UNIT – V

TWO-PORT NETWORKS

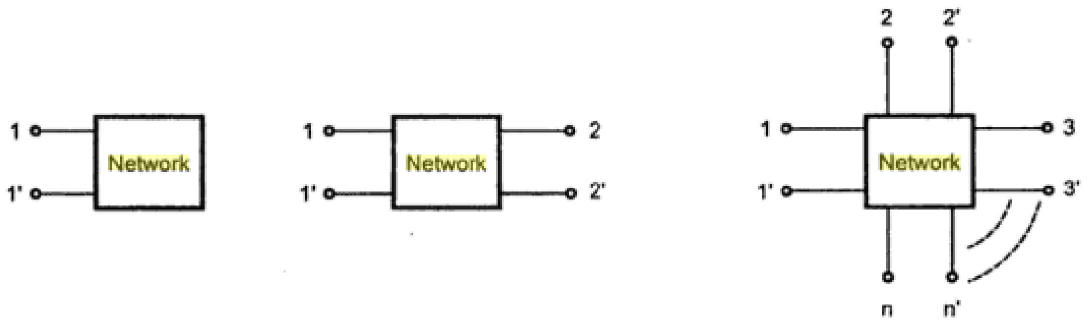
Introduction

A pair of terminals at which an electrical signal may enter or leave a network is called a **port**. The terminals or port is required for connecting input excitation to the network. It is also required for connecting some other networks such as load. The terminals are most useful for making measurements. In general, the minimum number of terminals required is two.

A network having only one pair of terminals or one port is called **one port network.** The Fig. 7.1 (a) shows a one port network.

A network consisting two pairs of terminals is called **two port network** as shown in the Fig. 7.1 (b). The terminals are generally named as 1-1′ and 2-2′ In general, a port designated 1-1′, is connected to the driving energy source while the other port designated 2-2′ is connected to the load. A port at which energy source is connected is called **driving point** of the network or **input port**. A port at which load is connected is called **output port**.

Fig. 7.1 (c) represents a network with n-port called **n-port network**. In such networks, generally one port is connected to energy source, one port is connected to load and other ports may be connected to the different networks.



(a) One port network **(b) Two port network** **(c) Multiport network**

Fig. 7.1

Two Port Network Parameters

Consider a two port network as shown in the Fig. 7.2. In all there are four variables; two voltages and two currents. In general, any two port network has one pair of voltage and current at each port.

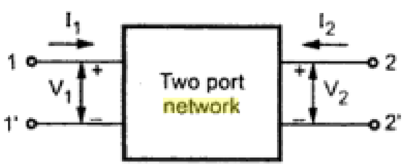


Fig. 7.2 Two port network

Assuming the variables at input and output ports as transformed quantities, the voltage and current at input terminals are V_1 and I_1 while at output terminals are V_2 and I_2, as shown in the Fig. 7.2. The directions of both the currents I_1 and I_2 are assumed to be flowing into the network. Such currents entering the network are assumed to be positive. Also the voltage have the positive. Also the voltages have the positive reference polarities.

Before starting the discussion on the two port network parameters, let us make certain assumptions necessary in the analysis. The assumptions are as follows.

1. The voltages and currents in the network present inside a box are not available for the measurements.
2. The network should consists only **linear elements** along with **dependent sources** if any. But independent or active sources should not be present in the network inside box.

3. If the network inside box consists energy storing elements such as inductor and capacitor, then **initial conditions** of such elements should be zero.

In order to describe the relationship between port voltages and currents, one requires the linear equations equal to the number of ports. So in two port network analysis, we will require two linear equations interms of four above mentioned variables. We can obtain these equations by considering two variables as dependent variables while other as independent variables. As the network consists only linear elements, the linear relationship can be obtained by writing two variables interms of other two variables. There are six possible ways of selecting two independent variables out of four variables. Thus there are six different pairs of equations defining their own sets of parameters such as impedance (z), admittance (y), hybrid (h), inverse hybrid (g), transmission and inverse transmission parameters. In next sections, we will discuss z-parameters, y-parameters, hybrid parameters and transmission parameters in detail.

z-Parameters

These are also called **impedance parameters.** These are obtained by expressing voltages at two ports in terms of currents at two ports. Thus, **currents I_1 and I_2 are independent variables**; while **V_1 and V_2 are dependent variables.**

$$V_1 = f_1(I_1, I_2)$$

$$V_2 = f_2(I_1, I_2)$$

In equation form, above relations can be written as,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \ldots (1)$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \ldots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

or $$[V] = [z]\,[I]$$

The individual z-parameters can be obtained by assigning values of independent variables to be zero. The z-parameters can be defined as follows.

[A] Let $I_2 = 0$; port - 2 is open circuited.

From equation (1),

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2 = 0} \Omega$$

The parameter z_{11} is called **open circuit driving point input impedance.**

From equation (2),

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2 = 0} \Omega$$

The parameters z_{21} is called **open circuit forward transfer impedance.**

[B] Let $I_1 = 0$; i.e. **port - 1 is open circuited.**

Form equation (1),

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1 = 0} \quad \Omega$$

The parameter z_{12} is called **open circuit reverse transfer impedance.**

From equation (2),

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1 = 0} \quad \Omega$$

The parameter z_{22} is called **open circuit driving point output impedance.**

these parameters are defined only when the current in one of the ports is zero. This corresponds to the conditions that the one of the ports is open circuited. Hence z-parameters are named as **open circuit impedance parameters.**

Example 7.1 : *Find the z-parameters for the network shown in following Fig. 7.4.*

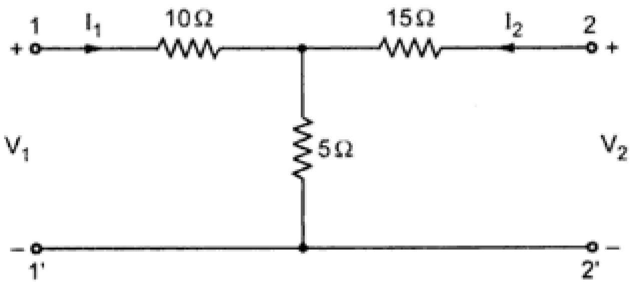


Fig. 7.4

Solution : By definition z-parameters are given as,

$$V_1 = z_{11} I_1 + z_{12} I_2$$

$$V_2 = z_{21} I_1 + z_{22} I_2$$

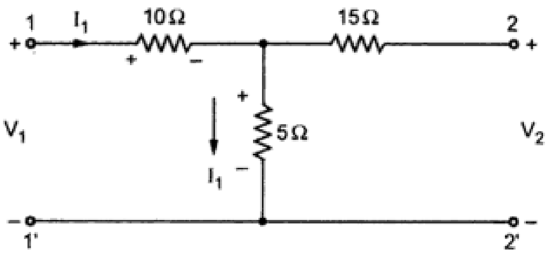


Fig. 7.4 (a)

(A) Let $I_2 = 0$, i.e. port 2 is open circuited as shown in the Fig. 7.4 (a).

As port 2 is open circuited, the current flowing through 5 Ω is also I_1. Note that no current will flow through 15 Ω as it is connected to open terminals.

Applying KVL at input side, we get,

$$-10 I_1 - 15 I_1 + V_1 = 0$$

$$\therefore \quad V_1 = 15 I_1$$

$$\therefore \quad z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = 15\ \Omega$$

From the Fig. 7.4 (a), we can write,

$$V_2 = 5 I_1$$

$$\therefore \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = 5\ \Omega$$

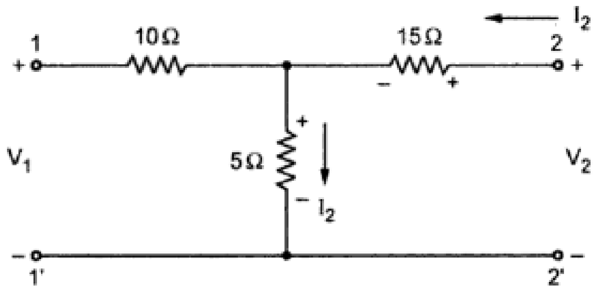


Fig. 7.4 (b)

(B) Let $I_1 = 0$, i.e. port 1 is open circuited as shown in the Fig. 7.4 (b).

As port 1 is open circuited, the current flowing through 5 Ω is also I_2. Note that no current will flow through 10 Ω as it is connected to open terminals.

Applying KVL at output side, we get,

$$-15 I_2 - 5 I_2 + V_2 = 0$$

$$\therefore \quad V_2 = 20 I_2$$

$$\therefore \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = 20\ \Omega$$

From the Fig. 7.4 (b), we can write,

$$V_1 = 5 I_2$$

$$\therefore \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = 5\ \Omega$$

Hence z-parameters of the given network are

$$[z] = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ 5 & 20 \end{bmatrix} \Omega$$

y-Parameters

These are also called **admittance parameters**. These are obtained by expressing currents at two ports in terms of voltages at two ports. Thus, **voltages V_1 and V_2 are independent variables, while I_1 and I_2 are dependent variables**. Thus, we have

$$I_1 = f_1(V_1, V_2)$$

$$I_2 = f_2(V_1, V_2)$$

In equation form, above relations can be written as,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{... (1)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{... (2)}$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

or $\quad [I] = [y]\,[V]$

The individual y-parameters are defined as follows,

[A] Let $V_2 = 0$ **i.e. port-2 is short circuited.**

From equation (1)

$$\boxed{y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \ \mho}$$

The parameter y_{11} is called **short circuit driving point input admittance.**

From equation (2)

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} \ \mho$$

The parameter y_{21} is called **short circuit forward transfer admittance.**

[B] Let $V_1 = 0$ i.e. **port-1 is short circuited.**

From equation (1),

$$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} \ \mho$$

The parameter y_{12} is called **short circuit reverse transfer admittance.**

From equation (2),

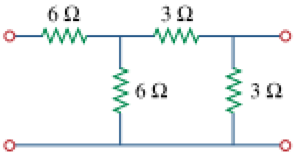
$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} \ \mho$$

The parameter y_{22} is called **short circuit driving point output admittance.**

These parameters are defined individually only when the voltage in any one of the ports is zero. This corresponds to the condition that one of the ports is short circuited. Hence y-parameters are also called **short circuit admittance parameters.**

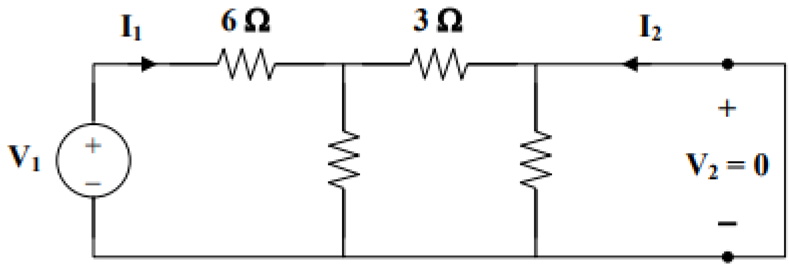
Problem

Calculate the *y* parameters for the two-port in Fig.



