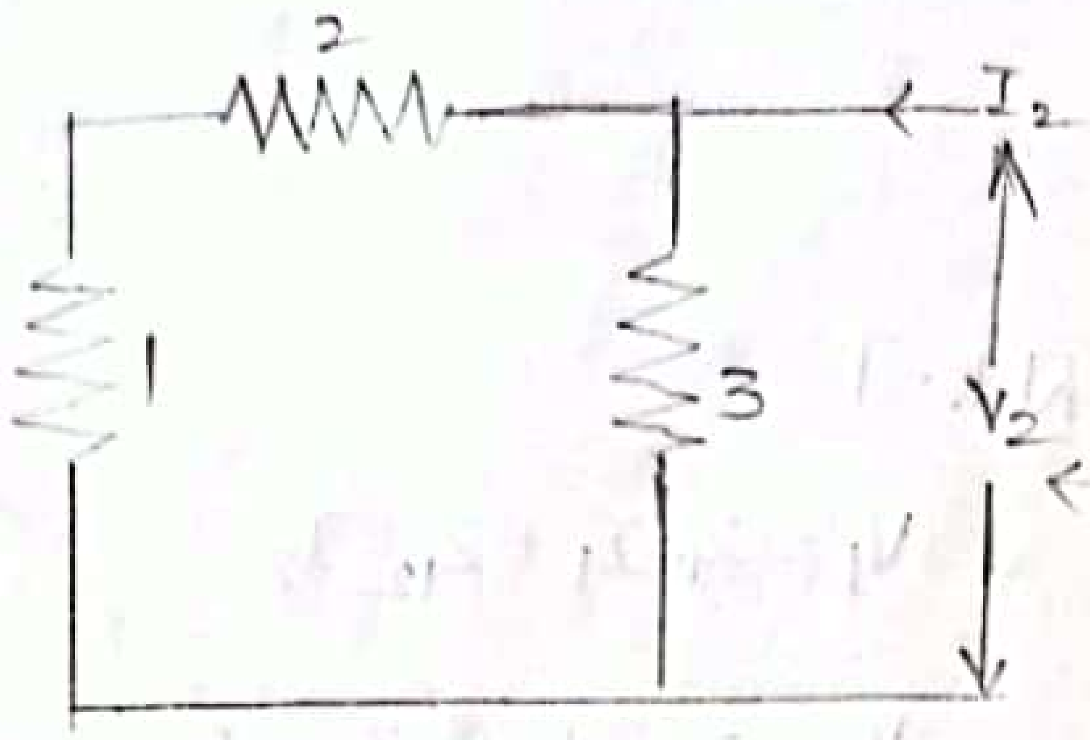
$$V_{3\Omega} = V_2 = 1/2\ I_1$$



$$Z_{21} = \frac{V_2}{I_1} = \frac{\frac{1}{2} I_1}{I_1} = \frac{1}{2}.$$

When $I_1 = 0$.

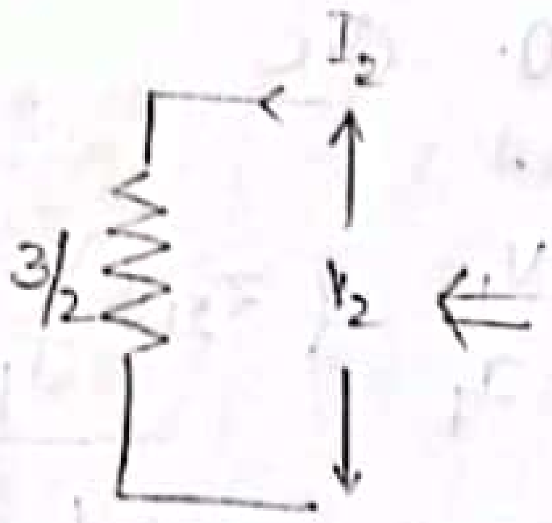
$$Z_{12} = \frac{V_1}{I_2} \qquad Z_{22} = \frac{V_2}{I_2}$$

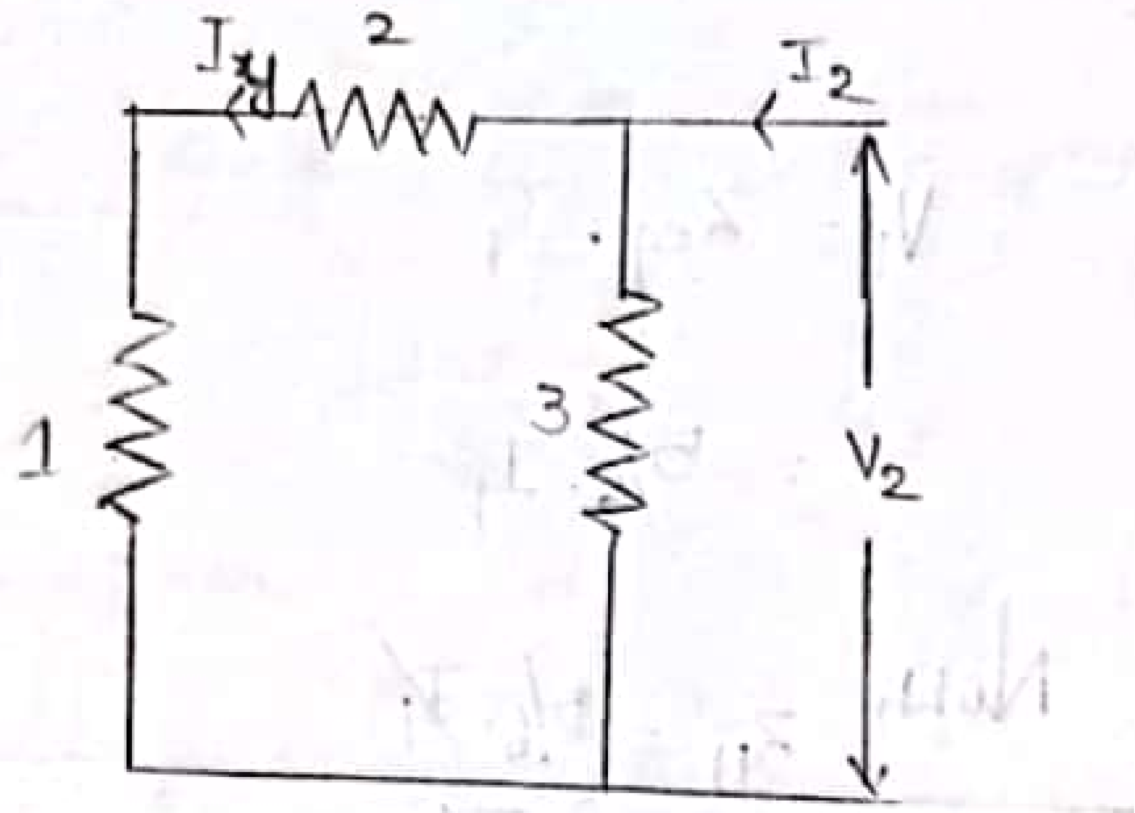


$$V_2 = \frac{3}{2} I_2.$$

Now,

$$Z_{22} = \frac{V_2}{I_2} = \frac{\frac{3}{2}\cancel{I_2}}{\cancel{I_2}} = \frac{3}{2}$$