Figure

To get $\mathbf{y}_{11}$ and $\mathbf{y}_{21}$, consider the circuit in Fig.(a).



(a)

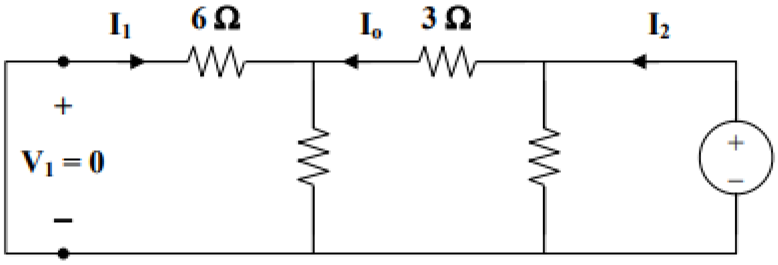
$$\mathbf{V}_1 = (6 + 6 \parallel 3)\mathbf{I}_1 = 8\mathbf{I}_1$$

$$\mathbf{y}_{11} = \frac{\mathbf{I}_1}{\mathbf{V}_1} = \frac{1}{8}$$

$$\mathbf{I}_2 = \frac{-6}{6+3}\mathbf{I}_1 = \frac{-2}{3}\frac{\mathbf{V}_1}{8} = \frac{-\mathbf{V}_1}{12}$$

$$\mathbf{y}_{21} = \frac{\mathbf{I}_2}{\mathbf{V}_1} = \frac{-1}{12}$$

To get $\mathbf{y}_{22}$ and $\mathbf{y}_{12}$, consider the circuit in Fig.(b).



(b)

$$\mathbf{y}_{22} = \frac{\mathbf{I}_2}{\mathbf{V}_2} = \frac{1}{3 \,\|\, (3 + 6 \,\|\, 6)} = \frac{1}{3 \,\|\, 6} = \frac{1}{2}$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_o}{2}, \qquad \mathbf{I}_o = \frac{3}{3+6}\mathbf{I}_2 = \frac{1}{3}\mathbf{I}_2$$

$$\mathbf{I}_1 = \frac{-\mathbf{I}_2}{6} = \left(\frac{-1}{6}\right)\left(\frac{1}{2}\mathbf{V}_2\right) = \frac{-\mathbf{V}_2}{12}$$

$$\mathbf{y}_{12} = \frac{\mathbf{I}_1}{\mathbf{V}_2} = \frac{-1}{12} = \mathbf{y}_{21}$$

Thus,

$$[\mathbf{y}] = \begin{bmatrix} \frac{1}{8} & \frac{-1}{12} \\ \frac{-1}{12} & \frac{1}{2} \end{bmatrix} \mathbf{S}$$

h-Parameters

These are also called **hybrid parameters.** These parameters are very useful in constructing models for transistors. The transistor parameters cannot be calculated using by either short circuit admittance parameter or open circuit impedance parameter measurement. These parameters are obtained by expressing voltage at input port and the current at output port in terms of the current at the input port and the voltage at the output port. Thus, the **current I_1 and voltage V_2 are independent variables;** while **current I_2 and voltage V_1 are the dependent variables.** Thus, we have,

$$V_1 = f_1(I_1, V_2)$$

$$I_2 = f_2(I_1, V_2)$$

In equation form, above relations can be written as,

$$V_1 = h_{11} I_1 + h_{12} V_2 \qquad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \text{... (2)}$$

In matrix form, the above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h-parameters can be defined as follows

[A] Let $V_2 = 0$; i.e. **port - 2 is short circuited.**

From equation (1),

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2 = 0} \ \Omega$$

The parameter h_{11} is called **short circuit input impedance.**

From equation (2),

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2 = 0}$$

The parameter h_{21} is called **short circuit forward current gain.** It is unitless.

[B] Let $I_1 = 0$; i.e. **port - 1 is circuited.**

From equation (1),

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1 = 0}$$

The parameter h_{12} is called **open circuit reverse voltage gain.** It is also unitless.

From equations (2),

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1 = 0} \quad \mho$$

The parameter h_{22} is called **open circuit output admittance.**

All above parameters are having different units such as ohm for short circuit impedance and mho for open circuit output admittance, the name of the parameter is **hybrid parameter.**

Problems-

Determine the h-parameter with the following data:

i. with the output terminals short circuited, $V_1 = 25$ V, $I_1 = 1$ A, $I_2 = 2$ A

ii. with the input terminals open circuited, $V_1 = 10$ V, $V_2 = 50$ V, $I_2 = 2$ A

Solution

The *h*-parameter equations are,

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

a. With output short-circuited, $V_2 = 0$, given: $V_1 = 25$ V, $I_1 = 1$ A and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 25 &= h_{11} \times 1 \\ \text{and} \quad 2 &= h_{21} \times 1 \end{aligned}\right\} \Rightarrow h_{11} = 25\ \Omega, \text{ and } h_{21} = 2$$

b. With input open-circuited, $I_1 = 0$, given: $V_1 = 10$ V, $V_2 = 50$ V and $I_2 = 2$ A.

$$\therefore \quad \left.\begin{aligned} 10 &= h_{12} \times 50 \\ \text{and} \quad 2 &= h_{22} \times 50 \end{aligned}\right\} \Rightarrow h_{12} = \frac{1}{5} = 0.2 \text{ and } h_{23} = \frac{1}{25}\ \mho = 0.04\ \mho$$

Thus, the *h*-parameters are:

$$[h] = \begin{bmatrix} 25\ \Omega & 0.2 \\ 2 & 0.04\ \Omega^{-1} \end{bmatrix}$$

ABCD Parameters or Transmission Parameters or Chain Parameters

These parameters are known as transmission parameters. These are generally used in the analysis of power transmission in which the input port is referred as the sending end while the output port is referred as receiving end. These are obtained by expressing voltage V_1 and current I_1 at input port interms of voltage V_2 and current I_2 at output port. Thus, **voltage V_2 and current I_2 are independent variables** while **voltage V_1 and current I_1 are dependent variables**. Thus, we have,

$$V_1 = f_1(V_2, -I_2)$$
$$I_1 = f_2(V_2, -I_2)$$

Generally we have considered the currents in both the ports are entering the port and both are positive. The negative sign with I_2 indicates that, for the ABCD parameters the current I_2 is leaving the port-2.

In equation form, above relations can be written as,

$$V_1 = A\,V_2 + B\,(-I_2) \quad \dots (1)$$
$$I_1 = C\,V_2 + D\,(-I_2) \quad \dots (2)$$

In matrix form, above equations can be written as,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The individual ABCD parameters can be defined as follows.

[A] Let $-I_2 = 0$, i.e. **port - 2 is circuited.**

From equation (1),

$$A = \left.\frac{V_1}{V_2}\right|_{-I_2 = 0}$$

The parameter A is called **open circuit reverse voltage gain.** It is unitless.

From equation (2),

$$C = \left.\frac{I_1}{V_2}\right|_{-I_2 = 0} \quad \mho$$

The parameter C is called **open circuit reverse transfer admittance.**

[B] Let $V_2 = 0$, i.e. **port - 2 is short circuited.**

From equation (1),

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = 0} \quad \Omega$$

The parameter B is called **short circuit reverse transfer impedance.**

From equation (2),

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0}$$

The parameter D is called **short circuit reverse current gain.** It is also unitless.

ABCD or Transmission parameters are also called **chain parameters.**

These parameter are effectively used for the analysis of power transmission line, so commonly known as **transmission parameters.**

Example : *Find the transmission or general parameters for the circuit shown in Fig. 4.10.*

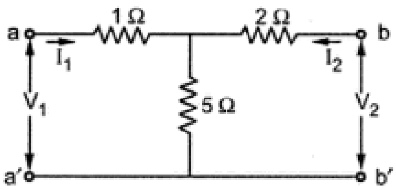


Fig. 4.10

Solution : By definition, transmission parameters are given by,

$$V_1 = A V_2 + B(-I_2)$$
$$I_1 = CV_2 + D(-I_2)$$

A) Let $-I_2 = 0$ **i.e. open circuit terminals b - b′** (i.e. **port-2**) as shown in the Fig. 4.10 (a).

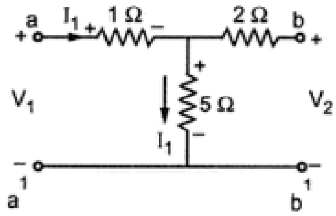


Fig. 4.10 (a)

From circuit drawn above, we can write,

$$V_2 = 5 I_1 \qquad \text{... (i)}$$

$$\therefore \quad C = \left.\frac{I_1}{V_2}\right|_{-I_2 = 0} = \frac{1}{5} \ ℧$$

Applying KVL at the input side, we get,

$$-I_1 - 5 I_1 + V_1 = 0$$

$$\therefore \quad 6 I_1 = V_1 \qquad \text{... (ii)}$$

From equation (i),

$$I_1 = \frac{V_2}{5} \qquad \text{... (iii)}$$

Substituting value of I_1 in equation (ii) , we get,

$$\therefore \quad 6\left(\frac{V_2}{5}\right) = V_1$$

$$\therefore \quad V_2 = \frac{5}{6} V_1$$

$$\therefore \quad A = \left.\frac{V_1}{V_2}\right|_{-I_2 = 0} = \frac{6}{5}$$

B) Let $V_2 = 0$ **i.e. short circuit port-2** (terminals b - b′) as shown in the Fig. 4.10 (b).

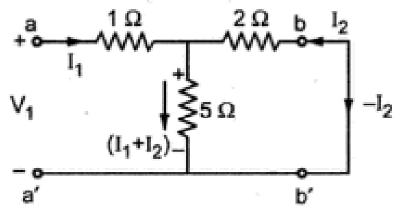


Fig. 4.10 (b)

Applying current divider rule, we get,

$$I_2 = -I_1\left[\frac{5}{2+5}\right]$$

$$\therefore \quad -I_2 = I_1\left(\frac{5}{7}\right)$$

$$\therefore \quad D = \left.\frac{I_1}{-I_2}\right|_{V_2 = 0} = \frac{7}{5}$$

Applying KVL at the input side, we get,

$$-I_1 - 5(I_1 + I_2) + V_1 = 0$$

$$\therefore \quad -6 I_1 - 5 I_2 = -V_1$$

$$\therefore \quad 6 I_1 + 5 I_2 = V_1$$

$$\therefore \quad 6\left(\frac{-7}{5}I_2\right)+5I_2 = V_1$$

$$\therefore \quad \left(\frac{-42}{5}+5\right)I_2 = V_1$$

$$\therefore \quad \frac{-17}{5}I_2 = V_1$$

$$\therefore \quad \left(\frac{17}{5}\right)(-I_2) = V_1$$

$$\therefore \quad B = \left.\frac{V_1}{-I_2}\right|_{V_2=0} = +\frac{17}{5}\ \Omega$$

Hence transmission parameters matrix is given by,

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{6}{5} & \frac{17}{5} \\ \frac{1}{5} & \frac{7}{5} \end{bmatrix}$$

INVERSE HYBRID (OR g) PARAMETERS

If V_1 and I_2 are chosen as independent variables, the two-port network equations may be written as

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2.$$

In matrix form, these equations are written as

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}.$$

The constants g_{11}, g_{12}, g_{21}, and g_{22} are known as inverse hybrid parameters or g-parameters. The g-parameters are defined as follows by using Equations

If $I_2 = 0$ the output port is open circuit.

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$ open circuit input admittance.

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ open circuit forward voltage gain.

If

$V_1 = 0$ the input port is short circuit.

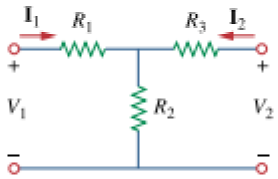
$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$ short circuit reverse current gain.

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ short circuit output impedance.

From the definitions of the g-parameters, it is seen that g_{11} has the dimensions of admittance, g_{21} and g_{12} are dimensionless, and g_{22} has the dimensions of impedance.

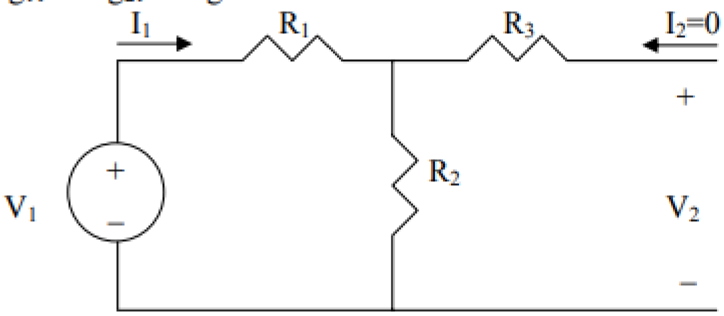
Problem

Obtain the g parameters for the circuit of Fig.



Figure

We obtain g_{11} and g_{21} using the circuit below.

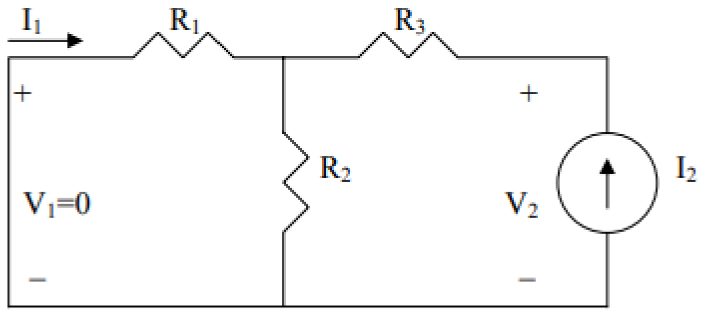


$$I_1 = \frac{V_1}{R_1 + R_2} \longrightarrow g_{11} = \frac{I_1}{V_1} = \frac{1}{R_1 + R_2}$$

By voltage division,

$$V_2 = \frac{R_2}{R_1 + R_2} V_1 \longrightarrow g_{21} = \frac{V_2}{V_1} = \frac{R_2}{R_1 + R_2}$$

We obtain g_{12} and g_{22} using the circuit below.



By current division,

$$I_1 = -\frac{R_2}{R_1 + R_2} I_2 \longrightarrow g_{12} = \frac{I_1}{I_2} = -\frac{R_2}{R_1 + R_2}$$

Also,

$$V_2 = I_2(R_3 + R_1 // R_2) = I_2\left(R_3 + \frac{R_1 R_2}{R_1 + R_2}\right) \quad g_{22} = \frac{V_2}{I_2} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

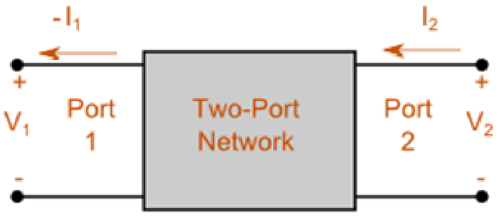
$$g_{11} = \frac{1}{R_1 + R_2}, g_{12} = -\frac{R_2}{R_1 + R_2}$$

$$g_{21} = \frac{R_2}{R_1 + R_2}, g_{22} = R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

Inverse Transmission Parameters

The transmission parameter and the inverse transmission parameter are duals of each other.

If, instead of quantities V_1 and I_1, quantities V_2 and I_2 are expressed in terms of V_1 and I_1, the resulting parameter (A', B', C', D') are called inverse transmission parameter.



The inverse transmission parameters of the two port network in figure having direction of voltages and current as shown,are given by

$$V_2 = A'V_1 + B'(-I_1)$$

$$I_2 = C'V_1 + D'(-I_1)$$

in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{pmatrix} A' & B' \\ C' & D' \end{pmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

The inverse transmission parameters can be defined as

$A' = \frac{V_2}{V_1}; I_1 = 0$ forward voltage ratio with sending end open circuited.

$C' = \frac{I_2}{V_1}; I_1 = 0$ transfer admittance with sending end open circuited.

$B' = \frac{V_2}{-I_1}; V_1 = 0$ transfer impedance with sending end short circuited.

$D' = \frac{I_2}{-I_1}; V_1 = 0$ forward current ratio with sending end short circuited.

Name of the parameters	Port variables		Equations
	Express (Dependent)	Interms of (Independent)	
Open circuit impedance i.e. z-parameters	V_1 , V_2	I_1 , I_2	$V_1 = z_{11} I_1 + z_{12} I_2$ $V_2 = z_{21} I_1 + z_{22} I_2$
Short circuit admittance i.e. y-parameters	I_1 , I_2	V_1 , V_2	$I_1 = y_{11} V_1 + y_{12} V_2$ $I_2 = y_{21} V_1 + y_{22} V_2$
h-parameters	V_1 , I_2	I_1 , V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
ABCD or transmission parameters	V_1 , I_1	V_2 , $-I_2$	$V_1 = AV_2 + B(-I_2)$ $I_1 = CV_2 + D(-I_2)$
Inverse hybrid or g-parameters	I_1 , V_2	V_1 , I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$
Inverse transmission parameters	V_2 , I_2	V_1 , $-I_1$	$V_2 = A' V_1 + B'(-I_1)$ $I_2 = C' V_1 + D'(-I_2)$

Table 4.1 Summary of two port network parameters

Interrelationships between the Parameters

During analysis of a two port network it is often required to obtain more than one set of parameters for a given two port network. Then it becomes time consuming and tedious to obtain different sets of parameter by using their respective basic definitions. Hence to make the analysis easier, any set of parameters is expressed interms of all the remaining sets of parameters. In other words, parameters are derived from each other. In this section, we will derive such interrelationships between the parameters.

z-Parameters Interms of other Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{... (A)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{... (B)}$$

[A] Interms of y-Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{... (1)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{... (2)}$$

Writing equations in matrix form, we have,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Solving above equations, using Cramer's rule for V_1 and V_2. We can write,

$$V_1 = \frac{\begin{vmatrix} I_1 & y_{12} \\ I_2 & y_{22} \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{22} I_1 - y_{12} I_2}{y_{11} y_{22} - y_{12} y_{21}}$$

$$= \frac{y_{22}}{y_{11} y_{22} - y_{12} y_{21}} \cdot I_1 - \frac{y_{12}}{y_{11} y_{22} - y_{12} y_{21}} I_2 \quad \text{... (3)}$$

Similarly,

$$V_2 = \frac{\begin{vmatrix} y_{11} & I_1 \\ y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}} = \frac{y_{11} I_2 - y_{21} I_1}{y_{11} y_{22} - y_{12} y_{21}}$$

$$= \frac{-y_{21}}{y_{11} y_{22} - y_{12} y_{21}} I_1 + \frac{y_{11}}{y_{11} y_{22} - y_{12} y_{21}} I_2 \quad \text{... (4)}$$

Let $\quad y_{11} y_{22} - y_{12} y_{21} = \Delta y$

Rewriting equations (3) and (4), we have,

$$V_1 = \left[\frac{y_{22}}{\Delta y}\right] I_1 + \left[\frac{-y_{12}}{\Delta y}\right] I_2 \quad \text{... (5)}$$

and

$$V_2 = \left[\frac{-y_{21}}{\Delta y}\right] I_1 + \left[\frac{y_{11}}{\Delta y}\right] I_2 \quad \text{... (6)}$$

Comparing equations (5) and (6) to equations (A) and (B), we have,

$$z_{11} = \frac{y_{22}}{\Delta y} \qquad z_{12} = \frac{-y_{12}}{\Delta y}$$

$$z_{21} = \frac{-y_{21}}{\Delta y} \qquad z_{22} = \frac{y_{11}}{\Delta y}$$

[B] Interms of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (2)}$$

From equation (2) we can write,

$$h_{22} V_2 = -h_{21} I_1 + I_2$$

$$\therefore \quad V_2 = \left[\frac{-h_{21}}{h_{22}}\right] I_1 + \left[\frac{1}{h_{22}}\right] I_2 \quad \text{... (3)}$$