$$\cancel{I_x} = \frac{\cancel{2}3}{3}\cancel{I_2}$$

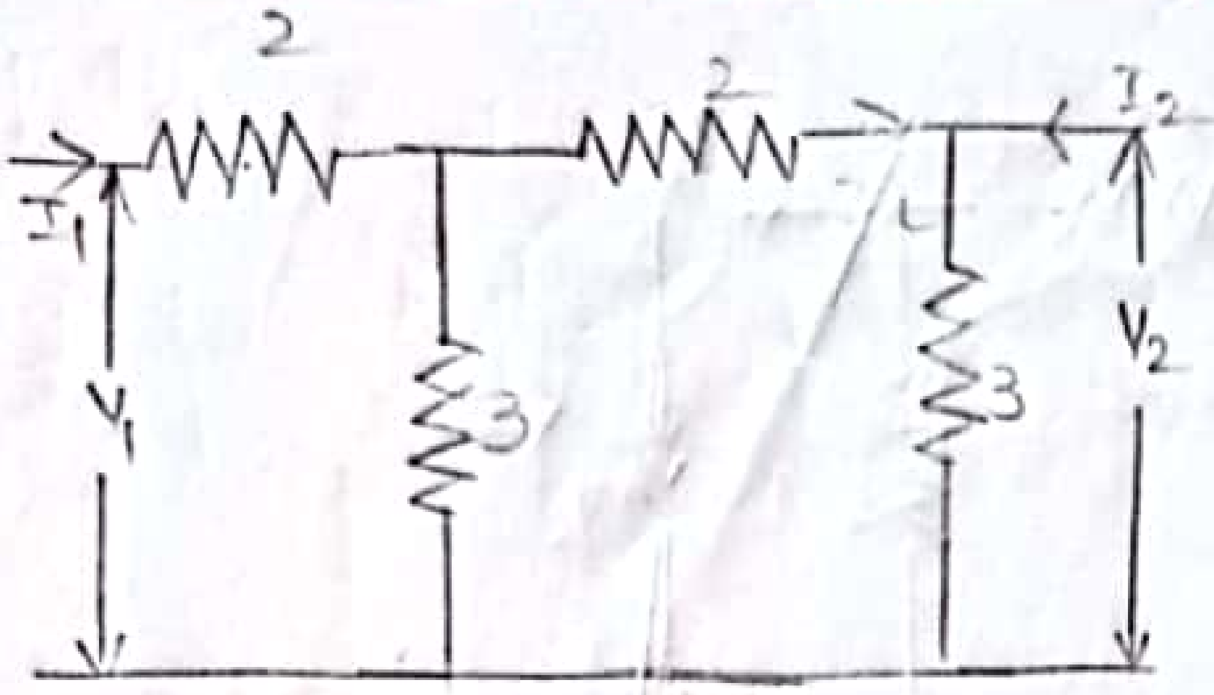
$$I_x = \frac{3}{6} I_2$$

$$I_x = \frac{\cancel{3}\,1}{\cancel{6}\,2} I_2 = \frac{1}{2} I_2$$

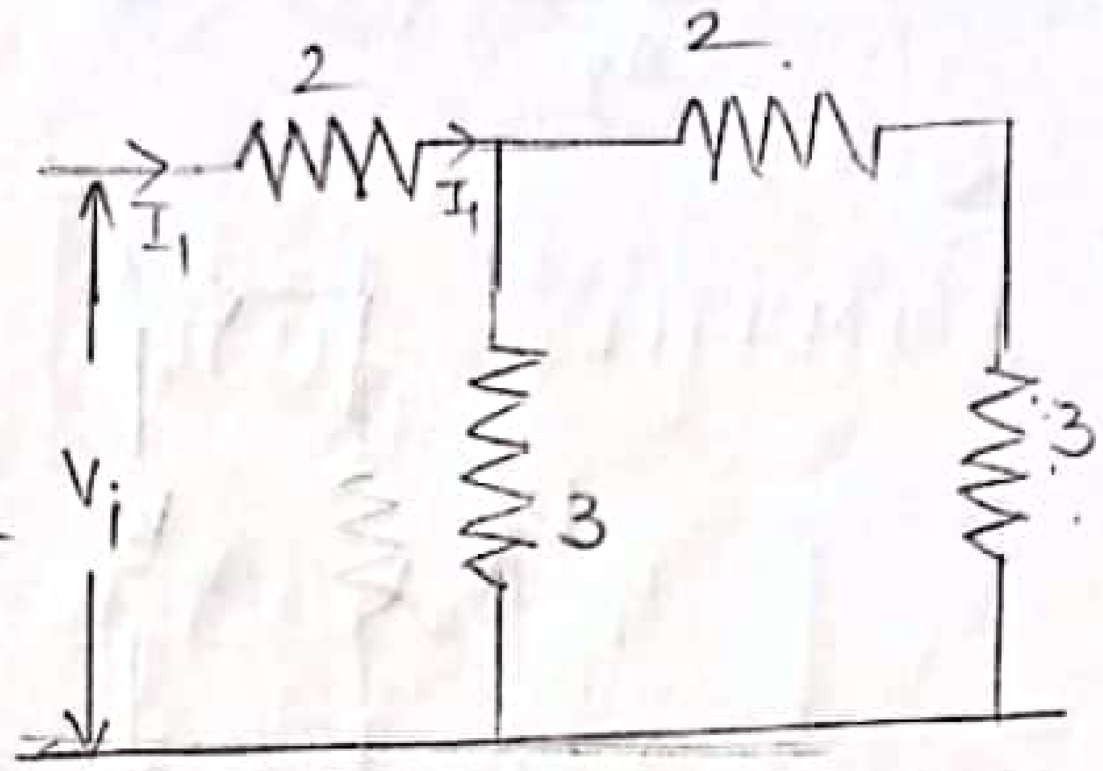
$$V_{1\Omega} = \frac{\cancel{3}\,1}{\cancel{6}\,2} I_2 \times 1 = \frac{1}{2} I_2$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{\frac{1}{2}\cancel{I_2}}{\cancel{I_2}} = \frac{1}{2}.$$

Q)



Ans:-

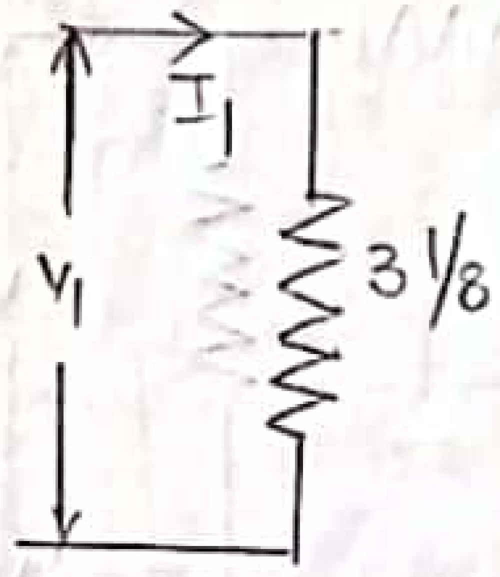


when $I_2 = 0$.

$Z_{11} = \frac{V_1}{I_1}$ & $Z_{21}' = \frac{V_2}{I_1}$

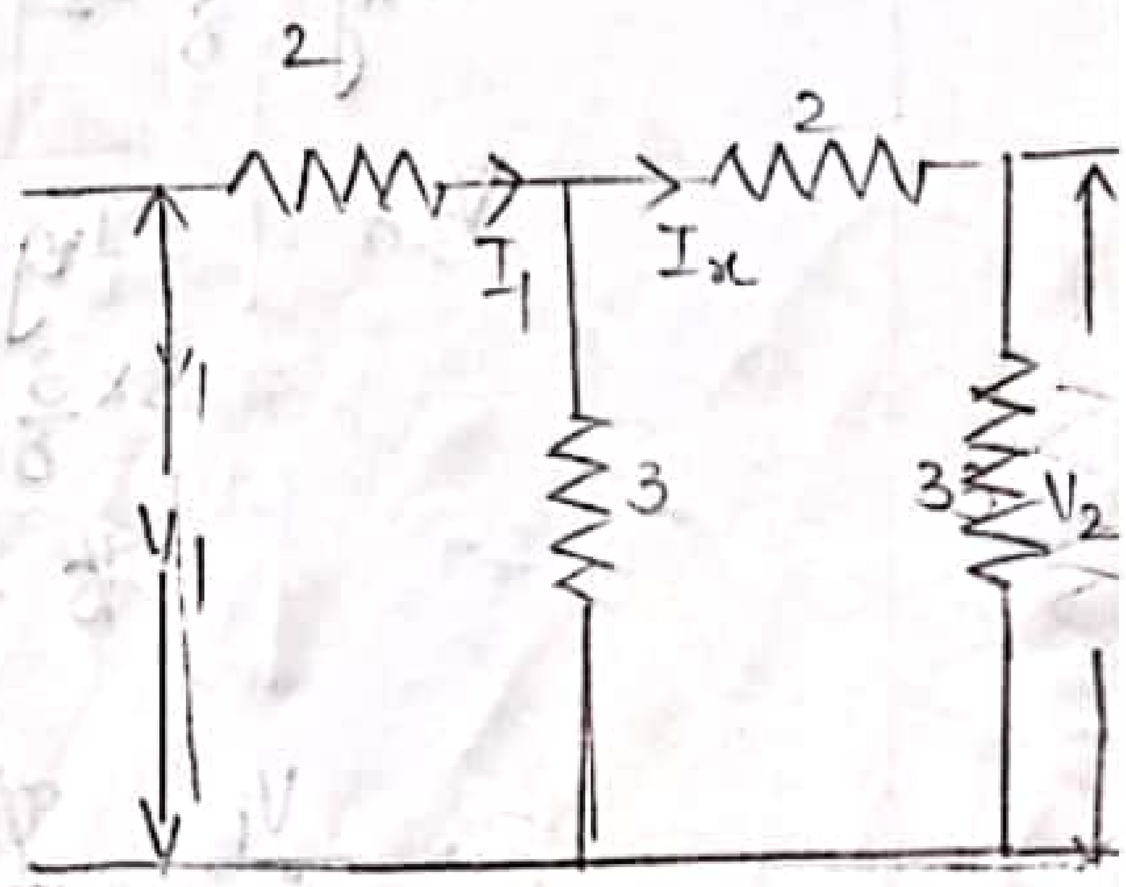
I_2 is open circuit

$V_1 = 31/8\, I_1$



$Z_{11} = \frac{31/8\, \cancel{I_1}}{\cancel{I_1}}$

$Z_{11} = 31/8$.