Substituting value of V_2 in equation (1) we have,

$$V_1 = h_{11} I_1 + h_{12}\left[\frac{-h_{21}}{h_{22}} I_1 + \frac{1}{h_{22}} I_2\right]$$

$$\therefore \quad V_1 = \left[h_{11} - \frac{h_{12} h_{21}}{h_{22}}\right] I_1 + \frac{h_{12}}{h_{22}} I_2$$

$$\therefore \quad V_1 = \left[\frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}}\right] I_1 + \left[\frac{h_{12}}{h_{22}}\right] I_2 \quad \text{... (4)}$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$$z_{11} = \frac{h_{11} h_{22} - h_{12} h_{21}}{h_{22}} \qquad z_{12} = \frac{h_{12}}{h_{22}}$$

$$z_{21} = \frac{-h_{21}}{h_{22}} \qquad z_{22} = \frac{1}{h_{22}}$$

[C] Interms of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A V_2 + B(-I_2) \quad \text{... (1)}$$

$$I_1 = C V_2 + D(-I_2) \quad \text{... (2)}$$

We can rewrite equation (2) as follows,

$$C V_2 = I_1 + D I_2$$

$$\therefore \quad V_2 = \left[\frac{1}{C}\right] I_1 + \left[\frac{D}{C}\right] I_2 \quad \text{... (3)}$$

Substituting value of V_2 in equation (1), we have,

$$V_1 = A\left[\left(\frac{1}{C}\right) I_1 + \left(\frac{D}{C}\right) I_2\right] - B \cdot I_2$$

$$\therefore \quad V_1 = \left[\frac{A}{C}\right] I_1 + \left[\frac{AD}{C} - B\right] I_2$$

$$\therefore \quad V_1 = \left[\frac{A}{C}\right] I_1 + \left[\frac{AD - BC}{C}\right] I_2 \quad \text{... (4)}$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$$z_{11} = \frac{A}{C} \qquad z_{12} = \frac{AD - BC}{C}$$

$$z_{21} = \frac{1}{C} \qquad z_{22} = \frac{D}{C}$$

y-Parameters Interms of other Parameters

The equations for y-parameters are as follows,

$$I_1 = y_{11} V_1 + y_{12} V_2 \quad \text{... (A)}$$

$$I_2 = y_{21} V_1 + y_{22} V_2 \quad \text{... (B)}$$

[A] Interms of z-Parameters

The equations for z-parameters are as follows,

$$V_1 = z_{11} I_1 + z_{12} I_2 \quad \text{... (1)}$$

$$V_2 = z_{21} I_1 + z_{22} I_2 \quad \text{... (2)}$$

Writing equations in matrix form,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}.$$

Solving above equations using Cramer's rule for I_1 and I_2, we can write,

$$I_1 = \frac{\begin{vmatrix} V_1 & z_{12} \\ V_2 & z_{22} \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{22} \cdot V_1 - z_{12} \cdot V_2}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$

$$= \frac{z_{22}}{z_{11}\ z_{22} - z_{12}\ z_{22}} \cdot V_1 + \frac{-z_{12}}{z_{11}\ z_{22} - z_{12}\ z_{21}} \cdot V_2 \quad \text{...(3)}$$

Similarly,

$$I_2 = \frac{\begin{vmatrix} z_{11} & V_1 \\ z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{vmatrix}} = \frac{z_{11} \cdot V_2 - z_{21} \cdot V_1}{z_{11} \cdot z_{22} - z_{12} \cdot z_{21}}$$

$$= \frac{-z_{21}}{z_{11} \cdot z_{22} - z_{12}\ z_{21}} \cdot V_1 + \frac{z_{11}}{z_{11}\ z_{22} - z_{12}\ z_{21}} \cdot V_2 \quad \text{... (4)}$$

Let $\Delta z = z_{11} \cdot z_{22} - z_{12} \cdot z_{21}$

Rewriting equations (3) and (4),

$$I_1 = \left[\frac{z_{22}}{\Delta z}\right] V_1 + \left[\frac{-z_{12}}{\Delta z}\right] V_2 \quad \text{... (5)}$$

and

$$I_2 = \left[\frac{-z_{21}}{\Delta z}\right] V_1 + \left[\frac{z_{11}}{\Delta z}\right] V_2 \quad \text{... (6)}$$

Comparing equations (5) and (6) with the equations (A) and (B) respectively, we have,

$$y_{11} = \frac{z_{22}}{\Delta z} \qquad y_{12} = \frac{-z_{12}}{\Delta z}$$

$$y_{21} = \frac{-z_{21}}{\Delta z} \qquad y_{22} = \frac{z_{11}}{\Delta z}$$

[B] Interms of h-Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad \text{... (1)}$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad \text{... (2)}$$

We can rewrite equation (1) as follows,

$$h_{11} I_1 = V_1 - h_{12} V_2$$

$$\therefore \quad I_1 = \left[\frac{1}{h_{11}}\right] V_1 + \left[\frac{-h_{12}}{h_{11}}\right] V_2 \quad \text{...(3)}$$

Substituting equation (3) in equation (2), we have,

$$I_2 = h_{21}\left[\frac{1}{h_{11}} V_1 - \frac{h_{12}}{h_{11}} V_2\right] + h_{22} V_2$$

$$\therefore \quad I_2 = \left[\frac{h_{21}}{h_{11}}\right] V_1 + \left[h_{22} - \frac{h_{12}\, h_{21}}{h_{11}}\right] V_2$$

$$\therefore \quad I_2 = \left[\frac{h_{21}}{h_{11}}\right] V_1 + \left[\frac{h_{11}\, h_{22} - h_{12}\, h_{21}}{h_{11}}\right] V_2 \qquad \dots (4)$$

Comparing equations (3) and (4) with the equations (A) and (B) respectively, we have,

$y_{11} = \frac{1}{h_{11}}$	$y_{12} = \frac{-h_{12}}{h_{11}}$
$y_{21} = \frac{h_{21}}{h_{11}}$	$y_{22} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{11}}$

[C] Interms of Transmission (ABCD) Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\, V_2 + B\,(-I_2) \qquad \dots (1)$$

$$I_1 = C\, V_2 + D\,(-I_2) \qquad \dots (2)$$

We can rewrite equation (1) as follows,

$$-B \cdot I_2 = V_1 - A\, V_2$$

$$\therefore \quad I_2 = \left[-\frac{1}{B}\right] V_1 + \left[\frac{A}{B}\right] V_2 \qquad \dots (3)$$

Substituting value of I_2 in equation (2), we have,

$$I_1 = C \cdot V_2 + D\left[\frac{+1}{B} V_1 - \frac{A}{B}\right] V_2$$

$$\therefore \quad I_1 = \left[\frac{D}{B}\right] V_1 + \left[C - \frac{AD}{B}\right] V_2$$

$$\therefore \quad I_1 = \left[\frac{D}{B}\right] V_1 + \left[\frac{BC - AD}{B}\right] V_2 \qquad \dots (4)$$

Comparing equations (4) and (3) with the equations (A) and (B) respectively, we have,

$y_{11} = \frac{D}{B}$	$y_{12} = \frac{BC - AD}{B}$
$y_{21} = \frac{-1}{B}$	$y_{22} = \frac{A}{B}$

h-Parameters interms of other Parameters

The equations for h-parameters are as follows,

$$V_1 = h_{11}\, I_1 + h_{12}\, V_2 \qquad \dots (A)$$

$$I_2 = h_{21}\, I_1 + h_{22}\, V_2 \qquad \dots (B)$$

[A] Interms of z-Parameters

The equations for z-parameter are as follows,

$$V_1 = z_{11}\, I_1 + z_{12}\, I_2 \qquad \dots (1)$$

$$V_2 = z_{21}\, I_1 + z_{22}\, I_2 \qquad \dots (2)$$

We can rewrite equations (2) as follows,

$$z_{22}\, I_2 = -z_{21}\, I_1 + V_2$$

$$I_2 = \left[\frac{-z_{21}}{z_{22}}\right] I_1 + \left[\frac{+1}{z_{22}}\right] V_2 \quad \text{... (3)}$$

Substituting value of I_2 in equation (1), we have,

$$V_1 = z_{11}\, I_1 + z_{12}\left(\left[\frac{-z_{21}}{z_{22}}\right] I_1 + \left[\frac{+1}{z_{22}}\right] V_2\right)$$

$$\therefore \quad V_1 = \left[\frac{z_{11}\, z_{22} - z_{12}\, z_{21}}{z_{22}}\right] I_1 + \left[\frac{z_{12}}{z_{22}}\right] V_2 \quad \text{... (4)}$$

Comparing equations (4) and (3) with equations (A) and (B) respectively, we have,

$$h_{11} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} \qquad h_{12} = \frac{z_{12}}{z_{22}}$$

$$h_{21} = \frac{-z_{21}}{z_{22}} \qquad h_{22} = \frac{1}{z_{22}}$$

[B] Interms of y-Parameters

The equations of y-parameters are as follows,

$$I_1 = y_{11}\, V_1 + y_{12}\, V_2 \quad \text{... (1)}$$

$$I_2 = y_{21}\, V_1 + y_{22}\, V_2 \quad \text{... (2)}$$

We can rewrite equation (1) as follows,

$$y_{11}\, V_1 = I_1 - y_{12}\, V_2$$

$$\therefore \quad V_1 = \left[\frac{1}{y_{11}}\right] I_1 + \left[\frac{-y_{12}}{y_{11}}\right] V_2 \quad \text{... (3)}$$

Substituting value of V_1 in equation (2), we have,

$$I_2 = y_{21}\left(\left[\frac{1}{y_{11}}\right] I_1 + \left[\frac{-y_{12}}{y_{11}}\right] V_2\right) + y_{22}\, V_2$$

$$\therefore \quad I_2 = \left[\frac{y_{21}}{y_{11}}\right] I_1 + \left[y_{22} - \frac{y_{12}y_{21}}{y_{11}}\right] V_2$$

$$\therefore \quad I_2 = \left[\frac{y_{21}}{y_{11}}\right] I_1 + \left[\frac{y_{11}y_{22} - y_{12}y_{21}}{y_{11}}\right] V_2 \quad \text{... (4)}$$

Comparing equations (3) and (4) with equation (A) and (B) respectively, we have,

$$h_{11} = \frac{1}{y_{11}} \qquad h_{12} = \frac{-y_{12}}{y_{11}}$$

$$h_{21} = \frac{y_{21}}{y_{11}} \qquad h_{22} = \frac{y_{11}\, y_{22} - y_{12}\, y_{21}}{y_{11}}$$

[C] Interms of Tramission Parameters

The equations for transmission parameters are as follows,

$$V_1 = A\, V_2 + B\,(-I_2) \quad \text{... (1)}$$

$$I_2 = C\, V_2 + D\,(-I_2) \quad \text{... (2)}$$

We can rewrite equations (2) as follows,

$$D\,(I_2) = -I_1 + C\, V_2$$

$$\therefore \quad I_2 = \left[\frac{-1}{D}\right] I_1 + \left[\frac{C}{D}\right] V2 \quad \text{... (3)}$$

Substituting value of I_2 in equation (1), we have,

$$V_1 = A V_2 - B\left(\left[\frac{-1}{D}\right] I_1 + \left[\frac{C}{D}\right] V_2\right)$$

$$\therefore \quad V_1 = \left[A - \frac{BC}{D}\right] V_2 + \left[\frac{B}{D}\right] I_1$$

$$\therefore \quad V_1 = \left[\frac{B}{D}\right] I_1 + \left[\frac{AD - BC}{D}\right] V_2 \qquad \text{... (4)}$$

Comparing equations (4) and (3) with equations (A) and (B) respectively we have,

$$h_{11} = \frac{B}{D} \qquad h_{12} = \frac{AD - BC}{D}$$
$$h_{21} = \frac{-1}{D} \qquad h_{22} = \frac{C}{D}$$

T-parameters in terms of other parameters

(*i*) T-parameters in terms of Z-parameters:

Step-I: We know that T-parameter equations;

$$V_1 = AV_2 - BI_2 \qquad \text{...(1)}$$
$$I_1 = CV_2 - DI_2 \qquad \text{...(2)}$$

and Z-parameters equations;

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \qquad \text{...(3)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \qquad \text{...(4)}$$

Step-II: In order to obtain as equation (1), elimated I_1 from equations (3) and (4).