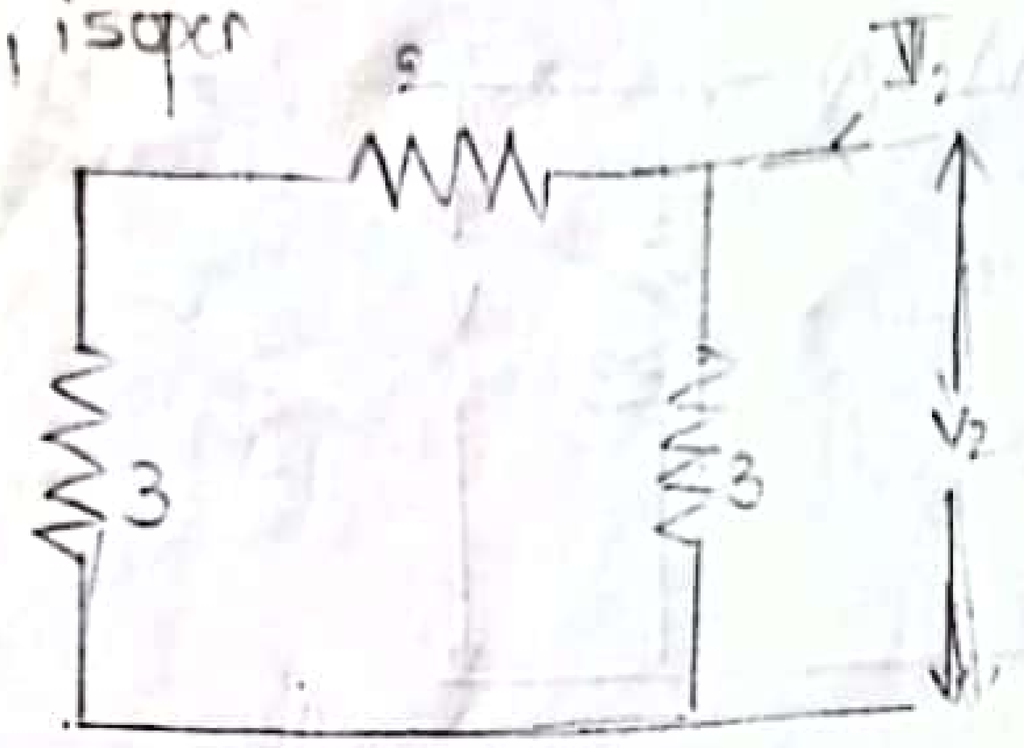
$I_x = \frac{3}{8} I_1$

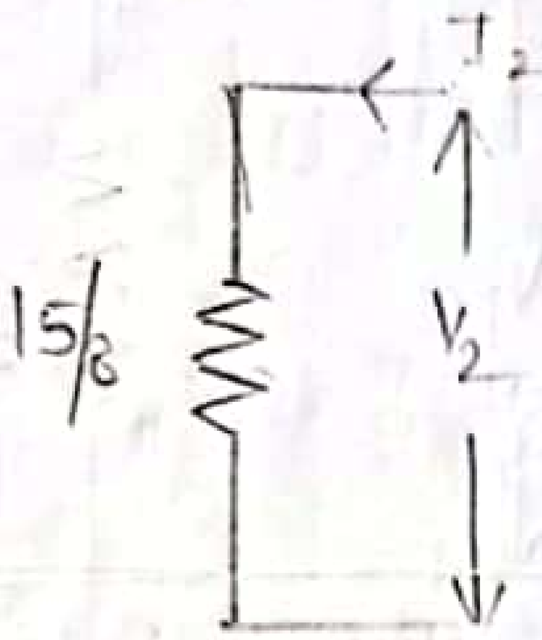
$V_{3\Omega} = \frac{3}{8} I_1 \times 3$

$= \frac{9}{8} I_1 = V_2$

$Z_{21}' = \frac{V_2}{I_1} = \frac{9/8\, \cancel{I_1}}{\cancel{I_1}} = 9/8$.

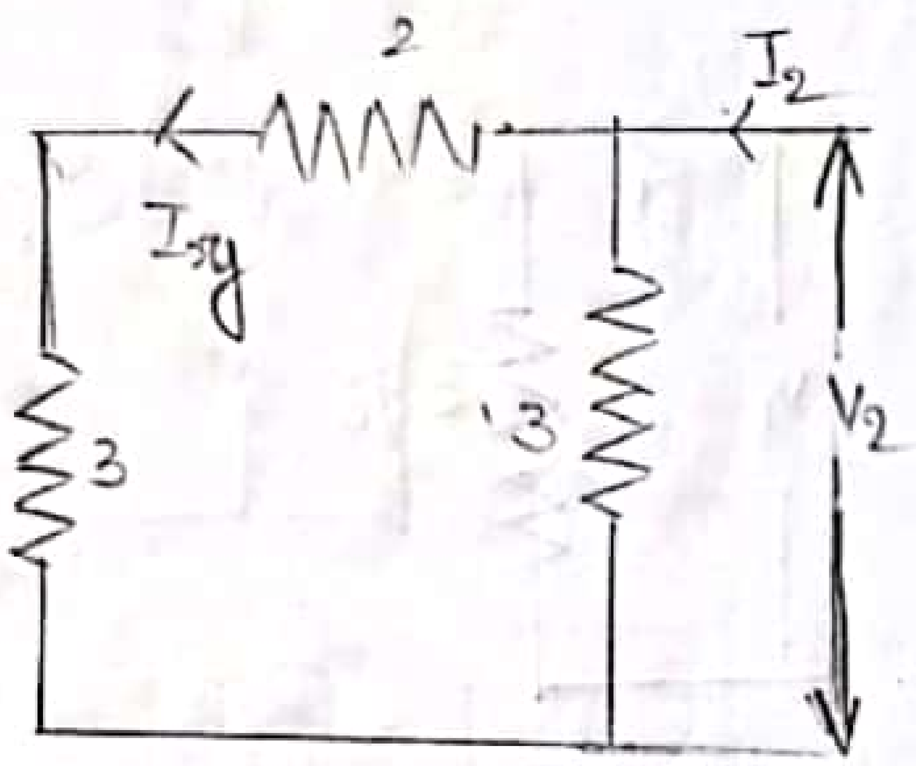
When I_1 is open





$$\frac{V_2}{I_2} = 15/8\,\Omega.$$

$$Z_{22} = 15/8\,\Omega.$$



$$I_x = \frac{3}{8} I_2$$

$$I_{xy} = \frac{3}{8} I_2$$

$$V_{3\Omega} = 3 I_{xy}$$
$$= 3 \times \frac{3}{8} I_2$$
$$= 9/8\, I_2. = V_1$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{9/8\, \cancel{I_2}}{\cancel{I_2}}$$
$$= 9/8\,\Omega.$$

Relation Ships:-

Z - Parameters Interms Of Y Parameters :-

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \longrightarrow ①$$

$$V_2 = Z_{21} I_1 + Z_{22} I_1 \longrightarrow ②.$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Delta Y = \begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix} = Y_{11} Y_{22} - Y_{12} Y_{21}$$

$$\Delta Y_1 = \begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix} = Y_{22} I_1 - Y_{12} I_2.$$

$$\Delta Y_2 = \begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix} = Y_{11} I_2 - Y_{21} I_1$$

$$V_1 = \frac{\Delta Y_1}{\Delta y} = \frac{Y_{22} I_1 - Y_{12} I_2}{\Delta y} \longrightarrow ③.$$

$$V_1 = \frac{Y_{22}}{\Delta y} I_1 - \frac{Y_{12}}{\Delta y} I_2 \longrightarrow ③.$$

$$V_2 = \frac{\Delta Y_2}{\Delta y} = \frac{Y_{11} I_2 - Y_{21} I_1}{\Delta y}$$

$$= \frac{Y_{11}}{\Delta y} I_2 - \frac{Y_{21}}{\Delta y} I_1 \longrightarrow ④.$$

from ①, ②, ③ & ④.

$$Z_{11} = \frac{Y_{22}}{\Delta y} \qquad Z_{21} = \frac{-Y_{21}}{\Delta y}.$$

$$Z_{12} = \frac{-Y_{12}}{\Delta y} \qquad Z_{22} = \frac{Y_{11}}{\Delta y}$$

Y- PARAMETERS INTERMS OF Z- PARAMETERS.

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—①}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—②.}$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

Now,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix} = Z_{11} Z_{22} - Z_{12} Z_{21}$$

$$\Delta Z_1 = \begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix} = V_1 Z_{22} - V_2 Z_{12}$$

$$\Delta Z_2 = \begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix} = Z_{11} V_2 - Z_{21} V_1$$

$$I_1 = \frac{\Delta Z_1}{\Delta Z} = \frac{V_1 Z_{22} - V_2 Z_{12}}{\Delta Z}$$

$$= \frac{V_1 Z_{22}}{\Delta Z} - \frac{V_2 Z_{12}}{\Delta Z} \quad \text{---} ③.$$

$$I_2 = \frac{\Delta Z_2}{\Delta Z} = \frac{Z_{11} V_2 - Z_{21} V_1}{\Delta Z}$$

$$= \frac{Z_{11} V_2}{\Delta Z} - \frac{Z_{21} V_1}{\Delta Z} \quad \text{---} ④.$$

Comparing ①, ② with ③, ④ respectively we get,

$$Y_{11} = \frac{Z_{22}}{\Delta Z} \qquad Y_{12} = \frac{-Z_{12}}{\Delta Z}.$$

$$Y_{21} = \frac{-Z_{21}}{\Delta Z} \qquad Y_{22} = \frac{+Z_{11}}{\Delta Z}.$$

Z-PARAMETERS IN TERMS OF H-PARAMETERS:-

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2.$$

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{---} ①$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{---} ②.$$

From ②

$$V_2 = \frac{I_2 - h_{21} I_1}{h_{22}} \quad \text{---} ③.$$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$$

put ③ in eqn ①.

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{I_2 - h_{21} I_1}{h_{22}}\right]$$

$$= h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1$$

$$= \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2.$$

$$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2. \quad \text{—④.}$$

$$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \quad \text{—⑤.}$$

Comparing z & h parameters we get.

$$Z_{11} = \frac{\Delta h}{h_{22}} \qquad Z_{12} = \frac{h_{12}}{h_{22}}$$

$$Z_{21} = \frac{-h_{21}}{h_{22}} \qquad Z_{22} = \frac{1}{h_{22}}.$$

H - PARAMETERS IN TERMS OF Z - PARAMETERS:-

$$V_1 = h_{11} I_1 + h_{12} V_2$$

$$I_2 = h_{21} I_1 + h_{22} V_2.$$

$$V_1 = Z_{11} I_1 + Z_{12} I_2. \quad \text{—①}$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—②.}$$

$$I_2 = \frac{V_2 - Z_{21} I_1}{Z_{22}} \quad \text{—③}$$

Substitute (3) in (1)

$$V_1 = Z_{11} I_1 + Z_{12}\left[\frac{V_2 - Z_{21} I_1}{Z_{22}}\right]$$

$$= Z_{11} I_1 + \frac{Z_{12} V_2}{Z_{22}} - \frac{Z_{12} Z_{21} I_1}{Z_{22}}$$

$$= \left[\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{22}}\right] I_1 + \frac{Z_{12}}{Z_{22}} V_2 .$$

$$\therefore V_1 = \frac{\Delta Z}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2 \quad \text{—(4)}$$

$$I_2 = \frac{1}{Z_{22}} V_2 - \frac{Z_{21}}{Z_{22}} I_1 \quad \text{—(5)}.$$

Comparing equation (5) & (4) with h-parameters

$$h_{11} = \frac{\Delta Z}{Z_{22}} \qquad h_{12} = \frac{Z_{12}}{Z_{22}}.$$

$$h_{21} = \frac{-Z_{21}}{Z_{22}} \qquad h_{22} = \frac{1}{Z_{22}}.$$

H-PARAMETERS IN TERMS OF Y-PARAMETERS:-

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{—(1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{—(2)}$$

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad \text{—(3)}$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad \text{—(4)}$$

from (3)

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$V_1 = \frac{I_1 - Y_{12} V_2}{Y_{11}} \quad \text{—(5)}.$$

Put substitute ⑤ in ④

$$I_2 = Y_{21}\left[\frac{I_1 - Y_{12}V_2}{Y_{11}}\right] + Y_{22}V_2.$$

$$= \frac{Y_{21}}{Y_{11}}I_1 - \frac{Y_{21}Y_{12}V_2}{Y_{11}} + Y_{22}V_2$$

$$I_2 = \frac{Y_{21}}{Y_{11}}I_1 + V_2\left[\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{11}}\right] \quad \text{— ⑥}$$

$$⑤ \Rightarrow V_1 = \frac{1}{Y_{11}}I_1 - \frac{Y_{12}}{Y_{11}}V_2$$

Comparing ⑤ & ⑥ with ① & ②. we get,

$$h_{11} = \frac{1}{Y_{11}} \qquad h_{12} = \frac{-Y_{12}}{Y_{11}}$$

$$h_{21} = \frac{Y_{21}}{Y_{11}} \qquad h_{22} = \frac{\Delta Y}{Y_{11}}$$

Y- parameters Interms Of H- parameters:-

$$I_1 = Y_{11}V_1 + Y_{12}V_2 \quad \text{— ①}$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2 \quad \text{— ②}$$

$$V_1 = h_{11}I_1 + h_{12}V_2 \quad \text{— ③}$$

$$I_2 = h_{21}I_1 + h_{22}V_2 \quad \text{— ④}$$

From ③.

$$I_1 = \frac{V_1 - h_{12}V_2}{h_{11}}$$

$$= \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2 \quad \text{— ⑤}$$

Substitute (5) in (4)

$$I_2 = h_{21}\left[\frac{V_1}{h_{11}} - \frac{h_{12}}{h_{11}}V_2\right] + h_{22}V_2$$

$$= \frac{h_{21}}{h_{11}}V_1 - \frac{h_{12}h_{21}}{h_{11}}V_2 + h_{22}V_2$$

$$I_2 = \frac{h_{21}}{h_{11}}V_1 + V_2\left[\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}\right] \quad -(6)$$

$$(5) \Rightarrow I_1 = \frac{1}{h_{11}}V_1 - \frac{h_{12}}{h_{11}}V_2$$

comparing (5) & (6) with (1) & (2) we get,

$$Y_{11} = \frac{1}{h_{11}} \qquad Y_{12} = \frac{-h_{12}}{h_{11}}$$

$$Y_{21} = \frac{h_{21}}{h_{11}} \qquad Y_{22} = \frac{\Delta h}{h_{11}}$$

* Relation between A,B,C,D , Z And Y-PARAMETERS :-

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

when $I_2 = 0$ $\quad A = \frac{V_1}{V_2}$

$$V_1 = Z_{11}I_1 \qquad A = \frac{V_1}{V_2} = \frac{Z_{11}\cancel{I_1}}{Z_{21}\cancel{I_1}}$$

$$V_2 = Z_{21}I_1 \qquad = \frac{Z_{11}}{Z_{21}}.$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{[Y_{22}/\Delta Y]}{[-Y_{21}/\Delta Y]} = \frac{-Y_{22}}{Y_{21}}.$$

$$\therefore A = \frac{Z_{11}}{Z_{21}} = \frac{-Y_{22}}{Y_{21}}.$$

$$C = \frac{I_1}{V_2} = \frac{\cancel{I_1}}{Z_{21}\cancel{I_1}} = \frac{1}{Z_{21}}$$

$$C = \frac{1}{Z_{21}} = \frac{1}{\frac{-Y_{21}}{\Delta Y}} = \frac{-\Delta Y}{Y_{21}}$$

when $V_2 = 0$ $\quad B = \frac{-V_1}{I_2}$

$$Y_{21} = \frac{I_2}{V_1} \Rightarrow V_1 = \frac{I_2}{Y_{21}}$$

Now,

$$B = \frac{-V_1}{I_2} = -\frac{\cancel{I_2}/Y_{21}}{\cancel{I_2}}$$

$$B = -\frac{1}{Y_{21}}$$

$$\therefore B = -1/Y_{21}$$

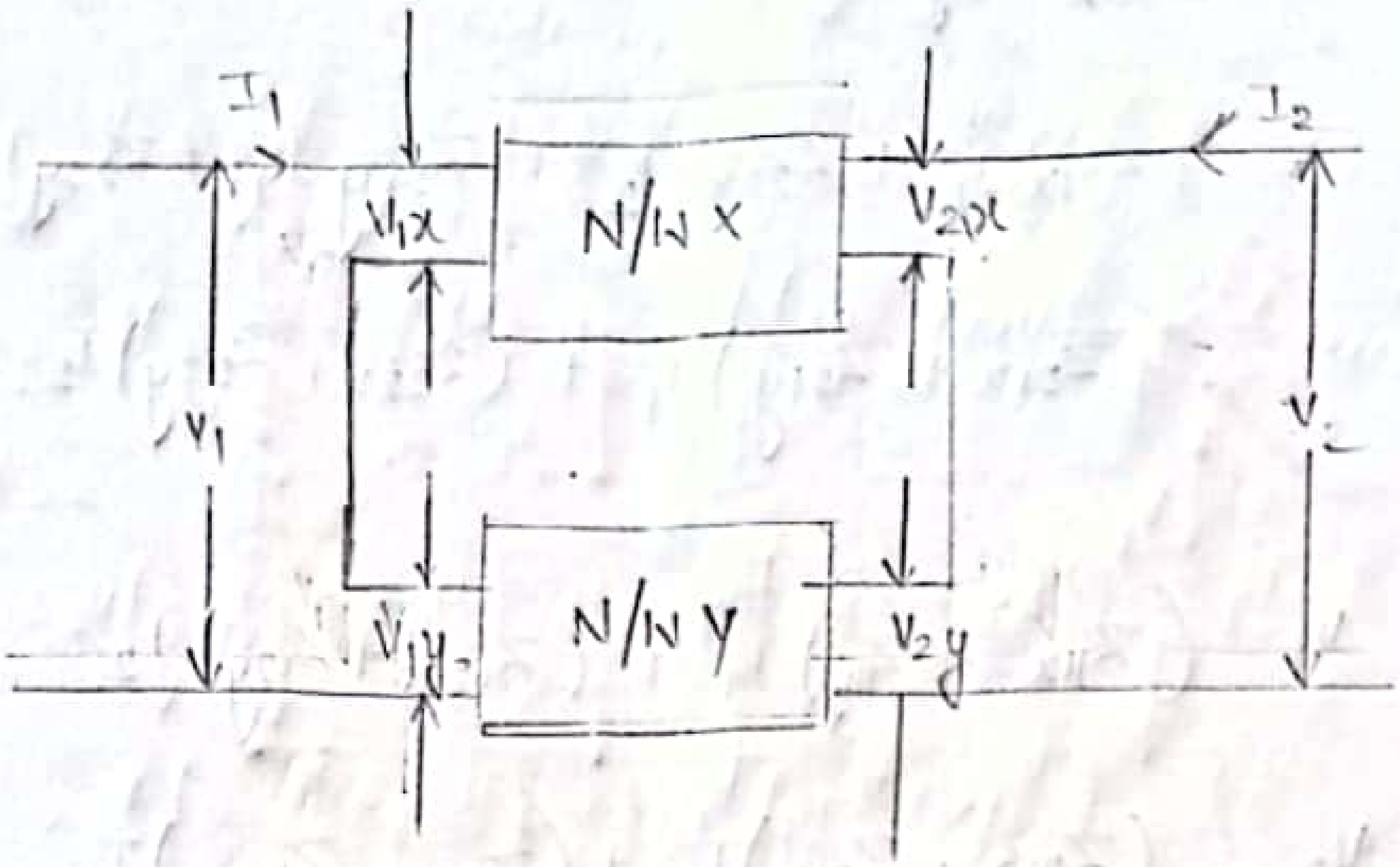
$$D = \frac{-I_1}{I_2} = \frac{-Y_{11}}{Y_{21}} = \frac{-Z_{22}/\Delta Z}{-Z_{21}/\Delta Z} = \frac{Z_{22}}{Z_{21}}$$

$* A = \frac{Z_{11}}{Z_{21}} = \frac{-Y_{22}}{Y_{21}}$

$* B = -\frac{1}{Y_{21}}$ $\qquad * D = \frac{-Y_{11}}{Y_{21}} = \frac{Z_{22}}{Z_{21}}$

$* C = \frac{-\Delta Y}{Y_{21}}$

Series Connection:-



When the 2 two ports networks are connected in sieres then,

$$V_{1x} = Z_{11x} I_1 + Z_{12x} I_2$$

$$V_{2x} = Z_{21x} I_1 + Z_{22x} I_2$$

for N/w y.