From equation (3), we have

$$I_1 = \frac{V_1 - Z_{12} I_2}{Z_{11}}$$

Put this value in equation (4), then we get

$$V_2 = Z_{21}\left(\frac{V_1 - Z_{12} I_2}{Z_{11}}\right) + Z_{22} I_2$$

or
$$V_2 = \frac{Z_{21}}{Z_{11}} V_1 + I_2\left(Z_{22} - \frac{Z_{12} Z_{21}}{Z_{11}}\right)$$

or
$$\frac{Z_{21}}{Z_{11}} V_1 = V_2 - I_2\left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{11}}\right)$$

or
$$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}\right) I_2 \qquad \text{...(5)}$$

For obtaining as equations (2), rewrite equation (4), then we get

$$Z_{21} I_1 = V_2 - Z_{22} I_2$$

or
$$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \qquad \text{...(6)}$$

Step-III: Comparing equations (1) and (5), then we get

$$A = \frac{Z_{11}}{Z_{21}} \text{ and } B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

Similary, comparing equation (2) and (6), then we get

$$C = \frac{1}{Z_{21}} \text{ and } D = \frac{Z_{22}}{Z_{21}}$$

(*ii*) *T*-parameters in terms of *Y*-parameters:

Step-I: We know that *T*-parameter equations:

$$V_1 = AV_2 - BI_2 \quad ...(1)$$

$$I_1 = CV_2 - DI_2 \quad ...(2)$$

and Y-parameters equations:

$$I_1 = Y_{11} V_1 + Y_{12} V_2 \quad ...(3)$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2 \quad ...(4)$$

$$V_1 = \frac{I_2 - Y_{22}V_2}{Y_{21}}$$

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2$$

$$V_1 = -\frac{Y_{22}}{Y_{21}} V_2 + \frac{1}{Y_{21}} I_2 \quad —(5)$$

compare equ (5) with (1)

$$A = -\frac{Y_{22}}{Y_{21}} \; ; \; B = -\frac{1}{Y_{21}}$$

$$I_1 = Y_{11}\left[\frac{I_2 - Y_{22}V_2}{Y_{21}}\right] + Y_{12}V_2$$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{Y_{11}Y_{22}V_2}{Y_{21}} + Y_{12}V_2$$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{[Y_{11}Y_{22} - Y_{12}Y_{21}]}{Y_{21}} V_2 \quad —(6)$$

I_1 Compare equ (6) with (2)

$$D = \frac{-Y_{11}}{Y_{21}} \; ; \; C = -\frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}$$

(*iv*) *T*-parameters in terms of *h*-parameters:

Step-I: We know that *T*-parameter equations;

$$V_1 = AV_2 - BI_2 \quad ...(1)$$

$$I_1 = CV_2 - DI_2 \quad ...(2)$$

and h-parameters equations:

$$V_1 = h_{11} I_1 + h_{12} V_2 \quad ...(3)$$

$$I_2 = h_{21} I_1 + h_{22} V_2 \quad ...(4)$$

Step-II: In order to obtain as equation (1), eliminated I_1 from equations (3) and (4).

From equation (3), we have

$$I' = \frac{V_1 - h_{12} V_2}{h_{11}}$$

Put this value in equation (4), then we get

$$I_2 = h_{21}\left(\frac{V_1 - h_{12} V_2}{h_{11}}\right) + h_{22} V_2$$

or
$$I_2 = \frac{h_{21}}{h_{11}} V_1 + V_2\left(h_{22} - \frac{h_{12}\, h_{21}}{h_{11}}\right)$$

or
$$\frac{h_{21}}{h_{11}} V_1 = \left(\frac{h_{12}\, h_{21} - h_{11}\, h_{22}}{h_{11}}\right) V_2 + I_2$$

or
$$V_1 = \left(\frac{h_{12}\, h_{21} - h_{11}\, h_{22}}{h_{21}}\right) V_2 + \frac{h_{11}}{h_{21}} I_2 \quad ...(5)$$

For obtaining as equation (2), rewrite equation (4), then we get

$$h_{21}\, I_1 = -\, h_{22}\, V_2 + I_2$$

or
$$I_1 = -\frac{h_{22}}{h_{21}} V_2 + \frac{l}{h_{21}} I_2 \quad ...(6)$$

Step-III: Comparing equations (1) and (5), then we get

$$A = \frac{h_{12}\, h_{21} - h_{11}\, h_{22}}{h_{21}} = \frac{-\Delta h}{h_{21}} \text{ and } B = \frac{-h_{11}}{h_{21}}$$

Similarly, comparing equations (2) and (6), then we get

$$C = \frac{-h_{22}}{h_{21}} \text{ and } D = -\frac{1}{h_{21}}$$

INTERCONNECTIONS OF TWO-PORT NETWORKS

Two-port networks may be interconnected in various configurations, such as series, parallel, cascade, series-parallel, and parallel-series connections. For each configuration a certain set of parameters may be more useful than others to describe the network.

Series Connection

Figure 10.19 shows a series connection of two-port networks N_a and N_b.

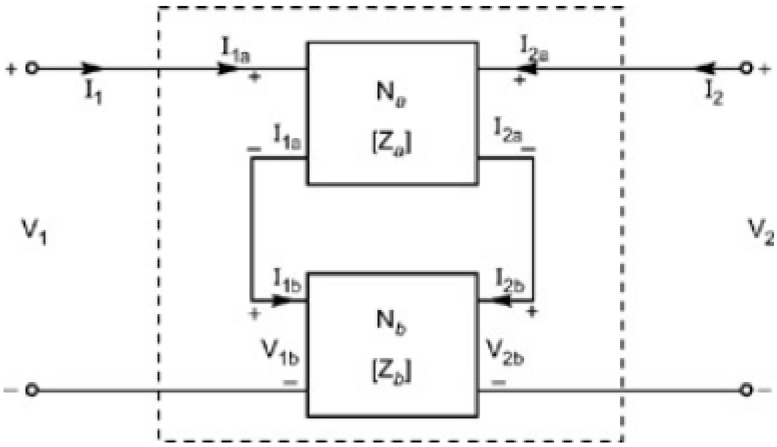


Figure 10.19: Series connection of two two-port networks

For network N_a,

$$\begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix} = \begin{bmatrix} Z_{11a} & Z_{12a} \\ Z_{21a} & Z_{22a} \end{bmatrix} \begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix}$$

$$(10.62) V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$$

$$(10.63) V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}.$$

For network N_b,

$$\begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix} = \begin{bmatrix} Z_{11b} & Z_{12b} \\ Z_{21b} & Z_{22}b \end{bmatrix} \begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix}$$

$$(10.64) V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$$

$$(10.65) V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$$

The condition for series connection is

$$I_{1a} = I_{1b} = I_1, \ \textit{and } I_2 = I_{2b} = I_2 \ (\textit{current same})$$

$$(10.66) V_1 = V_{1a} + V_{1b}$$

$$(10.67) V_2 = V_{2a} + V_{2b}$$

Putting the values of V_{1a} and V_{1b} from Equation (10.62) and Equation (10.64),

$$V_1 = Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b}$$

$$(10.68) \quad = Z_{11a}I_1 + Z_{12a}I_2 + Z_{11b}I_1 + Z_{12b}I_2 \; [I_{1a} = I_{1b} = I_1, \; I_{2a} = I_{2b} = I_2]$$

$$V_1 = (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2.$$

Putting the values of V_{2a} and V_{2b} from Equation (10.63) and Equation (10.65) into Equation (10.67), we get

$$(10.69) V_2 = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

The Z-parameters of the series-connected combined network can be written as

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2,$$

where

$$Z_11 = Z_{11a} + Z_{11b}$$

$$Z_12 = Z_{12a} + Z_{12b}$$

$$Z_21 = Z_{21a} + Z_{21b}$$

$$Z_22 = Z_{22a} + Z_{22b}$$

or in the matrix form,

$$[Z] = [Z_a] + [Z_b].$$

The overall Z-parameter matrix for series connected two-port networks is simply

the sum of Z-parameter matrices of each individual two-port network connected in series.

Parallel Connection

Figure 10.20 shows a parallel connection of two two-port networks N_a and N_b.

The resultant of two admittances connected in parallel is $Y_1 + Y_2$. So in parallel connection, the parameters are Y-parameters.