$$V_{1y} = Z_{11y} I_1 + Z_{12y} I_2$$

$$V_{2y} = Z_{21y} I_1 + Z_{22y} I_2$$

Voltage across V_1 is $V_1 = V_{1x} + V_{1y}$

across V_2 is $\therefore V_2 = V_{2x} + V_{2y}$

Now

$$V_1 = V_{1x} + V_{1y}$$

$$= Z_{11x} I_1 + Z_{12x} I_2 + Z_{11y} I_1 + Z_{12y} I_2$$

$$= (Z_{11x} + Z_{11y}) I_1 + (Z_{12x} + Z_{12y}) I_2$$

$$V_2 = V_{2x} + V_{2y}$$

$$= Z_{21x} I_1 + Z_{22x} I_2 + Z_{21y} I_1 + Z_{22y} I_2$$

$$= (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2.$$

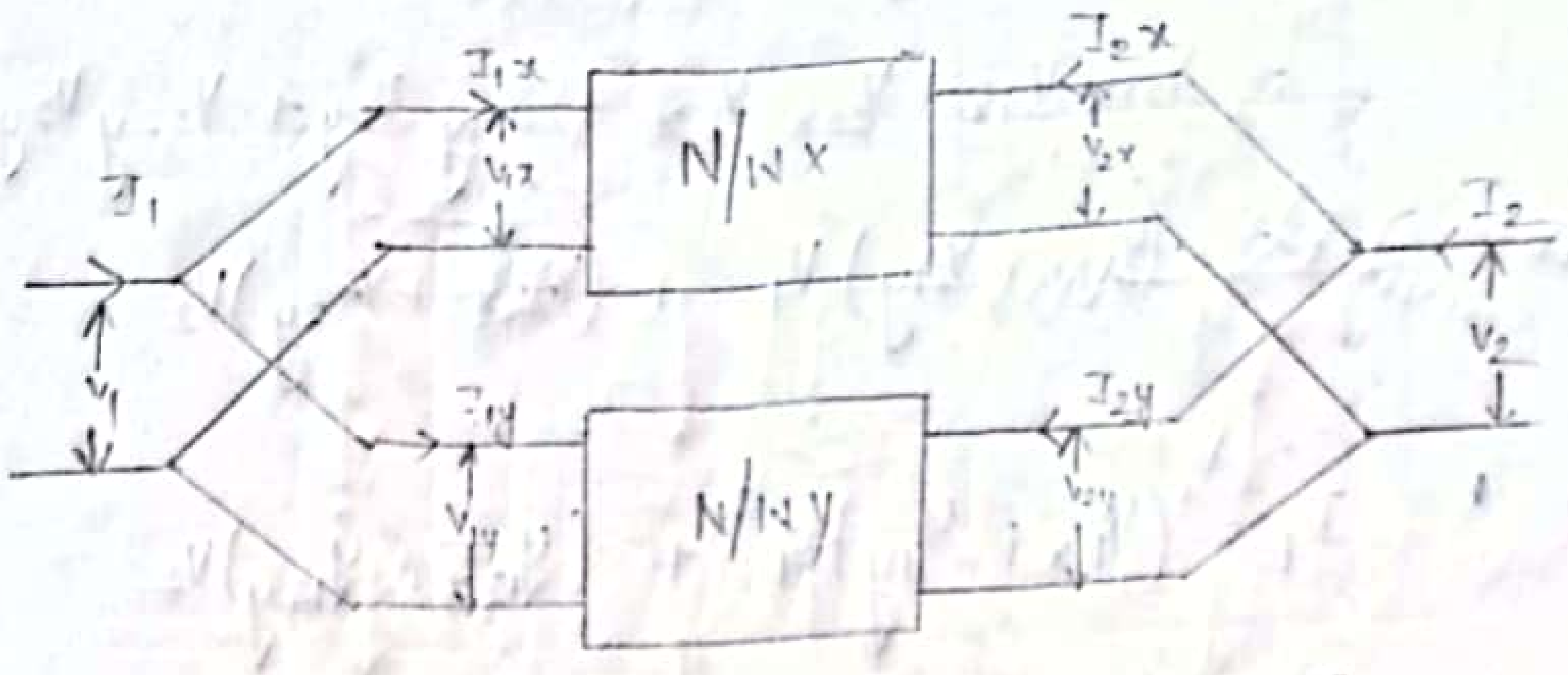
$$V_1 = (Z_{11x} + Z_{11y}) I_1 + (Z_{12x} + Z_{12y}) I_2$$

$$V_2 = (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2.$$

$Z_{11} = Z_{11x} + Z_{11y}$	$Z_{21} = Z_{21x} + Z_{12y}$
$Z_{22} = Z_{12x} + Z_{12y}$	$Z_{22} = Z_{22x} + Z_{22y}.$

From the above expressions we can say that, the impedence is equal to the individual impedences of x and y.

PARALLEL CONNECTION :-



When the 2 two ports networks are connected in parallel, then

$$I_1 = I_1x + I_1y \qquad V_1 = V_1x = V_1y$$

$$I_2 = I_2x + I_2y \qquad V_2 = V_2x = V_2y$$

from N/w x.

$$I_{1x} = Y_{11x} V_{1x} + Y_{12x} V_{2x}$$

$$I_{2x} = Y_{21x} V_{1x} + Y_{22x} V_{2x}$$

from N/w y

$$I_{1y} = Y_{11y} V_{1y} + Y_{12y} V_{2y}$$

$$I_{2y} = Y_{21y} V_{1y} + V_{22y} V_{2y}.$$

Now,

$$I_1 = I_1x + I_1y.$$

$$= Y_{11x} V_{1x} + Y_{12x} V_{2x} + Y_{11y} V_{1y} + Y_{12y} V_{2y}$$

$$= (Y_{11x} + Y_{11y}) V_1 + (Y_{12x} + Y_{12y}) V_2$$

$$I_2 = I_{2x} + I_{2y}$$

$$= Y_{21x} V_{1x} + Y_{22x} V_{2x} + Y_{21y} V_{1y} + Y_{22y} V_{2y}$$

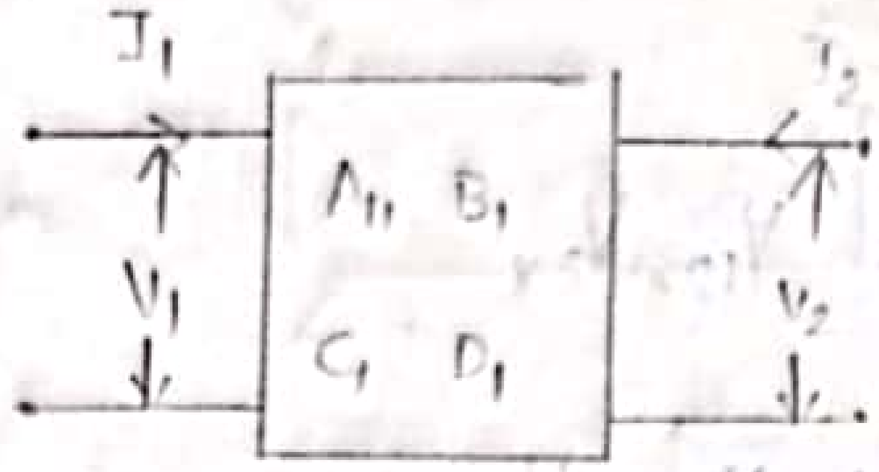
$$= (Y_{21x} + Y_{21y}) V_1 + (Y_{22x} + Y_{22y}) V_2$$

$$I_1 = (Y_{11x} + Y_{11y}) V_1 + (Y_{12x} + Y_{12y}) V_2$$

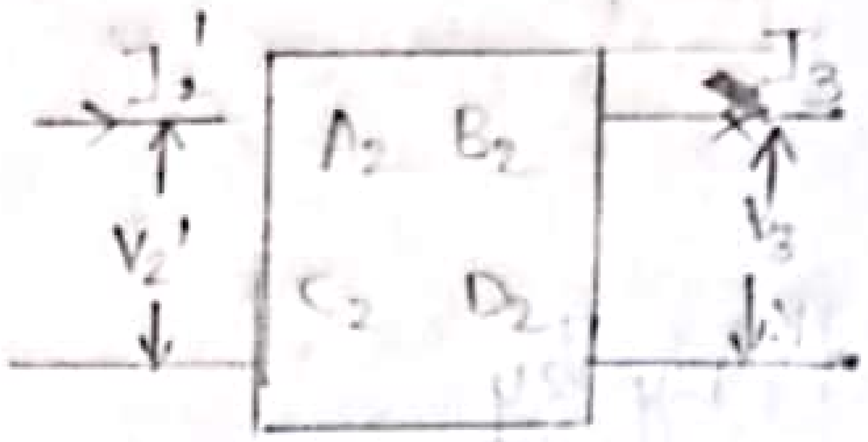
$$I_2 = (Y_{21x} + Y_{21y}) V_1 + (Y_{22x} + Y_{22y}) V_2.$$

$Y_{11} = Y_{11x} + Y_{11y}$	$Y_{21} = Y_{21x} + Y_{21y}$
$Y_{12} = Y_{12x} + Y_{12y}$	$Y_{22} = Y_{22x} + Y_{22y}$.