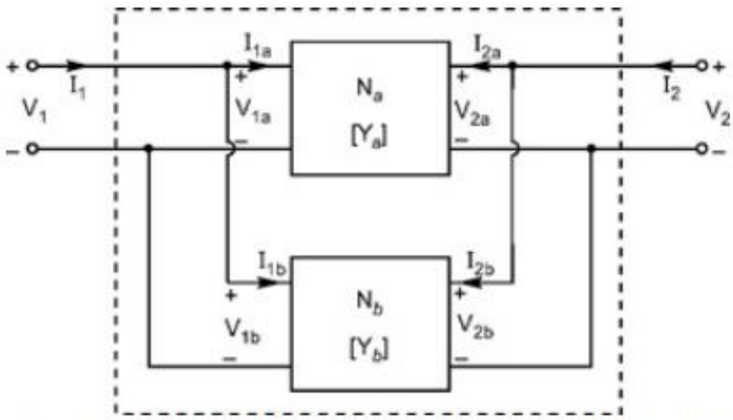


Figure 10.20: Parallel connections for two two-port networks

For network N_a,

$$\begin{bmatrix} I_{1a} \\ I_{2a} \end{bmatrix} = \begin{bmatrix} Y_{11a} & Y_{12a} \\ Y_{21a} & Y_{22a} \end{bmatrix} \begin{bmatrix} V_{1a} \\ V_{2a} \end{bmatrix}$$

or

$$(10.70) I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$

$$(10.71) I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}.$$

For network N $_b$,

$$\begin{bmatrix} I_{1b} \\ I_{2b} \end{bmatrix} = \begin{bmatrix} Y_{11b} & Y_{12b} \\ Y_{22b} & Y_{22b} \end{bmatrix} \begin{bmatrix} V_{1b} \\ V_{2b} \end{bmatrix}$$

$$(10.72) I_{1b} = Y_{11b} V_{1b} + Y_{12b} V_{2b}$$

$$(10.73) I_{2b} = Y_{21b} V_{1b} + Y_{22b} V_{2b}$$

Now the condition for paralel,

$$V_{1a} = V_{1b} = V_1,\ V_{2a} = V_{2b} = V_2\ [\textit{Same voltage}]$$

and

$$(10.74) I_1 = I_{1a} + I_{1b}$$

$$(10.75) I_2 = I_{2a} + I_{2b}$$

$$\begin{aligned} I_1 &= Y_{11a} V_{1a} + Y_{12a} V_{2a} + Y_{11b} V_{1b} + Y_{12b} V_{2b} \\ &= Y_{11a} V_1 + Y_{12a} V_2 + Y_{11b} V_1 + Y_{12b} V_2 \end{aligned}$$

$$(10.76) I_1 = (Y_{11a} + Y_{11b}) V_1 + (Y_{12a} + Y_{12b}) V_2$$

Similarly,

$$(10.77) I_2 = (Y_{21a} + Y_{21b}) V_1 + (Y_{22a} + Y_{22b}) V_2$$

The Y-parameters of the parallel connected combined network can be written as

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$Y_{11} = Y_{11a} + Y_{11b}$$

$$Y_{12} = Y_{12a} + Y_{12b}$$

$$Y_{21} = Y_{21a} + Y_{21b}$$

$$Y_{22} = Y_{22a} + Y_{22b}$$

V - Two port networks

General Two Port networks:-

* we will consider a general two port network composed of linear, bilateral elements and no independent sources.
* Dependent sources are permitted.
* It is represented as a black box with two accessible terminals pairs as shown bellow.

* The terminal pair 1-1' represents port 1 and is called i/p port or sending end
* The terminal pair (2-2') represent port 2 and is called out put port or receiving end.
* The voltage and current at port (1) are V_1 and I_1 and at Port (2) are V_2 and I_2.
* The polarities of V_1 and V_2 and the directions of I_1 and I_2 are customarily selected as shown above figure.
* out of the 4 variables V_1, I_1, V_2 and I_2 only two are independent.
* The other two are expressed in terms of the independent variables in terms of n/w parameters

This can be done in number of ways as shown in bellow Table.

Name of parameters	Expressed (dependent)	In terms of (independent)	Equations
1. open ckt impedance parameters (or) (Z-parameters)	V_1, V_2	I_1 and I_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
2. short ckt admittance parameters or Y-parameters	I_1, I_2	V_1 and V_2	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
3. Transmission Parameters (ABCD)	V_1, I_1	V_2 and I_2	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
4. Hybrid parameters (h-parameters)	V_1, I_2	I_1 and V_2	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$

open circuit Impedance (Z) parameters:-

A general linear two-port network defined in bellow, which does not contain any independent sources is shown bellow.

* The Z parameters of a two-port for the positive directions of voltages and currents may be defined by expressing the port voltages V_1 and V_2 in terms of the currents I_1 and I_2

* V_1 and V_2 are dependent variables, and I_1 and I_2 are independent variables.

The voltages at port 1-1' and port 2-2' are

~~V_1~~ =

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \rightarrow (1)$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2 \rightarrow (2)$$

* Here Z_{11}, Z_{12}, Z_{21} and Z_{22} are the network functions, and are called impedance (Z) parameters,

* These parameters can be represented by matrices we may write the matrix equation

$$[V] = [Z][I]$$

where V is the column matrix = $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

Z is the square matrix = $\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

and we may write I in the column matrix = $\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Thus, $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

* The individual Z-parameters for a given network can be defined by setting each of the port current equal to zero

suppose port 2-2' is left open-circuited, then $I_2 = 0$

Thus $$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

where Z_{11} is the driving-point impedance at port 1-1' with port 2-2' open circuited. It is called the open circuit input impedance.

Similarly $Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ where Z_{21} is transfer impedance & also called open ckt forward transfer impedance.

Suppose port 1-1' is left open circuited, then $I_1 = 0$

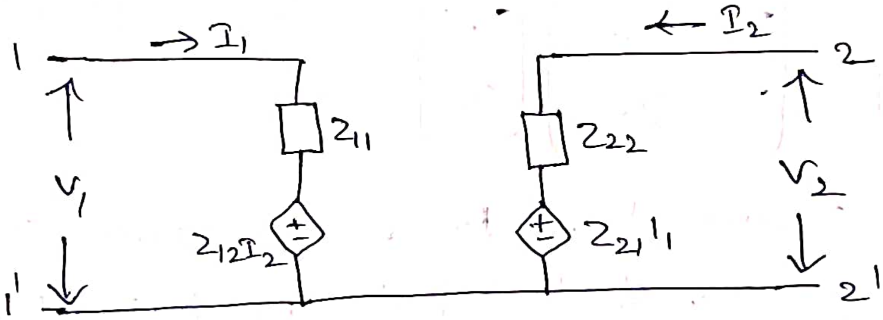
Thus $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

where Z_{12} is the transfer impedance and it is also called the open circuit reverse transfer impedance

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

where Z_{22} is the open ckt driving point impedance and it is also called the open ckt output impedance.

The equivalent circuit of the two-port networks governed by eqn (1) & (2)



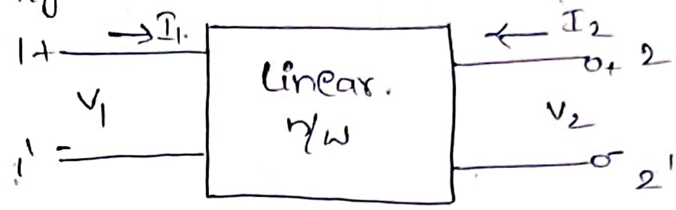
If the network under study is reciprocal or bilateral then in accordance with the reciprocity principle

$$\frac{V_2}{I_1}\Big|_{I_2=0} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

or $Z_{21} = Z_{12}$.

Short circuit Admittance (Y) Parameters:-

A general two-port network which is considered as shown in bellow figure.



The y parameters of a two-port network may be defined by expressing the two-port currents I_1 and I_2 in terms of the two port voltages V_1 and V_2

Thus,
$$I_1 = Y_{11}V_1 + Y_{12}V_2$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

In Matrix form, we can write

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$[I] = [Y][V]$$

The individual y parameters for a given network can be defined by setting each of the port voltages equal to zero.

Case 1:- when the output port is short circuited i.e $V_2 = 0$

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

where Y_{11} is the driving point admittance with the output port short-circuited. It is also called short circuit input admittance.

Similarly, $y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$

where y_{21} is the transfer admittance with the output port short-circuited. It is also called short-circuit forward transfer admittance.

Case 2:- when the i/p port is short-circuited i.e. $V_1 = 0$

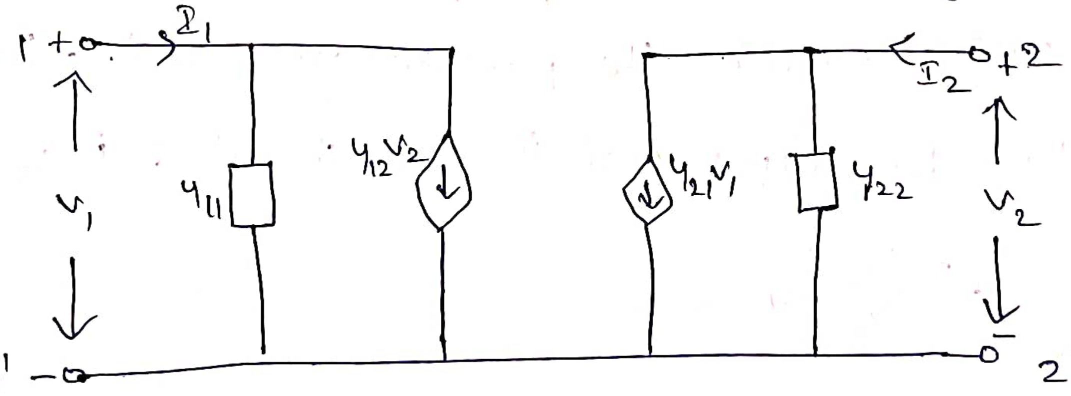
$$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

where y_{12} is the transfer admittance with the i/p port short-circuited. It is also called short-circuit reverse transfer admittance.

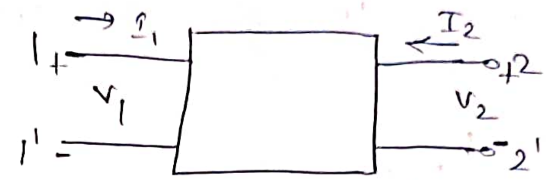
Similarly, $y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$

where y_{22} is the short circuit driving-point admittance with the input port short circuited. It is also called the short circuit output admittance.

The equivalent circuit of the two-port n/w in terms of y parameters as shown bellow figure.



Transmission Parameters (ABCD Parameters):-



The Transmission parameters or chain parameters or ABCD parameters serve to relate the voltage and current at the input port to voltage and current at the output port.

In equation form,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

* Here, the negative sign is used with I_2 and not for parameters B and D.

* The reason is the current I_2 carries a negative sign is that in transmission field, the output current is assumed to be comming out of the output port instead of going in to the port.

In Matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

where, Matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called transmission Matrix.

These parameters are determined as follows.

Case 1:- when the output port is open-circuited i.e $I_2 = 0$

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}$$

Where A is the reverse voltage gain with the output port open- circuited.

Similarly, $$C = \frac{I_1}{V_2}\Big|_{I_2=0}$$

Where C is the transfer admittance with the output port open circuited.