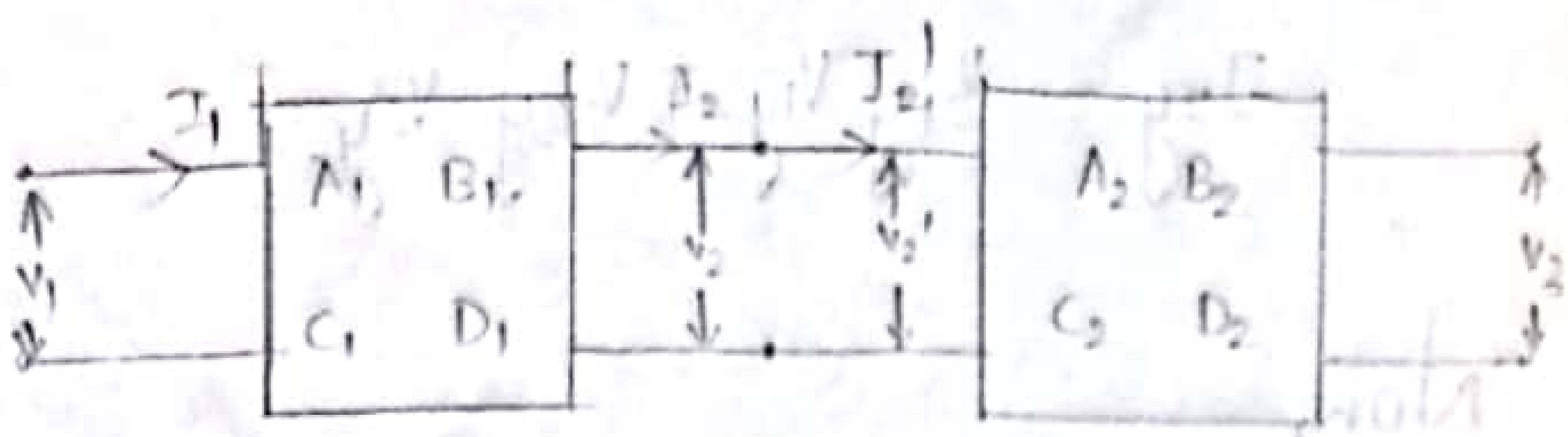
CASCADE CONNECTION:-



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$



$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$



from figure, $I_2 = I_2'$

$V_2 = V_2'$

Now,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1A_2 + B_1C_2 & A_1B_2 + B_1D_2 \\ C_1A_2 + D_1C_2 & B_2C_1 + D_1D_2 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

comparing above Matrix with

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_3 & B_3 \\ C_3 & D_3 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

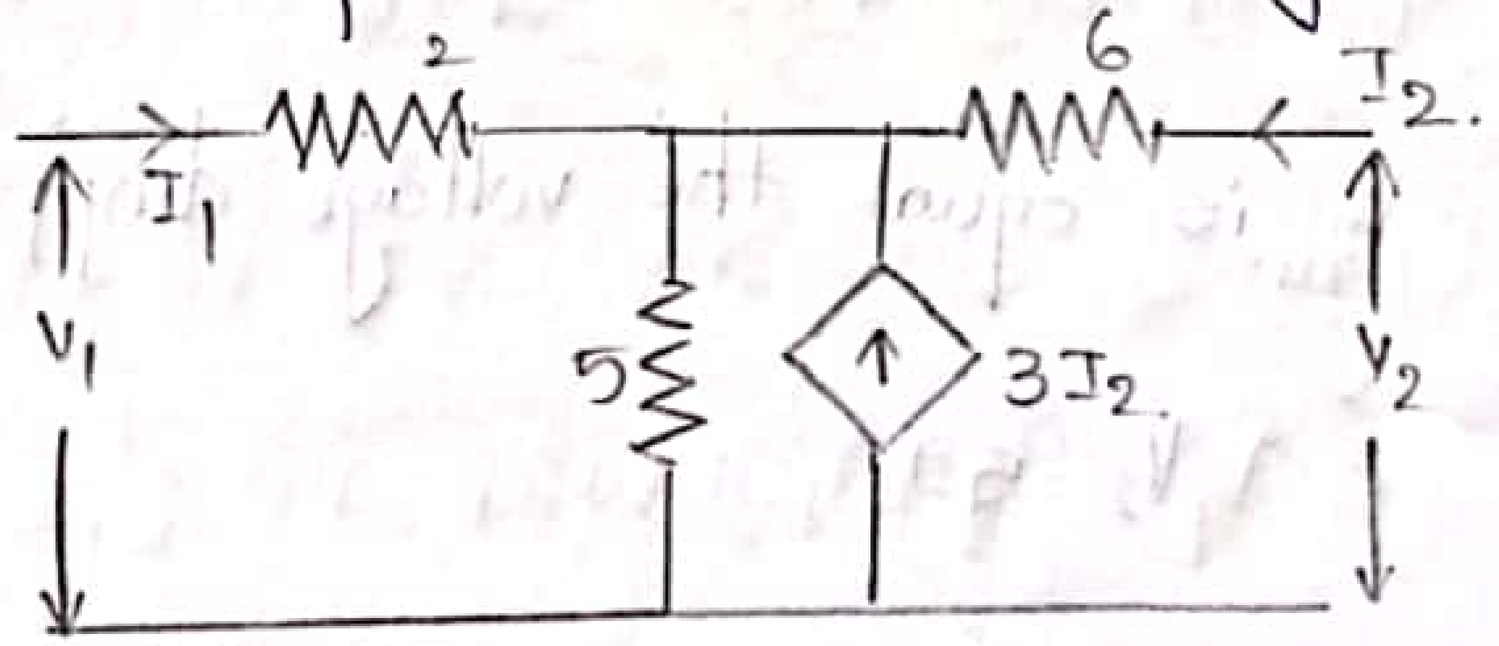
$$A_3 = A_1A_2 + B_1C_2$$

$$B_3 = A_1B_2 + B_1D_2$$

$$C_3 = C_1A_2 + D_1C_2$$

$$D_3 = B_2C_1 + D_1D_2$$

1. Find the z-parameters for the given network.



Ans: W.K.T

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

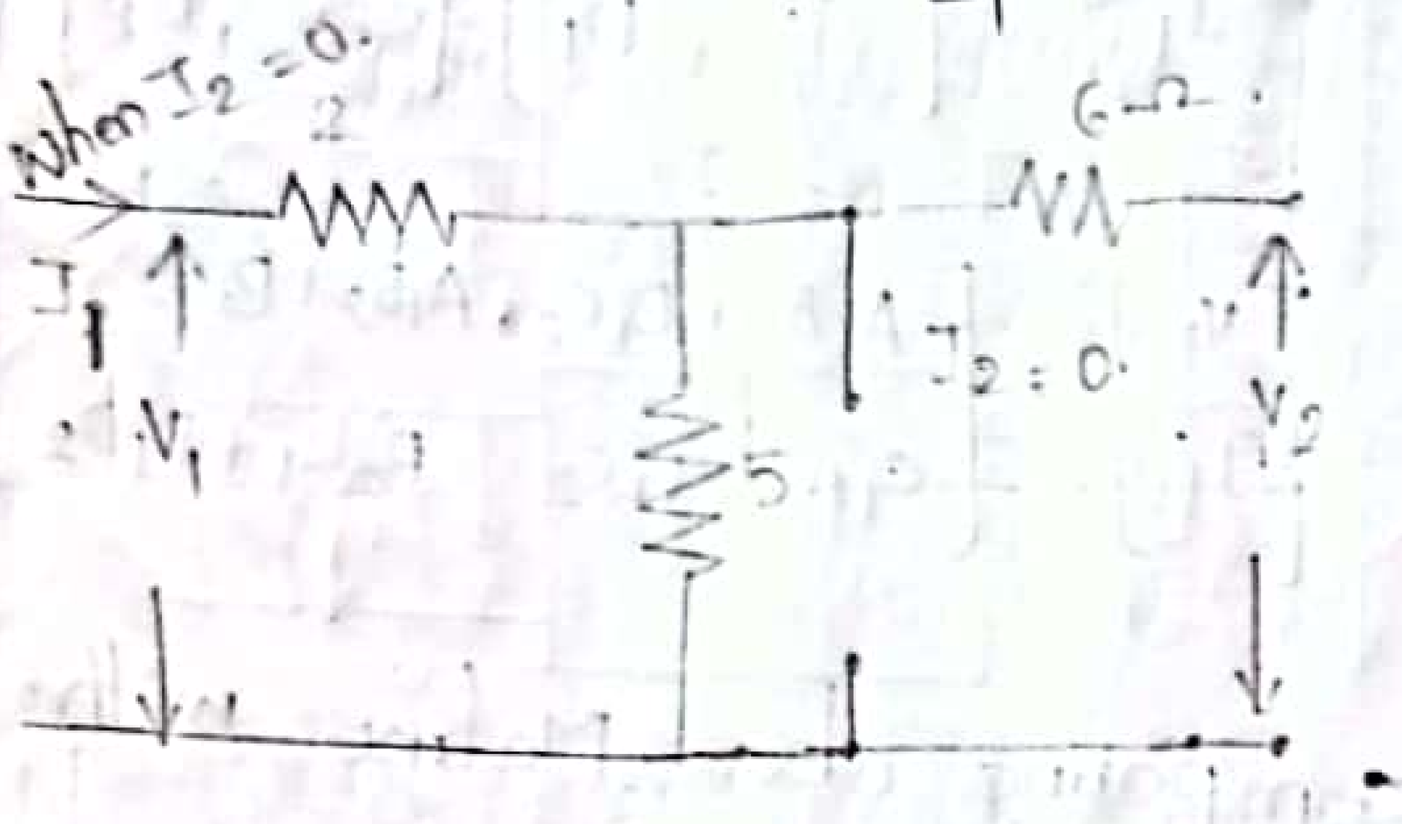
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

When port$_2$ is opencircuited. $I_2 = 0$

$$V_1 = Z_{11} I_1 \quad V_2 = Z_{21} I_1$$

$$Z_{11} = \frac{V_1}{I_1} \qquad Z_{21} = \frac{V_2}{I_1}$$

Now, circuit when $I_2 = 0$.