Case 2: - when output port is short circuited, i.e $V_2=0$

$$B = -\frac{V_1}{I_2}\Big|_{V_2=0}$$

where B is the transfer impedance with the output port short- circuited.

Similarly $$D = -\frac{I_1}{I_2}\Big|_{V_2=0}$$

Where D is the reverse current gain with the output port short - circuited.

Inverse Transmission parameters (A'B'C'D' parameters):-

The inverse transmission parameters serve to relate the voltage and current at the output port to the voltage and current at the input port.

In equation form,

$$V_2 = A'V_1 - B'I_1$$
$$I_2 = C'V_1 - D'I_1$$

In Matrix form, we can write

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

where Matrix $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix}$ is called the inverse transmission Matrix.

These parameters are determined as follows

Case 1:- when the input port is open-circuited i.e $I_1 = 0$

$$A' = \frac{V_2}{V_1} \Big|_{I_1 = 0}$$

where A' is the Forward voltage gain with the input port open circuited

Similarly, $$C' = \frac{I_2}{V_1} \Big|_{I_1 = 0}$$

where C' is the transfer admittance with the input port open circuited

Case 2:- when the input port is short-circuited i.e $V_1 = 0$

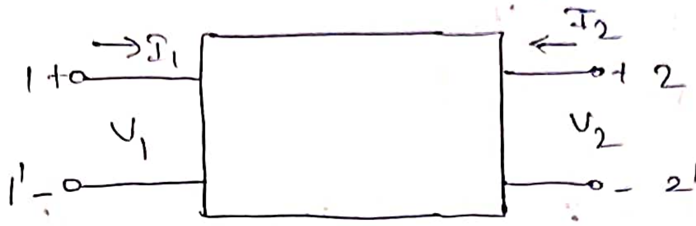
$$B' = -\frac{V_2}{I_1} \Big|_{V_1 = 0}$$

where B' is the transfer impedance with the input port short-circuited.

Similarly, $$D' = \frac{I_2}{I_1} \Big|_{V_1 = 0}$$

where D' is the forward current gain with the input port short-circuited.

Hybrid parameters (h parameters):-



The hybrid parameters of a two-port network may be defined by expressing the voltages of input port V_1 and current of output port I_2 in terms of current of input port I_1 and voltage of output port V_2.

In equation form
$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2$$

In matrix form, we can write

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

The individual h parameters can be defined by setting $I_1 = 0$ and $V_2 = 0$.

Case 1:- when the output port is short circuited i.e $V_2 = 0$

$$h_{11} = \frac{V_1}{I_1} \Big|_{V_2 = 0}$$

Where h_{11} is the short-circuit input impedance.

$$h_{21} = \frac{I_2}{I_1} \Big|_{V_2 = 0}$$

where h_{21} is the short-circuit forward current gain.

Case ii:- When the input port is open-circuited i.e $I_1 = 0$

$$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$$

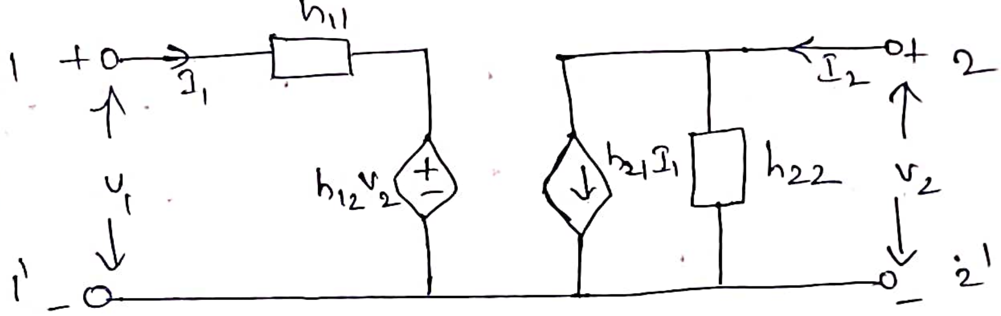
Where h_{12} is the open-circuit reverse voltage gain.

$$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$$

Where h_{22} is the open-circuit output admittance.

Since h parameters represent dimensionally an impedance, an admittance, a voltage gain and a current gain, these are called hybrid parameters.

The equivalent circuit of a two-port network in terms of hybrid parameters as shown bellow figure or Network.



Inverse hybrid parameters (g parameters):-

The inverse hybrid parameters of a ~~net~~ two-port network may be defined by expressing the current of the input port I_1 and voltage of the output port V_2 in terms of the voltage of the input port V_1 and the current of the output port I_2.

In equation form

$$I_1 = g_{11} V_1 + g_{12} I_2$$

$$V_2 = g_{21} V_1 + g_{22} I_2$$

In matrix form, we can write

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

The individual g parameters can be defined by setting $V_1 = 0$ and $I_2 = 0$

Case 1:- when the output port is open-circuited i.e $I_2 = 0$

$$g_{11} = \frac{I_1}{V_1}\Big|_{I_2=0}$$

where g_{11} is the open-circuit input admittance.

$$g_{21} = \frac{V_2}{V_1}\Big|_{I_2=0}$$

where g_{21} is the open-circuit forward voltage gain

Case 2:- when the input port is short-circuited i.e $V_1 = 0$

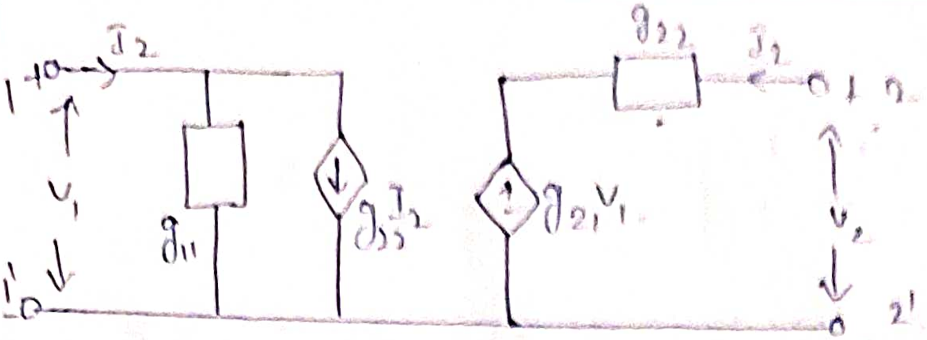
$$g_{12} = \frac{I_1}{I_2}\Big|_{V_1=0}$$

where g_{12} is the short-circuit reverse current gain.

$$g_{22} = \frac{V_2}{I_2}\Big|_{V_1=0}$$

where g_{22} is the short-circuit output impedance.

The equivalent circuit of a two-port network in terms of inverse hybrid parameters is shown bellow N/w.



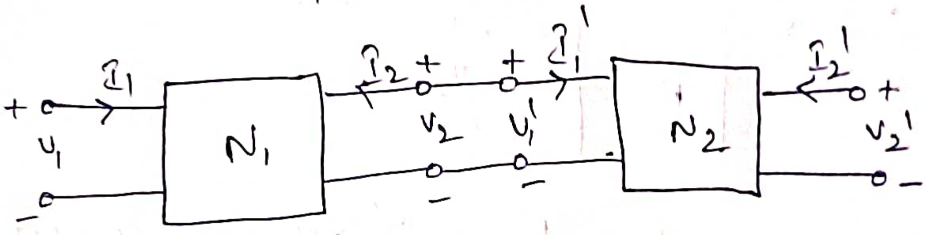
Interconnection of two-port networks:-

Cascade connection:-

1. Transmission Parameter Representation:-

* The bellow network shows the two-port networks connected in cascade.
* In the cascade connection, the output port of the first network becomes the input of the second network.
* Since it is assumed that input and output currents are positive when they enter the network, we have

$$I_1' = -I_2$$



Let A_1, B_1, C_1, D_1 be the transmission parameters of the network N_1 and A_2, B_2, C_2, D_2 be the transmission parameters of the Network N_2.

For the network N_1,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_1' \\ -I_2 \end{bmatrix} \rightarrow ①$$

For the network N_2,

$$\begin{pmatrix} V_1' \\ I_1' \end{pmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \rightarrow ②$$

Since $V_1' = V_2$ and $I_1' = -I_2$, Then we can write

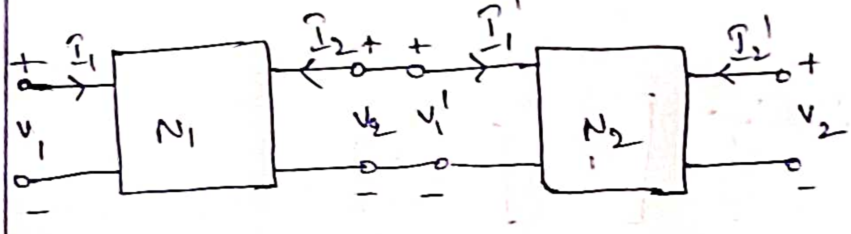
$$\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} \rightarrow ③$$

Combining equation ① and ③

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2' \\ -I_2' \end{bmatrix}$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$

Inverse Transmission Parameter Representation:-



The above network shows two-port networks connected in cascade. Since it is assumed that input and output currents are positive when they enter the network,

we have $-I_1' = I_2$

Let A_1', B_1', C_1', D_1' be the transmission parameters of the n/w N_1 and A_2', B_2', C_2', D_2' be the transmission parameters of the n/w N_2.

For the n/w N_1,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1' & B_1' \\ C_1' & D_1' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \rightarrow (1)$$

for the networks N_2,

$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2' & B_2' \\ C_2' & D_2' \end{bmatrix} \begin{bmatrix} V_2' \\ -I_1' \end{bmatrix} \rightarrow (2)$$

Since $V_1' = V_2$ and $-I_1' = I_2$, we can write

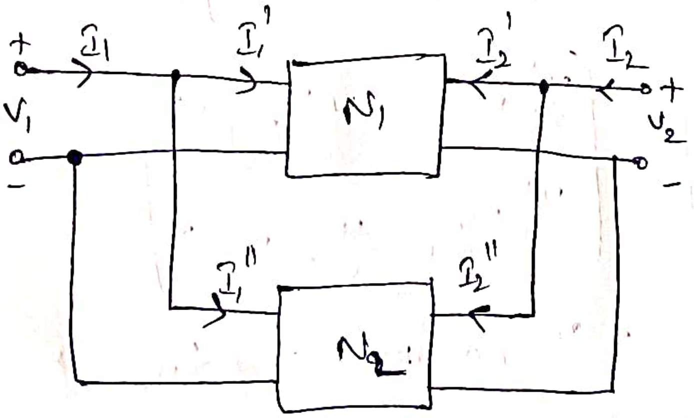
$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2' & B_2' \\ C_2' & D_2' \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} \rightarrow (3)$$

Combining equations (1) and (3)

$$\begin{bmatrix} V_2' \\ I_2' \end{bmatrix} = \begin{bmatrix} A_2' & B_2' \\ C_2' & D_2' \end{bmatrix} \begin{bmatrix} A_1' & B_1' \\ C_1' & D_1' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

where $\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} A_2' & B_2' \\ C_2' & D_2' \end{bmatrix} \begin{bmatrix} A_1' & B_1' \\ C_1' & D_1' \end{bmatrix}$

Parallel connection:-



Above figure shows two-port networks connected in parallel.