from figure

$$V_1 = I_1 Z_{eq} = I_1 (7)$$

$$V_1 = 7 I_1$$

$$Z_{11} = \frac{V_1}{I_1} = \frac{7 \cancel{I_1}}{\cancel{I_1}} = (7\ \Omega)$$

When

$$Z_{21} = \frac{V_2}{I_1}.$$

V_2 is equal the voltage dropped across 5Ω

$$V_2 = 5 I_1$$

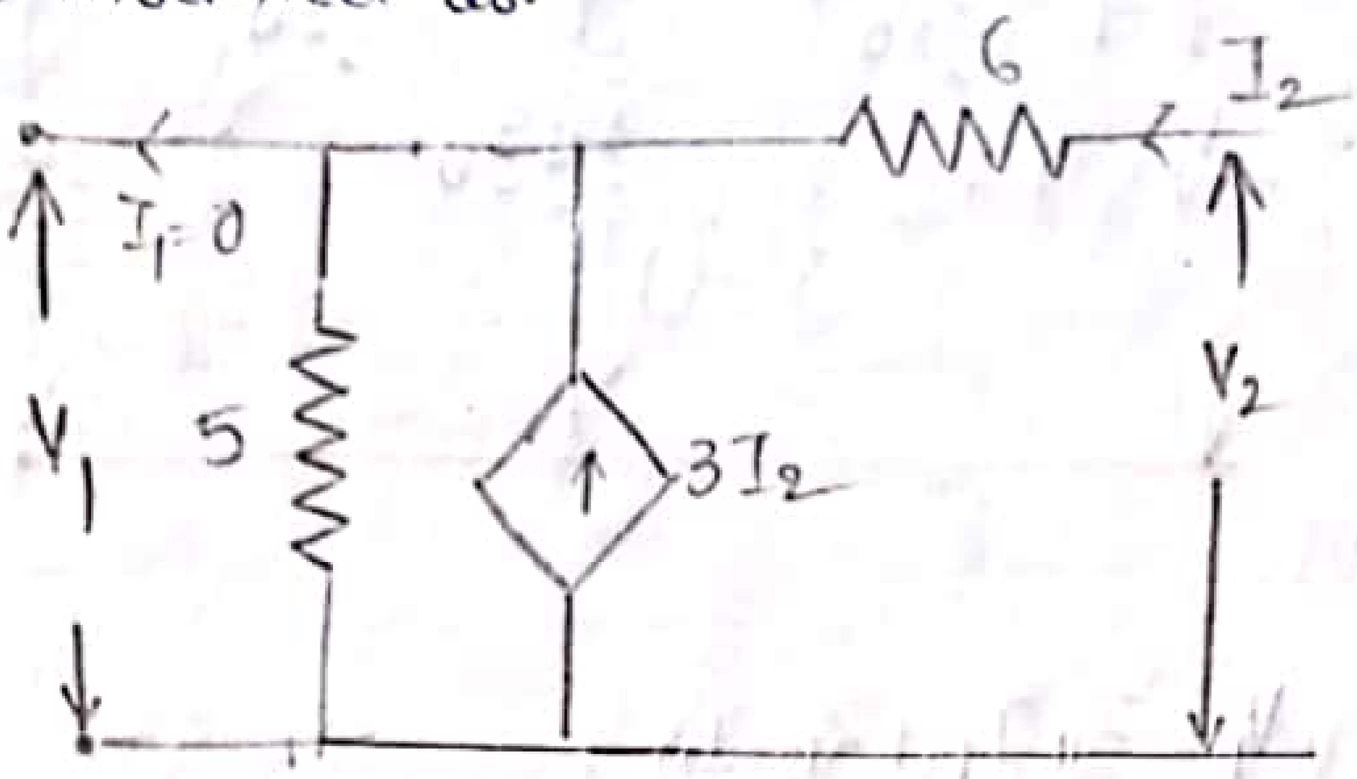
$$\frac{V_2}{I_1} = 5\ \Omega.$$

$$\therefore Z_{21} = 5\ \Omega.$$

When I_1 is made $= 0$

i.e., port 1 is open circuited.

circuit is modified as.



Now, $Z_{12} = \frac{V_1}{I_2}$; $Z_{22} = \frac{V_2}{I_2}$

when the voltage developed across 5Ω is V_1. voltage.

Now,
$$V_1 = 5(3I_2 + I_2)$$
$$= 5(4I_2)$$
$$V_1 = 20 I_2$$

$$Z_{12} = \frac{V_1}{I_2} = \frac{20\cancel{I_2}}{\cancel{I_2}} = 20\Omega.$$

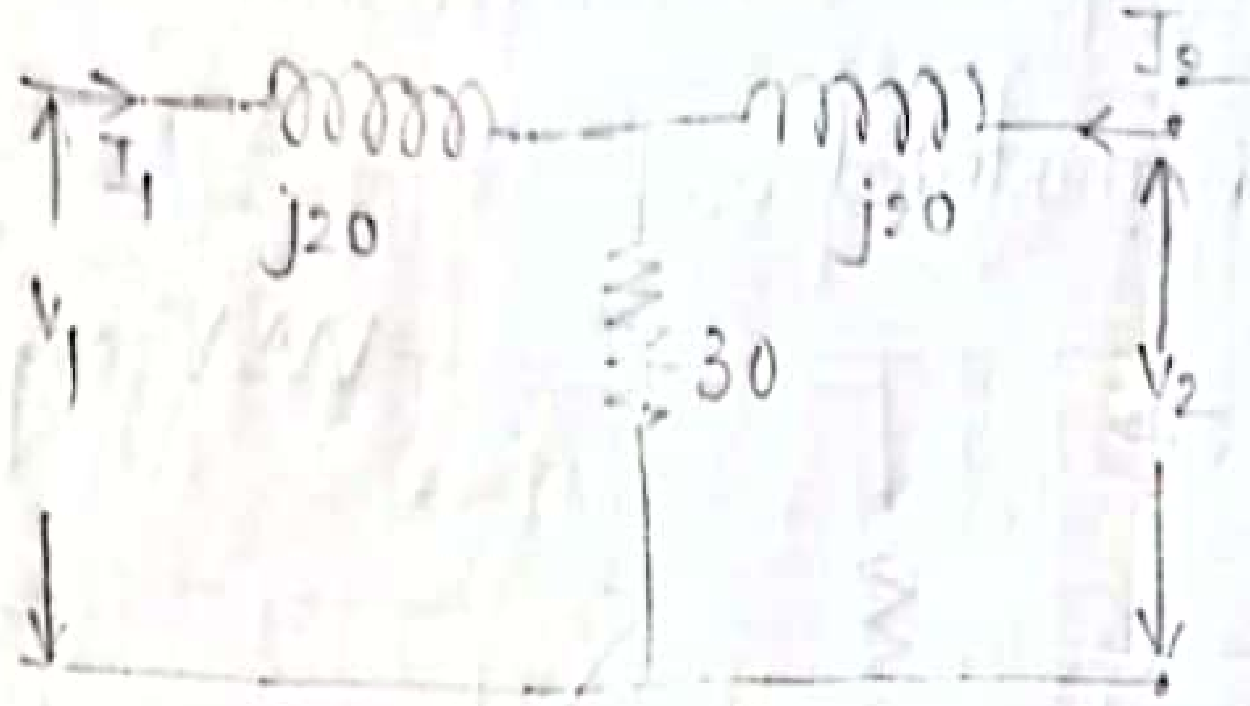
V_2 is the voltage drop across 6Ω & $3I_2$ (or) 5Ω resistor.

$$\therefore V_2 = 20 I_2 + 6 I_2.$$
$$V_2 = 26 I_2$$

$$\therefore Z_{22} = \frac{V_2}{I_2} = \frac{26\cancel{I_2}}{\cancel{I_2}} = 26\Omega.$$

$\therefore Z_{11} = 7\Omega$, $Z_{12} = 20\Omega$, $Z_{21} = 5\Omega$, $Z_{22} = 26\Omega$.

Find out Z- parameters:



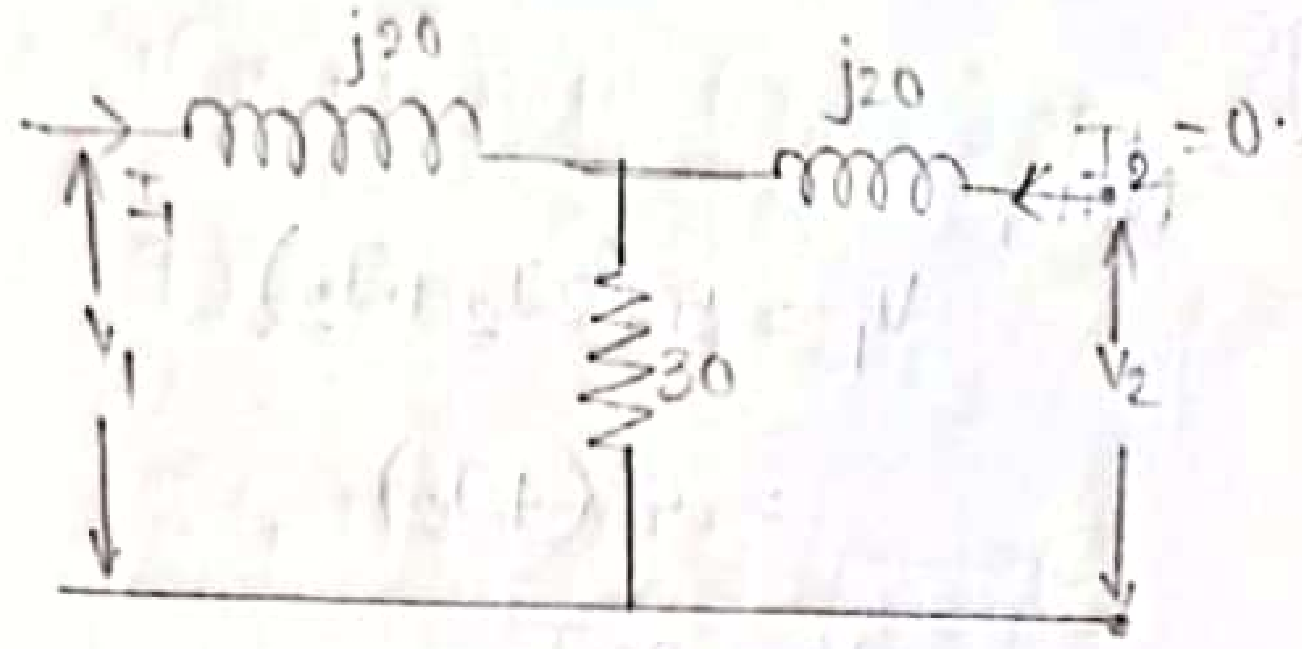
Ans

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2.$

when port 2 is open closed

$I_2 = 0.$



$V_1 = Z_{11} I_1 \quad \& \quad V_2 = Z_{21} I_1$

from figure

$V_1 = j20 I_1 + 30 I_1$

$Z_{11} = \frac{V_1}{I_1} = \frac{(30+j20)\cancel{I_1}}{\cancel{I_1}}$

$= 30 + j20.$

V_2 ~~Z_{21}~~ is voltage drop across 30Ω.