In the parallel connection, the two po networks have the same input voltages and the same output voltages.

Let $y_{11}', y_{12}', y_{21}', y_{22}'$ be the y-parameters of the network N_1 and $y_{11}'', y_{12}'', y_{21}'', y_{22}''$ be the y-parameters of the network N_2.

For the network N_1,

$$\begin{bmatrix} I_1' \\ I_2' \end{bmatrix} = \begin{bmatrix} y_{11}' & y_{12}' \\ y_{21}' & y_{22}' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

For the network N_2,

$$\begin{bmatrix} I_1'' \\ I_2'' \end{bmatrix} = \begin{bmatrix} y_{11}'' & y_{12}'' \\ y_{21}'' & y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

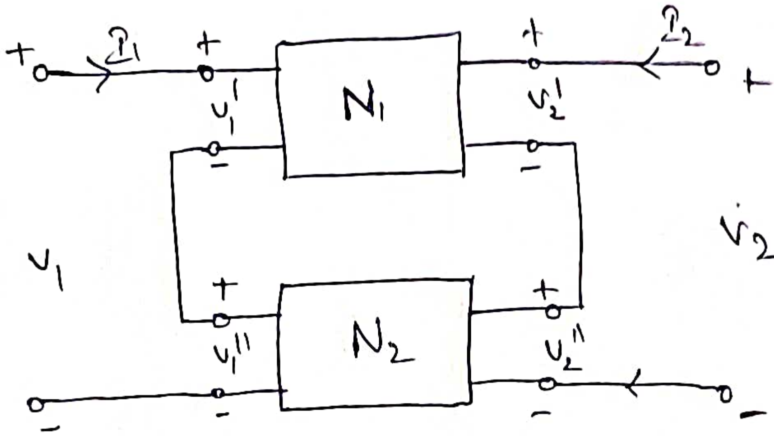
For the combined network, $I_1 = I_1' + I_1''$ and
$I_2 = I_2' + I_2''$

Hence,
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I_1' + I_1'' \\ I_2' + I_2'' \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

where
$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11}' + y_{11}'' & y_{12}' + y_{12}'' \\ y_{21}' + y_{21}'' & y_{22}' + y_{22}'' \end{bmatrix}$$

Thus, the resultant y-parameter matrix for parallel connected networks is the sum of y-matrices of each individual two-port networks.

Series connection :-



The above figure shows two-port networks connected in Series. In a series connection, both the networks carry the same input current. Their output currents are also equal.

Let $z_{11}', z_{12}', z_{21}', z_{22}'$ be the z-parameters of the network N_1 and

$z_{11}'', z_{12}'', z_{21}'', z_{22}''$ be the z-parameters of the network N_2

For the network N_1,
$$\begin{bmatrix} v_1' \\ v_2' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}'' \\ z_{21}' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the network N_2,
$$\begin{bmatrix} v_1'' \\ v_2'' \end{bmatrix} = \begin{bmatrix} z_{11}' & z_{12}'' \\ z_{21}' & z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

for the combined network

$$v_1 = v_1' + v_1'' \quad \text{and} \quad V_2 = V_2' + V_2''$$

Hence,
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} v_1' + v_1'' \\ v_2' + v_2'' \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

where
$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11}' + z_{11}'' & z_{12}' + z_{12}'' \\ z_{21}' + z_{21}'' & z_{22}' + z_{22}'' \end{bmatrix}$$

$$g_{21} = -\frac{h_{21}}{\Delta h}$$

$$g_{22} = \frac{h_{11}}{\Delta h}$$

Table 13.3 *Inter-relationship between parameters*

$$\Delta X = X_{11} X_{22} - X_{12} X_{21}$$

	In terms of [Z]	[Y]	[T]	[T′]	[h]	[g]
[Z]	Z_{11} Z_{12}	$\frac{Y_{22}}{\Delta Y}$ $-\frac{Y_{12}}{\Delta Y}$	$\frac{A}{C}$ $\frac{\Delta T}{C}$	$\frac{D'}{C'}$ $\frac{1}{C'}$	$\frac{\Delta h}{h_{22}}$ $\frac{h_{12}}{h_{22}}$	$\frac{1}{g_{11}}$ $-\frac{g_{12}}{g_{11}}$
	Z_{21} Z_{22}	$-\frac{Y_{21}}{\Delta Y}$ $\frac{Y_{11}}{\Delta Y}$	$\frac{1}{C}$ $\frac{D}{C}$	$\frac{\Delta T'}{C'}$ $\frac{A'}{C'}$	$-\frac{h_{21}}{h_{22}}$ $\frac{1}{h_{22}}$	$\frac{g_{21}}{g_{11}}$ $\frac{\Delta g}{g_{11}}$
[Y]	$\frac{Z_{22}}{\Delta Z}$ $-\frac{Z_{12}}{\Delta Z}$	Y_{11} Y_{12}	$\frac{D}{B}$ $-\frac{\Delta T}{B}$	$\frac{A'}{B'}$ $-\frac{1}{B'}$	$\frac{1}{h_{11}}$ $-\frac{h_{12}}{h_{11}}$	$\frac{\Delta g}{g_{22}}$ $\frac{g_{12}}{g_{22}}$
	$-\frac{Z_{21}}{\Delta Z}$ $\frac{Z_{11}}{\Delta Z}$	Y_{21} Y_{22}	$-\frac{1}{B}$ $\frac{A}{B}$	$\frac{\Delta T'}{B'}$ $\frac{D'}{B'}$	$\frac{h_{21}}{h_{11}}$ $\frac{\Delta h}{h_{11}}$	$-\frac{g_{21}}{g_{22}}$ $\frac{1}{g_{22}}$
[T]	$\frac{Z_{11}}{Z_{21}}$ $\frac{\Delta Z}{Z_{21}}$	$-\frac{Y_{22}}{Y_{21}}$ $-\frac{1}{Y_{21}}$	A B	$\frac{D'}{\Delta T'}$ $\frac{B'}{\Delta T'}$	$-\frac{\Delta h}{h_{21}}$ $-\frac{h_{11}}{h_{21}}$	$\frac{1}{g_{21}}$ $\frac{g_{22}}{g_{21}}$
	$\frac{1}{Z_{21}}$ $\frac{Z_{22}}{Z_{21}}$	$-\frac{\Delta Y}{Y_{21}}$ $-\frac{Y_{11}}{Y_{21}}$	C D	$\frac{C'}{\Delta T'}$ $\frac{A'}{\Delta T'}$	$-\frac{h_{22}}{h_{21}}$ $-\frac{1}{h_{21}}$	$\frac{g_{11}}{g_{21}}$ $\frac{\Delta g}{g_{21}}$
[T′]	$\frac{Z_{22}}{Z_{12}}$ $\frac{\Delta Z}{Z_{12}}$	$-\frac{Y_{11}}{Y_{12}}$ $-\frac{1}{Y_{12}}$	$\frac{D}{\Delta T}$ $\frac{B}{\Delta T}$	A' B'	$\frac{1}{h_{12}}$ $\frac{h_{11}}{h_{12}}$	$-\frac{\Delta g}{g_{12}}$ $-\frac{g_{22}}{g_{12}}$
	$\frac{1}{Z_{12}}$ $\frac{Z_{11}}{Z_{12}}$	$-\frac{\Delta Y}{Y_{12}}$ $-\frac{Y_{22}}{Y_{12}}$	$\frac{C}{\Delta T}$ $\frac{A}{\Delta T}$	C' D'	$\frac{h_{22}}{h_{12}}$ $\frac{\Delta h}{h_{12}}$	$-\frac{g_{11}}{g_{12}}$ $-\frac{1}{g_{12}}$
[h]	$\frac{\Delta Z}{Z_{22}}$ $\frac{Z_{12}}{Z_{22}}$	$\frac{1}{Y_{11}}$ $-\frac{Y_{12}}{Y_{11}}$	$\frac{B}{D}$ $\frac{\Delta T}{D}$	$\frac{B'}{A'}$ $\frac{1}{A'}$	h_{11} h_{12}	$\frac{g_{22}}{\Delta g}$ $-\frac{g_{12}}{\Delta g}$
	$-\frac{Z_{21}}{Z_{22}}$ $\frac{1}{Z_{22}}$	$\frac{Y_{21}}{Y_{11}}$ $\frac{\Delta Y}{Y_{11}}$	$-\frac{1}{D}$ $\frac{C}{D}$	$-\frac{\Delta T'}{A'}$ $\frac{C'}{A'}$	h_{21} h_{22}	$-\frac{g_{21}}{\Delta g}$ $\frac{g_{11}}{\Delta g}$
[g]	$\frac{1}{Z_{11}}$ $-\frac{Z_{12}}{Z_{11}}$	$\frac{\Delta Y}{Y_{22}}$ $\frac{Y_{12}}{Y_{22}}$	$\frac{C}{A}$ $-\frac{\Delta T}{A}$	$\frac{C'}{D'}$ $-\frac{1}{D'}$	$\frac{h_{22}}{\Delta h}$ $-\frac{h_{12}}{\Delta h}$	g_{11} g_{12}
	$\frac{Z_{21}}{Z_{11}}$ $\frac{\Delta Z}{Z_{11}}$	$-\frac{Y_{21}}{Y_{22}}$ $\frac{1}{Y_{22}}$	$\frac{1}{A}$ $\frac{B}{A}$	$\frac{\Delta T'}{D'}$ $\frac{B'}{D'}$	$-\frac{h_{21}}{\Delta h}$ $\frac{h_{11}}{\Delta h}$	g_{21} g_{22}

Example 13.26 *The Z parameters of a two-port network are $Z_{11} = 20\ \Omega$, $Z_{22} = 30\ \Omega$, $Z_{12} = Z_{21}$ $= 10\ \Omega$. Find Y and ABCD parameters.*