$V_{30\Omega} = 30 I_1.$

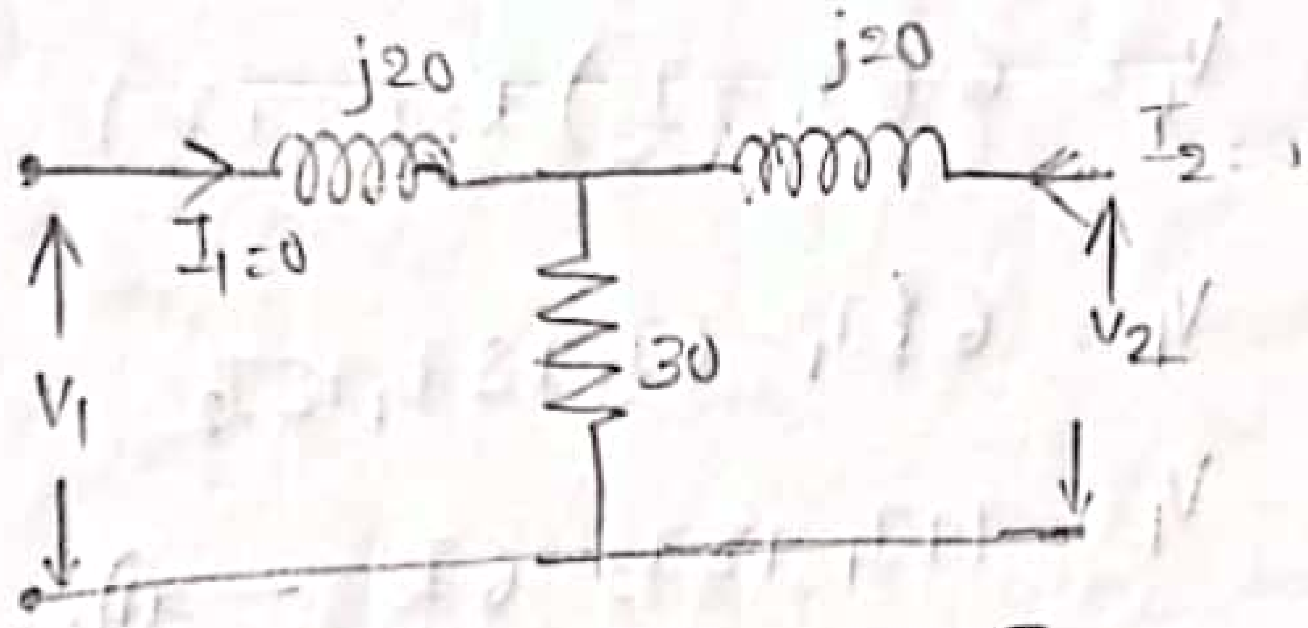
$V_2 = 30 I_1$

$$z_{21} = \frac{V_2}{I_1} = \frac{30 \cancel{I_1}}{\cancel{I_1}}$$

$$= 30.$$

When port 1 is closed.

$I_1 = 0.$



$V_1 = z_{12} I_2$ $\qquad$ $V_2 = z_{22} I_2$

from figure.

$$V_2 = j20 I_2 + 30 I_2$$

$$z_{22} = \frac{V_2}{I_2} = \frac{(j20+30)\cancel{I_2}}{\cancel{I_2}}$$

$$z_{22} = (30 + 20j)\,\Omega$$

V_1 is voltage drop across $30\,\Omega$.

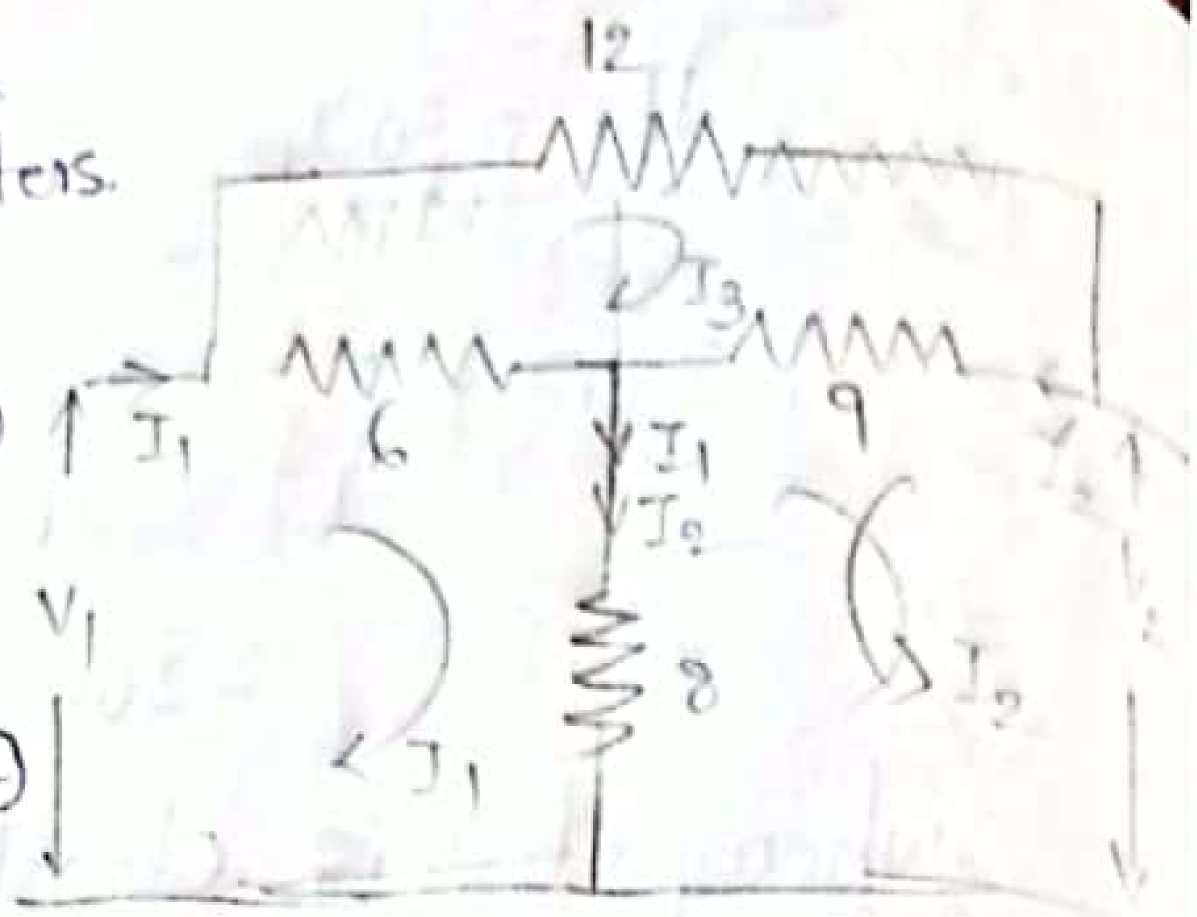
$$V_1 = 30 I_2$$

$$z_{12} = \frac{V_1}{I_2}$$

$$= \frac{30 \cancel{I_2}}{\cancel{I_2}}$$

$$= 30\,\Omega$$

Find out z-parameters.



Sol:- W.K.T

$V_1 = z_{11} I_1 + z_{12} I_2 \rightarrow ①$

$V_2 = z_{21} I_1 + z_{22} I_2 \rightarrow ②$

from KVL.

$V_1 = 6(I_1 - I_3) + 8(I_1 + I_2)$

$V_1 = 6 I_1 - 6 I_3 + 8 I_1 + 8 I_2$

$V_1 = 14 I_1 + 8 I_2 - 6 I_3 \longrightarrow ①.$

$V_2 = 9(I_2 + I_3) + 8(I_1 + I_2)$

$= 9 I_2 + 9 I_3 + 8 I_1 + 8 I_2$

$V_2 = 8 I_1 + 17 I_2 + 9 I_3 \longrightarrow ②$

$-12 I_3 - 9(I_3 + I_2) - 6(I_3 - I_1) = 0.$

$-12 I_3 - 9 I_3 - 9 I_2 - 6 I_3 + 6 I_1 = 0.$

$-27 I_3 - 9 I_2 + 6 I_1 = 0$

$6 I_1 = 9 I_2 + 27 I_3$

$I_1 = \frac{3}{2} I_2 + \frac{9}{2} I_3.$

$I_3 = -\frac{I_2}{3} + \frac{2 I_1}{9}$

Substitute I_3 in V_1

$$V_1 = 14I_1 + 8I_2 - 6\left(-\frac{I_2}{3} + \frac{2I_1}{9}\right)$$

$$= 14I_1 + 8I_2 + 2I_2 - \frac{4I_1}{3}$$

$$= \frac{38I_1}{3} + 10I_2.$$

Substitute I_3 in V_2

$$V_2 = 8I_1 + 17I_2 + 9I_3$$

$$= 8I_1 + 17I_2 + 9\left(\frac{2I_1}{9} - \frac{I_2}{3}\right)$$

$$= 8I_1 + 17I_2 + 2I_1 - 3I_2$$

$$= 10I_1 + 14I_2.$$

$$\therefore V_1 = \frac{38}{3}I_1 + 10I_2 \quad \text{---} \quad (3)$$

$$V_2 = 10I_1 + 14I_2. \quad \text{---} \quad (4).$$

Comparing with (1), (2). and (3) & (4)

We get

$$Z_{11} = \frac{38}{3} \quad ; \quad Z_{12} = 10$$

$$Z_{21} = 10 \quad ; \quad Z_{22} = 14.$$